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Application of Least Absolute Sum (LAS) Technique for Detecting Deformation of Structures

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Abstract

This study focuses on the deformation analysis using a geodetic method known as the Least Absolute Sum. The method consists mainly of independent adjustment of each epoch data, compatibility test on their a posteriori variances, followed by determination of trend of movements for all the common points in the network. A triangulation network was designed consisting of 45 Ytt series Second Order control points within the study area resulting in a total of 63 triangles,189 observations and 90 unknown parameters with 99 degrees of freedom. The network adjustment was done using the method of least squares observation equations. The estimated variance factors for the 2D (horizontal) network were 7.82989325645394e-08 and 7.7207636996395e-08 while 0.03944 and 0.052339 represent the estimated variance factors for the 1D (height) for the first and second epochs respectively. The compatibility of the two epoch data was tested with the variance ratio and compatibility test passed. Actual displacement vectors were computed and transformed into the same computational base using S-transformation by Least Absolute Sum (LAS), stable and unstable points were determined using Single Point displacement test, the displacement vector magnitude was computed, represented graphically to indicate possible trend of movements that might have occurred. This study finds Least Absolute Sum (LAS) Technique useful in studying the deformation of large engineering structures such as high rise buildings, bridges, dams, oil exploration zones, mining sites and land slide monitoring.

Key Words: *Epoch, Coordinates, Least Square Adjustment, Compatibility Test, Deformation Analysis, Least Absolute Sum (LAS).*

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Introduction

ANY object, when acted upon by external forces, deforms, or exhibits changes in its size or shape. These observable changes are manifestations of internal stresses or pressures produced by the physical interaction of the external forces and the material itself. Materials either fail or tear when stresses exceed certain critical values [7]. It is this risk of failure which practically necessitates deformation monitoring surveys, which allow the implementation of mitigating constructive procedures or evacuations to take place early enough, to prevent loss of life and material. Generally, the deformation measurement techniques can be divided into geotechnical, structural and geodetic methods. Geotechnical and structural methods are direct measurement methods, which uses special equipment to measure changes in length, inclination, relative height, strain, etc.,[32; 7]. On the other hand, in the geodetic method there are two basic types of geodetic monitoring networks namely the reference and relative networks [7].
In a reference network, some of the points or stations are assumed to be located outside of the deformable body or object, thus serving as reference points for the determination of the absolute displacements of the object points. However, in a relative network, all surveyed points are assumed to be located on the deformable body. This study will focus only on the geodetic method using a relative network. In a geodetic monitoring network, the object or area under investigation is usually represented by a number of points which are permanently monumented or marked. All the points are then observed in two or more epochs of time.

The geodetic monitoring network can be either a conventional (terrestrial) network, a photogrammetry (i.e., aerial or close-range) network, Global Positioning System (GPS) network or a combination of these network types. Deformation analysis using the geodetic method mainly consists of a two-step analysis via independent adjustment of the network of each epoch which involves testing coordinate differences for significance, by comparison to the accuracy of their determination, followed by deformation detection between the two epochs.

During deformation analysis it is important to determine the trend of movements (displacements) for all the common points in a monitoring network. The trend of movements, then form a basis for preliminary identification of the actual deformation models. Although deformation analysis is applicable to one-dimensional (1-D), two dimensional (2-D) and three-dimensional (3-D) monitoring networks, for this study a 2D (horizontal) and 1D (vertical) networks of secondary controls located around the study area were investigated using Least Absolute Sum Technique (LAS).

Methodology

The data used were point coordinates of second order control point obtained from the office of the Surveyor-General of Lagos State serving as the first epoch data, the second epoch data was observed while the Orthometric heights for these selected stations in the network are derived from EGM 2008. A total of 45 common stations coordinates were used for the two epochs. Deformation analysis using geodetic methods consists of independent network adjustment of each epoch data followed by deformation detection

Network Adjustment of Each Epoch Data

Adjustment of observations for each epoch data separately to obtain the estimate of the point coordinates X , Y and Z and their full covariance matrix using observation equation method of Least Square .

$$L^a = f(X^a) \tag{2.1.1}$$

After Linearization the observation equation is written as;

$$L = AX + \hat{V} \tag{2.1.2}$$

$$L^a = L^b + V, \quad \text{vector of adjusted observation} \tag{2.1.3}$$

$$X^a = X^o + X \text{ adjusted parameter} \tag{2.1.4}$$

$$L^o = F(X^o) \tag{2.1.5}$$

$$L = L^o - L^b \text{, the misclosure vector} \tag{2.1.6}$$

L^o = Approximate vector of observation, L^b = Vector of original observation
X^a = Vector of adjusted parameter
X = Vector of corrections to the approximate values
V = Vector of residuals

$A = \frac{dF(Xo)}{d(Xo)}$; the design matrix A was obtained by differentiating the observations with respect to the unknown parameters of each station

$$QX^a = (A^T PA)^{-1} \text{ co factor matrix of } X^a \tag{2.1.8}$$

$$QL^a = A(A^T PA)^{-1}A^T \text{ co factor matrix of } L^a \tag{2.1.9}$$

$$\sigma_o^2 = \frac{V^T PV}{n-m} \text{ a posteriori variance factor} \tag{2.1.10}$$

The programming for the network adjustment was done using Matlab. The adjustment module is based on Observation equation method of least squares. The point coordinates was used to design a reference network of 63 triangles, consisting 189 observations and 90 unknown parameters with 99 degrees of freedom. Table 2.1 below shows the adjusted coordinates of the first and second epoch data.

With σ_{o1}^2 *and* σ_{o2}^2. being the a-posteriori variance factors for the first and second campaigns respectively

The test statistic is $$T = \frac{\sigma_{oj}^2}{\sigma_{oi}^2} \sim \mathbf{F}(\alpha, \mathbf{d}f_j, \mathbf{d}f_i) \tag{2.2.1}$$

With j and i representing the larger and smaller variance factors, $\mathbf{F}$ is the Fisher's distribution, α is the chosen significance level (typically $\alpha = 0.05$) and $\mathbf{d}f_i$ and $\mathbf{d}f_j$ are the degrees of freedom for i and j observation campaigns respectively. The above test is accepted if $T < F(\alpha, df_j, \mathbf{d}f_i)$ at a significance level α. The failure of the above test may be caused by incompatible weighting between the two campaign observations or incorrect weighting scheme and any further analysis is stopped at such stage.

Trend Analysis

After the test on the variance ratio, the test is accepted, the displacement vector (coordinates differences) and its cofactor matrix is then computed as follows;

$$\mathbf{d} = \hat{\mathbf{x}}_2 - \hat{\mathbf{x}}_1 \tag{2.3.1}$$

$$\mathbf{Q}_{\mathrm{d}} = \mathbf{Q}_{\overline{x1}} + \mathbf{Q}_{\overline{x2}} \tag{2.3.2}$$

$\mathbf{d}$ is the displacement vector, $\mathbf{Q_d}$ is the cofactor matrix of $\mathbf{d}$, $\hat{x}_1$and $\hat{x}_2$ are the estimated coordinates of all the common points in the first and second observation epochs respectively (with same datum definition), $\mathbf{Q}_{\overline{x1}}$ and $\mathbf{Q}_{\overline{x2}}$ are the cofactor matrix of the estimated coordinates $\hat{x}_1$ and $\hat{x}_2$.

Least Absolute Sum (LAS)

A robust method known as Least Absolute Sum (LAS) was proposed by [1]. In the LAS method, some points in a reference network cannot be accepted as stable. In other words not every point has equal importance. Hence in the beginning, the weight matrix (W) is accepted as W = I. This indicates that all points in the network have the same importance. Therefore, the solution is similar to the Helmert transformation, if only some points are given unit weight and the others a zero weight, that is, $W = diag\ (I, 0)$ [2].

$$\mathbf{d}^{\mathbf{k+1}} = [\mathbf{I} - \mathbf{H}(\mathbf{H}^{\mathrm{T}}\mathbf{W}^{(\mathbf{k})}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{W}^{(\mathbf{k})}]\mathbf{d}^{\mathbf{k}} = \mathbf{S}^{(\mathbf{k})}\mathbf{d}^{(\mathbf{k})} \tag{2.4.1}$$

Where $\mathbf{I}$ = identity matrix, $\mathbf{k}$ = number of iterations, $\mathbf{d}$ = displacement vector, $\mathbf{S}$ = S-transformations matrix, and $\mathbf{W}$ = weight matrix

Then displacement values (d) are calculated as:

$$\mathbf{d_1} = \mathbf{S_1}d \tag{2.4.2}$$

$$\mathbf{Q}_{\mathrm{d1}} = \mathbf{S}\mathbf{Q_d}\mathbf{S}^{\mathrm{T}} \tag{2.4.3}$$

$$\mathrm{S} = [\mathbf{I} - \mathbf{H}(\mathbf{H}^{\mathrm{T}}\mathrm{WH})^{(-1)}\mathbf{H}^{\mathrm{T}}\mathrm{W} \tag{2.4.4}$$

$$\mathbf{d_2} = \mathbf{S_2}\mathbf{d_1} \tag{2.4.5}$$

$$\mathbf{Q}_{\mathrm{d2}} = \mathbf{S_2}\mathbf{Q_d}\mathbf{S}_2^T \tag{2.4.6}$$

Table 2.1: Adjusted Coordinates of the First and Second Epoch Data

		FIRST EPOCH			SECOND EPOCH		
S/N		EASTINGS(M)	NORTHINGS(M)	HEIGHT(M)	EASTINGS(M)	NORTHINGS(M)	HEIGHT(M)
1	YTT1	512770.871400334	718266.132200109	22.69139892	512770.871403101	718266.132201002	22.6714836
2	YTT2	514506.700499577	718531.839799538	22.69178864	514506.700502712	718531.839892683	22.6177766
3	YTT3	512893.348699673	714574.324598699	22.32421379	512893.348696706	714574.324682748	22.0283972
4	YTT4	515558.463298852	713569.142998863	22.31915793	515558.463313656	713569.143971483	22.1792805
5	YTT5	516586.611797575	714276.855800185	22.44819168	516586.611803276	714276.855808693	22.4573541
6	YTT6	518643.696295812	713094.787300631	22.27584535	518643.696301644	713094.787305278	22.1277046
7	YTT7	514352.907099268	714685.214899466	22.40743677	514352.90709294	714685.215000243	22.3007829
8	YTT8	517061.729398256	715437.606801309	22.54378808	517061.729400602	715437.606814866	22.6728747
9	YTT9	518422.044396225	714609.031901365	22.43557176	518422.044396833	714609.031912447	22.4370771
10	YTT10	520125.232796594	713647.970001077	22.37953996	520125.232796485	713647.97000797	22.3638133
11	YTT11	521363.15129068	715052.213702974	22.48641337	521363.151281339	715052.213730621	22.569482
12	YTT12	518498.663596491	716974.489604158	22.69806547	518498.663595936	716974.489641336	22.9511293
13	YTT13	514108.928199661	717481.663299636	22.61380567	514108.928201137	717481.663397287	22.5929253
14	YTT14	515601.588799851	717526.274999398	22.70542529	515601.588808979	717526.275961109	22.868333
15	YTT15	516950.750999607	716775.036400767	22.6656666	516950.7510017	716775.036407083	22.851861
16	YTT16	517138.43110192	717714.634600756	22.76961739	517138.431104485	717714.634610646	23.058185
17	YTT17	520079.581892186	717605.081806163	22.75625371	520079.58188775	717605.081862575	23.0669356
18	YTT18	521384.589782752	716820.772199095	22.65590067	521384.589767459	716820.772291492	22.8672022
19	YTT19	521584.838793279	713648.512600229	22.32802007	521584.838781533	713648.512604235	22.2474788
20	YTT20	523697.284691038	712610.341101032	22.17827755	523697.284674975	712610.341115527	21.9320408
21	YTT21	525256.684295581	712069.400902666	22.09307778	525256.684279801	712069.400939104	21.7478426
22	YTT22	523497.609891544	714124.578899686	22.3578494	523497.609877763	714124.578999448	22.304579
23	YTT23	525443.708593041	714191.748497196	22.32616293	525443.708582496	714191.748578731	22.212263
24	YTT24	527124.733799406	713617.755492013	22.25907939	527124.733796371	713617.755536764	22.0740095
25	YTT25	522501.845287421	715583.224899734	22.49919981	522501.845274277	715583.225000239	22.5282929
26	YTT26	526736.830187412	715474.552392427	22.45949366	526736.830178688	715474.552433881	22.4668134
27	YTT27	527887.037415264	714977.706471458	22.40016023	527887.037436763	714977.706532537	22.3521075
28	YTT28	518840.786597038	718875.794609559	22.89331219	518840.786598495	718875.794694225	23.3294166
29	YTT29	520145.435490858	718953.625408137	22.9183092	520145.435486309	718953.625481671	23.4036233
30	YTT30	522444.869566192	719783.514112478	22.97815103	522444.869536483	719783.514127727	23.494706
31	YTT31	522025.385573317	718114.274704924	22.79568104	522025.385549735	718114.274751322	23.1433067
32	YTT32	523186.583369713	717539.965614425	22.71579058	523186.583341969	717539.965648825	22.9760665
33	YTT33	528705.879517029	713817.503986232	22.26302379	528705.879534535	713817.504864103	22.0818816
34	YTT34	528043.110511926	712435.484798928	22.13055794	528043.110515779	712435.484909801	21.8191489
35	YTT35	528419.988315911	710633.958211361	21.92731111	528419.98831332	710633.958237693	21.4137506
36	YTT36	529967.93452679	711032.684607905	21.95829762	529967.934544663	711032.684711616	21.4742604
37	YTT37	528261.861876	717210.698619623	22.63104409	528261.861862215	717210.698704528	22.8097254
38	YTT38	526425.689061496	718724.127100844	22.81301857	526425.689028949	718724.127113802	23.1712292
39	YTT39	525076.468986405	719408.819474891	22.91308931	525076.468977532	719408.819551119	23.3689151
40	YTT40	526225.935350995	720282.574474673	22.98975953	526225.935307325	720282.574549655	23.5230335
41	YTT41	528493.426463876	718448.80777251	22.75950388	528493.426333629	718448.807829748	23.0647514
42	YTT42	527884.34385114	720371.80872944	22.9684489	527884.34360357	720371.80879708	23.481299
43	YTT43	523273.527400817	721154.484610349	23.12072684	523273.527204784	721154.484704536	23.78354
44	YTT44	524356.490404865	722381.886575353	23.23512038	524356.488142223	722381.886658528	24.0128619
45	YTT45	525882.380108575	722017.811261183	23.17551393	525882.380020785	722017.811315488	23.8941837

Initial Checking of Data and Test on Variance Ratio

Before deformation analysis can be carried out, it is important to perform initial checking on the input data and test on the *a-posterior* variance factors of both epochs [19; 20; 21], [22].This is to ensure that common points, same approximate coordinates and same point's names were used in the two campaigns. The *a posteriori* variance factors of both epochs were then tested for their compatibility. The null and alternative hypotheses used are as proposed by [27].

$$H_o: \sigma_{o1}^2 = \sigma_{o2}^2 \quad and\ H_a: \sigma_{o1}^2 > \sigma_{o2}^2\ or\ \sigma_{o2}^2 > \sigma_{o1}^2 \tag{2.2.0}$$

With σ_{01}^2 *and* σ_{02}^2 . being the a-posteriori variance factors for the first and second campaigns respectively

The test statistic is $$T = \frac{\sigma_{oj}^2}{\sigma_{oi}^2} \sim \mathbf{F}(\alpha, \mathbf{d}f_j, \mathbf{d}f_i) \tag{2.2.1}$$

With j and i representing the larger and smaller variance factors, **F** is the Fisher's distribution, α is the chosen significance level (typically $\alpha = 0.05$) and $\mathbf{d}f_i$ and $\mathbf{d}f_j$ are the degrees of freedom for i and j observation campaigns respectively. The above test is accepted if $T < F(\alpha, df_j, \mathbf{d}f_i)$ at a significance level α. The failure of the above test may be caused by incompatible weighting between the two campaign observations or incorrect weighting scheme and any further analysis is stopped at such stage.

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$$\mathbf{Q_d} = \mathbf{Q}_{\overline{x1}} + \mathbf{Q}_{\overline{x2}} \tag{2.3.2}$$

d is the displacement vector, $\mathbf{Q_d}$ is the cofactor matrix of **d** , $\hat{x}_1$ and $\hat{x}_2$ are the estimated coordinates of all the common points in the first and second observation epochs respectively (with same datum definition), $\mathbf{Q}_{\overline{x1}}$ and $\mathbf{Q}_{\overline{x2}}$ are the cofactor matrix of the estimated coordinates $\hat{x}_1$ and $\hat{x}_2$.

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$$\mathbf{d}^{k+1} = [\mathbf{I} - \mathbf{H}(\mathbf{H}^T\mathbf{W}^{(k)}\mathbf{H})^{-1}\mathbf{H}^T\mathbf{W}^{(k)}]\mathbf{d}^k = \mathbf{S}^{(k)}\mathbf{d}^{(k)} \tag{2.4.1}$$

Where $\mathbf{I}$ = identity matrix, $\mathbf{k}$ = number of iterations, $\mathbf{d}$ = displacement vector, $\mathbf{S}$ = S-transformations matrix, and $\mathbf{W}$ = weight matrix

Then displacement values (d) are calculated as:

$$\mathbf{d_1} = \mathbf{S_1}d \tag{2.4.2}$$
$$\mathbf{Q_{d1}} = \mathbf{S}\mathbf{Q_d}\mathbf{S}^T \tag{2.4.3}$$
$$\mathrm{S} = [\mathbf{I} - \mathbf{H}(\mathbf{H}^T\mathrm{W}\mathrm{H})^{(-1)}\mathbf{H}^T\mathrm{W} \tag{2.4.4}$$
$$\mathbf{d_2} = \mathbf{S_2}\mathbf{d_1} \tag{2.4.5}$$
$$\mathbf{Q_{d2}} = \mathbf{S_2}\mathbf{Q_d}\mathbf{S}_2^T \tag{2.4.6}$$

where $\mathbf{d}_1$ and $\mathbf{Q}_{d1}$ are the displacement vector and its cofactor matrix respectively based on the new datum or computational base, H , the inner constraints matrix constructed depending on the union of the datum defects in the two epochs and on the number of common points, and W is the weight matrix with diagonal value of one for datum points and zero elsewhere. Matrix S is symmetric only for the minimum trace solutions (i.e., all points in the network were defined as datum).The group of selected datum points is then tested for its stability by using Single Point displacement test. The LAS method is used when there is no previous information about the movement of points within the network.

Formation of matrix H for the final S-transformation

H is a configuration matrix for the datum defect, called inner constraint matrix. Basically, the matrix H depends on the type of network: 1D, 2D or 3D. For 1D, 2D and 3D networks, H is having maximum dimensions of (1**m** by 1), (2**m** by 4) and (3**m** by 7) respectively, where **m** is the number of stations. Equation (2.4.7) shows the components of the matrix H for a 1D network.

$$\mathbf{H}^{\mathbf{T}} = (1\,1\,1\,1\,1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .1m) \qquad (2.4.7)$$

For 2D surveying networks, the first two rows of the matrix H represent the translations in the x and y directions (tx and ty), the third row defines the rotation about the z axis (rz) and the last row is the scale of the network. Equation (3.2.4.1) shows the components of the matrix H for a 2D network.

$$H^T = \begin{bmatrix} 1 & 0 & 1 & 0 & \cdots & 0 & 1 \\ 0 & 1 & 0 & 1 & \cdots & 1 & 0 \\ y_1^o & -x_1^o & y_2^o & -x_2^o & \cdots & -y_m^o & -x_m^o \\ x_1^o & y_1^o & x_2^o & y_2^o & \cdots & x_m^o & y_m^o \end{bmatrix} \qquad (2.4.8)$$

Where x_i^o and y_i^o, z_i^o are the coordinates of point p_i which are reduced to the centroid or centre of gravity of the network, i.e.,

$$\mathrm{x}_i^o = \mathrm{x}_i - \frac{(\Sigma_{i=1}^{m} x_i)}{m} \qquad (2.4.9)$$

$$\mathrm{y}_i^o = \mathrm{y}_i - \frac{(\Sigma_{i=1}^{m} y_i)}{m} \qquad (2.4.10)$$

With x_i, y_i, the approximate coordinates of point p_i and m is the number of common points in the network. [16; 23; 28].The first two rows of the inner constraint matrix (H^T) take care of the translations in the x and y directions, while the third row defines the rotation about the vertical (z) axis and the last row defines the scale of the network. For a Trilateration network, the last row of H^T is omitted [25; 26; 27].

In the first transformation (k = 1) the weight matrix is taken as identity ($W^{(K)} = I$) for all the common points, this indicates that all the points in the network have the same importance. The weight matrix for Least Absolute Sum (LAS)

$$W^{(k)} = \text{diag}\left\{\frac{1}{\left|\sqrt{(dxi^{(k)} + dyi^{(k)})^2}\right|}\right\} \quad (2.4.11)$$

The iterative procedure continues until the absolute differences between the successive transformed displacements of all the common points, i.e.

$$\left|\mathbf{d}^{(k+1)} - \mathbf{d}^{(k)}\right| \quad (2.4.12)$$

are smaller than a tolerance value δ *(say 0.001m).* It is possible that during the iterations some *dxi, dyi, dzi may approach zero causing numerical instability because* W^K becomes very large .There are two ways to solve this problem, either by:

Setting a lower bound value e.g 0.0001m. If $d_j^{(k)}$ is smaller than the lower bound value, its weight is set to zero, or replacing equation (2.4.11) as:

$$W^{(K)} = diag\left\{\frac{1}{\sqrt{(dxi^{(k)} + \delta)^{(2)} + (dyi^{(k)} + \delta)^{(2)})^{(2)}}}\right\} \quad (2.4.13)$$

Where dicks is the δ component of the vector dk after kth iteration. In this study the Least Absolute Sum minimizes the sum of the lengths of the displacements i.e.

$$\sum\sqrt{(dxi)^2 + (dyi)^2} \longrightarrow \text{minimum} \quad (2.4.14)$$

In the final iteration, the cofactor matrix of the displacement vector is computed as

$$Q_d{}^{k+1} = S^{(k)} Q_d (S^{(k)})^T \quad (2.4.15)$$

For 1D networks, there are some differences for the calculation of $\mathbf{d}'$ and $Q_d{}'$. First, the displacements $\boldsymbol{d}$ are arranged in increasing order. The median is assigned unit weight 1 and zero weight is assigned to the other displacements $\boldsymbol{d}$. If the total number of $\boldsymbol{d}$ is an even number, the two middle (median) displacements $\boldsymbol{d}$ are assigned unit weight 1 and zero weight is assigned to the other displacements $\boldsymbol{d}$, Then, the new vector of displacements $\mathbf{d}'$ and its cofactor matrix $Q_d{}'$ are $\mathbf{d}' = min\sum |d_i - tz| \Rightarrow \quad Q_d{}' = \mathbf{S}\mathbf{Q}_\mathbf{d}\mathbf{S}^\mathrm{T}$ where $tz|$ is the mean value of the middle displacements and $\mathbf{d}'$ is the displacement of point i.

$$\mathbf{S} = [\mathbf{I} - \mathbf{H}(\mathbf{H}^\mathrm{T}\mathrm{W}\mathbf{H})^{(-1)}\mathbf{H}^\mathrm{T}\mathrm{W} \quad (2.4.16)$$

The stability information of each common point j is then determined through a single point test as below [25]; [27].

$$T_j = \frac{(d_j{}^{(k+1)})^T (Q_{dj}{}^{(k+1)})^{-1}\, d_j{}^{(k+1)})}{2\sigma_0^2} \sim F\,(\alpha, 2, \mathrm{df}) \quad (2.4.17)$$

Where

d_j, Q_{dj} = displacement vector and its cofactor matrix respectively for each common point j or pooled variance factor.

$\sigma_o^2 = \frac{[df_1(\sigma_{o1}^2)+df_2(\sigma_{o2}^2)]}{df}$, common or pool variance factor (2.4.18)

$(\sigma_{o1}^2), (\sigma_{o2}^2)$ = *a posteriori* variance factors of first and second epochs respectively

df_1, df_2 = degrees of freedom of first and second epochs

$df = df_1 + df_2$, sum of degrees of freedom of first and second epochs significance level (usually chosen as 0.05)

If the above test passes (i.e.,$T_j < F(\alpha, 2, df)$) then the point is assumed to be stable at a significance level α. Otherwise, if the test fails (i.e., $T_j \geq F(\alpha, 2, df)$) then the point is assumed to be deformed (moved).

Results and Data Analysis

Network Design

A triangulation network consisting of 45 YTT series second order control points within the study area resulting in a total of 63 triangles,189 observations and 90 unknown parameters with 99 degrees of freedom.

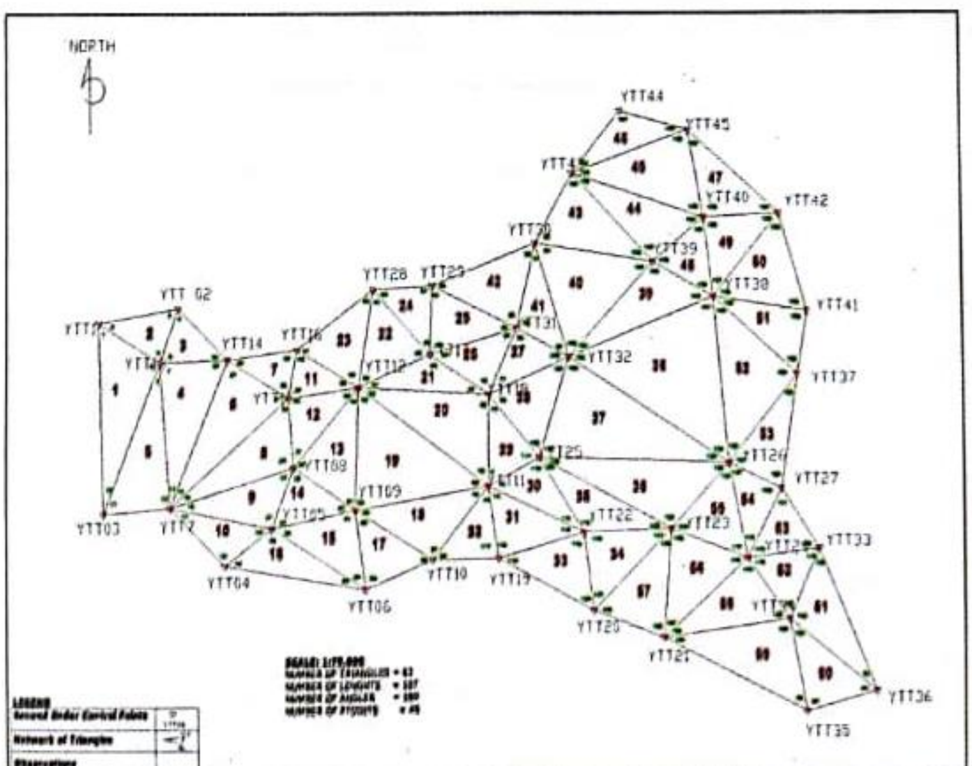

Figure 3.0: Network of Selected Control Points Across the Study Area

Adjustment Results

The results of the study are presented in the following sections. The network adjustment summaries for 2D (X, Y) and 1D (Height) are shown in Tables 3.1a and 3.2a respectively.

Table 3.1a: 2D (x, y) Network Adjustment Summary

Parameter	First Epoch	Second Epoch
No of Station	45	45
No of Observation (n)	189	189
No of Parameters (m)	90	90
Degree of Freedom (df=n-m)	99	99
Convergence Limit	0.00001	0.00001
A-posteriori Variance (σ)	7.82989325645394e-08	7.96836000130844e-08
Trace of the Covariance Matrix of the Adjusted parameter	5.183975843652210e-06	5.27565084002794e-06
Trace of the Adjusted Observation Matrix	7.04690393080854e-06	7.17152400117759e-06

Table 3.1b: 1D (Height) Network Adjustment Summary

Parameter	First Epoch	Second Epoch
No of Station	45	45
No of Observation (n)	107	107
No of Parameters (m)	45	45
Degree of Freedom (df=n-m)	62	62
A-posteriori Variance (σ)	0.0394472461577893	0.052339412620338
Trace of the Covariance Matrix of the Adjusted parameter	1.040613555969225	4.018695177022139
Trace of the Adjusted Observation Matrix	1.77512607710052	6.85527356791522

Deformation Analysis Result

After the network adjustment, the obtained results, especially the adjusted coordinates and the cofactor matrices were used for the computation of the displacement vector and the cofactor matrix of the displacement vector. The trend analysis and deformation detection were carried out using the LAS method. At the degrees of freedom of the epoch, the Fisher's critical value obtained at 0.05 (95%) significant level is 1.39. The result of the variance ratio test of the two epochs shows the test statistic (T) value is 1.020884677924254.

The displacement vector (d), cofactor matrix of the displacement vector ($\mathbf{Q_d}$), the inner constraint matrix (H), weight matrix (W), S-transformation matrix (S) and other parameters of the LAS were all computed. The results of the displacement vector (d) after adjustment of the network, the first iteration displacement vector $\mathbf{d_1}$ and the second iteration displacement vector $\mathbf{d_2}$ after transformation by Least Absolute Sum method the final single point displacement ($\mathbf{d_p}$) are as shown in Table 3.2 , Table 3.3, and Table 3.4.

Table 3.2: Displacement Vector of the 1D Network and Stable/Unstable Point Displacements

		Displacement Vector	Displacement Vector on a New Computational Base After S- Transformation		Single Point Displacement	PT<Fi (0.05,2,df) PT<1.550
S/N	Control Point Name	dZ(m)	$d_1 = S_1d$ dz1	$d_i = S_2d_1$ dz2	PTPz	PT<1.550
1	YTT1	-0.01992	-0.62106	-0.54265	0.811994	Stable
2	YTT2	-0.07401	-0.59154	-0.51313	0.988851	Stable
3	YTT3	-0.29582	-0.45274	-0.37433	0.592596	Stable
4	YTT4	-0.13988	-0.41891	-0.3405	0.648932	Stable
5	YTT5	0.009162	-0.40332	-0.32491	0.600531	Stable
6	YTT6	-0.14814	-0.35374	-0.27533	0.409578	Stable
7	YTT7	-0.10665	-0.29257	-0.21416	0.261906	Stable
8	YTT8	0.129087	-0.28865	-0.21024	0.306232	Stable
9	YTT9	0.001505	-0.25564	-0.17723	0.191986	Stable
10	YTT10	-0.01573	-0.24738	-0.16897	0.189104	Stable
11	YTT11	0.083069	-0.2214	-0.14299	0.147021	Stable
12	YTT12	0.253064	-0.21416	-0.13575	0.13268	Stable
13	YTT13	-0.02088	-0.18804	-0.10963	0.070051	Stable
14	YTT14	0.162908	-0.18152	-0.10311	0.068049	Stable
15	YTT15	0.186194	-0.16077	-0.08236	0.044635	Stable
16	YTT16	0.288568	-0.15556	-0.07715	0.040944	Stable
17	YTT17	0.310682	-0.12838	-0.04997	0.021027	Stable
18	YTT18	0.211302	-0.12742	-0.04901	0.021141	Stable
19	YTT19	-0.08054	-0.12323	-0.04482	0.017767	Stable
20	YTT20	-0.24624	-0.106	-0.02759	0.012284	Stable
21	YTT21	-0.34524	-0.10018	-0.02177	0.013732	Stable
22	YTT22	-0.05327	-0.09834	-0.01993	0.003962	Stable
23	YTT23	-0.1139	-0.07841	0	0	Stable
24	YTT24	-0.18507	-0.02444	0.053976	0.079391	Stable
25	YTT25	0.029093	0.021583	0.099994	0.108622	Stable
26	YTT26	0.00732	0.055404	0.133815	0.308231	Stable
27	YTT27	-0.04805	0.071178	0.149588	0.449503	Stable
28	YTT28	0.436104	0.078691	0.157101	0.183723	Stable
29	YTT29	0.485314	0.103798	0.182208	0.321928	Stable
30	YTT30	0.516555	0.14556	0.223971	0.528426	Stable
31	YTT31	0.347626	0.152772	0.231183	0.514461	Stable
32	YTT32	0.260276	0.181064	0.259474	0.756523	Stable
33	YTT33	-0.18114	0.197744	0.276154	1.883885	Moved
34	YTT34	-0.31141	0.203178	0.281589	2.125777	Moved
35	YTT35	-0.51356	0.240122	0.318533	2.76796	Moved
36	YTT36	-0.48404	0.250707	0.329118	2.839587	Moved
37	YTT37	0.178681	0.328601	0.407011	2.594411	Moved
38	YTT38	0.358211	0.348322	0.426733	2.418916	Moved
39	YTT39	0.455826	0.37781	0.456221	2.445382	Moved
40	YTT40	0.533274	0.405347	0.483757	2.887476	Moved
41	YTT41	0.305247	0.409051	0.487462	3.288789	Moved
42	YTT42	0.51285	0.42577	0.504181	3.274917	Moved
43	YTT43	0.662813	0.55531	0.63372	4.707015	Moved
44	YTT44	0.777741	0.611166	0.689577	5.687355	Moved
45	YTT45	0.71867	0.670238	0.748648	6.843486	Moved

Table 3.3: The Displacement Vector Pattern of the Epoch Data Using LAS

		Displacement Vecto (d)		Displacement Vector on a New Computational Base After S-Transformation By LAS						Single Point Displacement (PTp)
				Displacement Vector ($d_i = S_i d$)		Displacement Vector ($d_j = S_j d_j$)				
S/N	Control Point Name	X(m)	dY(m)	d1(X)	d1(Y)	d2(X)	d2(Y)	Magnitude $\sqrt{(d2(X)^2 + d2(Y)^2)}$	PTp(X)	PTP(Y)
1	YTT1	2.77E-06	2.59E-05	7.20E-06	-3.17E-05	4.25E-08	-3.88E-05	3.880E-05	0.000164	0.794432
2	YTT2	3.14E-06	-0.00011	2.26E-05	5.17E-05	1.54E-05	4.46E-05	4.718E-5	0.186669	1.069351
3	YTT3	-2.97E-06	0.000105	-2.08E-05	2.24E-05	-2.79E-05	1.52E-05	3.177E-05	0.456962	0.080737
4	YTT4	1.48E-05	-0.00031	1.11E-05	0.000886	3.96E-06	0.000879	8.79E-04	0.007146	4.246152
5	YTT5	5.70E-06	0.000124	1.44E-05	-7.86E-05	7.20E-06	-8.58E-05	8.61 E-05	0.024044	4.110841
6	YTT6	5.83E-06	4.10E-05	2.28E-05	-0.0001	1.57E-05	-0.00011	1.570E-05	0.111269	7.19099
7	YTT7	-6.33E-06	0.000162	-1.23E-05	3.08E-05	-1.94E-05	2.36E-05	3.053 E-05	0.16821	0.31783
8	YTT8	2.35E-06	0.000133	2.19E-05	-6.77E-05	1.48E-05	-7.48E-05	7.625 E-05	0.103594	3.158588
9	YTT9	6.08E-07	3.72E-05	2.54E-05	-8.51E-05	1.83E-05	-9.22E-05	9.399 E-05	0.157434	4.870269
10	YTT10	-1.09E-07	2.52E-05	3.17E-05	-0.00011	2.46E-05	-0.00011	1.127 E-05	0.293329	7.62088
11	YTT11	-9.34E-06	8.80E-05	4.08E-05	-8.36E-05	3.37E-05	-9.07E-05	9.675 E-05	0.565388	4.660503
12	YTT12	-5.55E-07	8.99E-05	3.97E-05	-4.13E-05	3.26E-05	-4.84E-05	5.835 E-05	0.521227	1.323015
13	YTT13	1.48E-06	1.26E-05	1.12E-05	5.07E-05	4.09E-06	4.35E-05	5.970 E-05	0.010728	0.872408
14	YTT14	9.13E-06	-0.00053	3.06E-05	0.000906	2.35E-05	0.000899	8.993 E-05	0.300271	4.536646
15	YTT15	2.09E-06	-6.81E-05	2.92E-05	-6.40E-05	2.21E-05	-7.11E-05	7.4455 E-05	0.236805	2.868241
16	YTT16	2.56E-06	1.83E-05	3.71E-05	-5.43E-05	2.99E-05	-6.15E-05	6.838 E-05	0.452714	2.130441
17	YTT17	-4.44E-06	0.00011	5.19E-05	-2.71E-05	4.48E-05	-3.43E-05	5.642 E-05	0.986544	0.649148
18	YTT18	-1.53E-05	-4.11E-6	4.62E-05	-5.38E-06	3.90E-05	-1.25E-05	2.337 E-05	0.760024	0.086324
19	YTT19	-1.17E-05	6.57E-05	3.13E-05	-0.00012	2.42E-05	-0.00013	1.3223 E-04	0.297002	9.061258
20	YTT20	-1.61E-05	0.00012	3.67E-05	-0.00013	2.95E-05	-0.00014	1.43E-04	0.466251	1.066337
21	YTT21	-1.58E-05	0.000264	4.55E-05	-0.00012	3.84E-05	-0.00013	1.3555 E-04	0.821701	9.233006
22	YTT22	-1.38E-05	5.46E-05	4.70E-05	-3.20E-05	3.98E-05	-3.91E-05	5.607 E-05	0.809365	0.851019
23	YTT23	-1.05E-05	7.04E-05	6.56E-05	-6.19E-05	5.84E-05	-6.91E-05	9.047 E-05	1.865367	2.618017
24	YTT24	-3.03E-06	0.000137	8.24E-05	-0.00011	7.52E-05	-0.00012	1.4161 E-04	3.129756	7.668884
25	YTT25	-1.31E-05	5.91E-05	4.91E-05	-1.38E-06	4.20E-05	-2.09E-05	4.691 E-05	0.899676	0.243445
26	YTT26	-8.72E-06	2.74E-05	8.54E-05	-0.0001	7.82E-05	-0.00011	1.3496 E-04	3.326455	6.232521
27	YTT27	2.15E-05	-0.00021	0.000121	-9.17E-05	0.000114	-9.89E-05	1.509 E-04	7.224628	5.070011
28	YTT28	1.46E-06	0.000112	5.63E-05	1.86E-05	4.92E-05	1.15E-05	5.0526 E-05	1.190501	0.07579
29	YTT29	-4.55E-06	0.000126	6.08E-05	-8.46E-08	5.36E-05	-7.24E-06	5.408 E-05	1.420155	0.028761
30	YTT30	-2.97E-05	0.000272	5.85E-05	-6.64E-05	5.14E-05	-7.36E-05	8.97714 E-05	1.386828	2.965948
31	YTT31	-2.36E-05	0.000111	5.09E-05	-4.55E-05	4.38E-05	-5.26E-05	5.661 E-05	0.975635	1.507281
32	YTT32	-2.77E-05	0.000331	5.21E-05	-6.92E-05	4.49E-05	-7.63E-05	8.853 E-05	1.042566	3.237843
33	YTT33	1.75E-05	-0.00018	0.000116	0.000711	0.000109	0.000704	1.297 E-04	6.784533	2.442919
34	YTT34	3.85E-06	0.000319	8.89E-05	-6.24E-05	8.17E-05	-6.96E-05	10.732 E-05	3.947721	2.469739
35	YTT35	-2.59E-06	0.000591	7.40E-05	-0.00016	6.68E-05	-0.00017	1.833 E-04	2.844835	1.500624
36	YTT36	1.79E-05	0.000657	0.000109	-9.24E-05	0.000102	-9.96E-05	1.4256 E-04	6.39048	4.741311
37	YTT37	-1.38E-05	-0.00113	0.000103	-5.31E-05	9.58E-05	-6.02E-05	11.314 E-05	5.015102	1.936708
38	YTT38	-3.25E-05	0.000115	7.96E-05	-0.0001	7.24E-05	-0.00011	7.240 E-05	2.885911	6.487339
39	YTT39	-8.87E-06	0.001186	9.72E-05	-2.49E-05	9.00E-05	-3.20E-05	9.5519 E-05	4.402552	0.566615
40	YTT40	-4.37E-05	0.002614	7.67E-05	-2.66E-05	6.95E-05	-3.38E-05	7.728 E-05	2.663538	0.634454
41	YTT41	-0.00013	-0.00189	-3.99E-06	-7.27E-05	-1.12E-05	-7.99E-05	8.068 E-05	0.068964	3.45412
42	YTT42	-0.00025	0.00043	0.000201	-0.00036	0.000194	-0.00037	4.1778 E-05	2.121703	7.28967
43	YTT43	-0.0002	-0.00133	-9.28E-05	1.78E-05	-1.00E-04	1.06E-05	1.00498 E-04	5.378697	0.062547
44	YTT44	-0.00226	0.016168	-0.00214	9.41E-06	-0.00215	2.25E-06	2.1500 E-03	2.257432	0.002949
45	YTT45	-8.78E-05	-0.00057	4.08E-05	-3.18E-05	3.37E-05	-3.90E-05	5.154 E-05	0.621957	0.823719

Table 3.4: The Stable and Unstable Point Detection

		Displacement Vector (d2)		Stable and Unstable Point (Single Point Displacement) Using LAS			
				Single Point Displacement PT=[(dp' * inv(Qdp) * dp)/ (2*pv)]		PT<Fi (0.05,2,df) PT<1.390	
S/N	Control Point Name	d2(X)	d2(Y)	PTp (X)	PTP(Y)	(X)	(Y)
1	YTT1	4.25E-08	-3.88E-05	0.000164	0.794432	Stable	Stable
2	YTT2	1.54E-05	4.46E-05	0.186669	1.069351	Stable	Stable
3	YTT3	-2.79E-5	1.52E-05	0.456962	0.080737	Stable	Stable
4	YTT4	3.96E-06	0.000879	0.007146	4.246152	Stable	Moved
5	YTT5	7.20E-06	-8.58E-05	0.024044	4.110841	Stable	Moved
6	YTT6	1.57E-05	-0.00011	0.111269	7.19099	Stable	Moved
7	YTT7	-1.94E-5	2.36E-05	0.16821	0.31783	Stable	Stable
8	YTT8	1.48E-05	-7.48E-05	0.103594	3.158588	Stable	Moved
9	YTT9	1.83E-05	-9.22E-05	0.157434	4.870269	Stable	Moved
10	YTT10	2.46E-05	-0.00011	0.293329	7.62088	Stable	Moved
11	YTT11	3.37E-05	-9.07E-05	0.565388	4.660503	Stable	Moved
12	YTT12	3.26E-05	-4.84E-05	0.521227	1.323015	Stable	Stable
13	YTT13	4.09E-06	4.35E-05	0.010728	0.872408	Stable	Stable
14	YTT14	2.35E-05	0.000899	0.300271	4.536646	Stable	Moved
15	YTT15	2.21E-05	-7.11E-05	0.236805	2.868241	Stable	Moved
16	YTT16	2.99E-05	-6.15E-05	0.452714	2.130441	Stable	Moved
17	YTT17	4.48E-05	-3.43E-05	0.986544	0.649148	Stable	Stable
18	YTT18	3.90E-05	-1.25E-05	0.760024	0.086324	Stable	Stable
19	YTT19	2.42E-05	-0.00013	0.297002	9.061258	Stable	Moved
20	YTT20	2.95E-05	-0.00014	0.466251	1.066337	Stable	Moved
21	YTT21	3.84E-05	-0.00013	0.821701	9.233006	Stable	Moved
22	YTT22	3.98E-05	-3.91E-05	0.809365	0.851019	Stable	Stable
23	YTT23	5.84E-05	-6.91E-05	1.865367	2.618017	Moved	Moved
24	YTT24	7.52E-05	-0.00012	3.129756	7.668884	Moved	Moved
25	YTT25	4.20E-05	-2.09E-05	0.899676	0.243445	Stable	Stable
26	YTT26	7.82E-05	-0.00011	3.326455	6.232521	Moved	Moved
27	YTT27	0.000114	-9.89E-05	7.224628	5.070011	Moved	Moved
28	YTT28	4.92E-05	1.15E-05	1.190501	0.07579	Stable	Stable
29	YTT29	5.36E-05	-7.24E-06	1.420155	0.028761	Moved	Stable
30	YTT30	5.14E-05	-7.36E-05	1.386828	2.965948	Stable	Moved
31	YTT31	4.38E-05	-5.26E-05	0.975635	1.507281	Stable	Moved
32	YTT32	4.49E-05	-7.63E-05	1.042566	3.237843	Stable	Moved
33	YTT33	0.00010	0.000704	6.784533	2.442919	Moved	Moved
34	YTT34	8.17E-05	-6.96E-05	3.947721	2.469739	Moved	Moved
35	YTT35	6.68E-05	-0.00017	2.844835	1.500624	Moved	Moved
36	YTT36	0.000102	-9.96E-05	6.39048	4.741311	Moved	Moved
37	YTT37	9.58E-05	-6.02E-05	5.015102	1.936708	Moved	Moved
38	YTT38	7.24E-05	-0.00011	2.885911	6.487339	Moved	Moved
39	YTT39	9.00E-05	-3.20E-05	4.402552	0.566615	Moved	Stable
40	YTT40	6.95E-05	-3.38E-05	2.663538	0.634454	Moved	Stable
41	YTT41	-1.12E-5	-7.99E-05	0.068964	3.45412	Stable	Moved
42	YTT42	0.000194	-0.00037	2.121703	7.28967	Moved	Moved
43	YTT43	-1.00E-4	1.06E-05	5.378697	0.062547	Moved	Stable
44	YTT44	-0.00215	2.25E-06	2.257432	0.002949	Moved	Stable
45	YTT45	3.37E-05	-3.90E-05	0.621957	0.823719	Stable	Stable

Analysis of Results

After the presentation of results, the results were analysed as shown in the sub session below.

Trend and Deformation Analysis of the Displacements Using LAS method

After the Least Square Estimation (LSE) of the data of the network, the compatibility of the two epochs data was tested with the variance ratio and compatibility test passed. The computed variance ratio of the campaigns is lesser than the F-distribution critical value for the specified confidence level. The critical value for the 0.05 (95%) significance level chosen for the Fisher's distribution (F) is 1.390. The test statistic (T), which is the ratio of the variances (the larger divided by the small passed. The test on the variance ratio passes at 0.05 significance level (i.e., 1.02088467792425< 1.390) of the Fisher's critical value, thus indicating the compatibility between the two epochs and permits further analysis to be carried out for deformation detection and analysis. For the 1D network, the critical value the 0.05(95%) significance level chosen for the Fisher's distribution (F) is 1.550 and it also passes the compatibility test.

The trends of movements and deformation analysis of the monitoring network was done using the adjusted coordinate differences and the cofactor matrices from both campaigns respectively and by applying the LAS method. The 1D and 2D point coordinates X, Y of each epoch and their cofactor matrices were calculated with two separate network adjustments. The Deformation program calculated displacement in X axis (dX) ,Y axis (dY) and (dZ).

The LAS determined the final displacement vector (dp). The data met the convergence criteria after two iterations. The displacement values obtained from the differences of the adjusted coordinates and their transformation by LAS method shows that virtually all the stations have undergone movements' overtime but this however did not result in deformation of all the point to a significant level.

The single point displacement test failed for some points thus confirming the existence of deformation for some of the group of selected control points. The summary of the parameters of the deformation detection and analysis for 2D and 1D are shown in Table 3.5 The results is emphasized by the plot of single point displacement vectors ,the stable and unstable points and the relative absolute error ellipse of the 45 stations in the network as represented in Figures 3.1, 3.2, and 3.3.

Table 3.5: Summary of Some Key Parameters of the Deformation Detection and Analysis (2D) and (1D)

Key Parameters	2D	1D
No of Iteration	2	2
Fisher's Distribution Critical Value for 95% Confidence Level (F)	1.390	1.550
Calculated Variance Ratio (T=rho1/rho2)	1.02088467792425	1.327053753
The Compatibility Test Passed (T<F)	1.02088467792425< 1.390)	1.327053753< 1.550)
Pooled Variance Factors	7.77532847804672e-08	0.0958933293890637
Combined Degree of Freedom	99	62

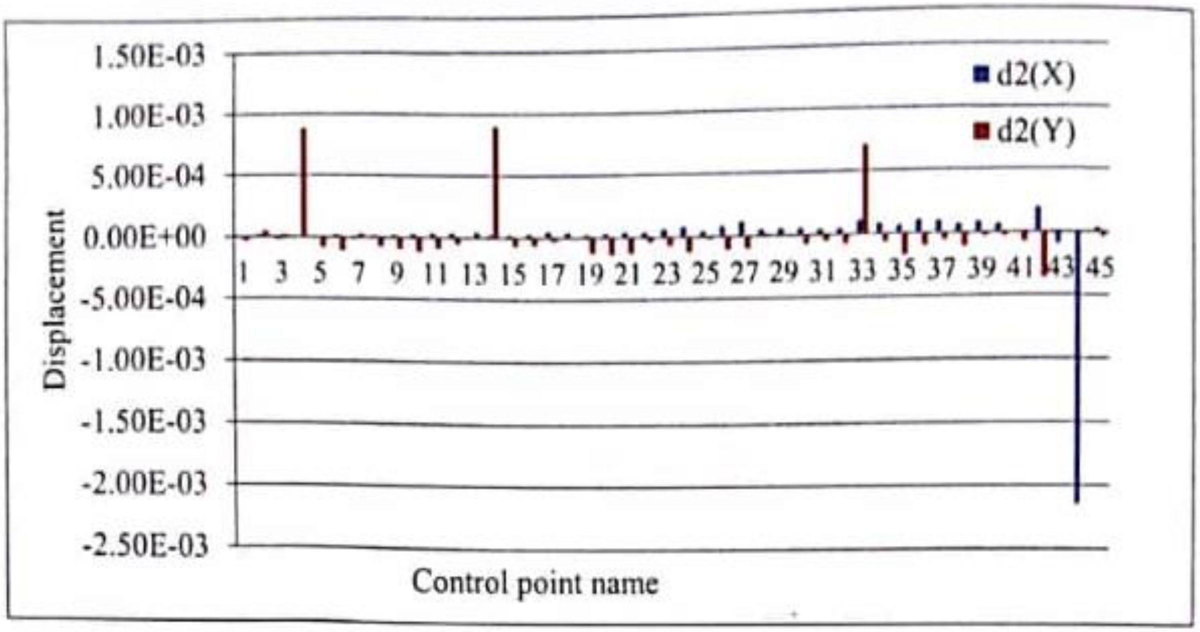

Figure 3.1: Displacement Vector Pattern after S-Transformation Using LAS

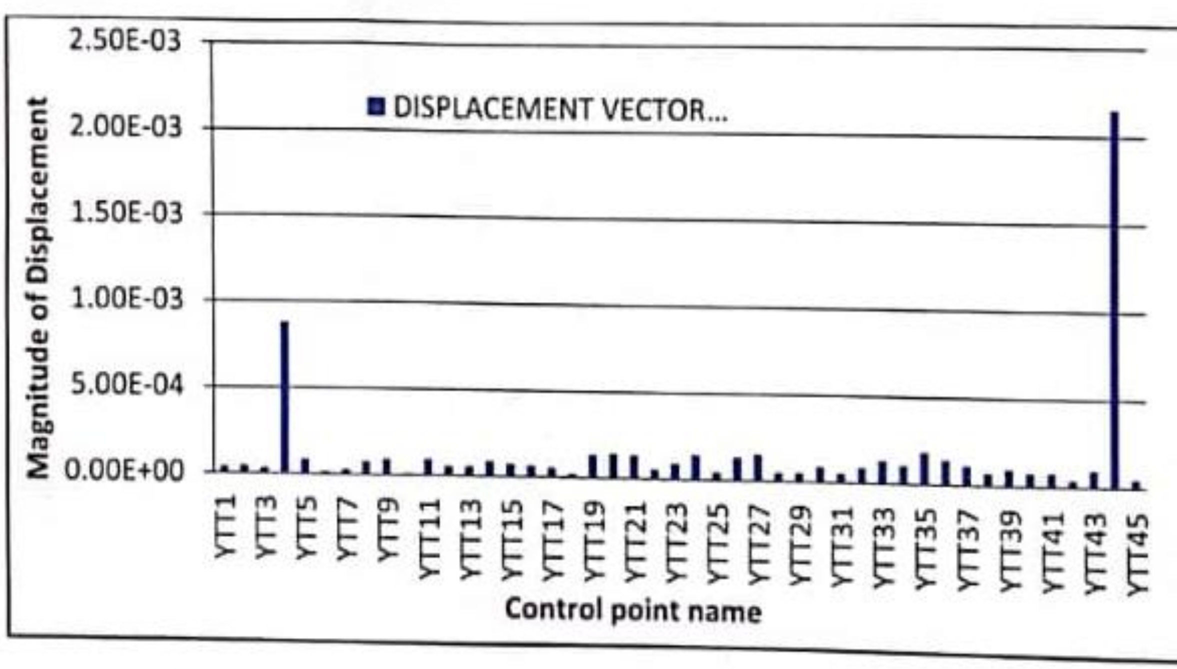

Figure 3.2: Displacement Vector Magnitude of the Stations Using LAS

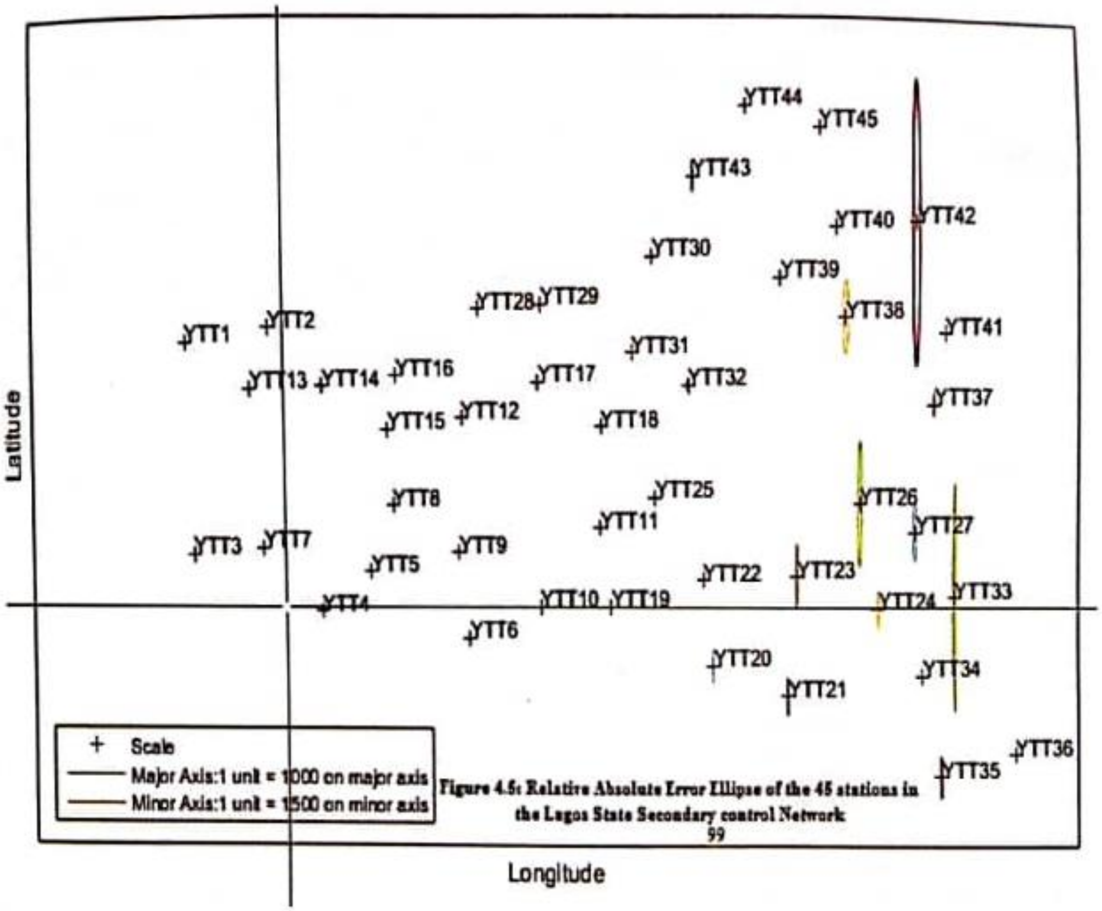

Figure 3.3: Relative Absolute Error Ellipse of the 45 stations in the Lagos State Secondary control Network

Conclusions

This study has presented successfully the application of Least Absolute Sum Technique in detecting deformation. The major focus has been on the identification of stable and unstable points in the network.

The two epoch data were adjusted by the least square adjustment technique and passed the compatibility test and are therefore compatible. The displacement vector obtained from the differences of the adjusted coordinates shows that virtually all the points have undergone movements overtime but this has not however resulted in deformation within the chosen significant level of 95% confidence limit.

The single point displacement test failed for some stations thus confirming the existence of deformation for some points. The determination of deformation status of reference points is very useful and can be applied for monitoring deformation trends in Dam Sites, Exploration areas, Tunnels and engineering structures. This shows that the Least Absolute Sum Technique (LAS) has the capacity to determine deformation of structures.

Recommendations

Based on the work done in this study, the following points are hereby recommended:

(i) Using data from more than two epochs will dramatically enhance the detection of any possible change in a deformation detection and analysis study.

(ii) As a future work, other robust and non-robust methods (e.g., Fredericton Approach, Danish Method, Total Least Square, Multi parameter Transformation, and Congruency testing methods) could be applied for the deformation detection and analysis. Furthermore dynamic model of deformation detection and prediction using the Kalman filtering methods for the velocity and acceleration determination of deformable body should be examined.

(iii) The Survey body in this country (Nigeria), should wakeup to determine how stable her platform is, in order to avert future hazards and disaster by carrying out observations on our network of controls regularly with advanced Differential Global Positioning System (DGPS) with reference to the continuously Operating reference stations (CORS) networks.

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