

Modelled Simulation of Epidemiological Differential Equations Using New Block Method

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Abstract

This study presents the development and application of a New Block Method (NBM) for the numerical simulation of first-order differential equations arising in epidemiological and real-life dynamical systems. The method is formulated using a power series polynomial approach combined with interpolation and collocation techniques to generate a high-order implicit block scheme. The resulting method is applied to three test problems, namely the susceptible-infected-recovered (SIR) model, a mixture (tank) model, and a logistic growth model. These models represent key applications in epidemiology, chemical engineering, and population dynamics, respectively. The performance of the proposed method is evaluated by comparing the computed solutions with exact analytical solutions across selected evaluation points. The results demonstrate that the NBM produces highly accurate approximations that closely match the exact solutions, as also illustrated by the graphical comparisons. The study further confirms that the method is stable, convergent, and efficient for solving nonlinear and linear differential equations. The NBM is shown to be a reliable and effective computational tool for modelling complex dynamical systems.

Keywords: New Block Method (NBM), epidemiological models, SIR model, logistic growth model, mixture model, differential equations, power series method.

Introduction

Numerical methods are essential for applied domains, which include scientific research, social science studies and engineering applications and medical practice, because exact solutions to their problems cannot be obtained. The methods enable various practical uses, which include spacecraft trajectory prediction and product safety testing and insurance risk assessment, through their combination with simulation methods that depend on solving PDEs (Adesanya, Onsachi and Odekunle, 2017). Numerical analysts develop methods to find approximate solutions, which they use to create solutions that approximate precise results, but they must overcome difficulties which need either extensive computation or limited time to achieve precise outcomes (Khalsaraei and Shokri, 2020; Kyagya, Raymond and Sabo, 2021; Ojo and Samuel, 2025). Numerical methods find applications in various fields, which include multidimensional root-finding and industry forecasts and computing and profit-loss calculations, while their theoretical and practical applications extend to engineering and

medical and business fields (Akinfenwa et al. 2020; Kwanamu et al. 2021; Ishaq et al. 2024).

The 1927 SIR model developed by Kermack and McKendrick serves as a key example for epidemiologists who use differential equations to study infectious disease transmission. The model divides populations into three groups which include susceptible (S) individuals, infective (I) individuals, and recovered (R) individuals while using oscillatory differential equations to show how diseases progress (Raymond and Sabo, 2023). Researchers create new numerical methods to study those models because they need better solutions than what existing methods can provide. The block hybrid methods use power series polynomials through interpolation and collocation to improve simulation accuracy and efficiency in epidemiological models which present difficulties because of their intricate disease spread patterns (Rizky, Mochammad and Sri, 2021; Raymond and Sabo, 2023; Bello et al. 2023; Udo and Awoyemi, 2017).

The Predictor-Corrector Method has several limitations that affect the numerical methods' efficiency and applicability in numerical computations. The method requires an initial prediction step which creates errors that spread through the entire computation process. The corrector needs multiple iterations to reach an accurate solution when the predictor delivers an incorrect result which results in higher computational expenses and longer processing durations. The problem occurs during stiff equation solution because inaccurate predictions lead to gradual convergence or complete method failure which reduces overall method stability (Adesanya, Odekunle and Alkali, 2012). The correction process uses implicit methods which require iterative methods like Newton's method. This requirement increases computational complexity especially in cases of nonlinear systems and systems that require high-dimensional processing (Tafida, 2015).

The Predictor-Corrector Method faces its main difficulty because it requires multiple function evaluations during large-scale computations. The method needs to solve equations during both prediction and correction steps which results in higher processing requirements than fully implicit methods that reduce extra work. The system loses its operational value because it cannot operate effectively to meet real-time requirements because its calculation process needs to be completed quicker (Kumleng et al., 2017). The process of selecting an appropriate step size proves essential for achieving both stability and accuracy while selecting an unsuitable step size results in unnecessary computation expenses and diminished accuracy. The implicit Runge-Kutta and Rosenbrock methods provide better stability and efficiency for solving stiff and large-scale problems than other numerical methods according to Tafida 2015. The method's accuracy and effectiveness face limitations because some implementations depend on lower-order predictors according to Adewale and Sabo 2024. The existing numerical techniques require additional development to achieve better performance across different disciplines according to Abdulrahman and Fatima 2016.

The Block Method was created as a new method to solve the problems that exist in the traditional Predictor-Corrector Method (Raymond and Sabo, 2023). The numerical method divides the time period into smaller sections which allow it to solve different schemes at various off-grid locations throughout each time section. The Block Method uses implicit integration schemes which include backward Euler and Rosenbrock methods to handle stiff and highly nonlinear systems through balanced computational distribution (Omar and Adeyeye, 2016). The primary advantages of block method is to enhanced efficiency as it divide into smaller partitions to reduces the computational burden (Khalsaraei and Shokri, 2020). The Block Method solves both the computational and stability problems of the Predictor-Corrector Method while providing research opportunities to develop better numerical methods which can effectively solve complex differential equations.

This research considers the simulation of first order oscillatory differential equation of the form

$$\frac{dy}{dx} = y, \quad y(x_0) = y(0), \quad x \in [a, b] \quad (1.1)$$

where $f : \mathfrak{R} \times \mathfrak{R}^m \rightarrow \mathfrak{R}^m$, $y, y_0 \in \mathfrak{R}^m$, f satisfy the Lipchitz condition.

Materials and Methodology

The New Block Method (NBM) derivation process in this section uses power series polynomial approximations to create a one-step solution method for the specific epidemic models that equation (1.1) defines. The method requires single-point power series interpolation together with multiple-point series collocation through alignment to partition points present in one interval.

The power series polynomial as defined by

$$y(x) = \sum_{i=0}^{\Psi} a_i \left(\frac{x - x_n}{h} \right)^i \quad (2.1)$$

The power series polynomial (2.1) is consider to simulate equation (1.1), for $x \in [x_n, x_{n+1}]$ where $\Psi = \nu + \nu - 1$, that is ν the number of interpolation and ν is the number of collocation, $n = 0, 1, 2, \dots, N - 1$, a 's the determination of the real coefficients is integral to this process. Additionally, a constant step size $h = x_n - x_{n-1}$ for partitioning the interval $[a, b]$ is prescribed by the partition itself $\pi_N : a = x_0 < x_1 < x_2 < \dots < x_n < x_{n+1} < \dots < x_N = b$

The one-step block hybrid method with ten partitions (nine grid points) were derived by using partitioned points $\left\{ 0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{9}{10}, 1 \right\}$.

Using equation (3.1) with $\nu = 1$, $\nu = 11$, we have a polynomial as

$$y(x) = \sum_{i=0}^{11} a_i \left(\frac{x - x_n}{h} \right)^i \quad (2.2)$$

Differentiating (2.2) once, to get

$$y'(x) = \sum_{i=0}^{11} i a_i \left(\frac{x - x_n}{h} \right)^{i-1} \quad (2.3)$$

Substituting (2.3) and (2.2) into (1.1) gives

$$\sum_{i=0}^{11} a_i \left(\frac{x - x_n}{h} \right)^i = f(x) \quad (2.4)$$

$$\sum_{i=0}^{11} i a_i \left(\frac{x - x_n}{h} \right)^{i-1} = f(x, y) \quad (2.5)$$

Now interpolating (2.4) at $\nu = 1$ and collocating (2.5) at

$$\nu = 0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{9}{10}, 1.$$

This leads to a system of equations expressed in matrix form as

$$\begin{bmatrix} 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6 & x_{n+1}^7 & x_{n+1}^8 & x_{n+1}^9 & x_{n+1}^{10} & x_{n+1}^{11} \\ 0 & 1 & 2x_n & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 & 6x_{n+1}^5 & 7x_{n+1}^6 & 8x_{n+1}^7 & 9x_{n+1}^8 & 10x_{n+1}^9 & 11x_{n+1}^{10} \\ 0 & 1 & 2x_{\frac{n+1}{10}} & 3x_{\frac{n+1}{10}}^2 & 4x_{\frac{n+1}{10}}^3 & 5x_{\frac{n+1}{10}}^4 & 6x_{\frac{n+1}{10}}^5 & 7x_{\frac{n+1}{10}}^6 & 8x_{\frac{n+1}{10}}^7 & 9x_{\frac{n+1}{10}}^8 & 10x_{\frac{n+1}{10}}^9 & 11x_{\frac{n+1}{10}}^{10} \\ 0 & 1 & 2x_{\frac{n+1}{5}} & 3x_{\frac{n+1}{5}}^2 & 4x_{\frac{n+1}{5}}^3 & 5x_{\frac{n+1}{5}}^4 & 6x_{\frac{n+1}{5}}^5 & 7x_{\frac{n+1}{5}}^6 & 8x_{\frac{n+1}{5}}^7 & 9x_{\frac{n+1}{5}}^8 & 10x_{\frac{n+1}{5}}^9 & 11x_{\frac{n+1}{5}}^{10} \\ 0 & 1 & 2x_{\frac{n+1}{10}} & 3x_{\frac{n+1}{10}}^2 & 4x_{\frac{n+1}{10}}^3 & 5x_{\frac{n+1}{10}}^4 & 6x_{\frac{n+1}{10}}^5 & 7x_{\frac{n+1}{10}}^6 & 8x_{\frac{n+1}{10}}^7 & 9x_{\frac{n+1}{10}}^8 & 10x_{\frac{n+1}{10}}^9 & 11x_{\frac{n+1}{10}}^{10} \\ 0 & 1 & 2x_{\frac{n+1}{5}} & 3x_{\frac{n+1}{5}}^2 & 4x_{\frac{n+1}{5}}^3 & 5x_{\frac{n+1}{5}}^4 & 6x_{\frac{n+1}{5}}^5 & 7x_{\frac{n+1}{5}}^6 & 8x_{\frac{n+1}{5}}^7 & 9x_{\frac{n+1}{5}}^8 & 10x_{\frac{n+1}{5}}^9 & 11x_{\frac{n+1}{5}}^{10} \\ 0 & 1 & 2x_{\frac{n+1}{2}} & 3x_{\frac{n+1}{2}}^2 & 4x_{\frac{n+1}{2}}^3 & 5x_{\frac{n+1}{2}}^4 & 6x_{\frac{n+1}{2}}^5 & 7x_{\frac{n+1}{2}}^6 & 8x_{\frac{n+1}{2}}^7 & 9x_{\frac{n+1}{2}}^8 & 10x_{\frac{n+1}{2}}^9 & 11x_{\frac{n+1}{2}}^{10} \\ 0 & 1 & 2x_{\frac{n+1}{5}} & 3x_{\frac{n+1}{5}}^2 & 4x_{\frac{n+1}{5}}^3 & 5x_{\frac{n+1}{5}}^4 & 6x_{\frac{n+1}{5}}^5 & 7x_{\frac{n+1}{5}}^6 & 8x_{\frac{n+1}{5}}^7 & 9x_{\frac{n+1}{5}}^8 & 10x_{\frac{n+1}{5}}^9 & 11x_{\frac{n+1}{5}}^{10} \\ 0 & 1 & 2x_{\frac{n+1}{10}} & 3x_{\frac{n+1}{10}}^2 & 4x_{\frac{n+1}{10}}^3 & 5x_{\frac{n+1}{10}}^4 & 6x_{\frac{n+1}{10}}^5 & 7x_{\frac{n+1}{10}}^6 & 8x_{\frac{n+1}{10}}^7 & 9x_{\frac{n+1}{10}}^8 & 10x_{\frac{n+1}{10}}^9 & 11x_{\frac{n+1}{10}}^{10} \\ 0 & 1 & 2x_{\frac{n+1}{5}} & 3x_{\frac{n+1}{5}}^2 & 4x_{\frac{n+1}{5}}^3 & 5x_{\frac{n+1}{5}}^4 & 6x_{\frac{n+1}{5}}^5 & 7x_{\frac{n+1}{5}}^6 & 8x_{\frac{n+1}{5}}^7 & 9x_{\frac{n+1}{5}}^8 & 10x_{\frac{n+1}{5}}^9 & 11x_{\frac{n+1}{5}}^{10} \\ 0 & 1 & 2x_{\frac{n+1}{10}} & 3x_{\frac{n+1}{10}}^2 & 4x_{\frac{n+1}{10}}^3 & 5x_{\frac{n+1}{10}}^4 & 6x_{\frac{n+1}{10}}^5 & 7x_{\frac{n+1}{10}}^6 & 8x_{\frac{n+1}{10}}^7 & 9x_{\frac{n+1}{10}}^8 & 10x_{\frac{n+1}{10}}^9 & 11x_{\frac{n+1}{10}}^{10} \\ 0 & 1 & 2x_{\frac{n+1}{5}} & 3x_{\frac{n+1}{5}}^2 & 4x_{\frac{n+1}{5}}^3 & 5x_{\frac{n+1}{5}}^4 & 6x_{\frac{n+1}{5}}^5 & 7x_{\frac{n+1}{5}}^6 & 8x_{\frac{n+1}{5}}^7 & 9x_{\frac{n+1}{5}}^8 & 10x_{\frac{n+1}{5}}^9 & 11x_{\frac{n+1}{5}}^{10} \\ 0 & 1 & 2x_{\frac{n+1}{10}} & 3x_{\frac{n+1}{10}}^2 & 4x_{\frac{n+1}{10}}^3 & 5x_{\frac{n+1}{10}}^4 & 6x_{\frac{n+1}{10}}^5 & 7x_{\frac{n+1}{10}}^6 & 8x_{\frac{n+1}{10}}^7 & 9x_{\frac{n+1}{10}}^8 & 10x_{\frac{n+1}{10}}^9 & 11x_{\frac{n+1}{10}}^{10} \\ 0 & 1 & 2x_{\frac{n+1}{5}} & 3x_{\frac{n+1}{5}}^2 & 4x_{\frac{n+1}{5}}^3 & 5x_{\frac{n+1}{5}}^4 & 6x_{\frac{n+1}{5}}^5 & 7x_{\frac{n+1}{5}}^6 & 8x_{\frac{n+1}{5}}^7 & 9x_{\frac{n+1}{5}}^8 & 10x_{\frac{n+1}{5}}^9 & 11x_{\frac{n+1}{5}}^{10} \\ 0 & 1 & 2x_{\frac{n+1}{10}} & 3x_{\frac{n+1}{10}}^2 & 4x_{\frac{n+1}{10}}^3 & 5x_{\frac{n+1}{10}}^4 & 6x_{\frac{n+1}{10}}^5 & 7x_{\frac{n+1}{10}}^6 & 8x_{\frac{n+1}{10}}^7 & 9x_{\frac{n+1}{10}}^8 & 10x_{\frac{n+1}{10}}^9 & 11x_{\frac{n+1}{10}}^{10} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \end{bmatrix} = \begin{bmatrix} y_{n+1} \\ f_n \\ f_{\frac{n+1}{10}} \\ f_{\frac{n+1}{5}} \\ f_{\frac{n+1}{10}} \\ f_{\frac{n+1}{5}} \\ f_{\frac{n+1}{2}} \\ f_{\frac{n+1}{5}} \\ f_{\frac{n+1}{10}} \\ f_{\frac{n+1}{5}} \\ f_{\frac{n+1}{10}} \\ f_{\frac{n+1}{5}} \\ f_{\frac{n+1}{10}} \\ f_{\frac{n+1}{5}} \\ f_{\frac{n+1}{10}} \end{bmatrix} \quad (2.6)$$

Employing the Gaussian elimination method on equation (2.6) yields the coefficients. These coefficients are subsequently substituted into equation (1.1) to obtain the implicit continuous method in the form:

$$y(x) = a_1 y_{n+1} + h \left(\beta_0 f_{n+0} + \beta_1 f_{\frac{n+1}{10}} + \beta_1 f_{\frac{n+1}{5}} + \beta_3 f_{\frac{n+1}{10}} + \beta_2 f_{\frac{n+1}{5}} + \beta_1 f_{\frac{n+1}{2}} + \beta_3 f_{\frac{n+1}{5}} + \beta_7 f_{\frac{n+1}{10}} + \beta_4 f_{\frac{n+1}{5}} + \beta_9 f_{\frac{n+1}{10}} + \beta_1 f_{n+1} \right) \quad (2.7)$$

$$\begin{aligned} \alpha_1 &= 1 \\ \beta_0 &= -\frac{16067}{598752} + t - \frac{7381}{504}t^2 + \frac{177133}{1512}t^3 - \frac{10511875}{18144}t^4 + \frac{8542325}{4536}t^5 - \frac{5369375}{1296}t^6 + \frac{4695625}{756}t^7 - \frac{9453125}{1512}t^8 + \frac{6875000}{1701}t^9 + \frac{859375}{567}t^{10} + \frac{1562500}{6232}t^{11} \\ \beta_{\frac{1}{10}} &= -\frac{26575}{149688} + 50t^2 - \frac{149688}{189}t^3 + \frac{1997825}{504}t^4 - \frac{16598525}{1134}t^5 + \frac{3775625}{108}t^6 - \frac{20965625}{378}t^7 + \frac{1046875}{18}t^8 - \frac{65937500}{1701}t^9 + \frac{312500}{21}t^{10} + \frac{15625000}{6232}t^{11} \\ \beta_{\frac{1}{5}} &= \frac{16175}{199584} - \frac{225}{2}t^2 + \frac{153025}{84}t^3 - \frac{2898075}{224}t^4 + \frac{6619175}{128}t^5 - \frac{57981875}{432}t^6 + \frac{56396875}{252}t^7 - \frac{17546875}{72}t^8 + \frac{94843750}{567}t^9 - \frac{4140625}{63}t^{10} + \frac{7812500}{693}t^{11} \\ \beta_{\frac{3}{10}} &= -\frac{5675}{12474} + 200t^2 - \frac{12474}{189}t^3 + \frac{10014475}{378}t^4 - \frac{21686950}{189}t^5 + \frac{8343125}{27}t^6 - \frac{33868750}{63}t^7 + \frac{5453125}{9}t^8 - \frac{242500000}{567}t^9 + \frac{32500000}{189}t^{10} + \frac{62500000}{2079}t^{11} \\ \beta_{\frac{1}{2}} &= \frac{4825}{11088} - \frac{525}{2}t^2 + \frac{168775}{36}t^3 - \frac{1792225}{48}t^4 + \frac{18214675}{108}t^5 - \frac{102070625}{216}t^6 + \frac{102070625}{126}t^7 - \frac{35640625}{36}t^8 + \frac{19375000}{27}t^9 - \frac{19375000}{9}t^{10} + \frac{15625000}{297}t^{11} \\ \beta_{\frac{3}{5}} &= -\frac{17807}{24948} + 252t^2 - \frac{252}{3}t^3 + \frac{149625}{4}t^4 - \frac{1563725}{9}t^5 + \frac{27074375}{54}t^6 - \frac{58608125}{63}t^7 + \frac{10000000}{9}t^8 - \frac{66875000}{81}t^9 + \frac{3125000}{9}t^{10} - \frac{6250000}{99}t^{11} \\ \beta_{\frac{7}{10}} &= \frac{4825}{561108800} - 175t^2 + \frac{174025}{54}t^3 - \frac{11544725}{432}t^4 + \frac{6835775}{54}t^5 - \frac{80666875}{216}t^6 + \frac{89384375}{126}t^7 - \frac{31234375}{36}t^8 + \frac{5937500}{9}t^9 - \frac{7656250}{27}t^{10} + \frac{15625000}{297}t^{11} \\ \beta_7 &= -\frac{5675}{12474} + \frac{600}{7}t^2 - \frac{600}{63}t^3 + \frac{600}{42}t^4 - \frac{12131950}{189}t^5 + \frac{5200625}{27}t^6 - \frac{1118750}{3}t^7 + \frac{29328125}{63}t^8 - \frac{29328125}{567}t^9 + \frac{10000000}{63}t^{10} - \frac{62500000}{2079}t^{11} \\ \beta_{\frac{9}{10}} &= \frac{1562500}{199584} - \frac{225}{8}t^2 + \frac{88325}{168}t^3 - \frac{996675}{224}t^4 + \frac{996675}{504}t^5 - \frac{28405625}{432}t^6 + \frac{28405625}{252}t^7 - \frac{11828125}{72}t^8 + \frac{73750000}{567}t^9 - \frac{73750000}{63}t^{10} + \frac{7812500}{693}t^{11} \\ \beta_9 &= -\frac{26575}{149688} + \frac{50}{9}t^2 - \frac{6575}{63}t^3 + \frac{4033825}{4536}t^4 - \frac{4943525}{1134}t^5 + \frac{4341875}{324}t^6 - \frac{10090625}{378}t^7 + \frac{1859375}{54}t^8 + \frac{1859375}{1701}t^9 - \frac{7187500}{567}t^{10} + \frac{15625000}{6232}t^{11} \\ \beta_1 &= -\frac{16067}{598752} + \frac{1}{2}t^2 - \frac{7129}{756}t^3 + \frac{162875}{2016}t^4 - \frac{226150}{567}t^5 + \frac{59375}{48}t^6 - \frac{1883125}{756}t^7 + \frac{78125}{24}t^8 - \frac{4531250}{1701}t^9 + \frac{78125}{63}t^{10} + \frac{1562500}{6232}t^{11} \end{aligned} \quad (2.8)$$

and

$$t = \frac{x - x_n}{h} \quad (2.9)$$

Simplify equation (2.7) at all the grid points to obtain the new Block Method (NBM) as

$$\left. \begin{aligned} y_{n+\frac{1}{10}} &= y_n + \frac{26842253}{958003200} h f_n + \frac{164046413}{1197504000} h f_{n+\frac{1}{10}} - \frac{296725183}{1596672000} h f_{n+\frac{1}{5}} + \frac{12051709}{39916800} h f_{n+\frac{3}{10}} - \frac{33765029}{88704000} h f_{n+\frac{2}{5}} \\ &+ \frac{2227571}{6237000} h f_{n+\frac{1}{2}} - \frac{21677723}{88704000} h f_{n+\frac{3}{5}} + \frac{23643791}{199584000} h f_{n+\frac{7}{10}} - \frac{12318413}{319334400} h f_{n+\frac{4}{5}} + \frac{9071219}{1197504000} h f_{n+\frac{9}{10}} - \frac{3250433}{4790016000} h f_{n+1} \\ y_{n+\frac{1}{5}} &= y_n + \frac{2046263}{74844000} h f_n + \frac{645431}{3742200} h f_{n+\frac{1}{10}} - \frac{2149811}{24948000} h f_{n+\frac{1}{5}} + \frac{355583}{1559250} h f_{n+\frac{3}{10}} - \frac{1258463}{4158000} h f_{n+\frac{2}{5}} + \frac{904403}{3118500} h f_{n+\frac{1}{2}} \\ &- \frac{166931}{831600} h f_{n+\frac{3}{5}} + \frac{21833}{222750} h f_{n+\frac{7}{10}} - \frac{800243}{24948000} h f_{n+\frac{4}{5}} + \frac{118291}{18711000} h f_{n+\frac{9}{10}} - \frac{8501}{14968800} h f_{n+1} \\ y_{n+\frac{3}{10}} &= y_n + \frac{108223}{3942400} h f_n + \frac{840607}{4928000} h f_{n+\frac{1}{10}} - \frac{879183}{19712000} h f_{n+\frac{1}{5}} + \frac{762497}{2464000} h f_{n+\frac{3}{10}} - \frac{670233}{1971200} h f_{n+\frac{2}{5}} + \frac{24399}{77000} h f_{n+\frac{1}{2}} \\ &- \frac{2136259}{9856000} h f_{n+\frac{3}{5}} + \frac{259071}{2464000} h f_{n+\frac{7}{10}} - \frac{675441}{19712000} h f_{n+\frac{4}{5}} + \frac{6637}{985600} h f_{n+\frac{9}{10}} - \frac{11899}{19712000} h f_{n+1} \\ y_{n+\frac{2}{5}} &= y_n + \frac{64121}{2338875} h f_n + \frac{400138}{2338875} h f_{n+\frac{1}{10}} - \frac{2159}{44550} h f_{n+\frac{1}{5}} + \frac{39752}{111375} h f_{n+\frac{3}{10}} - \frac{23423}{86625} h f_{n+\frac{2}{5}} + \frac{230788}{779625} h f_{n+\frac{1}{2}} \\ &- \frac{17879}{86625} h f_{n+\frac{3}{5}} + \frac{2248}{22275} h f_{n+\frac{7}{10}} - \frac{25754}{779625} h f_{n+\frac{4}{5}} + \frac{15226}{2338875} h f_{n+\frac{9}{10}} - \frac{547}{935550} h f_{n+1} \\ y_{n+\frac{1}{2}} &= y_n + \frac{1051285}{38320128} h f_n + \frac{1636625}{9580032} h f_{n+\frac{1}{10}} - \frac{599275}{12773376} h f_{n+\frac{1}{5}} + \frac{558725}{1596672} h f_{n+\frac{3}{10}} - \frac{461275}{2128896} h f_{n+\frac{2}{5}} + \frac{17807}{49896} h f_{n+\frac{1}{2}} \\ &- \frac{465125}{2128896} h f_{n+\frac{3}{5}} + \frac{167675}{1596672} h f_{n+\frac{7}{10}} - \frac{62275}{1824768} h f_{n+\frac{4}{5}} + \frac{64175}{9580032} h f_{n+\frac{9}{10}} - \frac{22997}{38320128} h f_{n+1} \\ y_{n+\frac{3}{5}} &= y_n + \frac{1689}{61600} h f_n + \frac{13169}{77000} h f_{n+\frac{1}{10}} - \frac{14787}{308000} h f_{n+\frac{1}{5}} + \frac{1363}{3850} h f_{n+\frac{3}{10}} - \frac{35229}{154000} h f_{n+\frac{2}{5}} + \frac{16083}{38500} h f_{n+\frac{1}{2}} \\ &- \frac{25373}{154000} h f_{n+\frac{3}{5}} + \frac{1887}{19250} h f_{n+\frac{7}{10}} - \frac{2007}{61600} h f_{n+\frac{4}{5}} + \frac{71}{11000} h f_{n+\frac{9}{10}} - \frac{179}{308000} h f_{n+1} \\ y_{n+\frac{7}{10}} &= y_n + \frac{18775351}{684288000} h f_n + \frac{5843887}{34214400} h f_{n+\frac{1}{10}} - \frac{10669897}{228096000} h f_{n+\frac{1}{5}} + \frac{9973607}{28512000} h f_{n+\frac{3}{10}} - \frac{2767667}{12672000} h f_{n+\frac{2}{5}} \\ &+ \frac{353633}{891000} h f_{n+\frac{1}{2}} - \frac{241129}{2534400} h f_{n+\frac{3}{5}} + \frac{4148249}{28512000} h f_{n+\frac{7}{10}} - \frac{8312311}{228096000} h f_{n+\frac{4}{5}} + \frac{1190357}{171072000} h f_{n+\frac{9}{10}} - \frac{84427}{136857600} h f_{n+1} \\ y_{n+\frac{4}{5}} &= y_n + \frac{12818}{467775} h f_n + \frac{400448}{2338875} h f_{n+\frac{1}{10}} - \frac{38176}{779625} h f_{n+\frac{1}{5}} + \frac{278272}{779625} h f_{n+\frac{3}{10}} - \frac{12184}{51975} h f_{n+\frac{2}{5}} + \frac{330368}{779625} h f_{n+\frac{1}{2}} \\ &- \frac{34432}{259875} h f_{n+\frac{3}{5}} + \frac{176896}{779625} h f_{n+\frac{7}{10}} - \frac{3998}{779625} h f_{n+\frac{4}{5}} + \frac{2368}{467775} h f_{n+\frac{9}{10}} - \frac{1184}{2338875} h f_{n+1} \\ y_{n+\frac{9}{10}} &= y_n + \frac{542331}{19712000} h f_n + \frac{837567}{4928000} h f_{n+\frac{1}{10}} - \frac{167427}{3942400} h f_{n+\frac{1}{5}} + \frac{829089}{2464000} h f_{n+\frac{3}{10}} - \frac{1880253}{9856000} h f_{n+\frac{2}{5}} + \frac{27459}{77000} h f_{n+\frac{1}{2}} \\ &- \frac{537219}{9856000} h f_{n+\frac{3}{5}} + \frac{10773}{70400} h f_{n+\frac{7}{10}} - \frac{2065743}{19712000} h f_{n+\frac{4}{5}} + \frac{199809}{4928000} h f_{n+\frac{9}{10}} - \frac{4671}{3942400} h f_{n+1} \\ y_{n+1} &= y_n + \frac{16067}{598752} h f_n + \frac{26575}{149688} h f_{n+\frac{1}{10}} - \frac{16175}{199584} h f_{n+\frac{1}{5}} + \frac{5675}{12474} h f_{n+\frac{3}{10}} - \frac{4825}{11088} h f_{n+\frac{2}{5}} + \frac{17807}{24948} h f_{n+\frac{1}{2}} \\ &- \frac{4825}{11088} h f_{n+\frac{3}{5}} + \frac{5675}{12474} h f_{n+\frac{7}{10}} + \frac{16175}{199584} h f_{n+\frac{4}{5}} + \frac{26575}{149688} h f_{n+\frac{9}{10}} + \frac{16067}{598752} h f_{n+1} \end{aligned} \right\} \quad (2.10)$$

Numerical Properties of New Block Method (NBM)

The analysis of the basic properties of NBM are analyzed numerically, such as order and error constant, consistency, zero-stability and the region of absolute stability.

Order and Error Constant of NBM

The order and error constant of NBM was carryout according to Kumleng et al. (2017), by using.

$$\left. \begin{aligned}
 &\sum_{j=0}^{\infty} \frac{\left(\frac{1}{10}\right)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n^{j+1} \left[\frac{164046413\left(\frac{1}{10}\right)^j}{1197504000} - \frac{296725183\left(\frac{1}{10}\right)^j}{1596672000} + \frac{12051709\left(\frac{3}{10}\right)^j}{39916800} - \frac{33765029\left(\frac{2}{5}\right)^j}{88704000} + \frac{2227571\left(\frac{1}{2}\right)^j}{6237000} \right. \\
 &\quad \left. - \frac{21677723}{88704000} h f_{n+\frac{3}{5}} + \frac{23643791}{199584000} h f_{n+\frac{7}{10}} - \frac{12318413}{319334400} h f_{n+\frac{4}{5}} + \frac{9071219}{1197504000} h f_{n+\frac{9}{10}} - \frac{3250433}{4790016000} h f_{n+1} \right] \\
 &\sum_{j=0}^{\infty} \frac{\left(\frac{1}{5}\right)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n^{j+1} \left[\frac{645431\left(\frac{1}{10}\right)^j}{3742200} - \frac{2149811\left(\frac{1}{10}\right)^j}{24948000} + \frac{355583\left(\frac{3}{10}\right)^j}{1559250} - \frac{1258463\left(\frac{2}{5}\right)^j}{4158000} + \frac{904403\left(\frac{1}{2}\right)^j}{3118500} \right. \\
 &\quad \left. - \frac{166931}{831600} h f_{n+\frac{3}{5}} + \frac{21833}{222750} h f_{n+\frac{7}{10}} - \frac{800243}{24948000} h f_{n+\frac{4}{5}} + \frac{118291}{18711000} h f_{n+\frac{9}{10}} - \frac{8501}{14968800} h f_{n+1} \right] \\
 &\sum_{j=0}^{\infty} \frac{\left(\frac{3}{10}\right)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n^{j+1} \left[\frac{840607\left(\frac{1}{10}\right)^j}{4928000} - \frac{879183\left(\frac{1}{10}\right)^j}{19712000} + \frac{762497\left(\frac{3}{10}\right)^j}{2464000} - \frac{670233\left(\frac{2}{5}\right)^j}{1971200} + \frac{24399\left(\frac{1}{2}\right)^j}{77000} \right. \\
 &\quad \left. - \frac{2136259}{9856000} h f_{n+\frac{3}{5}} + \frac{259071}{2464000} h f_{n+\frac{7}{10}} - \frac{675441}{19712000} h f_{n+\frac{4}{5}} + \frac{6637}{985600} h f_{n+\frac{9}{10}} - \frac{11899}{19712000} h f_{n+1} \right] \\
 &\sum_{j=0}^{\infty} \frac{\left(\frac{2}{5}\right)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n^{j+1} \left[\frac{400138\left(\frac{1}{10}\right)^j}{2338875} - \frac{2159\left(\frac{1}{5}\right)^j}{44550} + \frac{39752\left(\frac{3}{10}\right)^j}{111375} - \frac{23423\left(\frac{2}{5}\right)^j}{86625} + \frac{230788\left(\frac{1}{2}\right)^j}{779625} \right. \\
 &\quad \left. - \frac{17879}{86625} h f_{n+\frac{3}{5}} + \frac{2248}{22275} h f_{n+\frac{7}{10}} - \frac{25754}{779625} h f_{n+\frac{4}{5}} + \frac{15226}{2338875} h f_{n+\frac{9}{10}} - \frac{547}{935550} h f_{n+1} \right] \\
 &\sum_{j=0}^{\infty} \frac{\left(\frac{1}{2}\right)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n^{j+1} \left[\frac{1636625\left(\frac{1}{10}\right)^j}{9580032} - \frac{599275\left(\frac{1}{10}\right)^j}{12773376} + \frac{558725\left(\frac{3}{10}\right)^j}{1596672} - \frac{461275\left(\frac{2}{5}\right)^j}{2128896} + \frac{17807\left(\frac{1}{2}\right)^j}{49896} \right. \\
 &\quad \left. - \frac{465125}{2128896} h f_{n+\frac{3}{5}} + \frac{167675}{1596672} h f_{n+\frac{7}{10}} - \frac{62275}{1824768} h f_{n+\frac{4}{5}} + \frac{64175}{9580032} h f_{n+\frac{9}{10}} - \frac{22997}{38320128} h f_{n+1} \right] \\
 &\sum_{j=0}^{\infty} \frac{\left(\frac{3}{5}\right)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n^{j+1} \left[\frac{13169\left(\frac{1}{10}\right)^j}{77000} - \frac{14787\left(\frac{1}{5}\right)^j}{308000} + \frac{1363\left(\frac{3}{10}\right)^j}{3850} - \frac{35229\left(\frac{2}{5}\right)^j}{154000} + \frac{16083\left(\frac{1}{2}\right)^j}{38500} \right. \\
 &\quad \left. - \frac{25373\left(\frac{3}{5}\right)^j}{154000} + \frac{1887\left(\frac{7}{10}\right)^j}{19250} - \frac{2007\left(\frac{4}{5}\right)^j}{61600} + \frac{71\left(\frac{9}{10}\right)^j}{11000} - \frac{179}{308000} (1)^j \right] \\
 &\sum_{j=0}^{\infty} \frac{\left(\frac{7}{10}\right)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n^{j+1} \left[\frac{5843887\left(\frac{1}{10}\right)^j}{34214400} - \frac{10669897\left(\frac{1}{10}\right)^j}{228096000} + \frac{9973607\left(\frac{3}{10}\right)^j}{28512000} - \frac{2767667\left(\frac{2}{5}\right)^j}{12672000} + \frac{353633\left(\frac{1}{2}\right)^j}{891000} \right. \\
 &\quad \left. - \frac{241129\left(\frac{3}{5}\right)^j}{2534400} + \frac{4148249\left(\frac{7}{10}\right)^j}{28512000} - \frac{8312311\left(\frac{4}{5}\right)^j}{228096000} + \frac{1190357\left(\frac{9}{10}\right)^j}{171072000} - \frac{84427}{136857600} (1)^j \right] \\
 &\sum_{j=0}^{\infty} \frac{\left(\frac{4}{5}\right)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n^{j+1} \left[\frac{400448\left(\frac{1}{10}\right)^j}{2338875} - \frac{38176\left(\frac{1}{5}\right)^j}{779625} + \frac{278272\left(\frac{3}{10}\right)^j}{779625} - \frac{12184\left(\frac{2}{5}\right)^j}{51975} + \frac{330368\left(\frac{1}{2}\right)^j}{779625} \right. \\
 &\quad \left. - \frac{34432\left(\frac{3}{5}\right)^j}{259875} + \frac{176896\left(\frac{7}{10}\right)^j}{779625} - \frac{3998\left(\frac{4}{5}\right)^j}{779625} + \frac{2368\left(\frac{9}{10}\right)^j}{467775} - \frac{1184}{2338875} (1)^j \right] \\
 &\sum_{j=0}^{\infty} \frac{\left(\frac{9}{10}\right)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n^{j+1} \left[\frac{837567\left(\frac{1}{10}\right)^j}{4928000} - \frac{167427\left(\frac{1}{10}\right)^j}{3942400} + \frac{829089\left(\frac{3}{10}\right)^j}{2464000} - \frac{1880253\left(\frac{2}{5}\right)^j}{9856000} + \frac{27459\left(\frac{1}{2}\right)^j}{77000} \right. \\
 &\quad \left. - \frac{537219\left(\frac{3}{5}\right)^j}{9856000} + \frac{10773\left(\frac{7}{10}\right)^j}{70400} - \frac{2065743\left(\frac{4}{5}\right)^j}{19712000} + \frac{199809\left(\frac{9}{10}\right)^j}{4928000} - \frac{4671}{3942400} (1)^j \right] \\
 &\sum_{j=0}^{\infty} \frac{(1)^j}{j!} - y_n - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n^{j+1} \left[\frac{26575\left(\frac{1}{10}\right)^j}{149688} - \frac{16175\left(\frac{1}{5}\right)^j}{199584} + \frac{5675\left(\frac{3}{10}\right)^j}{12474} - \frac{4825\left(\frac{2}{5}\right)^j}{11088} + \frac{17807\left(\frac{1}{2}\right)^j}{24948} \right. \\
 &\quad \left. - \frac{4825\left(\frac{3}{5}\right)^j}{11088} + \frac{5675\left(\frac{7}{10}\right)^j}{12474} + \frac{16175\left(\frac{4}{5}\right)^j}{199584} + \frac{26575\left(\frac{9}{10}\right)^j}{149688} + \frac{16067}{598752} (1)^j \right]
 \end{aligned} \right\} = 0 \quad (3.1)$$

Thus, $\bar{C}_0 = 0, \bar{C}_1 = 0, \bar{C}_2 = 0, \bar{C}_3 = 0, \dots = 0, \bar{C}_p = 0$; implying that the order of NBM is 11. That is, NBM is of order eleven, with the error constant given as

$$C_{12} = \begin{bmatrix} 1.2343 \times 10^{-14} & 1.7861 \times 10^{-14} & 1.8763 \times 10^{-14} & 1.8821 \times 10^{-14} & 1.2301 \times 10^{-14} \\ 1.2226 \times 10^{-14} & 1.0752 \times 10^{-14} & 1.9328 \times 10^{-14} & 1.5213 \times 10^{-14} & 1.4439 \times 10^{-14} \end{bmatrix}^T$$

Consistency of NBM

According to numerical analyst, NBM is consistent, since the order is greater than one (Kwanamu, 2023).

Zero Stability of NBM

According to Kwanamu, (2023), the NBM is said to be zero-stable if as $h \rightarrow 0$, the root $z_j, j = 1(1)k$ of the first characteristic polynomial $\rho(z) = 0$ that is

$$\rho(z) = \det \left[\sum_{j=0}^k A^{(j)} z^{k-j} \right] = 0 \quad (3.2)$$

satisfies $|z_j| \leq 1$ and for those roots with $|z_j| = 1$, multiplicity must not exceed two. The NBM is expressed in the form

$$\rho(z) = z \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} = z^9(z-1) \quad (3.3)$$

Thus, solving for z in

$$\rho(z) = z^9(z-1) = 0 \quad (3.4)$$

Gives $z_1 = 0, z_2 = 0, z_3 = 0, z_4 = 0, z_5 = 0, z_6 = 0, z_7 = 0, z_8 = 0, z_9 = 0$ and $z_{10} = 1$

Therefore, NBM is zero-stable.

Convergence of NBM

Based on Dahlquist's Theorem which state that, the requisite and complete prerequisites for a NBM to achieve convergence entail both consistency and zero-stability (Yakubu & Markus, 2016).

Therefore, the NBMis convergence since it is consistency and zero-stability.

Absolute Stability Region of NBM

The absolute stability region discrib the region of complex z-plane. The boundary locus method was used on the NBM using Udo&Awoyemi, (2017) to obtain the stability polynomial as

$$\bar{h}(w) = h^{10} \left(\frac{12753803513931}{744943288200000} w^9 - \frac{58777}{4204890} w^{10} \right) - h^9 \left(- \frac{102184206471587}{633719116000000000000} w^9 + \frac{183517}{4752} w^{10} \right) + \left. \begin{aligned} & h^8 \left(- \frac{124078709442223}{405580234520000000000} w^9 + \frac{66528}{1181437} w^{10} \right) + h^7 \left(\frac{1338797113347}{14696043107843840000} w^9 - \frac{1303}{3214155168} w^{10} \right) + \\ & h^6 \left(- \frac{267567585251287}{9797362069562560000} w^9 + \frac{502211745}{4523} w^{10} \right) + h^5 \left(\frac{2174780359142223}{1216740702560000000000} w^9 - \frac{985}{5629856} w^{10} \right) + \\ & h^4 \left(- \frac{15542282539737}{5431878144000000000000} w^9 - \frac{986}{74640} w^{10} \right) + h^3 \left(\frac{16180995779906749}{391095226380000000000} w^9 - \frac{867}{90875} w^{10} \right) + \\ & h^2 \left(\frac{1441155772}{189684633000000000000} w^9 + \frac{332}{8524} w^{10} \right) + h \left(\frac{2400000}{10886400} w^9 - \frac{1}{8} w^{10} \right) - w^8 + w^9 \end{aligned} \right\} \quad (3.5)$$

The stability polynomial (3.5) is used to plot the absolute stability region as

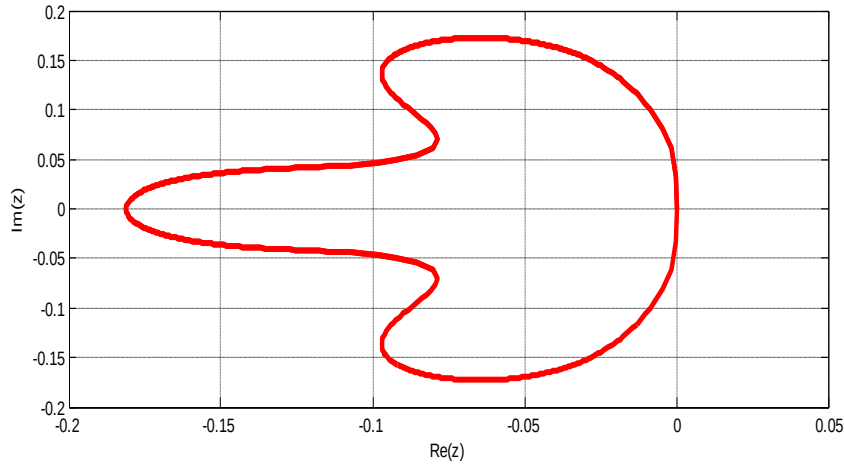


Figure 3.3: An A_α – Stability region of NBM.

Numerical Experiments

The NBM was numerically simulated on several epidemiological modelled differential equations of the form (1.1) as outlined below. The following notations shall be used in Tables and figure below.

t means Point of Evaluation

OA16 means Error in Omar and Adeyeye, (2016)

SOA13 means Error in Sunday, *et al.* (2013)

SYA16 means Error in Sunday et al. (2016)

Problem 4.1 (SIR Model):

The susceptible-infected-recovered (SIR) model tracks the progression of contagious diseases through its measurement of infected individuals in a population across different time points. The models are based on coupled equations which define three different population groups: $S(t)$ shows the number of people who still have not caught the disease, $I(t)$ displays the current count of infected cases and $R(t)$ shows the total number of people who have recovered from the illness. The model is coupled as:

$$\frac{dS}{dt} = \mu - \beta SI - \mu S \quad (4.1)$$

$$\frac{dI}{dt} = \beta SI - \gamma I - \mu I \quad (4.2)$$

$$\frac{dR}{dt} = \gamma I - \mu R \quad (4.3)$$

for μ, γ and β are positive parameters. y is given as,

$$y = S + I + R \quad (4.4)$$

Summing (4.1), (4.2) and (4.3) to get

$$y' = \mu(1 - y) \quad (4.5)$$

let $\mu = 0.5$ with initial condition as $y(0) = -0.5$, we get,

$$\frac{dy}{dt} = \frac{1}{2}(1-y), y(0) = \frac{1}{2}, h = 0.1 \quad (4.6)$$

with exact solution:

$$y(t) = 1 - \frac{1}{2}e^{-t} \quad (4.7)$$

Source: [Omar and Adeyeye, (2016) and Sunday *et al.* (2013)]

Problem 4.2 (Mixture Model)

The study observed a storage tank in the oil industry which 2000gal a certain amount of gasoline that originally has 100lb of an additive dissolved in it. When preparing for winter, gasoline containing 2lb of additive per gallon, which is pumped into the tank at a rate of 140gal/min. The well-mixed solution is pumped out at a rate of 45gal/min. Now, by numerical application, calculate the additive in the tank 0.1 min, 0.5 min and 1 min after the pumping process begins? operation starts.

We assumed y to be the amount (in pounds) of additive in the tank at time t . Taking $y = 100$, when the time $t = 0$. Therefore, after modeling the problem we have,

$$\frac{dy}{dt} = 80 - \frac{45}{(2000-5t)}y, y(0) = 100 \quad (4.8)$$

with the exact solution

$$2(2000-5t) - \frac{3900}{2000} \left(\frac{2000}{5t} \right) \quad (4.9)$$

Source: [Omar and Adeyeye, (2016) and Sunday *et al.* (2013)]

Problem 4.3 (Logistic Model)

The logistic model which extends the growth model functions as a law which accurately predicts population growth rates based on time. The model is established on Assuming the population grows within a confined environment where resources are limited and there is no migration in or out, the change in population over time can be described mathematically. Let $x(t)$ be the population size at time t be represented as x , which allows us to express its growth pattern through differential equation.

$$\frac{dx}{dT} = rx \left(1 - \frac{x}{k} \right) \quad (4.10)$$

Where $k > 0$ is the carrying capacity of the system/environment, $r > 0$ is a parameter called intrinsic growth rate ($r = b - d$, where b is the instantaneous birth rate and d the instantaneous death rate).

We will begin the process of non-dimensionalization through the scaling method for equation (4.10). The model in equation (4.10) needs its four parameters to be reduced through the application of an appropriate scaling method. We will establish the transformation through the following definition:

$$T = \frac{t}{r} \text{ and } x - ky \quad (4.11)$$

Substitute equation (4.11) in (4.10), we get

$$\frac{d(ky)}{d\frac{t}{r}} = rky \left(1 - \frac{ky}{k} \right) \quad (4.12)$$

Which reduce to

$$\frac{dy}{dt} = y(1 - y) \quad (4.13)$$

Equation (4.13) is called the logistic model. The equation can be solved by the method of separation of variable to give the exact solution,

$$y(t) = \frac{y_0}{y_0 + (1 - y_0)} \quad (4.14)$$

where $y(t_0) = y_0$.

It is significant to state that this equation possess a very simple asymptotic dynamics, all solutions with positive initial condition ($y_0 > 0$) will eventually approach the carrying capacity k .

Therefore, population size will eventually be stabilized to k in the long run if population dynamics initially either overshoot or undershoot the carrying capacity.

Here, we shall consider a special case of equation (4.14) given by substituting $r = k = 1$ in Equation (4.12). This gives,

$$\frac{dy}{dt} = y(1 - y), y(0) = 0.5 \quad (4.15)$$

With theoretical solution,

$$y(t) = \frac{0.5}{0.5(1 + e^{-t})} \quad (4.16)$$

Source: [Sunday et al. 2016].

Table 4.1: Showing the results of Problem 4.1

x	Exact Solution	Computed Solution	ENBM	EOA16	ESOA13
0.1	0.52438528774964299546	0.52438528774964299541	5.0000E-20	4.9562E-06	5.5744E-12
0.2	0.54758129098202021342	0.54758129098202021330	1.2000E-19	4.7260E-06	3.9462E-12
0.3	0.56964601178747109638	0.56964601178747109621	1.7000E-19	8.9799E-06	8.1832E-12
0.4	0.59063462346100907066	0.59063462346100907046	2.0000E-19	8.5524E-06	3.4361E-11
0.5	0.61059960846429756588	0.61059960846429756563	2.5000E-19	1.2193E-05	1.9294E-10
0.6	0.62959088965914106696	0.62959088965914106666	3.0000E-19	1.1608E-05	1.8790E-10
0.7	0.64765595514064328282	0.64765595514064328247	3.5000E-19	1.4713E-05	1.7768E-10
0.8	0.66483997698218034963	0.66483997698218034925	3.8000E-19	1.4004E-05	1.7247E-10
0.9	0.68118592418911335343	0.68118592418911335301	4.2000E-19	1.6643E-05	1.8476E-10
1.0	0.69673467014368328820	0.69673467014368328774	4.6000E-19	1.5839E-05	3.0058E-10

Table 4.2: Showing the results of Problem 4.2

x	Exact Solution	Computed Solution	ENBM	EOA16	ESOA13
0.1	107.7662301168309486000	107.76623011683094854000	1.0000E-17	2.5540E-06	2.7001E-13
0.2	115.5149409193028511000	115.51494091930285114800	1.0000E-17	2.5490E-06	1.2790E-13
0.3	123.2461630508845220000	123.24616305088452201000	1.0000E-17	5.0900E-06	1.4211E-13
0.4	130.9599271090910725000	130.95992710909107258000	4.0000E-17	5.0790E-06	4.2633E-13
0.5	138.6562636455413535000	138.65626364554135354000	2.0000E-17	7.6070E-06	1.1367E-13
0.6	146.3352031660153396000	146.33520316601533959000	2.0000E-17	7.5900E-06	1.7053E-13
0.7	153.9967761305114566000	153.99677613051145661000	1.0000E-17	1.0100E-05	8.5265E-14
0.8	161.6410129533038516000	161.64101295330385157700	1.0000E-17	1.0080E-05	8.5265E-14
0.9	169.2679440029996050000	169.26794400299960502000	1.0000E-17	1.2580E-05	8.5265E-14
1.0	176.8775996025958864000	176.87759960259588644000	4.0000E-17	1.2560E-05	2.2737E-13

Table 4.3: Showing the results of Problem 4.3

x	Exact Solution	Computed Solution	ENBM	SYA16
0.1	0.52497918747893998610	0.52497918747893998612	2.0000E-20	1.1102E-16
0.2	0.54983399731247790855	0.54983399731247790860	5.0000E-20	1.1102E-16
0.3	0.57444251681165898715	0.57444251681165898722	7.0000E-20	4.4489E-16
0.4	0.59868766011245200035	0.59868766011245200045	1.0000E-20	7.7716E-16
0.5	0.62245933120185456465	0.62245933120185456474	9.0000E-20	1.2213E-15
0.6	0.64565630622579545290	0.64565630622579545298	8.0000E-20	1.5543E-15
0.7	0.66818777216816610655	0.66818777216816610660	5.0000E-20	2.1094E-15
0.8	0.68997448112761244265	0.68997448112761244270	5.0000E-20	2.6645E-15
0.9	0.71094950262500396345	0.71094950262500396350	5.0000E-20	3.2197E-15
1.0	0.73105857863000487925	0.73105857863000487933	8.0000E-20	3.7748E-15

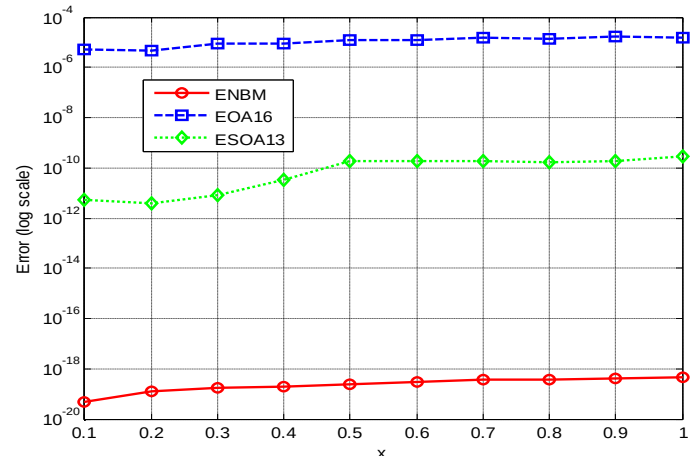


Figure 4.1: Textual curve of table 4.1

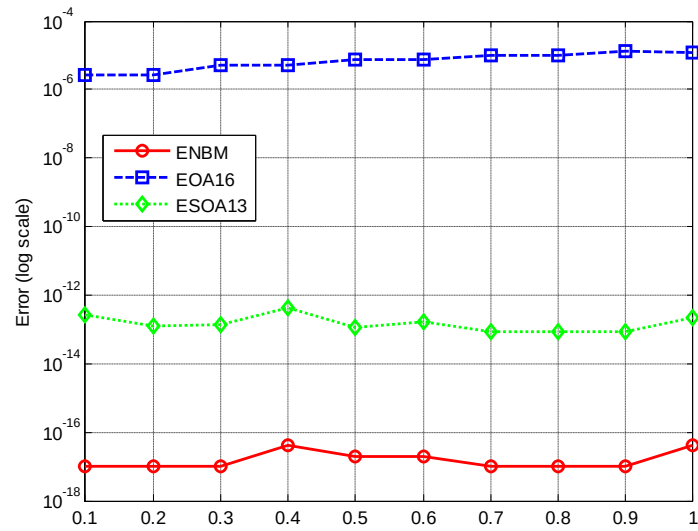


Figure 4.2: Textual curve of table 4.2

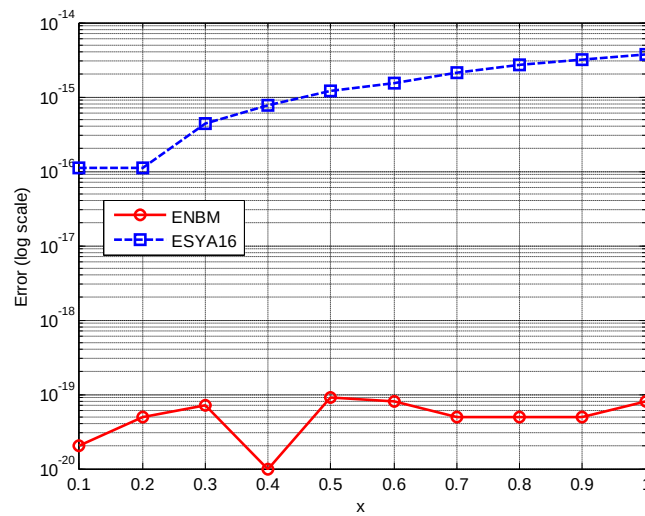


Figure 4.3: Textual curve of table 4.3

Discussion of Results

The numerical results of the Nonlinear Block Method (NBM) applied to the SIR model show an excellent agreement between the computed and exact solutions across all evaluation points. As presented in Table 4.1, the computed values closely follow the analytical solutions with no visible deviation at the displayed precision level. This indicates that the method is highly reliable for capturing the dynamics of infectious disease spread governed by the SIR system. The corresponding Figure 4.1 further illustrates this strong agreement, as both curves essentially overlap, confirming the accuracy and stability of the proposed method.

For the mixture model in Problem 4.2, the results in Table 4.2 also show a strong correspondence between the computed and exact solutions over the entire time interval considered. The numerical values increase steadily with time, reflecting the continuous accumulation of additive in the tank as dictated by the inflow and outflow rates. The NBM successfully reproduces this physical behaviour without noticeable deviation from the analytical solution. Figure 4.2 similarly shows that the computed curve follows the exact trajectory very closely, indicating that the method is well-suited for solving first-order linear transport and mixing problems in engineering applications.

In Problem 4.3, the logistic growth model results in Table 4.3 demonstrate that the computed solutions match the exact solutions throughout the evaluation points. The numerical results capture the characteristic sigmoidal growth pattern, where the solution increases rapidly at early times and gradually levels off as it approaches the carrying capacity. This behaviour is also clearly reflected in Figure 4.3, where the computed and exact curves are indistinguishable. The agreement confirms that the NBM accurately preserves the nonlinear dynamics inherent in population growth models.

The results across the three test problems demonstrate that the NBM is highly accurate and consistently reliable for solving nonlinear and linear differential equations arising in epidemiology and population dynamics. The close agreement between computed and exact solutions, as well as the overlapping nature of the graphical results, confirms the effectiveness of the method in capturing both transient and long-term system behaviour. This establishes the NBM as a robust computational technique for real-world mathematical modelling problems.

Summary and Conclusion

This study presents the development of a New Block Method (NBM) for solving first-order ordinary differential equations arising from real-life and epidemiological models. The method was constructed using a power series polynomial framework combined with interpolation and collocation techniques to generate a high-order implicit block scheme. It was applied to three benchmark problems: the SIR epidemiological model, a mixture (tank) model, and the logistic population growth model. These applications demonstrate the method's capability in handling both linear and nonlinear problems without reducing them to first-order systems.

The numerical experiments show that the NBM produces solutions that are in very close agreement with exact analytical solutions across all tested models. The results confirm that the method accurately captures the dynamic behaviour of each system, including disease transmission patterns in the SIR model, concentration variation in the tank model, and sigmoidal growth in the logistic model. Graphical and computational comparisons further indicate that the method performs with high precision and maintains the essential qualitative features of the models.

The study establishes that the New Block Method is stable, consistent, and convergent. It demonstrates strong reliability in solving initial value problems with high accuracy and computational efficiency. The method therefore serves as a robust

alternative to existing numerical techniques and is recommended for extension to more complex nonlinear systems and advanced epidemiological models requiring precise numerical approximation.

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