



**Reliability Assessment of the flexural Behavior of Reinforced  
Concrete Beam with embedded PVC Pipes**

**By**

**WASIU, John**

**13/68DB001**

**JULY, 2018.**

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Concrete Beam with embedded PVC Pipes**

**By**

**WASIU, John**

**B.Eng. (Bayero), M.Eng. (Bayero), R.Eng. (COREN)**

**A Thesis submitted in partial fulfillment of the requirements for the degree of  
Doctor of Philosophy (Civil Engineering)**

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## CERTIFICATION

This is to certify that this thesis has been read and approved as meeting the partial requirements of the Department of Civil Engineering and Postgraduate School, University of Ilorin, for the award of Doctor of Philosophy (Ph.D) in Civil Engineering.

.....

Prof. A. A. Adedeji

*Project Supervisor*

.....

Date

.....

Dr. I.T. Yusuf

*Postgraduate Coordinator*

.....

Date

.....

Dr. O. O. Adeleke

*Head of Department*

.....

Date

.....

*Internal Examiner*

.....

Date

.....

*External Examiner*

.....

Date

## **DEDICATION**

This project is dedicated to the Almighty God for his infinite wisdom and to the course of service to humanity.

## DECLARATION

I WASIU, John with matriculation number **13/68DB001** of Civil Engineering Department, University of Ilorin declare that this thesis **“Reliability Assessment of the flexural Behavior of Reinforced Concrete Beam with embedded PVC Pipes”** submitted to the Department of Civil Engineering, Faculty of Engineering and Technology, University of Ilorin, was carried out by me with the assistance of my supervisor and has never been a thesis submitted to any tertiary institution in Nigeria.

.....

WASIU, John

.....

Date

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## **LIST OF ABBREVIATIONS AND NOTATIONS**

C.S: Compressive strength

RCPVC1: Beam with one embedded PVC pipe

RCPVC2: Beam with two embedded PVC pipes

RCPVC3: Beam with three embedded PVC pipes

RCBM: Beam without PVC pipe

PVC pipe: Poly Vinyl Chloride pipe

$\sigma$ : Stress

$\epsilon$ : Strain

$\delta$ : Deflection

MoR: Modulus of Rupture

E: Modulus of elasticity

$\beta$ : Reliability Index

NRMCA: National Ready Mix Concrete Association

JCSS: Joint Committee of Structural Concrete

FORM: First Order Reliability Method

BS: British Standard

ASTM: American Society for testing and Materials

GUI: Graphical User Interface

MS: Microsoft

ANSYS: Analysis System for modeling and simulation

## ABSTRACT

Beams are major structural elements in structures against flexural failure. Increasing the span of building, increases the risk of deflection leading to increase in structural member thickness. Structural elements failures, due to the presence of electrical wire installation, are minimized by conducting them through the hollow polyvinyl chloride (PVC) pipes. In the tensile zone below the neutral axis, some concrete and reinforcing steel are eliminated by the presence of PVC pipes; hence, paving way for effective materials utilization. Therefore, the aim of this study was to investigate the reliability of the flexural strength of reinforced concrete beam with embedded PVC pipes below the neutral axis by experiment and modelling using ANSYS Pro 15.0. The objectives were to: (i) determine the compressive strength of concrete cubes with and without embedded PVC pipes and their stress-strain relationship; (ii) investigate the flexural behaviour of the beams with and without PVC pipes and their relationship with compressive strength; and (iii) evaluate the reliability of the beam with and without PVC pipes using the First Order Reliability Method (FORM).

Finite Element Method (FEM) was used and adopted for designing and modelling the beams (1000 x 150 x 150 mm) using a FORTRAN subroutine created and linked with the reliability software (FORM5). Experimental tests were carried out on conventional reinforced concrete beams with and without PVC pipes (diameter; 50 mm, density;  $1.41\text{g/cm}^3$ , thickness; 3 mm) below the neutral axis. Flexural strength tests were carried out using ASTM C 293 standard method. Stress, modulus of elasticity, peak and ultimate strain were measured and an improved analytical model was developed for the study of the flexural behaviour of reinforced concrete beams with and without PVC pipes using BioStat Pro and MS-Excel respectively. Finite Element Analysis and validation of experimental results of flexural strength were carried out using ANSYS Pro 15.0.

The findings of the study were that:

- (i) average compressive strength of all the non-embedded PVC concrete increased with curing age and ranges between 2-14% higher than specimen with embedded PVC pipes;
- (ii) flexural modulus of rupture (MoR) for beams without embedded PVC pipes were higher than PVC beams by 2.9% with similar failure pattern;
- (iii) proposed models using regression analysis at 5% level of significance for all beam specimens were in close agreement (within the range of 98%) with experimental data;
- (iv) finite element modeling and simulation show close agreement (within the range of 96%) with experimental results for all the specimens when subjected to the same test conditions; and
- (v) reliability analysis using the first order reliability method (FORM) for PVC beams revealed a safety index, within the range ( $3.7 \leq \beta \leq 4.7$ ) specified by ASTM C 293 with volume reduction between 6.1-9.9% compared to the non-embedded PVC beams.

The study concluded that beams with embedded PVC pipes below the neutral axis are economical and reliable. Therefore, it is recommended for same application with the conventional beam since their capacities recorded by experiment are in agreement with theoretical and simulation values.

# **CHAPTER ONE**

## **INTRODUCTION**

### **1.1 Background**

Beams are structural members that are mainly subjected to flexure and importance is given to the analysis of bending moment, shear, and deflection. When a beam is loaded, bending moment and bending strain are produced. The resistive bending moment is caused by internal stresses and compressive strains produced in the top of beam and tensile strains in the bottom under positive moment.

Beams have the capability of withstanding load primarily by resisting bending due to their capability of withstanding loads. Concrete beams are widely in use as roof supports not only in industrial and residential buildings but also in bridges. Their efficiency and economy made them widely used structural system.

Increasing the span of building, increases the risk of deflection leading to increase in structural member thickness. Thus, the construction of buildings consumes more materials such as concrete and steel. Amer, Nazar and Wissam (2013), Chung, Choi, Lee and Choi, (2009).

Structural elements failures, due to the presence of electrical wire installation, are minimized by conducting them through the hollow polyvinyl chloride (PVC) pipes. In the tensile zone below the neutral axis, some concrete and reinforcing steel are eliminated by the presence of PVC pipes; hence, paving way for effective materials utilization.

The disadvantages caused by increasing self-weight of beams can be avoided; by assessing the reliability of a system consisting of PVC pipes cast into the concrete to create a grid of void inside the beam with the hope of given major support to the objective of sustainable buildings. The sizes

structural members like walls, columns and base can economize of by lightening the weight of beams.

One of the most important building materials in the world is reinforced concrete and it is widely used in many engineering structures. The advantage of being economical, efficient and durable makes it an attractive material for a wide range of structural applications. Chung, Kim, Choi, Lee and Choi (2010). The use of concrete is required to satisfy the following:

The concrete must be;

- i. Safe and strong
- ii. Stiff and unblemished
- iii. Economical.

The traditional method for design of beams attempt to guard against events not considered in the design due to lack of information or data by imposing safety factors in the design equations. These safety factors are empirical values that are subject to certain uncertainties and as such may not result in safe and economic design.

The design criteria are subjectively derived quantitative evidence of the uncertainty inherent in structural designs. Changes to either the tools or domains require a change to the design margin. Unfortunately, with less reliance on engineering Judgments, the traditional methods often lead to an undetermined level of safety and performance that experience has shown is not always adequate. This inadequacy will be on the sensitive with the use of new design approaches beyond the traditional design. The professional engineers and builders, while aiming safety and perfection in their work, has always been conscious of the limitation in human knowledge, the unreliability of data about materials, and the approximation inherent in modeling and mathematical methods. Unfortunately, demands for lower costs, higher performances, and enhanced efficiency, lower

weight- to power ratio, more sophisticated Computer-aided analytical techniques as well as the demand for higher reliability assurance have increased the pressure on engineering designers to increase their safety margin.

The traditional methods of safety factors which in reality does not represent the actual operating conditions but to provide a sensible reserve against uncertainties which was usually based on years of accumulated in-service experience. Today engineers are aware that reliability is associated with risk taking, cost levels, life pattern and life expectancy.

Reliability is a broad field of study encompasses all aspect of life, from our daily household to the most sophisticated structure of the modern world. The importance of structural reliability has been on the increase. In order to avoid unexpected failures, which might involve large amount of money for repairs or total collapse, every user looks for the most reliable structure to inhabit.

Reliability has been defined by well respected internationally bodies. The most prominent among these bodies are the United Kingdom ministry of Defense [UkMD],(1991) which defines reliability as "the ability of an item to perform, or to be capable of performing, a required function without Failure under Stated Condition for a stated period of time on unit of operation.

## **1.2 Flexural strength**

National Ready Mixed Concrete Association [NRMCA], (2000) defined Flexural strength as one measure of the tensile strength of concrete. It is defined as a measure of any concrete member to resist failure due to bending. It is achieved in the laboratory by loading concrete beams with length at least three times its depth expressed as Modulus of Rupture (MR) in psi (MPa) and is determined by either ASTM C 78 standard test methods (third-point loading) or ASTM C 293 (center-point loading). Usually the flexural modulus of rupture must lie in the range of 10 to 20 percent of

compressive strength depending on the type, size and volume of coarse aggregate used [NRMCA], (2000).

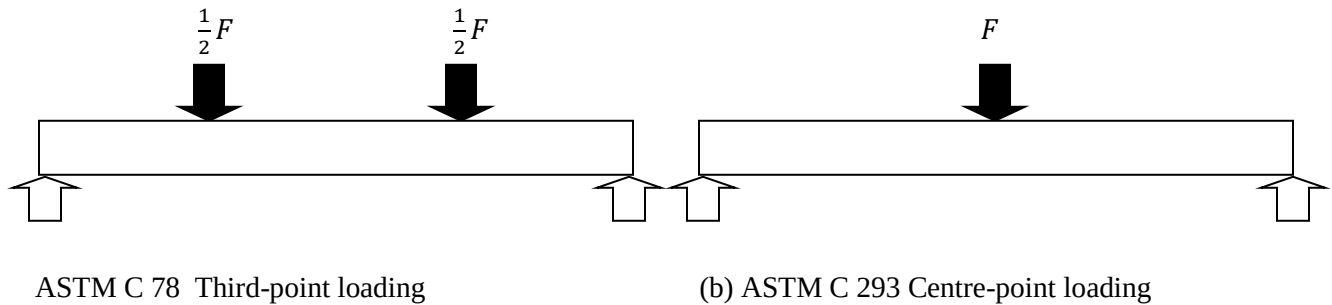


Figure 1.1: ASTM standard tests for Modulus of rupture

### 1.2.1 The purpose of Flexural Strength

Laboratory mix design tests may be required to obtain the needed design for flexural strength. Construction industry takes advantage of flexural strength for field quality control and acceptance of pavements. Other agencies find compressive strength suitable for checkmating the quality of concrete delivered at site.

## 1.3 Statement of the problem

The construction of reinforced concrete beams requires more materials such as concrete and steel, and therefore not economical. Also, in an attempt to solve the problem of deflection in beams, unnecessary increase in thickness result, which further increase the sizes of other structural members like slabs, columns and bases. The cracking of walls which are associated with electrical wire installation also leading to loss of aesthetics and structural element failure.

The design of a concrete member in flexure neglects the tensile resistance of the concrete below the neutral axis. Hence, the concrete below the neutral axis is theoretically ineffective, although it serves the purpose of resisting the shearing force surrounding the reinforcement bars in tension and

connecting the bars to the compression zone. It is therefore advantageous from the point of view of reducing the amount concrete below the neutral axis and thereby saving weight to omit the “ineffective” concrete while retaining enough to serve the purposes earlier stated. The PVC pipes are placed in the beam in grid form to omit the ineffective concrete.

In order to clarify these problems, this study therefore explores into the volume of concrete, cost, and reliability of reinforced concrete beams directed towards laboratory testing of the flexural behavior of the RC beams with and without PVC pipes and finite element modeling using ANSYS Pro 15.0.

## **1.4 Aim and objectives**

### **1.4.1 Aim**

The aim of this research is to investigate the Reliability of the flexural strength of reinforced concrete beams with and without PVC pipes by experiment and modeling using ANSYS Pro 15.0.

### **1.4.2 Objectives**

The objectives are to:

- (i) determine the compressive strength of concrete cubes with and without embedded PVC pipes and their stress-strain relationship;
- (ii) investigate the flexural behaviour of the beams with and without PVC pipes and their relationship with compressive strength;
- (iii) develop an improved analytical model for the study of the flexural behaviour of reinforced concrete beams with and without PVC pipes using BioStat Pro and MS-Excel respectively;
- (iv) validate experimental results of flexural strength of beam using ANSYS Pro 15.0; and

- (v) evaluate the reliability of the beam with and without PVC pipes using the First Order Reliability Method (FORM).

### **1.5 Significance of research**

The aim of structural design is to ensure safe, unblemished and economical structural elements. Concrete is the most important building material used in the construction industry. Strength, cost and durability of building is highly depended upon it. The use of PVC pipes as a component in concrete gives a sustainable alternative to the volume of concrete and therefore makes it economically viable.

This research examines the reliability of the flexural behavior of PVC pipes embedded in reinforced concrete beams with the following advantages; a reliable reinforced concrete beam, light weight beam structures, easy handling of plastics than steel and concrete, elimination of the use of conduit pipes and cracking of walls for electrical wire installation and cost effective section that will ensure minimum investment as compared to the conventional beams without PVC pipes.

### **1.6 Scope**

The scope of this research is within the following:

- Investigating the flexural behavior of reinforced concrete beams with and without PVC pipes by experiment and modeling using finite element predictive tools.
- Carrying out reliability assessment of the beams using data obtained from the experiment.

## CHAPTER TWO

### LITERATURE REVIEW

#### 2.1 Introduction

Reinforced concrete is generally known to be a composite element made up of steel and concrete. Concrete is however, very good in compression but poor in tension. Under loads, compressive and tensile stresses are set up. Reinforcing steel as a material with very high tensile capacity is used to bear the tensile weakness of the concrete. The tensile stress of concrete is neglected because it is very small compared to compressive stress (Mosley et al., 2013).

Material such as steel is considered as homogeneous and properties are generally well defined. While concrete on the other hand, a heterogeneous material made up of cement, mortar and aggregates. Their mechanical properties are widely scattered and not easily defined.

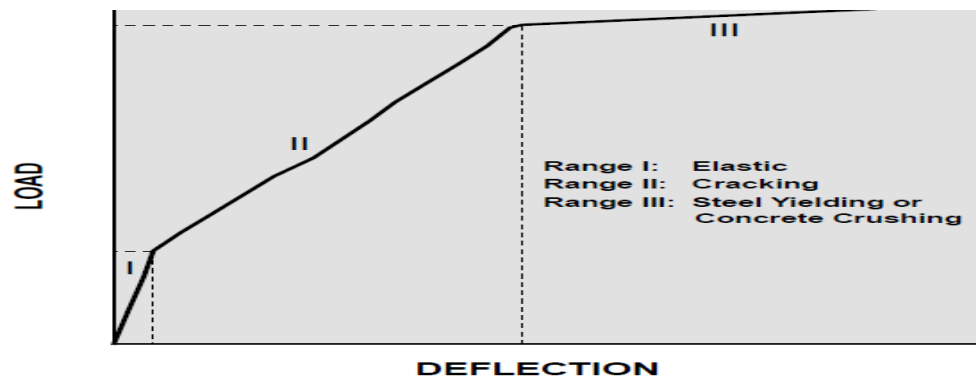


Figure 2.1: Typical load-displacement response of reinforced concrete element

The load-deformation behavior of a reinforced concrete simply supported beam is illustrated in Figure 2.1. Other reinforced concrete structural elements exhibit similar behavior. Structural beams encounters multitude of loads whose magnitude and times of occurrence are highly uncertain. Some of these load and combinations are capable of damaging the floors' structure, probably to a large extent. Damage often results in reduction or loss of structural integrity (Melchers and Harrington,

1984). However, the presence of uncertainties in the design of reinforced concrete structures requires a reliability based design optimization to be taken into account. The current study was therefore initiated to look statistically into the calculation and prediction of the probability of limit state violation, in the design of beams with and without embedded PVC pipes using ANSYS Pro.

15. Figure 2.2(a), (b), (c) and (d) shows reinforced concrete beam with and without PVC pipes.

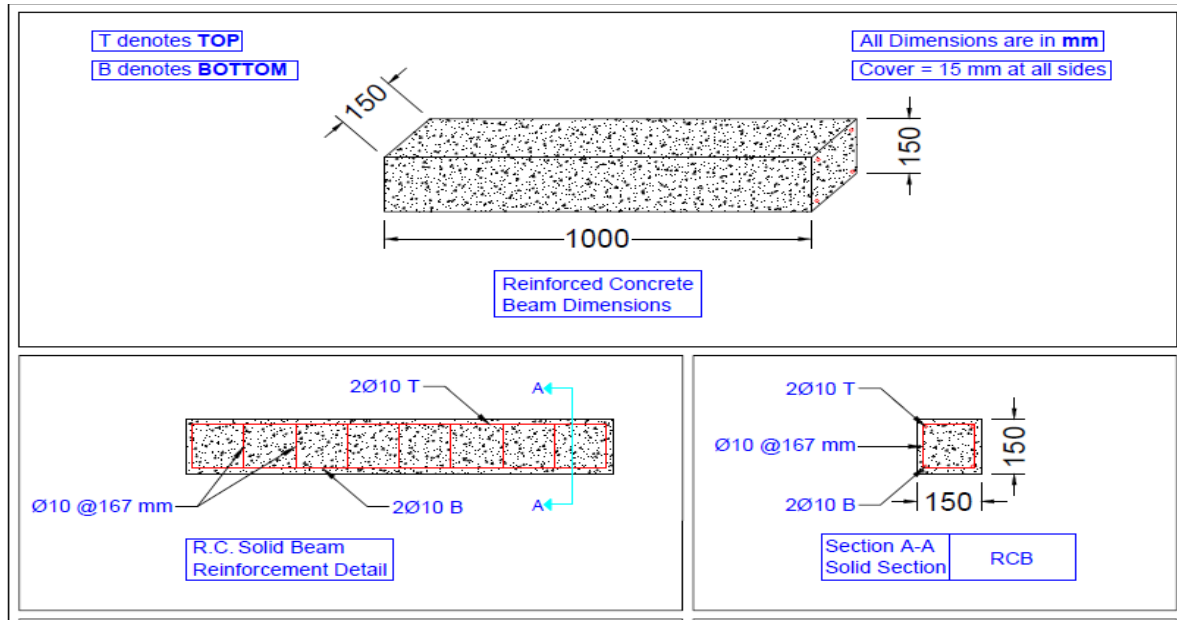


Figure 2.2 (a): Conventional Reinforced Concrete beam (RC Solid beam)

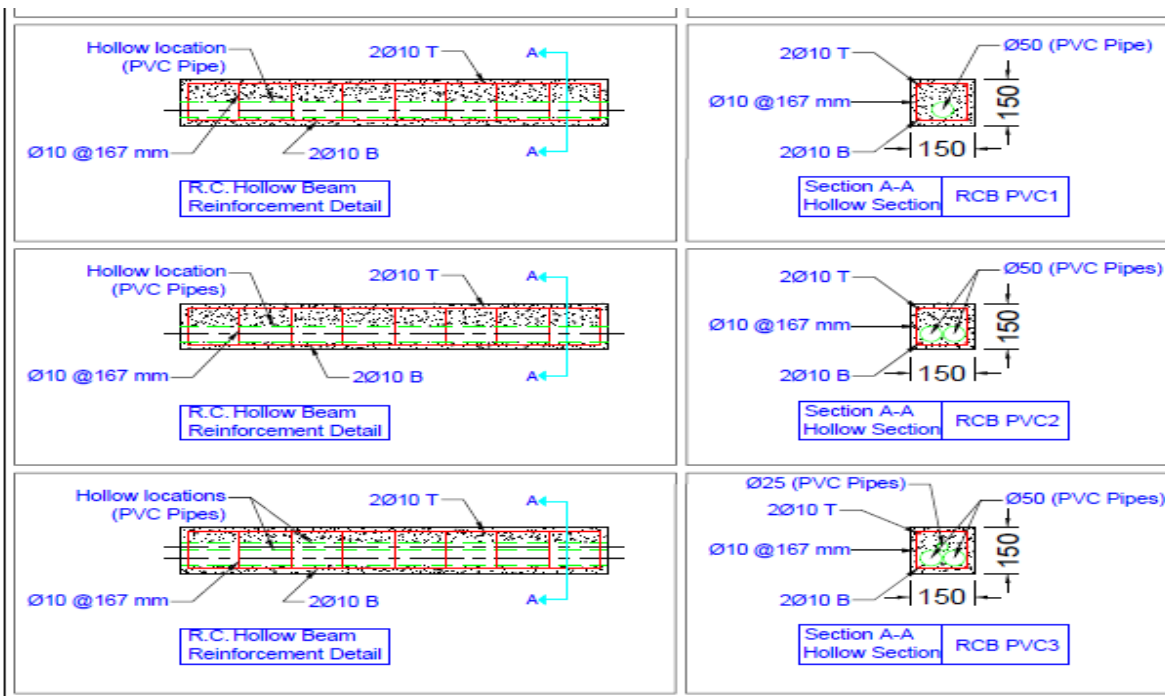


Figure 2.2 (b): Reinforced concrete beam with one, two and three embedded PVC pipes below the neutral axis

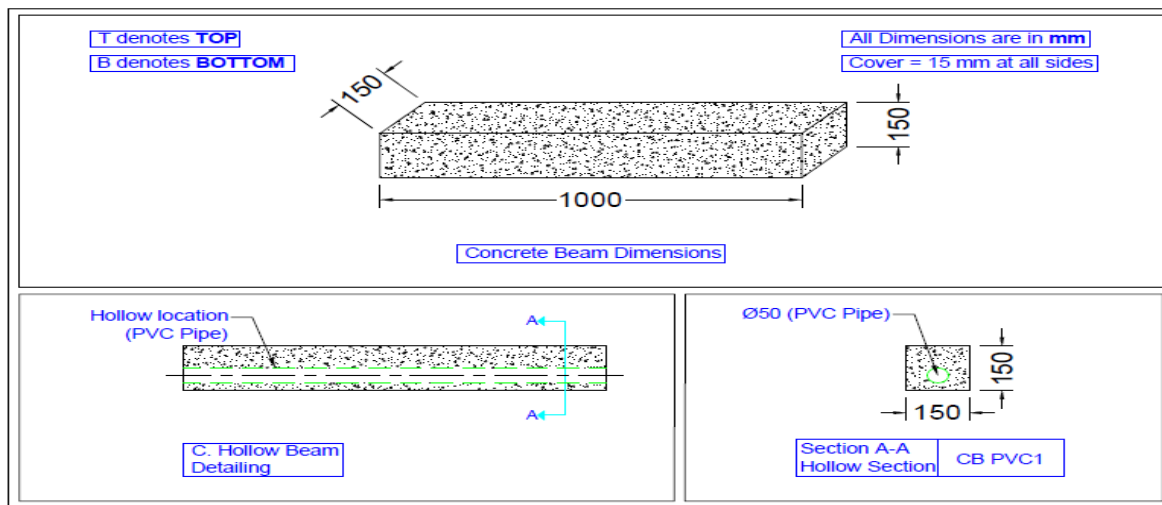


Figure 2.2 (c): Unreinforced concrete beam with one embedded PVC pipes and sections

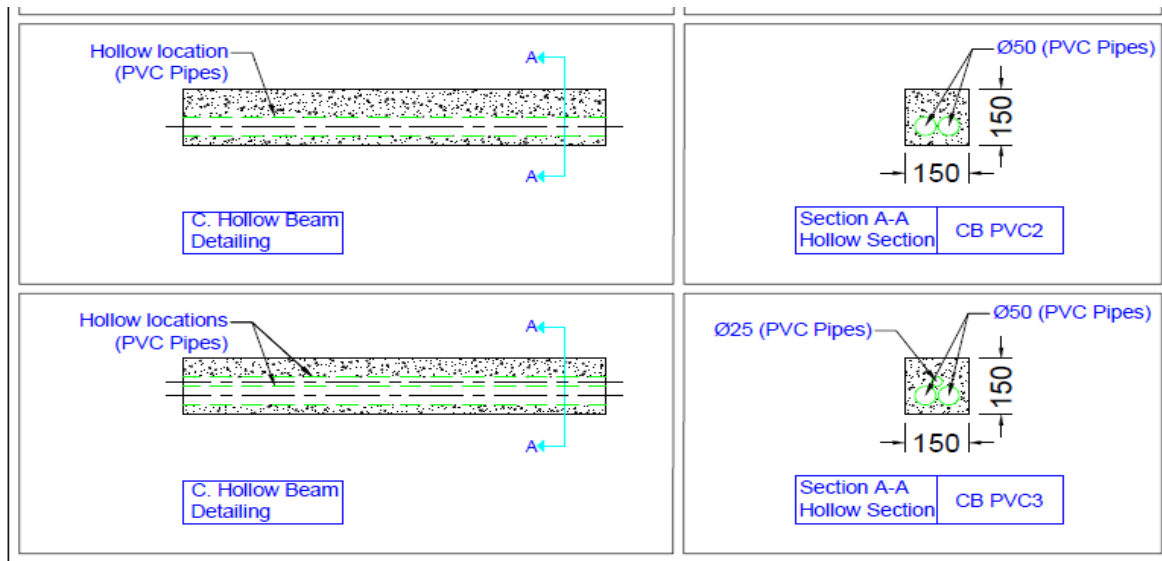


Figure 2.2 (d): Unreinforced concrete beam with two and three embedded PVC pipes

Mansur and Tan, (1999) reported that utility pipes and ducts are necessary to accommodate essential services (including air-conditioning, power supply, telephone line, computer network, sewerage and water supply) in a building. It is a common practice to hang pipes and ducts below the floor beams and covered by a suspended ceiling for its aesthetic purpose.

### 2.1.1 Stress-Strain behaviour of Conventional Reinforced Concrete

Structural behavior of concrete represented by a parabolic stress-strain relationship up to a strain of  $\epsilon_{c2}$ , from which the strain increases while stress remains constant is shown in Figure 2.3. The ultimate design stress is given by:

$$\frac{\alpha f_{ck}}{\gamma_c} = \frac{0.85 f_{ck}}{1.5} = 0.567 f_{ck} \quad (2.1)$$

0.85 safety factor caters for the difference between the bending strength and the cylinder crushing strength of the concrete, and  $\gamma_c = 1.5$  is the partial safety factor for the strength of concrete. The ultimate strain of  $\epsilon_{cu2} = 0.0035$  is typical for concrete classes  $\leq C50/60$ .



Figure 2.3: Parabolic-rectangular stress-strain diagram for concrete in compression

Source: Mosley et al., (2013)

### 2.1.2 Reinforcing Steel

A clear representation of short-term design stress-strain curve for reinforcement is shown in Figure 2.4. Structural material steel behaves in an identical manner in both tension and compression, being linear along the elastic range up to the design yield stress of  $\frac{f_{yk}}{\gamma_s}$  where  $f_{yk}$  is the characteristics yield stress and  $\gamma_s$  is partial factor of safety. Within the elastic range, the relationship between the stress and strain is given by;

$$\text{Stress} = \text{elastic modulus} \times \text{strain} = \quad (2.2)$$

$$\text{So that the design yield strain is } \frac{f_{yk}}{\gamma_s E_s} \quad (2.3)$$

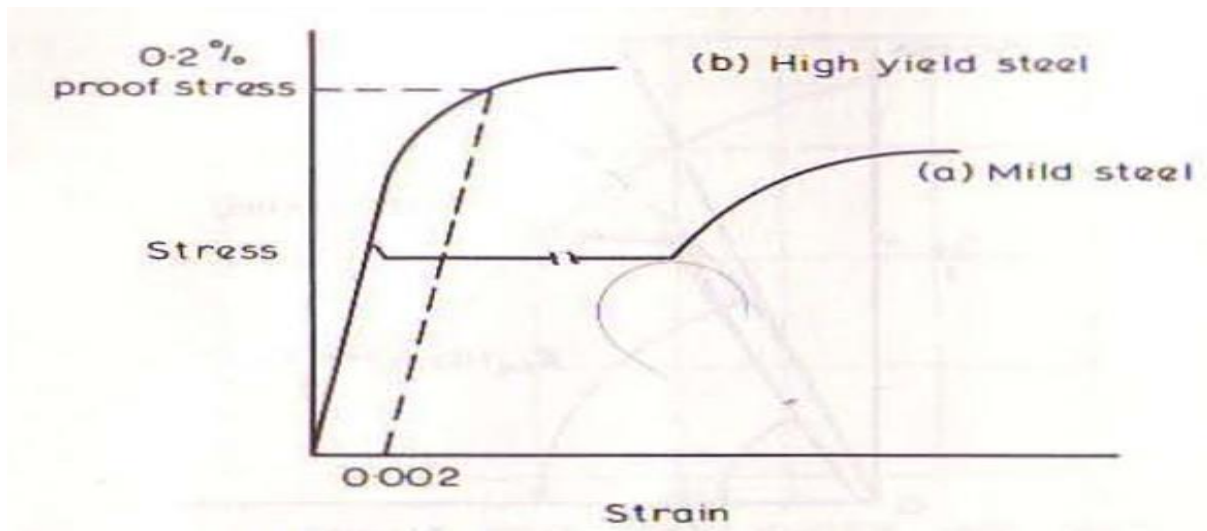


Figure 2.4: Short-term design stress-strain curve for reinforcement.

Source: Mosley et al., (2013)

Figure 2.4 shows the cross-section of a member subjected to bending, and the resultant strain diagram, together with three different types of stress distribution diagram in the concrete:

- (a) The triangular stress block is distribution used at the serviceability limit state.
- (b) The rectangular-parabolic stress block is associated with design at ultimate limit state.
- (c) The equivalent rectangular stress block. Mosley et al. (2013). See Figure 2.5.

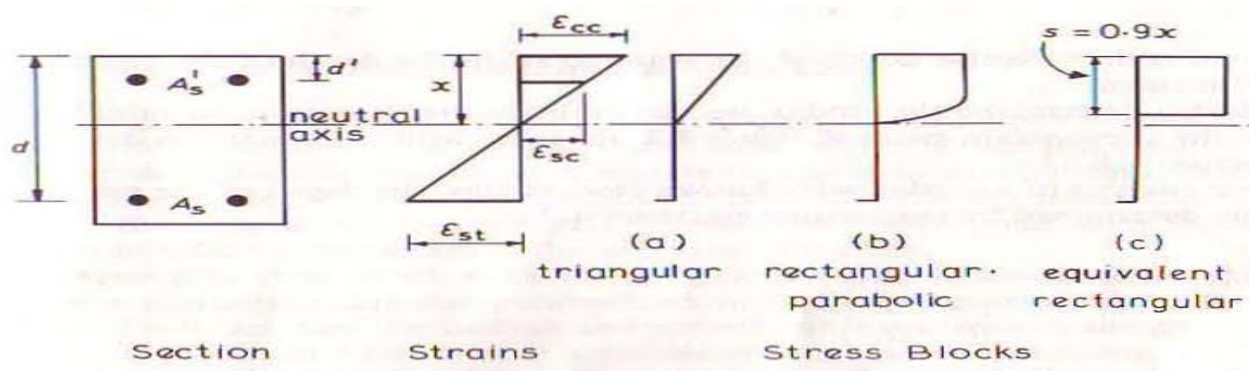


Figure 2.5: Section with strain diagram and stress blocks.

Source: Mosley et al., (2013)

The depth of the neutral axis ( $x$ ) and the maximum concrete strain ( $\varepsilon_{cu2}$ ) and the steel strains are related by;

$$\varepsilon_{st} = \varepsilon_{cu2} \frac{(d - x)}{x} \quad (2.4)$$

$$\varepsilon_{sc} = \varepsilon_{cu2} \frac{(x - d')}{x} \quad (2.5)$$

EC2 limits the depth of neutral axis to  $x \leq 0.45d$  for concrete class  $\leq C50/60$ .

Where,

$\varepsilon_{sc}$  = strain in compression

$\varepsilon_{st}$  = steel strain in tension

$\varepsilon_{cu2}$  = maximum concrete strain

$d$  = effective depth of section

$x$  = depth of neutral axis

$d'$  = distance between centre of reinforcement and top of section

### **2.1.3 Behaviour of Plastic Reinforced concrete**

Plastic is one of the most significant innovations in the recent time. The rate of consumption of plastic has increased the production of plastic related waste. They pose serious environmental threat to modern world. They are made up of chemicals that are harmful, pollutes soil, air and water.. Combustion of plastics releases poisonous toxic substances into the air.

#### **2.1.3.1 Stress strain curve for plastic**

An ideal plastic material experiences no work hardening during plastic deformation. Elastoplastic materials ideally will have liner elastic properties up to the yield stress and then exhibit the properties of ideal plastic. Unloading the material after reaching some deformation strain, will cause elastic strain recovery as shown by the red line in Figure 2.6. Reapplication of load will result in

retracing the red (elastic) line until the yield stress is reached and then lead to deformation along the black curve. The stress-strain diagram shows shear stress and shear strain, a situation that would apply to the torsional loading of a shaft.

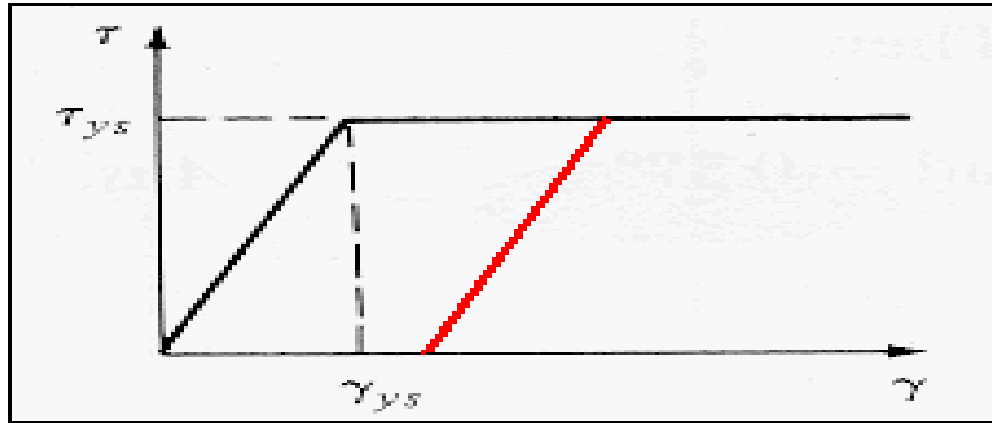


Figure 2.6: Stress-strain curve for Plastic.

Source: Kachlakev, D.I; and Mccurry, D.I. (2000).

### 2.1.3.2 Properties of PVC

PVC or “vinyl,” is one of the most common synthetic materials. Approximately 75% of all PVC manufactured is used in construction materials. Thomson (2002). Some its properties are presented in table 2.1

Table 2.1: Properties of PVC

| Property                                                                   | Rigid PVC           | Flexible PVC        |
|----------------------------------------------------------------------------|---------------------|---------------------|
| Density [ $\text{g}/\text{cm}^3$ ]                                         | 1.3 - 1.45          | 1.1 - 1.35          |
| Thermal Conductivity [ $\text{W}/(\text{m}\cdot\text{K})$ ]                | 0.14 - 0.28         | 0.14 – 0.17         |
| Yield Strength [ $\text{N}/\text{mm}^2$ ]                                  | 31.03 – 59.98       | 1450 – 3600         |
| Young Modulus [ $\text{N}/\text{mm}^2$ ]                                   | 3378.43             | -----               |
| Flexural Strength (yield) [ $\text{N}/\text{mm}^2$ ]                       | 72.40               | -----               |
| Compression Strength [ $\text{N}/\text{mm}^2$ ]                            | 65.50               | -----               |
| Coefficient Of Thermal Expansion [ $\text{mm}/(\text{mm}^\circ\text{C})$ ] | $50 \times 10^{-5}$ | $10^{12} - 10^{15}$ |
| Resistivity [ $\Omega \text{ m}$ ]                                         | $10^{16}$           | $10^{11} - 10^{12}$ |

Source: Thornton (2002)

## 2.2 Previous work on Reliability, flexural behavior of beams and Plastic

Afolayan and Abejide (2003) carried out a reliability analysis of a reinforced concrete one-way solid slab and concluded that the reliability design of such slab using FORM would ensure minimum investment and reduce failure of such reinforced concrete members.

Allam (2005) and Soroush et al., (2011) found out in their research that the presence of an opening in the web of a reinforced concrete beam may lead to reduction stiffness, unacceptable cracking, increased deflection and reduction in the beam strength. The change in the dimension of the beam will lead to high concentration of stress at the corners of opening leading to cracking.

Abubakar and Pius (2007) reported the results of the reliability analysis of a simply supported steel beam designed in accordance with BS5950 (1990) using First Order Reliability Method (FORM) at serviceability and ultimate limit states. The results show that BS5950 seemed conservative with respect to shear, unsafe with respect to bending and satisfactory in respect to deflection.

Afolayan and Opeyemi (2008) examined the assessment of the carrying capacities of pre-cast concrete piles in cohesive and cohesionless soils using FORM. The result shows that safety indices decreases with increase in length of pile in both soils. Static pile capacity is grossly conservative in cohesive soils and highly unsafe on cohesionless soils.

Balamuralikrishnan and Antony (2009) investigated the flexural behavior of rectangular beams with bonded CFRP fabric. The beams were subjected to the four point bending. Both found that, the finite element analysis was in good agreement with the test data.

Tamer and Bilal (2009), Mansur (1998) and Niea et al., (2006) reported in their research that the classification of opening is dependent on the structural response of beam. A circular opening may be considered as large when its diameter exceeds 40% of the depth of the web (Mansur, 1998 and Subhajit et al. 2011). When height exceed quarter of the depth of the web then, the opening is considered as large (Somes and Corley, 1974; Lee and Lee, 2008).

Sivaraja et al., (2007) and Thirumurugan (2010) described plastic waste as materials that can be used to produce new plastic like PVC pipes after processing. He further explained that the procedure is not an economical process as the recycled plastic degrades in quality and necessitates new plastic to make the original product.

Reza and Seyed (2010) experimentally and theoretically investigated six under-reinforced concrete beams using the finite method. Each concrete beam was reinforced with two 16mm diameter steel bars for tension. The finite element analysis was in agreement with the experimental design.

Mones and Brena (2013) performed push-off test on twenty four precast prestressed hollow-core units obtained from two different methods of hollow core unit production (dry mix and wet mix concrete), to examine the influence of surface preparation techniques on interfacial strength between the hollow-core slab and cast-in-place topping slab. The top surface condition are machine

finished surface, longitudinally raked surface, longitudinally or transversely broom surface and sand-blasted surface. The authors observed sudden and brittle failure in all specimens and failure along the interface plane. No cracks were observed on the hollow core slabs or topping slabs. All test specimens had an interface shear strength that far exceeded the 0.55 MPa limit specified in section 17.5.3.1 of ACI 318-11. It was also reported that dry-mix machine-finished specimen had lowest interface shear strength.

Amer et al., (2013) investigated the flexural capacities of reinforced concrete two - way hollow slabs with embedded plastic spherical voids, also known as Bubbledeck slab system. They concluded that the flexural capacities of the hollow slab are influenced by the void diameter to slab thickness ratio.

### **2.3 Beam design**

Reinforced concrete members in buildings can be categorised into horizontal and vertical members. The horizontal member includes slab, beams and staircases while the vertical member includes column, walls e.t.c. Loads from wall, slabs are transmitted through the beams to the column and finally to the foundation. Beams are designed to resist bending, shear, torsional moments if present. The design procedures for beams are;

- a) Design flexural reinforcement.
- b) Design shears reinforcement.
- c) Design torsion reinforcement.

### 2.3.1 Design of flexural Reinforcement

At each section along the beam, the flexural reinforcement is obtained following the steps below;

- a) Determine factored moments.
- b) Determine required flexural reinforcement.

#### 2.3.1.1 Determine Factored Moments

The factored moments are determined for each load combination by factoring the corresponding moments for each load cases, with the corresponding load factors.

#### 2.3.1.2 Determine Required Flexural Reinforcement

When the applied design moment exceeds the maximum moment capacity of singly reinforced section, compression reinforcement is added. The method of design is based on the simplified rectangular stress block. The code places a limitation on the neutral axis depth,  $x/d \leq 0.5$ , to cater for non-ductile failures.

### 2.4 Design of rectangular beams

For rectangular beams, the limiting moment capacity as a singly reinforced beam,  $M_{single}$ , is first calculated for a section. The reinforcement is determined based on  $M \leq M_{single}$ . See Figure 2.7. Calculate the ultimate limiting moment of resistance of the section as singly reinforced.

$$M_{single} = K' f_{cu} b d^2 \quad (2.6)$$

Where,

$$K' = 0.156 \quad (2.7)$$

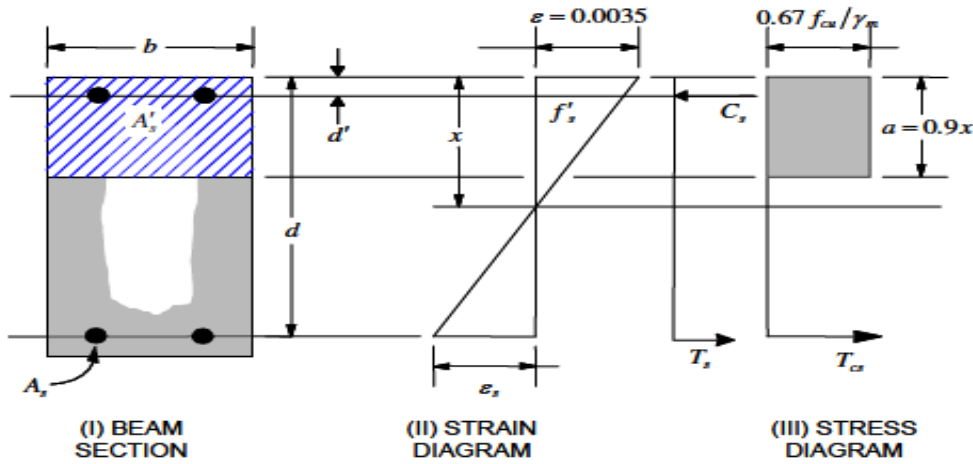


Figure 2.7: Rectangular Beam Design (Reinforced concrete design manual).

If  $M \leq M_{single}$ , the area of tension reinforcement is given by

$$A_s = \frac{M}{0.95f_y z} \quad (2.8)$$

$$z = d \left( 0.5 + \sqrt{0.25 - \frac{K}{0.9}} \right) \leq 0.95d \quad (2.9)$$

$$k = \frac{M}{f_{cu} b d^2} \quad (2.10)$$

The reinforcement is placed at the bottom if  $M$  is positive or at the top if  $M$  is negative.

If  $M > M_{single}$ , estimate the compression reinforcement is required as follows:

$$A'_s = \frac{M - M_{single}}{f'_s - \left( \frac{0.67 f_{cu}}{\gamma_c} \right) (d - d')} \quad (2.11)$$

Where  $d'$  is the depth of the compression reinforcement from the concrete compression face, and

$$f'_s = 0.95f_y \text{ if } d'/d \leq \frac{1}{2} \left( 1 - \frac{f_y}{800} \right) \quad (2.12)$$

$$f'_s = E_s \varepsilon_c \left(1 - \frac{2d'}{d}\right) \text{ if } d'/d > \frac{1}{2} \left(1 - \frac{f_y}{800}\right) \quad (2.13)$$

The tension reinforcement required for balancing the compression in the concrete and the compression reinforcement is calculated as:

$$A_s = \frac{M_{single}}{0.95 f_y z} + \frac{M - M_{single}}{0.95 f_y (d - d')} \quad (2.14)$$

$$z = d \left( 0.5 + \sqrt{0.25 - \frac{K'}{0.9}} \right) = 0.777d \quad (2.15)$$

## 2.5 Design of Flanged Beams

### 2.5.1 Flanged Beam under Negative Moment

In designing top reinforcement,  $M$ ; the calculation of the reinforcement area is exactly the same as described previously.

### 2.5.2 Flanged Beam under Positive Moment

The neutral axis is initially assumed to be located in the flange. Based on this assumption, the exact depth of the neutral axis is estimated. The contribution of the web to the flexural strength of the beam is taken into account as in figure 2.8.

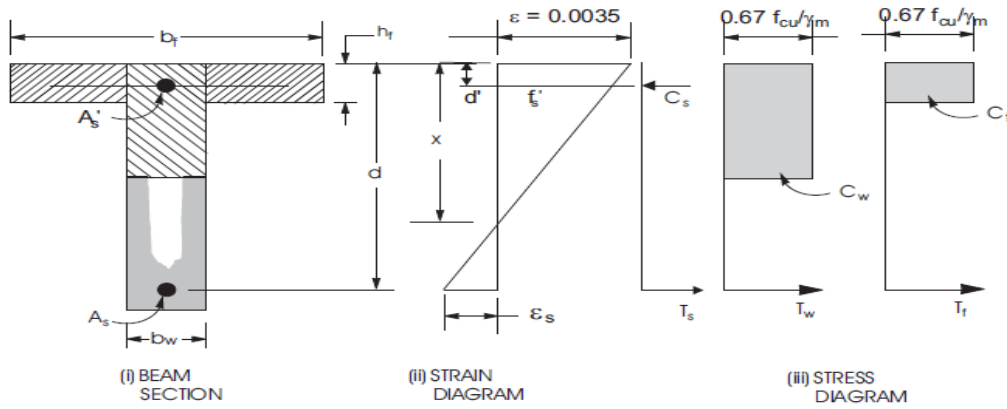


Figure 2.8: Design of a T-Beam Section (Reinforced concrete design manual).

If the neutral axis lie in the flange, the following are estimated:

$$k = \frac{M}{f_{cu} b d^2} \quad (2.16)$$

Then the moment arm is computed as:

$$z = d \left( 0.5 + \sqrt{0.25 - \frac{k}{0.9}} \right) \leq 0.95d \quad (2.17)$$

The depth of the neutral axis is computed as:

$$x = \frac{1}{0.45} (d - z) \quad (2.18)$$

And the depth of the compression block is given by;

$$a = 0.9x \quad (2.19)$$

If  $a \leq h_f$ , the subsequent calculations for  $A_s$ , are exactly the same as previously defined for the rectangular beam design. However, in that case, the width of the beam is taken as  $b_f$ . Compression reinforcement is required when  $K > K'$ .

If  $a > h_f$ , when  $M \leq \beta_f f_{cu} b d^2$  and  $\leq h_f 0.45d$ , then

$$A_s = \frac{M + 0.1 f_{cu} b d (0.45d - h_f)}{0.95 f_y (d - 0.5h_f)} \quad (2.20)$$

$$\beta_f = 0.45 \frac{h_f}{d} \left(1 - \frac{b_w}{b}\right) \left(1 - \frac{h_f}{2d}\right) = 0.15 \frac{b_w}{b} \quad (2.21)$$

The ultimate resistance moment of the flange is given by;

$$M_f = 0.45 f_{cu} (b_f - b_w) h_f (d - 0.5h_f) \quad (2.22)$$

The moment at the web given by:

$$M_w = M - M_f \quad (2.23)$$

And the normalized moment on web is estimated from:

$$K_w = \frac{M}{f_{cu} b_w d^2} \quad (2.24)$$

If  $K_w \leq 0.156$  designed as a singly RC beam. The steel reinforcement is given by:

$$A_s = \frac{M_f}{0.95 f_y (d - 0.5h_f)} + \frac{M_w}{0.95 f_y z}, \quad (2.25)$$

where

$$z = d \left( 0.5 + \sqrt{0.25 - \frac{k_w}{0.9}} \right) \leq 0.95d \quad (2.26)$$

If  $K_w > K'$  compression reinforcement is needed.

The moment of resistance of the web only is given by:

$$M_{uw} = K' f_{cu} b_w d^2 \quad (2.27)$$

The compression reinforcement is given by:

$$A'_s = \frac{M_w - M_{uw}}{f'_s - \left(\frac{0.67f_{cu}}{\gamma_c}\right)(d - d')} \quad (2.28)$$

Where,  $d'$  is the depth of the compression reinforcement from the concrete compression face, and

$$f'_s = 0.95f_y \text{ if } d'/d \leq \frac{1}{2} \left(1 - \frac{f_y}{800}\right) \quad (2.29)$$

$$f'_s = E_s \varepsilon_c \left(1 - \frac{2d'}{d}\right) \text{ if } d'/d > \frac{1}{2} \left(1 - \frac{f_y}{800}\right) \quad (2.30)$$

The area of tension reinforcement is obtained from equilibrium as:

$$A_s = \frac{M_f}{0.95f_y(d - 0.5h_f)} + \frac{M_{uw}}{0.95f_f(0.777d)} + \frac{M_w - M_{uw}}{0.95f_y(d - d')} \quad (2.31)$$

## 2.6 Minimum and Maximum Reinforcement

The minimum flexural tension reinforcement required in a beam section is given by the following table, which is taken from table 2.2 with interpolation for reinforcement of intermediate strength.

Table 2.2: Interpolation of Reinforcement of Intermediate Strength

| Section                          | Situation                  | Definition of percentage | Minimum percentage      |                         |
|----------------------------------|----------------------------|--------------------------|-------------------------|-------------------------|
|                                  |                            |                          | $f_y = 250 \text{ MPa}$ | $f_y = 500 \text{ MPa}$ |
| Rectangular                      | —                          | $100 \frac{A_s}{bh}$     | 0.24                    | 0.13                    |
| T- or L-Beam with web in tension | $\frac{b_w}{b_f} < 0.4$    | $100 \frac{A_s}{b_w h}$  | 0.32                    | 0.18                    |
|                                  | $\frac{b_w}{b_f} \geq 0.4$ | $100 \frac{A_s}{b_w h}$  | 0.24                    | 0.13                    |
| T-Beam with web in compression   | —                          | $100 \frac{A_s}{b_w h}$  | 0.48                    | 0.26                    |
| L-Beam with web in compression   | —                          | $100 \frac{A_s}{b_w h}$  | 0.36                    | 0.20                    |

Source: Reinforced concrete design manual.

The minimum flexural compression reinforcement, if it is required, provided in a rectangular or flanged beam is given by the following table, which is taken from BS Table 2.3.

Table 2.3: Minimum Flexural Compression Reinforcement.

| Section      | Situation          | Definition of percentage   | Minimum percentage |
|--------------|--------------------|----------------------------|--------------------|
| Rectangular  | —                  | $100 \frac{A'_s}{bh}$      | 0.20               |
| T- or L-Beam | Web in tension     | $100 \frac{A'_s}{b_f h_f}$ | 0.40               |
|              | Web in compression | $100 \frac{A'_s}{b_w h}$   | 0.20               |

Source: Reinforced concrete design manual

An upper limit of 0.04 times the gross web area on both tension and the compression reinforcement is imposed upon request as follows:

$$A_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w & \text{Flanged beam} \end{cases}$$

$$A'_s \leq \begin{cases} 0.04bd & \text{Rectangular beam} \\ 0.04b_w & \text{Flanged beam} \end{cases}$$

## 2.7 Design of beam Shear reinforcement

The shear reinforcement is designed for each load combination at each station along the length of the beam. In designing the shear reinforcement, the following steps are involved:

- Determine the shear stress,  $v$ .
- Determine the shear stress  $v_c$  that can be resisted by the concrete.
- Determine the shear reinforcement required to carry the balance.

### 2.7.1 Determine Shear Stress

In the design for the beam shear reinforcement, the shear forces for each load combination at a particular beam station are obtained by factoring the corresponding shear forces for different load cases with the corresponding load combination factors. The shear stress is then calculated as:

$$v = \frac{V}{b_w d} \quad (2.32)$$

The maximum allowable shear stress,  $v_{max}$  is defined as:

$$v_{max} = \min(0.8\sqrt{f_{cu}}, 5MPa) \quad (2.33)$$

For light-weight concrete,  $v_{max}$  is defined as:

$$v_{max} = \min(0.63\sqrt{f_{cu}}, 4MPa) \quad (2.34)$$

### 2.7.2 Determine Concrete Shear Capacity

The shear stress carried by the concrete,  $v_c$  is calculated as:

$$v'_c = v_c + 0.6 \frac{NVh}{A_c M} \leq v_c \sqrt{1 + \frac{N}{A_c v_c}} \quad (2.35)$$

$$v_c = \frac{0.79k_1k_2k_3}{\gamma_m} \left( \frac{100A_s}{bd} \right)^{1/3} \left( \frac{400}{d} \right)^{1/4} \quad (2.36)$$

$k_1$  -Is the enhancement factor for support compression, and is conservatively taken as 1.

$$k_2 = \left( \frac{f_{cu}}{25} \right)^{1/3}, 1 \leq k_2 \left( \frac{40}{25} \right)^{1/3} \quad (2.37)$$

$$\gamma_m = 1.25$$

However, the following limits also apply:

$$0.15 \leq \frac{100A_s}{bd} \leq 3 \quad (2.38)$$

$$\left( \frac{400}{d} \right)^{1/4} \geq 0.67 (\text{unreinforced}) \text{ or } \geq 1 (\text{reinforced}) \quad (2.39)$$

$$f_{cu} \leq 40 \text{ MPa (for calculation purposes only)} \quad (2.40)$$

$$\frac{Wh}{M} \leq 1 \quad (2.41)$$

$A_s$  is the area of tension reinforcement.

### 2.7.3 Determine Required Shear Reinforcement

If  $v$ ,  $v_c$ , and  $v_{max}$  are given, then; the required shear reinforcement is calculated as:

If  $v \leq (v'_c + 0.4)$ ,

$$\frac{A_{sv}}{s_v} = \frac{0.4b_w}{0.95f_{yv}} \quad (2.42)$$

If  $(v'_c + 0.4) < v \leq v_{max}$

$$\frac{A_{sv}}{s_v} = \frac{(v-v'_c)b_w}{0.95f_{yv}} \quad (2.43)$$

If  $v > v_{max}$ , then,  $f_y v \leq 500$  MPa.

## 2.8 Design of Beam Torsion Reinforcement

Torsion reinforcement is design for along the length of the beam. Longitudinal and shear reinforcement are obtained following the steps below;

- a) Determine the torsional shear stress,  $v_t$ .
- b) Determine special section properties.
- c) Determine critical torsion stress.
- d) Determine the torsion reinforcement required.

Note that this section refers to BS 8110-1985 instead of BS 8110-1997 code.

### 2.8.1 Determine Torsional Shear Stress

In the design of beam torsion reinforcement, the torsional shear stress,  $v_t$  for a rectangular section is computed as:

$$v_t = \frac{2T}{h_{min}^2(h_{max}-h_{min}/3)} \quad (2.44)$$

For flanged beam, the torsional shear stress is given by:

$$T_{seg} = T \left( \frac{h_{min}^3 h_{max}}{\sum (h_{min}^3 h_{max})} \right) \quad (2.45)$$

$h_{max}$  = rectangular section with greater dimension

$h_{min}$  = rectangular section with smaller dimension

If the computed torsional shear stress  $v_t$  exceeds the allowable, a failure condition is generated:

$$v_t \leq \min(0.8\sqrt{f_{cu}}, 5 \text{ K/mm}^2) \times \frac{y_1}{550} \quad (2.46)$$

### 2.8.2 Determine Critical Torsion Stress

The critical torsion stress,  $v_{t \min}$  for which the torsion in the section can be ignored, is given by:

$$v_{t, \min} = \min(0.067\sqrt{f_{cu}}, 0.4 \text{ N/mm}^2) \quad (2.47)$$

Where  $f_{cu}$  is the specified concrete compressive strength.

For light-weight concrete,  $v_{t \min}$  is given by:

$$v_{t, \min} = \min(0.067\sqrt{f_{cu}}, 0.4 \text{ N/mm}^2) \times 0.8 \quad .(2.48)$$

### 2.8.3 Determine Torsion Reinforcement

If the factored torsional shear stress  $v_t$  is less than the threshold limit,  $v_{t \min}$  torsion can safely be ignored (BS 8110-1985). However, if  $v_t$  exceed,  $v_{t \min}$  then torsional resistance is provided by closed stirrups and longitudinal bars (BS 8110-1985). If  $v_t > v_{t \min}$  the required closed stirrup area per unit spacing,  $A_{sv,t} / S_v$  is calculated as:

$$\frac{A_{sv,t}}{S_v} = \frac{T}{0.8x_1y_1(0.95f_{yv})} \quad (2.49)$$

And the required longitudinal reinforcement is calculated as:

$$A_l = \frac{A_{sv,t}f_{yv}(x_1+y_1)}{s_vf_y} \quad (2.50)$$

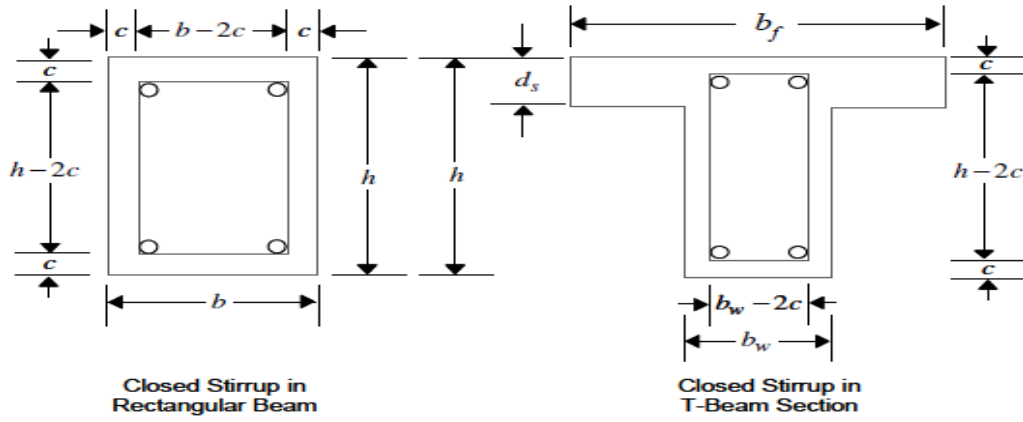


Figure 2.9: Closed Stirrup and Section Dimension for Torsion Design.

An upper limit of the combination of  $v$  and  $v_t$  that can be carried by the section is also checked using the equation:

$$v + v_t \leq \min(0.8\sqrt{f_{cu}}, 5 \text{ K/mm}^2) \quad (2.51)$$

For light-weight concrete,  $v_{max}$  is defined as:

$$v_{max} = \min(0.63\sqrt{f_{cu} + 4 \text{ MPa}}) \quad (2.52)$$

If the combination of shear stress,  $v$ , and torsional shear stress,  $v_t$  exceeds this limit, then; the concrete section should be increased.

## 2.9 Reliability Analysis

To begin reliability analysis, the state variables of the problem must be defined. The performance function is formulated using the state variables which include basic load and resistance parameters. The statistical parameters (means, variance, standard derivation) of these variables must be generated or calculated using either the Monte Carlo simulation technique or using formulae. Loads be represented by the variable  $M$  and total resistance (or capacity)  $R$ , then the limit state function equation is formulated as;

$$g(R, M) = 0 \quad (2.53)$$

Also,

$$g(R, M) \geq 0 \text{ (safe domain)} \quad (2.54)$$

or

$$g(R, M) \leq 0 \text{ (unsafe domain)} \quad (2.55)$$

Estimating this probability of failure or success is very difficult, so the concept of reliability index is used to quantify structural reliability.

## 2.9.1 Reliability index

### 2.9.1.1 Reduced variables

In reliability analysis, it is convenient to convert all random variables to their “standard form”, to a non-dimensional form of the variables. For the basic variable  $R$  and  $M$ , the standard forms is given by:

$$Z_R = \frac{R - \mu_R}{\sigma_R} \quad (2.56)$$

$$Z_M = \frac{R - \mu_M}{\sigma_M} \quad (2.57)$$

The limit state function  $(R, M) = R - M$  can be expressed in terms of the reduced variables as:

$$g(Z_R, Z_M) = \mu_R + Z_R \sigma_R - \mu_M - Z_M \sigma_M \quad (2.58)$$

$$g(Z_R, Z_M) = (\mu_R - \mu_M) + Z_R \sigma_R - Z_M \sigma_M \quad (2.59)$$

Reliability indeed is defined as the shortest distance from the origin of reduced variables to the line  $g(Z_R, Z_Q) = 0$  (Hasofer and Lind, 1974).

Therefore, reliability index,

$$\beta = \frac{(\mu_R - \mu_M)}{\sqrt{\sigma_R^2 - \sigma_M^2}} \quad (2.60)$$

Reliability index is related to the probability of failure by;

$$\beta = -\phi^{-1}(P_f) \quad (2.61)$$

$$P_f = \phi(-\beta) \quad (2.62)$$

Table 2.4: Variation of safety index ( $\beta$ ) with  $P_f$

| $P_f$ (probability of failure) | $\beta$ (safety indices) |
|--------------------------------|--------------------------|
| $10^{-1}$                      | 1.28                     |
| $10^{-2}$                      | 2.33                     |
| $10^{-3}$                      | 3.09                     |
| $10^{-4}$                      | 3.71                     |
| $10^{-5}$                      | 4.26                     |
| $10^{-6}$                      | 4.75                     |
| $10^{-7}$                      | 5.19                     |
| $10^{-8}$                      | 5.62                     |
| $10^{-9}$                      | 5.99                     |

Source: Andrzej and Kelvin (2000)

### 2.9.2 First order reliability methods (FORM)

First order reliability method (FORM) is a method used in estimating the safety index of structural elements. The first order reliability methods (FORM) includes:

- a) Hasofer – Lind reliability method
- b) Point estimate method
- c) Monte Carlo simulation method

**i. Hasofer – Lind reliability method**

In 1974, Hasofer and Lind proposed a modified way of estimating reliability index. The method evaluates the limit state at “design point” instead of the mean values. The design point is a point on the failure surface  $g = 0$ . Since this design point is generally not known, iteration must be used to solve for the reliability index.

Consider a limit state function  $g(X_1, X_2 \dots X_n)$  where the random variables  $X_i$  are all uncorrelated. The limit state function is rewritten in terms of the standard form of the variable (reduced variables) using

$$Z_i = \frac{X_i - \mu_{xi}}{\sigma_{xi}} \quad (2.63)$$

If the limit state function is linear, then the reliability index is still calculated as:

$$\beta = \frac{a_0 + \sum_{i=1}^n a_i \mu_{x_i}}{\sqrt{\sum_{i=1}^n (a_i \sigma_{x_i})^2}} \quad (2.64)$$

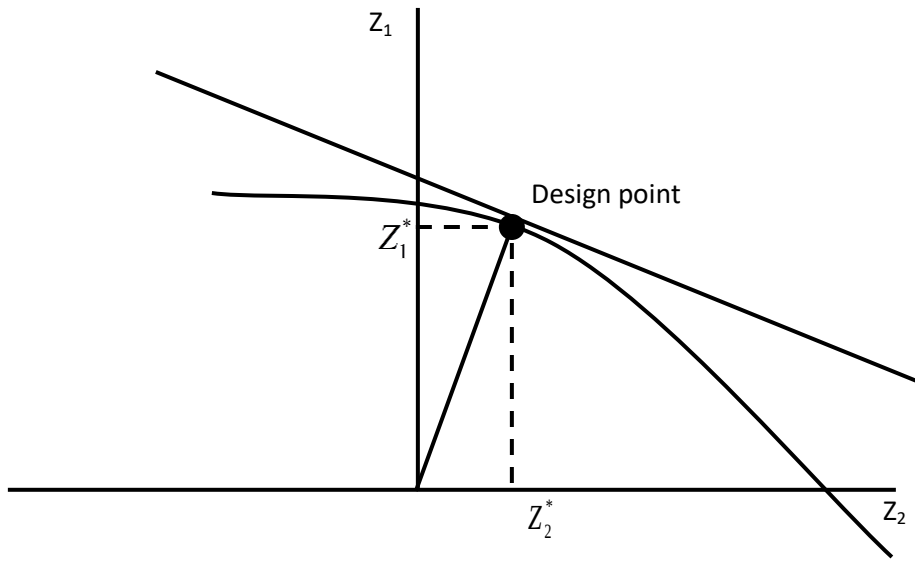


Figure 2.10: Hasofer - Lind reliability index. (Source: Melchers, 1985 )

If the limit state function is non linear, however, iteration is required to find the design point ( $Z_1^*$ ,  $Z_2^*$ , ...,  $Z_n^*$ ) in reduced variable space such that  $\beta$  still corresponds to the shortest distance as shown in figure 2.10 above. The iterative procedure requires the solution of  $(2n+1)$  simultaneous equations with  $(2n + 1)$  unknowns  $\beta, \alpha_1, \alpha_2, \dots, \alpha_n$ ,  $Z_1^*$ ,  $Z_2^*, \dots, Z_n^*$  where

$$\alpha_i = \frac{-\partial g / \partial z_i \text{ evaluated at design point}}{\sqrt{\sum_{i=1}^n \left( \partial g / \partial z_i \text{ evaluated at design point} \right)^2}} \quad (2.65)$$

Where,

$$\frac{\partial g}{\partial z_i} = \frac{\partial g}{\partial x_i} \frac{\partial x_i}{\partial z_i} = \frac{\partial g}{\partial x_i} \alpha_i \quad (2.66)$$

$$\sum_{i=1}^n (\alpha_i)^2 = 1 \quad (2.67)$$

$$Z_i^* = \beta \alpha_i \quad (2.68)$$

$$g(Z_1^*, Z_2^*, \dots, Z_n^*) = 0 \quad (2.69)$$

Where,

$\beta$  = reliability index

$Z$  = random variable in the reduced form

$\alpha$  = Constant ratios

$n$  = the number of variables present

Equation 2.66 is just an application of the chain rule of differentiation. Equation 2.67 is a requirement on the values of the  $\alpha_i$  variables, which can be confirmed by looking at Equation 2.67 a mathematical statement of the requirement that the design point must be on the failure boundary.

The procedures for performing the iterative analysis for Hasofer-lind reliability index are as follows:

- a) Formulate the limit state function and appropriate parameters for all random variables involved.
- b) Express the limit state function in terms of reduced variables  $Z_i$ .
- c) Use equation 2.68 to express the limit state function in terms of  $\beta$  and all  $\alpha_i$  noting that  $\alpha_i$  values must satisfy Eq. 2.67
- d) Substitute values of  $\beta$  and  $\alpha_i$  into the equation formed in step 3
- e) Solve the  $(n + 1)$  simultaneous equation for  $\beta$  and  $\alpha_i$ ,
- f) Go back to step 6 and repeat the iteration until the values of  $\beta$  and  $\alpha_i$  converge.

## ii. Point Estimate Method:

Complex problems may also be solved by using a point estimate method. Although there are many such methods, the 2K+1 method proposed by Rosenblueth (1985b) is one of the easiest to implement. Loosely speaking, this method may be thought of as simulation technique in which the number of simulations is  $N = 2k+1$ , where  $K$  is the number of input random variables. The basic idea is to evaluate a function of random variables at  $2k+1$ , where  $K$  is the number of input random variables. The basic idea is to evaluate a function of random variables at  $2k+1$  key points and then to use this information to estimate the mean and variance of the functions. Consider a function  $\gamma$  described by:

$$\gamma = f(\mu_{x1}, \mu_{x2}, \dots, \mu_x + \sigma_x, \dots, \mu_k) \quad (2.70)$$

Where  $f(\ )$  is some deterministic function and the  $x_i$  ( $i = 1, 2, \dots k$ ) are the random input variables.

The steps in point estimate method are as follows:

- a) Determine the mean value ( $\mu_{xi}$ ) and standard deviation ( $\sigma_{xi}$ ) for each of the  $k$  input random variables.
- b) Define  $y_o$  as the value of Eq. 2.23 when all input variable are equal to their mean values, that is,

$$y_o = f(\mu_{x1}, \mu_{x2}, \dots, \mu_x + \sigma_x, \dots, \mu_k) \quad (2.71)$$

- c) Evaluate the function  $\gamma$  at  $2k$  additional points. These values of the function will be referred to as  $y_1^+$  and  $y_1^-$

$$y_i^+ = f(\mu_{x1}, \mu_{x2}, \dots, \mu_x + \sigma_x, \dots, \mu_k) \quad (2.72)$$

$$y_i^- = f(\mu_{x1}, \mu_{x2}, \dots, \mu_x - \sigma_x, \dots, \mu_k) \quad (2.73)$$

The subscript denotes the variable which is shifted and the superscript indicates the direction of the shift.

d) Calculate

$$\bar{y}_1 = \frac{y_1^+ + y_1^-}{2} \quad (2.74)$$

$$V_y = \frac{y_1^+ + y_1^-}{y_1^+ - y_1^-} \quad (2.75)$$

e) Estimate the mean and coefficient of variation of  $\gamma$  as follows:

$$\bar{Y} = y_0 \prod_{i=1}^K \left( \frac{\bar{y}_1}{y_o} \right) \quad (2.76)$$

$$V_Y = \sqrt{\left\{ \prod (1 + V_y^2) \right\} - 1} \quad (2.77)$$

### iii. **Monte Carlo Simulation techniques**

The Monte Carlo method is a special technique that can be used to generate some results numerically without actually doing any physical testing.

The Monte Carlo method is often, applied in three situations

- a) It is used to solve complex problems for which closed – form solution are either not possible or extremely difficult.
- b) It is used to solve complex problems that can be solved (at least approximately) in closed form if many simplifying assumptions are made.
- c) It is used to check the result of other solution techniques.

### 2.9.3 First- order second-moment reliability index

#### 2.9.3.1 Linear Limit State Function

Consider a linear limit state of the form

$$g(X_1, X_2, \dots, X_n) = a_0 + \sum a_i X_i \quad (2.78)$$

Where the  $a_i$  terms ( $i = 0, 1, 2, \dots, n$ ) are constants and the  $x_i$  terms are uncorrelated random variables, the Hasofer - Lind reliability index is given by:

$$\beta = \frac{a_0 + \sum_{i=1}^n a_i \mu_{x_i}}{\sqrt{\sum_{i=1}^n (a_i \sigma_{x_i})^2}} \quad (2.79)$$

### 2.10 FORM5 Reliability Program

FORM has been designed to accommodate the computation of general probability integral over given domain occurring in structural reliability. The present version of FORM is FORM5. It has been in use in areas like hydrology, mathematical statistics, and control theory to mention just a few. The state function used in FORM5 is given by:

$g(X)$ . It is defined such that  $g(X)$  is greater than zero which corresponds to favorable (safe, intact, acceptable) state,  $g(X)=0$  denote the failure boundary,  $g(X) \leq 0$  defines the failure (unacceptable) domain.

FORM5 (written in FORTRAN) requires the users to specify the input parameters, the state function as a subroutine and the stochastic model in terms of the types of distribution function together with its distribution parameters.

The state function must be written as a subroutine (DOUBLE PRECISION) in FORTRAN. The present version FORM5 is an improvement and an extension of FORM4 and has been adopted to the PC environment. The user has to define the characteristics of the vector X (Stochastic model) in the main program or in EFORM and the state function  $g(X)$  in a subroutine. EFORM specifies the stochastic model interactively and facilitate the input/output handling. It can also perform up to 40 iterations to achieve convergence. Figure 2.11 shows the data flow for reliability software.

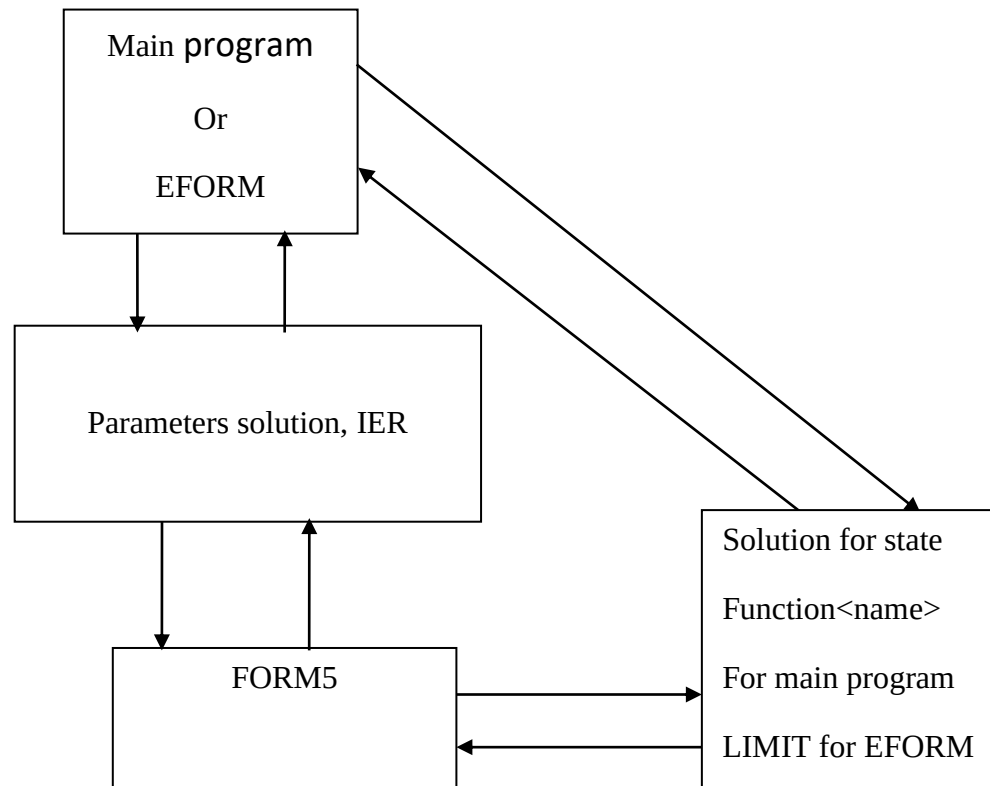


Figure 2.11: General data flow of reliability analysis

Some of the parameters used in FORM5 are defined below;

EX= vector of mean values of uncertain variables, SX= vector of standard deviation of uncertain variables, VP= matrix with 10 rows and N column, COV=Coefficient of variation of the random variables., IV= control parameter for iteration., NAUS= print control parameter,

GG= name of user subroutine, IRHO= control parameter for correlation, IER= error indicator,  
BETA= design solution, PF= probability of failure.

### 2.10.1 Fortran subroutine

FORTTRAN subroutine linked with FORM5 program is used to generate the safety index at different limit states. The flow chart is given in Figure 2.12.

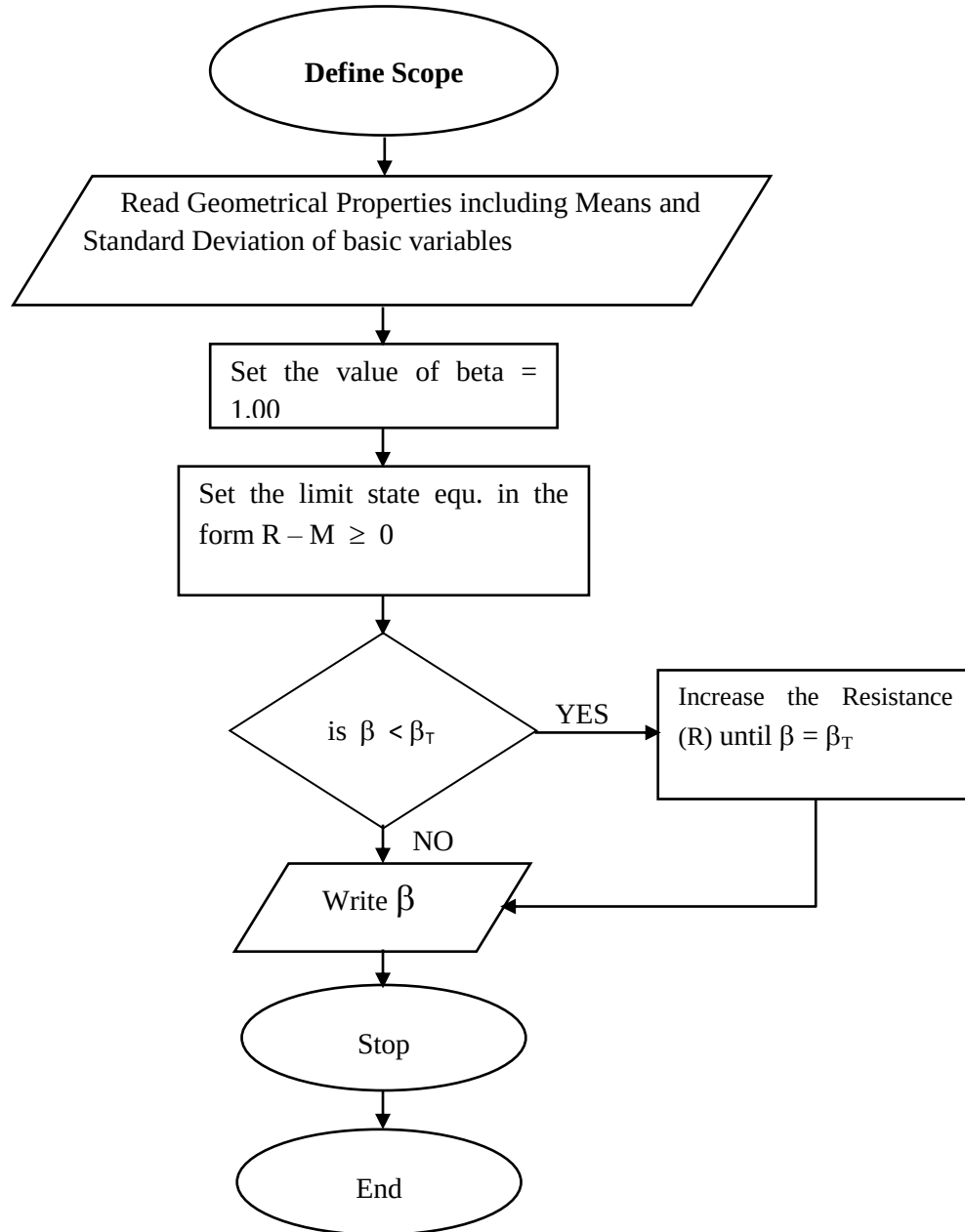


Figure 2.12: Flow Chart for FORTRAN subroutines

## 2.11 Finite Element

### 2.11.1 Historical background

Ritz in 1909 developed an approximate and effective solution of problems in structural mechanics of deformable solids. The fundamental concept involved the meshing of bodies under study into finite elements. Courant in 1943 advanced in his research by improving on the development of Ritz by introducing a special linear function defined over triangular regions. An important contribution was brought into FEM development by the papers of Argyris,(1955). Turner, Clough, Martin and Topp (1956), Hrennikov and many others. FEM finds suitable application in the field of structural mechanics; it has been successfully applied now for the solution of several other types of engineering problems like heat conduction, fluid dynamics, electric and magnetic fields, and others.

### 2.11.2 Comparison of FEM with other methods

The common methods of FEM available are classified as presented in Figure 2.13.

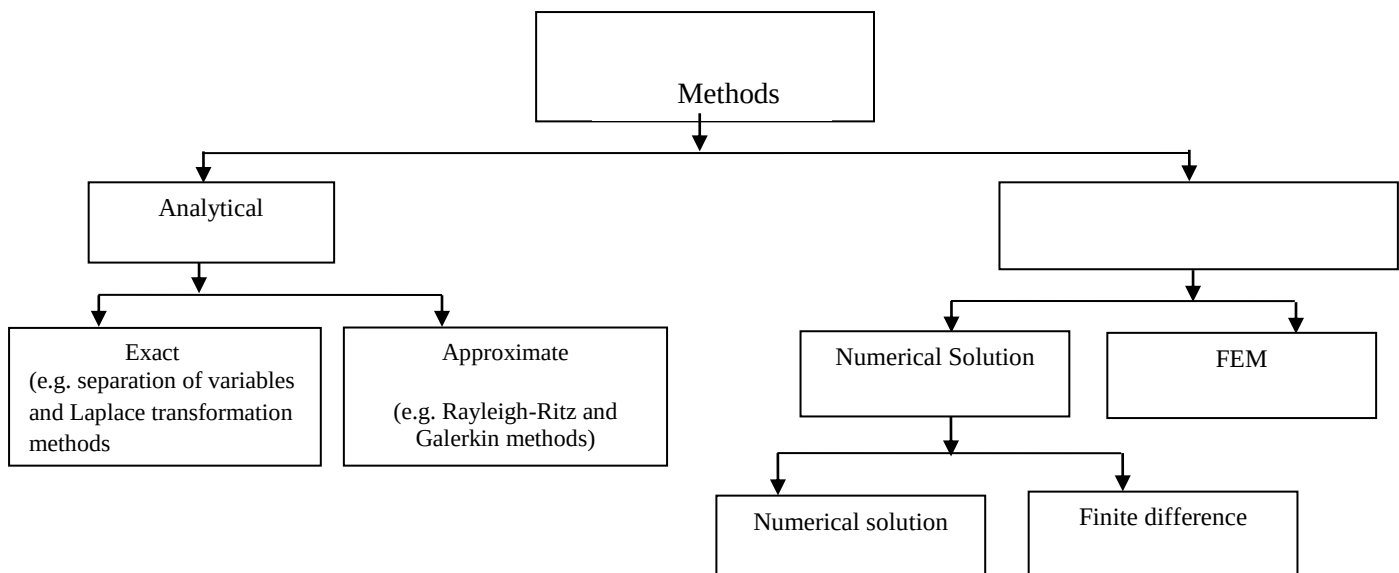


Figure 2.13: Classification of common methods of general field problems.

Source: Evgeny Barkanov (2001).

FEM of structural analysis approximate the solution in continuum mechanics by the analysis of an assemblage of finite elements interconnected by nodal points.

### **2.11.3 Finite Element Package**

The finite element packages include the following;

#### **2.11.3.1 ANSYS**

ANSYS is an abbreviation for analysis system for modeling and simulation. It is a program for the finite element analysis and design of engineering structures and materials. The ANSYS program is a general purpose program, since it can be used for almost any type of finite element analysis in virtually all industries.

- Improved performance through innovative developments of completely new solvers.
- Modeling of threaded bolt as a contact, rather than detailed geometry and
- Multiple finite element models that can be assembled, leveraging all the setup details of individual models.

The methods for simulation include;

- a) Building the model,
- b) Application loads on model and to obtain solution,
- c) Results verification.

#### **2.11.3.2 COMSOL**

Another finite element and simulation software is called COMSOL. They are used in solving various multiphysics problems in many engineering field. A COMSOL system allows the analysis

of partial differential equations (PDEs). The Physics Builder, which is included in, allows the creation of custom made physics interfaces accessible from the COMSOL desktop with the same look-and-feel as the built-in physics interfaces.

#### **2.11.4 Finite Element Modeling of Reinforced Concrete Beams**

FEM technique is used for the numerical solution of a variety of engineering problems. They find suitable application in stress analysis of solids, acoustical phenomena, neutron physics and fluid dynamic problems. A similar acceptance in nonlinear analysis problems depends on two major factors. These are;

- a) the increase in computational effort for nonlinear problems.
- b) the level of complexity of nonlinear analysis. Before application of nonlinear methods can become commonplace in design situations, the accuracy and reliability of the proposed models has to be established beyond doubt.

Several factors accounts for the non-linear behavior of concrete, they are;

- (1) Nonlinear material behavior of concrete and steel and their interaction through bond and dowel action;
- (2) Cracking of concrete; and
- (3) Time dependent effects such as creep, shrinkage, temperature and load history.

### 2.11.4.1 Finite Element Representation

The finite element analysis begins with the subdivision of elements under study. The displacement vector  $\{d\}$  at any point is approximated by interpolating functions associated with generalized,  $d_i$  coordinates which are the displacements of the nodes of the finite element discretization of the structure. For a finite element this approximation can be formally written as;

$$\{d\} = [N]\{d\}^{ele} = [N_1 \dots N_i \dots] \begin{Bmatrix} d_1 \\ \vdots \\ d_i \\ \vdots \end{Bmatrix} \quad (2.80)$$

Where the components of  $N$  are, in general, functions of position and  $\{d\}^{ele}$  is the vector of the node displacements of a particular element. Once the displacements at any point are known, the strains can be determined from the strain-displacement relation. In the discrete problem this can be formally written as;

$$\{\varepsilon\} = [B]\{d\}^{ele} \quad (2.81)$$

The stresses can now be determined from the material constitutive law

$$\{\sigma\} = [D](\{\varepsilon\} - \{\varepsilon_o\}) + \{\sigma_o\} \quad (2.82)$$

Where,  $[D]$  is the element material matrix,  $\{\varepsilon_o\}$  is the initial strain vector and  $\{\sigma_o\}$  is the initial stress vector. By applying the virtual work principle or the theorem of minimum potential energy to the assemblage of discrete elements the following equilibrium equations result;

$$[K] = \{d\} + \{F\}_p + \{F\}_g + \{F\}_{\varepsilon_o} + \{F\}_{\sigma_o} - \{R\} = 0 \quad (2.83)$$

The terms in Eq. 2.83 are derived as follows:

The stiffness matrix  $[K]$

$$[K] = \sum_{ele} \int [B]^T [D] [B] dV \quad (2.84)$$

The nodal forces due to surface traction,

$$\{F\}_p = - \sum_{ele} \int [N]^T \{p\} dV \quad (2.85)$$

The nodal forces due to body forces,

$$\{F\}_g = - \sum_{ele} \int [N]^T \{g\} dV \quad (2.86)$$

The nodal forces due to initial strains,

$$\{F\}_{\varepsilon_o} = - \sum_{ele} \int [B]^T [D] \{\varepsilon_o\} dV \quad (2.87)$$

And the nodal forces due to initial stresses

$$\{F\}_{\sigma_o} = - \sum_{ele} \int [B]^T [D] \{\sigma_o\} dV \quad (2.88)$$

In Eqns. 2.84-2.88,  $\{d\}$  is the vector of node displacements,  $\{R\}$  is the vector of applied nodal forces,  $\{p\}$  is the vector of surface forces and  $\{g\}$  is the vector of body forces. The node displacements  $\{d\}$  can be determined from the solution of the system of simultaneous algebraic equations in Eqn. 2.83, whereupon the strains and stresses at any point of the structure can be obtained from Eqns. 2.81 and 2.82. In a nonlinear problem the stiffness matrix  $[K]$  depends on the

displacement vector  $\{d\}$  and the nonlinear system of algebraic equations in Eqn. 2.83 has to be solved by one of the nonlinear solution methods.

#### 2.11.4.2 Finite Element Modeling of Reinforced Concrete Beams

The finite element idealization of reinforced concrete beams is accomplished by two dimensional elements which lie in the x-y plane of the beam elevation. Following the assumptions of the Timoshenko beam theory the resulting stresses are assumed to be constant through the thickness of each element and stresses  $\sigma_z$ ,  $\tau_{xz}$  and  $\tau_{yz}$  are ignored. These assumptions lead to a plane stress field. The displacement field is described by  $u$  and  $v$ , which represent the node displacements in the x and y-direction, respectively, and are in general functions of  $x$  and  $y$ . These functions must satisfy  $C^0$  continuity. The process of the finite element discretization of this displacement field can be formally written as;

$$\{d\} = \begin{Bmatrix} u \\ v \end{Bmatrix} = \sum_{j=1}^n \begin{bmatrix} N_j & 0 \\ 0 & N_j \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix}_j \quad (2.89)$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial N}{\partial x} & 0 \\ 0 & \frac{\partial N}{\partial y} \\ \frac{\partial N}{\partial y} & \frac{\partial N}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix}^{ele} \quad (2.90)$$

or, in compact form to

$$\{\varepsilon_p\} = - \sum_{ele} \int [B][d]^{ele} \quad (2.91)$$

The stiffness matrix of the composite reinforced concrete is given as;

$$[K]_{ele} = [K]_c + \sum_{i=1}^n [K_{GL}]_s \quad (2.92)$$

$n$  is the number of steel elements embedded in the concrete element and  $[K_{GL}]_s$  is the global stiffness matrix of the discrete steel element without or without bond-slip. The concrete element stiffness  $[K]_c$  is determined from;

$$[K]_c = \sum_V \int [B]^T [D_{GL}]_c [B] dV \quad (2.93)$$

Where  $[D_{GL}]_c$  is the concrete material matrix in global coordinates. The assembly of the element stiffness matrices  $[K]_{ele}$  results in the structural stiffness matrix  $[K]$ .

## 2.12 Material Properties and Constitutive Models

### 2.12.1 Element Type

Solid65 is an eight-node solid element, used to model the concrete with three degrees of freedom at each node. The element is capable of plastic deformation, cracking in three orthogonal directions, and crushing. The geometry and node locations for the element type are shown in Figure 2.14.

A Link 8 is used to model the steel reinforcement. Two nodes are required for this element. Each node has three degrees of freedom. The element is also capable of plastic deformation. The geometry and node locations for this element type are shown in Figure 2.16.

Solid45 is also eight-node solid element, used for the steel plates at the supports and applied load location in the beam models. The element has eight nodes with three degrees of freedom. The geometry and node locations for the element type are shown in Figure 2.15.

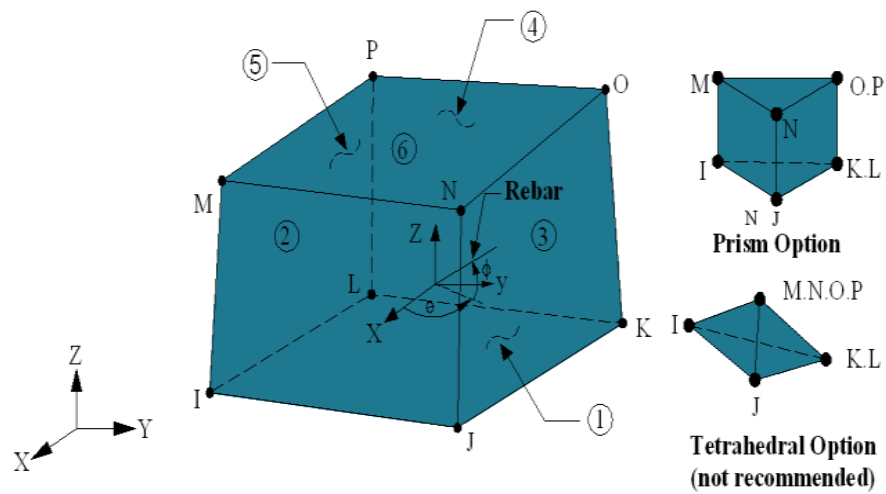


Figure 2.14: SOLID65 geometry (source: Amer et al, 2013)

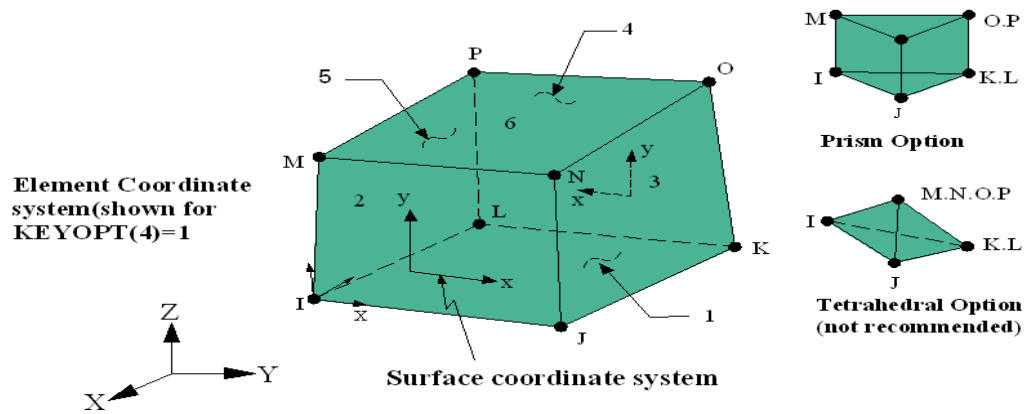


Figure 2.15: SOLID45 geometry (source: Amer et al, 2013)

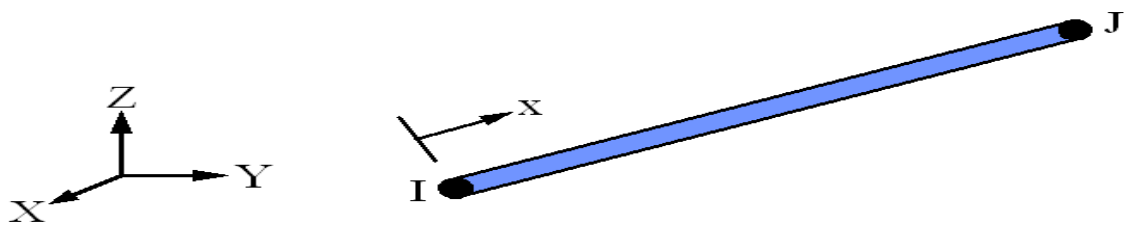


Figure 2.16: LINK8 geometry (source: Amer et al, 2013)

## 2.12.2 Modeling of Material Properties

### 2.12.2.1 Concrete

Microcracks are exhibited by concrete especially, at the interface between coarse aggregates and mortar, even before they are subjected any load. Microcracks in concrete has great effect on the mechanical behavior of concrete. The stress-strain curve for concrete in compression is linearly elastic up to about 30 percent of the maximum compressive strength. In tension, the stress-strain curve for concrete is approximately linearly elastic up to the maximum tensile strength. Figure 2.17 shows typical uniaxial compressive and tensile stress-strain curve for concrete. Desayi and Krishnan (1964); Gere and Timoshenko, 1997) see Figure 2.18.

$$f_c = \varepsilon E_c \quad \text{for } 0 \leq \varepsilon \leq \varepsilon_1 \quad (2.94)$$

$$f_c = \frac{\varepsilon E_c}{1 + \left[\frac{\varepsilon}{\varepsilon_o}\right]^2} \quad \text{for } \varepsilon_1 \leq \varepsilon \leq \varepsilon_o \quad (2.95)$$

$$f_c = f'_c \quad \text{for } \varepsilon_o \leq \varepsilon \leq \varepsilon_{cu} \quad (2.96)$$

$$\varepsilon_o = \frac{2f'_c}{E_c} \quad \text{for } \varepsilon_o \leq \varepsilon \leq \varepsilon_{cu} \quad (2.97)$$

Where,

$f_c$  = stress at any strain

$\varepsilon$  = strain

$\varepsilon_o$  = strain at ultimate compressive strength

$E_c$  = concrete modulus of Elasticity

The simplified stress-strain curve for each beam model is shown in figure 2.18.

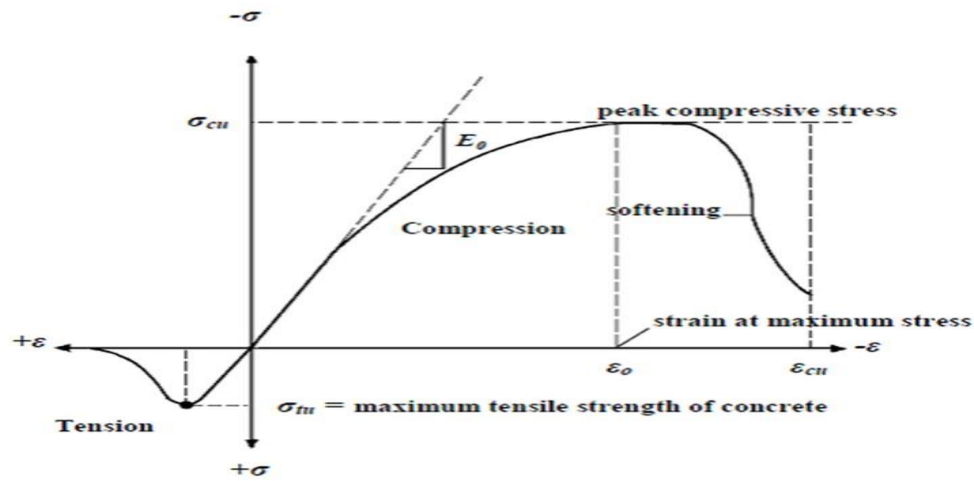


Figure 2.17: Typical Uniaxial Compressive and Tensile Stress-strain curve of concrete

Source: Kachlakev and McCurry, (2000)

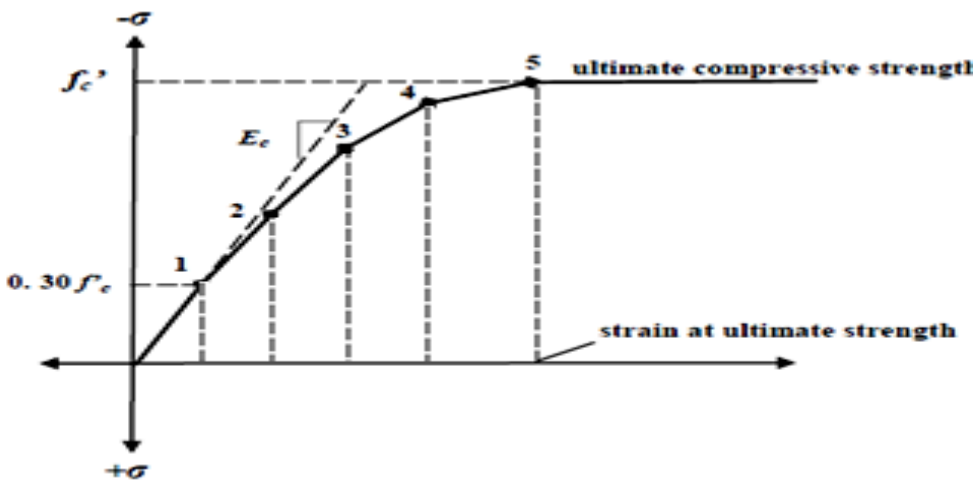


Figure 2.18: Simplified Compressive Uniaxial Stress- Strain Curve for Concrete.

Source: Desayi Krishnan, (1964); Gere et al., (1997).

### 2.12.2.2 Concrete Models

The simplest model use in reinforced concrete is the orthotropic models (Chen 1976, ASCE 1982, Meyer and Okamura, 1985). It is particularly suitable for the analysis of reinforced concrete beams, panels and shells. (Darwin and Pecknold, 1977).

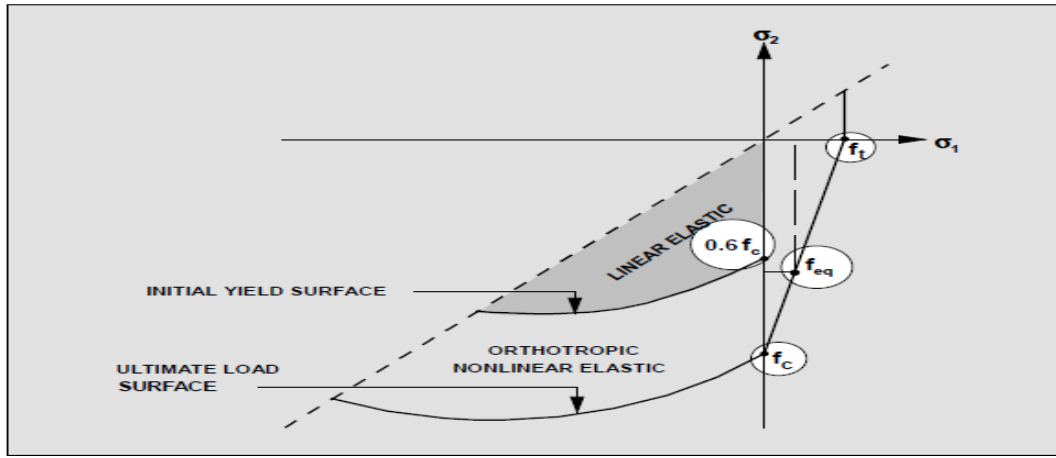


Figure 2.19: Strength failure envelope of concrete

### 2.12.2.3 Concrete Material Matrix

In the analysis of reinforced concrete structures plane stress problems make up a large majority of practical cases. In the following the constitutive relation for plane stress problems is derived for the concrete model of this study. For stress combinations in Figure 2.19, concrete is assumed to be a homogeneous, linear isotropic material. Thus, the stress-strain relation for plane stress problems has the simple form;

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (2.98)$$

where  $E$  is the initial elastic modulus of concrete and  $\nu$  is Poisson's ratio. (Desai and Siriwardance 1972; Chen 1976)

$$\begin{Bmatrix} d\sigma_{11} \\ d\sigma_{22} \\ d\tau_{12} \end{Bmatrix} = \frac{1}{1-\nu^2} \begin{bmatrix} E_1 & \nu\sqrt{E_1E_2} & 0 \\ \nu\sqrt{E_1E_2} & E_2 & 0 \\ 0 & 0 & (1-\nu^2)G \end{bmatrix} \begin{Bmatrix} d\varepsilon_{11} \\ d\varepsilon_{22} \\ d\gamma_{12} \end{Bmatrix} \quad (2.99)$$

Where  $E_1$  and  $E_2$  are the secant moduli of elasticity and  $\nu$  is Poisson's ratio and

$$(1-\nu^2)G = 0.25(E_1 + E_2 - 2\nu\sqrt{E_1E_2}) \quad (2.100)$$

#### **2.12.2.4 Steel**

Steel is assumed to be perfectly elastic material and identical in tension and compression. The properties of reinforcing steel are not dependent on environmental conditions or time as in the case of concrete. Therefore a single stress-strain relation is sufficient to define the material properties needed for the analysis. Figure. 2.20, shows the common idealization used depending on the level of accuracy (ASCE 1982; Cervenka et al. 1990). This model has been adopted in many analyses of reinforced concrete structures. (Ngo and Scordelis 1967; Vebo and Ghali 1977; Bashur and Darwin 1978).

#### **2.12.2.5 Steel Material Matrix**

Reinforcing steel can be modeled following the three approaches below:

- (1) The smeared or distributed model;
- (2) The embedded model; and
- (3) The discrete model (ASCE 1982).

Considering the smeared or distributed model, steel materials is assumed to be distributed over the concrete element at a certain orientation angle.

In an embedded steel materials model, the steel materials are considered as an axial member built into the isoparametric element such that its displacements are consistent with those of the element.

Discrete model is the most widely used model and represents a one dimensional truss element, assumed to be pin connected and possesses two degrees of freedom at each node. (Cervenka et al. 1990).

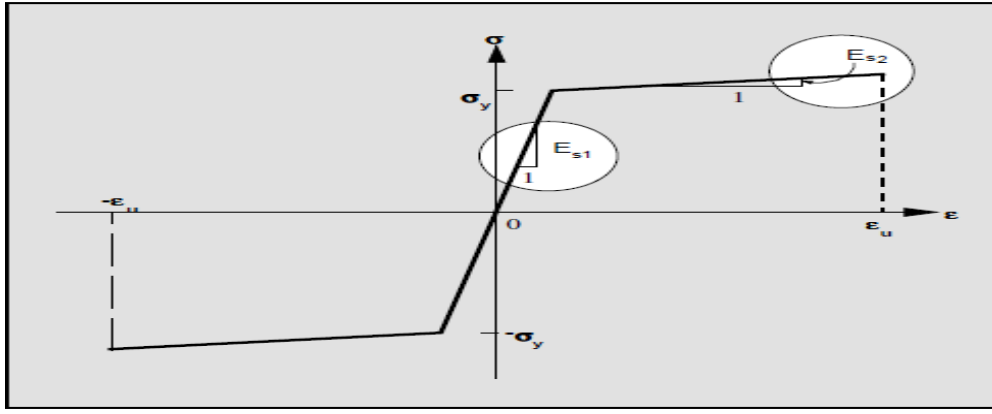


Figure 2.20: Steel stress-strain relation

For the one-dimensional truss element with constant strain the stiffness matrix is given by

$$\begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} \quad (2.101)$$

Or,

$$\{P\} = [K_{LO}]_c \{d\} \quad (2.102)$$

Where  $E$  is the modulus of elasticity,  $A$  the cross-sectional area and  $L$  the length of the bar element.  $P_1$  and  $P_2$  are axial end forces and  $d_1$  and  $d_2$  are axial end displacements of the reinforcing bar.  $E$  is equal to  $E_{s1}$  before yielding of the reinforcement and equal to  $E_{s2}$  after yielding (Figure 2.20). Eq. 2.102 can also be expressed by the following relation:

$$\begin{Bmatrix} P_{1x} \\ P_{1y} \\ P_{2x} \\ P_{2y} \end{Bmatrix} = [T_1]^T \cdot [K_{LO}]_c \cdot [T_1] \begin{Bmatrix} d_{1x} \\ d_{1y} \\ d_{2x} \\ d_{2y} \end{Bmatrix} \quad (2.103)$$

$$[T_1] = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{bmatrix} \quad (2.104)$$

1 and 2 refer to the end points of the steel element. Eq. 2.103 will be transformed before it can be assembled together with the concrete element stiffness matrix. This can be formally expressed by the following relation:

$$[K_{GL}]_s = [T_2]^T \cdot [T_1]^T \cdot [K_{LO}]_s \cdot [T_1] \cdot [T_2] \quad (2.105)$$

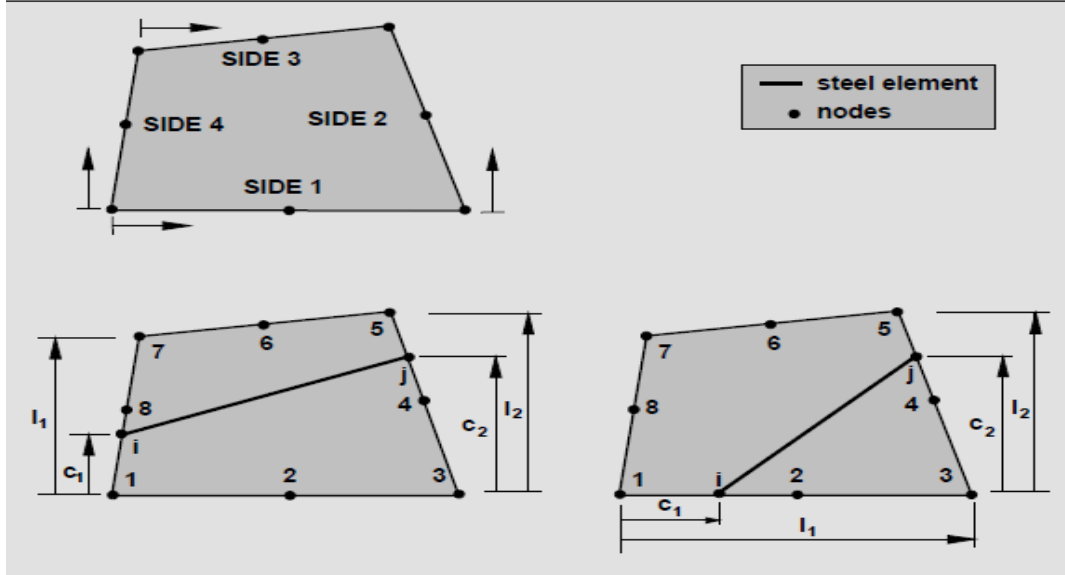


Figure 2.21: Steel element embedded in concrete

Transformation matrix  $[T_2]$  can be derived with the procedure used to establish the consistent nodal forces of the finite element method. The consistent nodal forces for node  $i$  are:

$$P_{xi} = \int_{-1}^1 N_i \left( P_t \frac{\partial x}{\partial \xi} - P_n \frac{\partial y}{\partial \xi} \right) d\xi \quad (2.106)$$

$$P_{yi} = \int_{-1}^1 N_i \left( P_n \frac{\partial x}{\partial \xi} - P_t \frac{\partial y}{\partial \xi} \right) d\xi \quad (2.107)$$

Where  $N_i$  is the shape function for node  $i$ ,  $P_n$  and,  $P_t$  are the normal and tangential boundary loads, respectively. If a concentrated load acts at a point  $\xi = \delta_i$  Eqs. 2.106 and 2.107 reduce to the following:

$$P_{xi} = N_i(\delta_i)P_x \quad (2.108)$$

$$P_{yi} = N_i(\delta_i)P_y \quad (2.109)$$

When the 8-node isoparametric element is used in the two-dimensional mesh representation of the member, the shape functions for nodes  $i, j$  and  $k$  in Figure 2.22 are

$$N_i = \frac{1}{2}\xi(\xi - 1) \quad (2.110)$$

$$N_j = (\xi - 1)(\xi + 1) \quad (2.111)$$

$$N_k = \frac{1}{2}\xi(\xi + 1) \quad (2.112)$$

To obtain  $N_i(\delta_i)$  in Eqs. 2.37 and 2.38  $\delta_i = \frac{2c_i}{l_i - 1}$  is substituted for  $\xi$  in Eqs. 2.110-2.112 where  $c_i$  characterizes the position of the steel node relative to the concrete element boundary, as shown in Figures 2.15 and 2.16. By setting  $r_i = \frac{c_i}{l_i}$  Eqs. 2.110-2.112 simplify to the following relations which are used to construct the transformation matrix  $[T_2]$ .

$$N_i = (r_i - 1)(2r_i - 1) \quad (2.113)$$

$$N_j = 4r_i(1 - r_i) \quad (2.114)$$

$$N_k = r_i(2r_i - 1) \quad (2.115)$$

Considering the reinforcing bar element crossing the concrete element boundary on sides 2 and 4 in Figure 2.21, then nodes  $i, j$  and  $k$  correspond to node numbers 1, 8 and 7, on side 4, and node numbers 3, 4 and 5 on side 2, respectively. With the notation of Figure 2.14 the transformation matrix  $[T_2]$  has the following form:

$$[T_2] = \begin{bmatrix} A_1 & 0 & 0 & 0 & 0 & 0 & A_3 & A_2 \\ 0 & 0 & B_1 & B_2 & B_3 & 0 & 0 & 0 \end{bmatrix} \quad (2.116)$$

Where,

$$A_1 = \begin{bmatrix} 2p^2 - 3p + 1 & 0 \\ 0 & 2p^2 - 3p + 1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 2q^2 - 3q + 1 & 0 \\ 0 & 2q^2 - 3q + 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -4p^2 + 4p & 0 \\ 0 & -4p^2 + 4p \end{bmatrix}$$

$$B_2 = \begin{bmatrix} -4q^2 + 4q & 0 \\ 0 & -4q^2 + 4q \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 2p^2 - 2p & 0 \\ 0 & 2p^2 - 2p \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 2q^2 - 2q & 0 \\ 0 & 2q^2 - 2q \end{bmatrix}$$

$$p = \frac{c_1}{l_1}$$

$$q = \frac{c_2}{l_2}$$

and  $\mathbf{0}$  is the 2x2 null matrix.

In Eq. 2.116 the position of submatrices  $\mathbf{A}$  and  $\mathbf{B}$  within the transformation matrix  $[T_2]$  is closely related to the concrete element which the reinforcing bar element crosses. If the element crosses the concrete element boundary on sides 1 and 2 in Figure 2.21, then the transformation matrix  $[T_2]$  takes the form;

$$[T_2] = \begin{bmatrix} A_1 & A_2 & A_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & B_1 & B_2 & B_3 & 0 & 0 & 0 \end{bmatrix} \quad (2.117)$$

noting that the submatrices **A** and **B** are the same as before, but  $c_1$  is now defined, as shown in Figure 2.21. When the 4-node isoparametric element is used in the two-dimensional mesh representation of the member, the shape functions for nodes  $i$ , and  $k$  change and the transformation matrix  $[T_2]$  simplifies considerably. The transformation matrix  $[T_2]$  takes the form;

$$[T_2] = \begin{bmatrix} 1-p & 0 & 0 & 0 & 0 & 0 & p & 0 \\ 0 & 1-p & 0 & 0 & 0 & 0 & 0 & p \\ 0 & 0 & 1-q & 0 & q & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-q & 0 & q & 0 & 0 \end{bmatrix} \quad (2.118)$$

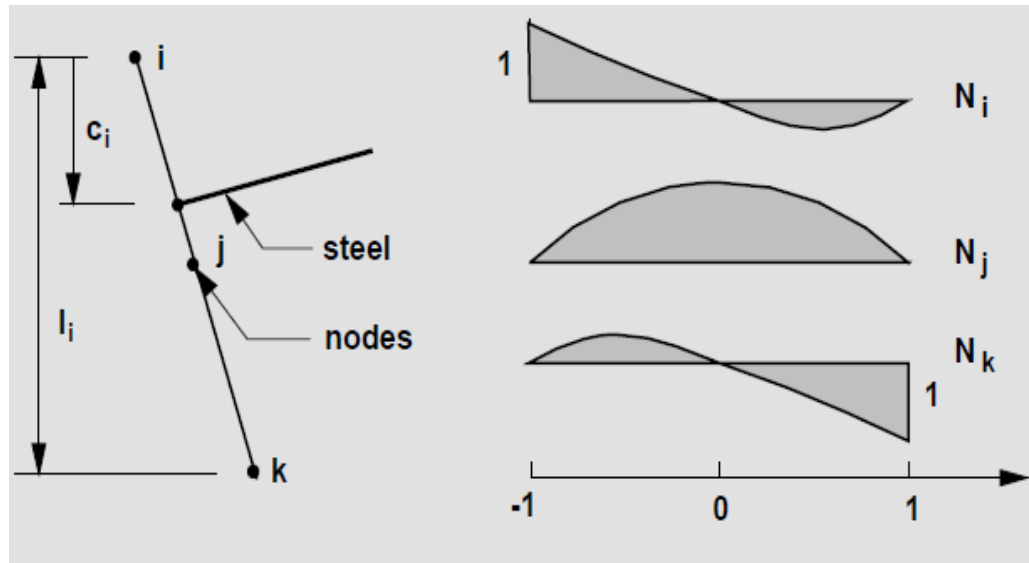


Figure 2.22: Parameters and shape functions of isoparametric concrete and embedded steel element

The steel stiffness matrix  $[K_{GL}]_s$  in Eqn. 2.115 can now be assembled together with the concrete element stiffness matrix to form the total stiffness of the structure.

## **CHAPTER THREE**

### **MATERIALS AND METHODS**

#### **3.1 Materials**

Materials required in this research include;

- (a) Coarse Aggregate
- (b) Fine Aggregate
- (c) Cement
- (d) Water
- (e) Steel rod
- (f) PVC pipes

##### **3.1.1 Coarse Aggregate**

Coarse aggregate are boulders of varying sizes used in concrete mix. They general crushed crushed rocks such as limestone or granite are commonly used. For the purpose of this research, aggregate size of 20 mm was used.

##### **3.1.2 Fine Aggregate**

One of the basic constituent of concrete is sand and it is found in abundance as a surface deposit along the course of rivers, on the shores of lakes, seas and in arid regions. Sharp sand used in the concrete mixture was obtained from Ado Ekiti, Ekiti state.

##### **3.1.3 Cement**

The cement commonly used in Nigerian construction industry is the general setting Portland cement. It has excellent binding properties for the coarse and fine aggregate and also solidifies and

hardens the concrete through a chemical process called hydration. The cement used in this research is the Lime Portland Cement (Dangote product) of grade 42.5.



Plate 3.1: Gravel



Plate 3.2: River sand

#### **3.1.4 Water**

The quality of water used in mixing concrete must be such that the chemical reactions which take place during mixing should not be impaired. The strength and workability of concrete depends greatly on the amount of water used in mixing. The purpose of using water is to cause the hydration of cement. Portable water from ABUAD was used for this research.

#### **3.1.5 PVC Pipes**

PVC (polyvinyl chloride) is a lightweight plastic used in the construction industry. Plasticizers are used to improve one or more properties in their physical state. The thickness of the PVC pipes used was 3mm, diameter of 50mm, density of  $1.41\text{g/cm}^3$  and length 2.75m. Other properties includes tensile strength of 52 MPa, Tensile modulus of 3-3.3 GPa and flexural modulus of 2.7-3GPa as reported by the manufacturer.

### 3.1.6 Steel bars

Concrete is known to be good in compression, but always has lower tensile strength. For this reason it is usually reinforced with steel materials to improve its weakness in tension. Steel rebar were used as a tensioning devise to reinforce concrete. The various sizes of reinforcement bar used includes Y8, Y10 and Y12. These bars were tested at the metallurgy laboratory, university of Ilorin and their strengths given in Table 3.1.



Plate 3.3: Beam reinforcement cage with shear



Plate 3.4: 50 mm PVC pipes links

Table 3.1: Bar sizes, yield and ultimate strength

| Bar Diameter ( $\emptyset$ ) | Yield stress( $f_o$ ) | Average ( $f_o$ ) | Ultimate stress( $f_u$ ) | Average ( $f_u$ ) |
|------------------------------|-----------------------|-------------------|--------------------------|-------------------|
| mm                           | N/mm <sup>2</sup>     |                   |                          |                   |
| 8                            | 367                   | 300.5             | 591                      | 524.5             |
|                              | 234                   |                   | 458                      |                   |
| 10                           | 438                   | 364.0             | 621                      | 539.5             |
|                              | 290                   |                   | 458                      |                   |
| 12                           | 432                   | 375.5             | 753                      | 649               |
|                              | 298                   |                   | 452                      |                   |
|                              | 397                   |                   | 742                      |                   |

## 3.2 METHODS

The proposed study was conducted in three phases namely; experimental approach, modelling and simulation and Reliability analysis.

### 3.2.1 Experimental approach

The experimental approach includes;

**i. Determination of the compressive strength of normal concrete and concrete with embedded PVC pipes**

The compressive strength for concrete cubes with and without embedded PVC pipes using the design mix ratio of 1:2:4 was determined using method proposed by BS 1881-108:1983 for testing concrete. 150 x 150 mm standard cube moulds were used. PVC pipes (50 mm diameter) were cut to 130 mm length and placed in the mould and occupying different grid positions before casting as shown in plates 3.5-3.12. C25 design mix was adopted and the mix proportioning is given Table 3.2.

Table 3.2: Material Proportioning (C25 design Mix)

| S/No             | Materials                 | Volume (Kg/m <sup>3</sup> ) |
|------------------|---------------------------|-----------------------------|
| 1                | Cement                    | 346.00                      |
| 2                | Fine Aggregates           | 664.65                      |
| 3                | Coarse aggregates         | 1234.35                     |
| 4                | Water                     | 190.00                      |
| 5                | Water/Cement Ratio        | 0.55                        |
| <b>Mix Ratio</b> | 0.55 : 1.00 : 1.92 : 3.57 |                             |

## **Procedure**

The procedure for concrete cube making following the British Standard (BS) 1881:1983 Testing Concrete was followed. Equipment used includes;

- Sample tray
- Mould for making test cube
- Spanners
- Scoop
- Steel float or trowel
- Compacting bar
- Vibrating hammer or vibrating table
- Cleaning rags
- A bucket or barrow for sample movement
- Polythene sheeting and curing tank.

## **Compaction**

The moulds (150 x 150 mm) were filled in three approximately equal layers (50 mm deep), and a compacting bar of length 380 mm with a 25 mm square end for ramming. The concrete was fully compacted and vibrated with the concrete vibrating table to allow consistency of constituent materials. The minimum number of strokes per layer was 35 strokes for maximum compaction. Surface finishing was done using the hand trowel. The compressive strength for both normal concrete and concrete with PVC pipes was determined at 7, 14, 21 and 28 days respectively. Figures 3.6-3.14 shows the casting, de-molding and testing of the cube specimen using the

universal compression testing machine (UCTM). Tables in appendix A gives the summary of compressive strength results for normal, PVC and reinforced concrete cubes.



Plate 3.5: Square grids of PVC at corners



Plate 3.6: Square grid of PVC at centre



Plate 3.7: Square grid after de-molding operation



Plate 3.8: Failure pattern for square grid at edges



Plate 3.9: Failure pattern at edges



Plate 3.10: Centre grid pattern



Plate 3.11: De-molding of centre grid pattern



Plate 3.12: Compression machine in crushing operation      Plate 3.13: Crushing of cube with PVC at edges



Plate 3.14: Crushing operation with PVC pipes at centre grid position

**ii. Casting of 56 nos. concrete beams of sizes 1000 x 150 x 150 mm with and without PVC pipes.**

Eight number (8 Nos.) of control concrete beams were produced with specimen label CTBM, eight number (8 Nos.) of reinforced concrete beams with one PVC pipe ( $0.00196\text{m}^3$ ) and specimen label RCPVC1, eight number (8 Nos.) of reinforced concrete beams with two PVC pipes ( $0.00393\text{m}^3$ ) and specimen label RCPVC2, eight number (8 Nos.) of reinforced concrete beams with three PVC pipes ( $0.00442\text{m}^3$ ) and specimen label RCPVC3, eight number (8 Nos.) of concrete beams with one PVC pipe ( $0.00196\text{m}^3$ ) and specimen label CBPVC1, eight number (8 Nos.) of concrete beams with two PVC pipe ( $0.00393\text{m}^3$ ) and specimen label CBPVC2 and eight number (8 Nos.) of reinforced concrete beams and specimen label RCBM respectively. Plates 3.15-3.18 shows the casting of the beams and curing process.



Plate 3.15: Formwork preparation for normal concrete and concrete with embedded PVC



Plate 3.16: Casting of Concrete with and without PVC pipes



Plate 3.17: Beams in curing Tank

**iii. Determination of flexural behavior using the ASTM C 293 centre point loading system.**

ASTM C 293 centre point loading system was used to carry out the flexural tests in the laboratory. Load at first crack, ultimate load at failure and their respective strain  $\epsilon_c$  and  $\epsilon_u$

were measured at 7, 14, 21 and 28 days respectively using the strain gauge attached to the beam specimen. See plate 3.18 and 3.19 respectively. The results obtained were used for the verification of the stress - strain relationship for both types of concrete under uniaxial compression. The results will also be compared with stress-strain relationship proposed by Desayi et al and Saenz, in equation 3.1-3.4 below:



Plate 3.18: Experimental set up (ASTM 293 C) with force units attached



Plate 3.19: Beam failure under maximum applied load

$$\frac{f}{f_o} = \frac{R \left( \frac{\epsilon}{\epsilon_o} \right)}{1 + (R - 1) \left( \frac{\epsilon}{\epsilon_o} \right)^\beta} \quad (3.1)$$

Where,

$$\beta = \frac{R}{R - 1} \quad (3.2)$$

$$R = \frac{E_c}{E_o} \quad (3.3)$$

$R$  = material parameter depending on the shape of the stress-strain curve

$E_c$  = modulus of elasticity of concrete,

$$E_o = \frac{f_o}{\epsilon_o}$$

$$\frac{f}{f_u} = \frac{A' \left( \frac{\epsilon}{\epsilon_u} \right)}{1 + B' \left( \frac{\epsilon}{\epsilon_u} \right)^2} \quad (3.4)$$

$$\epsilon = 0.000875 f_o^{0.25} \quad (3.5)$$

$$\epsilon_o = \frac{0.0078}{f_o^{0.25}} \quad (3.6)$$

$$\frac{f}{f_u} = \text{stress ratio}, \quad \frac{\epsilon}{\epsilon_u} = \text{strain ratio}$$

The Tables in appendix B and C gives the summary of laboratory results obtained for the stress-strain relationship and the flexural behavior using the ASTM 293 C.

### 3.2.2 Modelling and Simulation

The second phase involves numerical simulation i.e. finite element modelling of previously tested reinforced concrete beam specimen with and without PVC pipes using ANSYS. The primary objective is to validate the material models utilised in the numerical methods in predicting the experimental tests results.

Modelling and simulation of reinforced concrete beam requires some data inputs. For this research, these data will be obtained from previous experimental studies carried out on flexural behaviour of beams and from researchers available from literature. Some of the data include;

- a) Geometric data of beam specimens: length, depth, width and shear spans.
- b) Mechanical properties of the concrete: compressive strength, tensile strength, modulus of elasticity and poisson ratio of the concrete.
- c) Steel reinforcement quantities and arrangements: vertical reinforcements, horizontal reinforcements and tensile reinforcements and compression reinforcements.
- d) Mechanical properties of the steel reinforcement: yield strength, modulus of elasticity and poisson ratio.
- e) Beam reactions: cracking loads, ultimate loads, deflections as well as failure mode of the beam specimens.

When these data are inputted, the beam reactions are obtained and then compared with experimental results. The results from simulation analysis using ANSYS Pro 15 can be found in appendix D.

### 3.2.3 Reliability analysis using FORM.

Reliability analysis will be carried out in the third phase using the first order reliability method (FORM). FORTRAN subroutines will be created for the limit state in bending, shear and deflection of the beam and then linked with the reliability software (FORM5) for analysis.

The reliability analysis will be carried out by determining the means, standard deviations, coefficient of variations and the distributions of the basic variables. The probabilistic model code (JCSS, 2000) specified that:

- i. Material properties such as concrete strength characteristic strength e.t.c. are treated as log-normal distribution
- ii. Geometric properties are modeled as normal or log-normal distribution
- iii. Load considered in this research are permanent loading and are such treated as normal – distribution. The imposed load will be treated as lognormal.

#### 3.2.3.1 Limit States

The limit states considered in this research are:

- a) Bending
- b) Shear and
- c) Deflection on beams

##### 3.2.3.1.1 Bending

For a beam in bending, the nominal resistance is given by

$$R_n = A_s f_y \left( d - 0.59 \frac{A_s f_y}{f_c' b} \right) = A_s f_y d - 0.59 \frac{(A_s f_y)^2}{f_c' b} \quad (3.7)$$

Source: Ellinwood, Galambos, MacGregor, and Cornell, (1980)

The beam is examined for the limit state exceeding the beam capacity in bending. The performance function or limit state would be:

$$g(A_s, f_y, f_c, M, b, d) = A_s f_y d - 0.59 \frac{(A_s f_y)^2}{f_c' b} - M \quad (3.8)$$

Where,

M is the moment (load effect) due to the applied load.

$A_s$  = area of reinforcement

$f_y$  = characteristics yield strength of steel

$f_s$  = concrete strength.

### 3.2.3.1.2 Shear

For a beam in shear, the code (BS8110) specifies

$$v = \frac{V}{b_v d} \quad (3.9)$$

Where,

$v$  = shear stress

$V$  = Shear force

$b_v$  = width

$d$  = Effective depth of beam

The limit state is given by:

$$g(v_c, b_v, d, f_{yv}, A_{sv}, S_v, V) = v_c b_v d + 0.95 f_{yv} \frac{A_{sv}}{S_v} d - V \quad (3.10)$$

Where,

$v_c b_v d$  = shear resistance in the concrete

$0.95 f_{yv} \frac{A_{sv}}{S_v} d$  = shear resistance in the reinforcing steel

### 3.2.3.1.3 Deflection

The code specifies:

$$\text{Limiting deflection} \propto \frac{\text{Allowable span}}{\text{effective depth}} = 20 \times m.f_{TS} \times m.f_{CS}$$

$$\text{Actual deflection} \propto \frac{\text{Actual span}}{\text{effective depth}}$$

But,

$$m.f_{TS} = 0.55 + \frac{\left( 477 - \frac{2}{3} f_y \frac{A_s}{A_{sp}} \right)}{120 \left( 0.9 + \frac{M}{b_v d^2} \right)}$$

$$M = \frac{wl^2}{8} + \frac{Pl}{4}$$

Therefore, the limit state in deflection is given by:

$$g(d, f_y, A_s, A_{sp}, P, w, b_v) = d - \frac{l}{20 \times \left[ 0.55 + \frac{\left( 477 - \frac{0.67 f_y A_s}{A_{sp}} \right)}{\left( 108 + \frac{30Pl + 15wl^2}{b_v d^2} \right)} \right]} \quad (3.11)$$

The stochastic models for the basic variables in the different limit state will be calculated from equation 3.10-3.12.

$$COV = \frac{S(X)}{E(X)} \quad (3.12)$$

$$E(X) = N \times \lambda \quad (3.13)$$

$$S(X) = COV \times E(X) \quad (3.14)$$

Where,

COV = Coefficient of variation of the basic variables,

S(X) = Standard deviation of the basic variables,

E(X) = Mean of the basic variables,

$\lambda$  = Bias factor of the basic variables

N = Nominal values of the basic variables obtained from the deterministic analysis of the solid slab. The coefficient of variation and the bias factor are computed accordingly. The subroutine in FORTRAN used for the reliability analysis using the first order reliability method (FORM) is shown in appendix E and Tables in appendix F shows the computations of basic variables using equations 3.12-3.13 for the reliability analysis.

## **CHAPTER FOUR**

### **RESULTS AND DISCUSSION**

#### **4.1 Compressive strengths results and stress- strain analysis with and without PVC pipes**

##### **4.1.1 Compressive strength results for PVC Concrete cubes**

The compressive strength test results for PVC concrete (grade C25) following the mix design in appendix A, revealed that for concrete with 3 PVC embedded PVC pipes below the neutral axis (N.A), the mean strength increases steadily with age of concrete cubes (See Figures 4.1-4.3). The plots of all cube specimen shows that concrete cubes with three (3) PVC pipes below the neutral axis have similar strength values with the normal cubes as shown in Figures 4.4-4.7. The average strength values of concrete cubes with embedded PVC pipes increases with reduction in the number of PVC pipes within the specimen. The compressive strength for concrete cubes with four (4) embedded PVC pipes at centre grid position and at corners of cubes indicates similar trend as with concrete cubes having three (3) embedded PVC pipes but with lower strength within the range of 20-29%. The tables for strength values for the different cube specimen with embedded PVC pipes at different grid position are found in appendix B, Tables II-V.

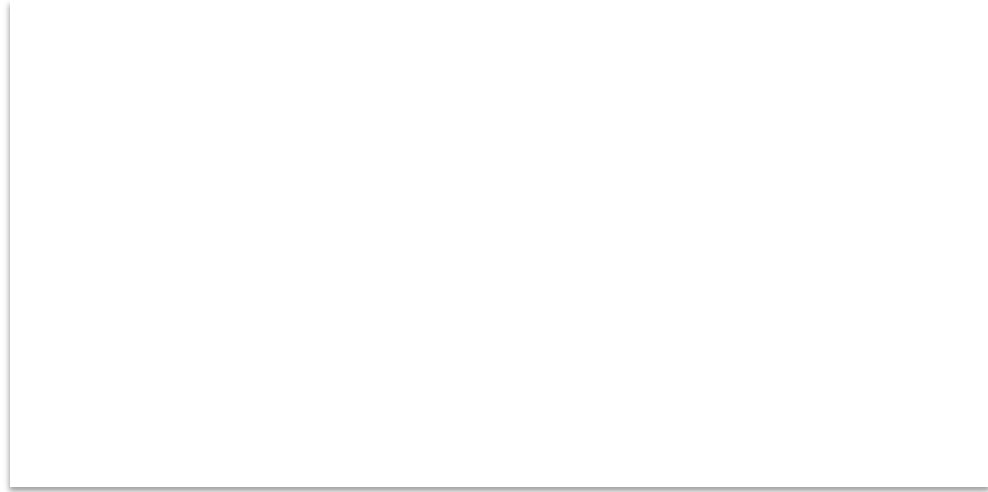


Figure 4.1: Compressive strength against age of concrete for PVC pipes at edge

Figure 4.2: Compressive strength against age of concrete for PVC pipes at middle

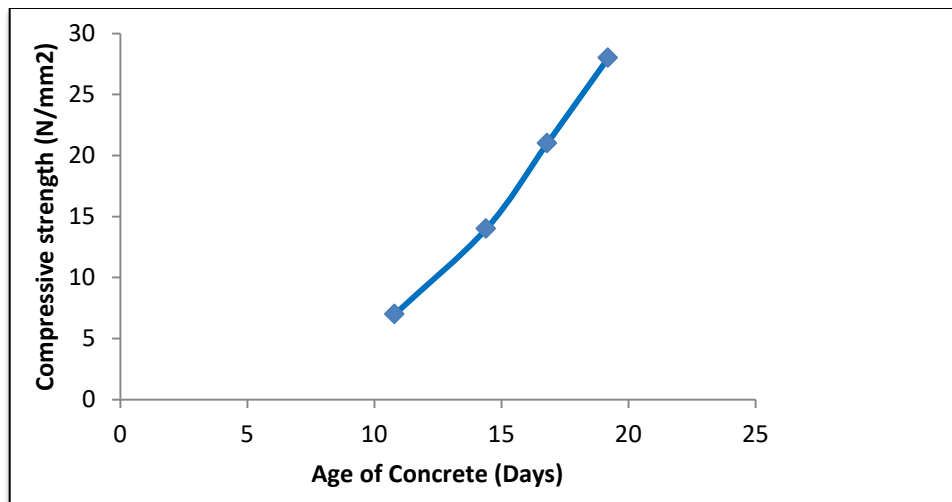


Figure 4.3: Compressive strength against age of concrete for PVC pipes below N.A

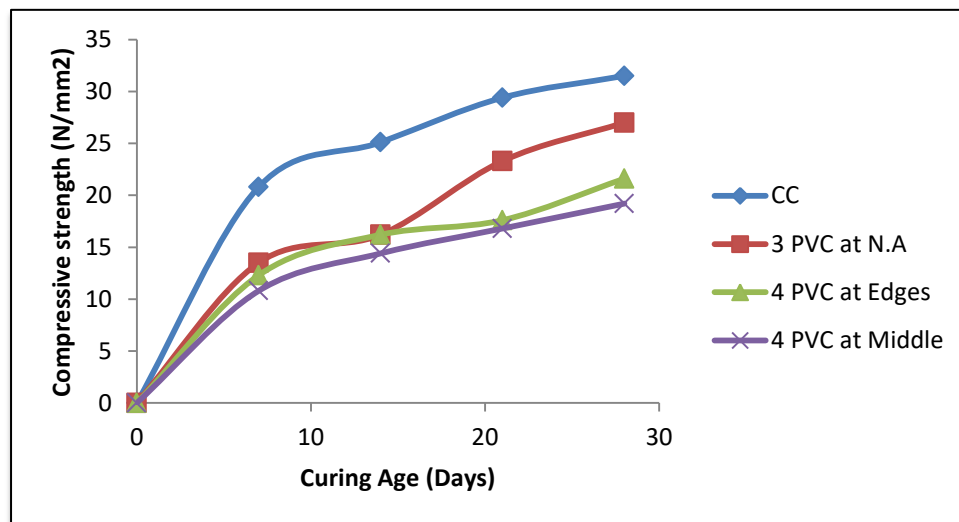


Figure 4.4: Compressive strengths against age of concrete with and without PVC pipes

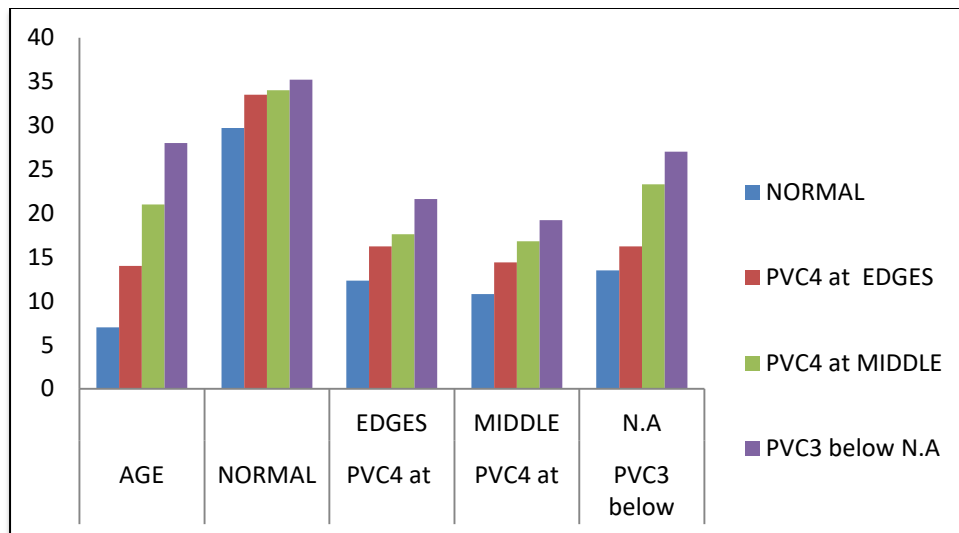


Figure 4.5: Variation of Compressive strengths with and without PVC pipes

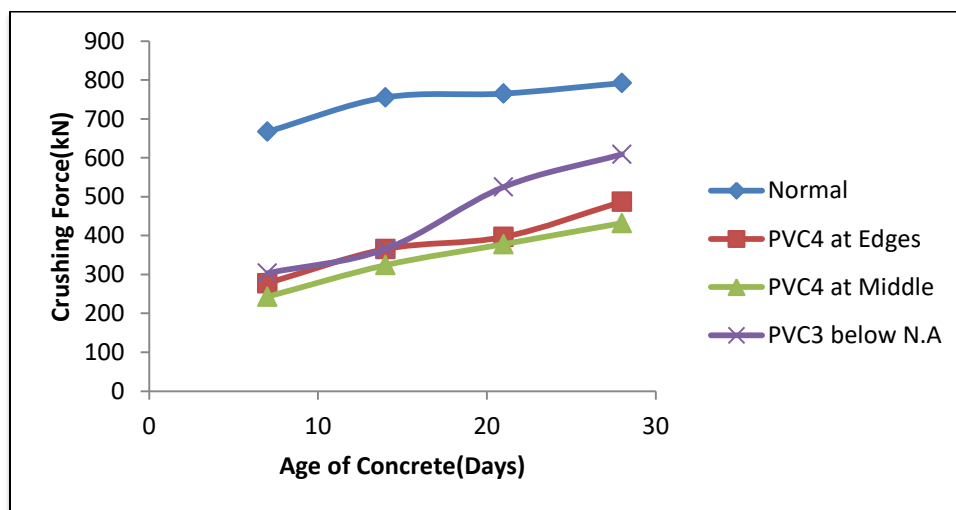


Figure 4.6: Crushing Force against age of concrete (with and without PVC pipes)

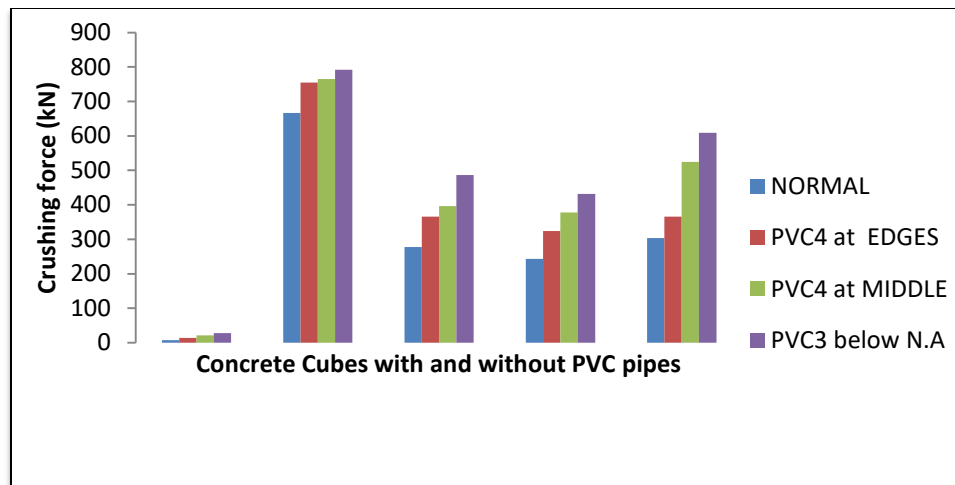


Figure 4.7: Variation of crushing Force with and without PVC pipes

#### 4.1.2 Compressive strength results for control Concrete cubes

b) The Compressive strength results for control cubes (unreinforced and non-PVC pipes) follows similar trend with that of PVC pipes but with higher strength value in the order of 14%, 31.4% and 39% for cubes for 3 embedded PVC pipes, 4 PVC pipes at corners and 4 PVC pipes at centre grid position respectively (See Figure 4.8).

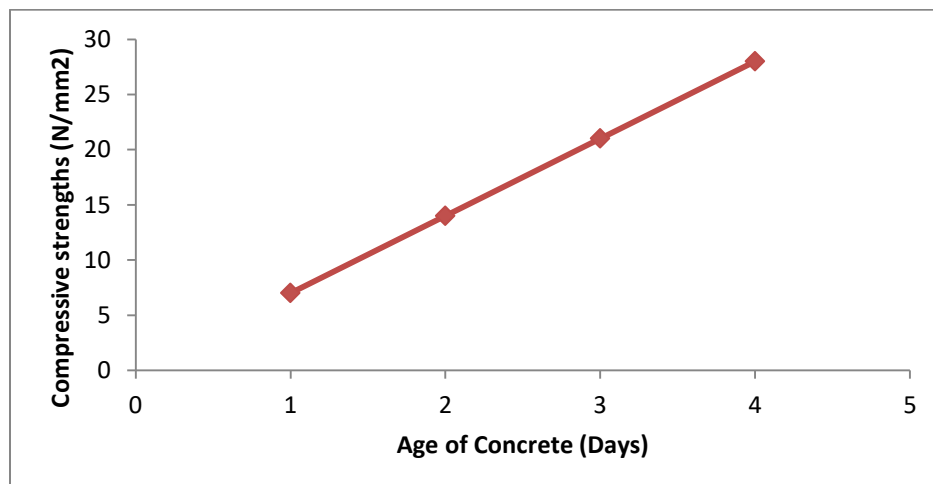


Figure 4.8: Compressive strength against age of concrete for normal cube

### 4.1.3 Stress-Strain Verification

For stress-strain verification in uniaxial compression, concrete cube were cast for grade C20, C25 and C30 with and without reinforcement (Y10) and cubes with PVC pipes respectively. Strain, young modulus of elasticity and R values were determined from experimental data and analyzed using BIOSTAT pro 5.9.8. Equation of best fit was proposed with  $n=4$  (i.e. polynomial of degree 4). The summary of the result is presented in Table 4.1 and also validated with stress-strain equations proposed by Desayi (1964) and Saenz (1964).

Experimental program was carried out on concrete cube specimens with and without reinforcement (Y10) and concrete cube specimens with PVC pipes at different grid pattern. The concrete cube specimens are of grade C20, C25 and C30 respectively. Statistical analysis was carried out at 5% level of significance.  $R, R^2$  and regression line equation determined. The proposed single polynomial equation of  $n=4$  fits the experimental results with close agreement.

Table 4.1: Comparison of stress from experiment, proposed model and other authors

| S/No         | Specimen               | Strain ratio | Desayi Model,<br>1964 | Saenz Model,<br>1964 | Expt.<br>2017 | Proposed equation,<br>2017 |
|--------------|------------------------|--------------|-----------------------|----------------------|---------------|----------------------------|
| Stress (MPa) |                        |              |                       |                      |               |                            |
| 1            | <b>C30 RC</b>          | 0.633        | 64.12                 | 82.90                | 82.20         | 82.85                      |
| 2            | <b>C25 RC</b>          | 0.580        | 50.68                 | 68.24                | 67.80         | 66.65                      |
| 3            | <b>C20 RC</b>          | 0.570        | 47.50                 | 64.53                | 64.50         | 65.13                      |
| 4            | <b>C 30</b>            | 0.639        | 28.60                 | 32.50                | 32.50         | 31.99                      |
| 5            | <b>C25</b>             | 0.630        | 27.50                 | 32.10                | 31.50         | 31.99                      |
| 6            | <b>C20</b>             | 0.560        | 20.60                 | 25.25                | 25.30         | 25.32                      |
| 7            | <b>3PVC below N.A</b>  | 0.590        | 23.20                 | 27.20                | 27.00         | 27.40                      |
| 8            | <b>4 PVC @ corners</b> | 0.520        | 17.23                 | 21.50                | 21.60         | 21.50                      |
| 9            | <b>4 PVC @ middle</b>  | 0.490        | 14.70                 | 19.30                | 19.20         | 19.00                      |

The Saenz stress-strain equation have close agreement with experimental and the proposed equation while stress-strain model by Desayi deviate by an average of 22% (accuracy increases as  $\frac{\varepsilon_o}{\varepsilon_u}$  tends to unity) from experimental results. This is shown in Figures 4.14-4.16. The proposed single polynomial equations are;

For RC cubes,

$$\sigma = -1.80\varepsilon^4 + 2.14\varepsilon^3 + 42.61\varepsilon^2 - 64.29\varepsilon \quad (4.1)$$

$$R = 0.9844, R^2 = 0.9690$$

For Control cubes,

$$\sigma = 6.10\varepsilon^4 - 59.90\varepsilon^3 + 183.57\varepsilon^2 - 162.82\varepsilon \quad (4.2).$$

$$R = 0.9952, R^2 = 0.9905$$

For PVC cubes,

$$\sigma = 3.13\varepsilon^4 - 32.15\varepsilon^3 + 100.45\varepsilon^2 - 84.79\varepsilon \quad (4.3)$$

$$R = 0.9906, R^2 = 0.9814$$

The R- values close to unity is an indication of best fit.

#### **4.1.3.1 Stress-Strain Verification for reinforced concrete cubes**

For reinforced concrete cubes, young's modulus of elasticity, yield strain, ultimate strain and stress were measured from experimental data using equations 3.1-3.6.

The compressive strength results for all grades of concrete (C20-C30) indicates higher strength due to the presence of reinforcement. The average strength varies from 64.5 - 82.2MPa and other parameters are within the following ranges: yield strain,  $\varepsilon_o = (2.48 - 2.64) \times 10^{-3}$ , ultimate

strain,  $\varepsilon_u = (2.61 - 2.75) \times 10^{-3}$ , moduli of elasticity  $E = 35 - 41 GPa$ . In all cases, the compressive strength increases with the age of concrete with equation of best fit (See equation 4.1). The stress-strain relationship for the different grades of cubes was predicted using a single polynomial equation of degree four (best fit) as shown in Figure 4.13.

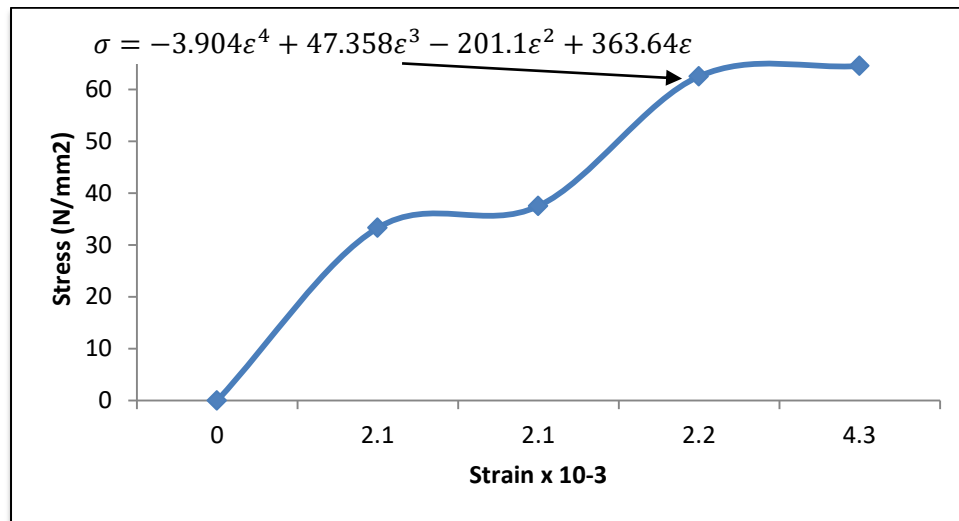


Figure 4.9: Experimental results of stress-strain relationship with polynomial equation (n=4) for Reinforced Concrete cubes (Y10 grade C20)

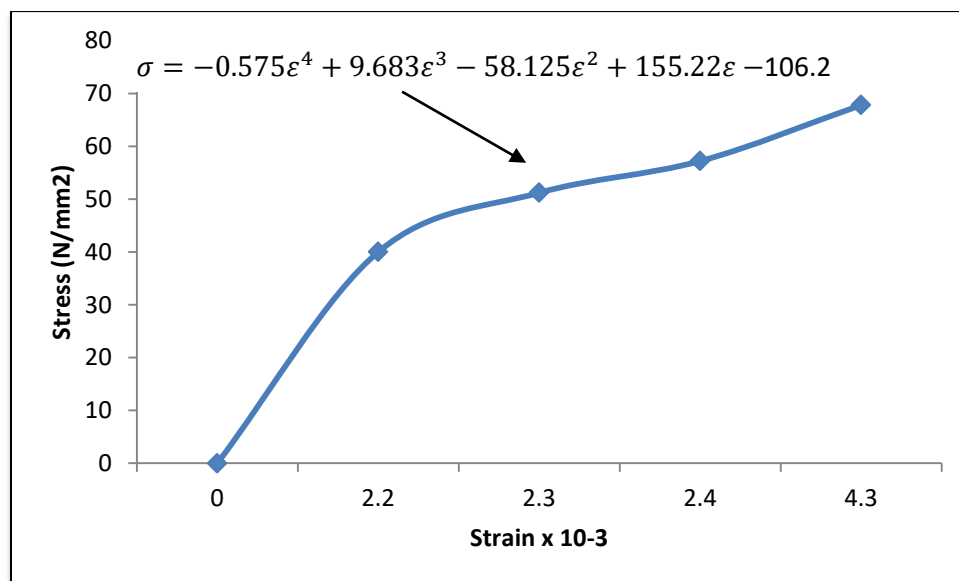


Figure 4.10: Experimental results of stress-strain relationship with polynomial equation (n=4) for Reinforced Concrete cubes (Y10 grade C25)

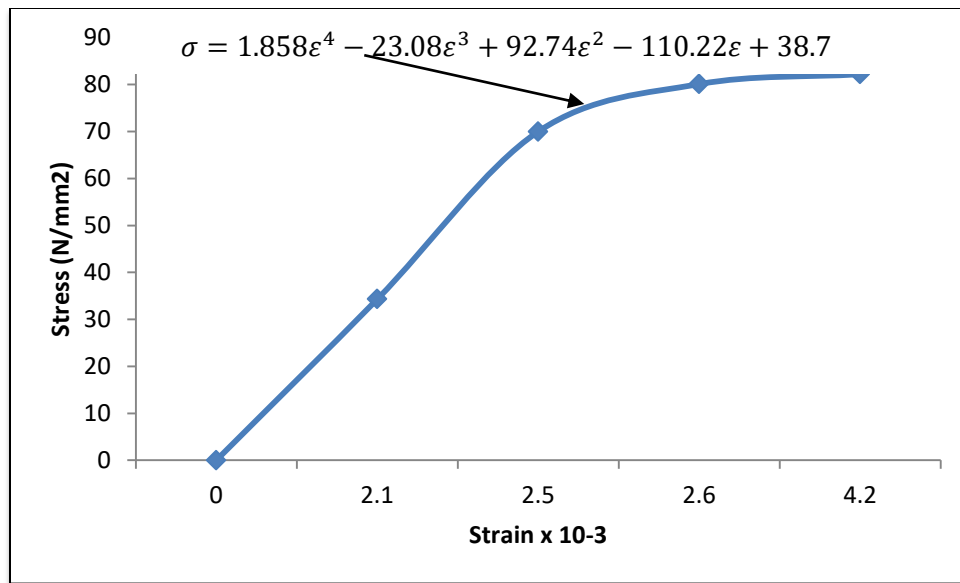


Figure 4.11: Experimental results of stress-strain relationship with polynomial equation (n=4) for Reinforced Concrete cubes (Y10 grade C30)

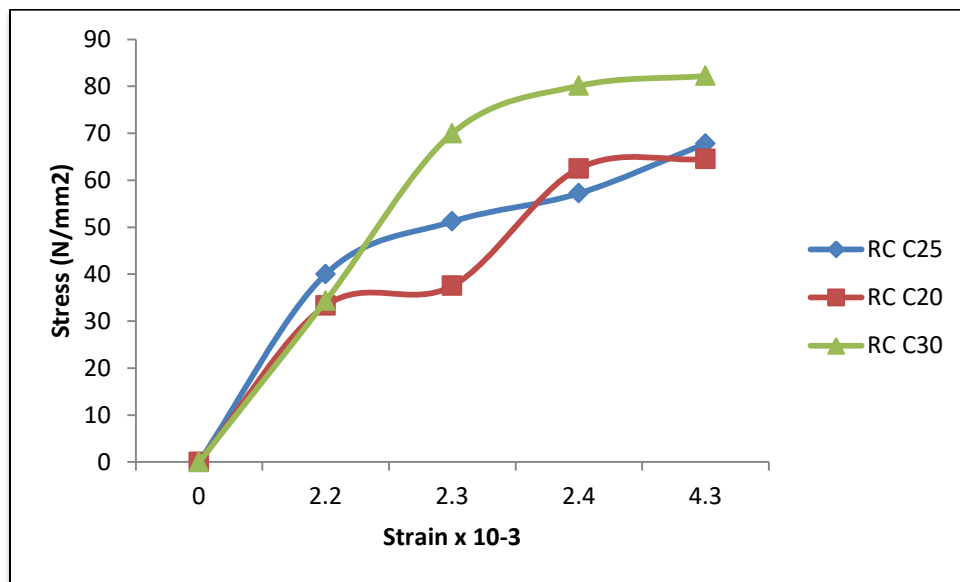


Figure 4.12: Stress-strain relationship for Reinforced cube for different concrete grade

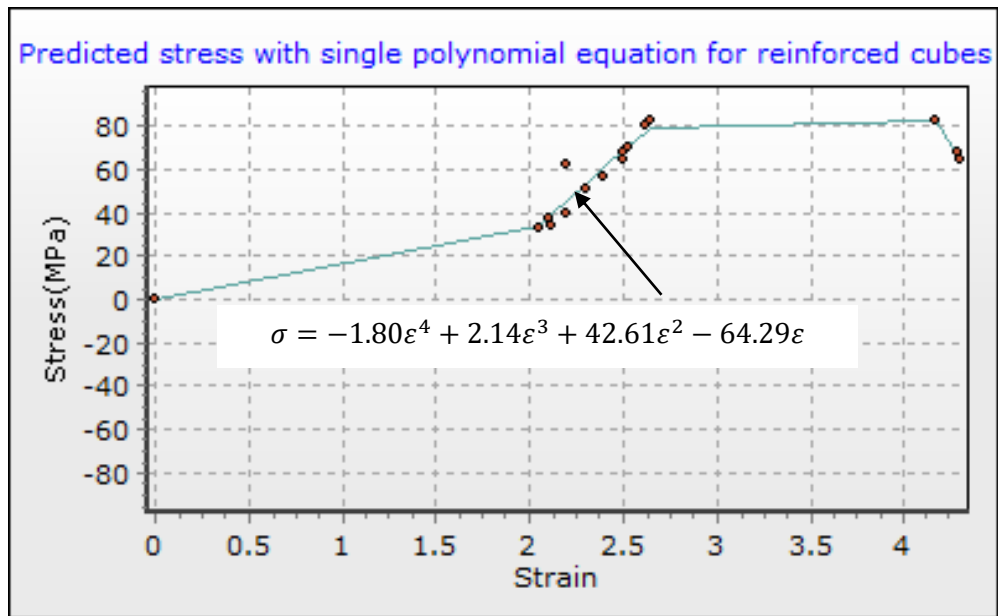


Figure 4.13: Predicted stress-strain relationship in a single polynomial equation (n=4) for RC concrete cubes.

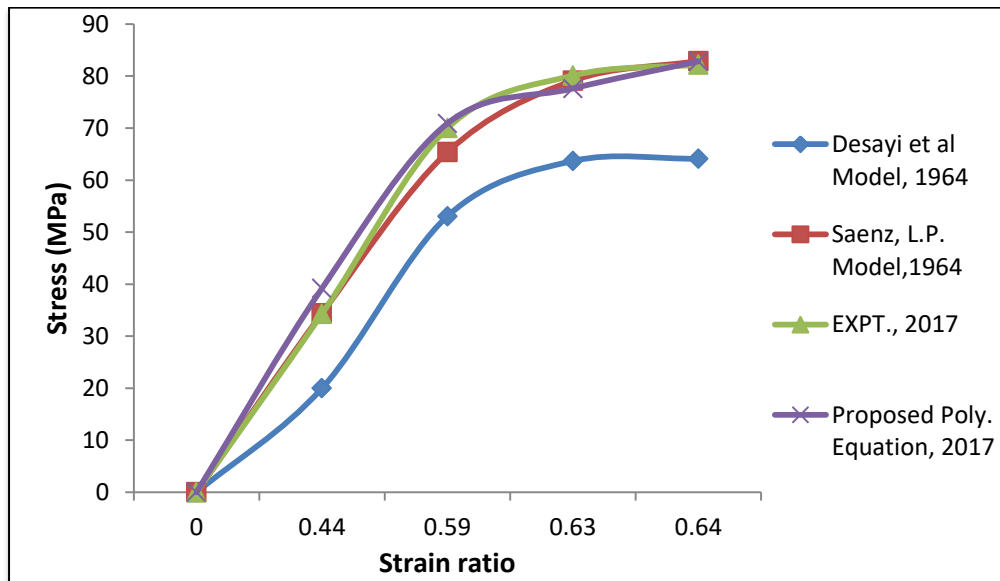


Figure 4.14: Stress models against strain ratio for RC cubes (C30)

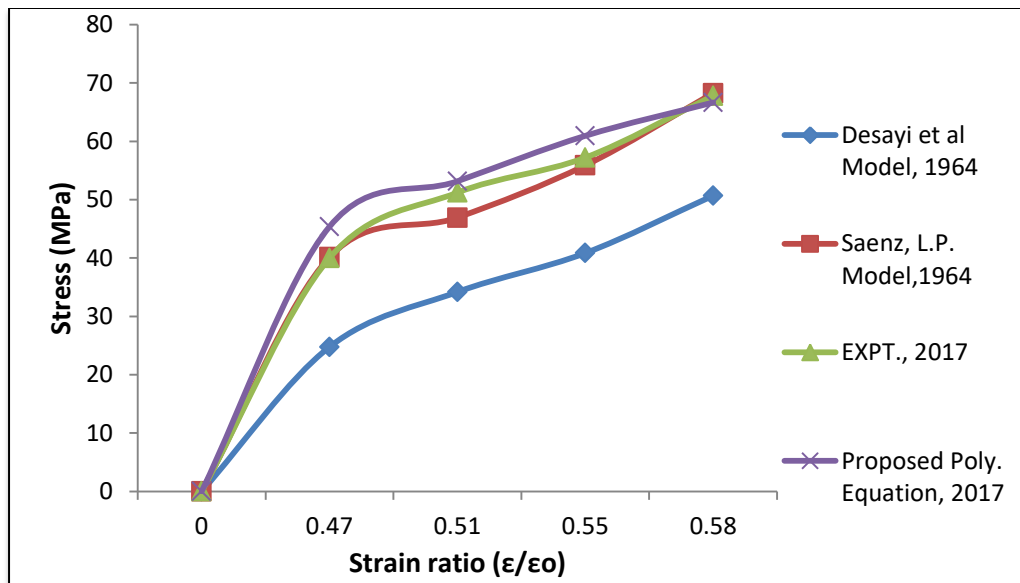


Figure 4.15: Graph showing different models of stress against strain ratio for RC cubes (C25)

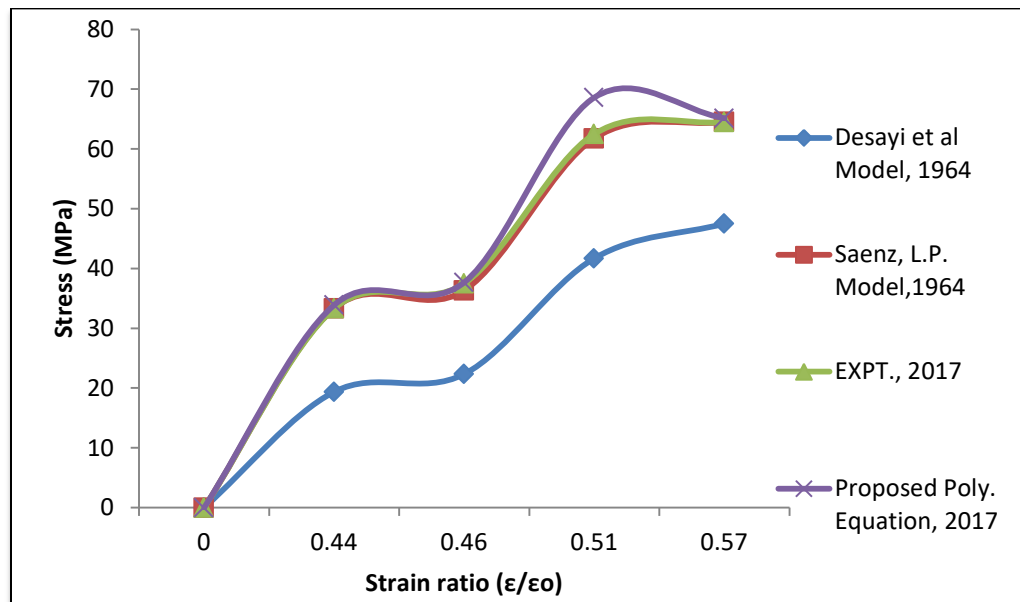


Figure 4.16: Graph showing different models of stress against strain ratio for RC cubes (C20)

#### 4.1.3.2 Stress-Strain Verification for Control Concrete Cubes

For stress-strain verification of control concrete cubes, young's modulus of elasticity, yield strain, ultimate strain and stress were measured from experimental data as explained earlier.

The yield strain, maximum strain, moduli of elasticity and compressive strength are  $\varepsilon_o = 2.07 \times 10^{-3}$ ,  $\varepsilon_u = 3.29 \times 10^{-3}$ ,  $E = 26.05 GPa$ ,  $C.S = 31.5 MPa$  for grade C25,  $\varepsilon_o = 1.96 \times 10^{-3}$ ,  $\varepsilon_u = 3.48 \times 10^{-3}$ ,  $E = 23.45 GPa$ ,  $C.S = 25.3 MPa$  for grade C20 and  $\varepsilon_o = 2.09 \times 10^{-3}$ ,  $\varepsilon_u = 3.27 \times 10^{-3}$ ,  $E = 27.4 GPa$ ,  $C.S = 32.5 MPa$  for grade C30. The summary of the results can be found in appendix BVII-BIX. The stress-strain behavior for the grade specimen (i.e C20-C30) is shown in Figures 4.17-4.19. The regression line equation in a single polynomial of best fit is indicated by equation 4.2 and its graphical representation in Figure 4.20. The validation using Desayi and saenz model are shown in Figures 4.21-4.23. The figures reveal deviation from Desayi's model while Saenz model have good fit with the experimental and proposed polynomial equation.

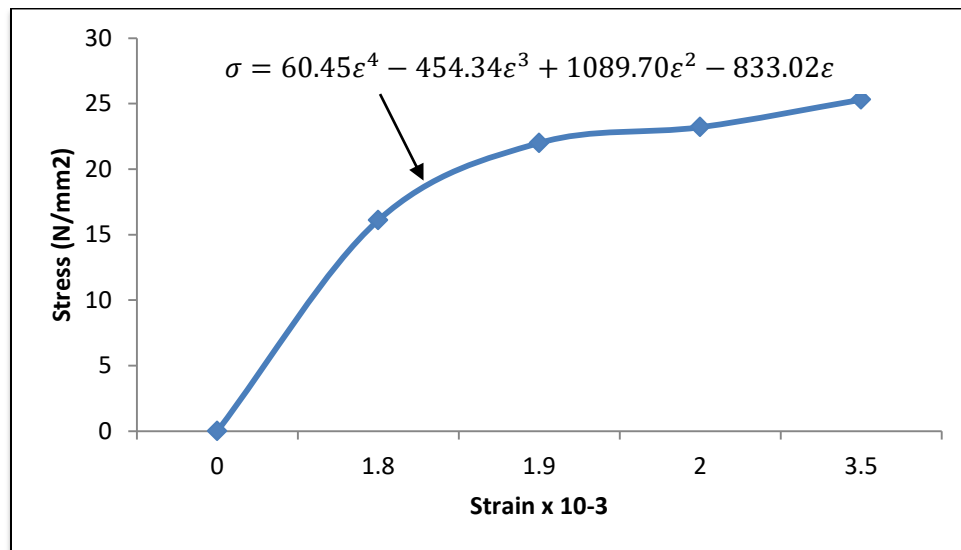


Figure 4.17: Experimental results of stress-strain relationship with polynomial equation (n=4) for Control concrete cubes (grade C20)

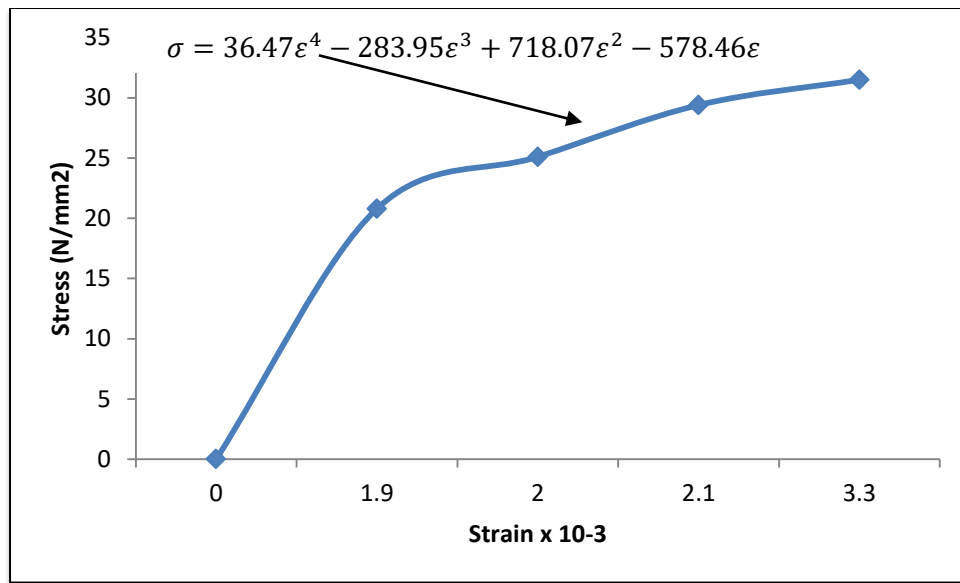


Figure 4.18: Experimental results of stress-strain relationship with polynomial equation (n=4) for Control concrete cubes (grade C25)

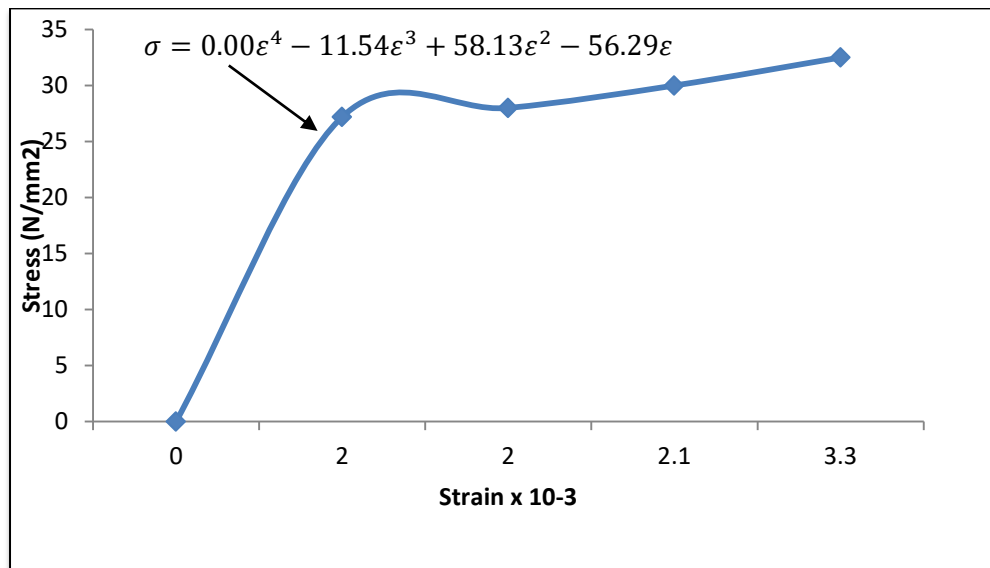


Figure 4.19: Experimental results of stress-strain relationship with polynomial equation (n=4) for Control concrete cubes (grade C30)

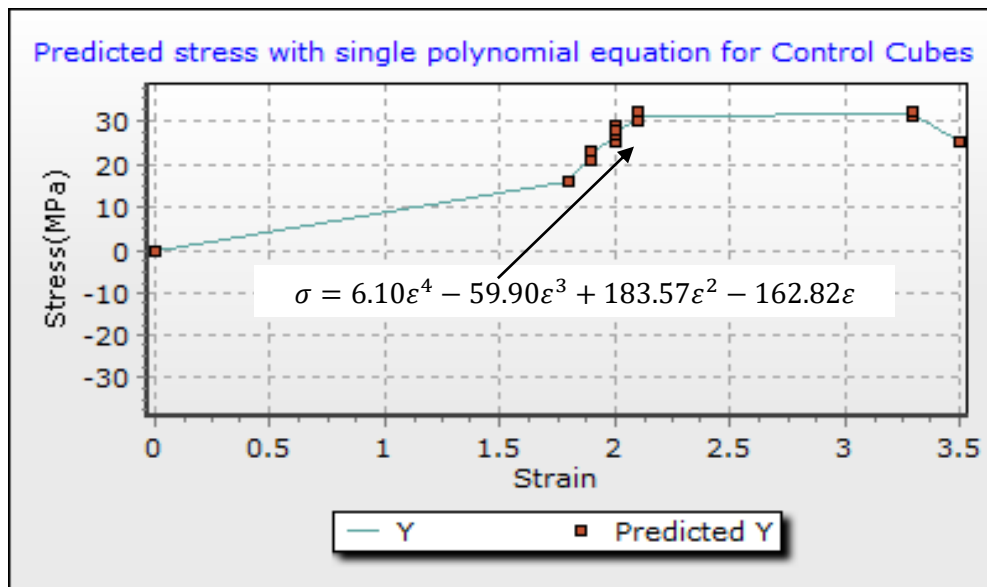


Figure 4.20: Experimental results of stress-strain relationship in a single polynomial equation (n=4) for Control concrete cubes.

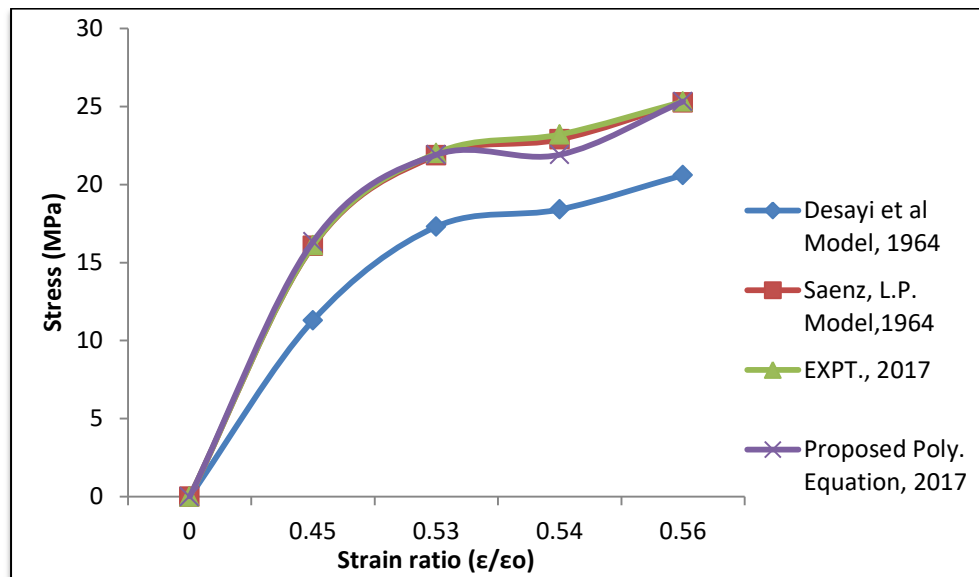


Figure 4.21: Different models of stress against strain ratio for control cubes (C20)

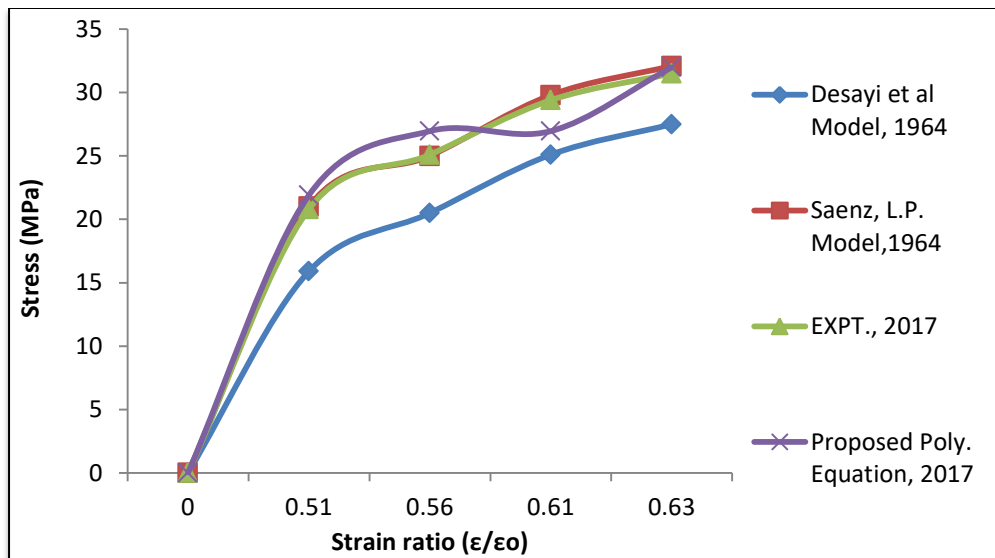


Figure 4.22: Different models of stress against strain ratio for control cubes (C25)

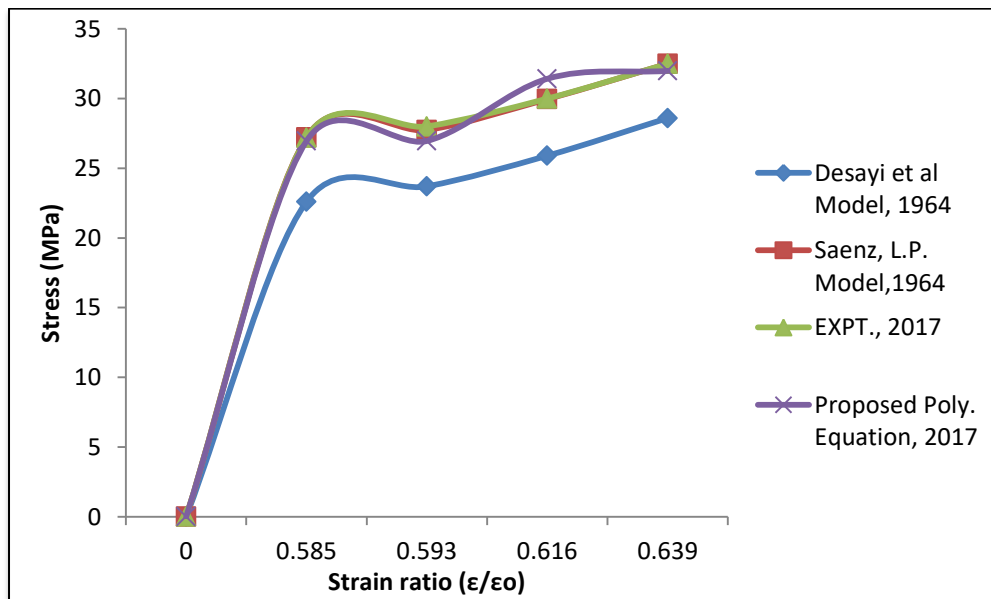


Figure 4.23: Different models of stress against strain ratio for control cubes (C20)

#### 4.1.3.3 Stress-Strain Verification for Concrete Cube with PVC Pipes

For stress-strain verification of concrete cubes (grade C25) with PVC pipes, young's modulus of elasticity, yield strain, ultimate strain and stress were measured from experimental data. The yield strain, maximum strain, moduli of elasticity and compressive strength are  $\varepsilon_o = 1.83 \times 10^{-3}$ ,  $\varepsilon_u = 3.73 \times 10^{-3}$ ,  $E = 19.7 GPa$ ,  $C.S = 19.2 MPa$  and  $\varepsilon_o = 1.89 \times 10^{-3}$ ,  $\varepsilon_u = 3.61 \times 10^{-3}$ ,  $E = 20.69 GPa$ ,  $C.S = 21.6 MPa$  respectively (See Tables in appendix A and B.). These results are in agreement with specification by AASHTO LRFD, 1994 and BS code.

The average strain at peak compression and modulus of elasticity are approximately  $\varepsilon_o = 2.00 \times 10^{-3}$ ,  $\varepsilon_u = (3.0 - 3.5) \times 10^{-3}$ ,  $E = 19 - 31 GPa$  respectively. The stress-strain behavior for the PVC concrete cubes is shown in Figures 4.24-4.26. The regression line equation in a single polynomial of best fit is indicated by equation 4.3 and its graphical representation in Figure 4.27. The validation of the stress-strain behavior using Desayi and saenz model is shown in Figures 4.28-4.30. These figures reveal deviation from Desayi's model while Saenz model have good fit with the experimental and proposed polynomial equations.

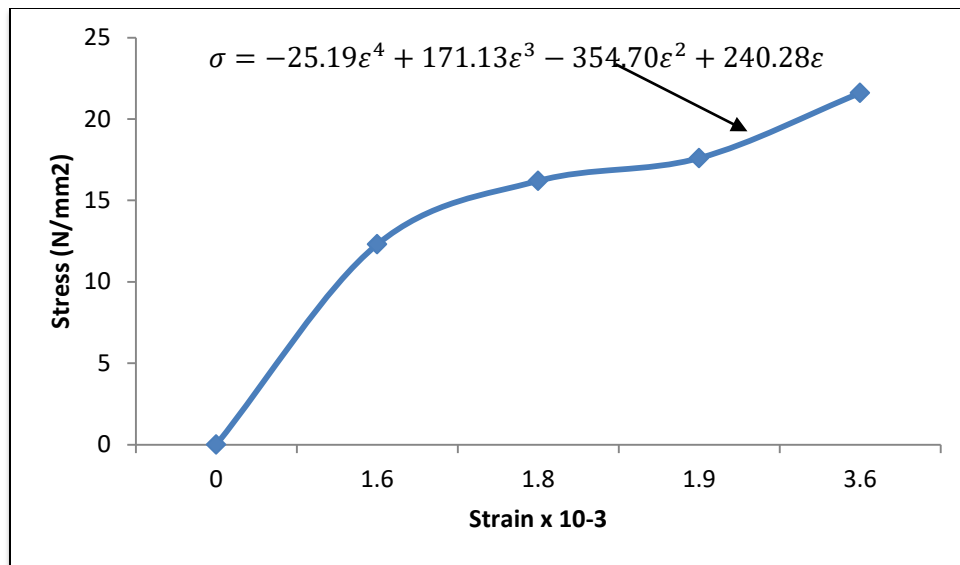


Figure 4.24: Experimental results of stress-strain relationship with polynomial equation (n=4) for 4PVC pipes@ edges cubes

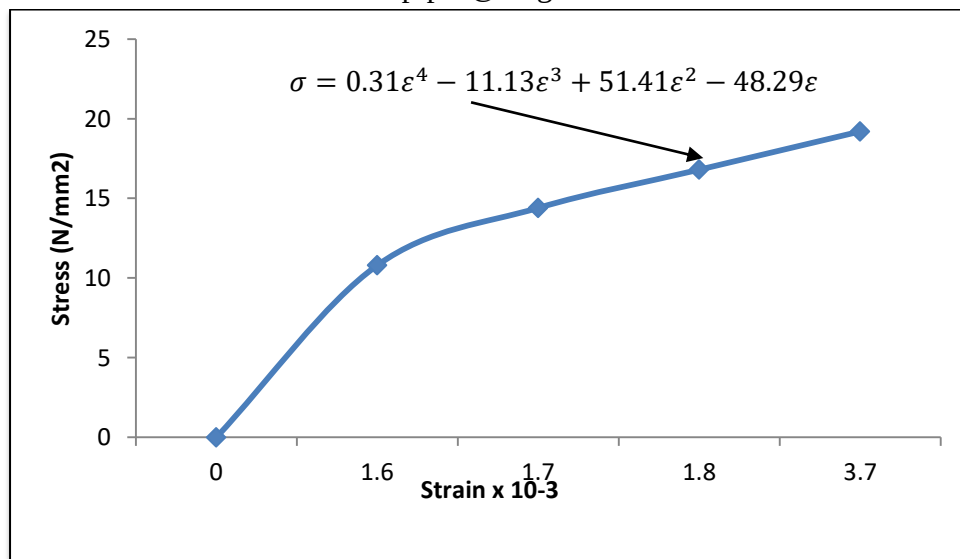


Figure 4.25: Experimental results of stress-strain relationship with polynomial equation (n=4) for concrete cubes with 4 PVC at middle

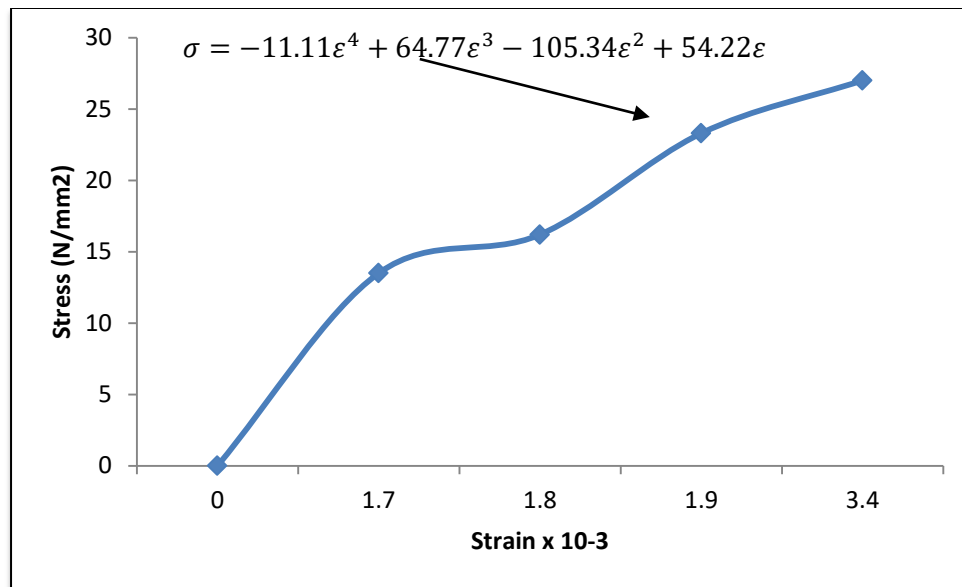


Figure 4.26: Experimental results of stress-strain relationship with polynomial equation (n=4) for with 3PVC concrete cubes

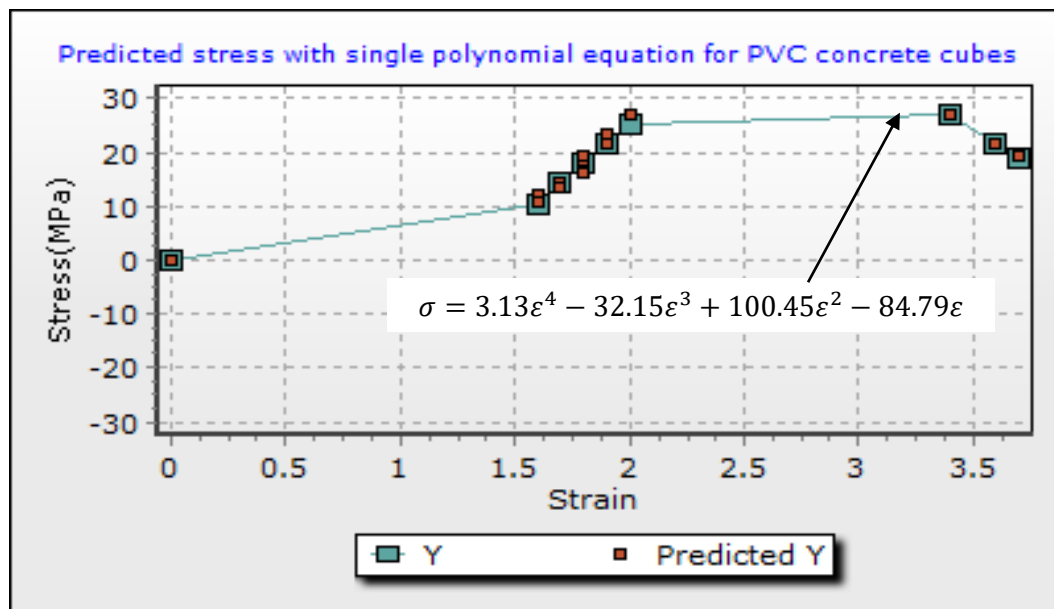


Figure 4.27: Experimental results of stress-strain relationship with polynomial equation (n=4) for concrete cubes

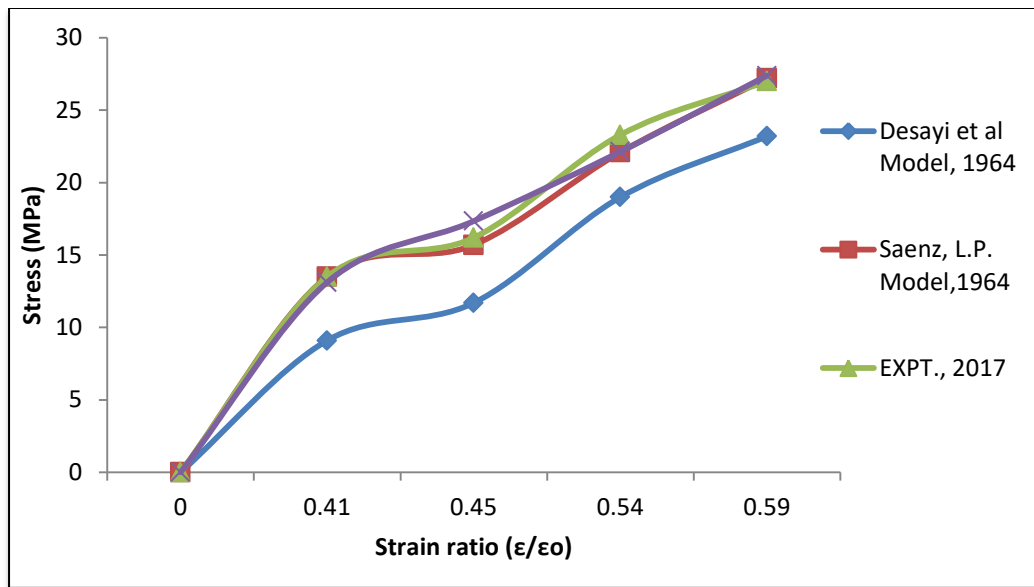


Figure 4.28: Different models of stress against strain ratio for cubes with 3 PVC below N.A

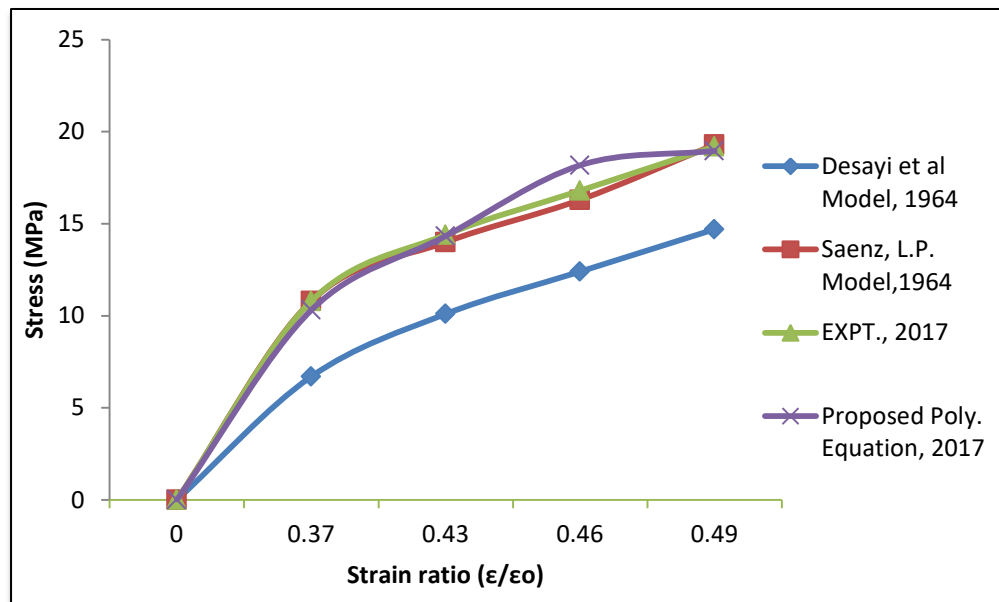


Figure 4.29: Different models of stress against strain ratio for cubes with 4 PVC @ middle

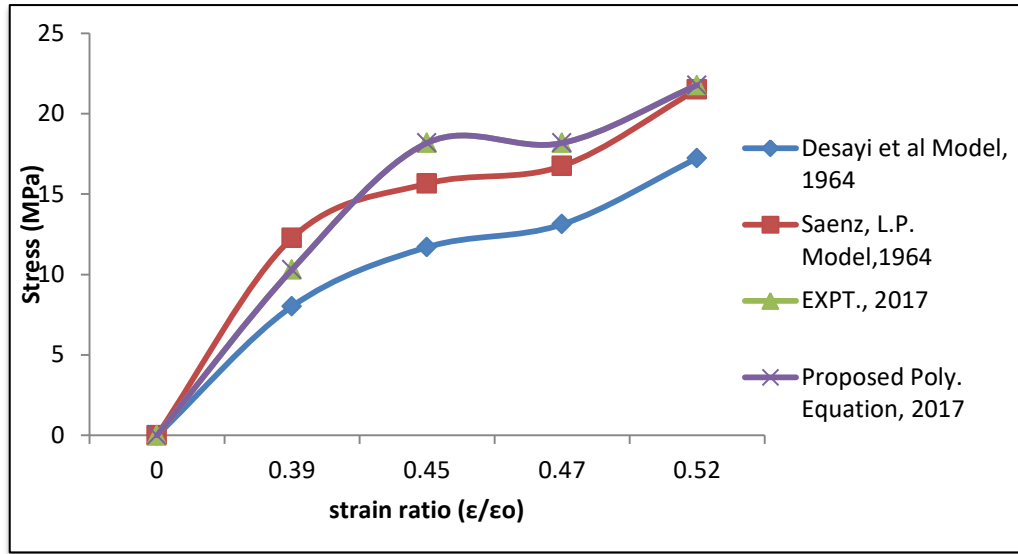


Figure 4.30: Different models of stress against strain ratio for cubes with 4 PVC@ edges

## 4.2 Experimental results on flexural strength with and without PVC pipes

### 4.2.1 Flexural strength of beams with PVC pipes

The results for the modulus of rupture, deflection and strain measured by experiment for reinforced concrete with PVC pipes are presented in the table 1.

The modulus of rupture (MoR) of reinforced concrete beams with one embedded PVC pipes (RCPVC1) indicates maximum deflection at the centre of magnitude  $\delta = 9\text{mm}$  (Figure 4.32) and modulus of rupture of  $f_{mr} = 6.7 \text{ MPa}$  and maximum stress of 62.3 MPa. The strain reading ranges between 0.0044-0.010 (See Figure 4.33) and maximum failure load of 25kN. The flexural modulus is 10.8% of the maximum stress which is within the range (20-30% of maximum stress) specified by National Ready Mixed Concrete Association [NRMCA]. The failure pattern observed was mainly bending with visible cracks at support and at loading point.

The flexural modulus of rupture ( $MoR$ ) of RCPVC2 and RCPVC3 indicates failure pattern similar in nature to that of RCPVC1 but with lesser cracks at support and at loading point. The beams failed in both shear and bending. Maximum deflection at the centre are;  $\delta = 7 \text{ mm}$ , modulus of rupture of  $f_{mr} = 4.0 \text{ MPa}$  (indicating 10.44% of maximum stress) and maximum stress of 38.32 MPa, strain ranges 0.0067-0.0078, maximum failure load of 15kN and  $\delta = 6 \text{ mm}$ , modulus of rupture of  $f_{mr} = 4.0 \text{ MPa}$  and maximum stress of 38.32 MPa, strain ranges between 0.0033-0.0067 and maximum failure load of 15kN respectively. This is shown graphically in Figures 4.34-4.39.

Table 4.2: Experimental data measurement of different beam samples

| SAMPLE | $F_0$<br>(kN) | $F_u$<br>(kN) | $\delta_o$<br>(mm) | $\delta_u$<br>(mm) | a<br>(mm) | b<br>(mm) | $F_{mr}$<br>(N/mm <sup>2</sup> ) |
|--------|---------------|---------------|--------------------|--------------------|-----------|-----------|----------------------------------|
| RCBM   | 15.0          | 26.0          | 1.5                | 14.0               | 31.0      | 35.5      | 6.9                              |
| RCPVC1 | 9.0           | 25.0          | 3.0                | 9.0                | 39.5      | 39.5      | 6.7                              |
| RCPVC2 | 7.5           | 15.0          | 2.5                | 9.5                | 13.5      | 42.0      | 4.0                              |
| RCPVC3 | 6.5           | 15.0          | 1.7                | 6.0                | 19.5      | 40.0      | 4.0                              |
| CBPVC1 | 0.0           | 7.0           | 0.0                | 9.0                | 41.0      | 36.0      | 1.9                              |
| CBPVC2 | 0.0           | 7.0           | 0.0                | 9.0                | 41.0      | 36.0      | 1.9                              |
| CTBM   | 0.0           | 8.5           | 0.0                | 4.3                | 34.5      | 43.5      | 2.3                              |

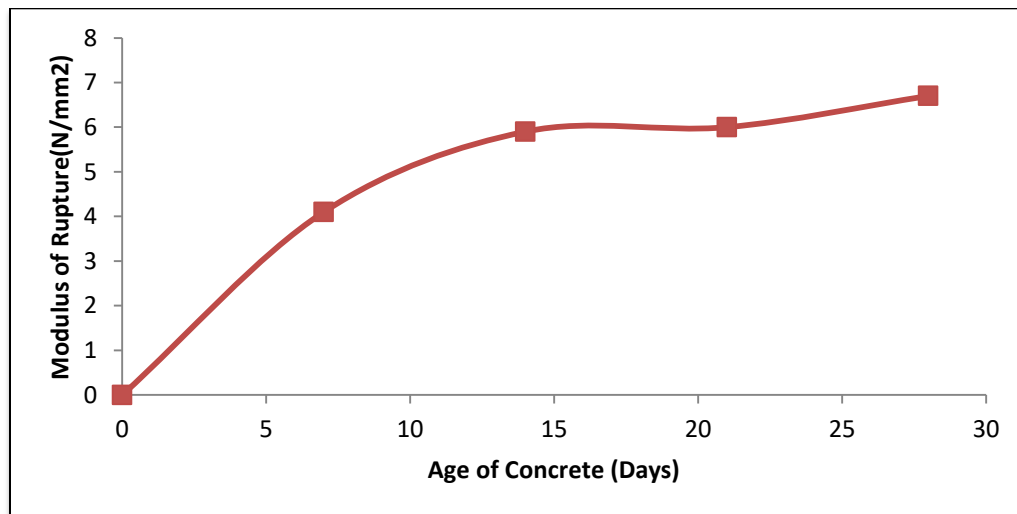


Figure 4.31: Modulus of rupture against age of concrete for RCPVC1-beam

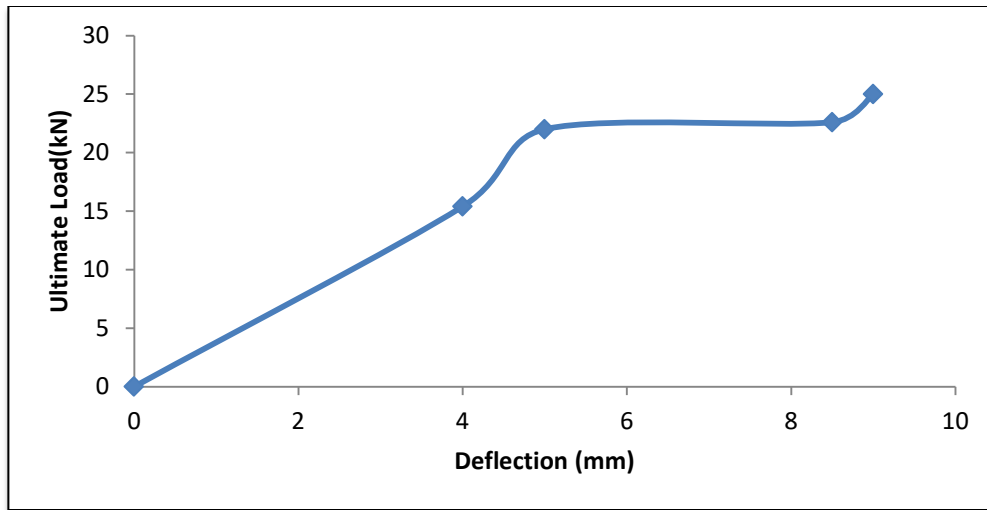


Figure 4.32: Ultimate load against deflection for RCPVC1-beam

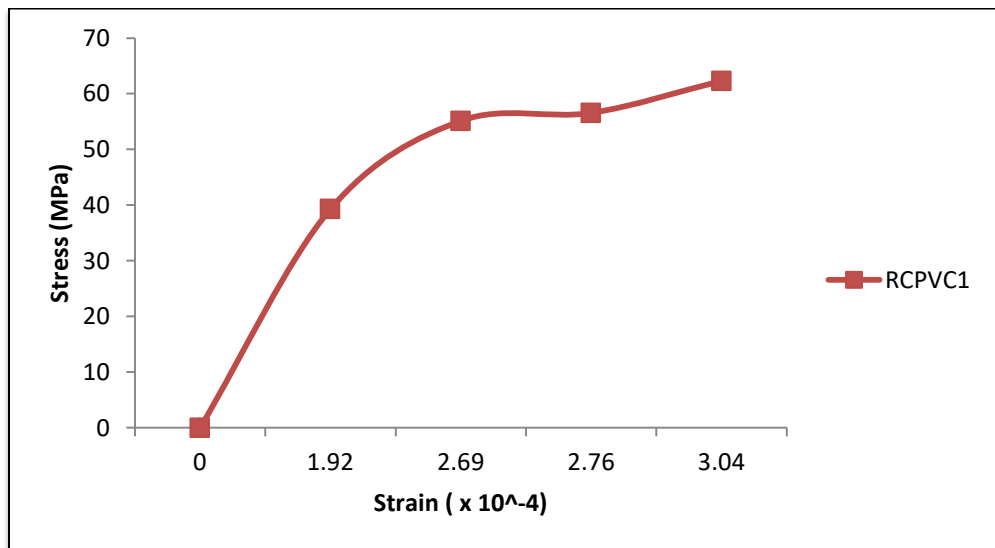


Figure 4.33: Stress- strain relationship for RCPVC1- beam

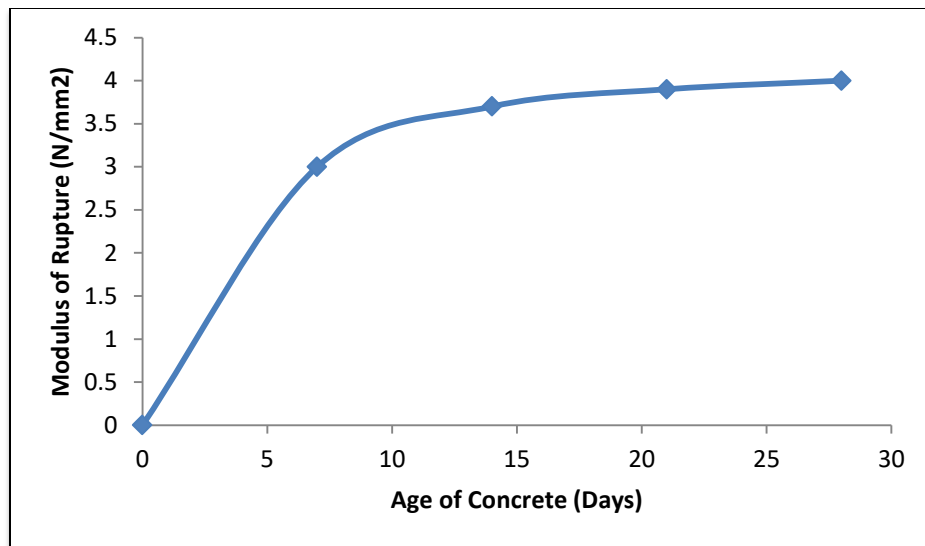


Figure 4.34: Modulus of rupture against age of concrete for RCPVC2- beam

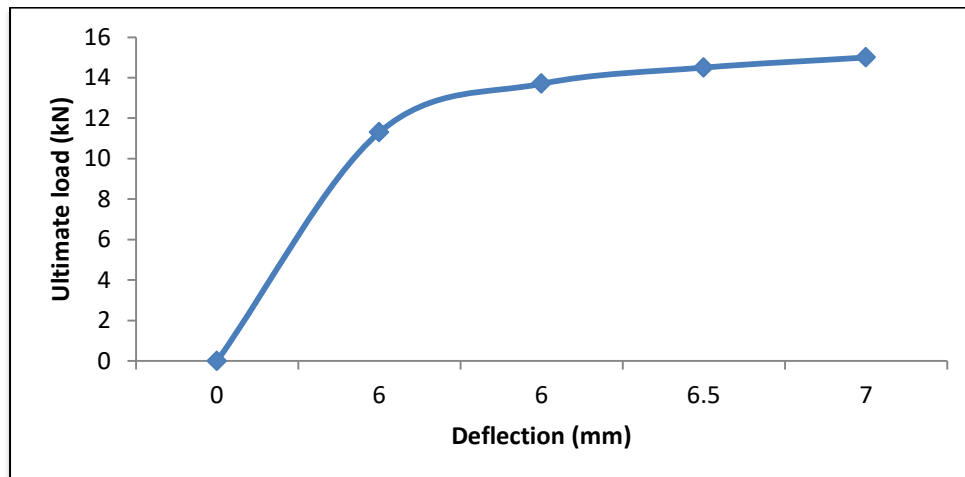


Figure 4.35: ultimate load against deflection for RCPVC2-beam

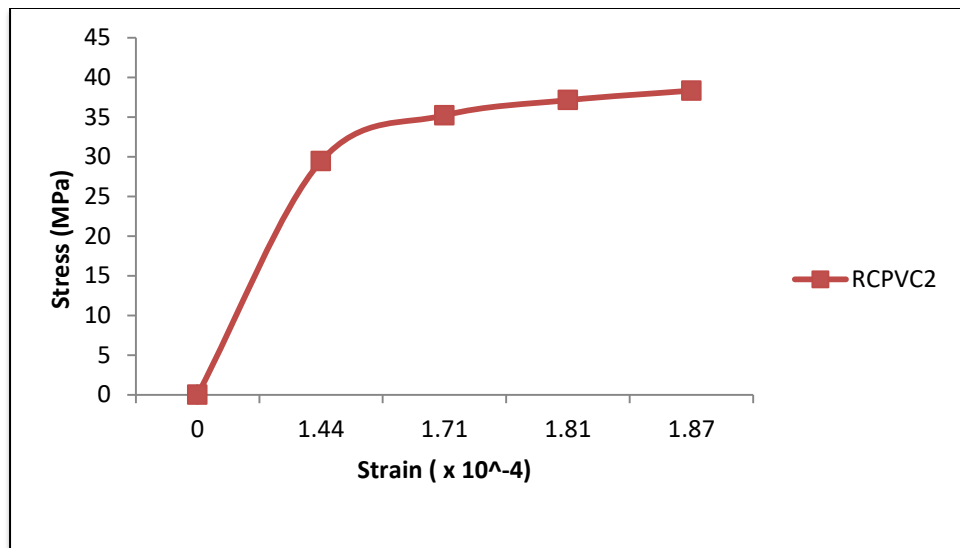


Figure 4.36: Stress- strain relationship for RCPVC2- beam

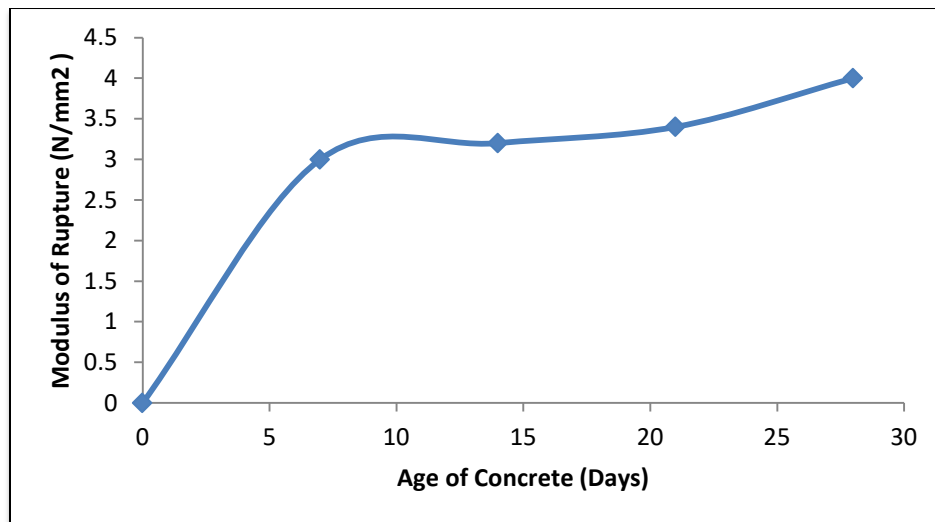


Figure 4.37: Modulus of rupture against age of concrete for RCPVC3- beam

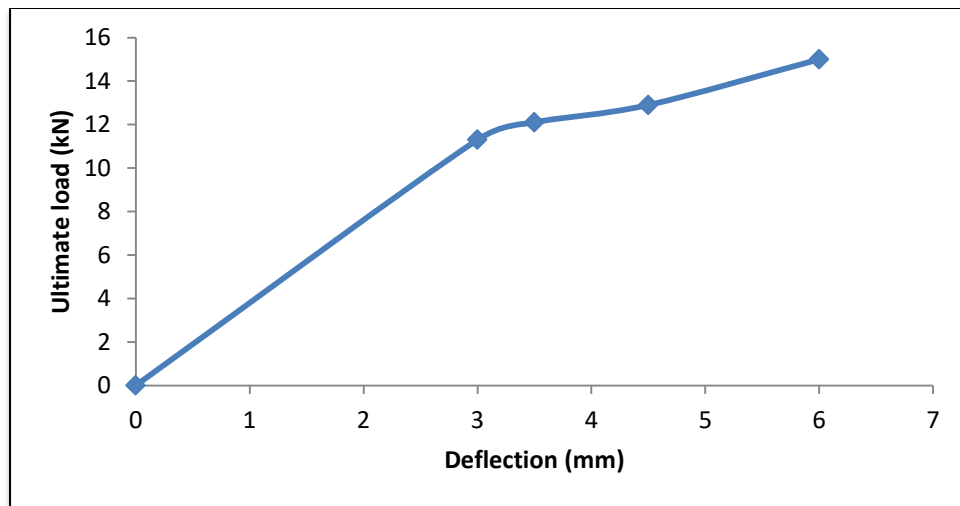


Figure 4.38: Ultimate load against deflection for RCPVC3-beam

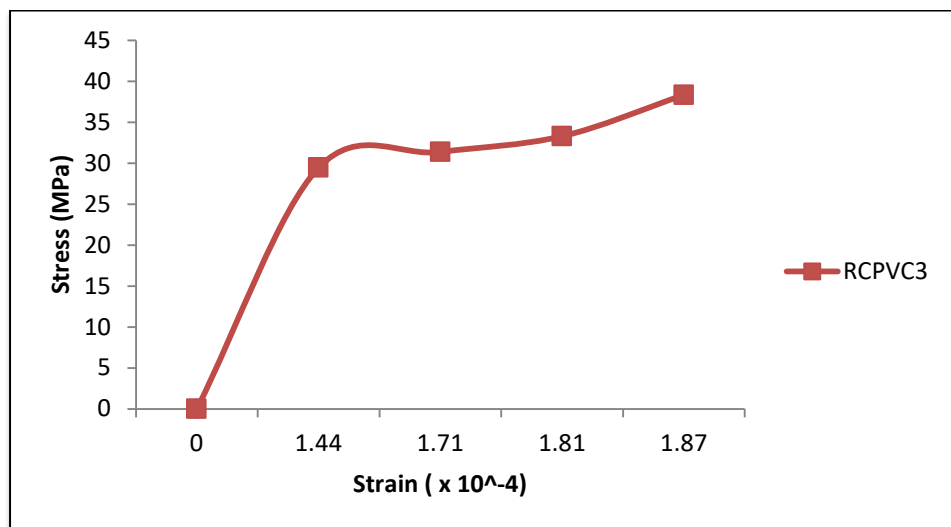


Figure 4.39: Stress- strain relationship for RCPVC3- beam

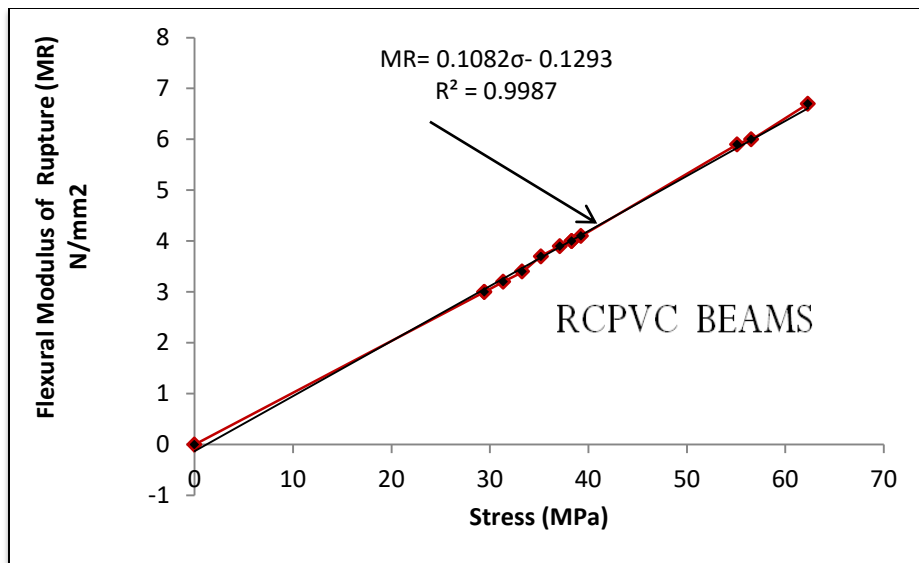


Figure 4.40: Stress- strain relationship for RCPVC beam

#### 4.2.2 Experimental results on flexural strength without PVC pipes

For reinforced concrete beams without PVC pipes (RCBM), results indicate maximum deflection of  $\delta = 14$  mm, modulus of rupture of  $f_{mr} = 6.9$  MPa (10.7% of maximum stress) and maximum stress of 64.71 MPa, strain ranges between 0.0067-0.0156 and maximum failure load of 26kN. Failure was mainly in bending with visible crack pattern at supports and at loading area.

All reinforced concrete beams with and without PVC pipes indicates failure mainly in bending with revealing cracks at supports and point of application of load. Figures 4.41-4.47 shows the variation of  $f_{mr}$  with age of concrete, ultimate with deflection and stress-strain curves. All graphs shows similar trends i.e modulus of rupture (MoR), ultimate load and stress increases steadily as the independent variables increases and up to the yield point. Thereafter, increment becomes almost constant before reaching the maximum point (See Figure 4.43). The stress-strain curve can also be compared with normal concrete curve of maximum compressive yield strain of 0.002. The modulus of rupture, ( $f_{mr}$ ) for all beams increases with age of concrete and values within specified range (10-20% of the maximum stress). Deflection and stress-strain curves for RCBM, RCPVC1, RCPVC2 and RCPVC3 similar pattern with maximum values of 14mm and 0.0156 recorded for RCBM. In each case, there is a close agreement in results obtained from RCBM and RCPVC1 compared with RCPVC2 and RCPVC3. The regression line equation in Figure 4.44 shows the relationship between modulus of rupture (MoR) and maximum stress in the beam specimen.

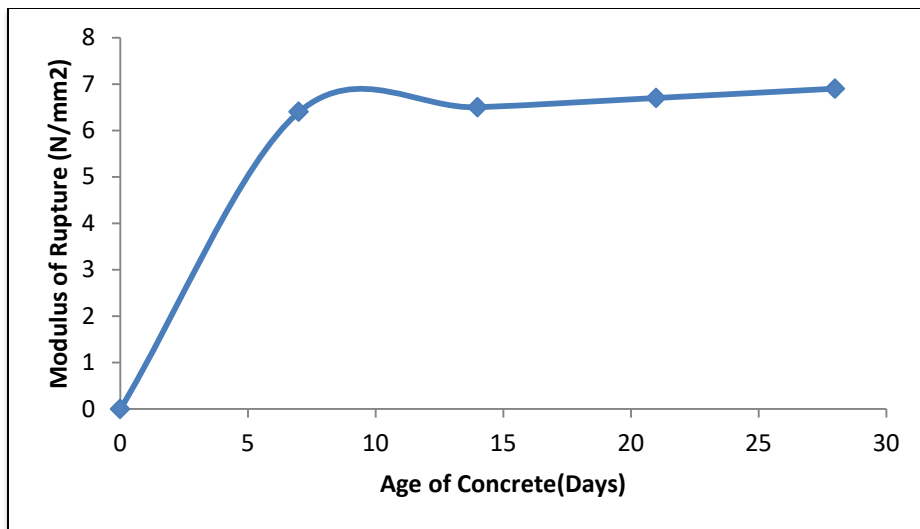


Figure 4.41: Modulus of rupture and concrete age for RCBM- beam

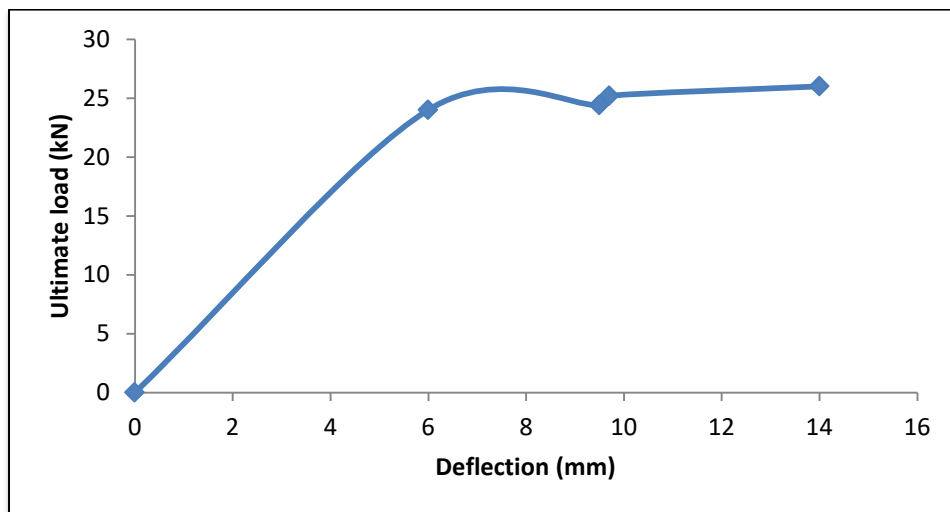


Figure 4.42: Ultimate load and deflection for RCBM- beam

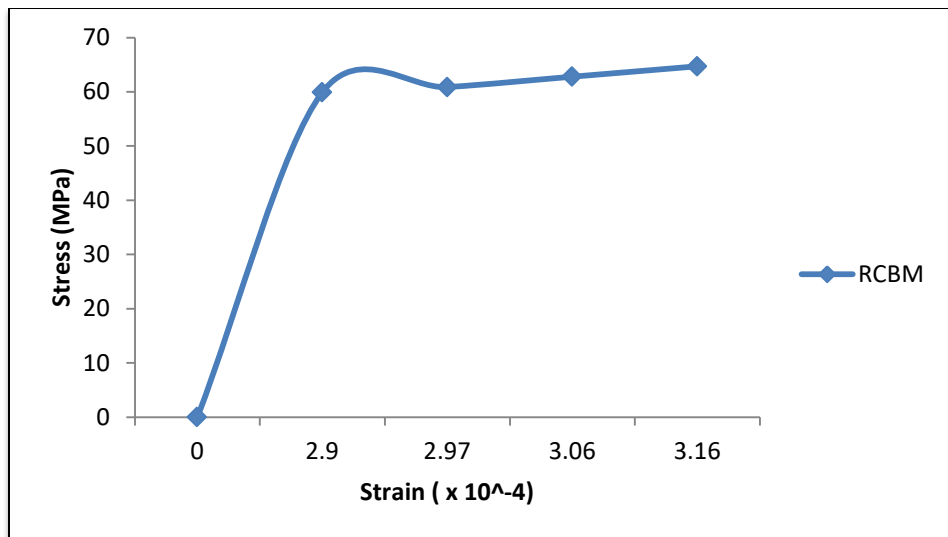


Figure 4.43: Stress-strain graph for RCBM

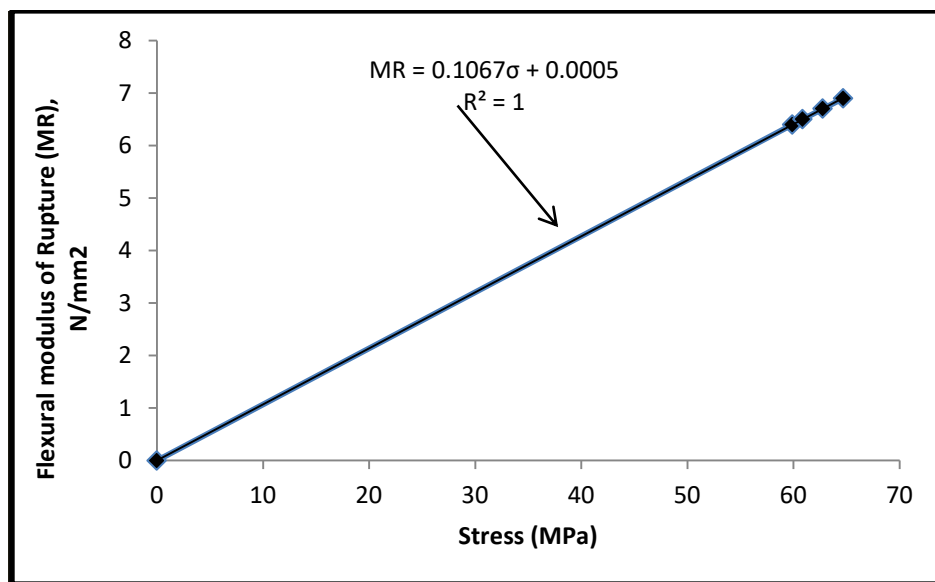


Figure 4.44: Flexural modulus against Stress for RCBM

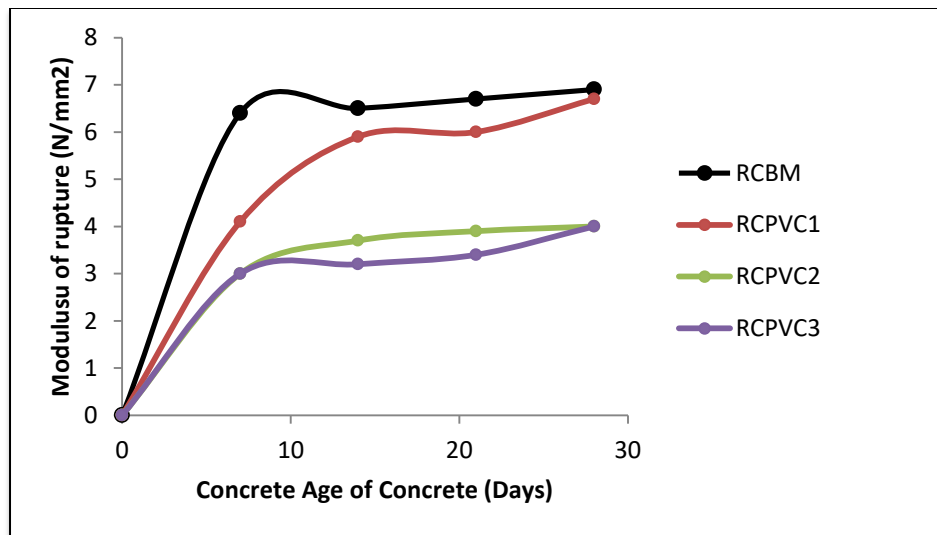


Figure 4.45: Modulus of rupture against concrete age for different beams

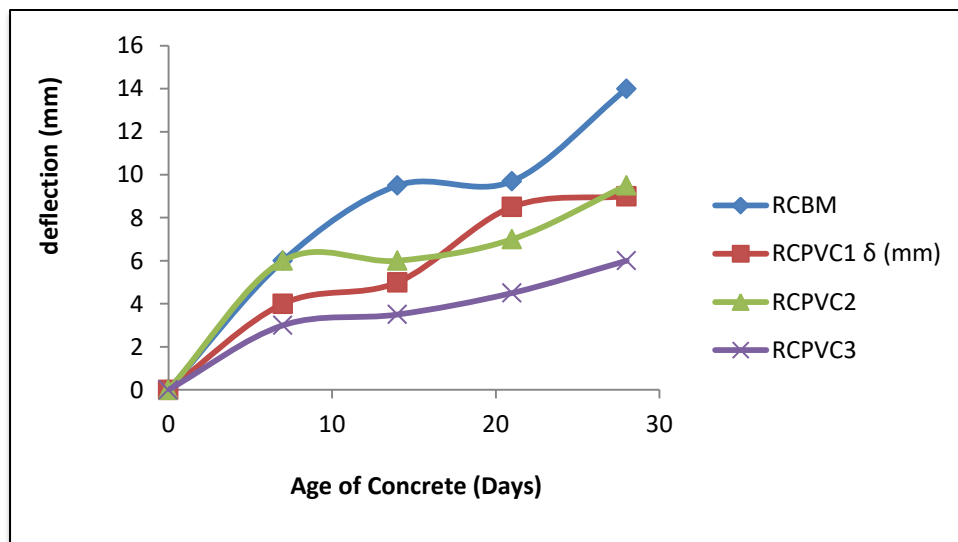


Figure 4.46: Deflection against concrete age for various beams

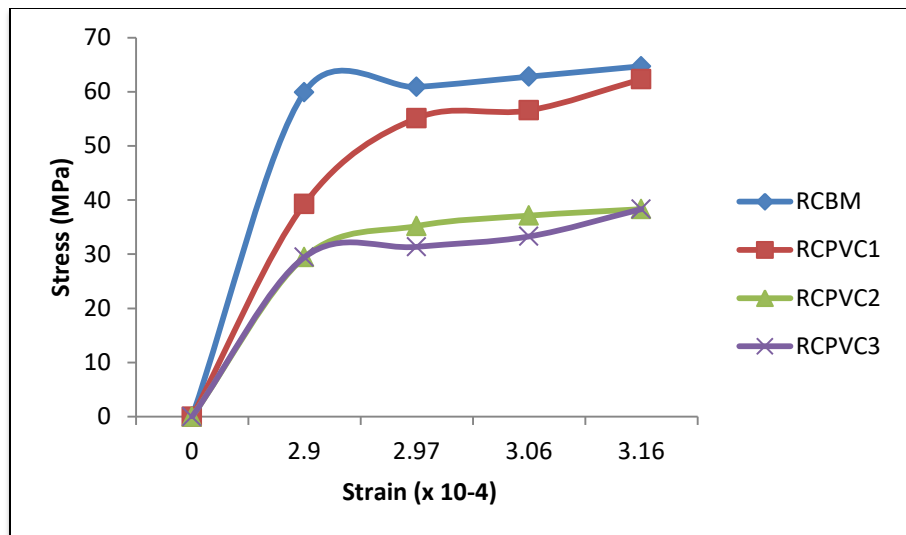


Figure 4.47: Strain against age of concrete for different beams

### 4.2.3 Flexural strength results for unreinforced concrete beams

For control beams with and without PVC pipes, results indicate similar trends with RC beams. The modulus of rupture ( MoR), strain and maximum stress are within specified range. All beams in this category failed mainly in shear upon loading. The failure was rapid for CTBM and slower for CBPVC2 due to the presence of pipes. Visible cracks were observed at the loading point and least at the supports. All curves are found to be in close agreement as shown in Figures 4.48-4.60.

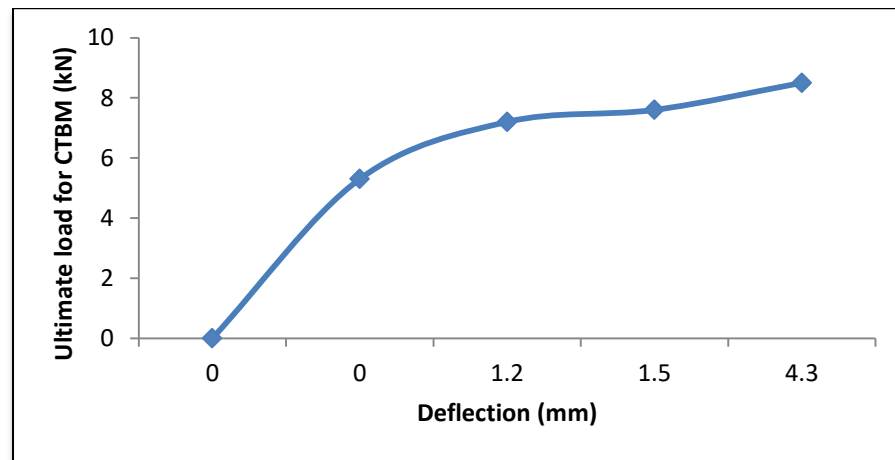


Figure 4.48: Ultimate load against deflection for CTBM

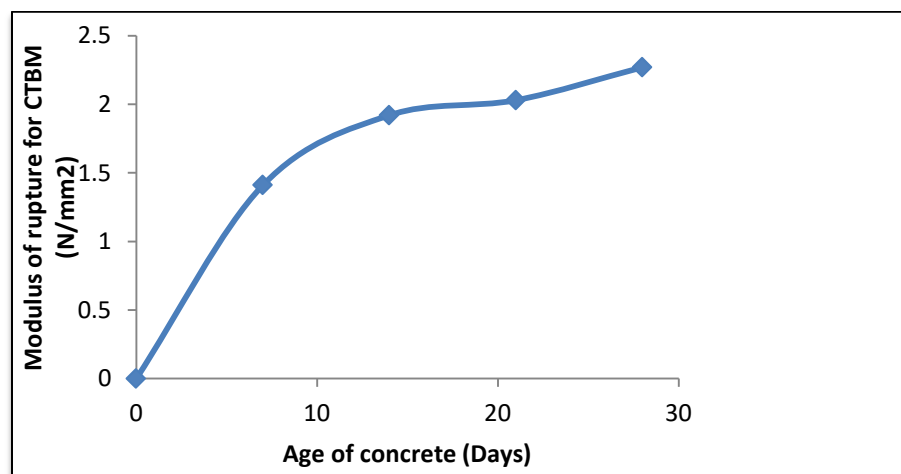


Figure 4.49: Modulus of rupture against age of concrete

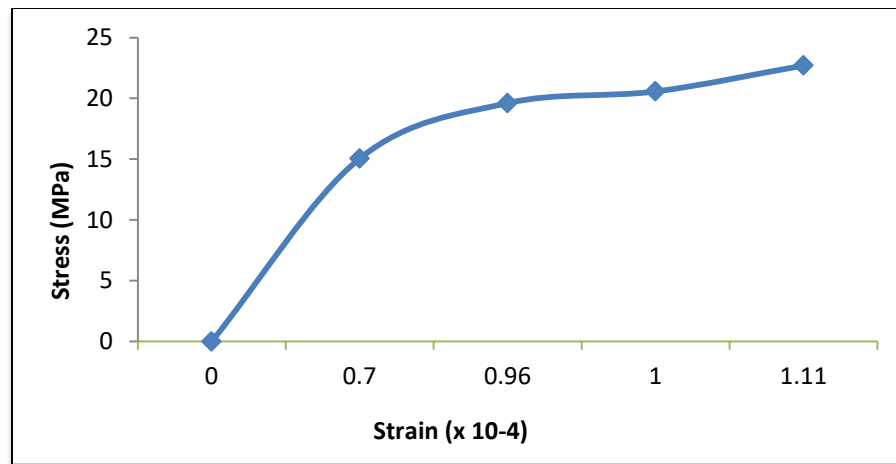


Figure 4.50: Graph of stress against strain for CTBM

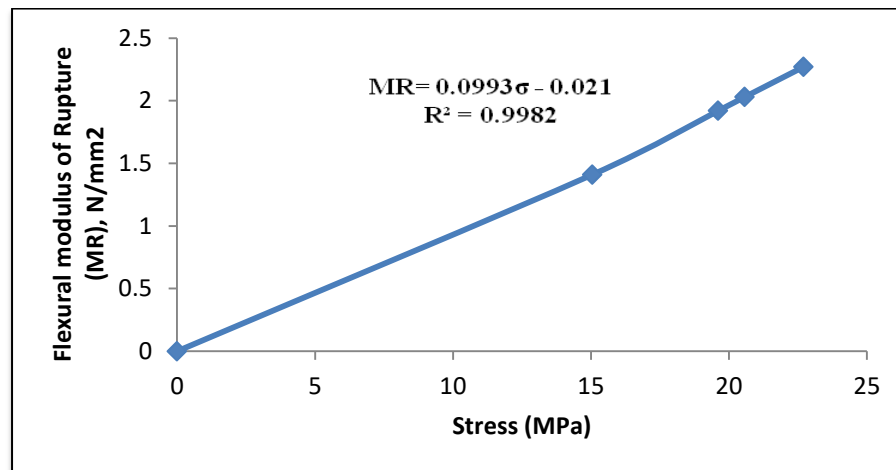


Figure 4.51: Flexural modulus against stress for unreinforced beam (CTBM)

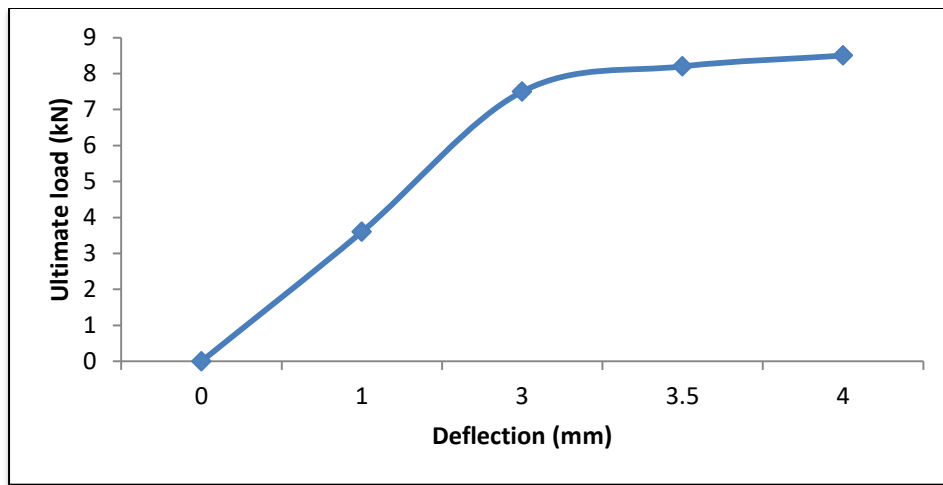


Figure 4.52: Ultimate load against deflection for CBPVC1

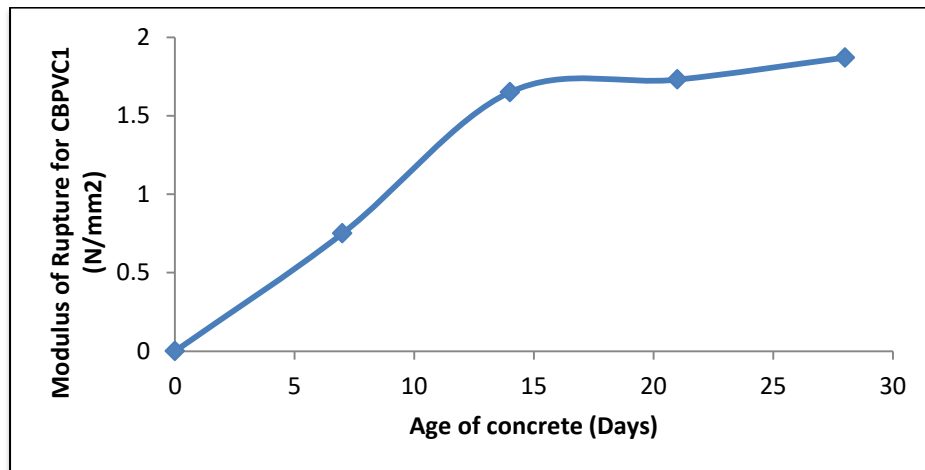


Figure 4.53: Modulus of rupture against age of concrete for CBPVC1

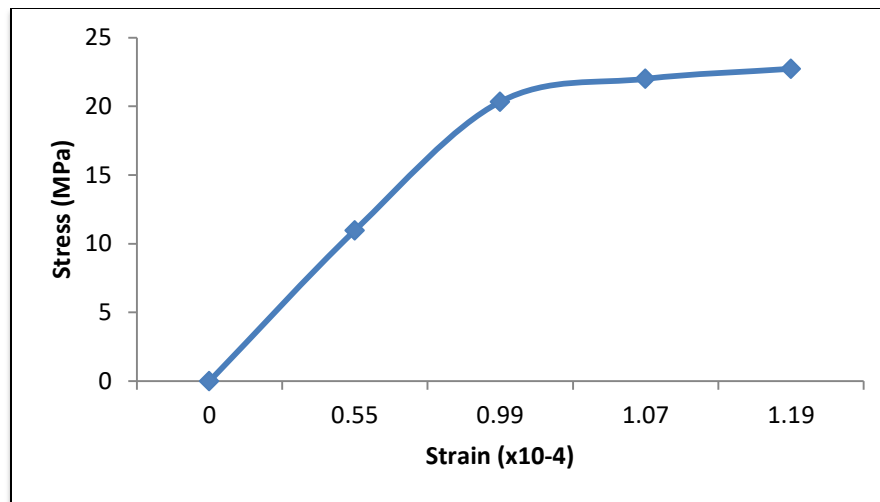


Figure 4.54: Stress against strain for CBPVC1

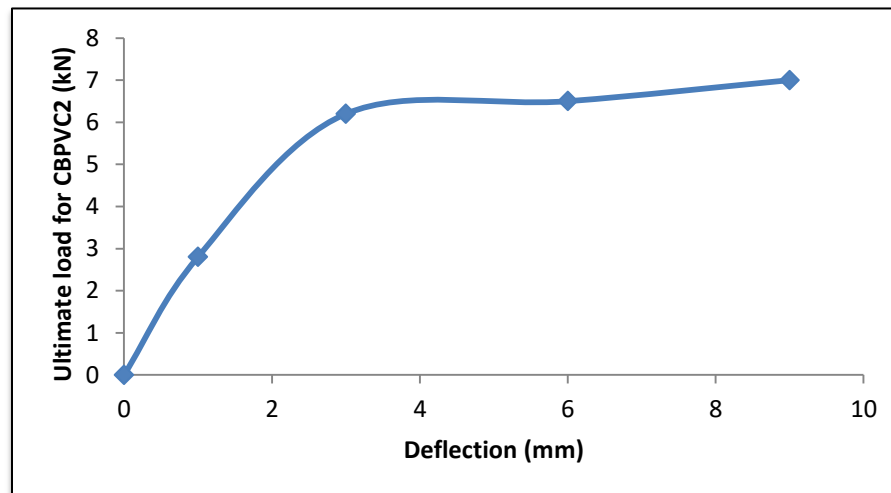


Figure 4.55: Ultimate load against deflection for CBPVC2

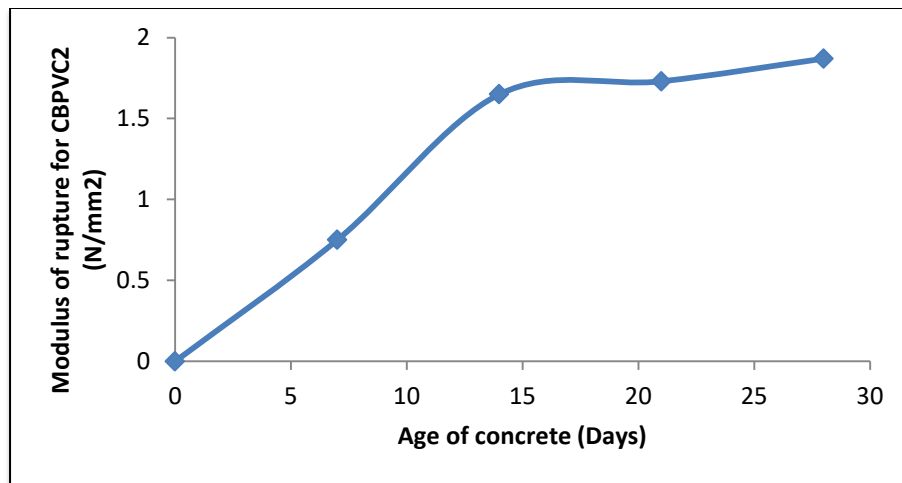


Figure 4.56: Modulus of rupture load against age of concrete for CBPVC2

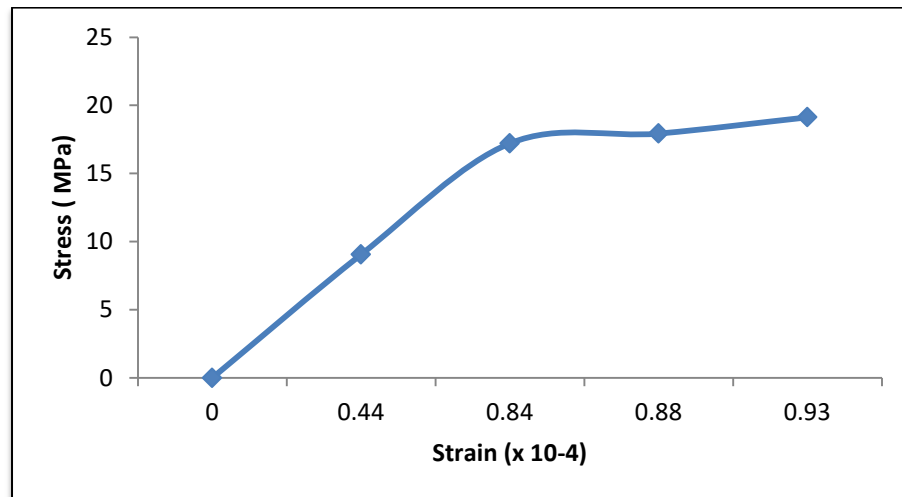


Figure 4.57: Stress against strain for CBPVC2

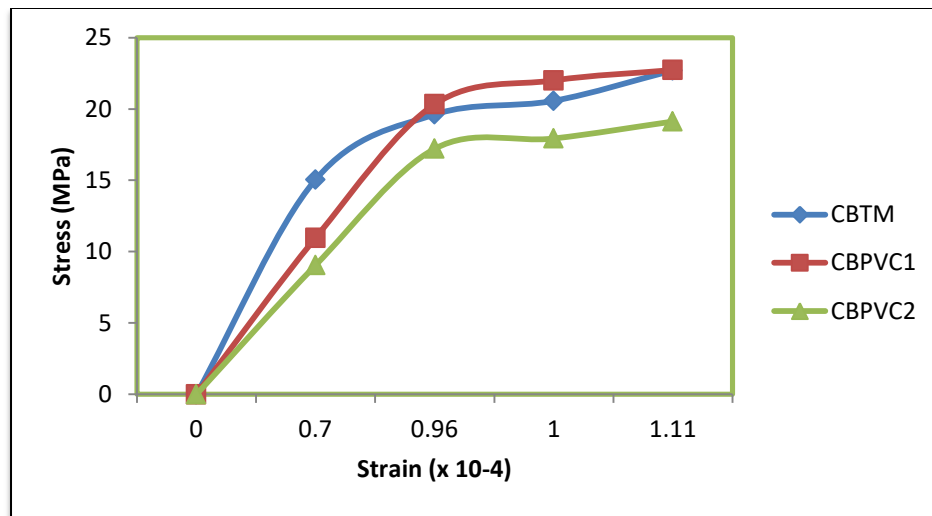


Figure 4.58: Strain against age of concrete for unreinforced beams

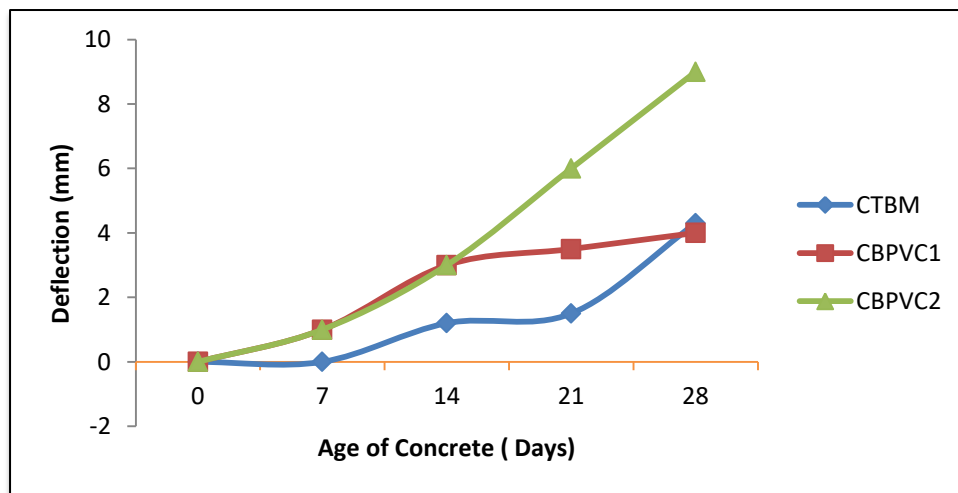


Figure 4.59: Deflection against age of concrete for unreinforced beams

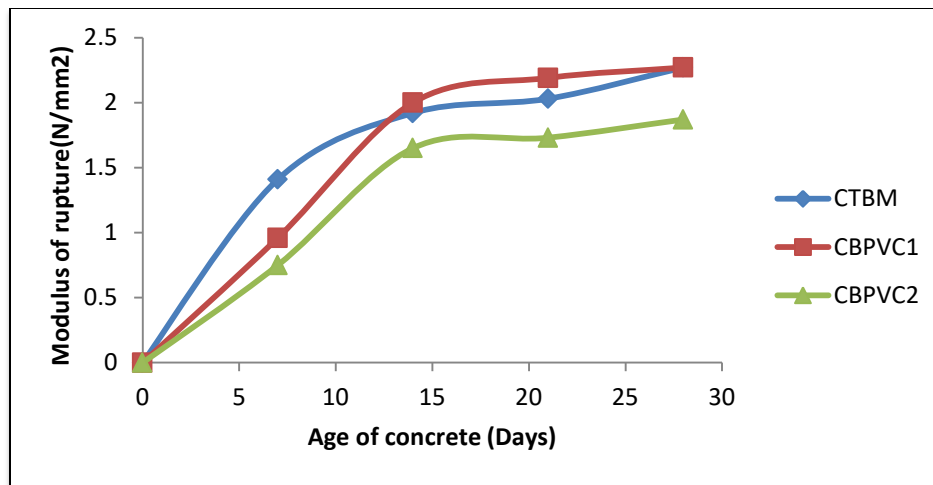


Figure 4.60: Modulus of rupture against age of concrete for unreinforced beams

### 4.3 Relationship between Compressive and Flexural strength

The relationship between flexural strength (MoR) and maximum stress ( $\sigma$ ) of the beams was established using BIOSTAT PRO 5.9.8 statistical software. Linear regression analysis was carried out at 5% level of significance with graphs in Figures 4.61-4.64. The following equations were proposed;

For RCPVC beams,

$$\text{Modulus of Rupture, } MoR = 0.1082\sigma - 0.1293, R^2 = 0.9987 \dots \dots \dots (4.4)$$

For RCBM beams,

$$\text{Modulus of Rupture, } MoR = 0.1067\sigma + 0.0005, R^2 = 1 \dots \dots \dots (4.5)$$

For Control (CTBM) beams,

$$\text{Modulus of Rupture, } MoR = 0.0993\sigma - 0.021, R^2 = 0.9982 \dots \dots \dots (4.6)$$

These equations were found to be in agreement with National Ready Mixed Concrete Association [NRMCA]. That is, flexural modulus between 10-20% of the maximum stress. The R-values close or equal to a unit indicates best fit.

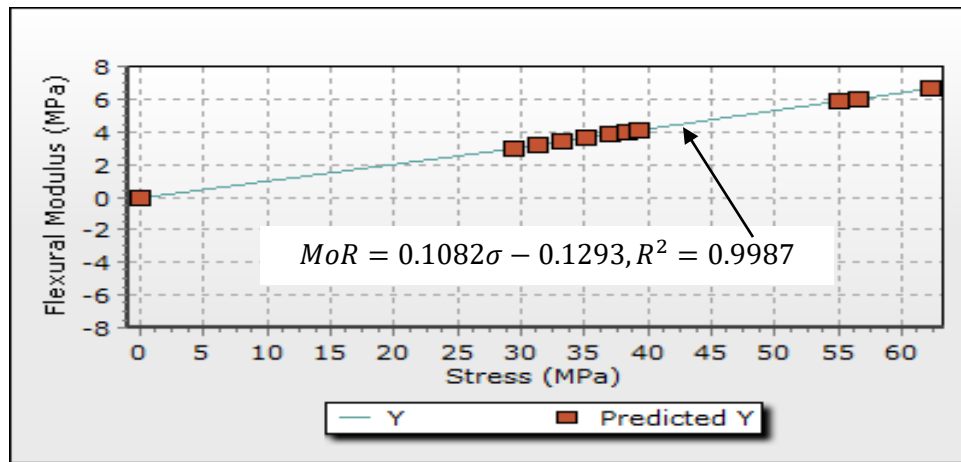


Figure 4.61: Stress against modulus of rupture for RCPVC beams in a single polynomial

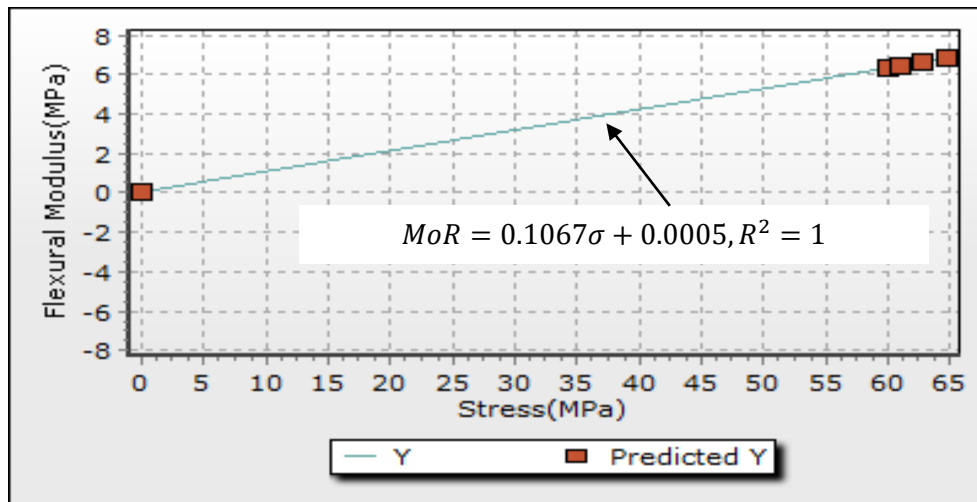


Figure 4.62: Stress against modulus of rupture for RCBM beams in a single polynomial

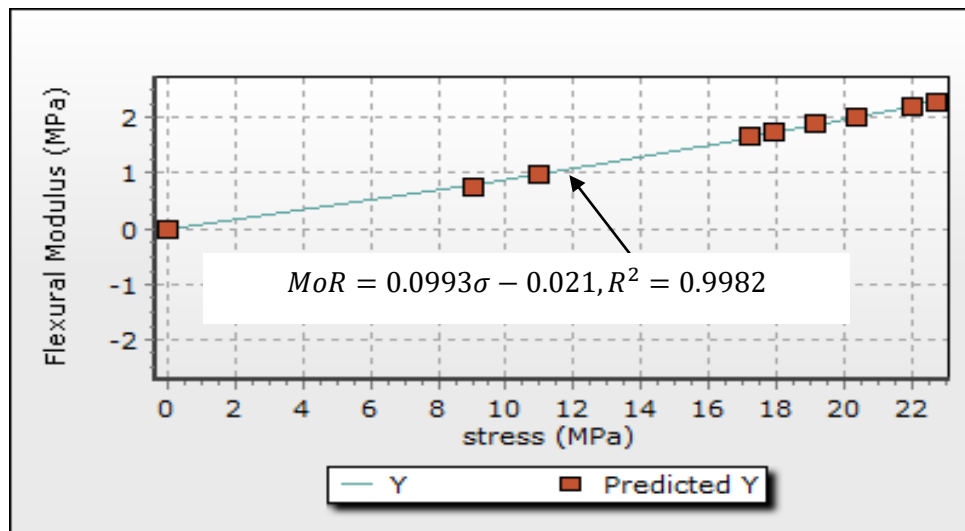


Figure 4.63: Stress with modulus of rupture for unreinforced PVC beams in a single polynomial

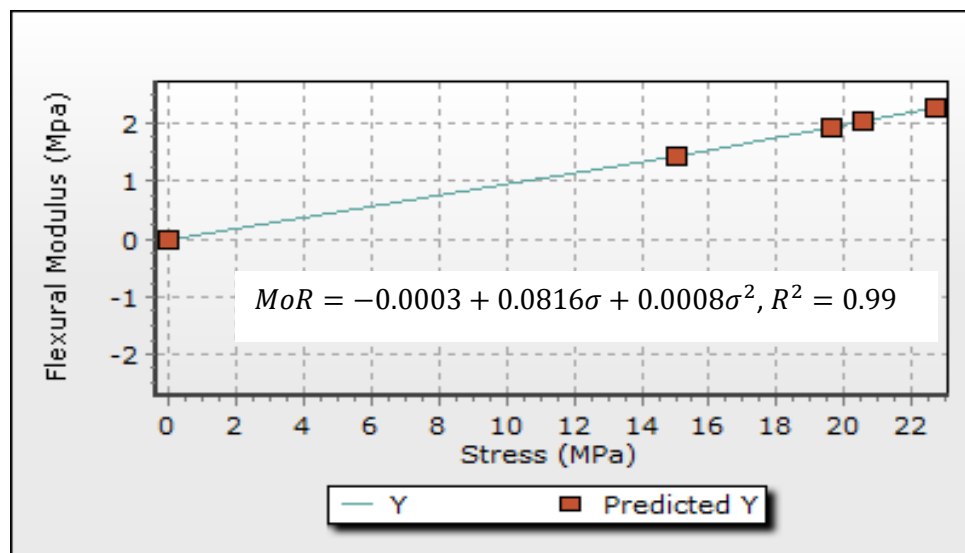


Figure 4.64: Stress against modulus of rupture for control beams in a single polynomial

## 4.4 Modelling and Simulation

Modelling and simulation involves numerical simulation i.e. finite element modelling of previously tested reinforced concrete beam specimen with and without PVC pipes using ANSYS Pro 15. The primary objective is to validate the material models utilised in the numerical methods in predicting the experimental tests results. The beams were subjected to same conditions as in experimental set-up. Reinforced concrete beams with one, two and three PVC pipes embedded below the neutral axis and that without PVC pipes were modelled and simulated as shown Figure 4.65. ANSYS modeling and simulation gave an excellent fit with experiment results in terms of stress. That is, failure occurs in ANSYS at approximately same load with the experimental set-up. The summary of stress obtained from simulation and theoretical estimation is shown in Table 4.3 while Table 4.4 shows comparison between theoretical stress, deflection and strain for the beam sample. Figures 4.66-4.69 shows the plot of theoretical estimate of stresses and stresses form ANSYS for the different beam specimen. The graphs indicates good match with theoretical estimation.

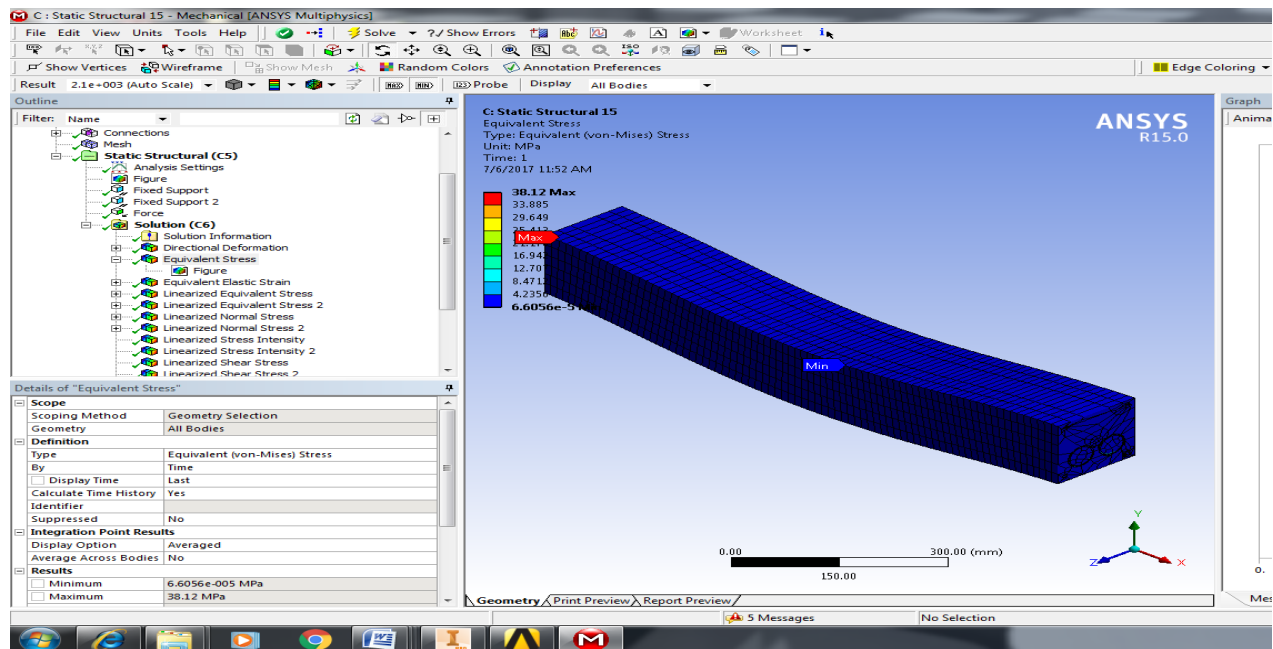


Figure 4.65: GUI Simulation using ANSYS 15.0

Table 4.3: Comparison between theoretical and simulation stress

| SAMPLES | Load applied | Theoretical         | Stress from ANSYS( $\sigma$ ) |
|---------|--------------|---------------------|-------------------------------|
|         | (F)          | stress ( $\sigma$ ) | MPa                           |
|         | kN           | MPa                 |                               |
| RCPVC1  | 0            | 0                   | 0                             |
|         | 15.4         | 39.3                | 39.9                          |
|         | 22           | 55.1                | 56.9                          |
|         | 22.6         | 56.6                | 58.5                          |
|         | 25.0         | 62.3                | 64.7                          |
| RCPVC2  | 0            | 0                   | 0                             |
|         | 11.3         | 29.4                | 28.7                          |
|         | 13.7         | 35.2                | 34.8                          |
|         | 14.5         | 37.1                | 36.9                          |
|         | 15.0         | 38.3                | 38.1                          |
| RCPVC3  | 0            | 0                   | 0                             |
|         | 11.3         | 29.44               | 28.4                          |
|         | 12.1         | 31.36               | 30.4                          |
|         | 12.9         | 33.28               | 32.4                          |
|         | 15.0         | 38.32               | 37.7                          |
| RCBM    | 0            | 0                   | 0                             |
|         | 24           | 59.92               | 66.7                          |
|         | 24.4         | 60.88               | 67.8                          |
|         | 25.2         | 62.79               | 70.0                          |
|         | 26.0         | 64.71               | 72.2                          |

Table 4.4: Comparison of stress, strain and deflection from theory and ANSYS

| S/N | Specimen | THEORETICAL                   |                                         |                            | ANSYS                         |                                         |                            |
|-----|----------|-------------------------------|-----------------------------------------|----------------------------|-------------------------------|-----------------------------------------|----------------------------|
|     |          | Stress<br>MPa<br>( $\sigma$ ) | Strain<br>( $\epsilon \times 10^{-2}$ ) | Deflection<br>( $\delta$ ) | Stress<br>MPa<br>( $\sigma$ ) | Strain<br>( $\epsilon \times 10^{-4}$ ) | Deflection<br>( $\delta$ ) |
| 1   | RCBM     | 59.9                          | 1.56                                    | 14                         | 66.7                          | 3.6                                     | 9.5                        |
| 2   | RCPVC1   | 62.3                          | 1.10                                    | 9                          | 64.7                          | 3.2                                     | 8.4                        |
| 3   | RCPVC2   | 38.3                          | 1.06                                    | 7                          | 38.1                          | 1.9                                     | 5.0                        |
| 4   | RCPVC3   | 38.3                          | 0.67                                    | 6                          | 37.7                          | 1.4                                     | 3.8                        |

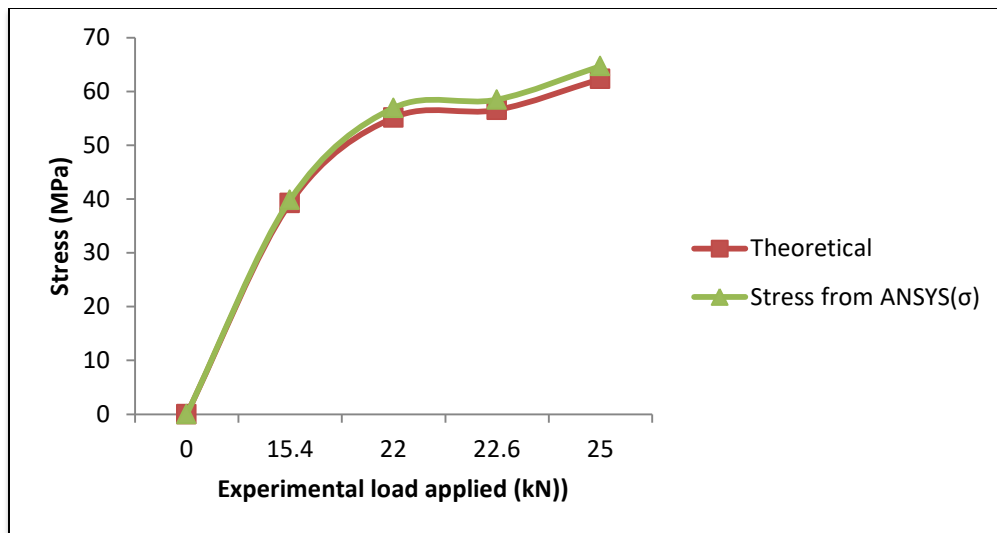


Figure 4.66: Graph showing theoretical and ANSYS stress for RCPVC1

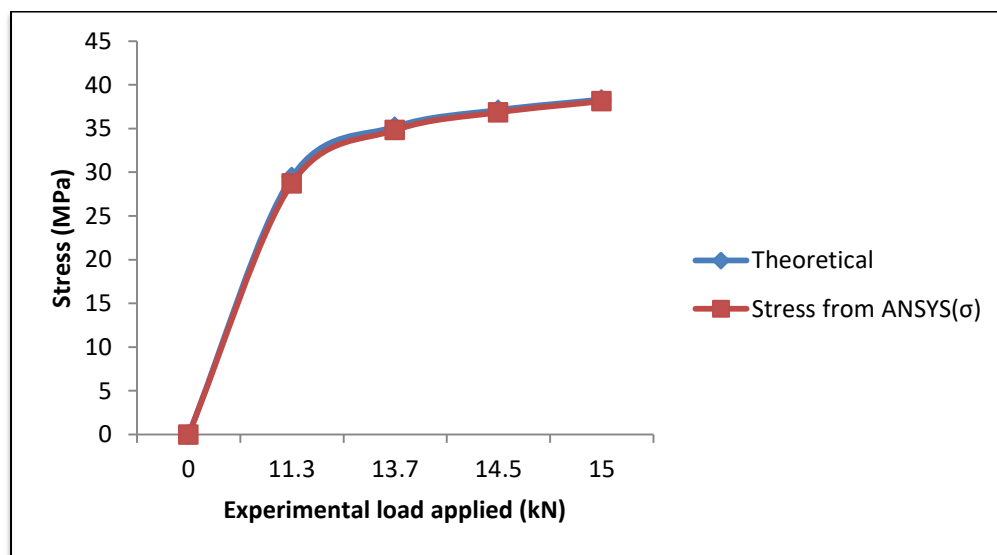


Figure 4.67: Theoretical and ANSYS stress for RCPVC2

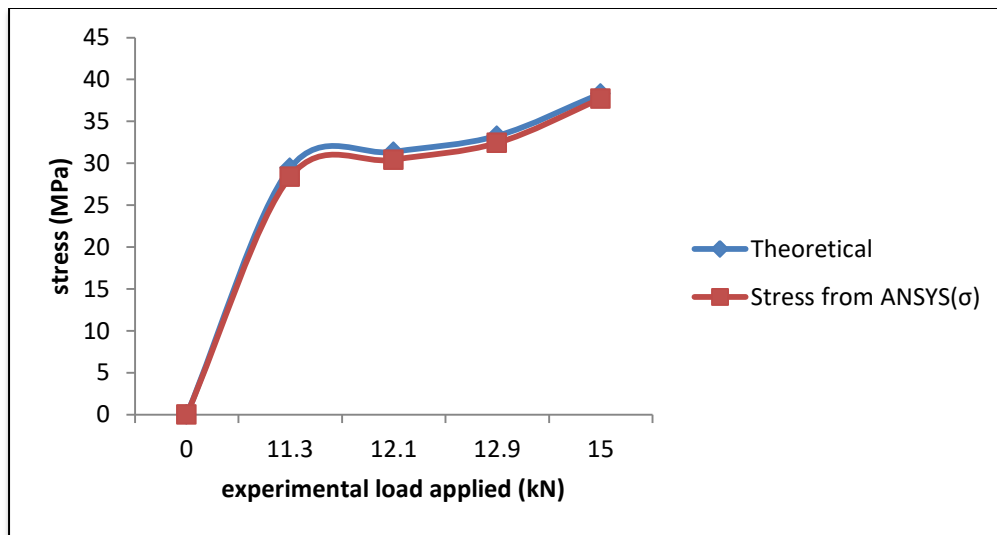


Figure 4.68: Graph showing theoretical and ANSYS stress for RCPVC3

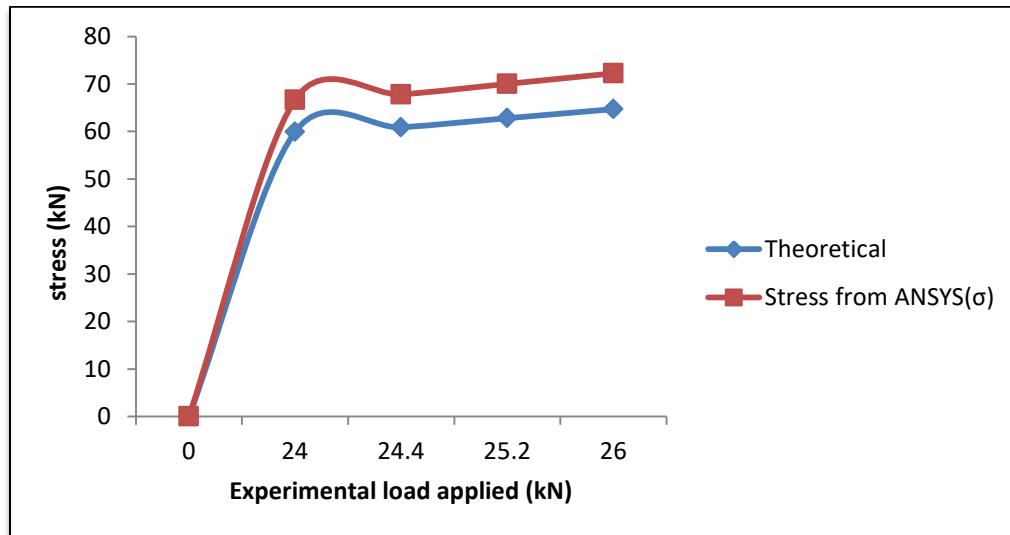


Figure 4.69: Graph showing theoretical and ANSYS stress for RCBM

## 4.5 Deterministic Design of Beam

### 4.5.1 Deterministic design of beams with and without PVC pipes

Deterministic designs were carried out on beams with and without PVC pipes using grade C25 concrete and in accordance with BS8110 (1997). The design was then compared with reliability analysis in section 4.6. The deterministic design is as follows:

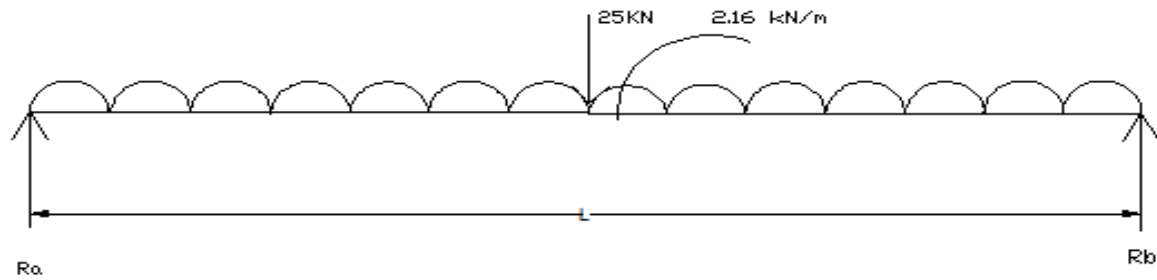


Figure 4.70: A simply supported loaded beam

#### Loading:

Beam self weight =  $0.15 \times 0.15 \times 24 = 0.54 \text{ kN/m}$

Finishes =  $1.0 \text{ kN/m}$

Total =  $1.54 \text{ kN/m}$

Design Load =  $1.4G_k = 1.4 \times 1.54 = 2.16 \text{ kN/m}$

Moment,  $M = \frac{PL}{4} + \frac{wL^2}{8} = \frac{25 \times 0.9}{4} + \frac{2.16 \times 0.9^2}{8} = 5.84 \text{ kNm}$

$R_a = R_b = \frac{P}{2} + \frac{wl}{2} = 13.47 \text{ kN}$

$$k = \frac{M}{bd^2f_{cu}} = 0.1046$$

$$d = 150 - 20 - 5 - 3 = 122$$

$$I_a = 0.5 + \sqrt{0.25 - \frac{0.1046}{0.9}} = 0.865$$

$$z = I_a d = 105.5 \text{ mm}$$

$$A_s = \frac{5.84 \times 10^6}{0.95 \times 410 \times 105.5} = 142 \text{ mm}^2$$

Provide

$$A_s = 157 \text{ mm}^2$$

$$A_{smin} = 0.15 bd/100 = 0.15 \times 150 \times 150/100 = 33.75 \text{ mm}^2$$

## **4.6 Reliability Based Design**

### **4.6.1 Reliability Analysis using First Order Reliability Method (FORM)**

Reliability analysis was carried out using FORM5 RELIABILITY software with subroutine programs RCPVC1, RCPVC2, RCPVC3 and RCBM for limit state in bending; NDEFLECT for limit state deflection and RCPVC1S, RCPVC2S, RCPVC3S, RCBMS for limit state in shear. All subroutines were written in FORTRAN LANGUAGE to generate the safety indices ( $\beta$ ) as shown in Figures 4.71-4.78

### **4.6.2 Reliability analysis using FORM5 (Reliability software)**

The design information and reliability analysis using FORM5 software for the various limit state in section 3.2.3.1(with and without PVC pipes) can be found in appendix D and F. Reliability analysis using the first order reliability method (FORM) revealed that safety index,  $\beta$  for all reinforced concrete beams with and without PVC pipes (RCBM, RCPVC1, RCPVC2 and RCPVC3), increases with increase in volume of beam considering moderate consequences of failure ( $\beta=3.3 - 4.4$ ) in Table 4.5. RCBM and RCPVC1 were subjected to the same load using the FORTRAN subroutine created; and they both display similar safety index within the code's specification (JCSS, 2000) compared to other reinforced concrete beams of similar arrangement. The safety indices increases with depth and decreases with length of beams as shown in Tables 4.6 -4.7 and Figures 4.83-4.84.

The reliability analysis using FORM5 yield an unsatisfactory safety index ( $\beta=1.93-2.16$ ) and resistive capacity ( $A_s= 226\text{mm}^2$ ) for RCBM and RCPVC1 by considering the limit state of bending. But, RCPVC2 and RCPVC3 gave a satisfactory safety index ( $\beta =4.81$ ) in bending and conservative values for shear and deflection. The analysis was repeated using FORM5 and resistive capacity of  $A_s= 402 \text{ mm}^2$ , all beam samples gave conservative safety index as shown in Table 4.8. It is important to get an optimum design by varying the resistance (R) of the beam samples and analysis repeated using FORM5. The results indicates

that RCBM and RCPVC1 performed satisfactorily well in bending with resistance,  $A_s=314 \text{ mm}^2$  (4Y1001) and safety index ( $\beta = 3.1-4.4$ ) which satisfies the code ( See Table 4.5) of the Joint Committee of Structural Safety requirements (JCSS, 2000). The conventional design (i.e deterministic approach) using the BS8110 (1997) gave  $A_s= 226 \text{ mm}^2$  (2Y1201) which seemed satisfactory based on BS 8110 (1997) but does not give the safety required based on JCSS (2000). On the other hand, RCPVC2 and RCPVC3 gave satisfactory area of steel ( $A_s= 226 \text{ mm}^2$ ) for both conventional and reliability based design which satisfies both BS and the JCSS code's requirements. The summary is can be found in Table 4.9.

Table 4.5: Tentative target reliability indices  $\beta$ , and probability of failure

| Relative cost of safety measure | Minor consequences of failure        | Moderate consequences of failure              | Large consequences of Failure                 |
|---------------------------------|--------------------------------------|-----------------------------------------------|-----------------------------------------------|
| Small (A)                       | $\beta=3.1$ ( $pF \approx 10^{-3}$ ) | $\beta=3.3$ ( $pF \approx 5 \times 10^{-4}$ ) | $\beta=3.7$ ( $pF \approx 1 \times 10^{-4}$ ) |
| Normal (B)                      | $\beta=3.7$ ( $pF \approx 10^{-4}$ ) | $\beta=4.2$ ( $pF \approx 10^{-5}$ )          | $\beta=4.4$ ( $pF \approx 5 \times 10^{-6}$ ) |
| Large (C)                       | $\beta=4.2$ ( $pF \approx 10^{-5}$ ) | $\beta=4.4$ ( $pF \approx 5 \times 10^{-6}$ ) | $\beta=4.7$ ( $pF \approx 10^{-6}$ )          |

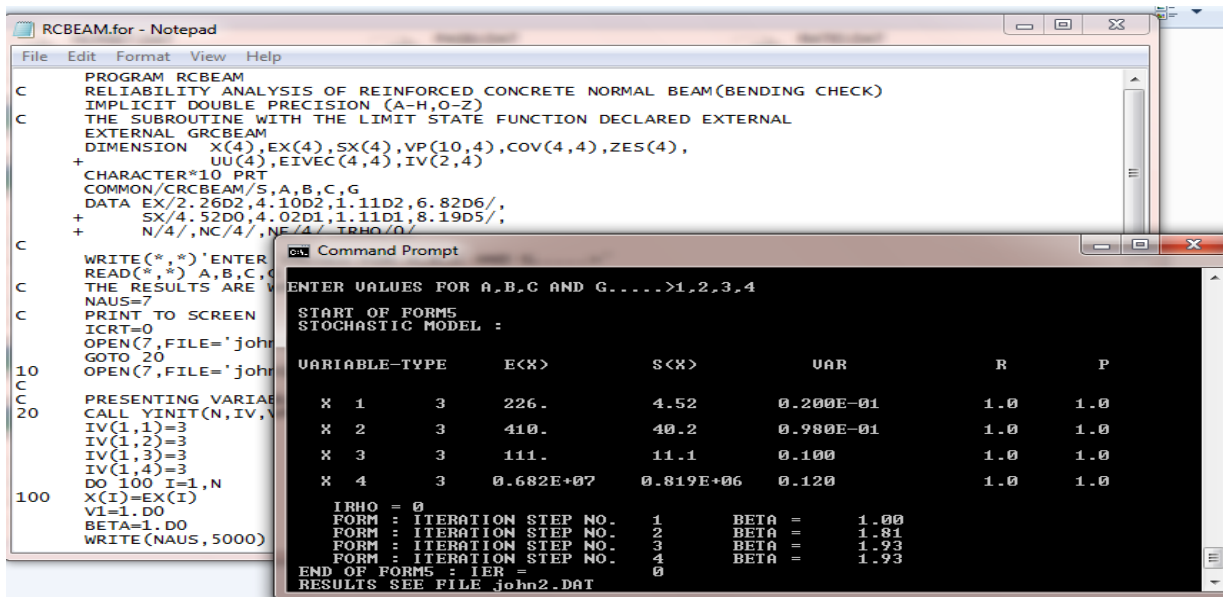


Figure 4.71: REL-analysis in bending using FORM5 for RCBM

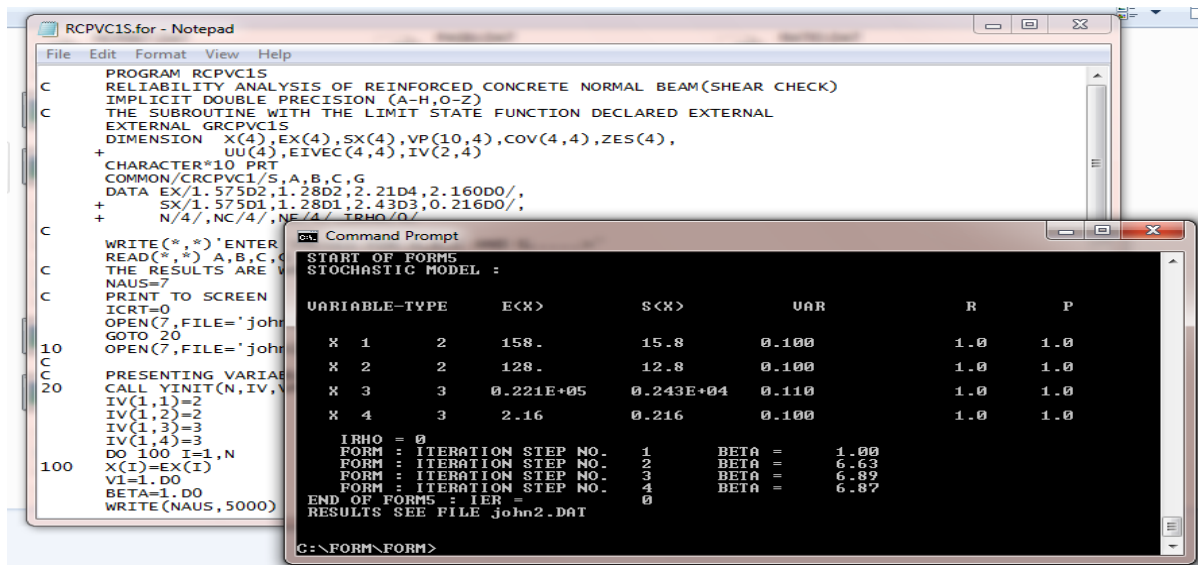


Figure 4.72: REL-analysis for RCBEAM in shear using FORM5

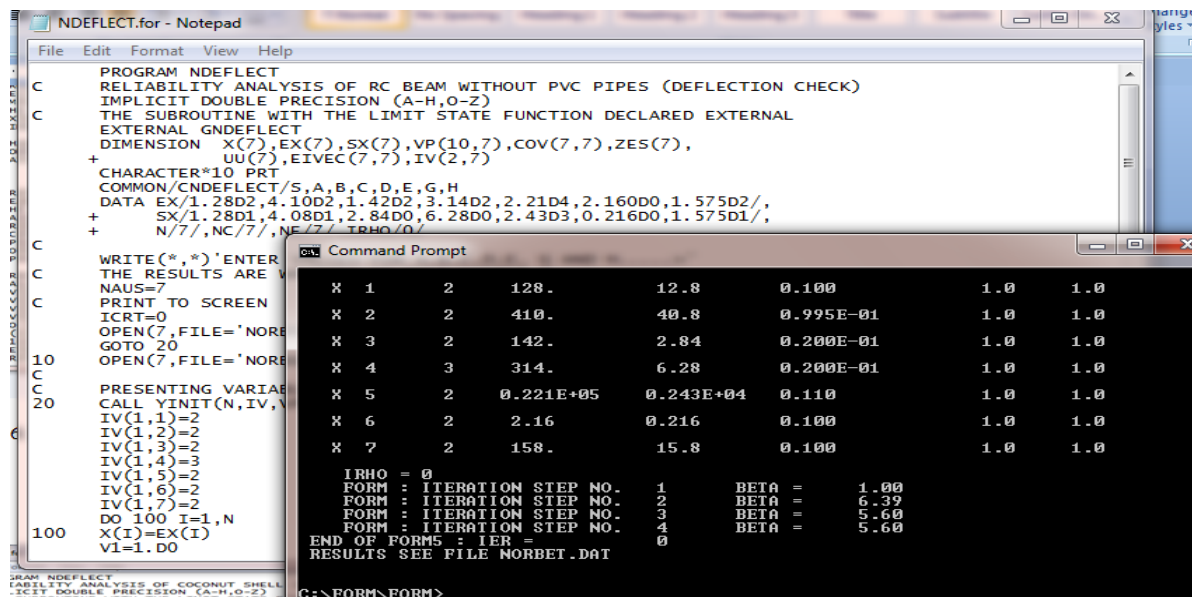


Figure 4.73: REL-analysis for RCBEAM in deflection using FORM5

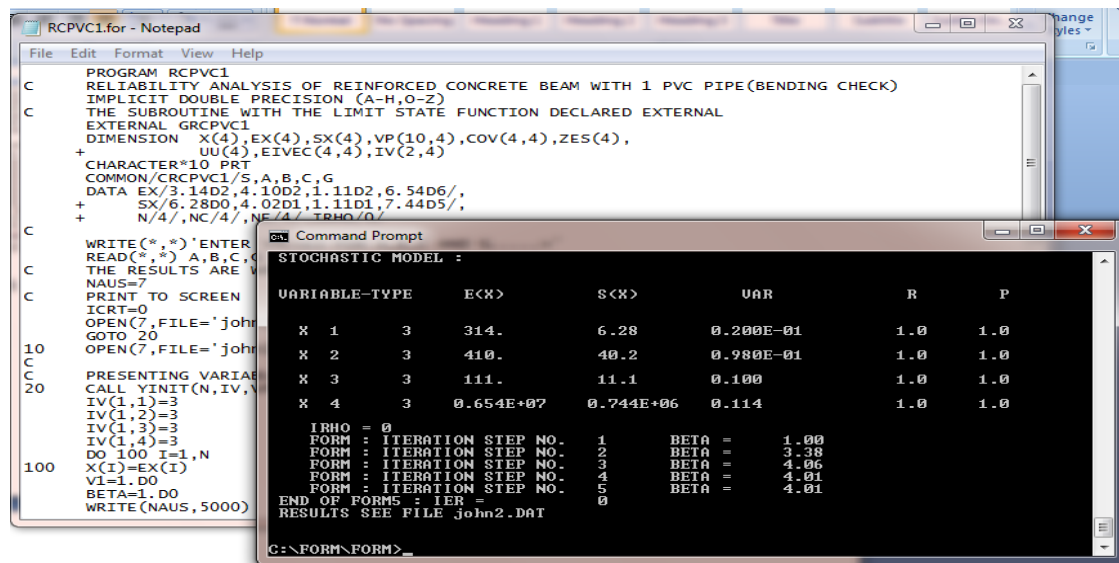


Figure 4.74: REL-analysis for RCPVC1 in bending using FORM5

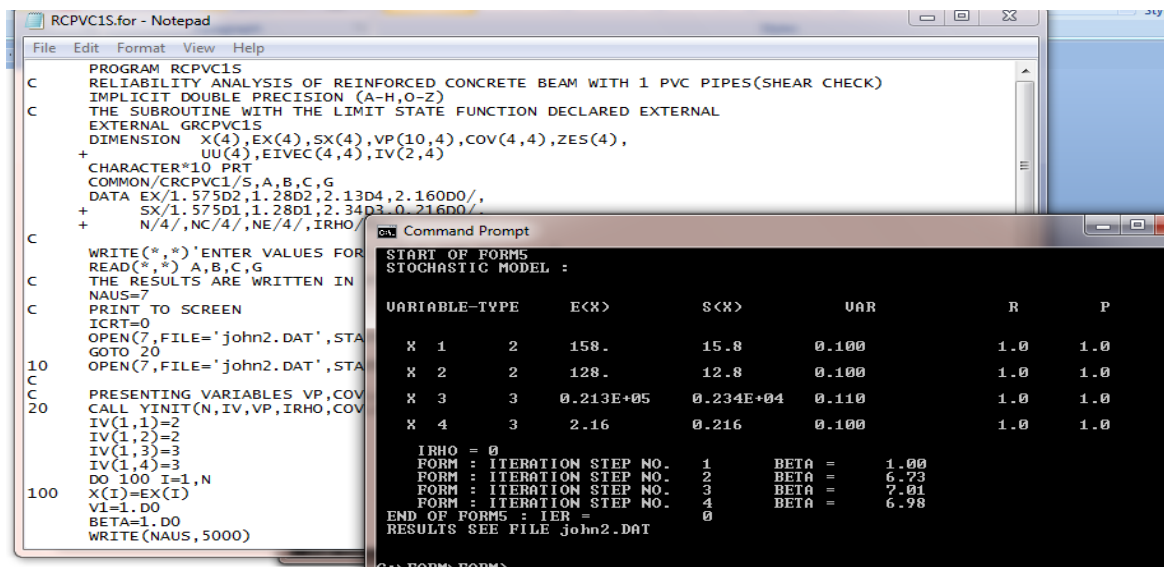


Figure 4.75: Showing REL-analysis for RCPVC1 in shear using FORM5

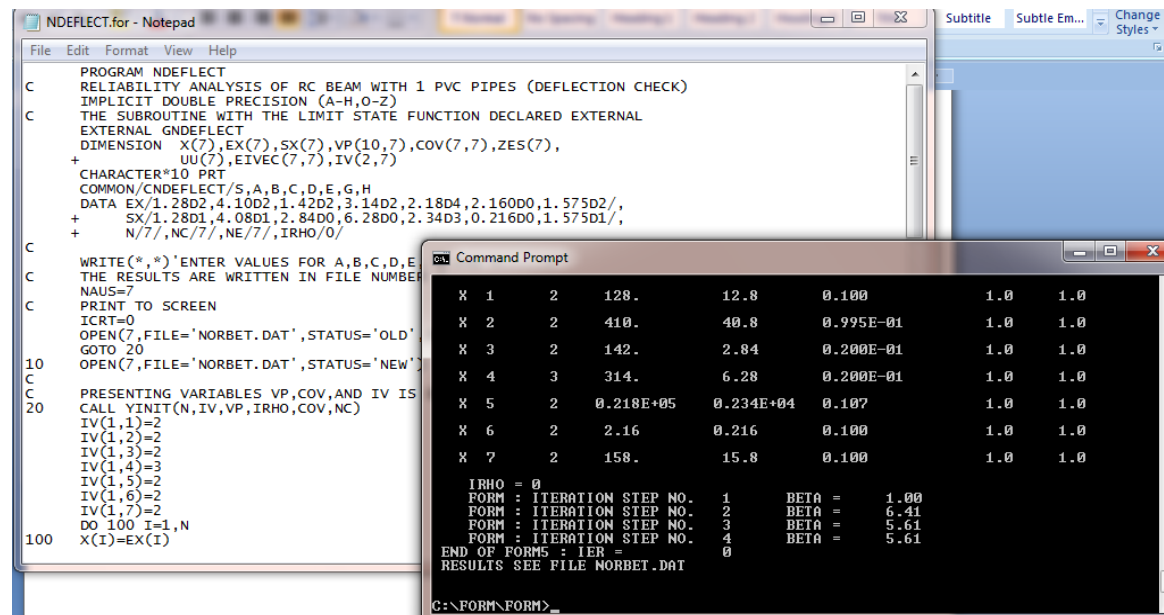


Figure 4.76: REL-analysis for RCPVC1 in deflection using FORM5

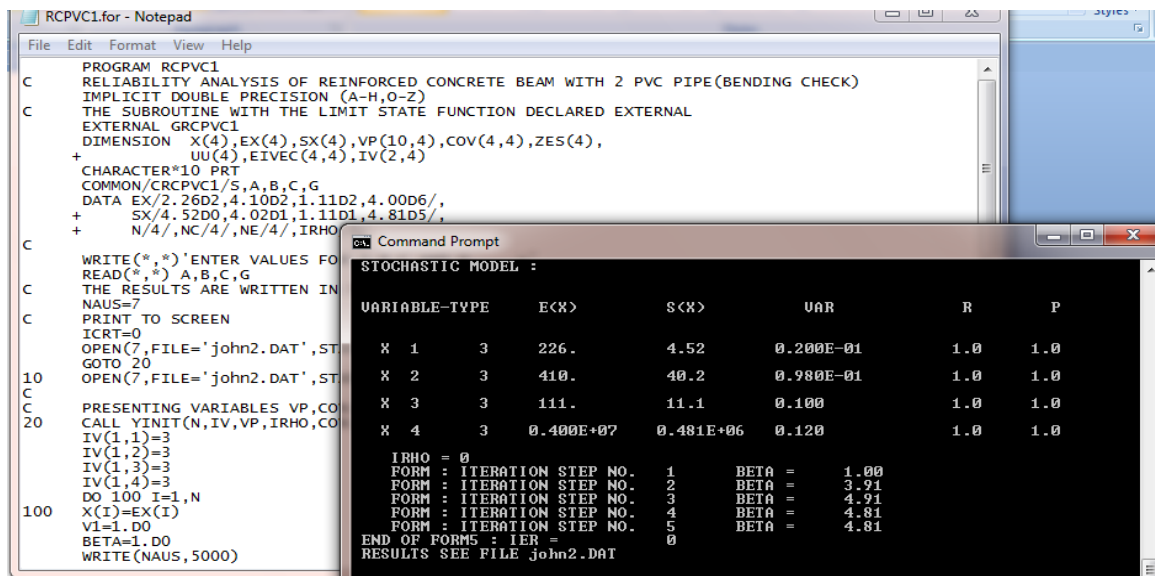


Figure 4.77: Showing REL-analysis for RCPVC2 in bending using FORM5

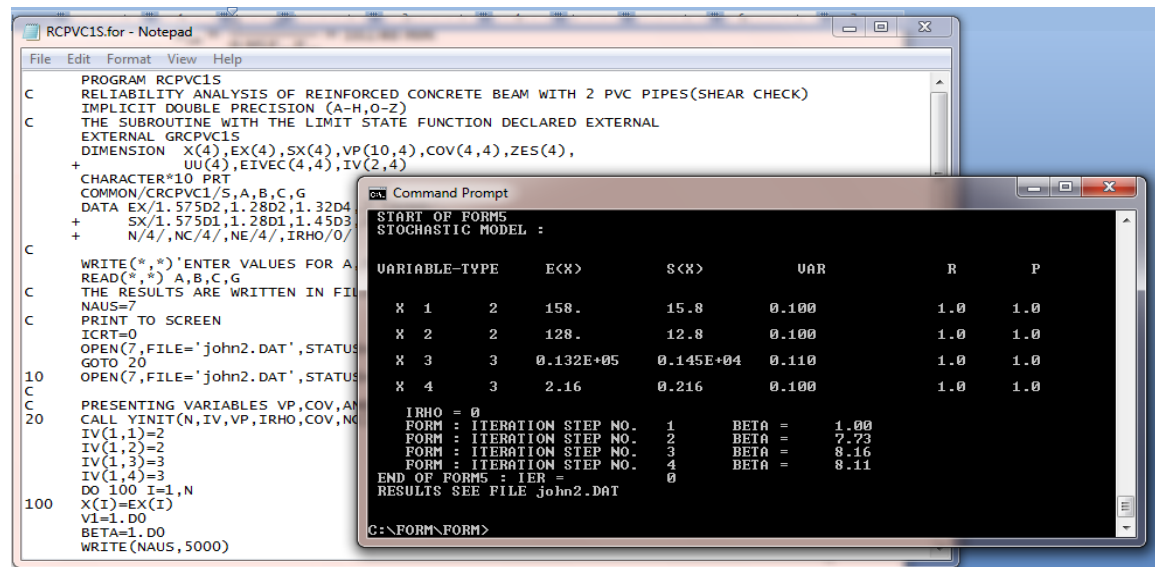


Figure 4.78: REL-analysis for RCPVC2 in shear using FORM5

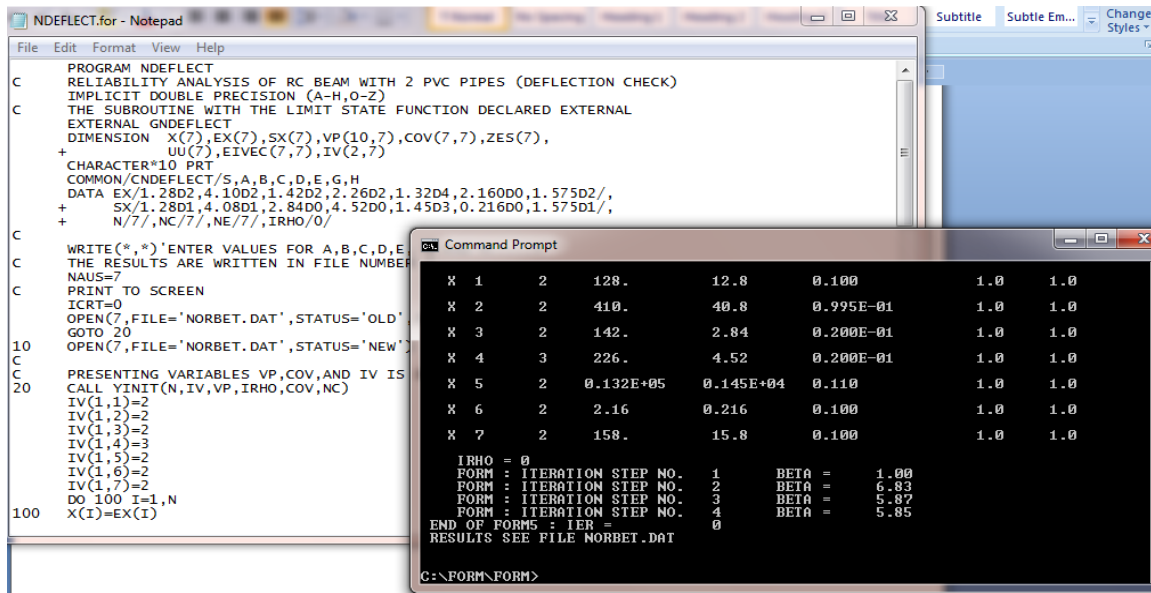


Figure 4.79: Showing REL-analysis for RCPVC2 in deflection using FORM5

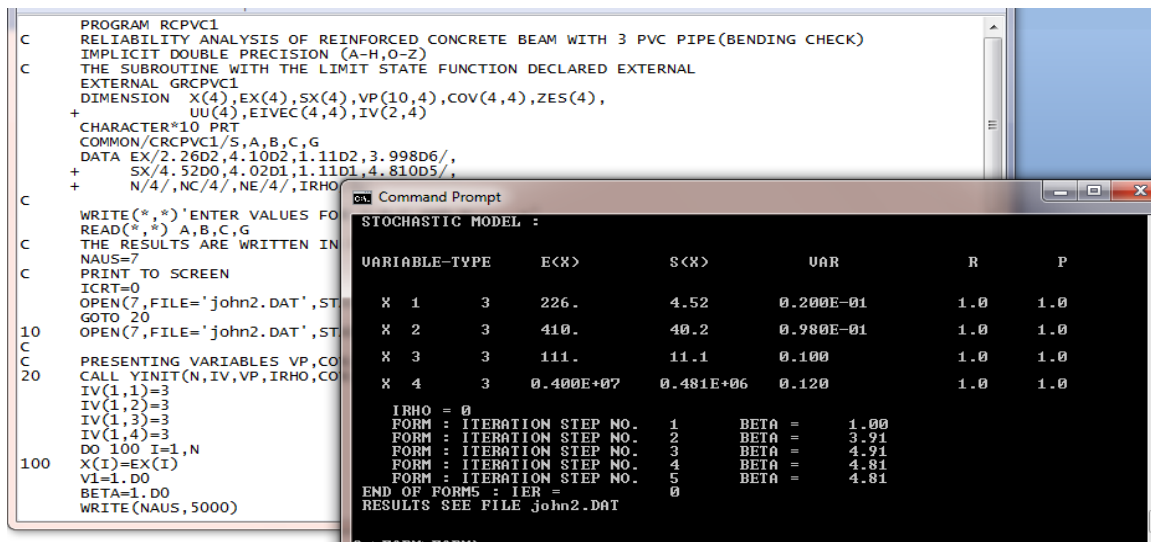


Figure 4.80: REL-analysis for RCPVC3 in bending using FORM5

```

RCPVC1S.for - Notepad
File Edit Format View Help
C PROGRAM RCPVC1S
C RELIABILITY ANALYSIS OF REINFORCED CONCRETE BEAM WITH 3 PVC PIPES(SHEAR CHECK)
C IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C THE SUBROUTINE WITH THE LIMIT STATE FUNCTION DECLARED EXTERNAL
C EXTERNAL GRCPVC1S
C DIMENSION X(4),EX(4),SX(4),VP(10,4),COV(4,4),ZES(4),
+ UU(4),EIVEC(4,4),IV(2,4)
C CHARACTER*10 PRT
C COMMON/CRCPVC1/S,A,B,C,G
C DATA EX/1.575D2,1.28D2,1.28D4
+ SX/1.575D1,1.28D1,1.40D3
+ N/4/,NC/4/,NE/4/,IRHO/0/
C WRITE(*,*)'ENTER VALUES FOR A,B,C,G
C READ(*,*) A,B,C,G
C THE RESULTS ARE WRITTEN IN FILE
C NAUS=7
C PRINT TO SCREEN
C ICRT=0
C OPEN(7,FILE='john2.DAT',STATUS='OLD')
C GOTO 20
C OPEN(7,FILE='john2.DAT',STATUS='NEW')
10 C
C PRESENTING VARIABLES VP,COV,AN
C CALL YINIT(N,IV,VP,IRHO,COV,NC)
20 C
C IV(1,1)=2
C IV(1,2)=2
C IV(1,3)=3
C IV(1,4)=3
C DO 100 I=1,N
C X(I)=EX(I)
100 C
C VI=1.D0
C BETA=1.D0
C WRITE(NAUS,5000)

```

```

Command Prompt
START OF FORM5
STOCHASTIC MODEL :
VARIABLE-TYPE      E(X)          S(X)          VAR          R          P
X 1      2      158.          15.8          0.100          1.0      1.0
X 2      2      128.          12.8          0.100          1.0      1.0
X 3      3      0.128E+05      0.140E+04      0.109          1.0      1.0
X 4      3      2.16          0.216          0.100          1.0      1.0
IRHO = 0
FORM : ITERATION STEP NO. 1      BETA = 1.00
FORM : ITERATION STEP NO. 2      BETA = 7.77
FORM : ITERATION STEP NO. 3      BETA = 8.22
FORM : ITERATION STEP NO. 4      BETA = 8.17
END OF FORMS : IER = 0
RESULTS SEE FILE john2.DAT
C:\FORM\FORM>

```

Figure 4.81: Showing REL-analysis for RCPVC3 in shear using FORM5

```

NDEFLECT.for - Notepad
File Edit Format View Help
C PROGRAM NDEFLECT
C RELIABILITY ANALYSIS OF RC BEAM WITH 3 PVC PIPES (DEFLECTION CHECK)
C IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C THE SUBROUTINE WITH THE LIMIT STATE FUNCTION DECLARED EXTERNAL
C EXTERNAL GDEFLECT
C DIMENSION X(7),EX(7),SX(7),VP(10,7),COV(7,7),ZES(7),
+ UU(7),EIVEC(7,7),IV(2,7)
C CHARACTER*10 PRT
C COMMON/CNDEFLECT/S,A,B,C,D,E,G,H
C DATA EX/1.28D2,4.10D2,1.42D2,2.26D2,1.28D4,2.160D0,1.575D2/,
+ SX/1.28D1,4.08D1,2.84D0,4.52D0,1.40D3,0.216D0,1.575D1/,
+ N/7/,NC/7/,NE/7/,IRHO/0/
C WRITE(*,*)'ENTER VALUES FOR A,B,C,D,E,G,H
C READ(*,*) A,B,C,D,E,G,H
C THE RESULTS ARE WRITTEN IN FILE NUMBER
C NAUS=7
C PRINT TO SCREEN
C ICRT=0
C OPEN(7,FILE='NORBET.DAT',STATUS='OLD')
C GOTO 20
C OPEN(7,FILE='NORBET.DAT',STATUS='NEW')
10 C
C PRESENTING VARIABLES VP,COV,AND IV IS
C CALL YINIT(N,IV,VP,IRHO,COV,NC)
20 C
C IV(1,1)=2
C IV(1,2)=2
C IV(1,3)=2
C IV(1,4)=3
C IV(1,5)=2
C IV(1,6)=2
C IV(1,7)=2
C DO 100 I=1,N
C X(I)=EX(I)
100 C

```

```

Command Prompt
X 1      2      128.          12.8          0.100          1.0      1.0
X 2      2      410.          40.8          0.995E-01          1.0      1.0
X 3      2      142.          2.84          0.200E-01          1.0      1.0
X 4      3      226.          4.52          0.200E-01          1.0      1.0
X 5      2      0.128E+05      0.140E+04      0.109          1.0      1.0
X 6      2      2.16          0.216          0.100          1.0      1.0
X 7      2      158.          15.8          0.100          1.0      1.0
IRHO = 0
FORM : ITERATION STEP NO. 1      BETA = 1.00
FORM : ITERATION STEP NO. 2      BETA = 6.86
FORM : ITERATION STEP NO. 3      BETA = 5.89
FORM : ITERATION STEP NO. 4      BETA = 5.88
END OF FORMS : IER = 0
RESULTS SEE FILE NORBET.DAT
C:\FORM\FORM>

```

Figure 4.82: REL-analysis for RCPVC3 in deflection using FORM5

Table 4.6: Variation of Reliability index with length for different samples of beams

| Samples |        | RCBM    | RCPVC1 | RCPVC2 | RCPVC3 |
|---------|--------|---------|--------|--------|--------|
| S/No    | Length | $\beta$ |        |        |        |
| 1       | 500    | 7.64    | 6.93   | 6.22   | 5.99   |
| 2       | 750    | 6.27    | 5.48   | 4.71   | 4.46   |
| 3       | 1000   | 4.68    | 3.94   | 3.2    | 2.96   |
| 4       | 1250   | 3.39    | 2.66   | 1.96   | 1.66   |
| 5       | 1500   | 2.31    | 1.62   | 0.92   | 0.7    |
| 6       | 1750   | 1.39    | 0.71   | 0.036  | -0.18  |
| 7       | 2000   | 0.59    | -0.86  | -0.73  | -0.95  |
| 8       | 2500   | -0.76   | -1.41  | -1.54  | -2.2   |

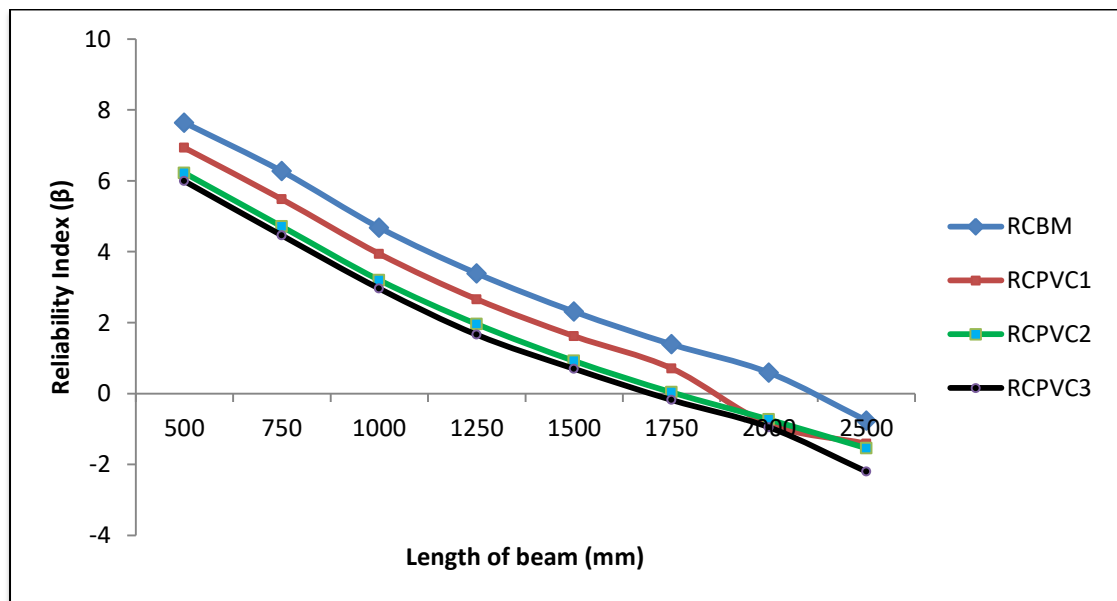


Figure 4.83: Variation of reliability index with length of beam using FORM

Table 4.7: Variation of Reliability index with depth of beams

| S/No | Depth of beam | $\beta$ |
|------|---------------|---------|
| 1    | 150           | 0.63    |
| 2    | 200           | 2.68    |
| 3    | 250           | 4.21    |
| 4    | 300           | 5.45    |
| 5    | 350           | 6.50    |
| 6    | 400           | 7.40    |
| 7    | 500           | 8.90    |
| 8    | 600           | 10.1    |

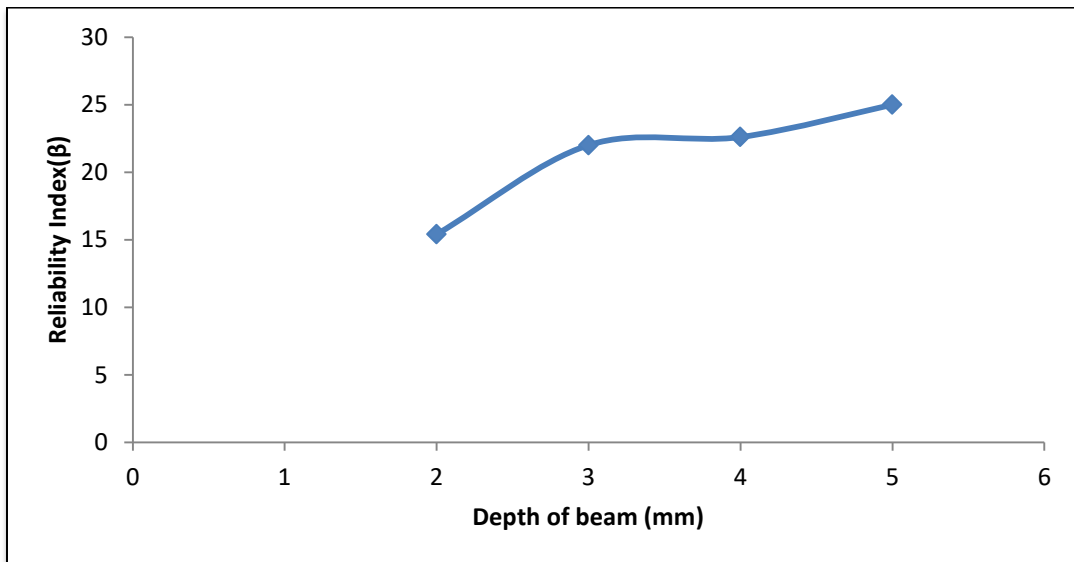


Figure 4.84: Variation of reliability index with depth of beam using FORM

### 4.6.3 Reliability Analysis using manual computation

Reliability analysis was carried out by manual computation following the Hasofer Lind reliability index and at limit state of bending, shear and deflection using the first order reliability method (FORM). The beams examined are RCBM, RCPVC1, RCPVC2 and RCPVC3. The values of the safety index ( $\beta$ ) obtained were found to be in close agreement with the analysis using the reliability software (FORM5). The manual analysis becomes complex when the limit state to be examined involves more than six variables. The comparison of results using the manual method and reliability software (FORM5) is shown in Table 4.8. The procedures for the manual analysis using FORM are;

#### Data

$$\mu_A = 201 \text{ mm}^2, \quad \sigma_{As} = \mu_{As} \cdot CoV_{As} = 201 \times 0.02 = 4.02$$

$$\mu_{fy} = 410 \text{ N/mm}^2, \quad \sigma_{fy} = \mu_{fy} \cdot CoV_{fy} = 410 \times 0.098 = 40.18$$

$$\mu_M = 6820000 \text{ N mm}, \quad \sigma_M = \mu_M \cdot CoV_M = 6820000 \times 0.12 = 818899 \text{ Nmm}$$

$$\mu_z = 111 \text{ mm}, \quad \sigma_z = \mu_z \cdot CoV_z = 111 \times 0.1 = 11.1 \text{ mm}$$

#### 1<sup>st</sup> iteration

The limit state in bending is given by;

$$g = (A_s, f_y, z, M) = 0.95 A_s A f_y z - M, \quad \text{evaluated at mean values}$$

At  $g=0$ ,

$$X_z = \frac{\mu_M}{0.95 \mu_{As} \mu_{fy}} = 87.11 \text{ mm}$$

a) Determining the reduce variates in the form

$$Z = \frac{X - \mu}{\sigma}$$

$$Z_{As} = \frac{X_{As} - \mu_{As}}{\sigma_{As}} = 0$$

$$Z_{fy} = \frac{X_{fy} - \mu_{fy}}{\sigma_{fy}} = 0$$

$$Z_M = \frac{X_M - \mu_M}{\sigma_M} = 0$$

$$Z_z = \frac{X_z - \mu_z}{\sigma_z} = -2.152$$

b) Determining the vector  $\{G\}$

$$G_1 = -\frac{\partial g}{\partial A_s} \sigma_{As} = -0.95 X_{fy} X_z \sigma_{As} = -0.95 \times 410 \times 87.11 \times 4.02 = -136400$$

$$G_2 = -\frac{\partial g}{\partial f_y} \sigma_{fy} = -668360$$

$$G_3 = -\frac{\partial g}{\partial M} \sigma_M = 818496$$

$$G_4 = -\frac{\partial g}{\partial z} \sigma_z = -869013$$

$$\text{Reliability index, } \beta = \frac{\{G^T\}\{z\}}{\sqrt{\{G^T\}\{G\}}} = 1.36$$

Iteration 2

$$\alpha_1 = \frac{G_1}{\sqrt{\{G^T\}\{G\}}} = -0.0992$$

$$\alpha_2 = \frac{G_2}{\sqrt{\{G^T\}\{G\}}} = -0.4861$$

$$\alpha_3 = \frac{G_3}{\sqrt{\{G^T\}\{G\}}} = 0.595$$

$$\alpha_4 = \frac{G_1}{\sqrt{\{G^T\}\{G\}}} = -0.6320$$

- c) Determining new design point in reduce variate

$$Z_{11} = \alpha\beta = -0.1349$$

$$Z_{12} = -0.6612$$

$$Z_{13} = 0.8097$$

$$Z_{14} = -0.8597$$

- d) Determining design point in original coordinates

$$X_{11} = \mu_{As} + Z_{11}\sigma_A = 200.1$$

$$X_{12} = 383.43$$

$$X_{13} = 6820000$$

$$X_{14} = \frac{X_{13}}{0.95X_{11}X_{12}} = 102.48 \text{ mm}$$

- e) Reliability index,  $\beta = \frac{\{G^T\}\{z\}}{\sqrt{\{G^T\}\{G\}}} = 1.33$

The procedures are continued until  $\beta$ -value converges.

Table 4.8: Reliability analysis using FORM and manual computation.

| Reliability Analysis (FORM) |            |       |            | Reliability Analysis (Manual) |       |            |                |           |
|-----------------------------|------------|-------|------------|-------------------------------|-------|------------|----------------|-----------|
| SAMPLE                      | $\beta$    |       |            | $\beta$                       |       |            | m <sup>3</sup> | %         |
|                             | Bending    | Shear | Deflection | Bending                       | Shear | Deflection | Volume         | reduction |
| <b>RCBM</b>                 | *1.93,5.04 | 6.87  | *5.40,5.66 | *1.98,5.05                    | 6.53  | *5.43,5.68 | 0.02025        | -         |
| <b>RCPVC1</b>               | *2.16,5.27 | 6.99  | *5.43,5.69 | *2.19,5.25                    | 6.62  | *5.48,5.71 | 0.01829        | 8.67      |
| <b>RCPVC2</b>               | *4.81,7.12 | 8.11  | *5.78,6.06 | *4.87,7.18                    | 7.53  | *5.76,6.09 | 0.01632        | 17.40     |
| <b>RCPVC3</b>               | *4.81,7.16 | 8.18  | *5.81,6.08 | *4.87,7.17                    | 7.57  | *5.83,6.12 | 0.01583        | 21.80     |

\* Safety index( $\beta$ ) obtained using  $A_s=226\text{mm}^2$ , - Safety index( $\beta$ ) obtained using  $A_s=402\text{mm}^2$

Table 4.9: Comparison between reliability and deterministic design

| Reliability Analysis (FORM) |         |       |            |                         | Deterministic design |       |            |                         |
|-----------------------------|---------|-------|------------|-------------------------|----------------------|-------|------------|-------------------------|
| SAMPLE                      | $\beta$ |       |            |                         |                      |       |            |                         |
|                             | Bending | Shear | Deflection | Area (mm <sup>2</sup> ) | Bending              | Shear | Deflection | Area (mm <sup>2</sup> ) |
| <b>RCBM</b>                 | 3.83    | 6.87  | 5.60       | 314(4Y1001)             | ok                   | ok    | ok         | 226(2Y1201)             |
| <b>RCPVC1</b>               | 3.31    | 6.98  | 5.61       | 314(4Y1001)             | ok                   | ok    | ok         | 226(2Y1201)             |
| <b>RCPVC2</b>               | 4.81    | 8.11  | 5.85       | 226(2Y1201)             | ok                   | ok    | ok         | 226(2Y1201)             |
| <b>RCPVC3</b>               | 4.81    | 8.17  | 5.88       | 226(2Y1201)             | ok                   | ok    | ok         | 226(2Y1201)             |

## CHAPTER FIVE

### CONCLUSIONS AND RECOMMENDATION

#### 5.1 Conclusions

- a) Compressive strength for all concrete cubes increases as the ages increases. Most importantly, the strength for normal and cubes with embedded PVC pipes below the neutral axis was found to be in close agreement than other cubes of different pattern of arrangement.
- b) Flexural modulus of rupture of reinforced concrete beams and reinforced concrete beams with one embedded PVC pipe are in close range (10-20% of the maximum stress). The stress, strain and failure pattern produced are similar compared to other beams. Therefore, both can accommodate approximately the same stress under same condition.
- c) The proposed model in the form of polynomial of degree four ( $n=4$ ) using a linear regression line indicates smooth correlation between reinforced concrete beams with and without PVC pipes.
- d) Experimental and finite element modeling revealed that beams with openings have reduced load carrying capacity compared with solid beams. The more the number of openings, the lower its load carrying capacity. RCPVC1 and RCBM have similar load carrying capacity.
- e) Stress-strain verification for reinforced, PVC and control cubes indicates that material component of concrete when loaded; stress and strain values are within specified range of the code with yield-peak strain between  $2.07 \times 10^{-3}$ - $3.5 \times 10^{-3}$ . Statistical analysis using MS-EXCEL spread sheet yield a regression line equation of degree four (4) and validated using Saenz model while Desayi's model deviated by 22%.

- f) Finite element modeling and simulation using ANSYS shows close relationship between RCBM and RCPVC1 than with other beams when subjected to the same test conditions. ANSYS on the other hand, gave best correlation with theoretical analysis.
- g) Reliability analysis using the first order reliability method (FORM) reveal that RCBM and RCPVC1 are capable of resisting the same applied load effects at the same time enhancing structural safety of the beams. RCPVC1 beams will yield an economical design compared to RCBM beams in terms of volume reduction (8.7-21.8%) at the same time satisfying the condition of safety ( $\beta = 3.1-4.7$ ) recommended by JCSS when considering moderate consequences of failure.

## 5.2 Recommendation

- a) Compressive strengths of Cubes with embedded PVC pipes below the neutral axis is recommended for quality control test for PVC concrete since their strength capacity and stress-strain characteristics is similar to that of the conventional cubes.
- b) A beam with a single opening containing PVC pipe below the neutral axis is recommended as floor or roof beams since its load carrying capacity is approximately equal to that of a solid beam. This will also lead to reduction in volume of structural elements, PVC pipes to replace electrical conduit pipes and eliminating cracking of wall during electrical installation.
- c) The statistical model obtained using BIOStat Pro can be used to study the strength properties of reinforced concrete beams with and without PVC pipes at construction sites.
- d) Finite element modeling using ANSYS is a predictive tool to support laboratory investigation. They are virtual laboratory machines, producing results by simulating the exact laboratory conditions within their graphical user interface (GUI) for structural

- elements. It enables designer to predict results of experimental work even before they are initiated. This is recommended for designers to minimize cost of laboratory work.
- e) Reliability analysis using FORM is very essential in engineering designs to predetermine safety of structural elements. They can act as backup tool for BS 8110, EU2 or Building code in design processes to ascertain safety level of structural elements even before they are constructed. It is therefore recommended in design processes to ascertain safety level of structural elements whether they satisfy code's requirements or not.

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# **APPENDICES**

## **APPENDIX A: MIX DESIGN FOR THE BEAMS USING C25**

Concrete design mix proposed by DOE (1975), was adopted and the procedure are as follows:

### **Stage one**

Characteristics strength of cube at 28 days,  $f_{ck} = 25N/mm^2$

Target mean =  $f_{ck} + 1.64\sigma = 25 + 1.64 \times 8 = 38N/mm^2$

Aggregates

Fine- Natural River Sand, relative density (RD) = 2.6

Coarse- Crushed rock, relative density (RD) = 2.7

Maximum water-cement ratio  $\left(\frac{w}{c}\right)_{max} = 0.55$

### **Stage two**

Slump = 10-30 mm

Free water (from concrete mix design table) = 0.53 (DOE, 1975)

Free water content =  $190Kg/mm^3$  (from concrete mix design table)

### **Stage three**

Cement content,  $CC = \frac{W}{0.55} = \frac{190}{0.55} = 346 Kg/mm^3$

Minimum cement content =  $300 Kg/mm^3$

### **Stage four**

Relative density of aggregates (SSD) = 2.7

Concrete Density = 2435 Kg/mm<sup>3</sup>

Total aggregates = 2435 - 346 - 190 = 1899 Kg/mm<sup>3</sup>

### **Stage five**

Grading of aggregates, Assume zone 2

Proportion of fine aggregates =  $31 - 39\% = \frac{31+39}{2} = 35\%$  (From concrete mix design table)

Fine content (sand) =  $0.35 \times 1899 = 664.65$  Kg/mm<sup>3</sup>

Coarse Content (granite) =  $1899 - 664.65 = 1234.35$  Kg/mm<sup>3</sup>

### **Quantities required**

Cement = 346 Kg/mm<sup>3</sup>

Water = 190 Kg/mm<sup>3</sup>

Fine aggregates (sand) = 664.65 Kg/mm<sup>3</sup>

Coarse aggregate (granite) = 1234.35 Kg/mm<sup>3</sup>

### **Proportion**

Water : Cement : Sand : Coarse aggregates

0.55 : 1 : 1.92 : 3.57

TABLE AI: Summary of Mix Design for normal Concrete cubes at different grade

| Concrete type | Mix         | W/C ratio | Mean strength<br>(N/mm <sup>2</sup> ) |
|---------------|-------------|-----------|---------------------------------------|
| C20           | 1:2:4       | 0.53      | 64.5                                  |
| C25           | 1:1.87:3.87 | 0.53      | 67.8                                  |
| C30           | 1:1.9:3.6   | 0.53      | 81.2                                  |

## APPENDIX B: COMPRESSIVES STRENGTH TESTS AND STRESS-STRAIN VERIFICATION FOR CUBES WITH AND WITHOUT PVC PIPES

Table BI: Compressive strength for normal Concrete Cubes

| S/N | Days | Area(mm <sup>2</sup> ) | Forces(kN) | Compressive strength (N/mm <sup>2</sup> ) |
|-----|------|------------------------|------------|-------------------------------------------|
| 1   | 7    | 22500                  | 667        | 20.8                                      |
| 2   | 14   | 22500                  | 755        | 25.1                                      |
| 3   | 21   | 22500                  | 765        | 29.4                                      |
| 4   | 28   | 22500                  | 792        | 31.5                                      |

Table BII: Concrete Cube with PVC4 pipes at the Edge

| S/N | Days | Area(mm <sup>2</sup> ) | Forces(kN) | Compressive strength (N/mm <sup>2</sup> ) |
|-----|------|------------------------|------------|-------------------------------------------|
| 1   | 7    | 22500                  | 277.7      | 12.3                                      |
| 2   | 14   | 22500                  | 365.4      | 16.2                                      |
| 3   | 21   | 22500                  | 396.5      | 17.6                                      |
| 4   | 28   | 22500                  | 486.9      | 21.6                                      |

Table BIII: Concrete Cube PVC4 pipes at the middle

| S/N | Days | Area(mm <sup>2</sup> ) | Forces(kN) | Compressive strength (N/mm <sup>2</sup> ) |
|-----|------|------------------------|------------|-------------------------------------------|
| 1   | 7    | 22500                  | 243.0      | 10.8                                      |
| 2   | 14   | 22500                  | 324.0      | 14.4                                      |
| 3   | 21   | 22500                  | 378.0      | 16.8                                      |
| 4   | 28   | 22500                  | 432.1      | 19.2                                      |

Table BIV: Concrete Cube with PVC3 pipes below neutral axis

| S/N | Days | Area(mm <sup>2</sup> ) | Forces(kN) | Compressive strength<br>(kN/mm <sup>2</sup> ) |
|-----|------|------------------------|------------|-----------------------------------------------|
| 1   | 7    | 22500                  | 303.3      | 13.5                                          |
| 2   | 14   | 22500                  | 365.4      | 16.2                                          |
| 3   | 21   | 22500                  | 525.0      | 23.3                                          |
| 4   | 28   | 22500                  | 609.0      | 27.0                                          |

Table BV: Variation of compressive strength (N/mm<sup>2</sup>) with and without PVC pipes

| S/N | Age | Normal | 4 PVC at Edges | 4 PVC at Middle | PVC3 below N.A |
|-----|-----|--------|----------------|-----------------|----------------|
| 1   | 7   | 20.8   | 12.3           | 10.8            | 13.5           |
| 2   | 14  | 25.1   | 16.2           | 14.4            | 16.2           |
| 3   | 21  | 29.4   | 17.6           | 16.8            | 23.3           |
| 4   | 28  | 31.5   | 21.6           | 19.2            | 27.0           |

Table BVI: Variation of crushing Force (kN) with and without PVC pipes

| S/No | Age | Normal | PVC4 at Edges | PVC4 at Middle | PVC3 below N.A |
|------|-----|--------|---------------|----------------|----------------|
| 1    | 7   | 667    | 277.7         | 243            | 303.3          |
| 2    | 14  | 755    | 365.4         | 324            | 365.4          |
| 3    | 21  | 765    | 396.5         | 378            | 525.0          |
| 4    | 28  | 792    | 486.9         | 432            | 609.0          |

Table BVII: Stress-Strain measurement for Reinforced Concrete cubes (Y10 grade C20)

| S/No | Age | Young's modulus (E) | Yield strain ( $\epsilon_o \times 10^{-3}$ ) | Ultimate strain ( $\epsilon_u \times 10^{-3}$ ) | Stress (MPa) |
|------|-----|---------------------|----------------------------------------------|-------------------------------------------------|--------------|
| 1    | 0   |                     | 0                                            | 0                                               | 0            |
| 2    | 7   | 35.18               | 2.1                                          | 4.8                                             | 33.3         |
| 3    | 14  |                     | 2.1                                          | 4.7                                             | 37.5         |
| 4    | 21  |                     | 2.2                                          | 4.4                                             | 62.5         |
| 5    | 28  |                     | 2.5                                          | 4.3                                             | 64.5         |

Table BVIII: Stress-Strain measurement for Control Concrete cubes (grade C20)

| S/No | Age | Young's modulus (E) | Yield strain ( $\epsilon_o \times 10^{-3}$ ) | Ultimate strain ( $\epsilon_u \times 10^{-3}$ ) | Stress (MPa) |
|------|-----|---------------------|----------------------------------------------|-------------------------------------------------|--------------|
| 1    | 0   |                     | 0                                            | 0                                               | 0            |
| 2    | 7   | 27.4                | 2.0                                          | 3.4                                             | 27.2         |
| 3    | 14  |                     | 2.0                                          | 3.4                                             | 28.0         |
| 4    | 21  |                     | 2.1                                          | 3.3                                             | 30.0         |
| 5    | 28  |                     | 2.1                                          | 3.3                                             | 32.5         |

Table BIX: Stress-Strain measurement for Reinforced Concrete cubes (Y10 grade C25)

| S/No | Age | Young's modulus<br>(E) | Yield strain<br>( $\epsilon_o \times 10^{-3}$ ) | Ultimate strain<br>( $\epsilon_u \times 10^{-3}$ ) | Stress (MPa) |
|------|-----|------------------------|-------------------------------------------------|----------------------------------------------------|--------------|
| 1    | 0   |                        | 0                                               | 0                                                  | 0            |
| 2    | 7   |                        | 2.2                                             | 4.7                                                | 40.0         |
| 3    | 14  | 37                     | 2.3                                             | 4.5                                                | 51.2         |
| 4    | 21  |                        | 2.4                                             | 4.4                                                | 57.2         |
| 5    | 28  |                        | 2.5                                             | 4.3                                                | 67.8         |

Table BX: Stress-Strain measurement for Reinforced Concrete cubes (Y10 grade C30)

| S/No | Age | Young's modulus<br>(E) | Yield strain<br>( $\epsilon_o \times 10^{-3}$ ) | Ultimate strain<br>( $\epsilon_u \times 10^{-3}$ ) | Stress (MPa) |
|------|-----|------------------------|-------------------------------------------------|----------------------------------------------------|--------------|
| 1    | 0   |                        | 0                                               | 0                                                  | 0            |
| 2    | 7   |                        | 2.1                                             | 4.8                                                | 34.3         |
| 3    | 14  | 40.75                  | 2.5                                             | 4.3                                                | 70.0         |
| 4    | 21  |                        | 2.6                                             | 4.2                                                | 80.1         |
| 5    | 28  |                        | 2.6                                             | 4.2                                                | 82.2         |

Table BXI: R-Values based on experimental results for Reinforced Concrete Cubes

| S/No | Stress<br>(MPa) | Young's modulus<br>( $E_c$ ) | Yield strain<br>( $\epsilon_o \times 10^{-3}$ ) | Young's modulus<br>( $E_o = \frac{\text{stress}}{\text{strain}}$ ) | $R = \frac{E_c}{E_o}$ |
|------|-----------------|------------------------------|-------------------------------------------------|--------------------------------------------------------------------|-----------------------|
| 1    | 82.2            | 40.8                         | 2.6                                             | 31.6                                                               | 1.29                  |
| 2    | 67.8            | 37.0                         | 2.5                                             | 27.0                                                               | 1.37                  |
| 3    | 64.5            | 35.2                         | 2.5                                             | 25.8                                                               | 1.36                  |

Table BXII: Stress-strain verification models for RC cubes (grade C30)

| S/No | Strain<br>Ratio | Desayi et al<br>Model, 1964 | Saenz, L.P.<br>Model, 1964<br>(N/mm <sup>2</sup> ) | EXPT., 2017 | Proposed Poly.<br>Equation, 2017 |
|------|-----------------|-----------------------------|----------------------------------------------------|-------------|----------------------------------|
| 1    | 0               | 0                           | 0                                                  | 0           | 0.0                              |
| 2    | 0.440           | 20.00                       | 34.40                                              | 34.3        | 39.21                            |
| 3    | 0.590           | 53.08                       | 65.39                                              | 70.0        | 70.91                            |
| 4    | 0.625           | 63.68                       | 79.12                                              | 80.1        | 77.62                            |
| 5    | 0.633           | 64.12                       | 82.9                                               | 82.2        | 82.85                            |

Table BXIII: Stress-strain verification for RC cubes (grade C25)

| S/No | Strain Ratio | Desayi et al Model, 1964 | Saenz, L.P. Model, 1964<br>(N/mm <sup>2</sup> ) | EXPT., 2017 | Proposed Poly. Equation, 2017 |
|------|--------------|--------------------------|-------------------------------------------------|-------------|-------------------------------|
| 1    | 0            | 0                        | 0                                               | 0           | 0.0                           |
| 2    | 0.47         | 24.76                    | 40.13                                           | 40          | 45.37                         |
| 3    | 0.51         | 34.17                    | 46.92                                           | 51.2        | 53.15                         |
| 4    | 0.55         | 40.84                    | 55.87                                           | 57.2        | 60.93                         |
| 5    | 0.58         | 50.68                    | 68.24                                           | 67.8        | 66.65                         |

Table BXIV: Stress-strain verification for RC cubes (grade C20)

| S/No | Strain Ratio | Desayi et al Model, 1964 | Saenz, L.P. Model, 1964<br>(N/mm <sup>2</sup> ) | EXPT., 2017 | Proposed Poly. Equation, 2017 |
|------|--------------|--------------------------|-------------------------------------------------|-------------|-------------------------------|
| 1    | 0            | 0                        | 0                                               | 0           | 0                             |
| 2    | 0.44         | 19.37                    | 33.33                                           | 33.3        | 33.89                         |
| 3    | 0.46         | 22.34                    | 36.34                                           | 37.5        | 37.68                         |
| 4    | 0.51         | 41.7                     | 61.77                                           | 62.5        | 68.60                         |
| 5    | 0.57         | 47.5                     | 64.53                                           | 64.5        | 65.13                         |

Table BXV: Stress-Strain measurement for Control Concrete cubes (grade C20)

| S/No | Age | Young's modulus (E) | Yield strain ( $\epsilon_o \times 10^{-3}$ ) | Ultimate strain ( $\epsilon_u \times 10^{-3}$ ) | Stress (MPa) |
|------|-----|---------------------|----------------------------------------------|-------------------------------------------------|--------------|
| 1    | 0   |                     | 0                                            | 0                                               | 0            |
| 2    | 7   |                     | 1.8                                          | 3.9                                             | 16.1         |
| 3    | 14  | 23.45               | 1.9                                          | 3.6                                             | 22.0         |
| 4    | 21  |                     | 1.9                                          | 3.6                                             | 23.2         |
| 5    | 28  |                     | 2.0                                          | 3.5                                             | 25.3         |

Table BXVI: Stress-Strain measurement for Control Concrete cubes (grade C25)

| S/No | Age | Young's modulus<br>(E) | Yield strain<br>( $\epsilon_o \times 10^{-3}$ ) | Ultimate strain<br>( $\epsilon_u \times 10^{-3}$ ) | Stress (MPa) |
|------|-----|------------------------|-------------------------------------------------|----------------------------------------------------|--------------|
| 1    | 0   | 26.05                  | 0                                               | 0                                                  | 0            |
| 2    | 7   |                        | 1.9                                             | 3.7                                                | 20.8         |
| 3    | 14  |                        | 2.0                                             | 3.5                                                | 25.1         |
| 4    | 21  |                        | 2.0                                             | 3.4                                                | 29.4         |
| 5    | 28  |                        | 2.1                                             | 3.3                                                | 31.5         |

Table BXVII: Stress-Strain measurement for Control Concrete cubes (grade C30)

| S/No | Age | Young's modulus<br>(E) | Yield strain<br>( $\epsilon_o \times 10^{-3}$ ) | Ultimate strain<br>( $\epsilon_u \times 10^{-3}$ ) | Stress (MPa) |
|------|-----|------------------------|-------------------------------------------------|----------------------------------------------------|--------------|
| 1    | 0   | 27.4                   | 0                                               | 0                                                  | 0            |
| 2    | 7   |                        | 2.0                                             | 3.4                                                | 27.2         |
| 3    | 14  |                        | 2.0                                             | 3.4                                                | 28.0         |
| 4    | 21  |                        | 2.1                                             | 3.3                                                | 30.0         |
| 5    | 28  |                        | 2.1                                             | 3.3                                                | 32.5         |

Table BXVIII: R-Values based on experimental results for Control Concrete cubes

| S/No | Stress<br>(MPa) | Young's modulus<br>( $E_c$ ) | Yield strain<br>( $\epsilon_o \times 10^{-3}$ ) | Young's modulus<br>( $E_o = \frac{\text{stress}}{\text{strain}}$ ) | $R = \frac{E_c}{E_o}$ |
|------|-----------------|------------------------------|-------------------------------------------------|--------------------------------------------------------------------|-----------------------|
| 1    | 32.5            | 27.4                         | 2.1                                             | 15.5                                                               | 1.77                  |
| 2    | 31.5            | 26.1                         | 2.1                                             | 15.0                                                               | 1.74                  |
| 3    | 27.0            | 22.4                         | 2.0                                             | 13.5                                                               | 1.66                  |

Table BXIX: Stress-strain verification for control concrete cubes (grade C20)

| S/No | Strain<br>Ratio | Desayi et al<br>Model, 1964 | Saenz, L.P.<br>Model, 1964<br>(N/mm <sup>2</sup> ) | EXPT., 2017 | Proposed Poly.<br>Equation, 2017 |
|------|-----------------|-----------------------------|----------------------------------------------------|-------------|----------------------------------|
| 1    | 0               | 0                           | 0                                                  | 0           | 0.0                              |
| 2    | 0.45            | 11.3                        | 16.07                                              | 16.1        | 16.34                            |
| 3    | 0.53            | 17.3                        | 21.86                                              | 22          | 21.90                            |
| 4    | 0.54            | 18.4                        | 22.89                                              | 23.2        | 21.90                            |
| 5    | 0.56            | 20.6                        | 25.25                                              | 25.3        | 25.32                            |

Table BXX: Stress-strain verification for control concrete cubes (grade C25)

| S/No | Strain Ratio | Desayi et al Model, 1964 | Saenz, L.P. Model, 1964 (N/mm <sup>2</sup> ) | EXPT., 2017 | Proposed Poly. Equation, 2017 |
|------|--------------|--------------------------|----------------------------------------------|-------------|-------------------------------|
| 1    | 0            | 0                        | 0                                            | 0           | 0.0                           |
| 2    | 0.51         | 15.9                     | 21                                           | 20.8        | 21.90                         |
| 3    | 0.56         | 20.5                     | 25                                           | 25.1        | 26.96                         |
| 4    | 0.61         | 25.1                     | 29.8                                         | 29.4        | 26.96                         |
| 5    | 0.63         | 27.5                     | 32.1                                         | 31.5        | 31.99                         |

Table BXXI: Stress-strain verification for control concrete cubes (grade C30)

| S/No | Strain Ratio | Desayi et al Model, 1964 | Saenz, L.P. Model, 1964 (N/mm <sup>2</sup> ) | EXPT., 2017 | Proposed Poly. Equation, 2017 |
|------|--------------|--------------------------|----------------------------------------------|-------------|-------------------------------|
| 1    | 0            | 0                        | 0                                            | 0           | 0.0                           |
| 2    | 0.585        | 22.6                     | 27.22                                        | 27.2        | 26.96                         |
| 3    | 0.593        | 23.7                     | 27.74                                        | 28          | 26.96                         |
| 4    | 0.616        | 25.9                     | 29.96                                        | 30          | 31.42                         |
| 5    | 0.639        | 28.6                     | 32.50                                        | 32.5        | 31.99                         |

Table BXXII: Stress-Strain measurement for Concrete cubes with 4PVC pipes@ edges

| S/No | Age | Young's modulus (E) | Yield strain ( $\epsilon_o \times 10^{-3}$ ) | Ultimate strain ( $\epsilon_u \times 10^{-3}$ ) | Stress (MPa) |
|------|-----|---------------------|----------------------------------------------|-------------------------------------------------|--------------|
| 1    | 0   |                     | 0                                            | 0                                               | 0            |
| 2    | 7   | 20.69               | 1.6                                          | 4.2                                             | 12.3         |
| 3    | 14  |                     | 1.8                                          | 3.9                                             | 16.2         |
| 4    | 21  |                     | 1.8                                          | 3.8                                             | 17.6         |
| 5    | 28  |                     | 1.9                                          | 3.6                                             | 21.6         |

Table BXXIII: Stress-Strain measurement for Concrete cubes with 4PVC pipes@ middle

| S/No | Age | Young's modulus<br>(E) | Yield strain<br>( $\epsilon_o \times 10^{-3}$ ) | Ultimate strain<br>( $\epsilon_u \times 10^{-3}$ ) | Stress (MPa) |
|------|-----|------------------------|-------------------------------------------------|----------------------------------------------------|--------------|
| 1    | 0   | 19.7                   | 0                                               | 0                                                  | 0            |
| 2    | 7   |                        | 1.6                                             | 4.3                                                | 10.8         |
| 3    | 14  |                        | 1.7                                             | 4.0                                                | 14.4         |
| 4    | 21  |                        | 1.8                                             | 3.9                                                | 16.8         |
| 5    | 28  |                        | 1.8                                             | 3.7                                                | 19.2         |

Table BXXIV: Stress-Strain measurement for Concrete cubes with 3PVC below neutral axis

| S/No | Age | Young's modulus<br>(E) | Yield strain<br>( $\epsilon_o \times 10^{-3}$ ) | Ultimate strain<br>( $\epsilon_u \times 10^{-3}$ ) | Stress (MPa) |
|------|-----|------------------------|-------------------------------------------------|----------------------------------------------------|--------------|
| 1    | 0   | 22.4                   | 0                                               | 0                                                  | 0            |
| 2    | 7   |                        | 1.7                                             | 4.1                                                | 13.5         |
| 3    | 14  |                        | 1.8                                             | 3.9                                                | 16.2         |
| 4    | 21  |                        | 1.9                                             | 3.6                                                | 23.3         |
| 5    | 28  |                        | 2.0                                             | 3.4                                                | 27.0         |

Table BXXV: R-Values based on experimental results for PVC cubes

| S/No | Stress<br>(MPa) | Young's<br>modulus<br>( $E_c$ ) | Yield strain<br>( $\epsilon_o \times 10^{-3}$ ) | Young's<br>modulus<br>( $E_o = \frac{\text{stress}}{\text{strain}}$ ) | $R = \frac{E_c}{E_o}$ |
|------|-----------------|---------------------------------|-------------------------------------------------|-----------------------------------------------------------------------|-----------------------|
| 1    | 25.3            | 23.5                            | 2.0                                             | 12.7                                                                  | 1.85                  |
| 2    | 21.6            | 20.7                            | 1.9                                             | 11.4                                                                  | 1.82                  |
| 3    | 19.2            | 19.7                            | 1.8                                             | 10.7                                                                  | 1.84                  |

Table BXXVI: Stress-strain verification for PVC concrete cubes (3 PVC @ edges)

| S/No | Strain<br>Ratio | Desayi et al<br>Model, 1964 | Saenz, L.P.<br>Model, 1964<br>(N/mm <sup>2</sup> ) | EXPT., 2017 | Proposed Poly.<br>Equation, 2017 |
|------|-----------------|-----------------------------|----------------------------------------------------|-------------|----------------------------------|
| 1    | 0               | 0                           | 0                                                  | 0           | 0.0                              |
| 2    | 0.41            | 9.1                         | 13.5                                               | 13.5        | 13.1                             |
| 3    | 0.45            | 11.7                        | 15.7                                               | 16.2        | 17.3                             |
| 4    | 0.54            | 19.0                        | 22.1                                               | 23.3        | 22.1                             |
| 5    | 0.59            | 23.2                        | 27.2                                               | 27.0        | 27.4                             |

Table BXXVII: Stress-strain verification for PVC concrete cubes (4 PVC @ middle)

| S/No | Strain Ratio | Desayi et al Model, 1964 | Saenz, L.P. Model, 1964 (N/mm <sup>2</sup> ) | EXPT., 2017 | Proposed Poly. Equation, 2017 |
|------|--------------|--------------------------|----------------------------------------------|-------------|-------------------------------|
| 1    | 0            | 0                        | 0                                            | 0           | 0.0                           |
| 2    | 0.37         | 6.7                      | 10.8                                         | 10.8        | 10.3                          |
| 3    | 0.43         | 10.1                     | 14                                           | 14.4        | 14.3                          |
| 4    | 0.46         | 12.4                     | 16.3                                         | 16.8        | 18.2                          |
| 5    | 0.49         | 14.7                     | 19.3                                         | 19.2        | 19.0                          |

Table BXXVIII: Stress-strain verification for PVC concrete cubes (4 PVC @ edges)

| S/No | Strain Ratio | Desayi et al Model, 1964 | Saenz, L.P. Model, 1964 (N/mm <sup>2</sup> ) | EXPT., 2017 | Proposed Poly. Equation, 2017 |
|------|--------------|--------------------------|----------------------------------------------|-------------|-------------------------------|
| 1    | 0            | 0                        | 0                                            | 0           | 0.0                           |
| 2    | 0.39         | 8.02                     | 12.27                                        | 12.3        | 10.3                          |
| 3    | 0.45         | 11.7                     | 15.65                                        | 16.2        | 18.2                          |
| 4    | 0.47         | 13.12                    | 16.75                                        | 17.6        | 18.2                          |
| 5    | 0.52         | 17.23                    | 21.5                                         | 21.6        | 21.8                          |

## APPENDIX C: EXPERIMENTAL RESULTS FOR FLEXURAL CAPACITY OF BEAMS

**Table CI: Reinforced Concrete beam with 1 embedded PVC pipe**

| S/N | AGE<br>(mm) | SAMPLE | $F_0(kN)$ | $F_u(kN)$ | $\delta_o$ | $\delta_u$ | a<br>(mm) | b<br>(mm) | $F_{mr}$<br>(N/mm <sup>2</sup> ) |
|-----|-------------|--------|-----------|-----------|------------|------------|-----------|-----------|----------------------------------|
| 1   | 7           | RCPVC1 | 12.2      | 15.4      | 2.0        | 4.0        | 40.5      | 39.5      | 4.1                              |
| 2   | 14          | RCPVC1 | 10.0      | 22.0      | 1.5        | 5.0        | 18.0      | 37.5      | 5.9                              |
| 3   | 21          | RCPVC1 | 4.0       | 22.6      | 1.0        | 8.5        | 39.0      | 36.5      | 6.0                              |
| 4   | 28          | RCPVC1 | 9.0       | 25.0      | 3.0        | 9.0        | 39.5      | 39.5      | 6.7                              |

**Table CII: Reinforced Concrete beam with 2 embedded PVC pipe**

| S/N | AGE<br>(mm) | SAMPLE | $F_0(kN)$ | $F_u(kN)$ | $\delta_o$ | $\delta_u$ | a<br>(mm) | b<br>(mm) | $F_{mr}$<br>(N/mm <sup>2</sup> ) |
|-----|-------------|--------|-----------|-----------|------------|------------|-----------|-----------|----------------------------------|
| 1   | 7           | RCPVC2 | 6.9       | 11.3      | 2.0        | 6.0        | 46.0      | 34.0      | 3.0                              |
| 2   | 14          | RCPVC2 | 5.6       | 13.7      | 2.0        | 6.0        | 22.0      | 54.5      | 3.7                              |
| 3   | 21          | RCPVC2 | 5.0       | 14.5      | 1.5        | 7.0        | 34.5      | 39.0      | 3.9                              |
| 4   | 28          | RCPVC2 | 7.5       | 15.0      | 2.5        | 9.5        | 13.5      | 42        | 4.0                              |

**Table CIII: Reinforced Concrete beam with 3 embedded PVC pipe**

| S/N | AGE<br>(mm) | SAMPLE | $F_0(kN)$ | $F_u(kN)$ | $\delta_o$ | $\delta_u$ | a<br>(mm) | b<br>(mm) | $F_{mr}$<br>(N/mm <sup>2</sup> ) |
|-----|-------------|--------|-----------|-----------|------------|------------|-----------|-----------|----------------------------------|
| 1   | 7           | RCPVC3 | 7.1       | 11.3      | 1.2        | 3.5        | 35.0      | 44.5      | 3.0                              |
| 2   | 14          | RCPVC3 | 7.4       | 12.1      | 2.0        | 3.7        | 34.0      | 49.0      | 3.2                              |
| 3   | 21          | RCPVC3 | 7.6       | 12.9      | 2.5        | 6.0        | 36.5      | 36.5      | 3.4                              |
| 4   | 28          | RCPVC3 | 6.5       | 15.0      | 1.7        | 6.0        | 19.5      | 40.0      | 4.0                              |

**Table CIV: Unreinforced Concrete beam with 1 embedded PVC pipe**

| S/N | AGE | SAMPLE | $F_0(kN)$ |     | $\delta_o$ | $\delta_u$ | A    | b    | $F_{mr}$ |
|-----|-----|--------|-----------|-----|------------|------------|------|------|----------|
|     |     |        |           |     |            |            |      |      |          |
| 1   | 7   | CBPVC1 | 0         | 2.8 | 0          | 1.0        | 36.5 | 37.5 | 0.75     |
| 2   | 14  | CBPVC1 | 0         | 6.2 | 0          | 3.0        | 41.0 | 34.0 | 1.65     |
| 3   | 21  | CBPVC1 | 0         | 6.5 | 0          | 6.0        | 38.0 | 41.0 | 1.73     |
| 4   | 28  | CBPVC1 | 0         | 7.0 | 0          | 9.0        | 41.0 | 36.0 | 1.87     |

**Table CV: Reinforced Concrete beam (Solid beam)**

| S/N | AGE | SAMPLE | $F_0(kN)$ |      | $\delta_o$ | $\delta_u$ | a    | b    | $F_{mr}$ |
|-----|-----|--------|-----------|------|------------|------------|------|------|----------|
|     |     |        |           |      |            |            |      |      |          |
| 1   | 7   | RCBM   | 8.5       | 24.0 | 1.0        | 6.0        | 37.0 | 44.0 | 6.4      |
| 2   | 14  | RCBM   | 12.0      | 24.4 | 1.1        | 9.5        | 41.0 | 16.0 | 6.5      |
| 3   | 21  | RCBM   | 18.0      | 25.2 | 4.0        | 9.7        | 20.0 | 28.0 | 6.7      |
| 4   | 28  | RCBM   | 15.0      | 26.0 | 1.5        | 14.0       | 31.0 | 35.5 | 6.9      |

**Table CVI: Unreinforced Concrete beam with 2 embedded PVC pipe**

| S/N | AGE | SAMPLE | $F_0(kN)$ |     | $\delta_o$ | $\delta_u$ | A    | b    | $F_{mr}$ |
|-----|-----|--------|-----------|-----|------------|------------|------|------|----------|
|     |     |        |           |     |            |            |      |      |          |
| 1   | 7   | CBPVC2 | 0         | 2.8 | 0          | 1.0        | 36.5 | 37.5 | 0.75     |
| 2   | 14  | CBPVC2 | 0         | 6.2 | 0          | 3.0        | 41.0 | 34.0 | 1.65     |
| 3   | 21  | CBPVC2 | 0         | 6.5 | 0          | 6.0        | 38.0 | 41.0 | 1.73     |
| 4   | 28  | CBPVC2 | 0         | 7.0 | 0          | 9.0        | 41.0 | 36.0 | 1.87     |

## APPENDIX CVII: Unreinforced Concrete beam without embedded PVC pipe

| S/N | AGE | SAMPLE | $F_0(kN)$ $F_0(kN)$ |     | $\delta_o$ | $\delta_u$ | A    | b    | $F_{mr}$ |
|-----|-----|--------|---------------------|-----|------------|------------|------|------|----------|
|     |     |        |                     |     |            |            |      |      |          |
| 1   | 7   | CTBM   | 4.0                 | 5.3 | 0.6        | 1.0        | 21.0 | 61.0 | 1.41     |
| 2   | 14  | CTBM   | 0.0                 | 7.2 | 0.0        | 0.5        | 21.5 | 43.0 | 1.92     |
| 3   | 21  | CTBM   | 0.0                 | 7.6 | 0.0        | 1.5        | 34.5 | 42.2 | 2.03     |
| 4   | 28  | CTBM   | 0.0                 | 8.5 | 0.0        | 4.3        | 34.5 | 43.5 | 2.27     |

Table CVIII: Variation of ultimate load with deflection, modulus of rupture, stress and strain

| Days | Sample | F (kN) | $\delta$ (mm) | $f_{mr}$ (N/mm <sup>2</sup> ) | $\sigma$ (N/mm <sup>2</sup> ) | $\epsilon$ |
|------|--------|--------|---------------|-------------------------------|-------------------------------|------------|
| 0    | RCPVC1 | 0      | 0             | 0                             | 0                             | 0          |
| 7    | RCPVC1 | 15.4   | 4.0           | 4.1                           | 39.28                         | 0.00019    |
| 14   | RCPVC1 | 22.0   | 5.0           | 5.9                           | 55.12                         | 0.00027    |
| 21   | RCPVC1 | 22.6   | 8.5           | 6.0                           | 56.56                         | 0.00028    |
| 28   | RCPVC1 | 25.0   | 9.0           | 6.7                           | 62.31                         | 0.00030    |

Table CIX: Variation of ultimate load with deflection, modulus of rupture, stress and strain

| DAYS | SAMPLE | F (kN) | $\delta$ (mm) | $f_{mr}$ (N/mm <sup>2</sup> ) | $\sigma$ (N/mm <sup>2</sup> ) | $\epsilon$ |
|------|--------|--------|---------------|-------------------------------|-------------------------------|------------|
| 0    | RCPVC2 | 0      | 0             | 0                             | 0                             | 0          |
| 7    | RCPVC2 | 11.3   | 6.0           | 3.0                           | 29.44                         | 0.000144   |
| 14   | RCPVC2 | 13.7   | 6.0           | 3.7                           | 35.2                          | 0.000171   |
| 21   | RCPVC2 | 14.5   | 6.5           | 3.9                           | 37.12                         | 0.000181   |
| 28   | RCPVC2 | 15.0   | 7.0           | 4.0                           | 38.32                         | 0.000187   |

Table CX: Variation of ultimate load with deflection, modulus of rupture, stress and strain

| DAYS | SAMPLE | F (kN) | $\delta$ (mm) | $f_{mr}$ (N/mm <sup>2</sup> ) | $\sigma$ (N/mm <sup>2</sup> ) | $\epsilon$ |
|------|--------|--------|---------------|-------------------------------|-------------------------------|------------|
| 0    | RCPVC3 | 0      | 0             | 0                             | 0                             | 0          |
| 7    | RCPVC3 | 11.3   | 3.0           | 3.0                           | 29.44                         | 0.00144    |
| 14   | RCPVC3 | 12.1   | 3.5           | 3.2                           | 31.36                         | 0.00153    |
| 21   | RCPVC3 | 12.9   | 4.5           | 3.4                           | 33.28                         | 0.00162    |
| 28   | RCPVC3 | 15.0   | 6.0           | 4.0                           | 38.32                         | 0.00187    |

Table CXI: Variation of ultimate load with deflection, modulus of rupture, stress and strain

| DAYS | SAMPLE | F (Kn) | $\delta$ (mm) | $f_{mr}$ (N/mm <sup>2</sup> ) | $\sigma$ (N/mm <sup>2</sup> ) | $\epsilon$ |
|------|--------|--------|---------------|-------------------------------|-------------------------------|------------|
| 0    | RCBM   | 0      | 0             | 0                             | 0                             | 0          |
| 7    | RCBM   | 24     | 6             | 6.4                           | 59.92                         | 0.00029    |
| 14   | RCBM   | 24.4   | 9.7           | 6.5                           | 60.88                         | 0.00030    |
| 21   | RCBM   | 25.2   | 9.5           | 6.7                           | 62.79                         | 0.00031    |
| 28   | RCBM   | 26.0   | 14            | 6.9                           | 64.71                         | 0.00032    |

Table CXII Variation of modulus of rupture with concrete age for different beams

|     |     | RCBM                          | RCPVC1 | RCPVC2 | RCPVC3 |
|-----|-----|-------------------------------|--------|--------|--------|
| S/N | Age | $f_{mr}$ (N/mm <sup>2</sup> ) |        |        |        |
| 1   | 0   | 0                             | 0      | 0      | 0      |
| 2   | 7   | 6.4                           | 4.1    | 3.0    | 3.0    |
| 3   | 14  | 6.5                           | 5.9    | 3.7    | 3.2    |
| 4   | 21  | 6.7                           | 6.0    | 3.9    | 3.4    |
| 5   | 28  | 6.9                           | 6.7    | 4.0    | 4.0    |

Table CXIII: Variation of deflection with concrete age for different beams

|     |     | RCBM          | RCPVC1 | RCPVC2 | RCPVC3 |
|-----|-----|---------------|--------|--------|--------|
| S/N | Age | $\delta$ (mm) |        |        |        |
| 1   | 0   | 0             | 0      | 0      | 0      |
| 2   | 7   | 6.0           | 4.0    | 6.0    | 4.5    |
| 3   | 14  | 9.7           | 5.0    | 6.0    | 6.0    |
| 4   | 21  | 9.5           | 8.5    | 9.5    | 3.5    |
| 5   | 28  | 14.0          | 9.0    | 7.0    | 6.0    |

Table CXIV: Variation of strain with age of concrete for different beams

| S/N | Age | RCBM | RCPVC1<br>$\epsilon ( \times 10^{-4})$ | RCPVC2 | RCPVC3 |
|-----|-----|------|----------------------------------------|--------|--------|
| 1   | 0   | 0    | 0                                      | 0      | 0      |
| 2   | 7   | 2.9  | 1.92                                   | 1.44   | 1.44   |
| 3   | 14  | 2.97 | 2.69                                   | 1.71   | 1.71   |
| 4   | 21  | 3.06 | 2.76                                   | 1.81   | 1.81   |
| 5   | 28  | 3.16 | 3.04                                   | 1.87   | 1.87   |

Table CXV: Variation of force, deflection, modulus of ruptures and strain with age of concrete

| DAYS | SAMPLE | F (kN) | $\delta$ (mm) | $f_{mr}$ (N/mm <sup>2</sup> ) | $\sigma$ (N/mm <sup>2</sup> ) | $\epsilon ( \times 10^{-4})$ |
|------|--------|--------|---------------|-------------------------------|-------------------------------|------------------------------|
| 0    | CTBM   | 0      | 0             | 0                             | 0                             | 0                            |
| 7    | CTBM   | 5.3    | 0             | 1.41                          | 15.05                         | 0.70                         |
| 14   | CTBM   | 7.2    | 1.2           | 1.92                          | 19.61                         | 0.96                         |
| 21   | CTBM   | 7.6    | 1.5           | 2.03                          | 20.57                         | 1.00                         |
| 28   | CTBM   | 8.5    | 4.3           | 2.27                          | 22.70                         | 1.11                         |

Table CXVI: Variation of force, deflection, modulus of rupture and strain with age of concrete

| DAYS | SAMPLE | F (kN) | $\delta$ (mm) | $f_{mr}$<br>(N/mm <sup>2</sup> ) | $\sigma$ (N/mm <sup>2</sup> ) | $\epsilon ( \times 10^{-4})$ |
|------|--------|--------|---------------|----------------------------------|-------------------------------|------------------------------|
| 0    | CBPVC1 | 0      | 0             | 0                                | 0                             | 0                            |
| 7    | CBPVC1 | 3.6    | 1.0           | 0.96                             | 10.97                         | 0.55                         |
| 14   | CBPVC1 | 7.5    | 3.0           | 2.00                             | 20.33                         | 0.99                         |
| 21   | CBPVC1 | 8.2    | 3.5           | 2.19                             | 22.00                         | 1.07                         |
| 28   | CBPVC1 | 8.5    | 4.0           | 2.27                             | 22.73                         | 1.19                         |

Table CXVII: Variation of force, deflection, modulus of ruptures and strain with age of concrete

| DAYS | SAMPLE | F (kN) | $\delta$ (mm) | $f_{mr}$<br>(N/mm <sup>2</sup> ) | $\sigma$ (N/mm <sup>2</sup> ) | $\epsilon ( \times 10^{-4})$ |
|------|--------|--------|---------------|----------------------------------|-------------------------------|------------------------------|
| 0    | CBPVC2 | 0      | 0             | 0                                | 0                             | 0                            |
| 7    | CBPVC2 | 2.8    | 1             | 0.75                             | 9.05                          | 0.44                         |
| 14   | CBPVC2 | 6.2    | 3             | 1.65                             | 17.21                         | 0.84                         |
| 21   | CBPVC2 | 6.5    | 6             | 1.73                             | 17.93                         | 0.88                         |
| 28   | CBPVC2 | 7.0    | 9             | 1.87                             | 19.12                         | 0.93                         |

Table CXVIII: Variation of strain with age of concrete

| S/NO | AGE | CTBM                               | CBPVC1 | CBPVC2 |
|------|-----|------------------------------------|--------|--------|
|      |     | $\varepsilon \text{ (x } 10^{-4})$ |        |        |
| 1    | 0   | 0                                  | 0      | 0      |
| 2    | 7   | 0.70                               | 0.55   | 0.44   |
| 3    | 14  | 0.96                               | 0.99   | 0.84   |
| 4    | 21  | 1.00                               | 1.07   | 0.88   |
| 5    | 28  | 1.11                               | 1.19   | 0.93   |

Table CXIX: Variation of deflection with age of concrete for unreinforced beams

| S/NO | AGE | CTBM                  | CBPVC1 | CBPVC2 |
|------|-----|-----------------------|--------|--------|
|      |     | $\delta \text{ (mm)}$ |        |        |
| 1    | 0   | 0                     | 0      | 0      |
| 2    | 7   | 0                     | 1.0    | 1.0    |
| 3    | 14  | 1.2                   | 3.0    | 3.0    |
| 4    | 21  | 1.5                   | 3.5    | 6.0    |
| 5    | 28  | 4.3                   | 4.0    | 9.0    |

Table CXX: Variation of modulus of rupture with age of concrete for unreinforced beams

| S/NO | AGE | CTBM                             | CBPVC1 | CBPVC2 |
|------|-----|----------------------------------|--------|--------|
|      |     | $f_{mr} \text{ (N/mm}^2\text{)}$ |        |        |
| 1    | 0   | 0                                | 0      | 0      |
| 2    | 7   | 1.41                             | 0.96   | 0.75   |
| 3    | 14  | 1.92                             | 2.00   | 1.65   |
| 4    | 21  | 2.03                             | 2.19   | 1.73   |
| 5    | 28  | 2.27                             | 2.27   | 1.87   |

## APPENDIX D: TABLES FOR THEORETICAL AND SIMULATION RESULTS OF STRESS

Table DI: Comparison between theoretical and simulation stress for RCPVC 1

| S/No | Load applied<br>(F)<br>kN | Theoretical<br>stress ( $\sigma$ )<br>MPa | Stress from<br>ANSYS( $\sigma$ )<br>MPa |
|------|---------------------------|-------------------------------------------|-----------------------------------------|
| 1    | 0                         | 0                                         | 0                                       |
| 2    | 15.4                      | 39.28                                     | 39.85                                   |
| 3    | 22.0                      | 55.12                                     | 56.93                                   |
| 4    | 22.6                      | 56.56                                     | 58.48                                   |
| 5    | 25.0                      | 62.31                                     | 64.70                                   |

Table DII: Comparison between theoretical and simulation stress for RCPVC 2

| S/No | Load applied<br>(F)<br>kN | Theoretical<br>stress ( $\sigma$ )<br>MPa | Stress from<br>ANSYS ( $\sigma$ )<br>MPa |
|------|---------------------------|-------------------------------------------|------------------------------------------|
| 1    | 0                         | 0                                         | 0                                        |
| 2    | 11.3                      | 29.44                                     | 28.72                                    |
| 3    | 13.7                      | 35.20                                     | 34.82                                    |
| 4    | 14.5                      | 37.12                                     | 36.85                                    |
| 5    | 15.0                      | 38.32                                     | 38.12                                    |

Table DIII: Comparison between theoretical and simulation stress for RCPVC 3

| S/No | Load applied (F)<br>kN | Theoretical stress ( $\sigma$ )<br>MPa | Stress from ANSYS<br>( $\sigma$ )<br>MPa |
|------|------------------------|----------------------------------------|------------------------------------------|
| 1    | 0                      | 0                                      | 0                                        |
| 2    | 11.3                   | 29.44                                  | 28.40                                    |
| 3    | 12.1                   | 31.36                                  | 30.42                                    |
| 4    | 12.9                   | 33.28                                  | 32.43                                    |
| 5    | 15.0                   | 38.32                                  | 37.70                                    |

Table DIV: Comparison between theoretical and simulation stress for RCBM

| S/No | Load applied (F)<br>kN | Theoretical<br>stress ( $\sigma$ )<br>MPa | Stress from ANSYS<br>( $\sigma$ )<br>MPa |
|------|------------------------|-------------------------------------------|------------------------------------------|
| 1    | 0                      | 0                                         | 0                                        |
| 2    | 24.0                   | 59.92                                     | 66.69                                    |
| 3    | 24.4                   | 60.88                                     | 67.80                                    |
| 4    | 25.2                   | 62.79                                     | 70.02                                    |
| 5    | 26.0                   | 64.71                                     | 72.24                                    |

## APPENDIX E: SUBROUTINE FOR RELIABILITY ANALYSIS USING FORM

### APPENDIX EI: PROGRAM RCBEAM

C RELIABILITY ANALYSIS OF REINFORCED CONCRETE NORMAL BEAM (BENDING CHECK)

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

C THE SUBROUTINE WITH THE LIMIT STATE FUNCTION DECLARED EXTERNAL

EXTERNAL GRCBEAM

DIMENSION X(4),EX(4),SX(4),VP(10,4),COV(4,4),ZES(4),

+ UU(4),EIVEC(4,4),IV(2,4)

CHARACTER\*10 PRT

COMMON/CRCBEAM/S,A,B,C,G

DATA EX/2.26D2,4.10D2,1.11D2,6.82D6/,

+ SX/4.52D0,4.02D1,1.11D1,8.19D5/,

+ N/4/,NC/4/,NE/4/,IRHO/0/

C

WRITE(\*,\*)'ENTER VALUES FOR A,B,C AND G.....>'

READ(\*,\*) A,B,C,G

C THE RESULTS ARE WRITTEN IN FILE NUMBER 7

NAUS=7

C PRINT TO SCREEN

ICRT=0

OPEN(7,FILE='john1.DAT',STATUS='OLD',ERR=10)

GOTO 20

10 OPEN(7,FILE='john1.DAT',STATUS='NEW')

C

C PRESENTING VARIABLES VP,COV,AND IV IS DONE USING YINIT

```

20  CALL YINIT(N,IV,VP,IRHO,COV,NC)

      IV(1,1)=3

      IV(1,2)=3

      IV(1,3)=3

      IV(1,4)=3

      DO 100 I=1,N

100  X(I)=EX(I)

      V1=1.D0

      BETA=1.D0

      WRITE(NAUS,5000)

5000  FORMAT (////,5X,70('*'),/,30X,'FORM5',/,5X,70('*'),/,5X,
      + 'EXAMPLE USING',/,5X,'john1.DAT',/,5X,'4VARIABLES:')

C

C   STOCHASTIC MODEL IS PRINTED USING 'YKOPF'

      CALL YKOPF(NAUS, N, IV, EX, SX, VP, IRHO)

C   PRINT ALSO TO SCREEN

      WRITE (ICRT,*) ' START OF FORM5 '

      WRITE (ICRT,*) ' STOCHASTIC MODEL :'

      CALL YKOPF (ICRT, N, IV, EX, SX, VP, IRHO)

C   THE MATRIX COV IS PRINTED USING 'YMAUS'

      PRT=' COV '

      CALL YMAUS (NAUS,NC,N,COV,PRT)

C   THE MAIN SUBROUTINE IN FORM IS CALLED

      CALL FORM5 (N, IV, EX, SX, VP, GRCBEAM, IRHO, COV, NC,
      +   EIVEC, NE, V1, NAUS, BETA, X, UU, ZES, IER)

C   THE COORDINATES OF THE BETA-POINT ARE PRINTED WITH TITLE

C   VECTOR UU

```

```

PRT=' UU '

CALL YFAUS (NAUS,N,UU,PRT)

C  THE VECTOR ZES IS PRINTED WITH THE TITLE VECTOR ZES

PRT=' ZES '

CALL YFAUS(NAUS,7,ZES,PRT)

WRITE (ICRT,*) ' END OF FORM5 : IER =',IER

WRITE (ICRT,*) ' RESULTS SEE FILE john1.DAT'

STOP

END


SUBROUTINE GRCBEAM (N, X, FX, IER)

C  LIMIT STATE FUNCTION (BENDING CHECK)OKAY

C

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

DIMENSION X(N)

COMMON /CRCBEAM/S,A,B,C,G

C

A=X(1)

B=X(2)

C=X(3)

D=X(4)

F1=0.95*A*B*C

F2=D

C  CHECK FOR ERRORS, CALCULATE FX

IF (F1.NE.0.)THEN

F=F1-F2

FX = F-0

```

```

IER = 0

ELSE

    FX = 1.0D+20

    IER = 1

END IF

RETURN

END

```

## APPENDIX EII: Results file from Reliability Analysis Using FORM5

\*\*\*\*\*

### FORM5

\*\*\*\*\*

EXAMPLE USING

john2.DAT

4VARIABLES:

| VARIABLE-TYPE   | E(X)      | S(X)      | VAR | R   | P |
|-----------------|-----------|-----------|-----|-----|---|
| X 1 3 226.      | 4.52      | 0.200E-01 | 1.0 | 1.0 |   |
| X 2 3 410.      | 40.2      | 0.980E-01 | 1.0 | 1.0 |   |
| X 3 3 111.      | 11.1      | 0.100     | 1.0 | 1.0 |   |
| X 4 3 0.400E+07 | 0.481E+06 | 0.120     | 1.0 | 1.0 |   |

IRHO = 0

MATRIX COV

|   | 1         | 2         | 3     | 4     |
|---|-----------|-----------|-------|-------|
| 1 | 0.601-153 | 0.000     | 0.000 | 0.000 |
| 2 | 0.601-153 | 0.000     | 0.000 | 0.000 |
| 3 | 0.175-309 | 0.175+213 | 0.000 | 0.000 |
| 4 | 0.000     | 0.108-304 | 0.000 | 0.000 |

FORM-5 by RCP-GmbH (Dec. 89)

G (M) = 0.58E+07

DURING ITERATION :

//U// G(X)

3.907 0.58E+07

4.907 0.11E+07

4.808 -0.71E+05

4.811 0.38E+04

4.811 11.

G (X-\*) = -0.16E-02 IT. NUMB. = 5

VECTOR X-\* :

224. 318. 85.3 0.577E+07

BETA = 4.811 PF = 0.752E-06

VECTOR UU

-0.520 -2.54 -2.59 3.11

VECTOR ZES

0.296E+06 0.543E+07 0.107E+07 0.207-314 0.601-153 0.601-153

0.175-309

## APPENDIX F: TABLES OF VALUES FOR RELIABILITY ANALYSIS

**Table FI: Stochastic variables for RCPVC1 in bending**

| VAR                    | N       | CoV    | $\mu$   | $\lambda$ | $\sigma$ | $\beta$          |
|------------------------|---------|--------|---------|-----------|----------|------------------|
| As(mm <sup>2</sup> )   | 201,402 | 2.00%  | 201     | 1         | 4.02     |                  |
| fy(N/mm <sup>2</sup> ) | 410     | 9.80%  | 410     | 1         | 40.18    | <b>1.53,5.27</b> |
| d(mm)                  | 150     | 10.00% | 157.5   | 1.05      | 15.75    |                  |
| M(kN/m)                | 5837000 | 12.00% | 6537440 | 1.12      | 784492.8 |                  |

**Table FII: Stochastic variables for RCPVC2 in bending**

| VAR                    | N       | CoV    | $\mu$   | $\lambda$ | $\sigma$ | $\beta$          |
|------------------------|---------|--------|---------|-----------|----------|------------------|
| As(mm <sup>2</sup> )   | 201,402 | 2.00%  | 201     | 1         | 4.02     |                  |
| fy(N/mm <sup>2</sup> ) | 410     | 9.80%  | 410     | 1         | 40.18    | <b>4.18,7.92</b> |
| d(mm)                  | 150     | 10.00% | 157.5   | 1.05      | 15.75    |                  |
| M(kN/m)                | 3580000 | 12.00% | 4000000 | 1.12      | 481000   |                  |

**Table FIII: Stochastic variables for RCPVC3 in bending**

| VAR                    | N       | CoV    | $\mu$   | $\lambda$ | $\sigma$ | $\beta$          |
|------------------------|---------|--------|---------|-----------|----------|------------------|
| As(mm <sup>2</sup> )   | 201,402 | 2.00%  | 201     | 1         | 4.02     |                  |
| fy(N/mm <sup>2</sup> ) | 410     | 9.80%  | 410     | 1         | 40.18    | <b>4.18,7.16</b> |
| d(mm)                  | 150     | 10.00% | 157.5   | 1.05      | 15.75    |                  |
| M(kN/m)                | 3570000 | 12.00% | 4000000 | 1.12      | 481000   |                  |

**Table FIV: Stochastic variables for RCBM in bending**

| VAR                    | N       | CoV    | $\mu$   | $\lambda$ | $\sigma$ | $\beta$          |
|------------------------|---------|--------|---------|-----------|----------|------------------|
| As(mm <sup>2</sup> )   | 201,402 | 2.00%  | 201     | 1         | 4.02     | <b>1.30,5.04</b> |
| fy(N/mm <sup>2</sup> ) | 410     | 9.80%  | 410     | 1         | 40.18    |                  |
| d(mm)                  | 150     | 10.00% | 157.5   | 1.05      | 15.75    |                  |
| M(kN/m)                | 6090000 | 12.00% | 6820000 | 1.12      | 818899   |                  |

**Table FV: Stochastic variables for RCBM in shear**

| VAR     | N     | CoV | $\mu$ | $\lambda$ | $\sigma$ | $\beta$     |
|---------|-------|-----|-------|-----------|----------|-------------|
| bv(mm)  | 150   | 10% | 157.5 | 1.05      | 15.75    | <b>6.87</b> |
| d(mm)   | 122   | 10% | 128.1 | 1.05      | 12.81    |             |
| P(kN)   | 26000 | 11% | 22100 | 0.85      | 2431     |             |
| w(kN/m) | 2.16  | 10% | 2.16  | 1.00      | 0.216    |             |

**Table FVI: Stochastic variables for RCPVC1 in shear**

| VAR     | N     | CoV | $\mu$ | $\lambda$ | $\sigma$ | $\beta$     |
|---------|-------|-----|-------|-----------|----------|-------------|
| bv(mm)  | 150   | 10% | 157.5 | 1.05      | 15.75    | <b>6.99</b> |
| d(mm)   | 122   | 10% | 128.1 | 1.05      | 12.81    |             |
| P(kN)   | 25000 | 11% | 21250 | 0.85      | 2337.5   |             |
| w(kN/m) | 2.16  | 10% | 2.16  | 1.00      | 0.216    |             |

**Table FVII: Stochastic variables for RCPVC2 in shear**

| VAR     | N     | CoV | $\mu$ | $\lambda$ | $\sigma$ | $\beta$     |
|---------|-------|-----|-------|-----------|----------|-------------|
| bv(mm)  | 150   | 10% | 157.5 | 1.05      | 15.75    | <b>8.11</b> |
| d(mm)   | 122   | 10% | 128.1 | 1.05      | 12.81    |             |
| P(kN)   | 15500 | 11% | 13175 | 0.85      | 1449.25  |             |
| w(kN/m) | 2.16  | 10% | 2.16  | 1.00      | 0.216    |             |

**Table FVIII: Stochastic variables for RCPVC3 in shear**

| VAR            | N     | CoV(%) | $\mu$ | $\lambda$ | $\sigma$ | $\beta$     |
|----------------|-------|--------|-------|-----------|----------|-------------|
| <b>bv(mm)</b>  | 150   | 10     | 157.5 | 1.05      | 15.75    |             |
| <b>d(mm)</b>   | 122   | 10     | 128.1 | 1.05      | 12.81    | <b>8.18</b> |
| <b>P(kN)</b>   | 15000 | 11     | 12750 | 0.85      | 1402.5   |             |
| <b>w(kN/m)</b> | 2.16  | 10     | 2.16  | 1.00      | 0.216    |             |

**Table FIX: Stochastic variables for RCPVC1 in deflection**

| VAR                                    | N       | CoV(%) | $\mu$ | $\lambda$ | $\sigma$ | $\beta$   |
|----------------------------------------|---------|--------|-------|-----------|----------|-----------|
| <b>d(mm)</b>                           | 150     | 10     | 157.5 | 1.05      | 15.75    |           |
| <b>f<sub>y</sub>(N/mm<sup>2</sup>)</b> | 410     | 9.8    | 410   | 1.00      | 40.18    |           |
| <b>A<sub>s</sub>(mm<sup>2</sup>)</b>   | 142     | 2.0    | 142   | 1.00      | 2.84     |           |
| <b>A<sub>sp</sub>(mm<sup>2</sup>)</b>  | 201,402 | 2.0    | 201   | 1.00      | 4.02     | 5.43,5.69 |
| <b>P(kN)</b>                           | 25000   | 11     | 21250 | 0.85      | 2337.5   |           |
| <b>w(kN/m)</b>                         | 2.16    | 10     | 2.16  | 1.00      | 0.216    |           |
| <b>b(mm)</b>                           | 150     | 10     | 157.5 | 1.05      | 15.75    |           |

**Table FX: Stochastic variables for RCPVC2 in deflection**

| VAR                                    | N     | CoV(%) | $\mu$ | $\lambda$ | $\sigma$ | $\beta$   |
|----------------------------------------|-------|--------|-------|-----------|----------|-----------|
| <b>d(mm)</b>                           | 150   | 10     | 157.5 | 1.05      | 15.75    |           |
| <b>f<sub>y</sub>(N/mm<sup>2</sup>)</b> | 410   | 9.8    | 410   | 1.00      | 40.18    |           |
| <b>A<sub>s</sub>(mm<sup>2</sup>)</b>   | 142   | 2.0    | 142   | 1.00      | 2.84     |           |
| <b>A<sub>sp</sub></b>                  | 201   | 2.0    | 201   | 1.00      | 4.02     | 5.78,6.06 |
| <b>P(kN)</b>                           | 15500 | 11     | 13175 | 0.85      | 1449.25  |           |
| <b>w(kN/m)</b>                         | 2.16  | 10     | 2.16  | 1.00      | 0.216    |           |
| <b>b(mm)</b>                           | 150   | 10     | 157.5 | 1.05      | 15.75    |           |

**Table FXI: Stochastic variables for RCPVC3 in deflection**

| <b>VAR</b>                             | <b>N</b> | <b>CoV(%)</b> | <b><math>\mu</math></b> | <b><math>\lambda</math></b> | <b><math>\sigma</math></b> | <b><math>\beta</math></b> |
|----------------------------------------|----------|---------------|-------------------------|-----------------------------|----------------------------|---------------------------|
| <b>d(mm)</b>                           | 150      | 10            | 157.5                   | 1.05                        | 15.75                      |                           |
| <b><math>f_y(\text{N/mm}^2)</math></b> | 410      | 9.8           | 410                     | 1.00                        | 40.18                      |                           |
| <b><math>A_s(\text{mm}^2)</math></b>   | 142      | 2.0           | 142                     | 1.00                        | 2.84                       |                           |
| <b><math>A_{sp}</math></b>             | 201      | 2.0           | 201                     | 1.00                        | 4.02                       | 5.81,6.08                 |
| <b>P(kN)</b>                           | 15000    | 11            | 12750                   | 0.85                        | 1402.5                     |                           |
| <b>w(kN/m)</b>                         | 2.16     | 10            | 2.16                    | 1.00                        | 0.216                      |                           |
| <b>b(mm)</b>                           | 150      | 10            | 157.5                   | 1.05                        | 15.75                      |                           |

**Table FXII: Stochastic variables for RCBM in deflection**

| <b>VAR</b>                             | <b>N</b> | <b>CoV</b> | <b><math>\mu</math></b> | <b><math>\lambda</math></b> | <b><math>\sigma</math></b> | <b><math>\beta</math></b> |
|----------------------------------------|----------|------------|-------------------------|-----------------------------|----------------------------|---------------------------|
| <b>d(mm)</b>                           | 150      | 10         | 157.5                   | 1.05                        | 15.75                      |                           |
| <b><math>f_y(\text{N/mm}^2)</math></b> | 410      | 9.8        | 410                     | 1                           | 40.18                      |                           |
| <b><math>A_s(\text{mm}^2)</math></b>   | 142      | 2          | 142                     | 1                           | 2.84                       |                           |
| <b><math>A_{sp}</math></b>             | 201      | 2          | 201                     | 1                           | 4.02                       | 5.40,5.66                 |
| <b>P(kN)</b>                           | 26000    | 11         | 22100                   | 0.85                        | 2431                       |                           |
| <b>w(kN/m)</b>                         | 2.16     | 10         | 2.16                    | 1                           | 0.216                      |                           |
| <b>b(mm)</b>                           | 150      | 10         | 157.5                   | 1.05                        | 15.75                      |                           |

## APPENDIX G: REGRESSION ANALYSIS FOR CONCRETE CUBES BIOSTAT PRO

| Polynomial Regression for reinforced and unreinforced concrete cubes of grade C25       |                    |                 |                           |            |          |
|-----------------------------------------------------------------------------------------|--------------------|-----------------|---------------------------|------------|----------|
| Regression Statistics                                                                   |                    |                 |                           |            |          |
| R                                                                                       | 0.999922459        |                 |                           |            |          |
| R Square                                                                                | 0.999844924        |                 |                           |            |          |
| Adjusted R Square                                                                       | 0.999782894        |                 |                           |            |          |
| Standard Error                                                                          | 0.295918905        |                 |                           |            |          |
| Total number of observations                                                            | 15                 |                 |                           |            |          |
| stress = + 0.0000 + 14.5321 * fmr - 3.0187 * fmr**2 + 0.6000 * fmr**3 - 0.0385 * fmr**4 |                    |                 |                           |            |          |
| ANOVA                                                                                   |                    |                 |                           |            |          |
|                                                                                         | d.f.               | SS              | MS                        | F          | p-level  |
| Regression                                                                              | 4                  | 5645.90961      | 1411.477403               | 16118.6442 | 5.42E-19 |
| Residual                                                                                | 10                 | 0.87567998      | 0.087567998               |            |          |
| Total                                                                                   | 14                 | 5646.78529      |                           |            |          |
|                                                                                         |                    |                 |                           |            |          |
| Residuals                                                                               |                    |                 |                           |            |          |
| <b>Observation</b>                                                                      | <b>Predicted Y</b> | <b>Residual</b> | <b>Standard Residuals</b> |            |          |
| 1                                                                                       | 2.52E-05           | -2.52E-05       | -0.000100763              |            |          |
| 2                                                                                       | 39.30910661        | 0.54089339      | 2.162733356               |            |          |
| 3                                                                                       | 57.22164435        | -0.2916444      | -1.166124388              |            |          |
| 4                                                                                       | 58.20996421        | 0.27003579      | 1.079723683               |            |          |
| 5                                                                                       | 64.70865963        | -0.0086596      | -0.034625066              |            |          |
| 6                                                                                       | 2.52E-05           | -2.52E-05       | -0.000100763              |            |          |
| 7                                                                                       | 29.5095394         | -0.0695394      | -0.278049567              |            |          |
| 8                                                                                       | 35.6183256         | -0.4183256      | -1.672652618              |            |          |
| 9                                                                                       | 37.4443128         | -0.3243128      | -1.296747449              |            |          |
| 10                                                                                      | 38.371873          | -0.051873       | -0.207411417              |            |          |
| 11                                                                                      | 2.52E-05           | -2.52E-05       | -0.000100763              |            |          |
| 12                                                                                      | 29.5095394         | -0.0695394      | -0.278049567              |            |          |
| 13                                                                                      | 31.21491185        | 0.14508815      | 0.580127246               |            |          |
| 14                                                                                      | 32.95017454        | 0.32982546      | 1.318789492               |            |          |
| 15                                                                                      | 38.371873          | -0.051873       | -0.207411417              |            |          |

| <b>Polynomial Regression for reinforced Concrete cubes grade C20 Y10</b>                |                    |                 |                           |            |          |
|-----------------------------------------------------------------------------------------|--------------------|-----------------|---------------------------|------------|----------|
| Regression Statistics                                                                   |                    |                 |                           |            |          |
| R                                                                                       | 0.999999998        |                 |                           |            |          |
| R Square                                                                                | 0.999999996        |                 |                           |            |          |
| Adjusted R Square                                                                       | 0.999999984        |                 |                           |            |          |
| Standard Error                                                                          | 0.003498741        |                 |                           |            |          |
| Total number of observations                                                            | 5                  |                 |                           |            |          |
| stress = - 0.0000 + 9.2424 * fmr + 0.0076 * fmr**2 + 0.0018 * fmr**3                    |                    |                 |                           |            |          |
|                                                                                         |                    |                 |                           |            |          |
| <b>ANOVA</b>                                                                            |                    |                 |                           |            |          |
|                                                                                         | d.f.               | SS              | MS                        | F          | p-level  |
| Regression                                                                              | 3                  | 3096.17099      | 1032.056996               | 84310211.6 | 8.01E-05 |
| Residual                                                                                | 1                  | 1.22E-05        | 1.22E-05                  |            |          |
| Total                                                                                   | 4                  | 3096.171        |                           |            |          |
| Residuals                                                                               |                    |                 |                           |            |          |
| <b>Observation</b>                                                                      | <b>Predicted Y</b> | <b>Residual</b> | <b>Standard Residuals</b> |            |          |
| 1                                                                                       | -7.16E-08          | 7.16E-08        | 2.13E-05                  |            |          |
| 2                                                                                       | 59.92143371        | -0.0014337      | -0.819574719              |            |          |
| 3                                                                                       | 60.87735305        | 0.00264695      | 1.513069933               |            |          |
| 4                                                                                       | 62.79171189        | -0.0017119      | -0.9785945                |            |          |
| 5                                                                                       | 64.70950126        | 0.00049874      | 0.285077989               |            |          |
| <b>Polynomial Regression normal concrete cube grade C20 –C30</b>                        |                    |                 |                           |            |          |
| Regression Statistics                                                                   |                    |                 |                           |            |          |
| R                                                                                       | 0.999996909        |                 |                           |            |          |
| R Square                                                                                | 0.999993819        |                 |                           |            |          |
| Adjusted R Square                                                                       | 0.999991346        |                 |                           |            |          |
| Standard Error                                                                          | 0.024885211        |                 |                           |            |          |
| Total number of observations                                                            | 15                 |                 |                           |            |          |
| stress = + 0.0005 + 15.4156 * fmr - 6.2525 * fmr**2 + 2.6156 * fmr**3 - 0.4016 * fmr**4 |                    |                 |                           |            |          |
| <b>ANOVA</b>                                                                            |                    |                 |                           |            |          |
|                                                                                         | d.f.               | SS              | MS                        | F          | p-level  |
| Regression                                                                              | 4                  | 1001.82838      | 250.4570951               | 404436.82  | 0        |
| Residual                                                                                | 10                 | 0.00619274      | 0.000619274               |            |          |

|                    |                    |                 |                               |  |  |
|--------------------|--------------------|-----------------|-------------------------------|--|--|
| Total              | 14                 | 1001.83457      |                               |  |  |
| Residuals          |                    |                 |                               |  |  |
| <b>Observation</b> | <b>Predicted Y</b> | <b>Residual</b> | <b>Standard<br/>Residuals</b> |  |  |
| 1                  | 0.00047808         | -0.0004781      | -0.022731244                  |  |  |
| 2                  | 15.05076519        | -0.0007652      | -0.036382595                  |  |  |
| 3                  | 19.60502799        | 0.00497201      | 0.236404053                   |  |  |
| 4                  | 20.58949833        | -0.0194983      | -0.927086125                  |  |  |
| 5                  | 22.70772707        | -0.0077271      | -0.367398424                  |  |  |
| 6                  | 0.00047808         | -0.0004781      | -0.022731244                  |  |  |
| 7                  | 11.01018359        | -0.0401836      | -1.910607174                  |  |  |
| 8                  | 20.32141111        | 0.00858889      | 0.408375507                   |  |  |
| 9                  | 22.00880957        | -0.0088096      | -0.418868045                  |  |  |
| 10                 | 22.70772707        | 0.02227293      | 1.059010169                   |  |  |
| 11                 | 0.00047808         | -0.0004781      | -0.022731244                  |  |  |
| 12                 | 9.021532523        | 0.02846748      | 1.35354179                    |  |  |
| 13                 | 17.18696817        | 0.02303183      | 1.095093362                   |  |  |
| 14                 | 17.90217703        | 0.02782297      | 1.32289762                    |  |  |
| 15                 | 19.15673814        | -0.0367381      | -1.746786406                  |  |  |

## Modeling and simulation report using ANSYS

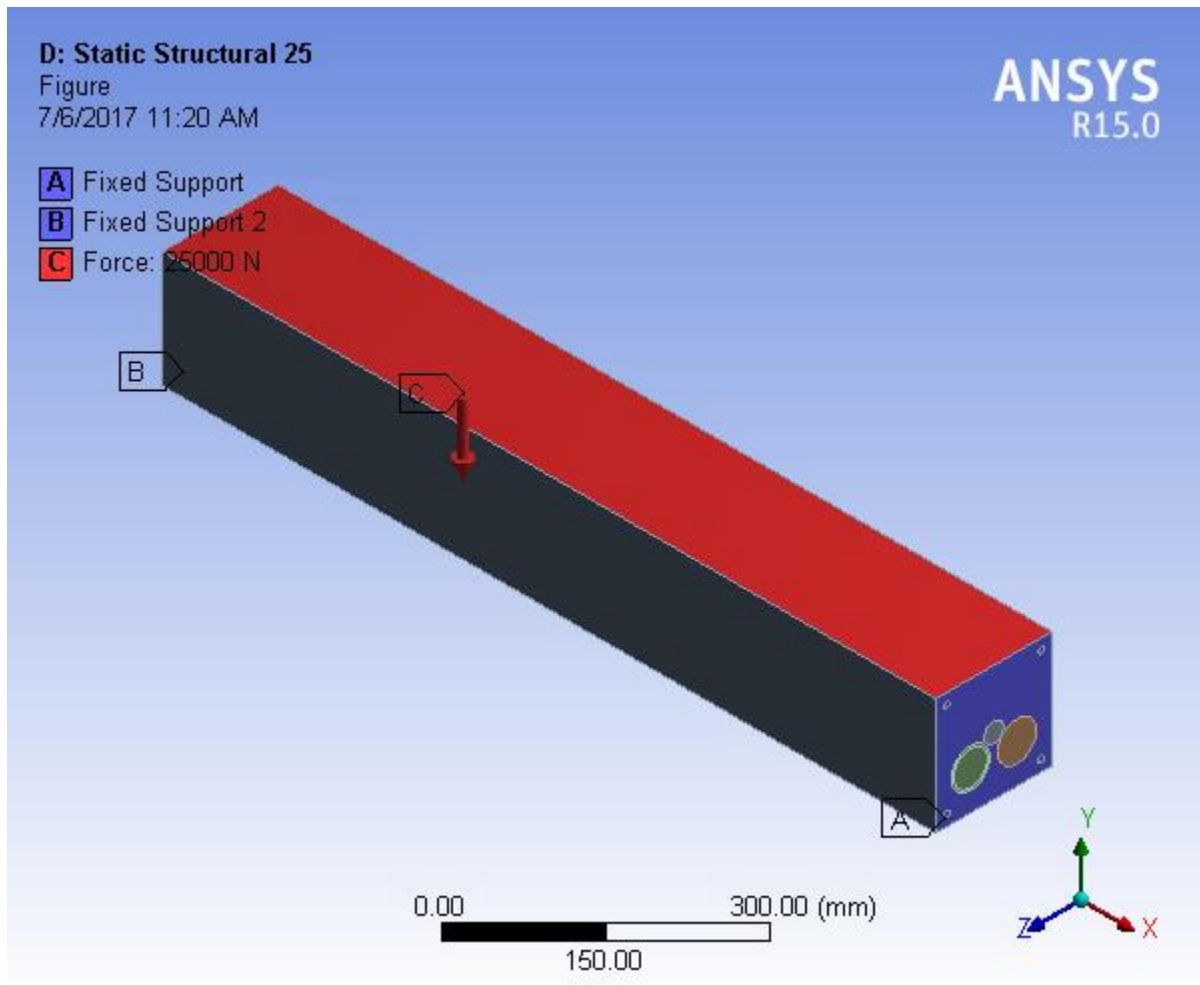
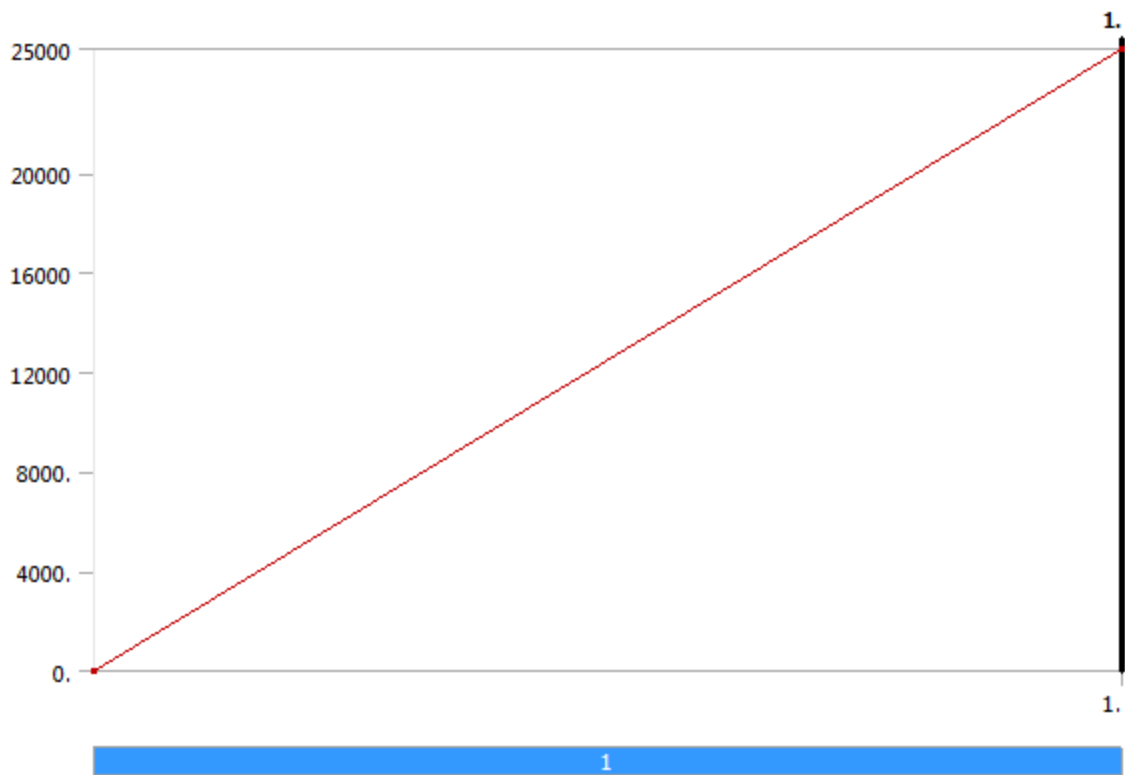


**TABLE 17**  
**Model (D4) > Static Structural (D5) > Loads**

|                |                    |                 |                  |
|----------------|--------------------|-----------------|------------------|
| Object Name    | Fixed Support      | Fixed Support 2 | Force            |
| State          | Fully Defined      |                 |                  |
| Scope          |                    |                 |                  |
| Scoping Method | Geometry Selection |                 |                  |
| Geometry       | 5 Faces            | 1 Face          |                  |
| Definition     |                    |                 |                  |
| Type           | Fixed Support      | Force           |                  |
| Suppressed     | No                 |                 |                  |
| Define By      |                    |                 | Vector           |
| Magnitude      |                    |                 | 25000 N (ramped) |
| Direction      |                    |                 | Defined          |

**FIGURE 2**  
**Model (D4) > Static Structural (D5) > Force**



## Solution (D6)

**TABLE 18**  
Model (D4) > Static Structural (D5) > Solution

|                                 |                      |
|---------------------------------|----------------------|
| Object Name                     | <i>Solution (D6)</i> |
| State                           | Solved               |
| <b>Adaptive Mesh Refinement</b> |                      |
| Max Refinement Loops            | 1.                   |
| Refinement Depth                | 2.                   |
| <b>Information</b>              |                      |
| Status                          | Done                 |

**TABLE 19**  
Model (D4) > Static Structural (D5) > Solution (D6) > Solution Information

|                                 |                             |
|---------------------------------|-----------------------------|
| Object Name                     | <i>Solution Information</i> |
| State                           | Solved                      |
| <b>Solution Information</b>     |                             |
| Solution Output                 | Solver Output               |
| Newton-Raphson Residuals        | 0                           |
| Update Interval                 | 2.5 s                       |
| Display Points                  | All                         |
| <b>FE Connection Visibility</b> |                             |
| Activate Visibility             | Yes                         |
| Display                         | All FE Connectors           |
| Draw Connections Attached To    | All Nodes                   |
| Line Color                      | Connection Type             |
| Visible on Results              | No                          |

|                |        |
|----------------|--------|
| Line Thickness | Single |
| Display Type   | Lines  |

**TABLE 20**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Results**

| Object Name            | Directional Deformation  | Equivalent Stress             | Equivalent Elastic Strain | Linearized Equivalent Stress | Linearized Equivalent Stress 2 | Linearized Normal Stress | Linearized Normal Stress 2 | Linearized Stress Intensity | Linearized Stress Intensity 2 | Linearized Shear Stress | Linearized Shear Stress 2 |
|------------------------|--------------------------|-------------------------------|---------------------------|------------------------------|--------------------------------|--------------------------|----------------------------|-----------------------------|-------------------------------|-------------------------|---------------------------|
| State                  | Solved                   |                               |                           |                              |                                |                          |                            |                             |                               |                         |                           |
| Scope                  |                          |                               |                           |                              |                                |                          |                            |                             |                               |                         |                           |
| Scoping Method         | Geometry Selection       |                               |                           | Path                         |                                |                          |                            |                             |                               |                         |                           |
| Geometry               | All Bodies               |                               |                           |                              |                                |                          |                            |                             |                               |                         |                           |
| Path                   |                          |                               |                           | Path                         | Path 2                         | Path                     | Path 2                     | Path                        | Path 2                        | Path                    | Path 2                    |
| Definition             |                          |                               |                           |                              |                                |                          |                            |                             |                               |                         |                           |
| Type                   | Directional Deformation  | Equivalent (von-Mises) Stress | Equivalent Elastic Strain | Linearized Equivalent Stress |                                | Linearized Normal Stress |                            | Linearized Stress Intensity |                               | Linearized Shear Stress |                           |
| Orientation            | X Axis                   |                               |                           |                              |                                | X Axis                   |                            |                             |                               | XY Plane                |                           |
| By                     | Time                     |                               |                           |                              |                                |                          |                            |                             |                               |                         |                           |
| Display Time           | Last                     |                               |                           |                              |                                |                          |                            |                             |                               |                         |                           |
| Coordinate System      | Global Coordinate System |                               |                           | Global Coordinate System     |                                |                          |                            |                             |                               |                         |                           |
| Calculate Time History | Yes                      |                               |                           |                              |                                |                          |                            |                             |                               |                         |                           |
| Identifier             |                          |                               |                           |                              |                                |                          |                            |                             |                               |                         |                           |
| Suppressed             | No                       |                               |                           |                              |                                |                          |                            |                             |                               |                         |                           |
| Subtype                |                          |                               |                           | All                          |                                |                          |                            |                             |                               |                         |                           |
| 2D Behavior            |                          |                               |                           | Planar                       |                                |                          |                            |                             |                               |                         |                           |
| Results                |                          |                               |                           |                              |                                |                          |                            |                             |                               |                         |                           |
| Minimum                | - 8.4038e-003 mm         | 1.1125e-004 MPa               | 8.6647e-008 mm/mm         |                              |                                |                          |                            |                             |                               |                         |                           |
| Maximum                | 8.4079e-003 mm           | 64.695 MPa                    | 3.236e-004 mm/mm          |                              |                                |                          |                            |                             |                               |                         |                           |
| Minimum Occurs On      | Part4-1                  | Solid                         | Part3-2                   |                              |                                |                          |                            |                             |                               |                         |                           |
| Maximum Occurs On      | Part4-1                  | Part2-4                       |                           |                              |                                |                          |                            |                             |                               |                         |                           |
| Membrane               |                          |                               |                           | 0.18151 MPa                  | 9.3511e-002 MPa                | - 5.6932e-003 MPa        | - 0.12341 MPa              | 0.20455 MPa                 | 9.8144e-002 MPa               | 1.5112e-005 MPa         | 1.77e-005 MPa             |
| Bending                |                          |                               |                           | 0.1130                       | 1.8632                         | 1.2239                   | 2.4275                     | 0.1305                      | 2.1515                        | -                       | -                         |

|                             |                 |                  |                     |             |                  |                   |                   |             |                  |                   |                   |
|-----------------------------|-----------------|------------------|---------------------|-------------|------------------|-------------------|-------------------|-------------|------------------|-------------------|-------------------|
| (Inside)                    |                 |                  |                     | 8 MPa       | MPa              | e-005 MPa         | e-004 MPa         | 7 MPa       | MPa              | 6.5225 e-002 MPa  | 1.0757 MPa        |
| Bending (Outside)           |                 |                  |                     | 0.11308 MPa | 1.8632 MPa       | -1.2239 e-005 MPa | -2.4275 e-004 MPa | 0.13057 MPa | 2.1515 MPa       | 6.5225 e-002 MPa  | 1.0757 MPa        |
| Membrane+B ending (Inside)  |                 |                  |                     | 0.21386 MPa | 1.8655 MPa       | -5.6809 e-003 MPa | -0.12317 MPa      | 0.23019 MPa | 2.1515 MPa       | -6.521e-002 MPa   | -1.0756 MPa       |
| Membrane+B ending (Center)  |                 |                  |                     | 0.18151 MPa | 9.3511 e-002 MPa | -5.6932 e-003 MPa | -0.12341 MPa      | 0.20455 MPa | 9.8144 e-002 MPa | 1.5112 e-005 MPa  | 1.77e-005 MPa     |
| Membrane+B ending (Outside) |                 |                  |                     | 0.21385 MPa | 1.8656 MPa       | -5.7054 e-003 MPa | -0.12365 MPa      | 0.23018 MPa | 2.1516 MPa       | 6.524e-002 MPa    | 1.0757 MPa        |
| Peak (Inside)               |                 |                  |                     | 4.1807 MPa  | 2.8865 MPa       | 4.8259 MPa        | -0.3983 MPa       | 4.3142 MPa  | 3.3139 MPa       | -1.0899 MPa       | 1.5993 MPa        |
| Peak (Center)               |                 |                  |                     | 1.5798 MPa  | 1.039 MPa        | -1.6177 MPa       | -1.0356 MPa       | 1.5978 MPa  | 1.0636 MPa       | -1.6629 e-004 MPa | -1.4646 e-004 MPa |
| Peak (Outside)              |                 |                  |                     | 4.1807 MPa  | 2.8821 MPa       | 4.8255 MPa        | -0.39554 MPa      | 4.3142 MPa  | 3.3089 MPa       | 1.0907 MPa        | -1.5967 MPa       |
| Total (Inside)              |                 |                  |                     | 4.271 MPa   | 1.2064 MPa       | 4.8202 MPa        | -0.52147 MPa      | 4.5027 MPa  | 1.3697 MPa       | -1.1551 MPa       | 0.52362 MPa       |
| Total (Center)              |                 |                  |                     | 1.5468 MPa  | 1.0778 MPa       | -1.6234 MPa       | -1.159 MPa        | 1.6239 MPa  | 1.1006 MPa       | -1.5118 e-004 MPa | -1.2876 e-004 MPa |
| Total (Outside)             |                 |                  |                     | 4.271 MPa   | 1.2023 MPa       | 4.8198 MPa        | -0.51919 MPa      | 4.5028 MPa  | 1.3651 MPa       | 1.156 MPa         | -0.52099 MPa      |
| Minimum Value Over Time     |                 |                  |                     |             |                  |                   |                   |             |                  |                   |                   |
| Minimum                     | -8.4038e-003 mm | 1.1125 e-004 MPa | 8.6647 e-008 mm/m m |             |                  |                   |                   |             |                  |                   |                   |
| Maximum                     | -8.4038e-003 mm | 1.1125 e-004 MPa | 8.6647 e-008 mm/m m |             |                  |                   |                   |             |                  |                   |                   |
| Maximum Value Over Time     |                 |                  |                     |             |                  |                   |                   |             |                  |                   |                   |
| Minimum                     | 8.4079e-003 mm  | 64.695 MPa       | 3.236e-004 mm/m m   |             |                  |                   |                   |             |                  |                   |                   |
| Maximum                     | 8.4079e-003 mm  | 64.695 MPa       | 3.236e-004 mm/m     |             |                  |                   |                   |             |                  |                   |                   |

|                           |      |          |   |  |
|---------------------------|------|----------|---|--|
|                           |      |          | m |  |
| Information               |      |          |   |  |
| Time                      | 1. s |          |   |  |
| Load Step                 | 1    |          |   |  |
| Substep                   | 1    |          |   |  |
| Iteration Number          | 1    |          |   |  |
| Integration Point Results |      |          |   |  |
| Display Option            |      | Averaged |   |  |
| Average Across Bodies     |      | No       |   |  |

**FIGURE 3**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Directional Deformation > Figure**

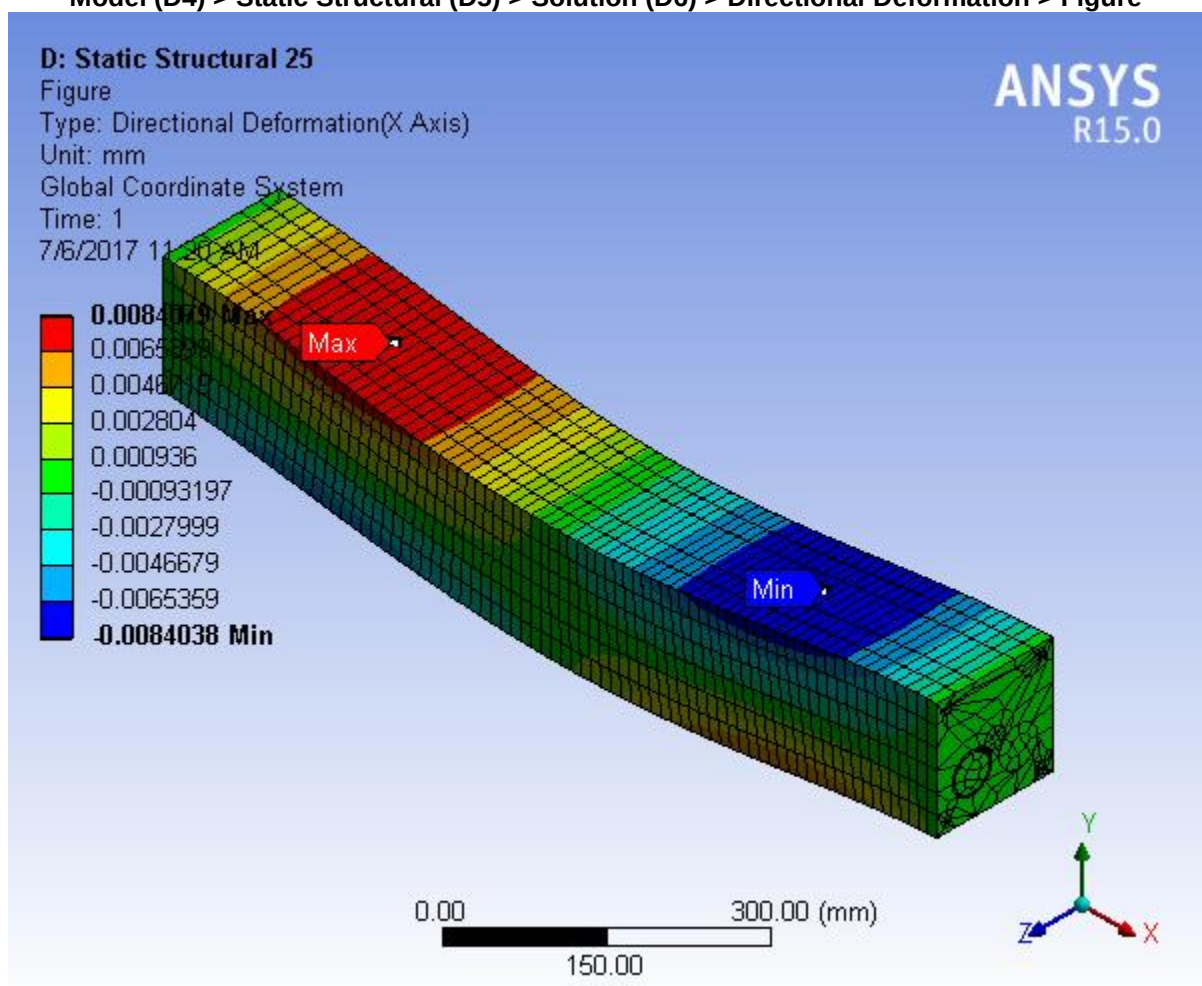


**FIGURE 4**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Equivalent Stress > Figure**

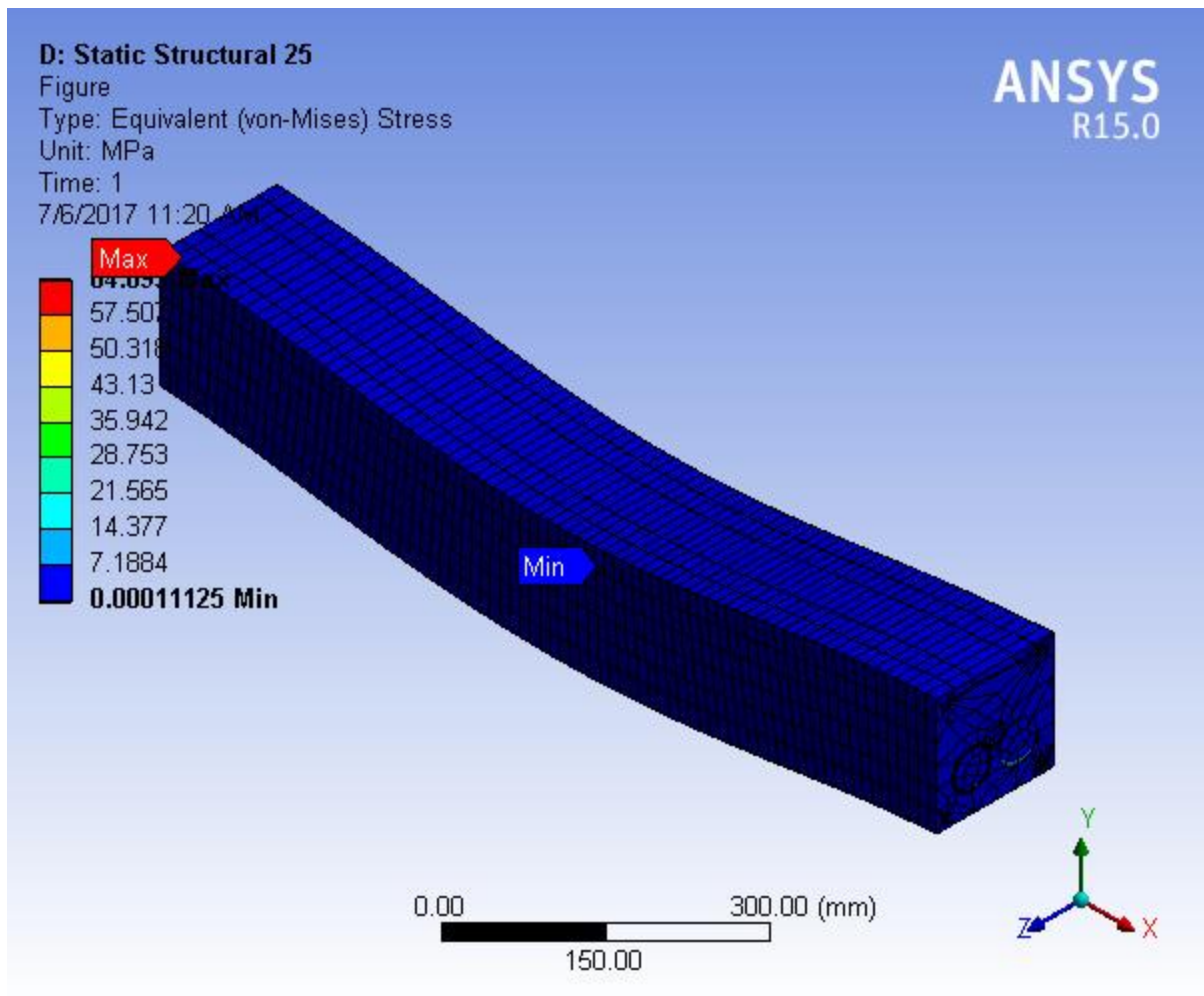


**FIGURE 5**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Equivalent Elastic Strain > Figure**

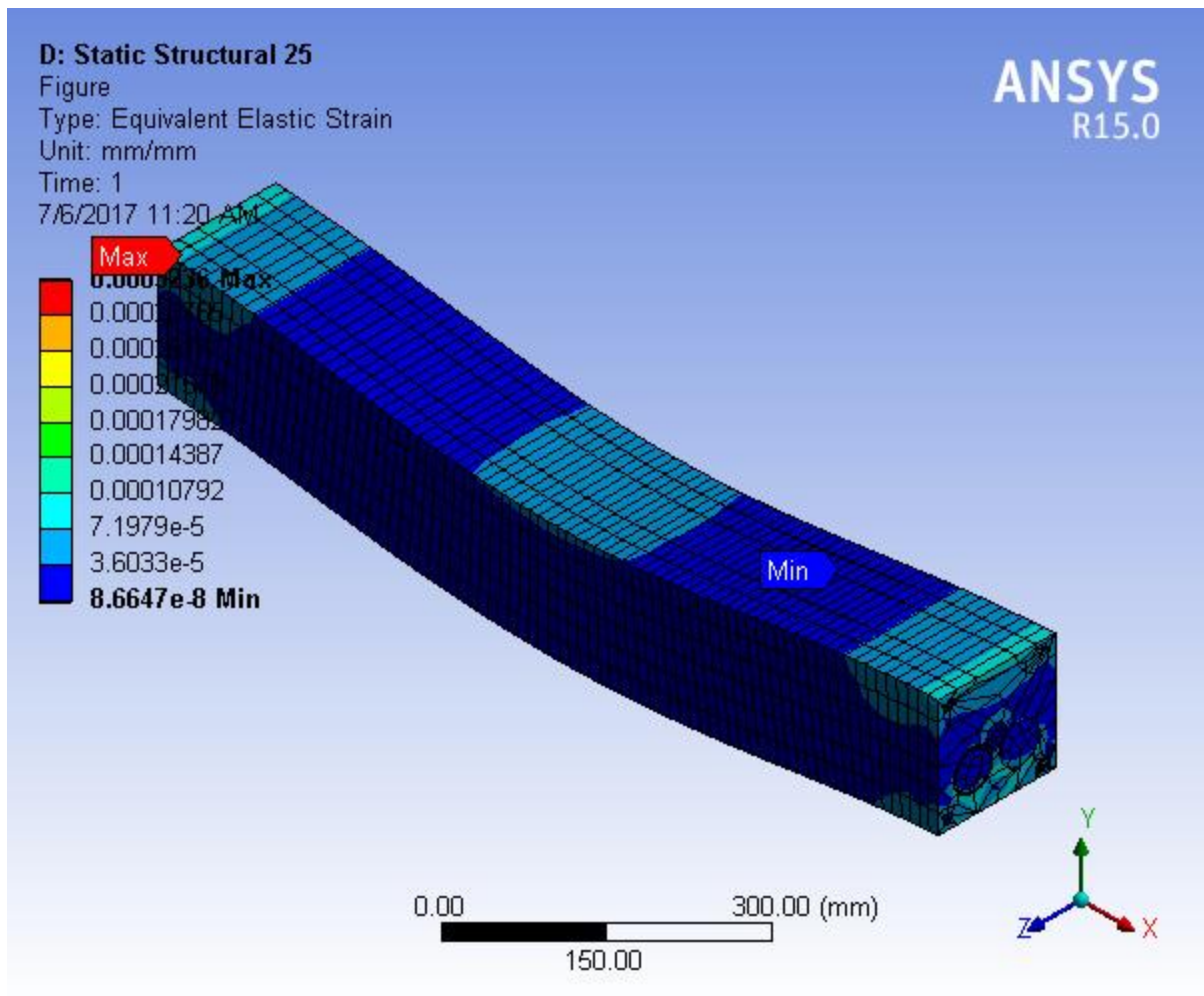


**FIGURE 6**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Equivalent Stress**

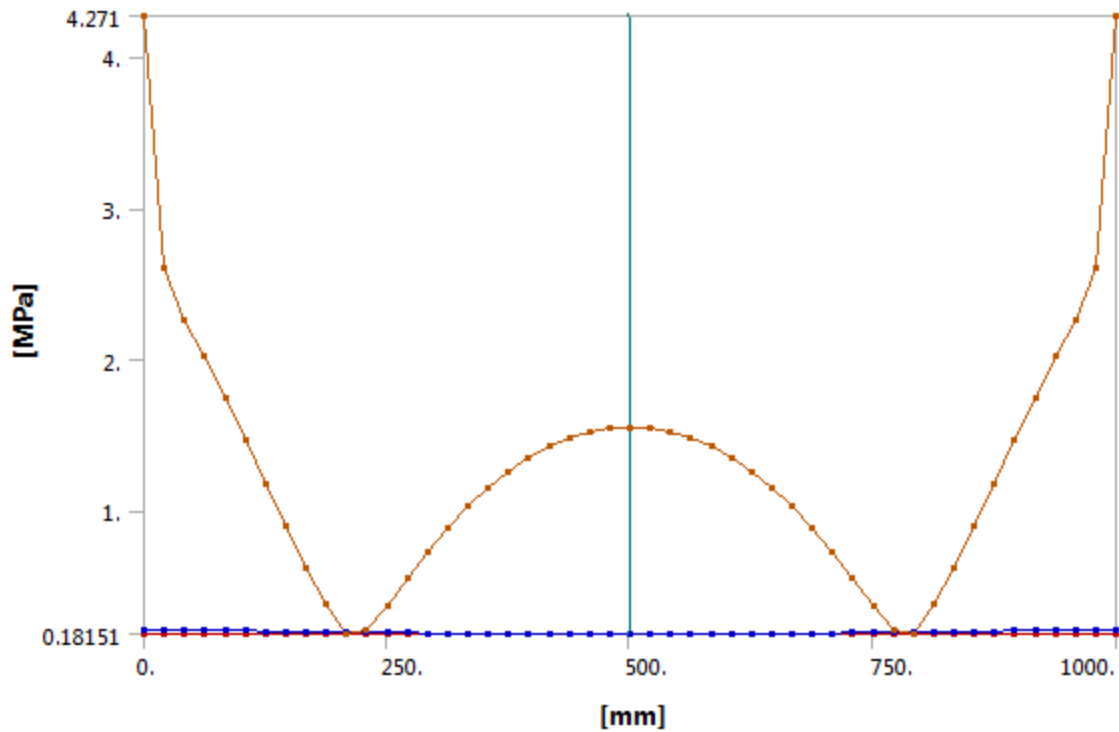


**TABLE 21**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Equivalent Stress**

| Length [mm] | Membrane [MPa] | Bending [MPa] | Membrane+Bending [MPa] | Peak [MPa]  | Total [MPa] |
|-------------|----------------|---------------|------------------------|-------------|-------------|
| 0.          | 0.18151        | 0.11308       | 0.21386                | 4.1807      | 4.271       |
| 20.833      |                | 0.10837       | 0.2114                 | 2.5509      | 2.6074      |
| 41.667      |                | 0.10366       | 0.20903                | 2.2075      | 2.2587      |
| 62.5        |                | 9.8944e-002   | 0.20673                | 1.9687      | 2.0161      |
| 83.333      |                | 9.4232e-002   | 0.20452                | 1.7007      | 1.7469      |
| 104.17      |                | 8.9521e-002   | 0.20239                | 1.4149      | 1.4614      |
| 125.        |                | 8.4809e-002   | 0.20035                | 1.1245      | 1.1723      |
| 145.83      |                | 8.0098e-002   | 0.1984                 | 0.84023     | 0.89075     |
| 166.67      |                | 7.5386e-002   | 0.19655                | 0.5675      | 0.62304     |
| 187.5       |                | 7.0674e-002   | 0.19479                | 0.31033     | 0.37714     |
| 208.33      |                | 6.5963e-002   | 0.19313                | 8.8929e-002 | 0.18685     |
| 229.17      |                | 6.1251e-002   | 0.19157                | 0.17854     | 0.20249     |
| 250.        |                | 5.6539e-002   | 0.19012                | 0.38085     | 0.37051     |
| 270.83      |                | 5.1828e-002   | 0.18877                | 0.57033     | 0.54959     |
| 291.67      |                | 4.7116e-002   | 0.18753                | 0.74796     | 0.72349     |
| 312.5       |                | 4.2405e-002   | 0.1864                 | 0.91019     | 0.88369     |
| 333.33      |                | 3.7693e-002   | 0.18539                | 1.053       | 1.0251      |
| 354.17      |                | 3.2981e-002   | 0.18449                | 1.1792      | 1.1503      |
| 375.        |                | 2.827e-002    | 0.1837                 | 1.2891      | 1.2594      |
| 395.83      |                | 2.3558e-002   | 0.18304                | 1.3808      | 1.3504      |
| 416.67      |                | 1.8846e-002   | 0.18249                | 1.4562      | 1.4253      |
| 437.5       |                | 1.4135e-002   | 0.18206                | 1.515       | 1.4838      |
| 458.33      |                | 9.4232e-003   | 0.18176                | 1.5544      | 1.5226      |
| 479.17      |                | 4.7116e-003   | 0.18157                | 1.5752      | 1.5428      |

|        |             |         |             |         |
|--------|-------------|---------|-------------|---------|
| 500.   | 0.          | 0.18151 | 1.5798      | 1.5468  |
| 520.83 | 4.7116e-003 | 0.18157 | 1.5752      | 1.5428  |
| 541.67 | 9.4232e-003 | 0.18176 | 1.5544      | 1.5226  |
| 562.5  | 1.4135e-002 | 0.18206 | 1.515       | 1.4838  |
| 583.33 | 1.8846e-002 | 0.18249 | 1.4562      | 1.4253  |
| 604.17 | 2.3558e-002 | 0.18303 | 1.3808      | 1.3504  |
| 625.   | 2.827e-002  | 0.1837  | 1.2891      | 1.2594  |
| 645.83 | 3.2981e-002 | 0.18448 | 1.1792      | 1.1503  |
| 666.67 | 3.7693e-002 | 0.18538 | 1.053       | 1.0251  |
| 687.5  | 4.2405e-002 | 0.1864  | 0.9102      | 0.88369 |
| 708.33 | 4.7116e-002 | 0.18753 | 0.74795     | 0.72347 |
| 729.17 | 5.1828e-002 | 0.18877 | 0.57032     | 0.54956 |
| 750.   | 5.6539e-002 | 0.19011 | 0.38084     | 0.37049 |
| 770.83 | 6.1251e-002 | 0.19157 | 0.17854     | 0.20248 |
| 791.67 | 6.5963e-002 | 0.19313 | 8.8925e-002 | 0.18684 |
| 812.5  | 7.0674e-002 | 0.19479 | 0.31032     | 0.37713 |
| 833.33 | 7.5386e-002 | 0.19654 | 0.56749     | 0.62304 |
| 854.17 | 8.0098e-002 | 0.1984  | 0.84022     | 0.89074 |
| 875.   | 8.4809e-002 | 0.20035 | 1.1245      | 1.1723  |
| 895.83 | 8.9521e-002 | 0.20239 | 1.4149      | 1.4614  |
| 916.67 | 9.4232e-002 | 0.20451 | 1.7007      | 1.7469  |
| 937.5  | 9.8944e-002 | 0.20673 | 1.9687      | 2.0161  |
| 958.33 | 0.10366     | 0.20902 | 2.2075      | 2.2587  |
| 979.17 | 0.10837     | 0.2114  | 2.5509      | 2.6074  |
| 1000.  | 0.11308     | 0.21385 | 4.1807      | 4.271   |

**FIGURE 7**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Equivalent Stress > Figure**

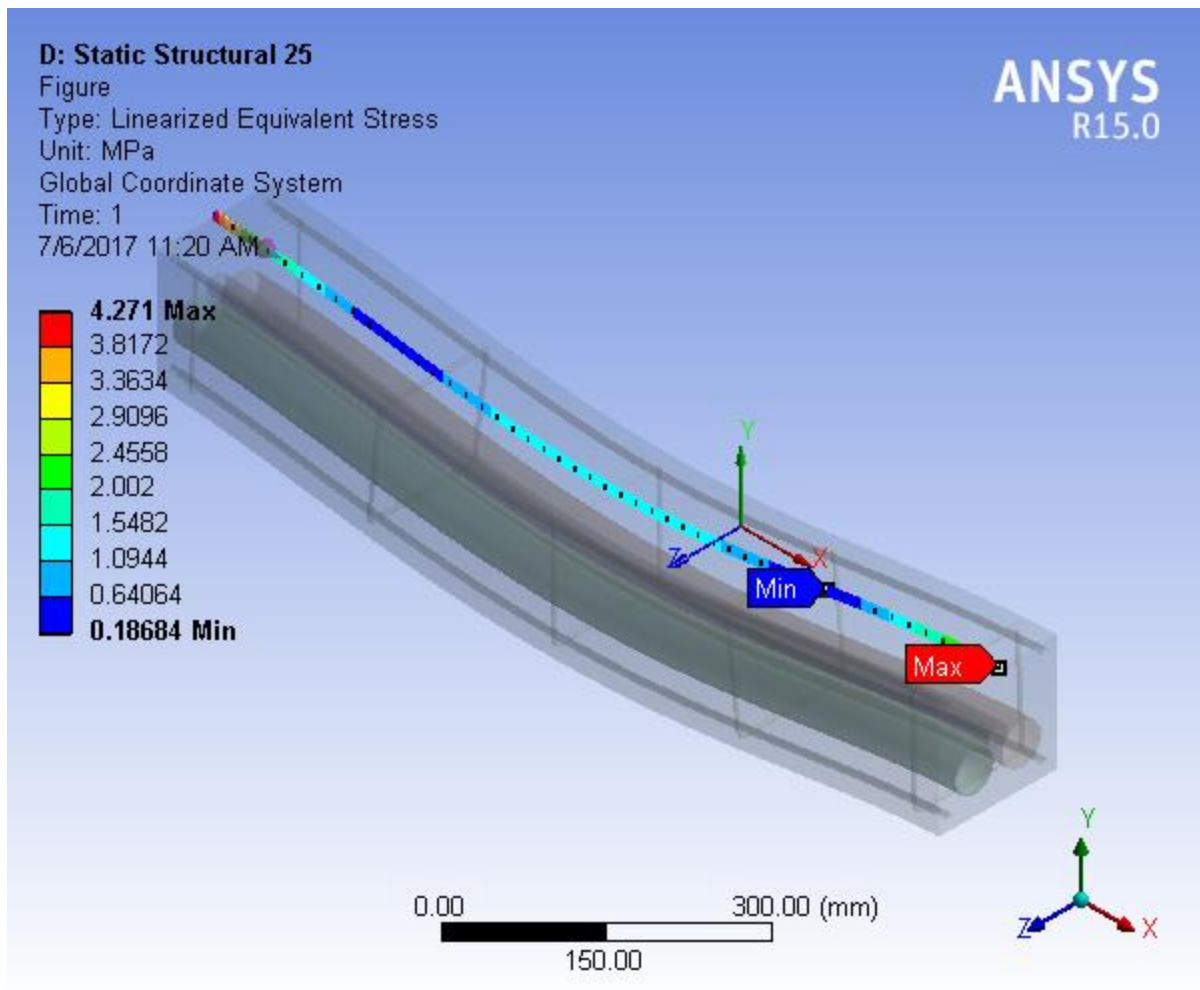


**FIGURE 8**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Equivalent Stress 2**

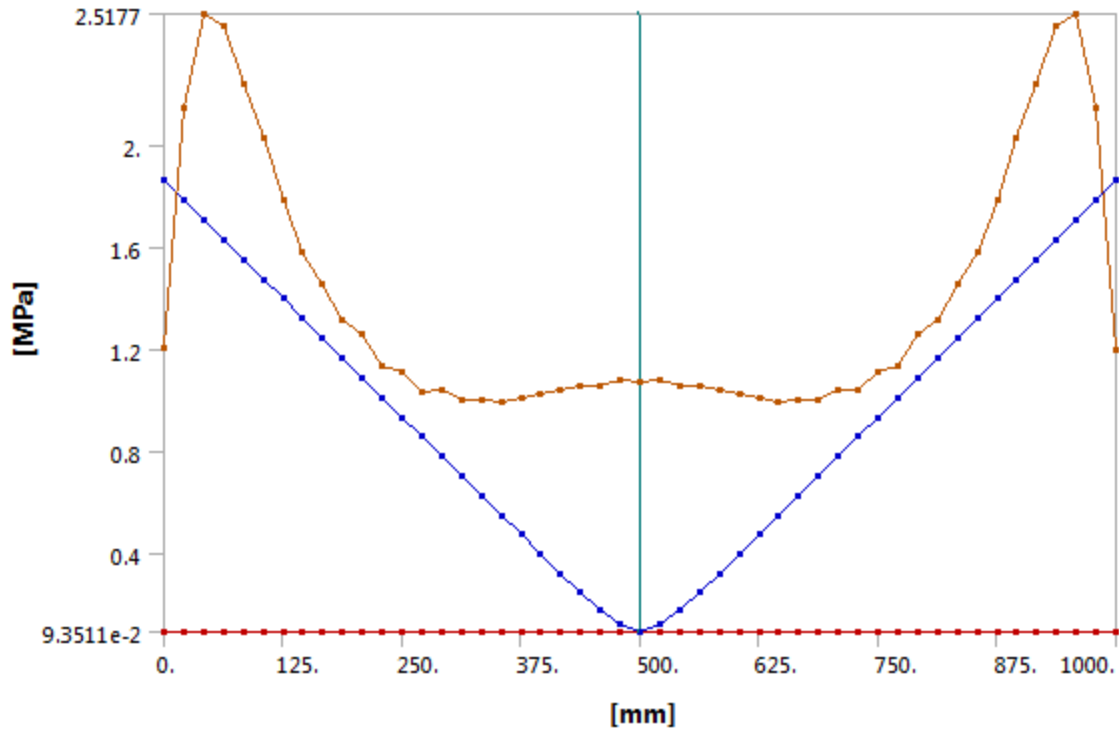


**TABLE 22**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Equivalent Stress 2**

| Length [mm] | Membrane [MPa] | Bending [MPa] | Membrane+Bending [MPa] | Peak [MPa] | Total [MPa] |
|-------------|----------------|---------------|------------------------|------------|-------------|
| 0.          | 9.3511e-002    | 1.8632        | 1.8655                 | 2.8865     | 1.2064      |
| 20.833      |                | 1.7856        | 1.788                  | 1.5861     | 2.146       |
| 41.667      |                | 1.708         | 1.7105                 | 1.7979     | 2.5177      |
| 62.5        |                | 1.6303        | 1.633                  | 1.6462     | 2.4682      |
| 83.333      |                | 1.5527        | 1.5555                 | 1.3693     | 2.246       |
| 104.17      |                | 1.4751        | 1.478                  | 1.0974     | 2.0282      |
| 125.        |                | 1.3974        | 1.4005                 | 0.822      | 1.788       |
| 145.83      |                | 1.3198        | 1.3231                 | 0.59116    | 1.5842      |
| 166.67      |                | 1.2422        | 1.2456                 | 0.39119    | 1.4595      |
| 187.5       |                | 1.1645        | 1.1682                 | 0.22512    | 1.3199      |
| 208.33      |                | 1.0869        | 1.0909                 | 0.17441    | 1.2615      |
| 229.17      |                | 1.0093        | 1.0135                 | 0.18018    | 1.1391      |
| 250.        |                | 0.93162       | 0.93627                | 0.31082    | 1.1124      |
| 270.83      |                | 0.85398       | 0.85906                | 0.40662    | 1.0388      |
| 291.67      |                | 0.77635       | 0.78193                | 0.52431    | 1.0408      |
| 312.5       |                | 0.69871       | 0.70491                | 0.61125    | 1.0026      |
| 333.33      |                | 0.62108       | 0.62805                | 0.69844    | 1.0039      |
| 354.17      |                | 0.54344       | 0.5514                 | 0.77152    | 0.99375     |
| 375.        |                | 0.46581       | 0.47507                | 0.84086    | 1.0086      |
| 395.83      |                | 0.38817       | 0.39925                | 0.90145    | 1.0291      |
| 416.67      |                | 0.31054       | 0.32428                | 0.94886    | 1.0413      |
| 437.5       |                | 0.2329        | 0.25095                | 0.98779    | 1.0571      |
| 458.33      |                | 0.15527       | 0.18123                | 1.0111     | 1.0623      |
| 479.17      |                | 7.7635e-002   | 0.12152                | 1.038      | 1.0802      |

|        |             |             |         |         |
|--------|-------------|-------------|---------|---------|
| 500.   | 4.0332e-016 | 9.3511e-002 | 1.039   | 1.0778  |
| 520.83 | 7.7635e-002 | 0.12156     | 1.038   | 1.0803  |
| 541.67 | 0.15527     | 0.18128     | 1.011   | 1.0623  |
| 562.5  | 0.2329      | 0.251       | 0.98776 | 1.0571  |
| 583.33 | 0.31054     | 0.32434     | 0.94884 | 1.0413  |
| 604.17 | 0.38817     | 0.39931     | 0.90142 | 1.0291  |
| 625.   | 0.46581     | 0.47513     | 0.84088 | 1.0087  |
| 645.83 | 0.54344     | 0.55146     | 0.77152 | 0.99377 |
| 666.67 | 0.62108     | 0.62811     | 0.69843 | 1.0039  |
| 687.5  | 0.69871     | 0.70497     | 0.61122 | 1.0027  |
| 708.33 | 0.77635     | 0.78199     | 0.52431 | 1.0409  |
| 729.17 | 0.85398     | 0.85912     | 0.4066  | 1.0389  |
| 750.   | 0.93162     | 0.93633     | 0.31075 | 1.1123  |
| 770.83 | 1.0093      | 1.0136      | 0.18003 | 1.139   |
| 791.67 | 1.0869      | 1.0909      | 0.17415 | 1.2614  |
| 812.5  | 1.1645      | 1.1683      | 0.22506 | 1.3198  |
| 833.33 | 1.2422      | 1.2457      | 0.39111 | 1.4594  |
| 854.17 | 1.3198      | 1.3231      | 0.59108 | 1.584   |
| 875.   | 1.3974      | 1.4006      | 0.82192 | 1.7877  |
| 895.83 | 1.4751      | 1.478       | 1.0974  | 2.0281  |
| 916.67 | 1.5527      | 1.5555      | 1.3692  | 2.2459  |
| 937.5  | 1.6303      | 1.633       | 1.6462  | 2.4681  |
| 958.33 | 1.708       | 1.7105      | 1.798   | 2.5177  |
| 979.17 | 1.7856      | 1.7881      | 1.5865  | 2.1464  |
| 1000.  | 1.8632      | 1.8656      | 2.8821  | 1.2023  |

**FIGURE 9**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Equivalent Stress 2 > Figure**

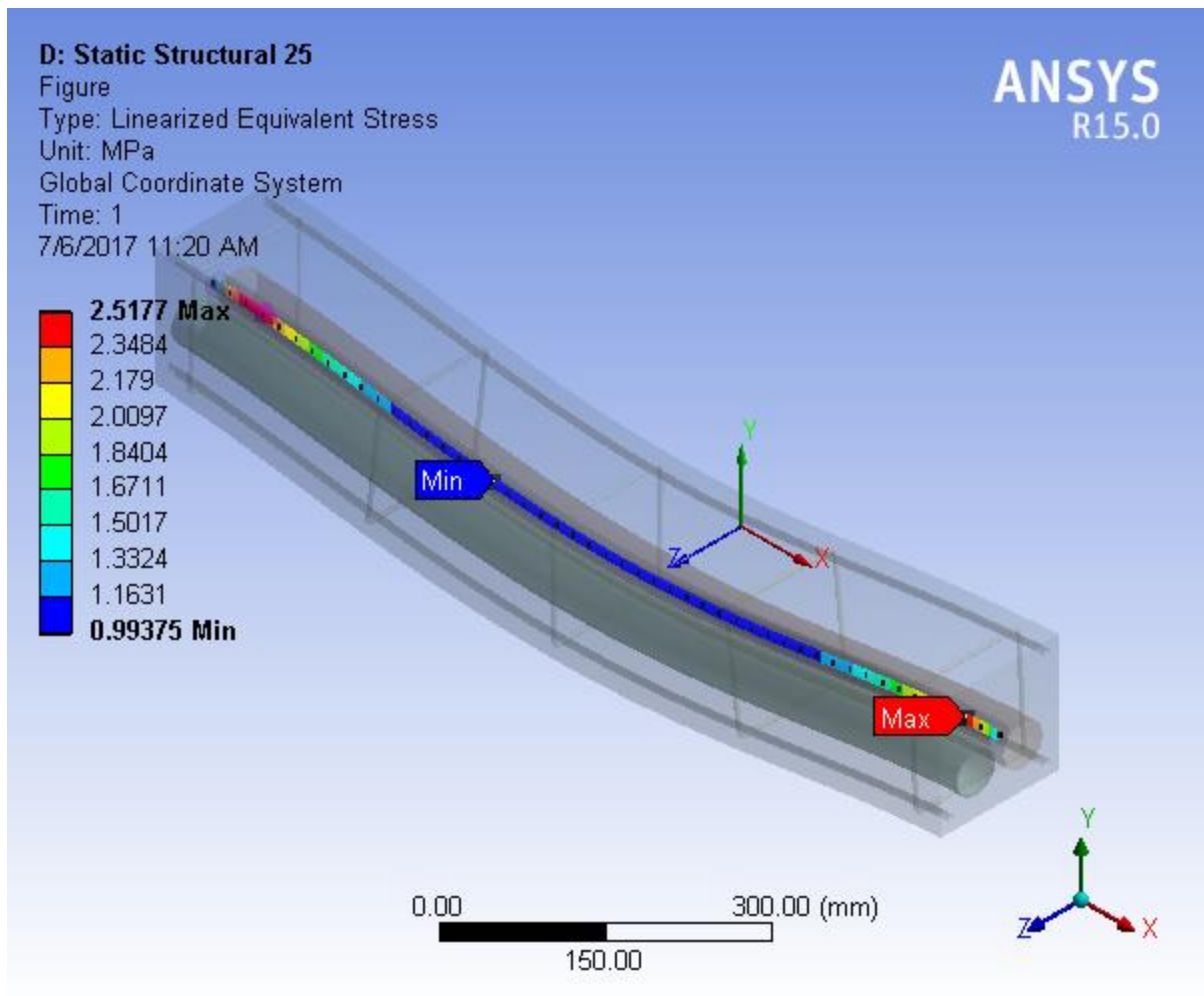


**FIGURE 10**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Normal Stress**

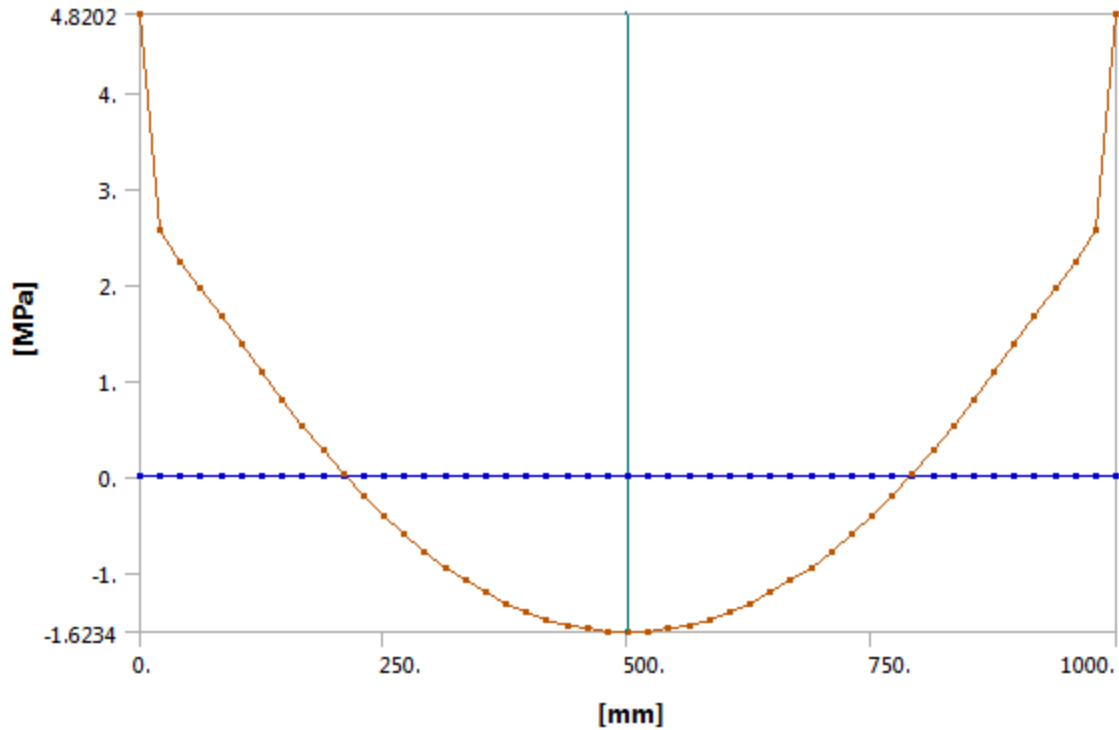


**TABLE 23**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Normal Stress**

| Length [mm] | Membrane [MPa] | Bending [MPa] | Membrane+Bending [MPa] | Peak [MPa]  | Total [MPa] |
|-------------|----------------|---------------|------------------------|-------------|-------------|
| 0.          | -5.6932e-003   | 1.2239e-005   | -5.6809e-003           | 4.8259      | 4.8202      |
| 20.833      |                | 1.1729e-005   | -5.6814e-003           | 2.578       | 2.5723      |
| 41.667      |                | 1.1219e-005   | -5.682e-003            | 2.2299      | 2.2242      |
| 62.5        |                | 1.0709e-005   | -5.6825e-003           | 1.9621      | 1.9564      |
| 83.333      |                | 1.0199e-005   | -5.683e-003            | 1.6795      | 1.6738      |
| 104.17      |                | 9.6891e-006   | -5.6835e-003           | 1.387       | 1.3813      |
| 125.        |                | 9.1791e-006   | -5.684e-003            | 1.0935      | 1.0878      |
| 145.83      |                | 8.6692e-006   | -5.6845e-003           | 0.80747     | 0.80178     |
| 166.67      |                | 8.1592e-006   | -5.685e-003            | 0.53293     | 0.52724     |
| 187.5       |                | 7.6493e-006   | -5.6855e-003           | 0.27217     | 0.26648     |
| 208.33      |                | 7.1393e-006   | -5.686e-003            | 2.8639e-002 | 2.2953e-002 |
| 229.17      |                | 6.6294e-006   | -5.6865e-003           | -0.19866    | -0.20435    |
| 250.        |                | 6.1194e-006   | -5.6871e-003           | -0.40908    | -0.41477    |
| 270.83      |                | 5.6095e-006   | -5.6876e-003           | -0.60157    | -0.60725    |
| 291.67      |                | 5.0995e-006   | -5.6881e-003           | -0.77878    | -0.78447    |
| 312.5       |                | 4.5896e-006   | -5.6886e-003           | -0.94006    | -0.94575    |
| 333.33      |                | 4.0796e-006   | -5.6891e-003           | -1.0824     | -1.0881     |
| 354.17      |                | 3.5697e-006   | -5.6896e-003           | -1.2084     | -1.2141     |
| 375.        |                | 3.0597e-006   | -5.6901e-003           | -1.3183     | -1.324      |
| 395.83      |                | 2.5498e-006   | -5.6906e-003           | -1.4101     | -1.4158     |
| 416.67      |                | 2.0398e-006   | -5.6911e-003           | -1.4859     | -1.4915     |
| 437.5       |                | 1.5299e-006   | -5.6916e-003           | -1.5455     | -1.5512     |
| 458.33      |                | 1.0199e-006   | -5.6922e-003           | -1.5864     | -1.5921     |
| 479.17      |                | 5.0995e-007   | -5.6927e-003           | -1.6102     | -1.6159     |

|        |              |              |             |             |
|--------|--------------|--------------|-------------|-------------|
| 500.   | 0.           | -5.6932e-003 | -1.6177     | -1.6234     |
| 520.83 | -5.0995e-007 | -5.6937e-003 | -1.6102     | -1.6159     |
| 541.67 | -1.0199e-006 | -5.6942e-003 | -1.5864     | -1.5921     |
| 562.5  | -1.5299e-006 | -5.6947e-003 | -1.5455     | -1.5512     |
| 583.33 | -2.0398e-006 | -5.6952e-003 | -1.4859     | -1.4915     |
| 604.17 | -2.5498e-006 | -5.6957e-003 | -1.4101     | -1.4158     |
| 625.   | -3.0597e-006 | -5.6962e-003 | -1.3183     | -1.324      |
| 645.83 | -3.5697e-006 | -5.6967e-003 | -1.2084     | -1.2141     |
| 666.67 | -4.0796e-006 | -5.6973e-003 | -1.0824     | -1.0881     |
| 687.5  | -4.5896e-006 | -5.6978e-003 | -0.94006    | -0.94575    |
| 708.33 | -5.0995e-006 | -5.6983e-003 | -0.77877    | -0.78447    |
| 729.17 | -5.6095e-006 | -5.6988e-003 | -0.60156    | -0.60726    |
| 750.   | -6.1194e-006 | -5.6993e-003 | -0.40907    | -0.41477    |
| 770.83 | -6.6294e-006 | -5.6998e-003 | -0.19866    | -0.20436    |
| 791.67 | -7.1393e-006 | -5.7003e-003 | 2.8646e-002 | 2.2945e-002 |
| 812.5  | -7.6493e-006 | -5.7008e-003 | 0.27217     | 0.26647     |
| 833.33 | -8.1592e-006 | -5.7013e-003 | 0.53294     | 0.52724     |
| 854.17 | -8.6692e-006 | -5.7018e-003 | 0.80748     | 0.80178     |
| 875.   | -9.1791e-006 | -5.7024e-003 | 1.0935      | 1.0878      |
| 895.83 | -9.6891e-006 | -5.7029e-003 | 1.387       | 1.3813      |
| 916.67 | -1.0199e-005 | -5.7034e-003 | 1.6795      | 1.6738      |
| 937.5  | -1.0709e-005 | -5.7039e-003 | 1.9622      | 1.9565      |
| 958.33 | -1.1219e-005 | -5.7044e-003 | 2.23        | 2.2242      |
| 979.17 | -1.1729e-005 | -5.7049e-003 | 2.5781      | 2.5724      |
| 1000.  | -1.2239e-005 | -5.7054e-003 | 4.8255      | 4.8198      |

**FIGURE 11**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Normal Stress > Figure**

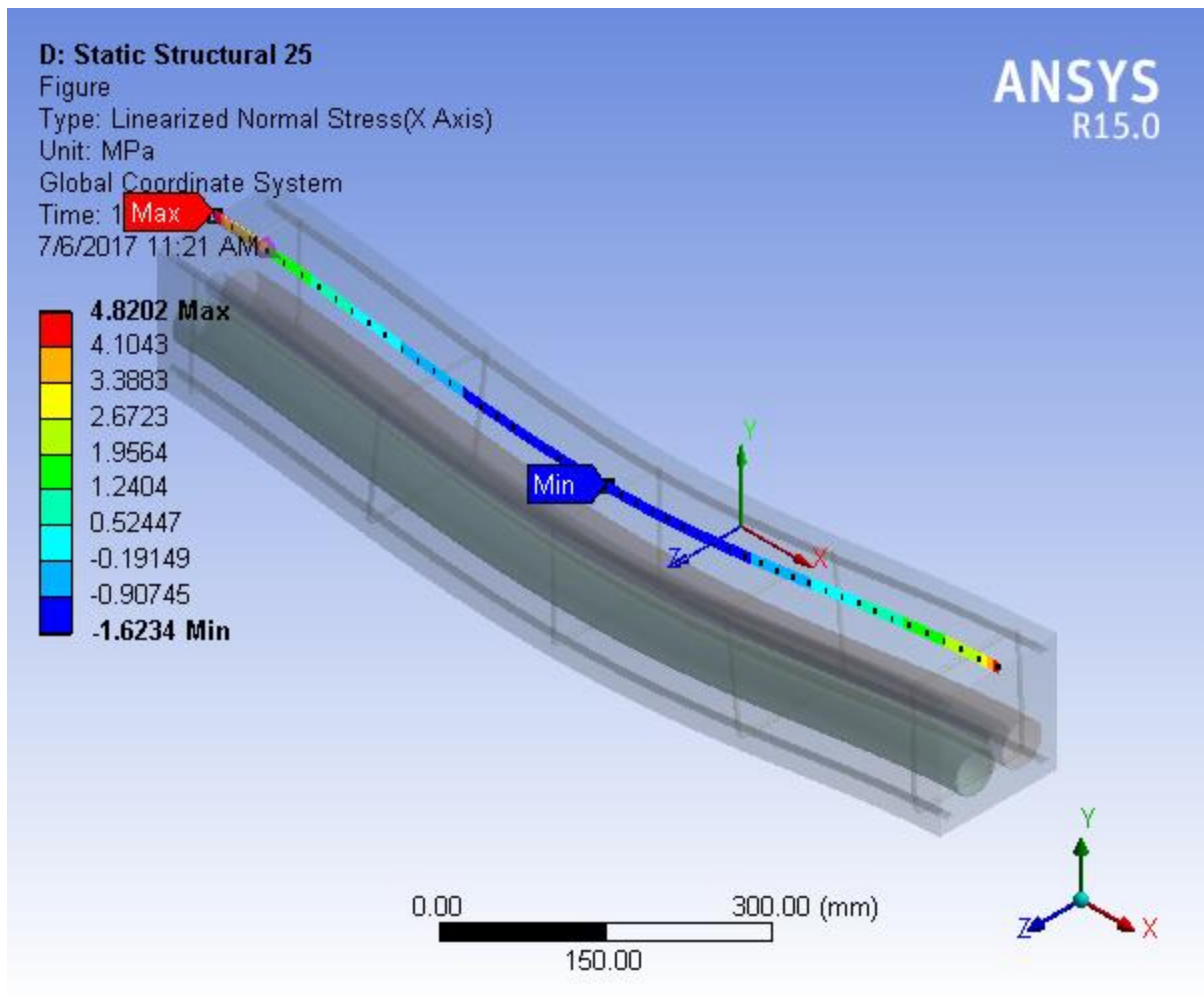


**FIGURE 12**  
 Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Normal Stress 2

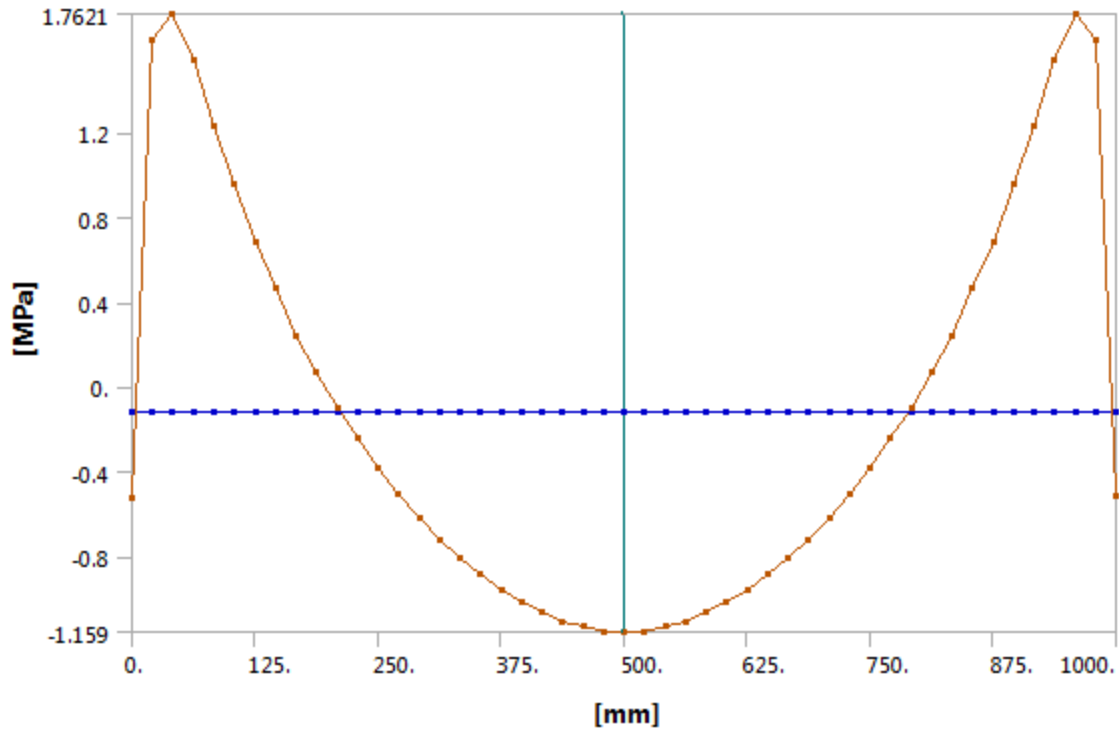


**TABLE 24**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Normal Stress 2**

| Length [mm] | Membrane [MPa] | Bending [MPa] | Membrane+Bending [MPa] | Peak [MPa]  | Total [MPa] |
|-------------|----------------|---------------|------------------------|-------------|-------------|
| 0.          | -0.12341       | 2.4275e-004   | -0.12317               | -0.3983     | -0.52147    |
| 20.833      |                | 2.3264e-004   | -0.12318               | 1.7589      | 1.6358      |
| 41.667      |                | 2.2252e-004   | -0.12319               | 1.8853      | 1.7621      |
| 62.5        |                | 2.1241e-004   | -0.1232                | 1.6679      | 1.5447      |
| 83.333      |                | 2.0229e-004   | -0.12321               | 1.3577      | 1.2344      |
| 104.17      |                | 1.9218e-004   | -0.12322               | 1.0836      | 0.96042     |
| 125.        |                | 1.8206e-004   | -0.12323               | 0.80499     | 0.68177     |
| 145.83      |                | 1.7195e-004   | -0.12324               | 0.58967     | 0.46643     |
| 166.67      |                | 1.6183e-004   | -0.12325               | 0.36548     | 0.24223     |
| 187.5       |                | 1.5172e-004   | -0.12326               | 0.19554     | 7.2288e-002 |
| 208.33      |                | 1.416e-004    | -0.12327               | 2.3032e-002 | -0.10024    |
| 229.17      |                | 1.3149e-004   | -0.12328               | -0.11819    | -0.24147    |
| 250.        |                | 1.2138e-004   | -0.12329               | -0.25974    | -0.38303    |
| 270.83      |                | 1.1126e-004   | -0.1233                | -0.38161    | -0.5049     |
| 291.67      |                | 1.0115e-004   | -0.12331               | -0.49841    | -0.62172    |
| 312.5       |                | 9.1032e-005   | -0.12332               | -0.59661    | -0.71993    |
| 333.33      |                | 8.0917e-005   | -0.12333               | -0.68758    | -0.81091    |
| 354.17      |                | 7.0802e-005   | -0.12334               | -0.76554    | -0.88888    |
| 375.        |                | 6.0688e-005   | -0.12335               | -0.83584    | -0.95919    |
| 395.83      |                | 5.0573e-005   | -0.12336               | -0.8972     | -1.0206     |
| 416.67      |                | 4.0459e-005   | -0.12337               | -0.94558    | -1.069      |
| 437.5       |                | 3.0344e-005   | -0.12338               | -0.98518    | -1.1086     |
| 458.33      |                | 2.0229e-005   | -0.12339               | -1.0095     | -1.1329     |
| 479.17      |                | 1.0115e-005   | -0.1234                | -1.0338     | -1.1572     |

|        |              |          |            |             |
|--------|--------------|----------|------------|-------------|
| 500.   | 5.6843e-020  | -0.12341 | -1.0356    | -1.159      |
| 520.83 | -1.0115e-005 | -0.12342 | -1.0338    | -1.1572     |
| 541.67 | -2.0229e-005 | -0.12343 | -1.0095    | -1.1329     |
| 562.5  | -3.0344e-005 | -0.12344 | -0.98518   | -1.1086     |
| 583.33 | -4.0459e-005 | -0.12345 | -0.9456    | -1.0691     |
| 604.17 | -5.0573e-005 | -0.12346 | -0.89722   | -1.0207     |
| 625.   | -6.0688e-005 | -0.12347 | -0.8359    | -0.95937    |
| 645.83 | -7.0802e-005 | -0.12348 | -0.7656    | -0.88908    |
| 666.67 | -8.0917e-005 | -0.12349 | -0.68765   | -0.81114    |
| 687.5  | -9.1032e-005 | -0.1235  | -0.59668   | -0.72018    |
| 708.33 | -1.0115e-004 | -0.12351 | -0.49851   | -0.62202    |
| 729.17 | -1.1126e-004 | -0.12352 | -0.38169   | -0.50521    |
| 750.   | -1.2138e-004 | -0.12353 | -0.25985   | -0.38338    |
| 770.83 | -1.3149e-004 | -0.12354 | -0.11829   | -0.24183    |
| 791.67 | -1.416e-004  | -0.12355 | 2.291e-002 | -0.10064    |
| 812.5  | -1.5172e-004 | -0.12356 | 0.19541    | 7.1855e-002 |
| 833.33 | -1.6183e-004 | -0.12357 | 0.36532    | 0.24175     |
| 854.17 | -1.7195e-004 | -0.12358 | 0.58949    | 0.46591     |
| 875.   | -1.8206e-004 | -0.12359 | 0.80478    | 0.68119     |
| 895.83 | -1.9218e-004 | -0.1236  | 1.0834     | 0.95976     |
| 916.67 | -2.0229e-004 | -0.12361 | 1.3574     | 1.2338      |
| 937.5  | -2.1241e-004 | -0.12362 | 1.6675     | 1.5439      |
| 958.33 | -2.2252e-004 | -0.12363 | 1.8852     | 1.7615      |
| 979.17 | -2.3264e-004 | -0.12364 | 1.7593     | 1.6356      |
| 1000.  | -2.4275e-004 | -0.12365 | -0.39554   | -0.51919    |

**FIGURE 13**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Normal Stress 2 > Figure**

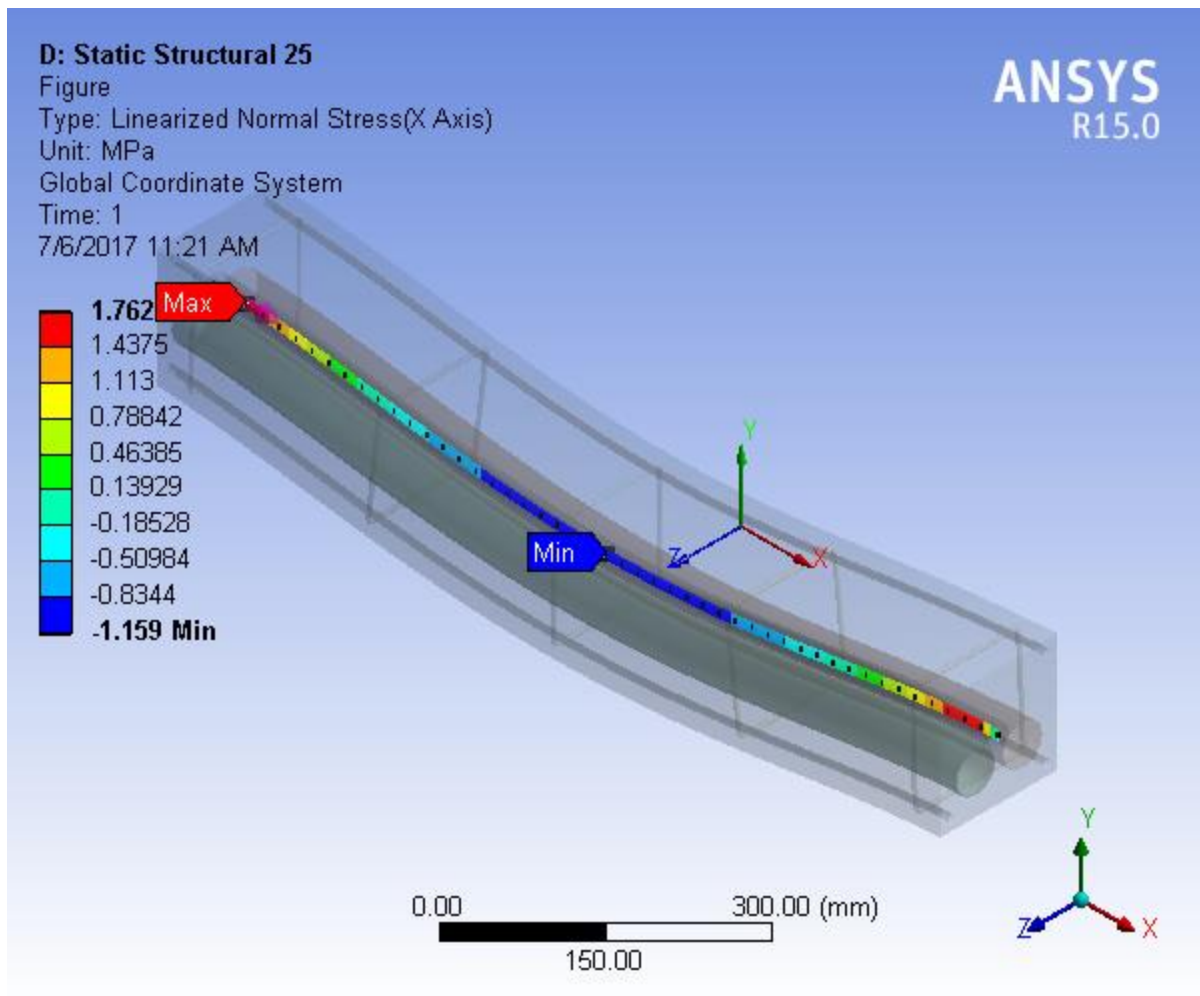


**FIGURE 14**  
Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Stress Intensity

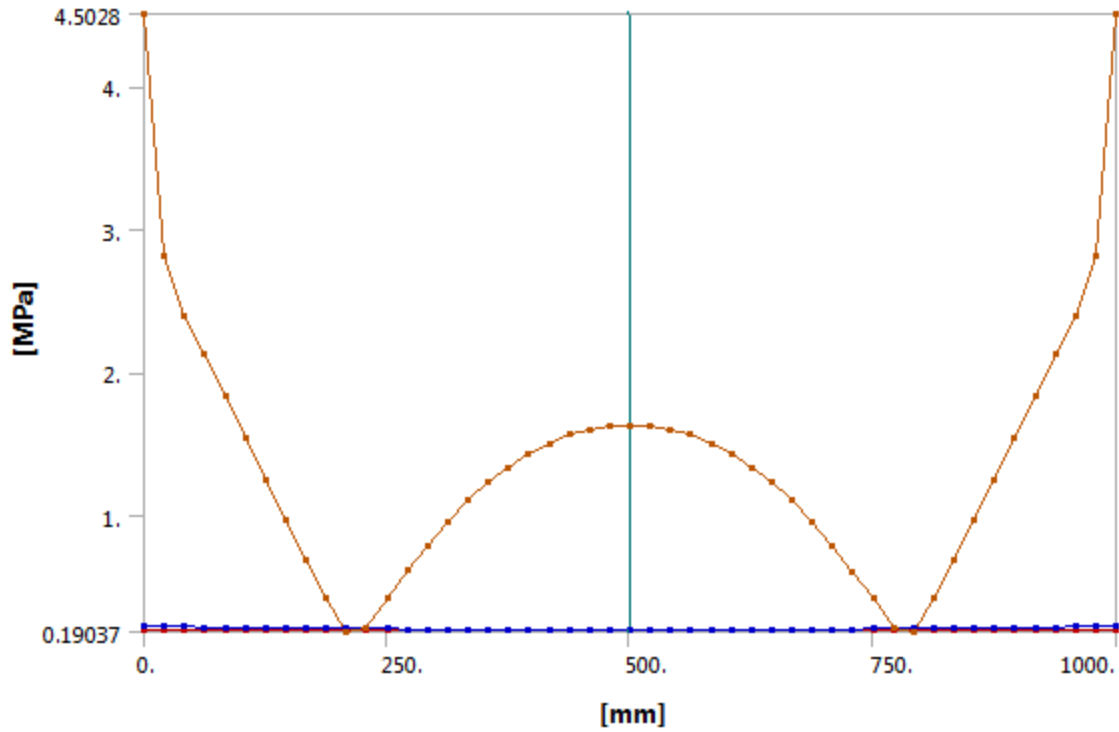


**TABLE 25**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Stress Intensity**

| Length [mm] | Membrane [MPa] | Bending [MPa] | Membrane+Bending [MPa] | Peak [MPa]  | Total [MPa] |
|-------------|----------------|---------------|------------------------|-------------|-------------|
| 0.          | 0.20455        | 0.13057       | 0.23019                | 4.3142      | 4.5027      |
| 20.833      |                | 0.12513       | 0.22835                | 2.6856      | 2.8196      |
| 41.667      |                | 0.11969       | 0.22655                | 2.2619      | 2.4         |
| 62.5        |                | 0.11425       | 0.2248                 | 1.9852      | 2.1238      |
| 83.333      |                | 0.10881       | 0.22311                | 1.7023      | 1.8408      |
| 104.17      |                | 0.10337       | 0.22146                | 1.4198      | 1.5483      |
| 125.        |                | 9.7929e-002   | 0.21988                | 1.1318      | 1.2549      |
| 145.83      |                | 9.2489e-002   | 0.21835                | 0.84826     | 0.96901     |
| 166.67      |                | 8.7048e-002   | 0.21689                | 0.57561     | 0.69454     |
| 187.5       |                | 8.1608e-002   | 0.21549                | 0.31769     | 0.43383     |
| 208.33      |                | 7.6167e-002   | 0.21416                | 8.9962e-002 | 0.19038     |
| 229.17      |                | 7.0727e-002   | 0.2129                 | 0.1914      | 0.21845     |
| 250.        |                | 6.5286e-002   | 0.21172                | 0.39427     | 0.42598     |
| 270.83      |                | 5.9846e-002   | 0.21062                | 0.58469     | 0.61605     |
| 291.67      |                | 5.4405e-002   | 0.2096                 | 0.76073     | 0.79626     |
| 312.5       |                | 4.8965e-002   | 0.20867                | 0.92139     | 0.96064     |
| 333.33      |                | 4.3524e-002   | 0.20782                | 1.0633      | 1.1046      |
| 354.17      |                | 3.8084e-002   | 0.20706                | 1.1891      | 1.2315      |
| 375.        |                | 3.2643e-002   | 0.2064                 | 1.2988      | 1.3417      |
| 395.83      |                | 2.7203e-002   | 0.20584                | 1.3904      | 1.4334      |
| 416.67      |                | 2.1762e-002   | 0.20538                | 1.4661      | 1.5086      |
| 437.5       |                | 1.6322e-002   | 0.20501                | 1.5256      | 1.5668      |
| 458.33      |                | 1.0881e-002   | 0.20475                | 1.5666      | 1.6045      |
| 479.17      |                | 5.4405e-003   | 0.2046                 | 1.5903      | 1.6224      |

|        |             |         |             |         |
|--------|-------------|---------|-------------|---------|
| 500.   | 0.          | 0.20455 | 1.5978      | 1.6239  |
| 520.83 | 5.4405e-003 | 0.2046  | 1.5903      | 1.6224  |
| 541.67 | 1.0881e-002 | 0.20475 | 1.5666      | 1.6045  |
| 562.5  | 1.6322e-002 | 0.20501 | 1.5256      | 1.5668  |
| 583.33 | 2.1762e-002 | 0.20538 | 1.4661      | 1.5086  |
| 604.17 | 2.7203e-002 | 0.20584 | 1.3904      | 1.4335  |
| 625.   | 3.2643e-002 | 0.2064  | 1.2988      | 1.3417  |
| 645.83 | 3.8084e-002 | 0.20706 | 1.1891      | 1.2315  |
| 666.67 | 4.3524e-002 | 0.20782 | 1.0634      | 1.1046  |
| 687.5  | 4.8965e-002 | 0.20866 | 0.9214      | 0.96063 |
| 708.33 | 5.4405e-002 | 0.2096  | 0.76073     | 0.79623 |
| 729.17 | 5.9846e-002 | 0.21062 | 0.58469     | 0.61601 |
| 750.   | 6.5286e-002 | 0.21172 | 0.39428     | 0.42596 |
| 770.83 | 7.0727e-002 | 0.2129  | 0.19141     | 0.21844 |
| 791.67 | 7.6167e-002 | 0.21416 | 8.9958e-002 | 0.19037 |
| 812.5  | 8.1608e-002 | 0.21549 | 0.31768     | 0.43382 |
| 833.33 | 8.7048e-002 | 0.21688 | 0.5756      | 0.69453 |
| 854.17 | 9.2489e-002 | 0.21835 | 0.84825     | 0.969   |
| 875.   | 9.7929e-002 | 0.21987 | 1.1318      | 1.2549  |
| 895.83 | 0.10337     | 0.22146 | 1.4198      | 1.5483  |
| 916.67 | 0.10881     | 0.2231  | 1.7023      | 1.8408  |
| 937.5  | 0.11425     | 0.2248  | 1.9852      | 2.1239  |
| 958.33 | 0.11969     | 0.22654 | 2.2619      | 2.4     |
| 979.17 | 0.12513     | 0.22834 | 2.6856      | 2.8196  |
| 1000.  | 0.13057     | 0.23018 | 4.3142      | 4.5028  |

**FIGURE 15**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Stress Intensity 2**

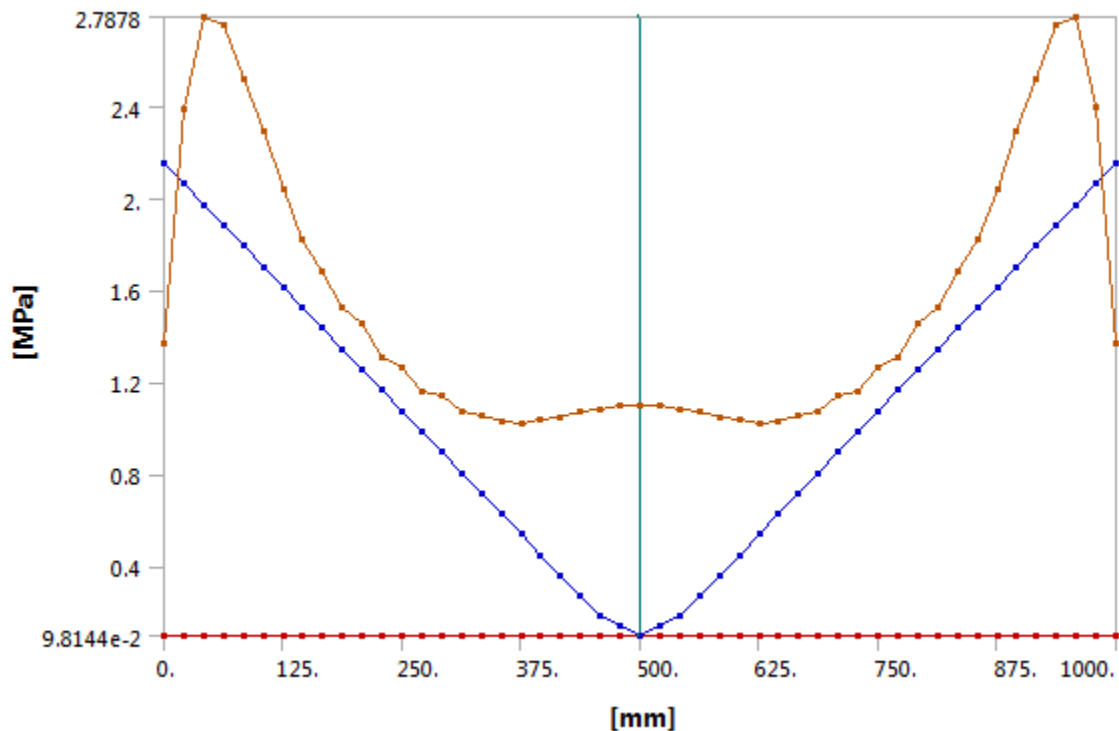


**TABLE 26**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Stress Intensity 2**

| Length [mm] | Membrane [MPa] | Bending [MPa] | Membrane+Bending [MPa] | Peak [MPa] | Total [MPa] |
|-------------|----------------|---------------|------------------------|------------|-------------|
| 0.          | 9.8144e-002    | 2.1515        | 2.1515                 | 3.3139     | 1.3697      |
| 20.833      |                | 2.0618        | 2.0618                 | 1.6696     | 2.3914      |
| 41.667      |                | 1.9722        | 1.9722                 | 1.8552     | 2.7877      |
| 62.5        |                | 1.8825        | 1.8826                 | 1.6967     | 2.7496      |
| 83.333      |                | 1.7929        | 1.7929                 | 1.4093     | 2.5208      |
| 104.17      |                | 1.7033        | 1.7033                 | 1.13       | 2.2951      |
| 125.        |                | 1.6136        | 1.6136                 | 0.84281    | 2.0382      |
| 145.83      |                | 1.524         | 1.524                  | 0.60592    | 1.8175      |
| 166.67      |                | 1.4343        | 1.4344                 | 0.40369    | 1.6826      |
| 187.5       |                | 1.3447        | 1.3447                 | 0.23761    | 1.5241      |
| 208.33      |                | 1.255         | 1.2551                 | 0.20075    | 1.4538      |
| 229.17      |                | 1.1654        | 1.1654                 | 0.19914    | 1.3046      |
| 250.        |                | 1.0757        | 1.0758                 | 0.33123    | 1.2608      |
| 270.83      |                | 0.98609       | 0.98617                | 0.42428    | 1.1592      |
| 291.67      |                | 0.89645       | 0.89654                | 0.54346    | 1.1421      |
| 312.5       |                | 0.8068        | 0.80691                | 0.63068    | 1.0773      |
| 333.33      |                | 0.71716       | 0.71728                | 0.71799    | 1.0576      |
| 354.17      |                | 0.62751       | 0.62766                | 0.79263    | 1.0261      |
| 375.        |                | 0.53787       | 0.53805                | 0.86197    | 1.0246      |
| 395.83      |                | 0.44822       | 0.44846                | 0.92461    | 1.0346      |
| 416.67      |                | 0.35858       | 0.3589                 | 0.97168    | 1.0489      |
| 437.5       |                | 0.26893       | 0.26949                | 1.0118     | 1.0706      |
| 458.33      |                | 0.17929       | 0.18512                | 1.035      | 1.0818      |
| 479.17      |                | 8.9645e-002   | 0.13823                | 1.0627     | 1.1018      |
| 500.        |                | 4.6572e-016   | 9.8144e-002            | 1.0636     | 1.1006      |
| 520.83      |                | 8.9645e-002   | 0.13827                | 1.0627     | 1.1018      |
| 541.67      |                | 0.17929       | 0.18515                | 1.035      | 1.0818      |
| 562.5       |                | 0.26893       | 0.26955                | 1.0118     | 1.0706      |
| 583.33      |                | 0.35858       | 0.35896                | 0.97165    | 1.0489      |
| 604.17      |                | 0.44822       | 0.44852                | 0.92458    | 1.0346      |
| 625.        |                | 0.53787       | 0.53811                | 0.86198    | 1.0247      |
| 645.83      |                | 0.62751       | 0.62772                | 0.79262    | 1.0261      |
| 666.67      |                | 0.71716       | 0.71734                | 0.71797    | 1.0576      |
| 687.5       |                | 0.8068        | 0.80697                | 0.63065    | 1.0773      |
| 708.33      |                | 0.89645       | 0.8966                 | 0.54346    | 1.1422      |
| 729.17      |                | 0.98609       | 0.98623                | 0.42425    | 1.1593      |
| 750.        |                | 1.0757        | 1.0759                 | 0.33114    | 1.2607      |
| 770.83      |                | 1.1654        | 1.1655                 | 0.19896    | 1.3045      |
| 791.67      |                | 1.255         | 1.2551                 | 0.20045    | 1.4536      |
| 812.5       |                | 1.3447        | 1.3448                 | 0.23754    | 1.524       |
| 833.33      |                | 1.4343        | 1.4344                 | 0.40358    | 1.6825      |
| 854.17      |                | 1.524         | 1.5241                 | 0.60582    | 1.8173      |
| 875.        |                | 1.6136        | 1.6137                 | 0.8427     | 2.0379      |
| 895.83      |                | 1.7033        | 1.7033                 | 1.1299     | 2.2951      |
| 916.67      |                | 1.7929        | 1.793                  | 1.4092     | 2.5207      |
| 937.5       |                | 1.8825        | 1.8826                 | 1.6967     | 2.7496      |
| 958.33      |                | 1.9722        | 1.9723                 | 1.8553     | 2.7878      |
| 979.17      |                | 2.0618        | 2.0619                 | 1.6699     | 2.3918      |
| 1000.       |                | 2.1515        | 2.1516                 | 3.3089     | 1.3651      |

**FIGURE 16**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Shear Stress**

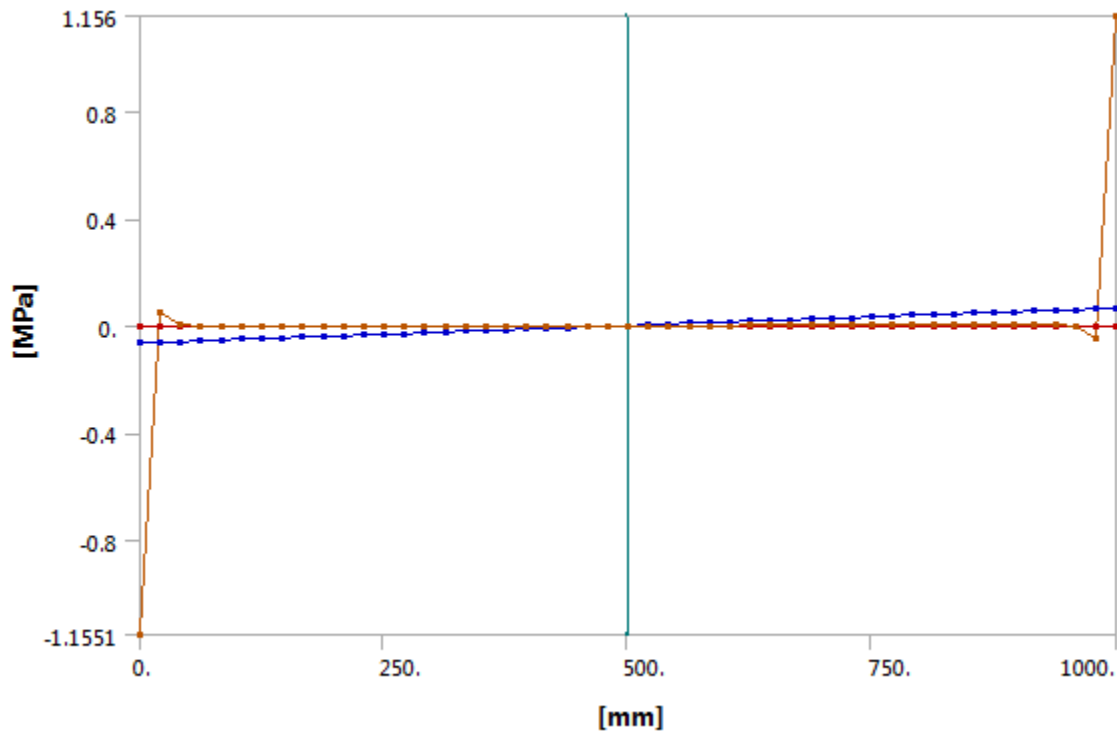


**TABLE 27**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Shear Stress**

| Length [mm] | Membrane [MPa] | Bending [MPa] | Membrane+Bending [MPa] | Peak [MPa]  | Total [MPa]  |
|-------------|----------------|---------------|------------------------|-------------|--------------|
| 0.          | 1.5112e-005    | -6.5225e-002  | -6.521e-002            | -1.0899     | -1.1551      |
| 20.833      |                | -6.2507e-002  | -6.2492e-002           | 0.11448     | 5.199e-002   |
| 41.667      |                | -5.9789e-002  | -5.9774e-002           | 6.5467e-002 | 5.6929e-003  |
| 62.5        |                | -5.7072e-002  | -5.7057e-002           | 5.6623e-002 | -4.3308e-004 |
| 83.333      |                | -5.4354e-002  | -5.4339e-002           | 5.3116e-002 | -1.2229e-003 |
| 104.17      |                | -5.1636e-002  | -5.1621e-002           | 5.0357e-002 | -1.2643e-003 |
| 125.        |                | -4.8919e-002  | -4.8903e-002           | 4.7568e-002 | -1.3352e-003 |
| 145.83      |                | -4.6201e-002  | -4.6186e-002           | 4.4831e-002 | -1.3549e-003 |
| 166.67      |                | -4.3483e-002  | -4.3468e-002           | 4.2131e-002 | -1.3372e-003 |
| 187.5       |                | -4.0765e-002  | -4.075e-002            | 3.9456e-002 | -1.2941e-003 |
| 208.33      |                | -3.8048e-002  | -3.8033e-002           | 3.6787e-002 | -1.2457e-003 |
| 229.17      |                | -3.533e-002   | -3.5315e-002           | 3.4102e-002 | -1.2133e-003 |
| 250.        |                | -3.2612e-002  | -3.2597e-002           | 3.1429e-002 | -1.1683e-003 |
| 270.83      |                | -2.9895e-002  | -2.988e-002            | 2.8823e-002 | -1.057e-003  |
| 291.67      |                | -2.7177e-002  | -2.7162e-002           | 2.6321e-002 | -8.4054e-004 |
| 312.5       |                | -2.4459e-002  | -2.4444e-002           | 2.3607e-002 | -8.3675e-004 |
| 333.33      |                | -2.1742e-002  | -2.1726e-002           | 2.0941e-002 | -7.8593e-004 |
| 354.17      |                | -1.9024e-002  | -1.9009e-002           | 1.8288e-002 | -7.2098e-004 |
| 375.        |                | -1.6306e-002  | -1.6291e-002           | 1.5644e-002 | -6.4691e-004 |
| 395.83      |                | -1.3588e-002  | -1.3573e-002           | 1.3001e-002 | -5.7274e-004 |
| 416.67      |                | -1.0871e-002  | -1.0856e-002           | 1.0357e-002 | -4.9907e-004 |
| 437.5       |                | -8.1531e-003  | -8.138e-003            | 7.698e-003  | -4.4002e-004 |
| 458.33      |                | -5.4354e-003  | -5.4203e-003           | 4.9965e-003 | -4.2378e-004 |

|        |              |              |              |              |
|--------|--------------|--------------|--------------|--------------|
| 479.17 | -2.7177e-003 | -2.7026e-003 | 2.3325e-003  | -3.7012e-004 |
| 500.   | 0.           | 1.5112e-005  | -1.6629e-004 | -1.5118e-004 |
| 520.83 | 2.7177e-003  | 2.7328e-003  | -2.6656e-003 | 6.7189e-005  |
| 541.67 | 5.4354e-003  | 5.4505e-003  | -5.3244e-003 | 1.2607e-004  |
| 562.5  | 8.1531e-003  | 8.1682e-003  | -8.0172e-003 | 1.5096e-004  |
| 583.33 | 1.0871e-002  | 1.0886e-002  | -1.0663e-002 | 2.225e-004   |
| 604.17 | 1.3588e-002  | 1.3604e-002  | -1.3292e-002 | 3.1198e-004  |
| 625.   | 1.6306e-002  | 1.6321e-002  | -1.5916e-002 | 4.0531e-004  |
| 645.83 | 1.9024e-002  | 1.9039e-002  | -1.8537e-002 | 5.0229e-004  |
| 666.67 | 2.1742e-002  | 2.1757e-002  | -2.1163e-002 | 5.9342e-004  |
| 687.5  | 2.4459e-002  | 2.4474e-002  | -2.3801e-002 | 6.7353e-004  |
| 708.33 | 2.7177e-002  | 2.7192e-002  | -2.6482e-002 | 7.1054e-004  |
| 729.17 | 2.9895e-002  | 2.991e-002   | -2.8947e-002 | 9.6233e-004  |
| 750.   | 3.2612e-002  | 3.2628e-002  | -3.1503e-002 | 1.1249e-003  |
| 770.83 | 3.533e-002   | 3.5345e-002  | -3.4139e-002 | 1.2057e-003  |
| 791.67 | 3.8048e-002  | 3.8063e-002  | -3.6777e-002 | 1.2855e-003  |
| 812.5  | 4.0765e-002  | 4.0781e-002  | -3.9396e-002 | 1.3846e-003  |
| 833.33 | 4.3483e-002  | 4.3498e-002  | -4.2016e-002 | 1.4821e-003  |
| 854.17 | 4.6201e-002  | 4.6216e-002  | -4.4659e-002 | 1.557e-003   |
| 875.   | 4.8919e-002  | 4.8934e-002  | -4.7337e-002 | 1.5969e-003  |
| 895.83 | 5.1636e-002  | 5.1651e-002  | -5.0064e-002 | 1.5872e-003  |
| 916.67 | 5.4354e-002  | 5.4369e-002  | -5.2763e-002 | 1.6058e-003  |
| 937.5  | 5.7072e-002  | 5.7087e-002  | -5.6202e-002 | 8.8467e-004  |
| 958.33 | 5.9789e-002  | 5.9805e-002  | -6.499e-002  | -5.1852e-003 |
| 979.17 | 6.2507e-002  | 6.2522e-002  | -0.11392     | -5.1403e-002 |
| 1000.  | 6.5225e-002  | 6.524e-002   | 1.0907       | 1.156        |

**FIGURE 17**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Shear Stress 2**

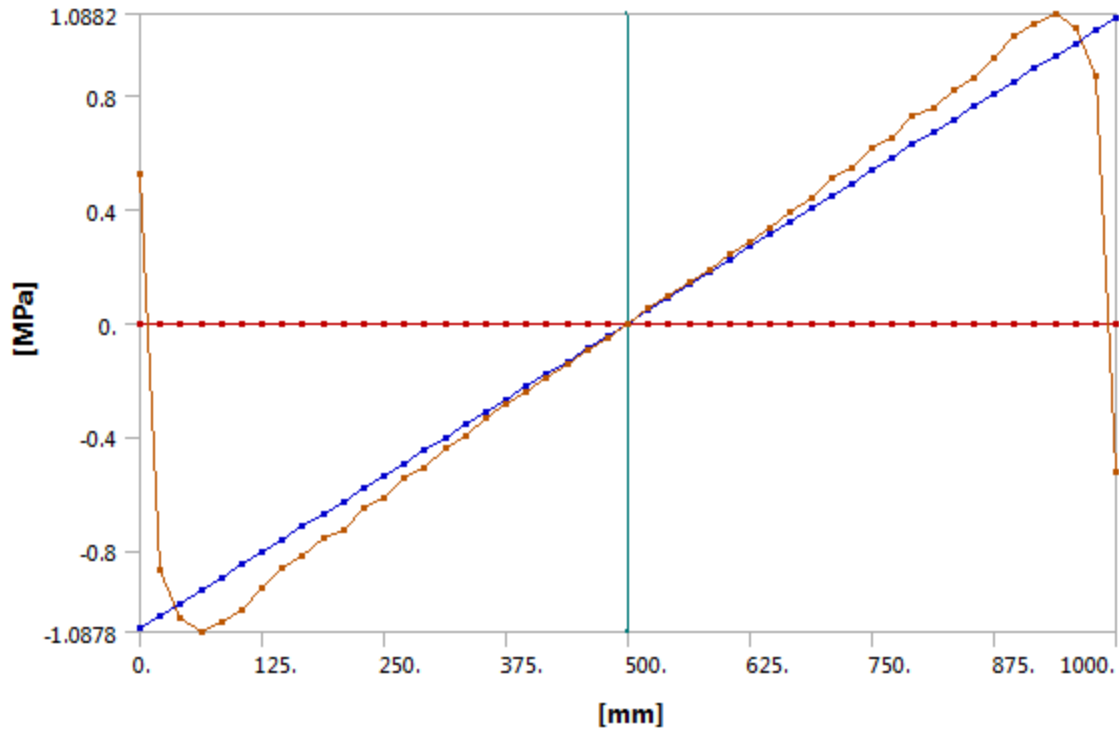


**TABLE 28**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Shear Stress 2**

| Length [mm] | Membrane [MPa] | Bending [MPa] | Membrane+Bending [MPa] | Peak [MPa]   | Total [MPa]  |
|-------------|----------------|---------------|------------------------|--------------|--------------|
| 0.          | 1.77e-005      | -1.0757       | -1.0756                | 1.5993       | 0.52362      |
| 20.833      |                | -1.0308       | -1.0308                | 0.16391      | -0.86691     |
| 41.667      |                | -0.98602      | -0.986                 | -5.0955e-002 | -1.037       |
| 62.5        |                | -0.9412       | -0.94119               | -0.14657     | -1.0878      |
| 83.333      |                | -0.89638      | -0.89637               | -0.15704     | -1.0534      |
| 104.17      |                | -0.85156      | -0.85155               | -0.15677     | -1.0083      |
| 125.        |                | -0.80675      | -0.80673               | -0.12767     | -0.9344      |
| 145.83      |                | -0.76193      | -0.76191               | -9.8818e-002 | -0.86073     |
| 166.67      |                | -0.71711      | -0.71709               | -0.10536     | -0.82245     |
| 187.5       |                | -0.67229      | -0.67227               | -8.4629e-002 | -0.7569      |
| 208.33      |                | -0.62747      | -0.62745               | -9.944e-002  | -0.72689     |
| 229.17      |                | -0.58265      | -0.58263               | -6.5985e-002 | -0.64862     |
| 250.        |                | -0.53783      | -0.53781               | -7.6514e-002 | -0.61433     |
| 270.83      |                | -0.49301      | -0.49299               | -5.0695e-002 | -0.54369     |
| 291.67      |                | -0.44819      | -0.44817               | -6.0432e-002 | -0.50861     |
| 312.5       |                | -0.40337      | -0.40335               | -3.8622e-002 | -0.44198     |
| 333.33      |                | -0.35855      | -0.35854               | -3.6013e-002 | -0.39455     |
| 354.17      |                | -0.31373      | -0.31372               | -1.8531e-002 | -0.33225     |
| 375.        |                | -0.26892      | -0.2689                | -1.6007e-002 | -0.2849      |
| 395.83      |                | -0.2241       | -0.22408               | -1.7966e-002 | -0.24204     |
| 416.67      |                | -0.17928      | -0.17926               | -1.1173e-002 | -0.19043     |
| 437.5       |                | -0.13446      | -0.13444               | -1.1537e-002 | -0.14598     |
| 458.33      |                | -8.9638e-002  | -8.9621e-002           | -3.9193e-003 | -9.354e-002  |
| 479.17      |                | -4.4819e-002  | -4.4801e-002           | -5.146e-003  | -4.9947e-002 |

|        |              |             |              |              |
|--------|--------------|-------------|--------------|--------------|
| 500.   | -2.3283e-016 | 1.77e-005   | -1.4646e-004 | -1.2876e-004 |
| 520.83 | 4.4819e-002  | 4.4837e-002 | 4.8499e-003  | 4.9687e-002  |
| 541.67 | 8.9638e-002  | 8.9656e-002 | 3.6192e-003  | 9.3275e-002  |
| 562.5  | 0.13446      | 0.13448     | 1.1229e-002  | 0.1457       |
| 583.33 | 0.17928      | 0.17929     | 1.0879e-002  | 0.19017      |
| 604.17 | 0.2241       | 0.22411     | 1.7676e-002  | 0.24179      |
| 625.   | 0.26892      | 0.26893     | 1.578e-002   | 0.28471      |
| 645.83 | 0.31373      | 0.31375     | 1.8306e-002  | 0.33206      |
| 666.67 | 0.35855      | 0.35857     | 3.5813e-002  | 0.39438      |
| 687.5  | 0.40337      | 0.40339     | 3.8442e-002  | 0.44183      |
| 708.33 | 0.44819      | 0.44821     | 6.0334e-002  | 0.50854      |
| 729.17 | 0.49301      | 0.49303     | 5.0589e-002  | 0.54362      |
| 750.   | 0.53783      | 0.53785     | 7.6391e-002  | 0.61424      |
| 770.83 | 0.58265      | 0.58267     | 6.5846e-002  | 0.64851      |
| 791.67 | 0.62747      | 0.62749     | 9.929e-002   | 0.72678      |
| 812.5  | 0.67229      | 0.67231     | 8.4596e-002  | 0.7569       |
| 833.33 | 0.71711      | 0.71712     | 0.10532      | 0.82244      |
| 854.17 | 0.76193      | 0.76194     | 9.8784e-002  | 0.86073      |
| 875.   | 0.80675      | 0.80676     | 0.12767      | 0.93443      |
| 895.83 | 0.85156      | 0.85158     | 0.15699      | 1.0086       |
| 916.67 | 0.89638      | 0.8964      | 0.15732      | 1.0537       |
| 937.5  | 0.9412       | 0.94122     | 0.147        | 1.0882       |
| 958.33 | 0.98602      | 0.98604     | 5.1404e-002  | 1.0374       |
| 979.17 | 1.0308       | 1.0309      | -0.16338     | 0.86747      |
| 1000.  | 1.0757       | 1.0757      | -1.5967      | -0.52099     |

**TABLE 29**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Results**

| Object Name               | Linearized<br>Maximum<br>Principal<br>Stress | Linearized<br>Maximum<br>Principal<br>Stress 2 | Linearized<br>Minimum<br>Principal<br>Stress | Linearized<br>Minimum<br>Principal<br>Stress 2 | Directional<br>Deformation<br>2 | Directional<br>Deformation<br>3 |
|---------------------------|----------------------------------------------|------------------------------------------------|----------------------------------------------|------------------------------------------------|---------------------------------|---------------------------------|
| State                     | Solved                                       |                                                |                                              |                                                |                                 |                                 |
| Scope                     |                                              |                                                |                                              |                                                |                                 |                                 |
| Scoping Method            | Path                                         |                                                |                                              |                                                |                                 |                                 |
| Path                      | Path                                         | Path 2                                         | Path                                         | Path 2                                         | Path                            | Path 2                          |
| Geometry                  | All Bodies                                   |                                                |                                              |                                                |                                 |                                 |
| Definition                |                                              |                                                |                                              |                                                |                                 |                                 |
| Type                      | Linearized Maximum<br>Principal Stress       |                                                | Linearized Minimum<br>Principal Stress       |                                                | Directional Deformation         |                                 |
| Subtype                   | All                                          |                                                |                                              |                                                |                                 |                                 |
| By                        | Time                                         |                                                |                                              |                                                |                                 |                                 |
| Display Time              | Last                                         |                                                |                                              |                                                |                                 |                                 |
| Coordinate System         | Global Coordinate System                     |                                                |                                              |                                                |                                 |                                 |
| 2D Behavior               | Planar                                       |                                                |                                              |                                                |                                 |                                 |
| Suppressed                | No                                           |                                                |                                              |                                                |                                 |                                 |
| Orientation               |                                              |                                                |                                              |                                                | X Axis                          |                                 |
| Calculate Time<br>History |                                              |                                                |                                              |                                                | Yes                             |                                 |
| Results                   |                                              |                                                |                                              |                                                |                                 |                                 |
| Membrane                  | 5.6983e-002<br>MPa                           | -3.5348e-<br>002 MPa                           | -0.14756<br>MPa                              | -0.13349<br>MPa                                |                                 |                                 |

|                            |                  |                  |                  |              |                 |                 |
|----------------------------|------------------|------------------|------------------|--------------|-----------------|-----------------|
| Bending (Inside)           | 6.5286e-002 MPa  | 1.0757 MPa       | -6.5287e-002 MPa | -1.0758 MPa  |                 |                 |
| Bending (Outside)          | 6.5287e-002 MPa  | 1.0758 MPa       | -6.5286e-002 MPa | -1.0757 MPa  |                 |                 |
| Membrane+Bending (Inside)  | 5.7201e-002 MPa  | 0.94741 MPa      | -0.17299 MPa     | -1.2041 MPa  |                 |                 |
| Membrane+Bending (Center)  | 5.6983e-002 MPa  | -3.5348e-002 MPa | -0.14756 MPa     | -0.13349 MPa |                 |                 |
| Membrane+Bending (Outside) | 5.7192e-002 MPa  | 0.94752 MPa      | -0.17299 MPa     | -1.204 MPa   |                 |                 |
| Peak (Inside)              | 5.1216 MPa       | 0.84179 MPa      | 0.80741 MPa      | -2.4721 MPa  |                 |                 |
| Peak (Center)              | -1.9887e-002 MPa | 2.8017e-002 MPa  | -1.6177 MPa      | -1.0356 MPa  |                 |                 |
| Peak (Outside)             | 5.1216 MPa       | 0.84184 MPa      | 0.80742 MPa      | -2.4671 MPa  |                 |                 |
| Total (Inside)             | 5.1393 MPa       | -0.25734 MPa     | 0.63653 MPa      | -1.627 MPa   |                 |                 |
| Total (Center)             | 5.0297e-004 MPa  | -5.8341e-002 MPa | -1.6234 MPa      | -1.159 MPa   |                 |                 |
| Total (Outside)            | 5.1393 MPa       | -0.25703 MPa     | 0.63653 MPa      | -1.6221 MPa  |                 |                 |
| Minimum                    |                  |                  |                  |              | -8.4008e-003 mm | -1.1074e-003 mm |
| Maximum                    |                  |                  |                  |              | 8.4047e-003 mm  | 1.1113e-003 mm  |
| Minimum Occurs On          |                  |                  |                  |              | Part4-1         |                 |
| Maximum Occurs On          |                  |                  |                  |              | Part4-1         |                 |
| Information                |                  |                  |                  |              |                 |                 |
| Time                       | 1. s             |                  |                  |              |                 |                 |
| Load Step                  | 1                |                  |                  |              |                 |                 |
| Substep                    | 1                |                  |                  |              |                 |                 |
| Iteration Number           | 1                |                  |                  |              |                 |                 |
| Graph Controls             |                  |                  |                  |              |                 |                 |
| X-Axis                     |                  |                  |                  |              | S               |                 |
| Minimum Value Over Time    |                  |                  |                  |              |                 |                 |
| Minimum                    |                  |                  |                  |              | -8.4008e-003 mm | -1.1074e-003 mm |
| Maximum                    |                  |                  |                  |              | -8.4008e-003 mm | -1.1074e-003 mm |
| Maximum Value Over Time    |                  |                  |                  |              |                 |                 |
| Minimum                    |                  |                  |                  |              | 8.4047e-003 mm  | 1.1113e-003 mm  |
| Maximum                    |                  |                  |                  |              | 8.4047e-003 mm  | 1.1113e-003 mm  |

**FIGURE 18**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Maximum Principal Stress**

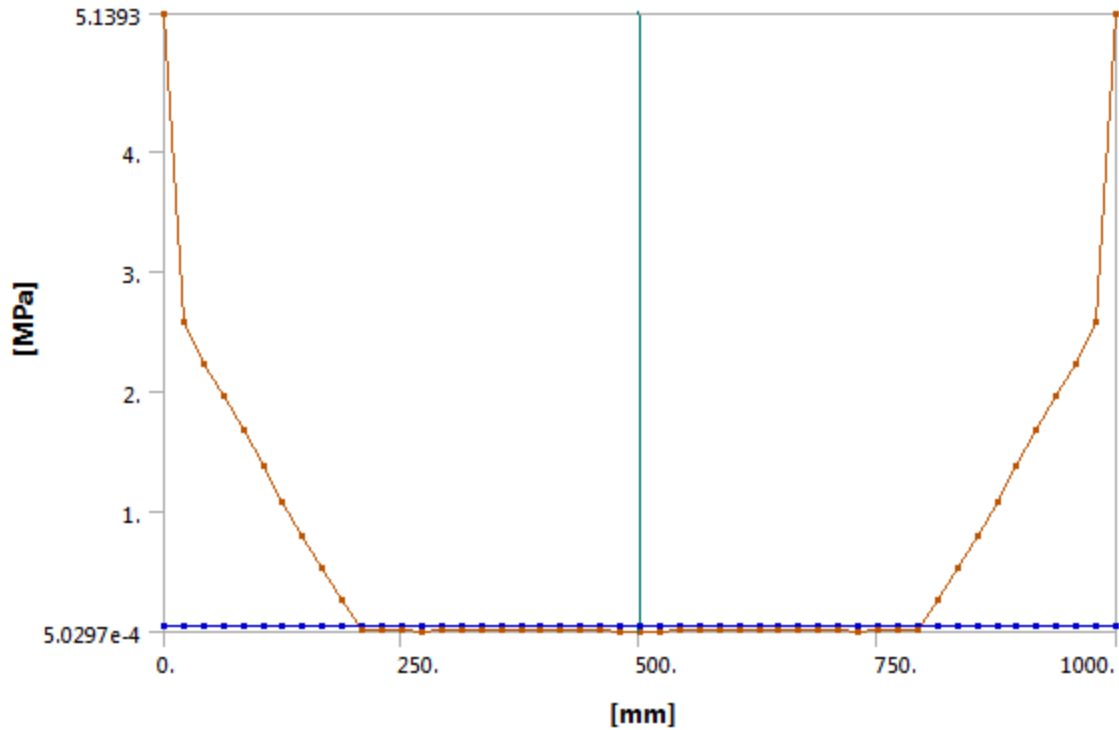


**TABLE 30**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Maximum Principal Stress**

| Length [mm] | Membrane [MPa] | Bending [MPa] | Membrane+Bending [MPa] | Peak [MPa]   | Total [MPa] |
|-------------|----------------|---------------|------------------------|--------------|-------------|
| 0.          | 5.6983e-002    | 6.5286e-002   | 5.7201e-002            | 5.1216       | 5.1393      |
| 20.833      |                | 6.2565e-002   | 5.7176e-002            | 2.5829       | 2.5733      |
| 41.667      |                | 5.9845e-002   | 5.7154e-002            | 2.2318       | 2.2242      |
| 62.5        |                | 5.7125e-002   | 5.7134e-002            | 1.9637       | 1.9564      |
| 83.333      |                | 5.4405e-002   | 5.7116e-002            | 1.6811       | 1.6738      |
| 104.17      |                | 5.1684e-002   | 5.71e-002              | 1.3888       | 1.3813      |
| 125.        |                | 4.8964e-002   | 5.7086e-002            | 1.0955       | 1.0878      |
| 145.83      |                | 4.6244e-002   | 5.7073e-002            | 0.80989      | 0.80178     |
| 166.67      |                | 4.3524e-002   | 5.7061e-002            | 0.53612      | 0.52725     |
| 187.5       |                | 4.0803e-002   | 5.705e-002             | 0.27741      | 0.26648     |
| 208.33      |                | 3.8083e-002   | 5.7041e-002            | 4.8473e-002  | 2.2995e-002 |
| 229.17      |                | 3.5363e-002   | 5.7032e-002            | -1.3546e-002 | 1.4061e-002 |
| 250.        |                | 3.2643e-002   | 5.7024e-002            | -1.733e-002  | 1.1209e-002 |
| 270.83      |                | 2.9923e-002   | 5.7018e-002            | -1.8301e-002 | 8.7917e-003 |
| 291.67      |                | 2.7202e-002   | 5.7011e-002            | -1.8967e-002 | 1.179e-002  |
| 312.5       |                | 2.4482e-002   | 5.7006e-002            | -1.9271e-002 | 1.4887e-002 |
| 333.33      |                | 2.1762e-002   | 5.7001e-002            | -1.9475e-002 | 1.6539e-002 |
| 354.17      |                | 1.9042e-002   | 5.6997e-002            | -1.9609e-002 | 1.7395e-002 |
| 375.        |                | 1.6321e-002   | 5.6993e-002            | -1.9707e-002 | 1.7745e-002 |
| 395.83      |                | 1.3601e-002   | 5.699e-002             | -1.978e-002  | 1.767e-002  |
| 416.67      |                | 1.0881e-002   | 5.6988e-002            | -1.9836e-002 | 1.7093e-002 |
| 437.5       |                | 8.1607e-003   | 5.6986e-002            | -1.9873e-002 | 1.5673e-002 |
| 458.33      |                | 5.4405e-003   | 5.6985e-002            | -1.9858e-002 | 1.2407e-002 |
| 479.17      |                | 2.7202e-003   | 5.6984e-002            | -1.9959e-002 | 6.5169e-003 |

|        |             |             |              |             |
|--------|-------------|-------------|--------------|-------------|
| 500.   | 0.          | 5.6983e-002 | -1.9887e-002 | 5.0297e-004 |
| 520.83 | 2.7203e-003 |             | -1.9959e-002 | 6.5137e-003 |
| 541.67 | 5.4406e-003 | 5.6984e-002 | -1.9858e-002 | 1.2405e-002 |
| 562.5  | 8.1608e-003 | 5.6985e-002 | -1.9873e-002 | 1.5671e-002 |
| 583.33 | 1.0881e-002 | 5.6986e-002 | -1.9836e-002 | 1.7093e-002 |
| 604.17 | 1.3601e-002 | 5.6989e-002 | -1.978e-002  | 1.7669e-002 |
| 625.   | 1.6322e-002 | 5.6991e-002 | -1.9707e-002 | 1.7744e-002 |
| 645.83 | 1.9042e-002 | 5.6994e-002 | -1.9609e-002 | 1.7393e-002 |
| 666.67 | 2.1762e-002 | 5.6998e-002 | -1.9474e-002 | 1.6533e-002 |
| 687.5  | 2.4482e-002 | 5.7003e-002 | -1.9269e-002 | 1.4874e-002 |
| 708.33 | 2.7203e-002 | 5.7008e-002 | -1.8963e-002 | 1.1764e-002 |
| 729.17 | 2.9923e-002 | 5.7013e-002 | -1.8305e-002 | 8.7507e-003 |
| 750.   | 3.2643e-002 | 5.702e-002  | -1.7321e-002 | 1.1184e-002 |
| 770.83 | 3.5364e-002 | 5.7027e-002 | -1.3546e-002 | 1.4043e-002 |
| 791.67 | 3.8084e-002 | 5.7035e-002 | 4.8466e-002  | 2.2988e-002 |
| 812.5  | 4.0804e-002 | 5.7045e-002 | 0.2774       | 0.26648     |
| 833.33 | 4.3524e-002 | 5.7055e-002 | 0.53612      | 0.52724     |
| 854.17 | 4.6245e-002 | 5.7066e-002 | 0.80988      | 0.80178     |
| 875.   | 4.8965e-002 | 5.7079e-002 | 1.0955       | 1.0878      |
| 895.83 | 5.1685e-002 | 5.7093e-002 | 1.3888       | 1.3813      |
| 916.67 | 5.4406e-002 | 5.7109e-002 | 1.6811       | 1.6738      |
| 937.5  | 5.7126e-002 | 5.7126e-002 | 1.9637       | 1.9565      |
| 958.33 | 5.9846e-002 | 5.7146e-002 | 2.2318       | 2.2243      |
| 979.17 | 6.2566e-002 | 5.7167e-002 | 2.5829       | 2.5733      |
| 1000.  | 6.5287e-002 | 5.7192e-002 | 5.1216       | 5.1393      |

**FIGURE 19**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Maximum Principal Stress 2**

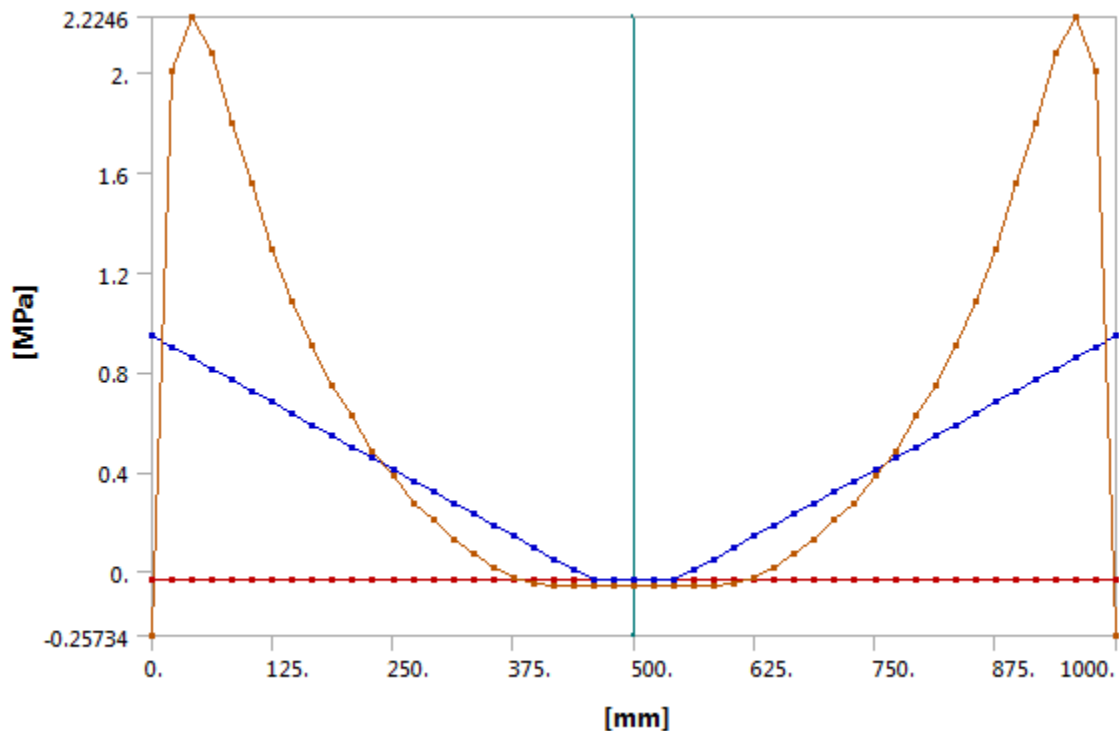


**TABLE 31**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Maximum Principal Stress 2**

| Length [mm] | Membrane [MPa] | Bending [MPa] | Membrane+Bending [MPa] | Peak [MPa]  | Total [MPa]  |
|-------------|----------------|---------------|------------------------|-------------|--------------|
| 0.          | -3.5348e-002   | 1.0757        | 0.94741                | 0.84179     | -0.25734     |
| 20.833      |                | 1.0309        | 0.90259                | 1.7755      | 2.0091       |
| 41.667      |                | 0.98606       | 0.85777                | 1.8867      | 2.2246       |
| 62.5        |                | 0.94124       | 0.81295                | 1.6807      | 2.0787       |
| 83.333      |                | 0.89642       | 0.76813                | 1.3754      | 1.8028       |
| 104.17      |                | 0.8516        | 0.72331                | 1.106       | 1.5601       |
| 125.        |                | 0.80677       | 0.67849                | 0.82484     | 1.2941       |
| 145.83      |                | 0.76195       | 0.63368                | 0.60626     | 1.0837       |
| 166.67      |                | 0.71713       | 0.58886                | 0.39538     | 0.90637      |
| 187.5       |                | 0.67231       | 0.54404                | 0.23113     | 0.74594      |
| 208.33      |                | 0.62749       | 0.49922                | 0.12767     | 0.62568      |
| 229.17      |                | 0.58267       | 0.45441                | 5.583e-002  | 0.48017      |
| 250.        |                | 0.53785       | 0.40959                | 5.2196e-002 | 0.38854      |
| 270.83      |                | 0.49303       | 0.36477                | 3.6487e-002 | 0.27558      |
| 291.67      |                | 0.44821       | 0.31996                | 3.7965e-002 | 0.20893      |
| 312.5       |                | 0.40339       | 0.27515                | 3.1659e-002 | 0.12663      |
| 333.33      |                | 0.35857       | 0.23034                | 2.8502e-002 | 6.9956e-002  |
| 354.17      |                | 0.31375       | 0.18553                | 2.6653e-002 | 1.5094e-002  |
| 375.        |                | 0.26892       | 0.14074                | 2.5828e-002 | -2.1095e-002 |
| 395.83      |                | 0.2241        | 9.5952e-002            | 2.7052e-002 | -4.61e-002   |
| 416.67      |                | 0.17928       | 5.12e-002              | 2.5963e-002 | -5.6426e-002 |
| 437.5       |                | 0.13446       | 6.5738e-003            | 2.6524e-002 | -5.8765e-002 |
| 458.33      |                | 8.9642e-002   | -3.3034e-002           | 2.5486e-002 | -5.9563e-002 |
| 479.17      |                | 4.4821e-002   | -3.526e-002            | 2.8931e-002 | -5.772e-002  |
| 500.        |                | 2.3286e-016   | -3.5348e-002           | 2.8017e-002 | -5.8341e-002 |
| 520.83      |                | 4.4824e-002   | -3.5254e-002           | 2.8929e-002 | -5.7714e-002 |
| 541.67      |                | 8.9648e-002   | -3.3021e-002           | 2.5468e-002 | -5.9563e-002 |
| 562.5       |                | 0.13447       | 6.613e-003             | 2.6492e-002 | -5.8769e-002 |
| 583.33      |                | 0.1793        | 5.1244e-002            | 2.5925e-002 | -5.6427e-002 |
| 604.17      |                | 0.22412       | 9.5999e-002            | 2.7004e-002 | -4.6133e-002 |
| 625.        |                | 0.26894       | 0.14079                | 2.5787e-002 | -2.1101e-002 |
| 645.83      |                | 0.31377       | 0.18559                | 2.6599e-002 | 1.5074e-002  |
| 666.67      |                | 0.35859       | 0.2304                 | 2.8434e-002 | 6.9932e-002  |
| 687.5       |                | 0.40342       | 0.27521                | 3.158e-002  | 0.12661      |
| 708.33      |                | 0.44824       | 0.32002                | 3.7889e-002 | 0.20895      |
| 729.17      |                | 0.49306       | 0.36484                | 3.6397e-002 | 0.27557      |
| 750.        |                | 0.53789       | 0.40966                | 5.2066e-002 | 0.38847      |
| 770.83      |                | 0.58271       | 0.45448                | 5.5653e-002 | 0.48005      |
| 791.67      |                | 0.62754       | 0.4993                 | 0.12742     | 0.62551      |
| 812.5       |                | 0.67236       | 0.54412                | 0.23098     | 0.74584      |
| 833.33      |                | 0.71718       | 0.58894                | 0.3952      | 0.90621      |
| 854.17      |                | 0.76201       | 0.63376                | 0.60607     | 1.0835       |
| 875.        |                | 0.80683       | 0.67858                | 0.82463     | 1.2939       |
| 895.83      |                | 0.85165       | 0.7234                 | 1.1058      | 1.56         |
| 916.67      |                | 0.89648       | 0.76823                | 1.3752      | 1.8027       |
| 937.5       |                | 0.9413        | 0.81305                | 1.6805      | 2.0786       |
| 958.33      |                | 0.98613       | 0.85787                | 1.8866      | 2.2245       |
| 979.17      |                | 1.031         | 0.90269                | 1.7757      | 2.0094       |
| 1000.       |                | 1.0758        | 0.94752                | 0.84184     | -0.25703     |

**FIGURE 20**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Minimum Principal Stress**

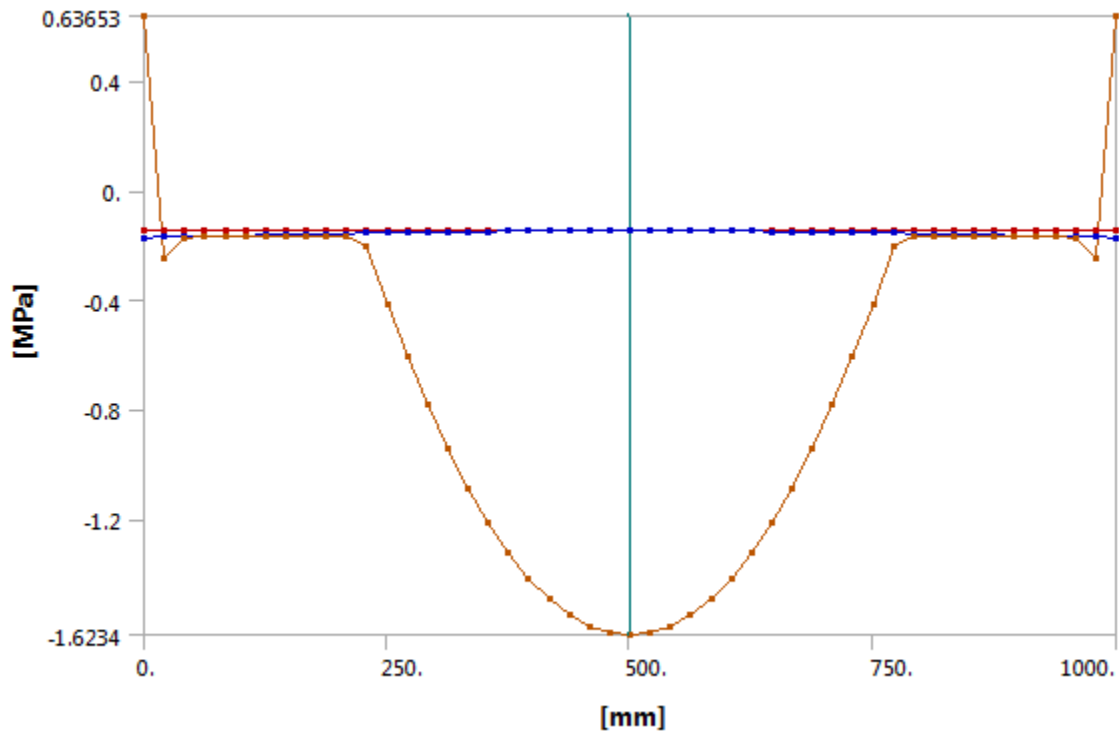


**TABLE 32**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Minimum Principal Stress**

| Length [mm] | Membrane [MPa] | Bending [MPa] | Membrane+Bending [MPa] | Peak [MPa]   | Total [MPa] |
|-------------|----------------|---------------|------------------------|--------------|-------------|
| 0.          | -0.14756       | -6.5287e-002  | -0.17299               | 0.80741      | 0.63653     |
| 20.833      |                | -6.2566e-002  | -0.17117               | -0.10266     | -0.2463     |
| 41.667      |                | -5.9846e-002  | -0.1694                | -3.0099e-002 | -0.17578    |
| 62.5        |                | -5.7126e-002  | -0.16767               | -2.1471e-002 | -0.16741    |
| 83.333      |                | -5.4406e-002  | -0.16599               | -2.1116e-002 | -0.16696    |
| 104.17      |                | -5.1685e-002  | -0.16436               | -3.0978e-002 | -0.16703    |
| 125.        |                | -4.8965e-002  | -0.16279               | -3.6252e-002 | -0.16713    |
| 145.83      |                | -4.6245e-002  | -0.16128               | -3.8372e-002 | -0.16723    |
| 166.67      |                | -4.3524e-002  | -0.15983               | -3.9489e-002 | -0.16729    |
| 187.5       |                | -4.0804e-002  | -0.15844               | -4.0277e-002 | -0.16734    |
| 208.33      |                | -3.8084e-002  | -0.15712               | -4.1489e-002 | -0.16738    |
| 229.17      |                | -3.5364e-002  | -0.15587               | -0.20495     | -0.20439    |
| 250.        |                | -3.2643e-002  | -0.1547                | -0.4116      | -0.41477    |
| 270.83      |                | -2.9923e-002  | -0.1536                | -0.60299     | -0.60726    |
| 291.67      |                | -2.7203e-002  | -0.15259               | -0.77969     | -0.78447    |
| 312.5       |                | -2.4482e-002  | -0.15166               | -0.94066     | -0.94575    |
| 333.33      |                | -2.1762e-002  | -0.15082               | -1.0828      | -1.0881     |
| 354.17      |                | -1.9042e-002  | -0.15007               | -1.2087      | -1.2141     |
| 375.        |                | -1.6322e-002  | -0.14941               | -1.3185      | -1.324      |
| 395.83      |                | -1.3601e-002  | -0.14885               | -1.4102      | -1.4158     |
| 416.67      |                | -1.0881e-002  | -0.14839               | -1.4859      | -1.4915     |
| 437.5       |                | -8.1608e-003  | -0.14803               | -1.5455      | -1.5512     |
| 458.33      |                | -5.4406e-003  | -0.14777               | -1.5865      | -1.5921     |

|        |              |          |              |          |
|--------|--------------|----------|--------------|----------|
| 479.17 | -2.7203e-003 | -0.14761 | -1.6102      | -1.6159  |
| 500.   | 0.           | -0.14756 | -1.6177      | -1.6234  |
| 520.83 | -2.7202e-003 | -0.14761 | -1.6102      | -1.6159  |
| 541.67 | -5.4405e-003 | -0.14777 | -1.5865      | -1.5921  |
| 562.5  | -8.1607e-003 | -0.14803 | -1.5455      | -1.5512  |
| 583.33 | -1.0881e-002 | -0.14839 | -1.4859      | -1.4915  |
| 604.17 | -1.3601e-002 | -0.14885 | -1.4102      | -1.4158  |
| 625.   | -1.6321e-002 | -0.14941 | -1.3185      | -1.324   |
| 645.83 | -1.9042e-002 | -0.15007 | -1.2087      | -1.2141  |
| 666.67 | -2.1762e-002 | -0.15082 | -1.0828      | -1.0881  |
| 687.5  | -2.4482e-002 | -0.15166 | -0.94067     | -0.94575 |
| 708.33 | -2.7202e-002 | -0.15259 | -0.77969     | -0.78447 |
| 729.17 | -2.9923e-002 | -0.1536  | -0.603       | -0.60726 |
| 750.   | -3.2643e-002 | -0.1547  | -0.4116      | -0.41477 |
| 770.83 | -3.5363e-002 | -0.15587 | -0.20496     | -0.2044  |
| 791.67 | -3.8083e-002 | -0.15712 | -4.1492e-002 | -0.16738 |
| 812.5  | -4.0803e-002 | -0.15844 | -4.0275e-002 | -0.16734 |
| 833.33 | -4.3524e-002 | -0.15983 | -3.9486e-002 | -0.16729 |
| 854.17 | -4.6244e-002 | -0.16128 | -3.8369e-002 | -0.16723 |
| 875.   | -4.8964e-002 | -0.16279 | -3.6251e-002 | -0.16713 |
| 895.83 | -5.1684e-002 | -0.16437 | -3.0979e-002 | -0.16703 |
| 916.67 | -5.4405e-002 | -0.16599 | -2.1114e-002 | -0.16696 |
| 937.5  | -5.7125e-002 | -0.16767 | -2.1484e-002 | -0.16743 |
| 958.33 | -5.9845e-002 | -0.1694  | -3.0097e-002 | -0.17578 |
| 979.17 | -6.2565e-002 | -0.17117 | -0.10267     | -0.24631 |
| 1000.  | -6.5286e-002 | -0.17299 | 0.80742      | 0.63653  |

**FIGURE 21**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Minimum Principal Stress 2**

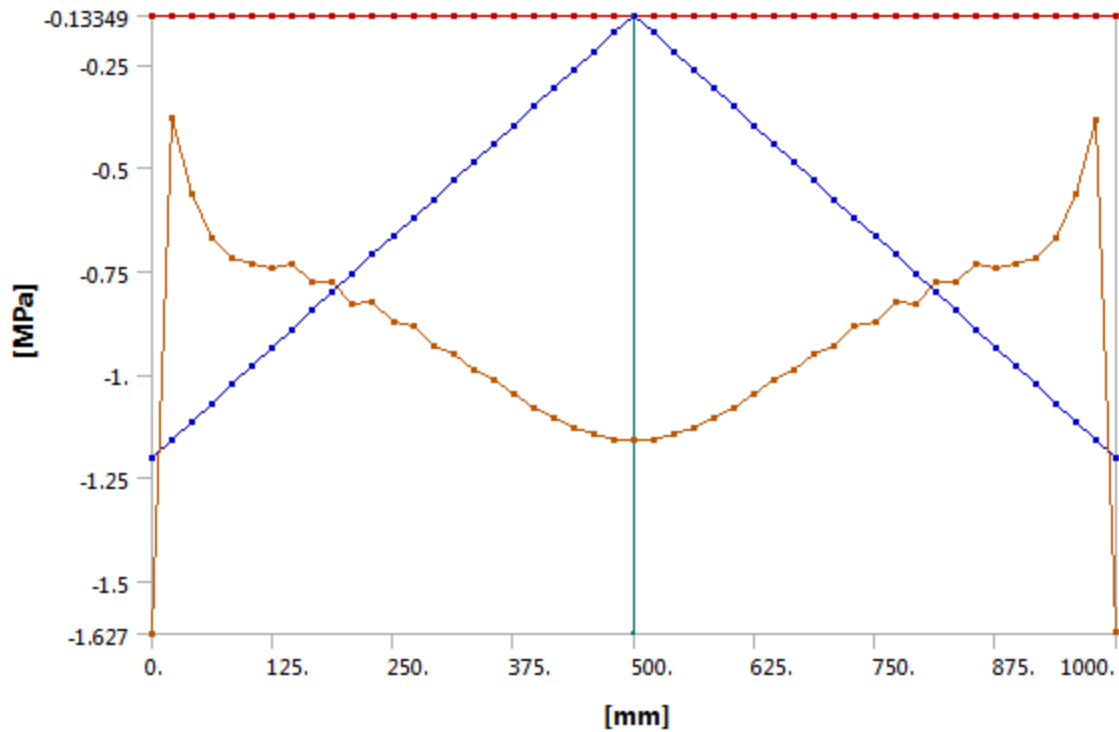


**TABLE 33**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Linearized Minimum Principal Stress 2**

| Length [mm] | Membrane [MPa] | Bending [MPa] | Membrane+Bending [MPa] | Peak [MPa]   | Total [MPa] |
|-------------|----------------|---------------|------------------------|--------------|-------------|
| 0.          | -0.13349       | -1.0758       | -1.2041                | -2.4721      | -1.627      |
| 20.833      |                | -1.031        | -1.1593                | 0.10593      | -0.3823     |
| 41.667      |                | -0.98613      | -1.1144                | 3.148e-002   | -0.56313    |
| 62.5        |                | -0.9413       | -1.0696                | -1.5982e-002 | -0.67085    |
| 83.333      |                | -0.89648      | -1.0248                | -3.3833e-002 | -0.71796    |
| 104.17      |                | -0.85165      | -0.97997               | -2.3984e-002 | -0.73502    |
| 125.        |                | -0.80683      | -0.93515               | -1.7973e-002 | -0.74405    |
| 145.83      |                | -0.76201      | -0.89032               | 3.4192e-004  | -0.73383    |
| 166.67      |                | -0.71718      | -0.8455                | -8.3105e-003 | -0.77627    |
| 187.5       |                | -0.67236      | -0.80068               | -6.4834e-003 | -0.77817    |
| 208.33      |                | -0.62754      | -0.75586               | -7.3084e-002 | -0.82815    |
| 229.17      |                | -0.58271      | -0.71104               | -0.14331     | -0.82448    |
| 250.        |                | -0.53789      | -0.66622               | -0.27903     | -0.87222    |
| 270.83      |                | -0.49306      | -0.6214                | -0.3878      | -0.88365    |
| 291.67      |                | -0.44824      | -0.57658               | -0.5055      | -0.9332     |
| 312.5       |                | -0.40342      | -0.53176               | -0.59902     | -0.9507     |
| 333.33      |                | -0.35859      | -0.48694               | -0.68949     | -0.98766    |
| 354.17      |                | -0.31377      | -0.44213               | -0.76598     | -1.011      |
| 375.        |                | -0.26894      | -0.39731               | -0.83615     | -1.0457     |
| 395.83      |                | -0.22412      | -0.3525                | -0.89756     | -1.0807     |
| 416.67      |                | -0.1793       | -0.3077                | -0.94572     | -1.1053     |
| 437.5       |                | -0.13447      | -0.26291               | -0.98531     | -1.1294     |
| 458.33      |                | -8.9648e-002  | -0.21815               | -1.0095      | -1.1414     |
| 479.17      |                | -4.4824e-002  | -0.17349               | -1.0338      | -1.1595     |

|        |              |          |              |          |
|--------|--------------|----------|--------------|----------|
| 500.   | -2.3286e-016 | -0.13349 | -1.0356      | -1.159   |
| 520.83 | -4.4821e-002 | -0.17352 | -1.0338      | -1.1595  |
| 541.67 | -8.9642e-002 | -0.21818 | -1.0095      | -1.1414  |
| 562.5  | -0.13446     | -0.26293 | -0.98531     | -1.1294  |
| 583.33 | -0.17928     | -0.30772 | -0.94573     | -1.1053  |
| 604.17 | -0.2241      | -0.35252 | -0.89757     | -1.0807  |
| 625.   | -0.26892     | -0.39733 | -0.8362      | -1.0458  |
| 645.83 | -0.31375     | -0.44214 | -0.76602     | -1.0111  |
| 666.67 | -0.35857     | -0.48695 | -0.68954     | -0.9877  |
| 687.5  | -0.40339     | -0.53176 | -0.59907     | -0.95073 |
| 708.33 | -0.44821     | -0.57658 | -0.50558     | -0.9333  |
| 729.17 | -0.49303     | -0.62139 | -0.38786     | -0.88372 |
| 750.   | -0.53785     | -0.66621 | -0.27907     | -0.87224 |
| 770.83 | -0.58267     | -0.71103 | -0.14331     | -0.82446 |
| 791.67 | -0.62749     | -0.75585 | -7.3029e-002 | -0.82808 |
| 812.5  | -0.67231     | -0.80066 | -6.5555e-003 | -0.77819 |
| 833.33 | -0.71713     | -0.84548 | -8.377e-003  | -0.77625 |
| 854.17 | -0.76195     | -0.8903  | 2.5356e-004  | -0.73378 |
| 875.   | -0.80677     | -0.93512 | -1.8074e-002 | -0.74399 |
| 895.83 | -0.8516      | -0.97994 | -2.413e-002  | -0.7351  |
| 916.67 | -0.89642     | -1.0248  | -3.3986e-002 | -0.71803 |
| 937.5  | -0.94124     | -1.0696  | -1.6174e-002 | -0.671   |
| 958.33 | -0.98606     | -1.1144  | 3.128e-002   | -0.56324 |
| 979.17 | -1.0309      | -1.1592  | 0.10579      | -0.38242 |
| 1000.  | -1.0757      | -1.204   | -2.4671      | -1.6221  |

**FIGURE 22**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Directional Deformation 2**

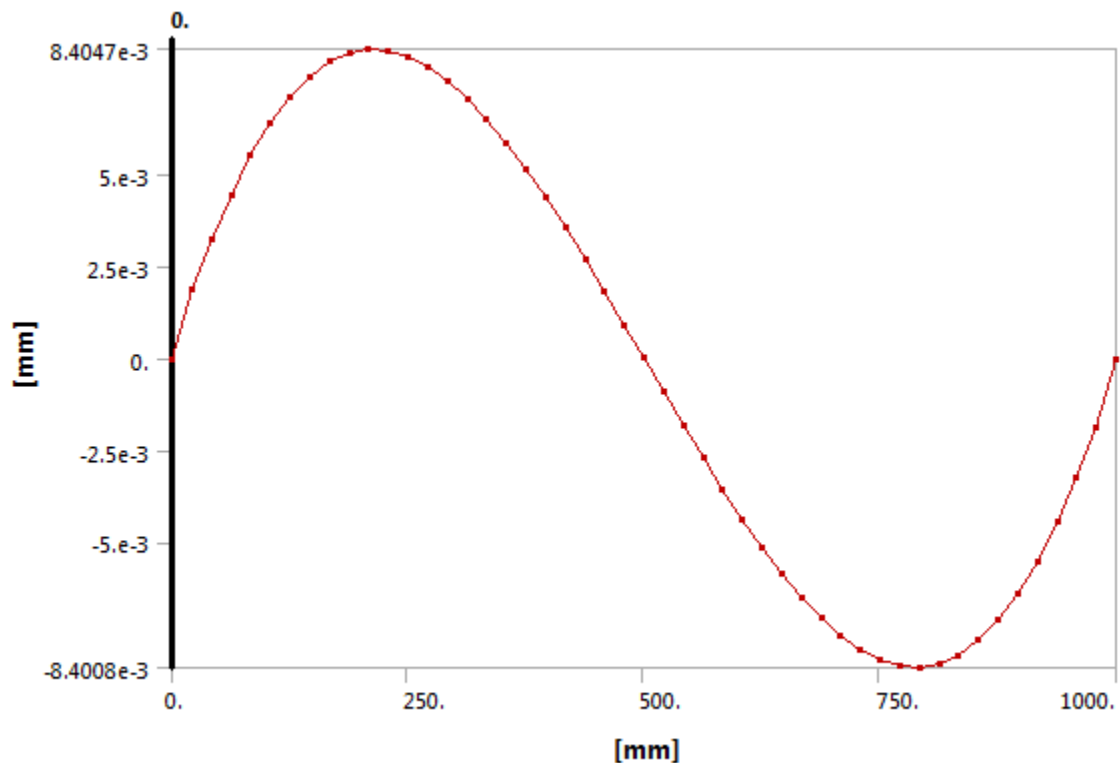


**TABLE 34**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Directional Deformation 2**

| Length [mm] | Value [mm]   |
|-------------|--------------|
| 0.          | -2.1027e-017 |
| 20.833      | 1.8877e-003  |
| 41.667      | 3.2472e-003  |
| 62.5        | 4.4457e-003  |
| 83.333      | 5.496e-003   |
| 104.17      | 6.3861e-003  |
| 125.        | 7.1095e-003  |
| 145.83      | 7.6668e-003  |
| 166.67      | 8.0631e-003  |
| 187.5       | 8.3061e-003  |
| 208.33      | 8.4047e-003  |
| 229.17      | 8.3683e-003  |
| 250.        | 8.207e-003   |
| 270.83      | 7.9303e-003  |
| 291.67      | 7.5476e-003  |
| 312.5       | 7.0679e-003  |
| 333.33      | 6.5011e-003  |
| 354.17      | 5.8569e-003  |
| 375.        | 5.1453e-003  |
| 395.83      | 4.3759e-003  |
| 416.67      | 3.5584e-003  |
| 437.5       | 2.7024e-003  |
| 458.33      | 1.818e-003   |
| 479.17      | 9.1534e-004  |
| 500.        | 4.1042e-006  |
| 520.83      | -9.0715e-004 |
| 541.67      | -1.8099e-003 |
| 562.5       | -2.6944e-003 |
| 583.33      | -3.5506e-003 |
| 604.17      | -4.3684e-003 |
| 625.        | -5.138e-003  |
| 645.83      | -5.8499e-003 |
| 666.67      | -6.4944e-003 |
| 687.5       | -7.0617e-003 |
| 708.33      | -7.5417e-003 |
| 729.17      | -7.925e-003  |
| 750.        | -8.202e-003  |
| 770.83      | -8.3639e-003 |
| 791.67      | -8.4008e-003 |
| 812.5       | -8.3027e-003 |
| 833.33      | -8.0602e-003 |
| 854.17      | -7.6643e-003 |
| 875.        | -7.1075e-003 |
| 895.83      | -6.3845e-003 |
| 916.67      | -5.4948e-003 |
| 937.5       | -4.4449e-003 |
| 958.33      | -3.2467e-003 |
| 979.17      | -1.8875e-003 |
| 1000.       | 8.097e-017   |

**FIGURE 23**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Directional Deformation 2 > Figure**

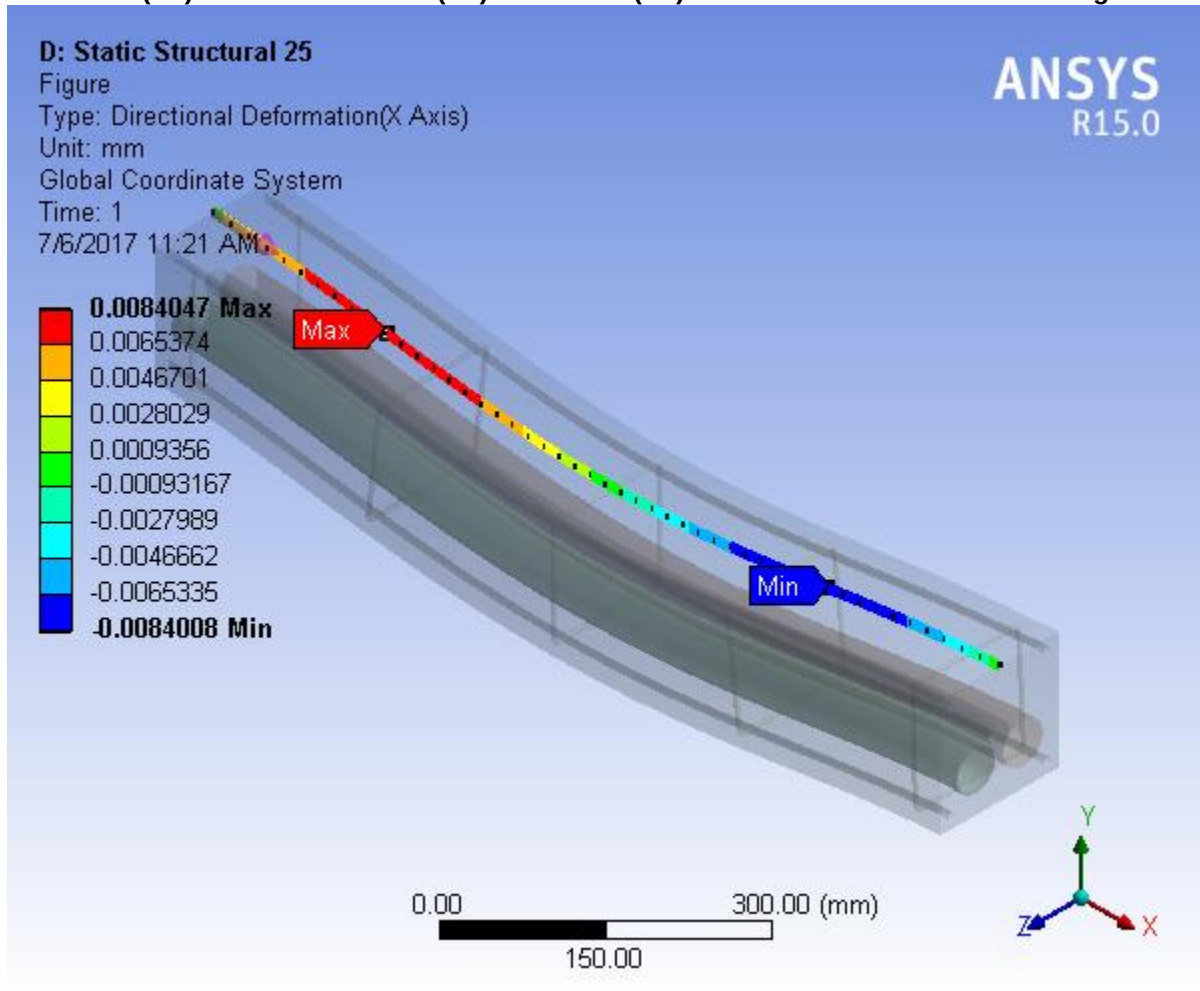


**FIGURE 24**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Directional Deformation 3**

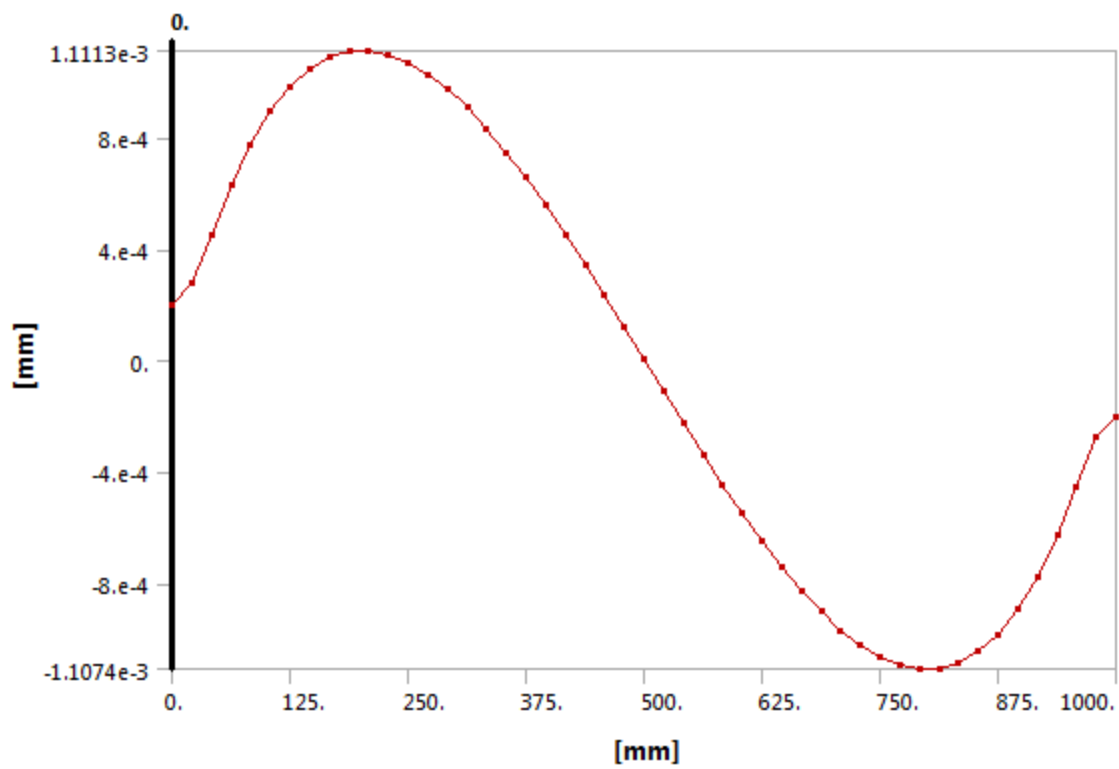


**TABLE 35**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Directional Deformation 3**

| Length [mm] | Value [mm]  |
|-------------|-------------|
| 0.          | 2.0259e-004 |
| 20.833      | 2.7726e-004 |
| 41.667      | 4.5417e-004 |
| 62.5        | 6.283e-004  |
| 83.333      | 7.7637e-004 |
| 104.17      | 8.9471e-004 |
| 125.        | 9.8403e-004 |
| 145.83      | 1.0475e-003 |
| 166.67      | 1.088e-003  |
| 187.5       | 1.1088e-003 |
| 208.33      | 1.1113e-003 |
| 229.17      | 1.0978e-003 |
| 250.        | 1.0695e-003 |
| 270.83      | 1.0277e-003 |
| 291.67      | 9.7343e-004 |
| 312.5       | 9.0799e-004 |
| 333.33      | 8.3259e-004 |
| 354.17      | 7.4843e-004 |
| 375.        | 6.5672e-004 |
| 395.83      | 5.5791e-004 |
| 416.67      | 4.5395e-004 |
| 437.5       | 3.4526e-004 |
| 458.33      | 2.3349e-004 |
| 479.17      | 1.1939e-004 |

|        |              |
|--------|--------------|
| 500.   | 4.1569e-006  |
| 520.83 | -1.1111e-004 |
| 541.67 | -2.2528e-004 |
| 562.5  | -3.3719e-004 |
| 583.33 | -4.4606e-004 |
| 604.17 | -5.5025e-004 |
| 625.   | -6.4933e-004 |
| 645.83 | -7.4137e-004 |
| 666.67 | -8.2589e-004 |
| 687.5  | -9.0168e-004 |
| 708.33 | -9.6756e-004 |
| 729.17 | -1.0223e-003 |
| 750.   | -1.0646e-003 |
| 770.83 | -1.0934e-003 |
| 791.67 | -1.1074e-003 |
| 812.5  | -1.1054e-003 |
| 833.33 | -1.0852e-003 |
| 854.17 | -1.0451e-003 |
| 875.   | -9.8217e-004 |
| 895.83 | -8.9331e-004 |
| 916.67 | -7.7541e-004 |
| 937.5  | -6.2773e-004 |
| 958.33 | -4.5391e-004 |
| 979.17 | -2.7722e-004 |
| 1000.  | -2.026e-004  |

**FIGURE 25**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Directional Deformation 3 > Figure**

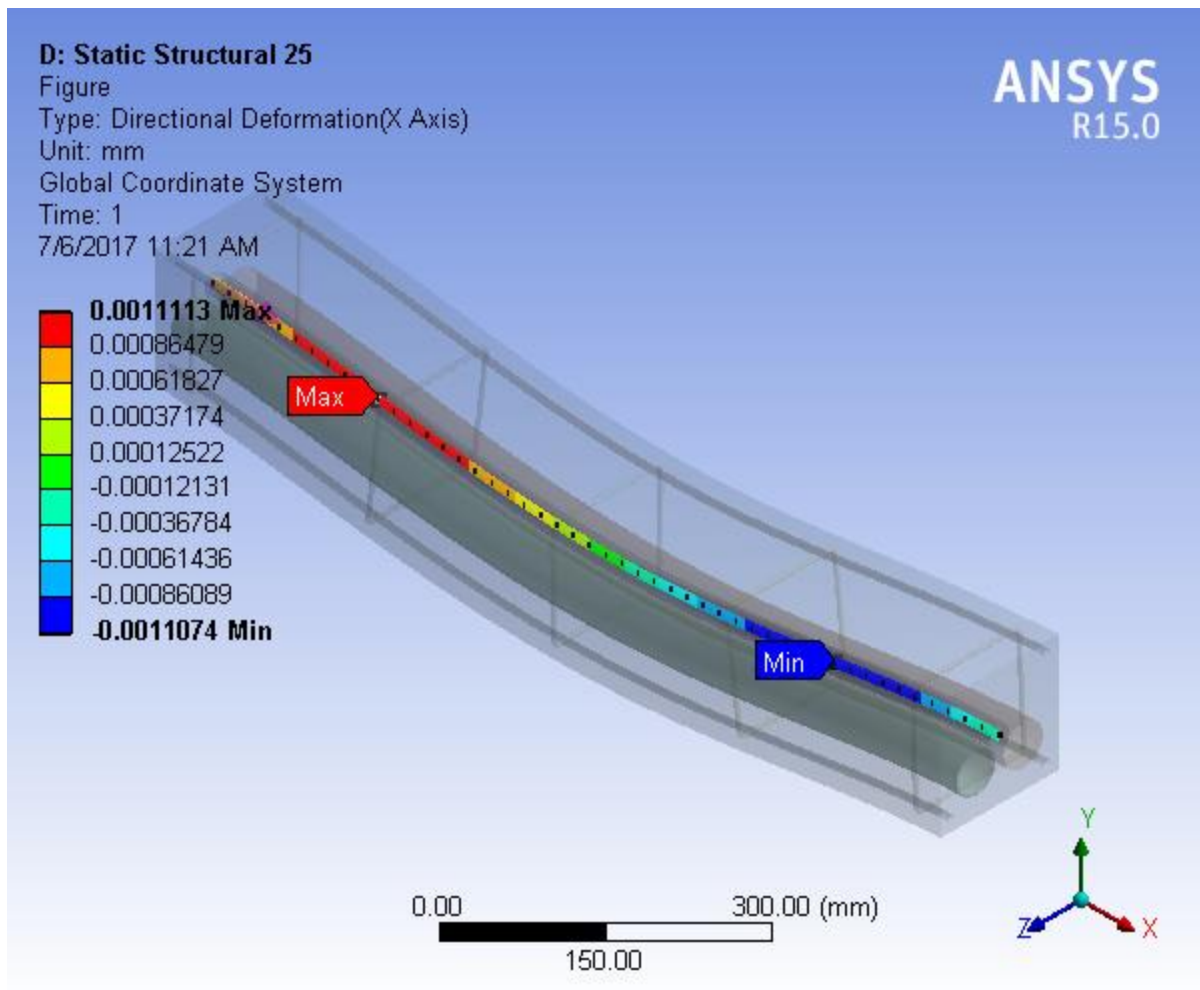


**TABLE 36**

**Model (D4) > Static Structural (D5) > Solution (D6) > Stress Safety Tools**

|                   |                            |
|-------------------|----------------------------|
| Object Name       | <i>Stress Tool</i>         |
| State             | Solved                     |
| <b>Definition</b> |                            |
| Theory            | Max Equivalent Stress      |
| Stress Limit Type | Tensile Yield Per Material |

**TABLE 37**

**Model (D4) > Static Structural (D5) > Solution (D6) > Stress Tool > Results**

|                        |                    |                 |                 |
|------------------------|--------------------|-----------------|-----------------|
| Object Name            | Safety Factor      | Safety Factor 2 | Safety Factor 3 |
| State                  | Solved             |                 |                 |
| Scope                  |                    |                 |                 |
| Scoping Method         | Geometry Selection | Path            |                 |
| Geometry               | All Bodies         |                 |                 |
| Path                   |                    | Path            | Path 2          |
| Definition             |                    |                 |                 |
| Type                   | Safety Factor      |                 |                 |
| By                     | Time               |                 |                 |
| Display Time           | Last               |                 |                 |
| Calculate Time History | Yes                |                 |                 |
| Identifier             |                    |                 |                 |

|                           |          |        |      |
|---------------------------|----------|--------|------|
| Suppressed                | No       |        |      |
| Integration Point Results |          |        |      |
| Display Option            | Averaged |        |      |
| Average Across Bodies     | No       |        |      |
| Results                   |          |        |      |
| Minimum                   | 1.292    | 1.6144 | > 10 |
| Minimum Occurs On         | Part4-1  |        |      |
| Minimum Value Over Time   |          |        |      |
| Minimum                   | 1.292    | 1.6144 | 15.  |
| Maximum                   | 1.292    | 1.6144 | 15.  |
| Maximum Value Over Time   |          |        |      |
| Minimum                   | 15.      |        |      |
| Maximum                   | 15.      |        |      |
| Information               |          |        |      |
| Time                      | 1. s     |        |      |
| Load Step                 | 1        |        |      |
| Substep                   | 1        |        |      |
| Iteration Number          | 1        |        |      |
| Graph Controls            |          |        |      |
| X-Axis                    |          | S      |      |

FIGURE 26

Model (D4) > Static Structural (D5) > Solution (D6) > Stress Tool > Safety Factor > Figure

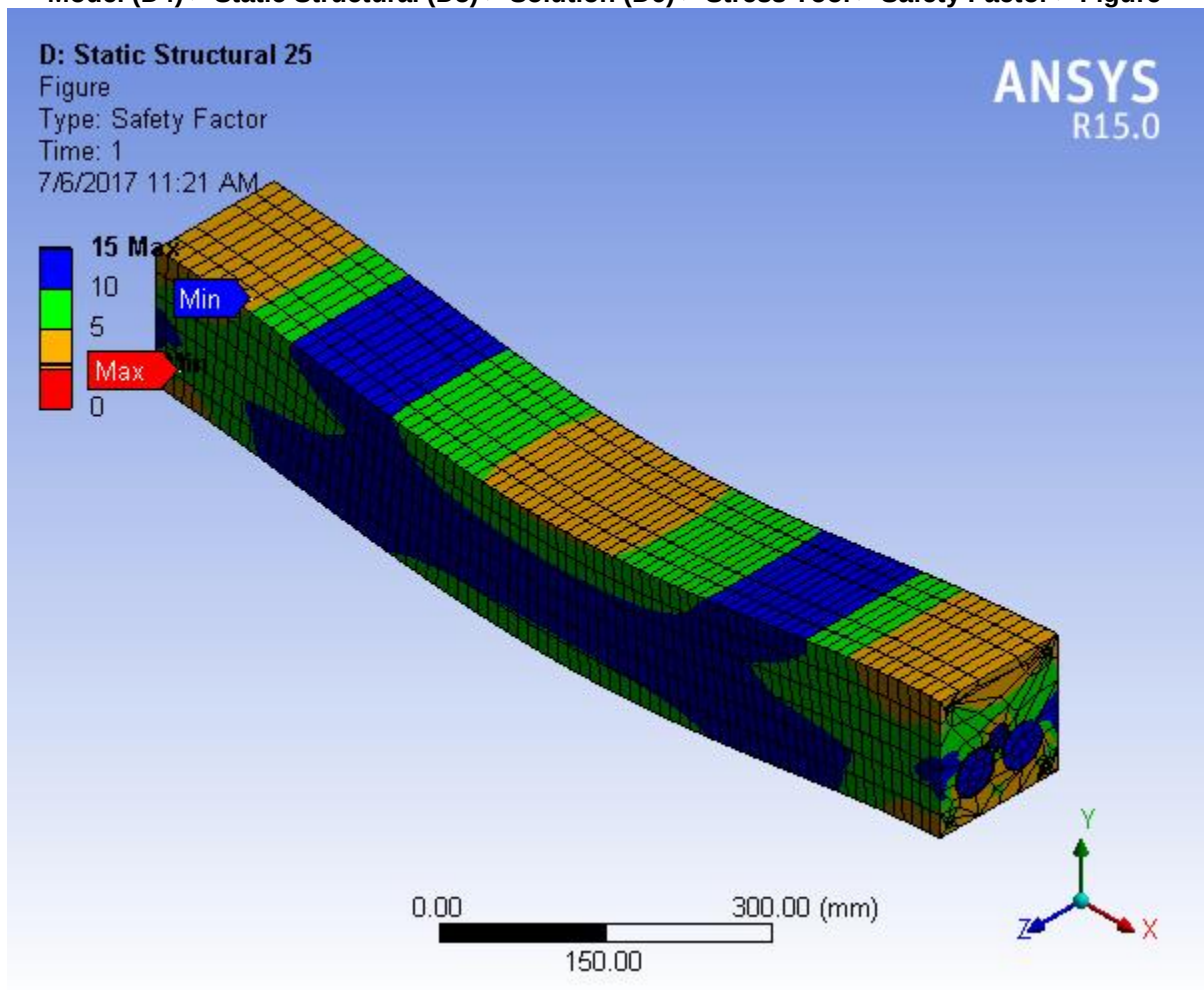


**FIGURE 27**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Stress Tool > Safety Factor 2**

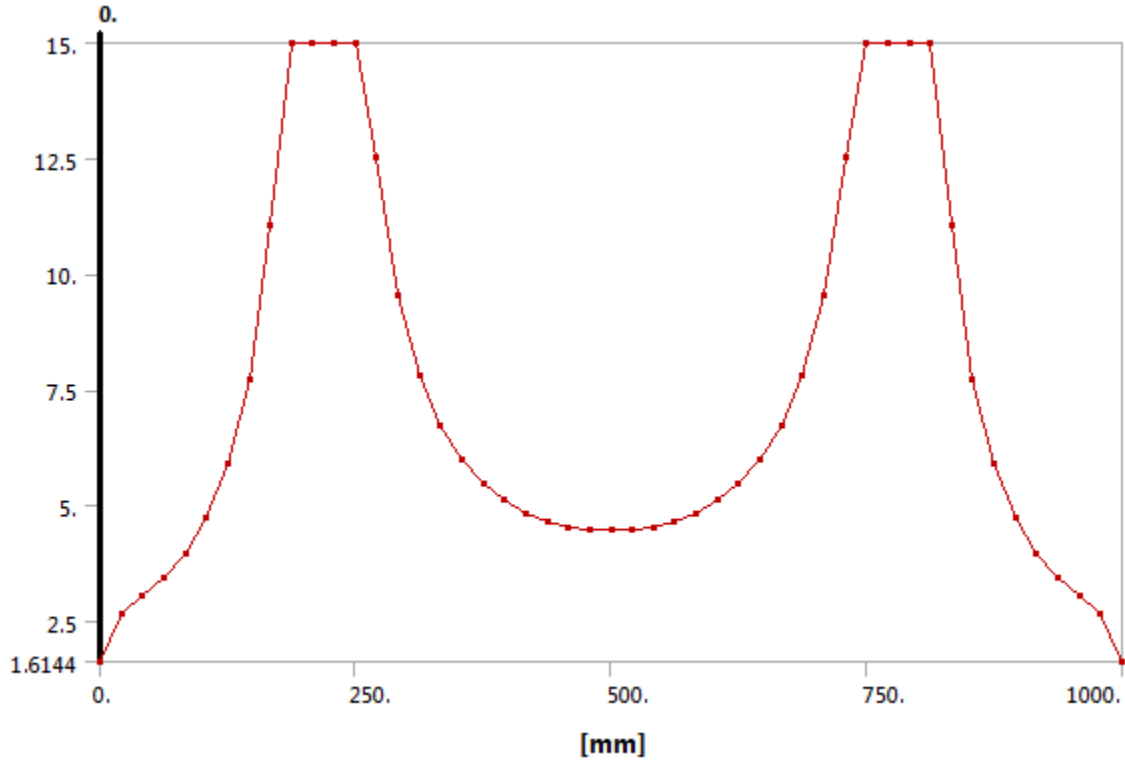


**TABLE 38**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Stress Tool > Safety Factor 2**

| Length [mm] | Value  |
|-------------|--------|
| 0.          | 1.6144 |
| 20.833      | 2.6445 |
| 41.667      | 3.0526 |
| 62.5        | 3.4201 |
| 83.333      | 3.9471 |
| 104.17      | 4.7178 |
| 125.        | 5.8816 |
| 145.83      | 7.7425 |
| 166.67      | 11.061 |
| 187.5       | 15.    |
| 208.33      |        |
| 229.17      |        |
| 250.        |        |
| 270.83      | 12.543 |
| 291.67      | 9.5312 |
| 312.5       | 7.8026 |
| 333.33      | 6.7262 |
| 354.17      | 5.9945 |
| 375.        | 5.4751 |
| 395.83      | 5.1059 |
| 416.67      | 4.8375 |
| 437.5       | 4.647  |
| 458.33      | 4.5285 |

|        |        |
|--------|--------|
| 479.17 | 4.4693 |
| 500.   | 4.4577 |
| 520.83 | 4.4693 |
| 541.67 | 4.5285 |
| 562.5  | 4.647  |
| 583.33 | 4.8375 |
| 604.17 | 5.1058 |
| 625.   | 5.4751 |
| 645.83 | 5.9944 |
| 666.67 | 6.7262 |
| 687.5  | 7.8027 |
| 708.33 | 9.5314 |
| 729.17 | 12.544 |
| 750.   | 15.    |
| 770.83 |        |
| 791.67 |        |
| 812.5  |        |
| 833.33 | 11.061 |
| 854.17 | 7.7425 |
| 875.   | 5.8816 |
| 895.83 | 4.7178 |
| 916.67 | 3.9471 |
| 937.5  | 3.42   |
| 958.33 | 3.0526 |
| 979.17 | 2.6445 |
| 1000.  | 1.6144 |

**FIGURE 28**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Stress Tool > Safety Factor 3**

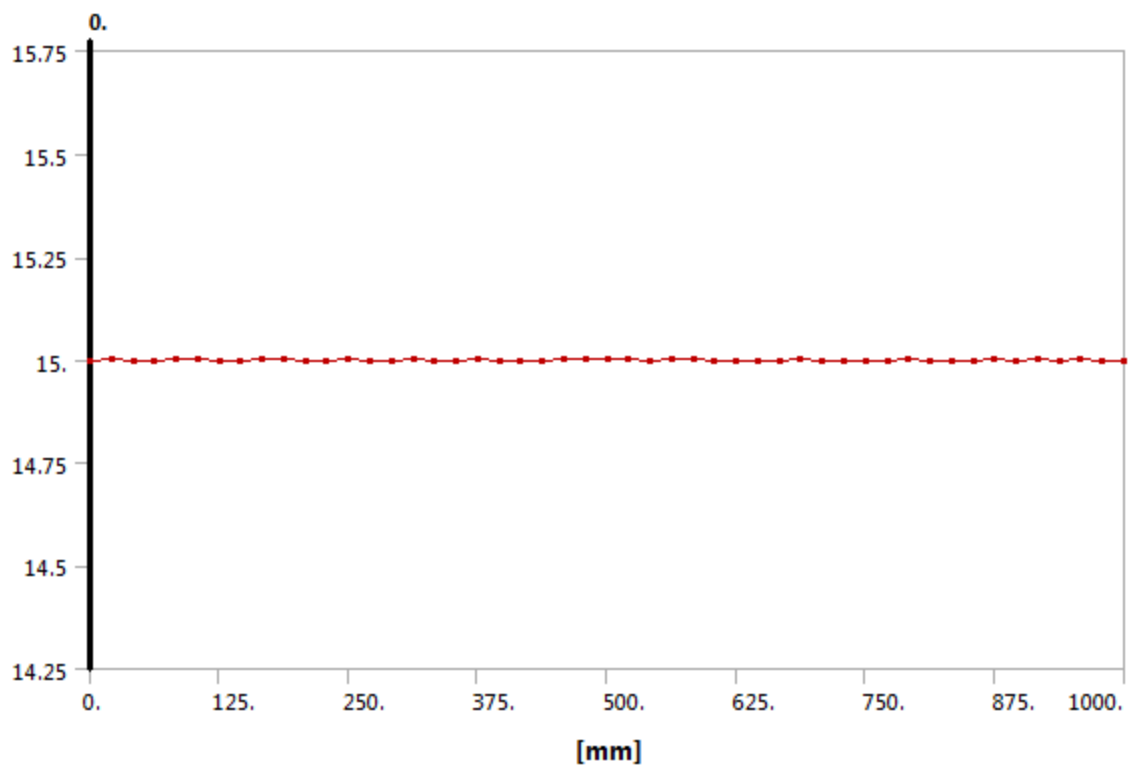


**TABLE 39**  
**Model (D4) > Static Structural (D5) > Solution (D6) > Stress Tool > Safety Factor 3**

| Length [mm] | Value |
|-------------|-------|
| 0.          | 15.   |
| 20.833      |       |
| 41.667      |       |
| 62.5        |       |
| 83.333      |       |
| 104.17      |       |
| 125.        |       |
| 145.83      |       |
| 166.67      |       |
| 187.5       |       |
| 208.33      |       |
| 229.17      |       |
| 250.        |       |
| 270.83      |       |
| 291.67      |       |
| 312.5       |       |
| 333.33      |       |
| 354.17      |       |
| 375.        |       |
| 395.83      |       |
| 416.67      |       |
| 437.5       |       |

