RUIN PROBABILITIES FOR SOME COLLECTIVE RISK RESERVE PROCESSES WITH STOCHASTIC INVESTMENT AND INFLATION

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Certification

This is to certify that this thesis has been read and approved as meeting the requirements of the Department of Statistics, University of Ilorin, Ilorin, Nigeria for the Award of Ph.D. degree.

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Dedication

This work is specially dedicated to everybody who lost their jobs due to recapitalization of Insurance Companies in 2005 and also to every individual with humble beginning who aspires to reach the summit of educational ladder.

Declaration

I, OSENI, Bamidele Mustapha hereby declare that this thesis entitled, "Ruin Probabilities for Some Collective Risk Reserve Processes with Stochastic Investment and Inflation" is a record of my research work. It has neither been presented nor accepted in any previous application for a higher degree. All sources of information have been specifically acknowledged.

In addition, the research work has been ethically approved by the University Ethical Review Committee.

_____ _____

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Abstract

Basically, Insurance firms collect premiums and pay claims. Not to go bankrupt in a highly competitive market, insurance companies engage in several investment portfolios which can be classified as either investments with fixed or stochastic returns but not both. Companies engaging in these investments need to monitor their investments and re-asses their strategy periodically in order not to be bankrupt. This study provided a model for the risk reserve process of the insurance firm involved in both investment portfolios at the same time, as against what is obtainable in several other works and investigated the ruin model through computation of ruin probabilities. The specific objectives were to: (i) develop a collective risk model with stochastic investment and inflation, (ii) determine the ruin probabilities for the model, (iii) compare various cases of the model and investigate the effect of stochastic investment and inflation and (iv) apply the model to real life data.

A collective risk model representing situations where ruin is caused by claims and/or investments respectively was formulated. The model was converted into differential equations and were resolved using the theory of confluence hypergeometric functions. R-statistical software was used with "fAsian Options" package to generate results. These results were generated at varying values of capital, claim size, premium rate, claims arrival rate, investment, interest, drift and volatility. The following were the findings of the study:

1. A collective risk model with stochastic investment and inflation was developed and resolved into ruin probabilities.
2. The results show that ruin probabilities for either claims or investments decrease as capital increases, irrespective of the values of premium rate, average claim size, arrival rate of claims and returns from investments.
3. Using various cases, ruin probabilities decrease when average claim size, claim arrival rate, volatility and fraction of investments are increased.
4. Ruin probability attributable to claims was found to be constant when claim arrival rate is set to zero while other parameters are kept constant.
5. Ruin probabilities for the model is independent of inflation and decrease as fraction of investments into risky asset decreases. The lowest value was obtained when there is no investment with stochastic returns while the highest value was obtained when all investments are assumed to be with stochastic returns.
6. The results from real life data, were comparable to the findings from empirical data.

It can be concluded from this study that investing in markets with stochastic returns is comparatively more risky than either investing part or all resources into fixed returned market. In particular, the lowest ruin probability from investment were obtained when all resources were plunged into markets with fixed returns. It is therefore recommended to invest in markets with both fixed and stochastic returns. This study also showed that ruin probabilities were independent of inflation.

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CHAPTER 1

INTRODUCTION

1.1 Preamble

In recent past, destructive events such as torrential downpour, floods, hurricane, tsunami etc are becoming ubiquitous, thus severe losses are being incurred by people in most of the affected areas. At times of high uncertainty, most people would greatly appreciate the potency of some institutions they would easily fall back on, to ameliorate their losses. Most often the government undertakes this responsibility especially when the affected population consist of the indigents. Nevertheless, a sizable number is willing to part with some amount at other times in exchange for certain measure in times of misfortune.

An established institution that takes this responsibility (apart from the government and NGOs) is the insurance company. Insurance companies require individuals (the insured) to pay premium in exchange for cover in times of crisis. This cover is given as claims to the affected individuals. Thus, there is a certain measure of risk inherent in the interaction between the insurance and the insured. For the insurance company, ruin is impending if the claims by the insured surpass total revenue generated from the premium. To avoid this, most insurance companies arrange for suitable types of reinsurance to reduce the possible damage due to large claims and some even set aside a certain amount of its funds for other lines of businesses and carefully monitor the level of their reserve throughout the operation.

A classical risk model was first introduced by Lundberg (1903) and extensively studied by Cramér (1930-1955). The model most often called Cramér–Lundberg model uses the compound Poisson process to model the claims. Basically, the model implies that the risk reserve over time is simply the

difference between the premiums collected and claims over the period plus the initial capital. That is, the risk reserve process $\{R(t), t \geq 0 \}$

$$R(t) = u + \pi(t) - \sum_{i=1}^{N(t)} X_i \tag{1.1}$$

where,

- u is the initial capital
- $\pi(t)$ is the accumulated premium collected over the period $(0, t]$
- X_i $(i = 1, ... N(t))$ are the claims over the period $(0, t]$
- $N(t)$ is the number of claims over the period $(0, t]$

Since the processes depend on the explicit form of $\pi(t)$ and the stochastic behavior of the accumulated claim process $\sum_{i=1}^{N(t)} X_i$, a number of assumptions were made among which are the assumptions of independence among claim sizes $X_1, X_2, \ldots$ and between claim size X_i and claim arrival epoch T_i. The claim process is assumed to be compound Poisson process while the premium $\pi(t)$ is assumed to be a linear function ct (Bühlmann, 1996), where c is the premium rate. Also, a necessary condition is the "net profit condition", which assumes that on the average, the premium rate must be greater than the claim rate. Mathematically,

$$c > \lambda E(X), \tag{1.2}$$

where λ is the intensity of $N(t)$. On the other hand, if $c = \lambda E(X)$ the company will be running at loss, as they can neither make profit nor take care of administrative costs. The trivial case is when $c < \lambda E(X)$, the company gets ruined by both claims and administrative costs. To avoid this scenario and to be able to determine the expected gain, most companies assume the premium rate of the form

$$c = (\varrho + 1)\lambda E(X) \tag{1.3}$$

Where ϱ is the "safety loading" which takes care of not only the administrative cost but also the gain the company intends to make on the long run. The safety loading also gives the company the flexibility to adjust the premium rate in accordance with the expected risk from the portfolio. The higher the risk, the higher the safety loading.

The Lundberg model has been criticized for its simplicity, by a number of authors. Notably among are Hipps, who said that the model is too simple to be realistic though simple enough to calculate the probabilities of interest (Back et al., 2003). This view is corroborated by the statement, *"Classical insurance principles are not necessarily valid anymore!"* (Albrecher, 2008). But the Cramér–Lundberg model is still relevant not only because of its flexibility but also because it separately models each component of risk process and also account for frequent and large claims, which are the two major causes of big losses.

Other more realistic risk processes are the Sparre-Andersen (Renewal) process and the Markov modulated risk process (Back et al., 2003). All the techniques developed for the Cramér– Lundberg model are still useful in these models. Sparre-Andersen model (which many author refer to as a more general model) was developed in 1957, by considering non-exponential inter-arrival times. Here, more attention is paid to the choice of the distribution of the inter-arrival times and a new theory is developed for the case "where the intensity of occurrence of claims depends on the space of time which has passed since the last claim", (Andersen, 1957). As the case with the Lundberg model, the Sparre-Andersen model also depends on the form of the premium.

The calculation of premium to be charged by an insurer constitutes an essential part of the actuarial know-how. An insurer must determine its premium in such a way that the solvability of the portfolio can be guaranteed. That is, the premium rate must increase fast enough to cope with the incoming claims and yet remain very competitive since rival insurance companies might attract clients by offering lower premiums while covering the same risk (Goovaerts et al., 1987). To remain competitive and maintain solvability of the portfolio, some insurers invest a part (or all) of their reserve in financial markets. This constitutes a source of uncertainty which could best be described by diffusion processes.

Description of financial markets using stochastic processes is one of the major breakthroughs of stochastic analysis. Central to this is the Brownian motion. Brownian motion takes both negative and positive values hence it is not appropriate for modeling all prices in the financial market. The non-negative variation of the Brownian motion introduced by Samuelson (1965) called geometric Brownian motion is often used in place of the Brownian motion when the prices can only be positive (e.g. stock prices, bonds etc). Due to the randomness of the (geometric) Brownian motion, it is used to model risky assets when the price could drop causing the investor to lose money. Among the researchers that have successfully investigated risk models with diffusion component are Paulsen and Gjessing (1997), Promislow and Young (2005), Paulsen (2008), Yuen and Wang (2005), Gerber and Yang (2007) and Yong and Xiang (2012) Yong and Xiang(2012) to mention a few.

The basic subjects of analysis of risk models are the probabilities of ruin, the infinite-horizon ruin probability $\psi(u)$ and the finite-horizon ruin probability $\psi(u,T)$. Some of the theoretically derived results may seem to have limited scope in practical situations, but they have been the major source of inspiration in development of most techniques in actuarial science (Rolski et al., 1999). Ruin occurs when the claims paid exceed the surplus available. That is, when $R(t)<0$. The random variable

$$\tau(u)=\inf\{t\geq 0: R(t)<0\} \tag{1.4}$$

is called the time of ruin of a portfolio, where $\tau(u)=\infty$ when no ruin ever occurs. The finite-horizon and infinite-horizon probabilities of ruin are defined respectively by

$$\psi(u)=P\left(\inf_{t\geq 0} R(t)<0\right)=P\left(\tau(u)<\infty\right) \tag{1.5}$$

$$\psi(u,T)=P\left(\inf_{0\geq t\geq T} R(t)<0\right)=P\left(\tau(u)<T\right) \tag{1.6}$$

When no safety loading is assumed $\psi(u)=1$, but with safety loading, $R(t)\longrightarrow\infty$ and the ruin probability satisfy a certain integro-differential equation. Alternatively, ruins can be expressed in terms of survival probabilities which can be defined by

$$\varphi(u)=\bar{\psi}(u)=1-\psi(u) \tag{1.7}$$

$$\varphi(u,T)=\bar{\psi}(u,T)=1-\psi(u,T) \tag{1.8}$$

Although the study of insurance risk models with stochastic interest has attracted a fair amount of attention in recent years, many significant ruin problems associated with these models remain to be investigated Yuen and Wang (2005). This work will be investigating Ruin models in economic environment. That is, the insurer invests its reserve in a risky asset that follows geometric Brownian motion in the presence of inflation. Cramér–Lundberg will specifically be assumed to be the base model.

1.2 Motivation

In 2005, the Central Bank of Nigeria (CBN) embarked on the second round of recapitalization of insurance companies in Nigeria with a view to strengthening them. The companies were requested to increase their capital base form N150 million to N2 billion for Life Insurance businesses, N300 million to N3 billion for Non-Life businesses and N350 million to N10 billion for the Re-Insurance companies. This resulted in a lot of liquidation and merger between some companies as the number of insurance companies reduced from 103 to 49. Some Insured were migrated from one company to another and some shareholders were subjected to considerable slash in their share holdings. As a result, Insurance companies in Nigeria diversified their investment and reduced reliance on premium

from insured considerably. Despite the recapitalization, companies can go bankrupt if their assets are not properly managed. Further more, there are clear indications that the current recapitalization levels are inadequate, as industry statistics reveal that insurance companies lose the opportunity of earning about N70 billion in premiums annually from the oil and gas sector as a result of premium flight and this is expected to double with the emergence of a functional power sector on the back of the current power reforms if nothing is done to position the industry (Oyelade, 2013).

These actually necessitate the need to investigate the survivability of insurance companies in the long ruin, given a specified asset and certain type of investments. This work, though borne out of curiosity in the first instance, is further motivated by the need to develop a model and subsequently derive the probability of ruin of such companies with a view to providing a platform for proper monitoring of the effect of capital, investment and inflation on survivability of insurance companies.

1.3 Aim and Objectives

The aim of this work is to provided a model for the risk reserve process of the insurance firm involved in risky and risk-less investments at the same time, as against what is obtainable in several other works and investigated the ruin model through computation of ruin probabilities while the specific objectives are to:

1. Develop a collective risk models with stochastic investment and inflation.
2. Determine the ruin probabilities for the models.
3. Compare various cases of the models and investigate the effect of stochastic investment and inflation.
4. Apply the models to real life data.

1.4 Research Problems

Recall the risk reserve process (1.1)

$$R(t)=u+\pi(t)-\sum_{i=1}^{N(t)}X_i$$

The claim surplus process $\{S(t),t\geq 0\}$ associated with (1.1) is given by

$$S(t)=U(t)-\pi(t) \tag{1.9}$$

where

$$U(t)=\sum_{i=1}^{N(t)} X_i. \tag{1.10}$$

The incoming flow of fund is through the premiums collected $\pi(t)$, while outgoing flow is through the claims. Suppose the variation in collective premium and claim is modeled as part of the risk reserve, thus the risk reserve $R(t)$ is given by

$$R(t)=u+\pi(t)-\sum_{i=1}^{N(t)} X_i+\varepsilon(t) \tag{1.11}$$

where $\varepsilon(t)$ is a Gaussian white noise process. Suppose the following assumptions are made about (1.11):

Assumption 1.4.1 *The accumulated premium collected $\pi(t)$ is a linear function ct as commonly found in literature. That is, the premium is continuously collected at a constant rate c over the time period $[0,t\ ]$ (Bühlmann, 1996).*

Assumption 1.4.2 *The Gaussian white noise process $\varepsilon(t)$ is a linear function $\sigma_p \varepsilon t$ and its values are independent at every distinct time interval t, where σ_p is the standard deviation of the change in the risk reserve process at time interval t and ε is normally distributed random error with zero mean and unity variance.*

Assumption 1.4.3 *$N(t),t\geq 0$ is a Poisson process with rate λ. That is, N(t) is a counting process with independent and stationary increment. Also $N(0)=0$ and $E[N(t)]=\lambda t$*

Assumption 1.4.4 $(X_i)_{i\in N}$ *(where N is the set of Natural numbers) are positive, independent, identically distributed random variables with finite mean and distribution function F(x).*

Assumption 1.4.5 *The distribution function F(x) above is light-tailed. That is, for some constants $a,b>0$, the tail $\bar{F}(x)=1-F(x)$ is such that*

$$\bar{F}<ae^{bx}.$$

Model (1.11) with the assumptions – forms the block upon which the research problems are derived.

The distribution of the claims $F(x)$ is assumed to follow certain distributions. Two cases considered are:

Case 1 *when the claim is exponentially distributed. This is the simplest case for single portfolio by a company.*

Case 2 *when the claim is a mixture of two exponential distributions. This represents the case where the company is involved in two portfolios each following exponential distribution. The combine claim size from the two portfolio is assumed to be mixture of two exponential distributions.*

For both cases above, the situation where the company invest its asset in only the two types of investment below:

- Investment with deterministic returns.
- Investment with stochastic returns.

is investigated while at the same time assuming inflation is present.

1.5 Justification

As noted by Constantinescu (2006), the basic question of actuarial science is “what is the fair premium to be charged so that insurance company may cover the risk of future losses and stay solvent, at the same time?” This question is fast being replaced by "what is the optimal investment strategies so that for a given premium rate, the company may cover future losses and stay solvent at the same time”. The common measure of solvency is the ruin probability. A number of research works have been carried out on ruin probabilities, with the intent of getting closer to what is obtainable in real life. However these ruin probabilities may not be exactly close to reality when the effect of inflation and interest is not being considered collectively in the model. Insurance companies these days could hardly exist without engaging in various types of investments. To engage in these investments, a sufficient reserve capable of catering for the incoming claim at that instant, and at the same time support investment must exist. The nominal value of this reserve at that instant is not exactly the same as the value at a future time due to inflation and return from investment. Thus it is necessary to integrate these two quantities into the risk model before obtaining the ruin probability. A risk model with stochastic investment and inflation is considered in this work. The solvency of a company that engages in the above will be investigated by computation of the ruin probabilities for the models

Chapter 2

Literature Review

The pioneering works of Lundberg (1903, 1909, 1926) and Cramér (1930, 1955) provided the sound mathematical background on which Stochastic modeling of insurance risk reserve processes are being developed. They introduce collective risk model with several assumptions among which are the assumption that claim arrivals is Poisson distributed. Some of these assumptions are mere mathematical simplification and quite unrealistic, yet Cramér-Lundberg model is still being used because it provides the skeletal frame work for more realistic models being developed recently.

Basically, the focus of investigation ab initio was on particular conditions on the claim size distribution (Cramér, 1930; Gerber, 1973; Constantinescu, 2006). With the Cramér-Lundberg condition on claim size, ruin probability decays exponential fast to zero as the initial capital tends to infinity (Cramér, 1930) and if the condition is relaxed, the asymptotic behavior of the ruin probability changes suddenly. For instance, in the case of sub-exponentially distributed claim sizes, the ruin is asymptotically determined by a large claim (Embrechts et al., 1997). Subsequently, the model was improved by considering randomness in premium income, time dependence and rate of returns on the investments through perturbations.

2.1 Ruin Models with Investments

The pioneering investigations on ruin models were devoid of investment incomes despite insurance firms actively engaging in various types of investments. An obvious reason for this is the simplicity and tractability of the resulting equations, although other reasons may exist, and back in the first half of the 20th century the theory of stochastic processes was far less developed, and also far less known, than it is today (Paulsen, 2008). The first work attempting to investigate ruin model with investment income was undertaken by Segerdahl (1942), were it was assumed that capital earns interest at a fixed rate. Further investigations followed afterwards from Gerber (1971), Harrison (1977) and Delbaen and Haezendonck (1987). Harrison (1977) investigated ruin problems with compounding assets, where the assets were represented with a simple path-wise integral with respect to the income process. A general characterization was obtained for the probability that the assets will ever fall to zero. In addition, an approximation theorem was proved showing that if the income process is well approximated by Brownian motion with drift, then the assets process is well approximated by a certain diffusion process and the probability of ruin is well approximated by the first passage probability corresponding to the certain diffusion process. Delbaen and Haezendonck (1987)

provided a simple generalization by considering the influence of macro-economic factors such as interest and inflation on the classical risk model. The interest and inflation were assumed to be time dependent and real interest force was obtained as the difference between inflation and interest. Bounds on the ruin probability were obtained for various premium calculations method, against the current trends of deriving the actual ruin probability.

A simpler method of solving ruin problems with compounding assets was introduced by Dassions and Embrechts (1989) where the model was re-classified as a piece-wise deterministic Markov process. The piece-wise deterministic Markov structure was essentially a vehicle for obtaining some interesting martingales, which were subsequently used to calculate relevant functionals of a process with Gamma claim sizes and investment policies, including borrowing.

Inspired by ideas from mathematical finance, Paulsen (1993) suggested a model where capital is allowed to be invested into risky assets in economic environment (Paulsen, 2008). The risky assets $\mathcal{R}_t$ were assumed to be of the form

$$\mathcal{R}_t = exp\{(r - \frac{1}{2}\sigma_R^2) + W_{R,t}\} \prod_{i=1}^{N_{R,t}} (1 + \tilde{S}_{R,i}). \tag{2.1}$$

where r is drift, σ_R is volatility, $W_{R,t}$ is diffusion process, $N_{R,t}$ is Poisson process and $\tilde{S}_{R,i}$ is surplus process. Inflation was also assumed to be stochastic and of the form,

$$\bar{\mathrm{I}} = exp\{(\bar{\imath} - \frac{1}{2}\sigma_I^2) + W_{I,t}\} \prod_{i=1}^{N_{I,t}} (1 + \tilde{S}_{I,i}). \tag{2.2}$$

where $\bar{\imath}$ is inflation rate, σ_I is volatility due to inflation, $W_{I,t}$ is diffusion process, $N_{I,t}$ is Poisson process and $\tilde{S}_{I,i}$ is surplus process. Theory of martingale was used to determine, in some cases exact probability of eventual ruin and in other cases, inequality for the probability of eventual ruin.

Paulsen and Gjessing (1997) extended the work of Paulsen (1993) to include the time of eventual ruin. The Laplace transform of the time of eventual ruin were obtained as solutions to certain boundary value problems containing integro-differential equations. The concept of infinitesimal generator was also used to derive some interesting integro-differential equations for the probability of eventual ruin which were subsequently solved for a number of special cases. Some interesting theorem which further simplifies the mathematics of risk processes where also proposed. The much celebrated work of Gerber and Shiu (1998) further broadened the scope of investigations in application of stochastic processes to risk processes of insurance firm. The work generalizes the

classical risk process by discounting with respect to the time of ruin. The calculation of expected discount penalty which is due at ruin and may depend on the deficit at ruin and on the surplus immediately before ruin also was explained. Basically, the joint distribution of the time of ruin, the surplus immediately before ruin, and the deficit at ruin was studied.

Nyrhinen (1999) provided a model which re-calculates capitals on annual bases and proceeded to compute the ruin probability in a general economic environment. The model assumed that working capitals of insurance firms changes after every financial year and denoted the capital at the end of year n by U_n and

$$U_n=(1+r_n)(U_{n-1}-B_n) \tag{2.3}$$

where $U_0=M$ is the initial capital, r_n describes the rate of return on investments in year n and B_n is the net payout of the company in the year n (excluding the return from investments). The risk reserve process by Nyrhinen (1999) with capital recalculated on annual bases is

$$Y_n=B_1+A_1B_2+A_1A_2B_3+\cdots+A_1\cdots A_{n-1}B_n \tag{2.4}$$

where $A_n=1/(1+r_n)$ is the discount factor. A crude estimate of ruin probability $P(T_M<\infty)$ was obtained as

$$P(T_M<\infty)\approx M^{-w} \tag{2.5}$$

where T_M is the time of ruin when the initial capital is M. Norberg (1999) introduced the concept of cash balance process;

$$X_t = e^t(X_0-\int_0^t e^{-\tau}dB_\tau) \tag{2.6}$$

$$t = \int_0^t \delta_t d\tau \tag{2.7}$$

where X_0 is the initial cash deposit by an insurer intended for offsetting possible future excess of claims over premiums, B_t is the total amount of excess of claims over premiums in the interval $[0,t]$ and δ_t is the instantaneous rate of return at time t. The investment of the insurance firm was assumed to be of diffusion type;

$$dX_t=(X_t(\delta+\frac{1}{2}\sigma_d^2)+\beta)dt+X_t\sigma_d dW_{d,t}+\sigma_b dW_{b,t} \quad (2.8)$$

where σ_d is the diffusion coefficient which measure the size of the liability risk, β represents the expected gain per time unit due to saftey loading in the premium and $W_{b,t}$ is a Brownian motion. Ruin was defined as the events that the cash balance X_t falls to zero. A generalization of the existing results on the probability distribution of the running maximum of a Brownian motion, relationship between the probability of ruin and the probability distribution of discounted total payments were determined.

Yeun et al. (2004), considered a risk process with stochastic return on investments where the investment generating process was assumed to be a compound Poisson process and a Brownian motion with positive drift. The lower bound for the ruin probability was derived for a twice continuously differentiable integro-differential equation for the ultimate ruin probability.

Paulsen (2002) established the asymptotics for the ruin probability for a risk process in a stochastic economic environment. Both the investment and the risk process was considered as a levy processes and it was established asymptotically that ruin probabilty essentially behave as $y^{-\kappa}$ as the initial capital increases. If the perturbation is considered and the risk process is assumed to be geometric Brownian motion, the asymptotic decay of ruin probability is algebraic at best. But if the perturbation remains unaltered as the Brownian motion, the asymptotic decay of ruin probability remains exponential (Schmidli, 1995). The former perturbation could be seen as the price of the risky asset bought with the capital at an instant (Frolova et al., 2002; Constantinescu, 2003). Kalashnikov and Norberg (2002) considered the situation where the reserve of an insurance business is invested in an asset that may yield negative interest. Upper and lower bounds for the probability of ruin were obtained in the case where the cash flow of premiums less claims and the logarithm of the asset price were both Lévy processes.

Ma and Sun (2003), extended the Cramer–Lundberg model to contain stochastic interest rates, reserve-dependent expense loading, diffusion perturbed models, and many others as special cases. The Cramer-Lundberg type estimations was put into a unified framework through the introduction of a new type exponential martingale parametrized by a general rate function. Using some examples, its was established that appropriate choice of rate functions could lead to recovery of many existing Lundberg-type bounds for ruin probabilities. Exponential and non- exponential upper bounds for probability of ruin for a risk model with stochastic investment was established by Zhang (2005). Zhang (2005) risk model was a generalization of four other types of model – classical discrete time risk model, Yang (1999) model, discrete time risk model with stochastic interest rate and discrete

time risk mode with stochastic investment return and dividend – which have been previously considered in literature.

Recursive and integral equations for ruin probabilities of a compound Poisson surplus process with investment into a stochastic interest process which is assumed to be a Lévy process was derived by Cai (2004). The expected discounted penalty function associated with the time of ruin was investigated when the interest process is a Brownian motion with drift. Yuen and Wang (2005) considered a risk process with stochastic interest in which the basic risk process is the classical risk process and the stochastic interest process is a compound Poisson process with positive drift. An exact solution of an integro-differential equation for the Gerber-Shiu expected discounted penalty function derived, was obtained.

A further investigation on ruin probability with stochastic investment was carried out by Pergamenshchikov and Zeitouny (2006). However, the premium rate was considered to be a bounded non-negative random function c_t while the investment was assumed to be a geometric Brownian motion. The asymptotic upper and lower bounds for ruin probability $\varphi(u)$ as the initial capital u tends to infinity was derived to be of the form

$$C_* u^{-\beta} < \varphi(u) < C^* u^{-\beta} \tag{2.9}$$

where $\beta := 2a/\sigma^2 - 1 > 0$, a is the mean return or drift and $\sigma > 0$ is the volatility of the geometric Brownian motion.

Constantinescu (2006) examined a renewal risk model where investment is made into an asset with stochastic returns. An integro-differential equation was derived for the model using the infinitesimal generator of the process. This integro-differential equation which was referred to as general is of the form

$$Z(t) = Z(0) exp\{(a - \frac{\sigma^2}{a})t + \sigma W_t\} + c \int_0^t exp\{(a - \frac{\sigma^2}{a})(t-u) + \sigma(W_t - W_u)\} du \tag{2.10}$$

Through the integro-differential equation (2.10), a particular integro-differential equation for the general conditions regarding the claim size, claim arrivals and the returns from investment was deduced.

Heyde and Wang (2009) considered a finite-time ruin probability for a Poisson risk model with an exponential Lévy process investment return and heavy-tailed claim size. The extreme of insurance risk was found to dominate the extreme of financial risk, but, for the case of dangerous risk, the extreme of financial risk has more and more of an effect on the total risk, and as time passes, the

extreme of financial risk finally dominates the extreme of insurance risk. Wang et al. (2010) considered the dividend payments in a compound Poisson risk model with credit and debit interests under absolute ruin. An integro-differential equations satisfied by the moment generating function and moments of the discounted aggregate dividend payments was obtained and subsequently used to get the explicit expressions of the absolute ruin for exponential claims.

A compound Poisson process with investment in risky assets, whose prices satisfy the Black-Scholes model was considered by Yong and Xiang (2012). The ruin probability of this process was decomposed into the sum of two ruin probabilities which are caused by the claim and the oscillations. The lower bound of the ruin probability was obtained from a third order integro-differential equation derived for the process. A compound Poisson surplus model with threshold dividend strategy and proportional investment was considered by Chen and Ou (2013). Study was carried out on the expected discounted dividend payments and the expected penalty–reward function through derivation of integro-differential equation with certain boundary conditions. A numerical sinc method was proposed since a closed form solution does not exist.

Risk models with slight variations from the classical Crammer-Lundberg model was also studied by some researchers. Zhang and Wang (2012) considered a bi-dimensional risk model with the possibility of investment. The upper bound of the infinite-time ruin probability with respect to the time of ruin $T_{max}(u1,u2)$ was obtained using the martingale technique. An explicit expression for the finite-time ruin probability was also derived through an integro-differential equation. Xu et al. (2014) investigated the ruin probability for a Cox risk model with intensity depending on premiums and stochastic investment returns, and proposed a model which allows the dependence between premiums and claims. Coupled integral equations for the Gerber–Shiu expected discounted penalty function (GS function) were derived for a situation where the surplus is invested in the bond market with constant interest force. Together with the initial value and Laplace transformation of the GS function, a numerical procedure was provided for obtaining the GS function. A minimal upper bound for the ruin probability was also found for a situation where the surplus can be invested in risky asset driven by a drifted Brownian motion.

A two-dimensional time-dependent risk model with stochastic investment returns was considered by Guo et al. (2017). In the model, an insurer operates two lines of insurance businesses sharing a common claim number process and can invest its surplus into some risky assets. The claim number process was assumed to be a renewal counting process and the investment return was modeled by a geometric Lévy process. Furthermore, claim sizes of the two insurance businesses and their common inter-arrival times correspondingly follow a three-dimensional Sarmanov distribution. Some uniform asymptotic formulas for ruin probability of the model over certain time horizon were established for situation when claim sizes of the two lines of insurance businesses were heavy tailed. Hussain and

Parvez (2017) considered the formulation of the wealth, analysis of the continuous- and discrete-time investment strategies in risky as well as risk-free markets and the probability of ruin for insurance companies. The solvency condition for each investment strategy was examined and an explicit expression for the probability of ruin of a particular investments strategy was obtained. Maximum claim size which would not produce ruin when investments are made was also identified.

Some researchers basically showed interest in finding the optimal investment strategy for insurance company under certain situations. Liu and Yang (2004) considered a model which allows the insurer to invest in both risky and risk-less assets, receives premium at a constant rate while at the same time allowing claims to arrive a exponential rate. The investment behavior was investigated numerically for various claim-size distributions. The optimal policy and the solution to the associated Hamilton-Jacobi-Bellman equation were then computed under each of the assumed distribution. Paulsen and Kasozi (2005) investigated the issue of maximization of dividends of an insurer whose portfolio is exposed to insurance risk. The risk considered arose from the classical Crammer-Lundberg type risk where investment were made into risk free assets whose price dynamics are governed by a constant force of interest. The solution to the derived Volterra integral equation were solved by numerical scheme which involved a block by block method and Simpson rule.

Kötter and Bäuerle (2008) investigated the problem of maximization of the adjustment coefficient of a periodic risk model with the possibility of investing into a risky asset, given by a geometrical Brownian motion. The optimal investment strategy was shown to only depends on the averaged data of the model and is constant over time, thus establishing that maximization of the adjustment coefficient is a very weak optimization criterion. Fernández et al. (2008) derived a closed form solution for an exponential risk preference as well as estimate the ruin probability when the optimal strategy is used. The risk model considered gives the company the opportunity to invest in a risky asset using stochastic control techniques. The classical model with risky investment, when interest is in the ruin probability that is minimized by a suitably chosen investment strategy for a capital market index was considered by Wei (2008). The claim sizes were assumed to be of common distribution of extended regular variation, starting from an integro-differential equation for the maximal survival probability. The corresponding ruin probability as a function of the initial surplus was also found to be of extended regular variation. Badaoui and Fernández (2013) considered an optimal investment problem of an insurance company with a Cramér–Lundberg risk process and investments portfolio consisting of a risky asset with stochastic volatility and a money market. A correlated economic factor, modeled as diffusion process was allowed to affect the asset price. A verification theorem, showing that any solution to the Hamilton–Jacobi–Bellman equation solves the optimization problem was also provided. An explicit form of the optimal strategy was also provided with a proof of existence of a smooth solution.

Zhang et al. (2015) examined a problem of minimizing ruin probability under optimal control for the continuous-time compound binomial model with investment. The jump mechanism considered differs from that of Liu et al. (2005) where investment, and hence, the additional Brownian motion term, was not considered. A technical difficulty which arose through the introduction of investment was overcome through the usage of the theory of change of measure and exponential martingale was obtained by virtue of the extended generator. The ruin probability was minimized through maximizing adjustment coefficient in the sense of Lundberg bounds. At the same time, the optimal investment strategy was obtained.

2.2 Perturbed Ruin models with Investments.

The uncertainty in the reserve due to fluctuation in the premium and or claims are sometimes accounted for by perturbing the reserve with appropriate diffusion process. The due difficulties in solving models that arose from perturbed risk processes, works dealing with perturbation of the reserve; especially when investment is considered; are not as many as those dealing with unperturbed risk processes. Among the few early researchers who investigated perturbation in risk processes are Dufresne and Gerber (1991), who extended the classical model of collective risk theory by adding diffusion process to the compound Poisson process and was able to show that the probabilities of ruin by oscillation or by a claim satisfy certain defective renewal equations. Interpretations in terms of record highs aggregate loss process was given to the convolution formula derived for probability of ruin. The probability of ruin was computed for individual claim amounts which was distributed as combinations of exponentials.

Paulsen (1998), considered a classical risk process which was assumed to be a levy process and compounded by another independent levy process. Sharp conditions were given on the parameters of these two independent levy processes to ensure when ruin is certain, and also when the time of ruin is of exponential type. A generalization of the result when ruin is not certain as shown in Paulsen (1993) was also derived. It was established that the ruin probability depends on the distribution function of a certain present value, and an integro-differential equation for the characteristic function for a special case when the two levy processes have only a finite number of jumps on a finite time interval.

Among those who have carried out works in this area in recent past is Gerber and Yang (2007) who considers a compound Poisson risk model perturbed by diffusion where the company can invest its surplus in both a risky asset and the risk-free asset according to a fixed proportion. In the model, a constant debit interest rate was applied, if the surplus is negative. Closed-form solutions were obtained for absolute ruin probability function.

Though few researchers have engaged in works considering rate of return and interest, none have actually considered the effect of inflation simultaneously. Aside Paulsen (1993), who considered a

model for the risk process that takes into account stochastic rates of return and inflation, most authors have been assuming deterministic rate of return (interest) and inflation. Thus this work is intended to bridge the gap by considering both stochastic rate of return, interest and inflation simultaneously.

2.3 Some Concepts and Definitions

Definition 2.3.1 (Itô Chain Rule) *Let X_t be an itô process given by*

$$dX_t = udt + vdW_t \tag{2.11}$$

Let g(t,x) be twice continuously differentiable on $[0,\infty)\times R$. Then

$$Y_t = g(t,X_t)$$

is again an itô process, and

$$dY_t = \frac{\partial g(t,X_t)}{\partial t}dt + \frac{\partial g(t,X_t)}{\partial x}dX_t + \frac{1}{2}\frac{\partial^2 g(t,X_t)}{\partial x^2}(dX_t)^2 \tag{2.12}$$

where $(dX_t)^2=(dX_t)\cdot(dX_t)$ is computed according to the rules $dt\cdot dt=dt\cdot dW_t=dW_t\cdot dt=0$ and $dW_t\cdot dW_t=dt$ (Øksendal, 2000).

Definition 2.3.2 *A Markov process X(t), $t\geq 0$ on a state space $S=\{(a,b), -\infty\leq a<b\leq\infty\}$ is said to be diffusion with drift coefficient $\mu(x,t)$ and diffusion coefficient $\sigma^2(x,t)>0$, if it has continuous sample path and the following relations hold for every $\varepsilon>0$:*

$$\mathrm{E}[(X_{s+t}-X_s)1_{|(X_{s+t}-X)|\leq\varepsilon} \mid X_s = x] = t\mu(x,t)+o(t) \tag{2.13}$$

$$\mathrm{E}[(X_{s+t}-X_s)^2 1_{|(X_{s+t}-X)|\leq\varepsilon} \mid X_s = x] = t\mu(x,t)+o(t) \tag{2.14}$$

$$P(|X_{s+t}-X_s|>\varepsilon \mid X_s = x) = o(t)] \tag{2.15}$$

as $t\longrightarrow 0^+$, where the functions $\mu(x,t)$ and $\sigma(x,t)$ are continuously differentiable, with bounded derivatives on S, σ'' exists and is continuous, and $\sigma^2(x,t)>0$ for all x (see Bhattacharya and Waymire, 2009, pg.368). The function o(t) is such that the ratio $\frac{o(t)}{t}$ approaches zero as t tends to zero.

Definition 2.3.3 *The infinitesimal generator A of $\{T_t:t>0\}$, or of the Markov process X(t), is the linear operator A defined by*

$$(Af)(x)=\lim_{s\to 0}\frac{(T_s f)(x)-f(x)}{s} \tag{2.16}$$

where

$$(T_s f)(x)=E[f(X(s))|X(0)=x]=\int f(y)p(s;x,dy) \qquad (s>0)$$

is a transition operator

Definition 2.3.4 *The family of the transition operators* $\{T_t:t>0\}$ *has a semigroup property*

$$T_{s+h}=T_s T_h \tag{2.17}$$

where the right side is a composition of two maps. This follows from

$$\begin{aligned}(T_{s+h})(x) &= E[f(X(s+h))|X(0)=x]\\ &= E[E(f(X(s+h))|X(s))|X(0)=x]\\ &= E[(T_h f(X_s)|X(0)=x]\\ &= T_s(T_h f)(x)\end{aligned} \tag{2.18}$$

Lemma 2.3.1 (Net Profit Condition) *Given model (1.1), Then the net profit condition is given by*

$$E\left[R(t)\right]=u+ct-\lambda tE[X]>0 \tag{2.19}$$

Proof. See Embrechts et al. (1997, pg. 25) □

Lemma 2.3.2 *The necessary condition for solvency of an insurance company is*

$$c-\lambda E[X]>0 \tag{2.20}$$

Proof. Given the net profit condition above, then

$$\begin{aligned}\lim_{t\to\infty}\frac{E\left[R(t)\right]}{t} &= \lim_{t\to\infty}\frac{u}{t}+c-\lambda E[X]\\ &= c-\lambda E[X]\end{aligned} \tag{2.21}$$

Thus on the long run $c-\lambda E[X]>0$ (see Constantinescu, 2006). □

Lemma 2.3.3 *Let X(t) be a right-continuous Markov process on a metric space S, and* $f\in D_A$. *Assume that* $s\longrightarrow(Af)\left(X(s)\right)$ *is right-continuous for all sample paths. Then the process Z(t), where*

$$Z(t)=f(X(t))-\int_0^t (Af)(X(s))ds \qquad (t\geq 0) \tag{2.22}$$

is a $\{F\}$*-martingale, with* $F=\sigma\{X(u):0\leq u\leq t\}$

Proof. See Bhattacharya and Waymire (2009, pg. 375). □

The following Lemma is from Theorem 2.1 of Paulsen and Gjessing (1997).

Lemma 2.3.4 *Let* $\tau_u = inf\{t:Y(t)<0\}$ *be the time of ruin where we set* $\tau_u = \infty$ *if* $Y(t) \geq 0$, *for all t.*

i) *Suppose g(u) is bounded and twice continuously differentiable on* $u \geq 0$ *with a bounded first derivative there, where g'(u) at u=0 implies the right-hand derivative. If g(u) is the solution of Ag(u)=0 on u>0, with the boundary conditions*

$$g(u) = 1 \quad on\ u<0$$

$$g(0) = 1 \quad if\ \sigma_p^2 > 0$$

$$\lim_{u\to\infty} g(u) = 0$$

then

$$g(u) = P(\tau_u < \infty)$$

ii) *Suppose* q_α *is bounded and twice continuously differentiable on* $u \geq 0$ *with a bounded first derivative there, where* $q_\alpha'(u)$ *at u=0 implies the right-hand derivative. If* $q_\alpha(u)$ *is the solution of* $Aq_\alpha(u) = \alpha q_\alpha(u)$ *on u>0, with the boundary conditions*

$$q_\alpha(u) = 1 \quad on\ u<0$$

$$q_\alpha(0) = 1 \quad if\ \sigma_p^2 > 0$$

$$\lim_{u\to\infty} q_\alpha(u) = 0$$

then

$$q_\alpha(u) = E(e^{-\alpha\tau u})$$

Proof. See Paulsen and Gjessing (1997, pg. 968). □

When α=0 in lemma part (*ii*) one obtains part (*i*). Thus part (*ii*) is a generalization.

Chapter 3

Methodology

3.1 Models Formulation

Consider model (1.11) equipped with assumptions – as described in section 1.4. If a company invests a fraction of its wealth, say "ϕ" in risky assets such as stock etc, and the return follow geometric Brownian motion. Then this can be modeled using the stochastic differential equation (Rolski et al., 1999)

$$d\left(\phi Y(t)\right)=a\phi Y(t)dt+\sigma_R\phi Y(t)dW \tag{3.1}$$

where

- a is the drift rate (which could be interpreted as the force of interest if the return is deterministic),
- σ_R is the volatility (which is the standard deviation of the returns from the investment),
- $W(t)$ is a standard Wiener process and
- ϕ is the fraction of the company asset invested in markets with stochastic return

Suppose the remaining fraction is invested in riskless bond or held in a savings account which earns interest at a constant rate r, then the resulting reserve will increase not only because the company receives premiums but also because the company earns return on investment. On the other hand, the decrease in reserve is brought about by the claims paid only. In a short time interval dt, the income will be

$$cdt+\sigma_p\varepsilon\, dt+rY(t)\left(1-\phi\right)dt+a\phi Y(t)dt+\sigma_R\phi Y(t)dW. \tag{3.2}$$

Since the time derivative of the standardized random error is the Wiener process, equation (3.2) can be expressed as

$$cdt+\sigma_p dW+rY(t)\left(1-\phi\right)dt+a\phi Y(t)dt+\sigma_R\phi Y(t)dW \tag{3.3}$$

Suppose the level of inflation $I(t)$ is deterministic and given by the solution of the equation (Paulsen, 1993).

$$\frac{dI(t)}{I(t)}=\delta dt; \qquad I(0)=1 \tag{3.4}$$

where δ is the inflation rate.

Subjecting the premiums in the interval dt to inflation gives $cI(t)dt$ (i.e. the company is expected to increase its premium to $cI(t)$ in a unit interval of time). The claims are jump processes since they do not occur continuously. Using the measure introduced by Delbaen and Haezendonck (1987), the inflated claims paid in the interval dt can be described as $X_{N_t} I(t)dN_t$. The change in the reserve is depicted in figure 3.1

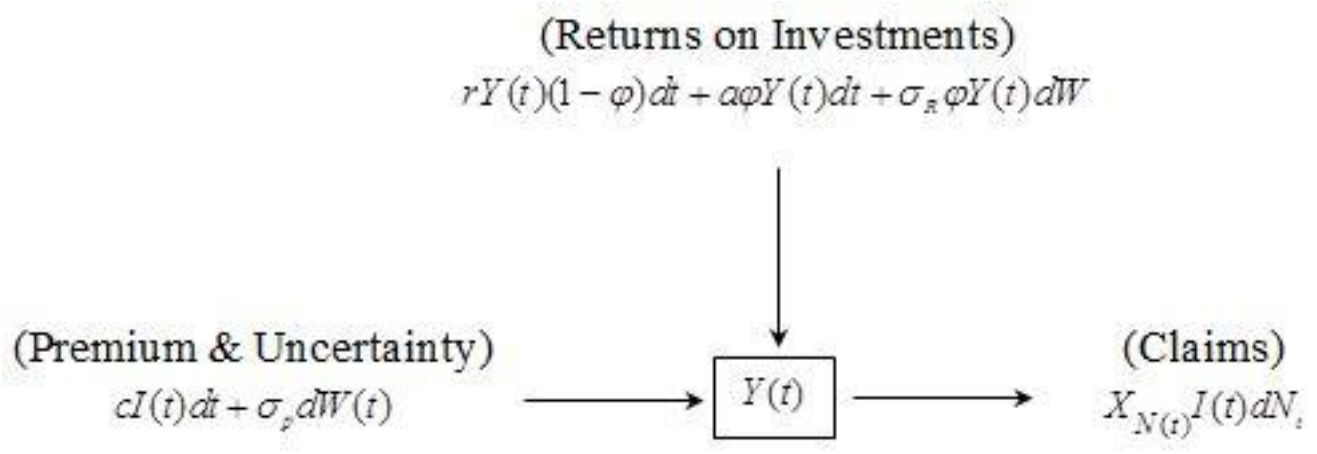


Figure 3.1: The Dynamics of Income and Claim Processes

The above can be represented mathematically as

$$dY(t)=cI(t)dt+\sigma_p dW(t)+rY(t)\,(1-\phi)dt+a\phi Y(t)dt+\sigma_R \phi Y(t)dW-X_{N_t} I(t)dN_t \quad (3.5)$$

The change in the incoming fund is given by the expression

$$cI(t)dt+\sigma_p dW(t)+r\,(1-\phi)Y(t)dt+a\phi Y(t)dt+\sigma_R \phi Y(t)dW(t) \quad (3.6)$$

Thus the total inflow of fund over the interval $[0,t]$ is

$$c\int_0^t I(s)ds+r\,(1-\phi)\int_0^t Y(s)ds+a\phi\int_0^t Y(s)ds+\int_0^t (\sigma_p+\sigma_R \phi Y(s))dW(s) \quad (3.7)$$

Similarly, the change in the outgoing fund is given by

$$X_{N_t} I(t)dN_t \quad (3.8)$$

and the total outflow of fund over the interval $[0,t]$ is given by

$$\sum_{i=1}^{N(t)} X_i I(T_i) \quad (3.9)$$

Thus the reserve process $\{Y(t), t\geq 0\}$ which is the net flow of fund is obtained as

$$Y(t)=u+c\int_0^t I(s)ds+r\,(1-\phi)\int_0^t Y(s)ds+a\phi\int_0^t Y(s)ds$$

$$+\int_0^t (\sigma_p+\sigma_R\phi Y(s))dW(s)-\sum_{i=1}^{N(t)} X_i I(T_i) \tag{3.10}$$

where u is the initial capital and the value of $I(t)$ is given by equation (3.4)

Model (3.1) is the implicit representation of the reserve of an insurance company that invests a fraction of its reserve in a risky asset which follows geometric Brownian motion, while the rest is either invested in a riskless bond or held in an account, with inflation taken into consideration. In other words, the model represents the worth of a company in a future time t, given that the company invests a fraction of its wealth in risky assets that follows geometric Brownian motion and that the premium and claims are inflated with time. The $Y(t)$ in the model is given in nominal units. Thus, the model gives the reserve of the company in money units but not in the real purchasing power. Since the price of the units of money never remains the same, rather it decays over time due to inflation. It is therefore necessary to obtain the real units equivalent of the model.

Theorem 3.1.1 *Suppose a company invests a fraction of its reserve in a risky asset which follows geometric Brownian motion, while the rest is either invested in a riskless bond or held in an account (with fixed interest rate), with inflation taken into consideration as given in the implicit representation expressed in nominal units in equation (3.1) shown below,*

$$\begin{aligned} Y(t)=u \quad &+c\int_0^t I(s)ds+r\,(1-\phi)\int_0^t Y(s)ds+a\phi\int_0^t Y(s)ds \\ &+\int_0^t (\sigma_p+\sigma_R\phi Y(s))dW(s)-\sum_{i=1}^{N(t)} X_i I(T_i) \end{aligned}$$

then the implicit representation of the reserve in real units is

$$\begin{aligned} \tilde{Y}(t)= \quad &e^{-\delta t}u+c\int_0^t e^{-\delta(t-s)}ds+r\,(1-\phi)e^{-\delta t}\int_0^t Y(s)ds \\ &+a\phi e^{-\delta t}\int_0^t Y(s)ds+e^{-\delta t}\int_0^t (\sigma_p+\sigma_R\phi Y(s))dW(s)-\sum_{i=1}^{N(t)} e^{-\delta(t-T_i)}X_i \end{aligned} \tag{3.11}$$

Proof. The reserve in terms of the real units at time t is given as (Paulsen, 1993)

$$\tilde{Y}(t)=e^{-\delta t}Y(t). \tag{3.12}$$

where δ is given as the inflation rate.

Thus, substituting equation (3.1) into equation (3.12) completes the proof. □

In Risk theory, it is necessary at times to obtain the value today of the reserve specified at the future time *t*. This is called the present value (discounted) risk reserve process.

Lemma 3.1.1 *Suppose the return from an investment is given as*

$$dY(t)=r(1-\phi)Y(t)dt+a\phi Y(t)dt-\sigma_R\phi Y(t)dW(t) \tag{3.13}$$

then, the discount factor is

$$f(t)=e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}; \tag{3.14}$$

where

$$-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t) \tag{3.15}$$

is the discount rate.

Proof. Equation (3.13) can be written as

$$\frac{dY(t)}{Y(t)}=[r+(a-r)\phi]dt+\sigma_R\phi dW(t) \tag{3.16}$$

Using Itô chain rule (Definition 2.3) on 3.16 gives

$$d(logY(t))=\left(r+(a-r)\phi-\frac{1}{2}\sigma^2\phi_R^2\right)dt+\sigma_R\phi dW(t) \tag{3.17}$$

replacing *t* by *z* and integrating from 0 to *t*, one obtains

$$logY(t)-log(Y(0))=\left(r+(a-r)\phi-\frac{1}{2}\sigma^2\phi_R^2\right)t+\sigma_R\phi W(t) \tag{3.18}$$

Consequently

$$\frac{Y(0)}{Y(t)}=e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)} \tag{3.19}$$

where $Y(0)$ is the present value and $Y(t)$ is the future value. Thus, substituting

$$f(t)=\frac{Y(0)}{Y(t)}$$

completes the proof. □

Theorem 3.1.2 *Suppose an insurance company's risk reserve process is represented by model (3.1), then the company's discounted risk reserve (Present value reserve) process at time t in nominal units is given by*

$$Z(t)= e^{\sigma_R^2\phi^2t}\left(u+c\int_0^t e^{-\left(r+(a-r)\phi+\frac{1}{2}\sigma^2\phi^2\right)s-\sigma\phi W(s)}ds\right.$$

$$+\sigma_p \int_0^t e^{-\left(r+(a-r)\phi+\frac{1}{2}\sigma_R^2\phi^2\right)s-\sigma_R\phi W(s)} dW(s) - \sum_{i=1}^{N(t)} X_i e^{-\left(r+(a-r)\phi+\frac{1}{2}\sigma^2\phi^2\right)t-\sigma\phi W(T_i)} \quad (3.20)$$

Proof. The discounted reserve (Present value reserve) is generally expressed as

$$Z(t)=Y(t)f(t) \quad (3.21)$$

where $Y(t)$ is the future reserve in nominal units and $f(t)$ is the discount factor. From Lemma , the discount factor is given in equation (3.14) as

$$f(t)=e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}$$

Consequently, the discounted reserve can be expressed as

$$Z(t)=Y(t)e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)} \quad (3.22)$$

Differentiating equation (3.22) above with respect to "t", one obtains

$$dZ(t)=e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)} dY(t)+Y(t)d\left(e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}\right) \quad (3.23)$$

Evaluating $d\left(e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}\right)$ using Itô chain rule (Definition 2.3) gives

$$\begin{aligned} d\left(e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)-\sigma_R\phi W(t)}\right) &= -\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dt \\ &\quad -\sigma_R\phi e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dW \\ &\quad +\frac{1}{2}\sigma_R^2\phi^2 e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dt \end{aligned} \quad (3.24)$$

That is,

$$\begin{aligned} d\left(e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}\right) &= \left(\sigma_R^2\phi^2+(a-r)\phi-r\right)e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dt \\ &\quad -\sigma_R\phi e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dW \end{aligned} \quad (3.25)$$

Consequently, equation (3.23) becomes

$dZ(t)=$

$$\begin{aligned} &e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)} dY(t)+Y(t)\left(\sigma_R^2\phi^2+(a-r)\phi-r\right)e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dt \\ &\quad -Y(t)\sigma_R\phi e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dW \end{aligned} \quad (3.26)$$

Recall from equation (3.5) that

$$dY(t)=cI(t)dt+\sigma_p dW+r(1-\phi)Y(t)dt+\left(a\phi Y(t)dt+\sigma_R\phi Y(t)dW(t)\right)-X_{N_t}I(t)dN_t$$

Substituting the equation above for $dY(t)$ in equation (3.26) and simplifying yields

$$dZ(t)=e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}cI(t)dt+\sigma_p e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dW$$

$$+\sigma_R^2\phi^2 Y(t)e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dt-X_{N_t}I(t)e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dN_t \quad (3.27)$$

But $I(t)=e^{\delta t}$ from equation 3.4, consequently

$$dZ(t)=ce^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dt+\sigma_p e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dW$$

$$+\sigma_R^2\phi^2 Y(t)e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dt-X_{N_t}e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dN_t \quad (3.28)$$

Substituting equation (3.22) into (3.28), one obtains

$$dZ(t)=ce^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dt$$

$$+\sigma_p e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dW+\sigma_R^2\phi^2 Z(t)dt-X_{N_t}e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dN_t \quad (3.29)$$

Equivalently, this can be expressed as

$$dZ(t)-\sigma_R^2\phi^2 Z(t)dt= ce^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dt$$

$$+\sigma_p e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dW-X_{N_t}e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dN_t \quad (3.30)$$

Using $e^{-\sigma_R^2\phi^2 t}$ as integrating factor gives

$$d\left(Z(t)e^{-\sigma_R^2\phi^2 t}\right)=e^{-\sigma_R^2\phi^2 t}(ce^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dt$$

$$+\sigma_p e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dW-X_{N_t}e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma_R^2\phi^2\right)t-\sigma_R\phi W(t)}dN_t) \quad (3.31)$$

Consequently, one obtains

$$Z(t)e^{-\sigma^2\phi^2 t}= Z(0)+c\int_0^t e^{-\left(r+(a-r)\phi+\frac{1}{2}\sigma^2\phi^2\right)s-\sigma\phi W(s)}ds$$

$$+\sigma_p\int_0^t e^{-\left(r+(a-r)\phi+\frac{1}{2}\sigma_R^2\phi^2\right)s-\sigma_R\phi W(s)}dW(s)-\sum_{i=1}^{N(t)}X_i e^{-\left(r+(a-r)\phi+\frac{1}{2}\sigma^2\phi^2\right)t-\sigma\phi W(T_i)} \quad (3.32)$$

Substituting u for $Z(0)$ and multiplying both sides by $e^{\sigma^2\phi^2 t}$ completes the proof. □

Theorem 3.1.3 *Suppose an insurance company's risk reserve process is represented by model (3.1), then the company's discounted risk reserve (Present value reserve) process at time t in real units is given by*

$$\tilde{Z}(t)= \quad e^{-(\delta-\sigma^2\phi^2)t}(u+c\int_0^t e^{-\left(r+(a-r)\phi+\frac{1}{2}\sigma^2\phi^2\right)s-\sigma\phi W(s)}ds$$

$$+\sigma_p\int_0^t e^{-\left(r+(a-r)\phi+\frac{1}{2}\sigma_R^2\phi^2\right)s-\sigma_R\phi W(s)}dW(s)-\sum_{i=1}^{N(t)}X_i e^{-\left(r+(a-r)\phi+\frac{1}{2}\sigma^2\phi^2\right)t-\sigma\phi W(T_i)}) \quad (3.33)$$

Proof. Since the real units equivalent of a reserve expressed in nominal units is obtained by adjusting for inflation. Recall(3.12)

$$\tilde{Y}(t)=e^{-\delta t}Y(t).$$

Thus, multiplying equation (3.20) by $e^{-\delta t}$ completes the proof. □

Theorem 3.1.4 *Suppose that a company's risk reserve process in nominal units is represented implicitly by model (3.1) shown below,*

$$Y(t)=u \quad +c\int_0^t I(s)ds+r(1-\phi)\int_0^t Y(s)ds+a\phi\int_0^t Y(s)ds$$

$$+\int_0^t(\sigma_p+\sigma_R\phi Y(s))dW(s)-\sum_{i=1}^{N(t)}X_i I(T_i)$$

then the explicit representation:

1. *in nominal units is given by*

$$Y(t)=e^{\left(r+(a-r)\phi+\frac{1}{2}\sigma^2\phi^2\right)t-\sigma\phi W(t)}(u+c\int_0^t e^{-\left(r+(a-r)\phi+\frac{1}{2}\sigma^2\phi^2\right)s-\sigma\phi W(s)}ds$$

$$+\sigma_p\int_0^t e^{-\left(r+(a-r)\phi+\frac{1}{2}\sigma_R^2\phi^2\right)s-\sigma_R\phi W(s)}dW(s)-\sum_{i=1}^{N(t)}X_i e^{-\left(r+(a-r)\phi+\frac{1}{2}\sigma^2\phi^2\right)t-\sigma\phi W(T_i)})$$

(3.34)

2. *in real units is given by*

$$\tilde{Y}(t)=e^{\left(r-\delta+(a-r)\phi+\frac{1}{2}\sigma^2\phi^2\right)t-\sigma\phi W(t)}(u+c\int_0^t e^{-\left(r+(a-r)\phi+\frac{1}{2}\sigma^2\phi^2\right)s-\sigma\phi W(s)}ds$$

$$+\sigma_p\int_0^t e^{-\left(r+(a-r)\phi+\frac{1}{2}\sigma_R^2\phi^2\right)s-\sigma_R\phi W(s)}dW(s)-\sum_{i=1}^{N(t)}X_i e^{-\left(r+(a-r)\phi+\frac{1}{2}\sigma^2\phi^2\right)t-\sigma\phi W(T_i)})$$

(3.35)

Proof. Recall from equation (3.22) in the proof of theorem on 40 that the discounted reserve for the implicit model in nominal units can be expressed as

$$Z(t)=Y(t)e^{-\left(r+(a-r)\phi-\frac{1}{2}\sigma^2\phi^2\right)t-\sigma\phi W(t)}$$

Recall also from theorem , on page 39 that the discounted reserve can equally be expressed as;

$$Z(t)= e^{\sigma_R^2\phi^2 t}(u+c\int_0^t e^{-\left(r+(a-r)\phi+\frac{1}{2}\sigma^2\phi^2\right)s-\sigma\phi W(s)}ds$$

$$+\sigma_p\int_0^t e^{-\left(r+(a-r)\phi+\frac{1}{2}\sigma_R^2\phi^2\right)s-\sigma_R\phi W(s)}dW(s)-\sum_{i=1}^{N(t)}X_i e^{-\left(r+(a-r)\phi+\frac{1}{2}\sigma^2\phi^2\right)t-\sigma\phi W(T_i)})$$

Substituting the later equation into the former equation and multiplying both sides by $e^{\left(r+(a-r)\phi-\frac{1}{2}\sigma^2\phi^2\right)t-\sigma\phi W(t)}$ completes the first part of the proof. To proof the second part, also recall from equation (3.12) in the proof of theorem on page 38 that

$$\tilde{Y}(t)=e^{-\delta t}Y(t).$$

Substituting equation (3.34) into the above equation completes the second part. □

Remark 3.1.1 *Equations (3.1) and (3.11) are the implicit representations of the risk model in nominal and real units respectively while equations (3.34) and (3.35) are the explicit representation of the risk model in nominal and real units respectively. The discounted risk model is given by (3.20) and (3.33) in nominal and real units respectively*

3.2 Special Cases

Equations (3.20),(3.33),(3.34) and (3.35) can be considered as mathematical generalizations of some of the existing models in literature. The models which can be considered as special cases of this are:

1. **Harrison (1977):** Suppose no wealth is invested into risky asset and there is no inflation. Then $\phi=0$ and $\delta=0$. Thus, if random variation in collective premiums and claims are not considered equation (3.34) can be written as:

$$Y(t)=e^{rt}u+c\int_0^t e^{r(t-s)}ds-\sum_{i=1}^{N(t)}X_i e^{r(t-T_i)} \tag{3.36}$$

   This is an explicit form of the expression obtained on pg. 67 of Harrison (1977).
2. **Delbaen and Haezendonck (1987):** Suppose no wealth is invested into risky asset, then $\phi=0$ and the expression in (3.20) reduces to

$$Z(t)=u+c\int_0^t e^{-(r-\delta)s}ds-\sum_{i=1}^{N(t)}X_i e^{-(r-\delta)T_i} \qquad (3.37)$$

where $r-\delta$ is the real interest rate. This is equation (1.4) obtained on pg. 86 of Delbaen and Haezendonck (1987). Thus, the model in Delbaen and Haezendonck (1987) expresses the discounted risk reserve in nominal units using the real interest rate.

3. **Paulsen (1993):** Suppose no wealth is investment into risky asset (that is $\phi=0$), then equation (3.35) gives.

$$\tilde{Y}(t)=ue^{(r-\delta)t}+c\int_0^t e^{(r-\delta)(t-s)}ds-\sum_{i=1}^{N(t)}X_i e^{(r-\delta)(t-T_i)} \qquad (3.38)$$

This the explicit form of the expression (1.5) obtained on pg. 328 of Paulsen (1993).

4. **Constantinescu (2006):** The equation (3.2) on pg. 26 of Constantinescu (2006) is also a specific case of the model where $\delta=0$ and the company invest all its wealth into risky asset (i.e. $\phi=1$). Thus substituting $\delta=0$ and $\phi=1$ into (3.1) which is the implicit form of (3.20) yields

$$Y(t)=u+ct+a\int_0^t Y(s)ds+\sigma_R\int_0^t Y(s)dW(s)-\sum_{i=1}^{N(t)}X_i \qquad (3.39)$$

Remark 3.2.1 *The following remarks are made:*

1. *The relationship between the real interest rate and nominal interest rate is obtained through the Fisher hypothesis which states that the nominal interest rates rise point-for-point with expected inflation, leaving the real rate unaffected (Fisher, 1930; Barsky, 1987). Real interest rate = Nominal interest units – Inflation*

2. *The event of ruin for both undiscounted risk reserve process and discounted risk reserve process is the same (Rolski et al., 1999).*

Theorem 3.2.1 *Given the risk reserve model (3.1)*

$$Y(t)=u+c\int_0^t I(s)ds+r(1-\phi)\int_0^t Y(s)ds+a\phi\int_0^t Y(s)ds+\int_0^t(\sigma_p-\sigma_R\phi Y(s))dW(s)+\sum_{i=1}^{N(t)}X_i I(T_i)$$

and (3.11)

$$\tilde{Y}(t)=e^{-\delta t}u+c\int_0^t e^{-\delta(t-s)}ds+r(1-\phi)e^{-\delta t}\int_0^t Y(s)ds$$

$$+a\phi e^{-\delta t}\int_0^t Y(s)ds+e^{-\delta t}\int_0^t(\sigma_p+\sigma_R\phi Y(s))dW(s)-\sum_{i=1}^{N(t)}e^{-\delta(t-T_i)}X_i$$

which are the implicit expressions of the model in nominal and real units respectively. The probabilities of ruin for both models are the same.

Proof. Recall (3.12) given below

$$\tilde{Y}(t)=e^{-\delta t}Y(t)$$

and the time of ruin for a reserve $Y(t)$ given in (1.4) as

$$\tau(u)=\inf\{t\geq:R(t)<0\}$$

Suppose $t>0$ and $Y(t)<0$ then $\tilde{Y}(t)<0$ and if $t=0$; $Y(0)=\tilde{Y}(0)=u$. Thus

$$\tau(u)=\inf\{t\geq 0:Y(t)<0\}\equiv\inf\{t\geq 0:\tilde{Y}(t)<0\} \tag{3.40}$$

The probability of ruin is defined in (1.1) as

$$\psi(u)=P\left(\tau(u)<\infty\right)=P\left(\inf_{t\geq 0}Y(t)<0\right)$$

Hence the ruin probability of both models are same □

Since the ruin probabilities of both models are same, model (3.1) would be considered for reasons of conveniences.

3.3 The Risk Models

In his famous Uppsala thesis of 1903, “Approximerad framslällning av sannolikhelsfunklionen. Ater-försäkring av kolleklivrisker”; Filip Lundberg laid the foundation of actuarial risk theory, where it was discovered that Poisson processes lie at the heart of non-life insurance models. Subsequently in 1930, Harald Cramér and his Stockholm School incorporated Lundberg’s ideas into the emerging theory of stochastic processes (Embrechts et al., 1997). These early works produced the classical risk model referred to in the sequel as the Cramér-Lundberg model. The structure of the model is described as follows:

Consider the risk reserve process (1.11) given by

$$R(t)=u+\pi(t)-\sum_{i=1}^{N(t)}X_i+\varepsilon(t)$$

where the variables are as previously defined.

Assumption 3.3.1 *Suppose the reserve satisfies the following conditions:*

1. ***The claim arrival process:*** *the number of claims arriving in the interval [0,t] denoted by N(t) is*

   $$\sup\{n\geq 1:T_n\leq t\};:t\geq 0$$

   where, by convention, $\sup\emptyset=0$
2. ***The claim times:*** *the claims occurs at the random instants of time*

$$0<T_1<T_2<\ldots \quad a.s.$$

3. ***The inter-arrival times:*** *the time between the arrival epoch*

$$\gamma_1=T_1,\gamma_i=T_i-T_{i-1} \quad i=2,3,\ldots$$

are iid exponentially distributed with finite mean $E[\gamma_i]=\frac{1}{\lambda}$

4. ***The claim size process:*** *the claim sizes* $(X_i)_{i\in N}$ *are positive iid random variables with common distribution function, finite mean* $\mu_c=E[X_i]$*, and variance* $\sigma_c^2=Var(X_1)<\infty$

5. *The sequences* (X_i) *and* (γ_i) *are independent of each other.*

From the above, it's obvious that Cramér-Lundberg model is a compound Poisson process. Since the basic model from which (3.1) is developed is model (1.1), axioms 1, 2, 4 and 5 is certified by default. Equipping model (3.1) with axiom 3 makes the model a classical risk model.

3.3.1 Integro-differential Equation

The derivation of the integro-differential equation for model (3.1) equipped with assumption 3.3 involves the determination of the infinitesimal generator A [1] and usage of martingale property. To do this, the model is re-written as

$$Y(t) = U(t)+\int_0^t \left(ce^{\delta s}+r(1-\phi)Y(s)+a\phi Y(s)\right)ds+(\sigma_p+\sigma_R\phi\int_0^t Y(s))dW(s) \tag{3.41}$$

$$U(t) = u-\sum_{i=1}^{N(t)}\bar{X}_i \qquad where \quad \bar{X}_i=e^{-\delta T_i}X_i \tag{3.42}$$

The process $U(t)$ is clearly a jump process with the claim process constituting its stochastic part. Between the jumps, $U(t)$ is deterministic.

Using the notation analogous to differential equation, equation (3.3.1) can be represented as

$$dY(t)=dU(t)+\left(ce^{\delta s}+r(1-\phi)Y(s)+a\phi Y(s)\right)ds+(\sigma_p+\sigma_R\phi Y(s))dW(s) \tag{3.43}$$

Between the instance when the asset is invested, and when the next premium and claim arrives, there is no change in $U(t)$, hence $dU(t)=0$. Clearly, equation (3.3.1) comprises two stochastic parts;

1. The income process

$$dY(t)=\left(ce^{\delta s}+r(1-\phi)Y(s)+a\phi Y(s)\right)ds+(\sigma_p+\sigma_R\phi Y(s))dW(s) \tag{3.44}$$

2. The jump process produced by the claims $U(t)$ as given in equation (3.42)

Thus, the infinitesimal generator A of $Y(t)$ is the sum of the infinitesimal generators of these two parts.

The income process (3.44) is clearly a diffusion process with drift coefficient

[1] See definition in page dfn_infinitesimalGen

$$ce^{\delta s}+r(1-\phi)Y(s)+a\phi Y(s)$$

and diffusion coefficient

$$\sigma_p^2+(\sigma_R\phi Y(s))^2$$

Thus equation (3.44) satisfy the property of diffusion process stated in definition .

Lemma 3.3.1 *Suppose the jump process U(t) is defined by equation (3.42);*

$$U(t)=u-\sum_{i=1}^{N(t)}X_i$$

Let g be a bounded twice continuously differentiable function, then

$$lim_{t\to 0}\frac{E[g(U(t))|U(0)=u]-g(u)}{t}=-\lambda g(u)+\lambda\int_0^\infty g(u-x)dF(x) \tag{3.45}$$

Proof. Substituting equation (3.42) into the expectation gives

$$E[g(U(t))|U(0)=u]=E\left[g(u-\sum_{i=1}^{N(t)}X_i)\right] \tag{3.46}$$

But $N(t)$ is Poisson distributed, thus one obtains

$$E[g(U(t))|U(0)=u]=\sum_{n=0}^{\infty}E\left[g(u-\sum_{i=1}^{n}X_i)|N(t)=n\right]\frac{(\lambda t)^n}{n!}e^{-\lambda t} \tag{3.47}$$

Expanding the summand, one obtains

$$\begin{aligned}E[g(U(t))|U(0)=u]= \quad & E[g(u)]e^{-\lambda t}+E\left[g(u-X_i)|N(t)=1\right]\lambda te^{-\lambda t}\\ & +E\left[g(u-\sum_{i=2}^{N(t)}X_i)|N(t)\geq 2\right]o(t)\end{aligned} \tag{3.48}$$

Subtracting $g(u)$ from both sides and dividing by t, one obtains;

$$\begin{aligned}\frac{E[g(U(t))|U(0)=u]-g(u)}{t}= \quad & \frac{E[g(u)]e^{-\lambda t}-g(u)}{t}\\ & +E\left[g(u-X_i)|N(t)=1\right]\lambda e^{-\lambda t}\\ & +E\left[g(u-\sum_{i=2}^{N(t)}X_i)|N(t)\geq 2\right]\frac{o(t)}{t}\end{aligned} \tag{3.49}$$

Expanding equation (3.49) further, gives

$$\begin{aligned}\frac{E[g(U(t))|U(0)=u]-g(u)}{t}= \quad & \frac{\left(1-\lambda t+\frac{(\lambda t)^2}{2}-\ldots\right)E[g(u)]-g(u)}{t}\\ & +\lambda\left(1-\lambda t+\frac{(\lambda t)^2}{2}-\ldots\right)E\left[g(u-X_i)|N(t)=1\right]\end{aligned}$$

$$+E\left[g(u-\sum_{i=2}^{N(t)}X_i)|N(t)\geq 2\right]\frac{o(t)}{t} \tag{3.50}$$

Taking the limits at $t\rightarrow 0$ completes the proof. □

The infinitesimal generator of the jump process given in equation (3.42) can be deduced from the lemma above. The theorem below gives the infinitesimal generator of model (3.1).

Theorem 3.3.1 *Let g be a bounded thrice continuously differentiable function of Y(t). If Y(t) is a diffusion process with drift coefficient* $ce^{\delta s}+r(1-\phi)Y(t)+a\phi Y(t)$ *and diffusion coefficient* $\left(\sigma_p+\sigma_R\phi Y(t)\right)^2$ *defined on S, then,*

$$(Af)(u)=\frac{1}{2}(\sigma_p^2+\left(\sigma_R\phi\right)^2u^2)g''(u)+\left(c+r(1-\phi)u+a\phi u\right)\qquad g'(u)-\lambda g(u)+\lambda\int_0^{\infty}g(u-x)dF(x) \tag{3.51}$$

Proof. [2], Suppose $Y(s)=y\in S$ is fixed. Let $\eta>0$ be an arbitrary small number. Since g is continuous, there exist $\varepsilon>0$ such that $|z-y|\leq\varepsilon$ whenever $|g''(z)-g''(y)|\leq\eta$ for all z. Thus

$$(T_tg)(y)=E[g(Y(t))1_{|Y(t)-Y(s)|\leq\varepsilon}|Y(s)=y]+E[g(Y(t))1_{|Y(t)-Y(s)|>\varepsilon}|Y(s)=y] \tag{3.52}$$

where $0\leq s<t$, $1_{|Y(t)-Y(s)|\leq\varepsilon}$ is an indicator function and $(T_tg)(y)$ is a transition operator[3]. Expanding the first term on the RHS of equation (3.52) into Taylor's series around y, one obtains;

$$(T_tg)(y)=E[(g(y)+(Y(t)-y)g'(y)+\frac{1}{2}(Y(t)-y)^2g''(y)+\frac{1}{6}(Y(t)-y)^3g'''(\zeta_t))$$
$$\times 1_{|Y(t)-Y(s)|\leq\varepsilon}|Y(s)=y]+E[g(Y(t))1_{|Y(t)-Y(s)|>\varepsilon}|Y(s)=y] \tag{3.53}$$

where $\frac{1}{6}(Y(t)-y)^3g'''(\xi_t)$ is the remainder term and ξ_t is some number between y and $Y(t)$.

Expanding equation (3.53) over the expectation gives

$$\begin{aligned}(T_tg)(y)= \quad & E[g(y)1_{|Y(t)-Y(s)|\leq\varepsilon}|Y(s)=y]+E[(Y(t)-y)g'(y)1_{|Y(t)-Y(s)|\leq\varepsilon}|Y(s)=y]\\ & +\frac{1}{2}E[(Y(t)-y)^2g''(y)1_{|Y(t)-Y(s)|\leq\varepsilon}|Y(s)=y]\\ & +o(t)+E[g(Y(t))1_{|Y(t)-Y(s)|>\varepsilon}|Y(s)=y]\end{aligned} \tag{3.54}$$

Using the properties of diffusion processes in definition , one obtains

$$(T_tg)(y)= \quad E[g(y)1_{|Y(t)-Y(s)|\leq\varepsilon}|Y(s)=y]+(ce^{\delta s}+r(1-\phi)Y(s)+a\phi Y(s))tg'(y)$$

[2] The proof of theorem follows steps similar to that of (Constantinescu, 2003)

[3] see definition

$$+\frac{1}{2}(\sigma_p^2+(\sigma_R\phi Y(s))^2)tg''(y)+o(t) \tag{3.55}$$

Using the semi-group property in definition , one obtains

$$(T_tg)(u)= \quad E[(E[g(y)1_{|Y(t)-Y(s)|\le\varepsilon}|Y(s)=y]+(ce^{\delta s}+r(1-\phi)Y(s)+a\phi Y(s))tg'(y)$$

$$+\frac{1}{2}(\sigma_p^2+(\sigma_R\phi Y(s))^2)tg''(y)+o(t))+o(t))|Y(0)=u] \tag{3.56}$$

Expanding over the outer expectation gives

$$\begin{aligned}(T_tg)(u)= \quad & E[E[g(y)1_{|Y(t)-Y(s)|\le\varepsilon}|Y(s)=y]|Y(0)=u] \\ & +E[(ce^{\delta s}+r(1-\phi)Y(s)+a\phi Y(s))tg'(y)|Y(0)=u] \\ & +\frac{1}{2}E[((\sigma_p^2+(\sigma_R\phi Y(s))^2)tg''(y)+o(t))|Y(0)=u]+o(t)\end{aligned} \tag{3.57}$$

Taking the limit as $t\to 0$, after some computation, one obtains

$$\begin{aligned}\lim_{t\to0}\frac{(T_tg)(u)-g(u)}{t}= \quad & \lim_{t\to0}\frac{E[E[g(y)1_{|Y(t)-Y(s)|\le\varepsilon}|Y(s)=y]|Y(0)=u]-g(u)}{t} \\ & +\lim_{t\to0}E[(ce^{\delta s}+r(1-\phi)Y(s)+a\phi Y(s))g'(y)|Y(0)=u] \\ & +\lim_{t\to0}\frac{1}{2}E[((\sigma_p^2+(\sigma_R\phi Y(s))^2)g''(y)+\frac{o(t)}{t})|Y(0)=u]+\lim_{t\to0}\frac{o(t)}{t}\end{aligned} \tag{3.58}$$

But $\lim_{t\to0}\frac{o(t)}{t}=0$, $Y(0)=U(0)=u$ and $t\to0$ implies $s\to0$, therefore

$$\begin{aligned}\lim_{t\to0}\frac{(T_tg)(u)-g(u)}{t}= \quad & \lim_{t\to0}\frac{E[E[g(U)|U(0)=u]-g(u)]}{t}+(c+r(1-\phi)u+a\phi u)g'(u) \\ & +\frac{1}{2}(\sigma_p^2+(\sigma_R\phi u)^2)g''(u)\end{aligned} \tag{3.59}$$

Using lemma on page 47, one obtains

$$\begin{aligned}\lim_{t\to0}\frac{(T_tg)(u)-g(u)}{t}= & \frac{1}{2}(\sigma_p^2+(\sigma_R\phi u)^2)g''(u)+(c+r(1-\phi)u+a\phi u)g'(u)-\lambda g(u) \\ & +\lambda\int_0^\infty g(u-x)dF(x)\end{aligned} \tag{3.60}$$

But, from definition that infinitesimal generator is defined as,

$$(Af)(u)=\lim_{t\to0}\frac{(T_tf)(u)-f(u)}{t}$$

Substituting the equation above into equation (3.60) completes the proof. □

Theorem 3.3.2 *Suppose {Y(t),t≥0} is a risk reserve process defined by model (3.1) as given below;*

$$Y(t)=u \quad +c\int_0^t I(s)ds+r(1-\phi)\int_0^t Y(s)ds+a\phi\int_0^t Y(s)ds$$

$$+\int_0^t (\sigma_p+\sigma_R\phi Y(s))dW(s)+\sum_{i=1}^{N(t)} X_i I(T_i)$$

and satisfying assumption 3.3. Then, the integro-differential equation for the model is given by

$$\frac{1}{2}(\sigma_p^2+\left(\sigma_R\phi\right)^2 u^2)g''(u)+\left(c+r\left(1-\phi\right)u+a\phi u\right)g'(u)-\lambda g(u)+\lambda\int_0^\infty g(u-x)dF(x)=0 \qquad (3.61)$$

where $I(s)=e^{\delta s}$ is the inflation factor defined by equation (3.4)

Proof. $\{Y(t)\}$ is right continuous Markov process defined on a metric $(-\infty,\infty)$. Using lemma , define a martingale $Z(t)$ by

$$Z(t)=f(Y(t))-\int_0^t (Af)(Y(s))ds \qquad (t\geq 0) \qquad (3.62)$$

Since $f(Y(t))$ is a martingale, $(Af)(Y(s))=0$ completes the proof. □

3.3.2 Ruin Probability

According to part (*i*) of Lemma , the function $g(u)$ is the probability of ruin for the model (3.1). Thus the integro-differential equation (3.61) can be written as

$$\left(c+r\left(1-\phi\right)u+a\phi u\right)\psi'(u)+\frac{1}{2}(\sigma_p^2+\left(\sigma_R\phi\right)^2 u^2)\psi''(u)-\lambda\psi(u)+\lambda\int_0^\infty \psi(u-x)dF(x)=0 \qquad (3.63)$$

where ψ is the ruin probability defined in equation (1.1). A more generalized equation from part (*ii*) of Lemma is given in equation 3.64:

$$\left(c+r\left(1-\phi\right)u+a\phi u\right)q_\alpha'(u)+\frac{1}{2}(\sigma_p^2+\left(\sigma_R\phi\right)^2 u^2)q_\alpha''(u)+\lambda\int_0^\infty (q_\alpha(u-x)-q_\alpha(u))dF(x)=\alpha q_\alpha(u) \qquad (3.64)$$

where $q_\alpha=E(e^{-\alpha\tau}u)$, with the boundary conditions

$$q_\alpha(u)= \quad 1 \quad on\ u<0$$

$$\lim_{u\to\infty} q_\alpha(u)= \quad 0$$

Equation (3.63) and (3.64) are Volterra integro-differential equations of the second kind.[4]

The integro-differential equations can be evaluated by either eliminating the terms with derivatives, thereby converting the equation into volterra integral equation or by eliminating the integral to convert the equation into differential equation. The former method results into equations with non-difference kernel which can only be solved through numerical procedures. Thus, the later method is adopted.

Two cases are considered depending on the claim size distibution as listed below

[4] see Volterra integro-differential equation in appendix

- Claim size is distributed as exponential
- Claim size is distributed as a mixture of two exponential.

Equation (3.64) is considered in all cases, since setting α=0 results in (3.63). The cases are considered below.

Claim Size Distributed as Exponential

Suppose the claim size is exponentially distributed, with density function $f(x)=\Theta e^{-\Theta x}$, where $x>0$ and $\Theta>0$, then the integro-differential equation (3.64) is converted into differential equation in the following theorem.

Theorem 3.3.3 *Given that the claim size distribution f(x) is exponentially distributed with mean 1/Θ, then the integro-differential equation (3.64) is equivalent to the third order differential equation.*

$$\frac{1}{2}(\sigma_p^2+(\sigma_R\phi)^2u^2)q_\alpha'''(u)+(c+[(\sigma_R\phi)^2+\bar{r}]u+\frac{1}{2}\Theta(\sigma_p^2+(\sigma_R\phi)^2u^2))q_\alpha''(u)$$
$$+(\bar{r}-\lambda-\alpha+c\Theta+\bar{r}\Theta u)q_\alpha'(u)-\alpha\Theta q(u)=0 \qquad (3.65)$$

where $\bar{r}=r(1-\phi)+a\phi$.

Proof. Recall equation (3.64) below,

$$(c+r(1-\phi)u+a\phi u)q_\alpha'(u)+\frac{1}{2}(\sigma_p^2+(\sigma_R\phi)^2u^2)q_\alpha''(u)+\lambda\int_0^\infty(q_\alpha(u-x)-q_\alpha(u))dF(x)=\alpha q_\alpha(u)$$

Since $f(x)$ is exponentially distributed, then

$$dF(x)=\Theta e^{-\Theta x}dx$$

thus, equation (3.64) becomes

$$(c+r(1-\phi)u+a\phi u)q_\alpha'(u)+\frac{1}{2}(\sigma_p^2+(\sigma_R\phi)^2u^2)q_\alpha''(u)=-\lambda\Theta\int_0^\infty(q_\alpha(u-x)-q_\alpha(u))e^{-\Theta x}dx+\alpha q_\alpha(u) \qquad (3.66)$$

Simplifying and rearranging equation (3.66), one obtains

$$\frac{1}{2}(\sigma_p^2+(\sigma_R\phi)^2u^2)q_\alpha''(u)+(c+r(1-\phi)u+a\phi u)q_\alpha'(u)-(\alpha)q_\alpha(u)=\lambda q_\alpha(u)-\lambda\Theta e^{-\Theta u}\int_0^u q_\alpha(x)e^{\Theta x}dx \qquad (3.67)$$

Differentiating both sides of (3.67) and rearranging gives

$$\frac{1}{2}(\sigma_p^2+(\sigma_R\phi)^2u^2)q_\alpha'''(u)+\left(c+[(\sigma_R\phi)^2+r(1-\phi)+a\phi]u\right)q_\alpha''(u)$$
$$+(r(1-\phi)+a\phi-\lambda-\alpha)q_\alpha'(u) \quad =-\Theta(\lambda q_\alpha(u)-\lambda\Theta e^{-\Theta u}\int_0^u q_\alpha(x)e^{\Theta x}dx) \qquad (3.68)$$

Substituting equation (3.67) into (3.68) one obtains

$$\frac{1}{2}(\sigma_p^2+(\sigma_R\phi)^2 u^2)q_\alpha'''(u)+\left(c+[(\sigma_R\phi)^2+r(1-\phi)+a\phi]u+\frac{\Theta}{2}(\sigma_p^2+(\sigma_R\phi)^2u^2)\right)q_\alpha''(u)$$

$$+(r(1-\phi)+a\phi-\lambda-\alpha+c\Theta+[r(1-\phi)+a\phi]\Theta u)q_\alpha'(u)-\alpha\Theta q(u)=0 \quad (3.69)$$

substituting $\bar{r}=r(1-\phi)+a\phi$ completes the proof. □

Equation (3.65) above is a complex equation which is very difficult to solve. Since the equation takes the same structure as equation (2.11) of Paulsen and Gjessing (1997), a similar approach to that of Paulsen and Gjessing (1997) is used to resolve the equation.

The boundary condition is very important to solving differential equations. According to lemma , the boundary condition and boundedness assumption is sufficient to determine a unique solution of equation (3.64) if it exist. But since the equation has been differentiated to eliminate the integral operator, it is reasonable to determine new sets of conditions which preserve the original information. Thus, using the original boundary conditions as specified in the lemma and letting $y\longmapsto 0$ in (3.64), we have

$$\sigma_p^2 q_\alpha''(0)+2cq_\alpha'(0)-2\alpha q_\alpha(0)=0 \qquad when\sigma_p^2>0 \quad (3.70)$$

and

$$cq_\alpha'(0)-\alpha q_\alpha(0)=0 \qquad when\sigma_p=0 \quad (3.71)$$

As pointed above, equation (3.69) is very difficult to solve, thus we simplify the equation to some specific cases by setting some parameters to zero. Consider the case when $\sigma_R=0$, the equation becomes

$$\frac{1}{2}\sigma_p^2 q_\alpha'''(u)+\left(\frac{\Theta}{2}\sigma_p^2+c+\bar{r}u\right)q_\alpha''(u)+(\bar{r}-\lambda-\alpha+c\Theta+\bar{r}\Theta u)q_\alpha'(u)-\alpha\Theta q(u)=0 \quad (3.72)$$

with boundary conditions given in lemma , equations (3.70) and (3.71). To overcome the problem of having to solve the complex differential equation (3.69), it is noted that ruin can either be caused by the claims or by oscillations. Thus, the equation is decomposed into two differential equations. Firstly, we set $\sigma_p=0$ to get the equation for ruin caused by claims, given as

$$(c+\bar{r}u)q_\alpha''(u)+(\bar{r}-\lambda-\alpha+c\Theta+\bar{r}\Theta u)q_\alpha'(u)-\alpha\Theta q(u)=0 \quad (3.73)$$

where

$$q_\alpha(u)= \quad 1 \quad on\ u<0$$

$$\lim_{t\to\infty} q_\alpha(u)= \quad 0$$

$$cq_\alpha''(0)+(\bar{r}-\lambda-\alpha+c\Theta)q_\alpha'(0)= \quad \alpha\Theta$$

To determine the equation for ruin due to oscillations, we set the rate of arrival of claims $\lambda=0$

$$\frac{1}{2}\sigma_p^2 q_\alpha'''(u)+\left(\frac{\Theta}{2}\sigma_p^2+c+\bar{r}u\right)q_\alpha''(u)+(\bar{r}-\alpha+c\Theta+\bar{r}\Theta u)q_\alpha'(u)-\alpha\Theta q(u)=0 \tag{3.74}$$

with the boundary conditions

$$\begin{aligned} q_\alpha(u) &= 1 \quad on\ u<0 \\ \lim_{t\to\infty} q_\alpha(u) &= 0 \\ cq_\alpha''(0)+(\bar{r}-\alpha)q_\alpha'(0) &= 0 \end{aligned}$$

Equation (3.73) can be reduced to hypergeometic equation[5] using appropriate transformation. We utilize the same transformation used by Paulsen and Gjessing (1997). Let

$$u=-x/\Theta-c/\bar{r} \quad and \quad v(x)=q_\alpha(u) \quad then$$

$$\begin{aligned} q_\alpha'(u) &= -\Theta v'(x) \\ q_\alpha''(u) &= \Theta^2 v''(x) \end{aligned} \tag{3.75}$$

Substituting the transformation into equation (3.73), gives

$$xv''(x)+(1-\frac{\lambda+\alpha}{\bar{r}}-x)v'(x)-(-\frac{\alpha}{\bar{r}})v(x)=0 \tag{3.76}$$

Equation (3.76) is a confluence hypergeometric equation, hence by Slater (1960),

$$\begin{aligned} v_0(x) &= x^{1-b}e^x M(1-a,2-b;-x) \\ v_1(x) &= e^x U(b-a,b;-x) \end{aligned} \tag{3.77}$$

are solutions of equation (3.76) where $a=-\alpha/\bar{r}$, $b=1-(\lambda+\alpha)/\bar{r}$, $M(1-a,2-b;-x)$ is a confluent hypergeometric function and $U(b-a,b;-x)$ is the second form of confluent hypergeometric function. Thus, we have,

$$\begin{aligned} v_0(x) &= x^{(\lambda+\alpha)/\bar{r}}e^x M(1+\frac{\alpha}{\bar{r}},1+\frac{\lambda+\alpha}{\bar{r}};-x) \\ v_1(x) &= e^x U(1-\frac{\lambda}{\bar{r}},1-\frac{\lambda+\alpha}{\bar{r}};-x) \end{aligned} \tag{3.78}$$

Furthermore, $v_0(x)$ and $v_1(x)$ are two linearly independent solutions. Following the same rational as in Cai and Yang (2005), it can be shown that if $v_0(x)$ is a solution of equation (3.76) which is linearly independent of $v_1(x)$, then so is

$$v_2(x)=-v_0(x)=-x^{(\lambda+\alpha)/\bar{r}}e^x M(1+\frac{\alpha}{\bar{r}},1+\frac{\lambda+\alpha}{\bar{r}};-x) \tag{3.79}$$

Thus, the general solution of (3.76) is

$$v(x)=C_1 v_1(x)+C_2 v_2(x)$$

[5] Hypergeometric equation (See Appendix)

$$=C_1 e^{x} U(1-\frac{\lambda}{\bar{r}},1-\frac{\lambda+\alpha}{\bar{r}};-x)-C_2 x^{(\lambda+\alpha)/\bar{r}} e^{x} M(1+\frac{\alpha}{\bar{r}},1+\frac{\lambda+\alpha}{\bar{r}};-x) \tag{3.80}$$

where C_1 and C_2 are constant. Re-applying the transformation above gives the solution of equation (3.73) as

$$q_{\alpha}(u)=C_1 e^{-(u\bar{r}+c)\Theta/\bar{r}} U(1-\frac{\lambda}{\bar{r}},1-\frac{\lambda+\alpha}{\bar{r}};\frac{(u\bar{r}+c)\Theta}{\bar{r}})$$
$$+C_2(\frac{(u\bar{r}+c)\Theta}{\bar{r}})^{(\lambda+\alpha)/\bar{r}} e^{-(u\bar{r}+c)\Theta/\bar{r}} M(1+\frac{\alpha}{\bar{r}},1+\frac{\lambda+\alpha}{\bar{r}};\frac{(u\bar{r}+c)\Theta}{\bar{r}}) \tag{3.81}$$

To determine the constants C_1 and C_2, we substitute the boundary condition to get the following two equations

$$C_1 U(1-\frac{\lambda}{\bar{r}},1-\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}})+C_2(\frac{c\Theta}{\bar{r}})^{(\lambda+\alpha)/\bar{r}} M(1+\frac{\alpha}{\bar{r}},1+\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}})=e^{-c\Theta/\bar{r}} \tag{3.82}$$

and

$$C_1(cH_{21}+(r-\lambda-\alpha-c\Theta)H_{11})+C_2(cH_{22}+(r-\lambda-\alpha-c\Theta)H_{12})=\alpha\Theta \tag{3.83}$$

where

$$H_{11}=-\Theta e^{-c\Theta/\bar{r}} U(1-\frac{\lambda}{\bar{r}},1-\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}})-\Theta(1-\frac{\lambda}{\bar{r}})e^{-c\Theta/\bar{r}} U(2-\frac{\lambda}{\bar{r}},2-\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}}), \tag{3.84}$$

$$H_{12}=\Theta(\frac{\lambda+\alpha}{r})(\frac{c\Theta}{\bar{r}})^{(\lambda+\alpha)/r-1} e^{-c\Theta/\bar{r}} M(1+\frac{\alpha}{\bar{r}},1+\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}})-\Theta(\frac{c\Theta}{\bar{r}})^{(\lambda+\alpha)/r} e^{-c\Theta/\bar{r}} M(1+\frac{\alpha}{\bar{r}},1+\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}})$$
$$+\Theta(\frac{c\Theta}{r})^{(\lambda+\alpha)/\bar{r}}(\frac{\lambda+\alpha+r}{\alpha+r}) e^{-c\Theta/\bar{r}} M(2+\frac{\alpha}{\bar{r}},2+\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}}) \tag{3.85}$$

$$H_{21}=\Theta^2 e^{-c\Theta/\bar{r}} U(1-\frac{\lambda}{\bar{r}},1-\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}})+2\Theta^2(1-\frac{\lambda}{\bar{r}})e^{-c\Theta/\bar{r}} U(2-\frac{\lambda}{\bar{r}},2-\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}})$$
$$-\Theta^2(1-\frac{\lambda}{\bar{r}})(2-\frac{\lambda}{\bar{r}})e^{-c\Theta/\bar{r}} U(3-\frac{\lambda}{\bar{r}},3-\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}}) \tag{3.86}$$

$$H_{22}=\Theta^2(\frac{\lambda+\alpha}{r})(\frac{\lambda+\alpha}{r}-1)(\frac{c\Theta}{\bar{r}})^{(\lambda+\alpha)/r-2} e^{-c\Theta/\bar{r}} M(1+\frac{\alpha}{\bar{r}},1+\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}})$$
$$-2\Theta^2(\frac{\lambda+\alpha}{r})(\frac{c\Theta}{\bar{r}})^{(\lambda+\alpha)/r-1} e^{-c\Theta/\bar{r}} M(1+\frac{\alpha}{\bar{r}},1+\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}})$$
$$+2\Theta^2(\frac{\lambda+\alpha}{r})(\frac{c\Theta}{\bar{r}})^{(\lambda+\alpha)/r-1}(\frac{\lambda+\alpha+r}{\alpha+r}) e^{-c\Theta/\bar{r}} M(2+\frac{\alpha}{\bar{r}},2+\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}})$$
$$+\Theta^2(\frac{c\Theta}{\bar{r}})^{(\lambda+\alpha)/r} e^{-c\Theta/\bar{r}} M(1+\frac{\alpha}{\bar{r}},1+\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}})$$
$$-2\Theta^2(\frac{c\Theta}{r})^{(\lambda+\alpha)/\bar{r}}(\frac{\lambda+\alpha+r}{\alpha+r}) e^{-c\Theta/\bar{r}} M(2+\frac{\alpha}{\bar{r}},2+\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}}))$$
$$+\Theta^2(\frac{c\Theta}{r})^{(\lambda+\alpha)/\bar{r}}(\frac{\lambda+\alpha+r}{\alpha+r})(\frac{\lambda+\alpha+2r}{\alpha+2r}) e^{-c\Theta/\bar{r}} M(3+\frac{\alpha}{\bar{r}},3+\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}})) \tag{3.87}$$

Equations (3.82) and (3.83) are then solved simultaneously to give

$$C_2=\frac{\alpha\Theta U(1-\frac{\lambda}{\bar{r}},1-\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}})-(cH_{21}+(r-\lambda-\alpha-c\Theta)H_{11})e^{-c\Theta/\bar{r}}}{K}$$

$$C_1=\frac{(cH_{22}+(r-\lambda-\alpha-c\Theta)H_{12})e^{-c\Theta/\bar{r}}-\alpha\Theta M(1+\frac{\alpha}{\bar{r}},1+\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}})}{K} \quad (3.88)$$

where

$$K=(cH_{22}+(r-\lambda-\alpha-c\Theta)H_{12})U(1-\frac{\lambda}{\bar{r}},1-\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}})$$
$$-(cH_{21}+(r-\lambda-\alpha-c\Theta)H_{11})M(1+\frac{\alpha}{\bar{r}},1+\frac{\lambda+\alpha}{\bar{r}};\frac{c\Theta}{\bar{r}})$$

$,H_{11},H_{12},H_{21}$ and H_{22} are as defined above.

Recall the equation for oscillation (3.90) as

$$\frac{1}{2}\sigma_p^2 q_\alpha'''(u)+\left(\frac{\Theta}{2}\sigma_p^2+c+\bar{r}u\right)q_\alpha''(u)+(\bar{r}-\alpha+c\Theta+\bar{r}\Theta u)q_\alpha'(u)-\alpha\Theta q(u)=0 \quad (3.89)$$

To determine the probability of ruin due to oscillation, say ψ_s, we set $q_\alpha=\psi_s$ and $\alpha=0$, thus we have the equation for ruin probability due to oscillation given below

$$\frac{1}{2}\sigma_p^2\ \psi_s'''(u)+\left(\frac{\Theta}{2}\sigma_p^2+c+\bar{r}u\right)\psi_s''(u)+(\bar{r}+c\Theta+\bar{r}\Theta u)\psi_s'(u)=0 \quad (3.90)$$

with the boundary conditions

$$\psi_s(u)= \quad 1 \quad on\ u<0$$
$$\lim_{u\to\infty}\psi_s(u)= \quad 0$$
$$c\psi_s''(0)+(\bar{r}-\alpha)\psi_s'(0)= \quad 0$$

This equation can be reduced to an easily solvable form after some transformation. Firstly, we reduce the third order differential equation (3.90) to second order differential equation using the transformation

$$\psi'(u)=v(z)e^{-\Theta z} \qquad where \quad z=u+c/r-\Theta\sigma^2/2r \quad (3.91)$$

The resulting equation is given as

$$\frac{1}{2}\sigma_p^2 v''(u)+rzv'(u)+rv(u)=0 \quad (3.92)$$

Then applying another transformation,

$$v(z)=h(x) \qquad where \quad x=-rz^2/\sigma^2 \quad (3.93)$$

results in the confluence hyper-geometric equation

$$xh''(x)+(\frac{1}{2}-x)h'(x)-\frac{1}{2}h(x)=0 \quad (3.94)$$

Using Slater (1960), the following solutions are obtained

$$h_0(x) = \sqrt{x}\,e^{x} M(\tfrac{1}{2}, \tfrac{3}{2}; -x)$$

$$h_1(x) = e^{x} U(0, \tfrac{1}{2}; -x) \tag{3.95}$$

Clearly, if $h_0(x)$ is a solution of equation (3.94), so also is $\sqrt{-1}\,h_0(x)$, thus

$$h_2(x) = \sqrt{-x}\,e^{x} M(\tfrac{1}{2}, \tfrac{3}{2}; -x) \tag{3.96}$$

is a solution of 3.94. Therefore the general solution is given by

$$h(x) = C_1 \sqrt{-x}\,e^{x} M(\tfrac{1}{2}, \tfrac{3}{2}; -x) + C_2 e^{x} U(0, \tfrac{1}{2}; -x) \tag{3.97}$$

where C_1 and C_2 are arbitrary constants. But,

$$v(z) = h(x) = C_1 \frac{\sqrt{rz}}{\sigma} e^{-rz^2/\sigma^2} M(\tfrac{1}{2}, \tfrac{3}{2}; \frac{rz^2}{\sigma^2}) + C_2 e^{-rz^2/\sigma^2} U(0, \tfrac{1}{2}; \frac{rz^2}{\sigma^2}) \tag{3.98}$$

where z is related to x as given (3.93), But

$$\psi'(u) = v(z) e^{-\Theta z} = C_1 \frac{\sqrt{r(u+\bar{a})}}{\sigma} e^{-\Theta \bar{a}} e^{-(\Theta u + r(u+\bar{a})^2/\sigma^2)} M(\tfrac{1}{2}, \tfrac{3}{2}; \frac{r(u+\bar{a})^2}{\sigma^2})$$

$$+ C_2 e^{-\Theta \bar{a}} e^{-(\Theta u + r(u+\bar{a})^2/\sigma^2)} U(0, \tfrac{1}{2}; \frac{r(u+\bar{a})^2}{\sigma^2}) \tag{3.99}$$

where $\bar{a} = c/r - \Theta\sigma^2/2r$. Thus,

$$\psi'(u) = C_1^{*} H_1' + C_2^{*} H_2' \tag{3.100}$$

where

$$C_1^{*} = C_1 \frac{\sqrt{r}}{\sigma} e^{-\Theta \bar{a}},$$

$$C_2^{*} = C_2 e^{-\Theta \bar{a}},$$

$$H_1' = (u+\bar{a})\, e^{-(\Theta u + r(u+\bar{a})^2/\sigma^2)} M(\tfrac{1}{2}, \tfrac{3}{2}; \frac{r(u+\bar{a})^2}{\sigma^2}) \quad \textit{and}$$

$$H_2' = e^{-(\Theta u + r(u+\bar{a})^2/\sigma^2)} U(0, \tfrac{1}{2}; \frac{r(u+\bar{a})^2}{\sigma^2})$$

Integrating both sides of (3.100), using equation (4.12) in Cai and Yang (2005) results to

$$\psi(u) = \int_u^{\infty} \psi'(x)\,dx = C_1^{*} H_1 + C_2^{*} H_2 \tag{3.101}$$

where

$$H_1(u)=\int_u^{\infty}(x+\bar{a})\,e^{-(\Theta x+r(x+\bar{a})^2/\sigma^2)}M(\tfrac{1}{2},\tfrac{3}{2};\frac{r(x+\bar{a})^2}{\sigma^2})dx \qquad \textit{and}$$

$$H_2(u)=\int_u^{\infty}e^{-(\Theta x+r(x+\bar{a})^2/\sigma^2)}U(0,\tfrac{1}{2};\frac{r(x+\bar{a})^2}{\sigma^2})dx$$

To determine C_1^* and C_2^* in terms of the parameters we proceed to obtain $\psi(u)''$,

$$\psi''(u)=C_1^*H_1''+C_2^*H_2'' \tag{3.102}$$

where

$$H_1''=\bar{a}\,e^{-(\Theta x+r(x+\bar{a})^2/\sigma^2)}M(\tfrac{1}{2},\tfrac{3}{2};\frac{r(x+\bar{a})^2}{\sigma^2})$$

$$-\frac{(\Theta+2r(x+\bar{a}))(x+\bar{a})}{\sigma^2}e^{-(\Theta x+r(x+\bar{a})^2/\sigma^2)}M(\tfrac{1}{2},\tfrac{3}{2};\frac{r(x+\bar{a})^2}{\sigma^2})$$

$$+\frac{2r(x+\bar{a})^2}{3\sigma^2}e^{-(\Theta x+r(x+\bar{a})^2/\sigma^2)}M(\tfrac{3}{2},\tfrac{5}{2};\frac{r(x+\bar{a})^2}{\sigma^2})$$

$$H_2''=-\frac{(\Theta+2r(x+\bar{a}))}{\sigma^2}e^{-(\Theta x+r(x+\bar{a})^2/\sigma^2)}U(0,\tfrac{1}{2};\frac{r(x+\bar{a})^2}{\sigma^2})$$

Using the boundary conditions as shown in equation (3.90) together with equations (3.100), (3.101) and (3.102), one gets

$$C_1^*H_1(0)+C_2^*H_2(0)=1$$

$$C_1^*(cH_1''(0)+(\bar{r}-\alpha)H_1'(0))+C_2^*(cH_2''(0)+(\bar{r}-\alpha)H_2'(0))=0 \tag{3.103}$$

where

$$H_1'(0)=\bar{a}\,e^{-r\bar{a}^2/\sigma^2}M(\tfrac{1}{2},\tfrac{3}{2};\frac{r\bar{a}^2}{\sigma^2})$$

$$H_2'(0)=e^{-r\bar{a}^2/\sigma^2}U(0,\tfrac{1}{2};\frac{r\bar{a}^2}{\sigma^2})$$

$$H_1''(0)=\bar{a}\,e^{-r\bar{a}^2/\sigma^2}M(\tfrac{1}{2},\tfrac{3}{2};\frac{r\bar{a}^2}{\sigma^2})-\frac{2r\bar{a}^2}{\sigma^2}e^{-r\bar{a}^2/\sigma^2}M(\tfrac{1}{2},\tfrac{3}{2};\frac{r\bar{a}^2}{\sigma^2})+\frac{2r\bar{a}^2}{3\sigma^2}e^{-r\bar{a}^2/\sigma^2}M(\tfrac{3}{2},\tfrac{5}{2};\frac{r\bar{a}^2}{\sigma^2})$$

$$H_2''(0)=-\frac{\Theta+2r\bar{a}}{\sigma^2}e^{-r\bar{a}^2/\sigma^2}U(0,\tfrac{1}{2};\frac{r\bar{a}^2}{\sigma^2})$$

Solving equation (3.103) simultaneously gives

$$C_1^* = -\frac{(cH_2''(0)+(\bar{r}-\alpha)H_2'(0))}{H_2(0)(cH_1''(0)+(\bar{r}-\alpha)H_1'(0))-H_1(0)(cH_2''(0)+(\bar{r}-\alpha)H_2'(0))}$$

$$C_2^* = \frac{(cH_1''(0)+(\bar{r}-\alpha)H_1'(0))}{H_2(0)(cH_1''(0)+(\bar{r}-\alpha)H_1'(0))-H_1(0)(cH_2''(0)+(\bar{r}-\alpha)H_2'(0))} \quad (3.104)$$

Thus the ruin probability due to oscillation for this model is given as

$$\psi(u) = \frac{H_2(u)(cH_1''(0)+(\bar{r}-\alpha)H_1'(0))-H_1(u)(cH_2''(0)+(\bar{r}-\alpha)H_2'(0))}{H_2(0)(cH_1''(0)+(\bar{r}-\alpha)H_1'(0))-H_1(0)(cH_2''(0)+(\bar{r}-\alpha)H_2'(0))} \quad (3.105)$$

where $H_1(u), H_2(u), H_1'(0), H_2'(0), H_1''(0)$ and $H_2''(0)$ are as defined in equations (3.101) and (3.103)

Claim Size Distributed as Mixture of Two Exponential

Suppose the claim size is distributed as mixture of two exponentials with density function (see Rider (1961)),

$$f(x) = \Theta\Theta_1 e^{-\Theta 1 x} + (1-(\Theta))\Theta_2 e^{-\Theta 2 x} \quad (3.106)$$

where $x>0, \Theta>0$ $\Theta_1>0$ and $\Theta_2>0$ are the proportion of investment into portfolio 1, reciprocal of mean claim size from portfolio 1 and 2 respectively. Then, the integro-differential equation (3.64) is converted into differential equation in the following theorem.

Theorem 3.3.4 *Given that the claim size distribution f(x) is mixture of two exponential distributions with means $1/\Theta_1$ and $1/\Theta_2$, then the integro-differential equation (3.64) is equivalent to the order differential equation below.*

$$\frac{1}{2}(\sigma_p^2+(\sigma_R\phi)^2u^2)q_\alpha''''(u)+(2(\sigma_R\phi)^2u+\bar{r}u+c+\frac{1}{2}(\Theta_1+\Theta_2)(\sigma_p^2+(\sigma_R\phi)^2u^2))q_\alpha'''(u)$$

$$+((\sigma_R\phi)^2+2\bar{r}-\lambda-\alpha+(\Theta_1+\Theta_2)((\sigma_R\phi)^2u+\bar{r}u+c)+\frac{1}{2}\Theta_1\Theta_2(\sigma_p^2+(\sigma_R\phi)^2u^2))q_\alpha''$$

$$+(\lambda(\Theta\Theta_1+(1-\Theta)\Theta_2)+(\Theta_1+\Theta_2)(\bar{r}-\lambda-\alpha)+\Theta_1\Theta_2(\bar{r}u+c))q_\alpha'(u)-\alpha\Theta_1\Theta_2q_\alpha=0 \quad (3.107)$$

where $\bar{r}=r(1-\phi)+a\phi$.

Proof. Recall equation (3.64) as shown below,

$$(c+r(1-\phi)u+a\phi u)q_\alpha'(u)+\frac{1}{2}(\sigma_p^2+(\sigma_R\phi)^2u^2)q_\alpha''(u)+\lambda\int_0^\infty(q_\alpha(u-x)-q_\alpha(u))dF(x)=\alpha q_\alpha(u)$$

Since $f(x)$ is distributed as mixture of two exponentials, then

$$dF(x)=(\Theta\Theta_1 e^{-\Theta_1 x}+(1-\Theta)\Theta_2 e^{-\Theta_2 x})dx$$

Substituting $\bar{r}=r(1-\phi)+a\phi$, simplifying and rearranging equation (3.64) shown above, one obtains

$$\frac{1}{2}(\sigma_p^2+(\sigma_R\phi)^2u^2)q_\alpha''(u)+(c+\bar{r}u)q_\alpha'(u)-(\lambda+\alpha)q_\alpha(u)$$

$$=-\lambda(\Theta\Theta_1 e^{-\Theta_1 u}\int_0^u q_\alpha(x)e^{\Theta_1 x}dx+(1-\Theta)\Theta_2 e^{-\Theta_2 u}\int_0^u q_\alpha(x)e^{\Theta_2 x}dx) \quad (3.108)$$

Differentiating both sides of (3.108) and rearranging gives

$$\frac{1}{2}(\sigma_p^2+(\sigma_R\phi)^2u^2)q_\alpha'''(u)+((\sigma_R\phi)^2u+\bar{r}u+c)q_\alpha''(u)+(\bar{r}-\lambda-\alpha)q_\alpha'(u)+\lambda(\Theta\Theta_1+(1-\Theta)\Theta_2)q_\alpha(x)$$

$$=\lambda(\Theta\Theta_1^2 e^{-\Theta_1 u}\int_0^u q_\alpha(x)e^{\Theta_1 x}dx+(1-\Theta)\Theta_2^2 e^{-\Theta_2 u}\int_0^u q_\alpha(x)e^{\Theta_2 x}dx) \quad (3.109)$$

Differentiating the equation once more, one obtains

$$\frac{1}{2}(\sigma_p^2+(\sigma_R\phi)^2u^2)q_\alpha''''(u)+(2(\sigma_R\phi)^2u+\bar{r}u+c)q_\alpha'''(u)+((\sigma_R\phi)^2+2\bar{r}-\lambda-\alpha)q_\alpha''(u)$$

$$+\lambda(\Theta\Theta_1+(1-\Theta)\Theta_2)q_\alpha'(x)-\lambda(\Theta\Theta_1^2+(1-\Theta)\Theta_2^2)q_\alpha(u)$$

$$=-\lambda(\Theta\Theta_1^3 e^{-\Theta_1 u}\int_0^u q_\alpha(x)e^{\Theta_1 x}dx+(1-\Theta)\Theta_2^3 e^{-\Theta_2 u}\int_0^u q_\alpha(x)e^{\Theta_2 x}dx) \quad (3.110)$$

To eliminate the integral part of equation (3.108), equation (3.108) and (3.109) are multipled by $\Theta_1\Theta_2$ and $\Theta_1+\Theta_2$ respectively. Summing the resulting equations completes the proof. □

Similar to the method adopted when the claim size distribution is exponential, we proceed to simplify the equation by setting $\sigma_R=0$. The resulting equation becomes

$$\frac{1}{2}\sigma_p^2 q_\alpha''''(u)+(\bar{r}u+c+\frac{1}{2}(\Theta_1+\Theta_2)\sigma_p^2)q_\alpha'''(u)+(2\bar{r}-\lambda-\alpha+(\Theta_1+\Theta_2)(\bar{r}u+c)+\frac{1}{2}\Theta_1\Theta_2\sigma_p^2)q_\alpha''$$

$$+(\lambda(\Theta\Theta_1+(1-\Theta)\Theta_2)+(\Theta_1+\Theta_2)(\bar{r}-\lambda-\alpha)+\Theta_1\Theta_2(\bar{r}u+c))q_\alpha'(u)-\alpha\Theta_1\Theta_2 q_\alpha(u)=0 \quad (3.111)$$

with boundary conditions given, in lemma , equations (3.70) and (3.71), as shown below.

$$g(u)= \quad 1 \quad on\ u<0$$

$$g(0)= \quad 1 \quad if\ \sigma_p^2>0$$

$$\lim_{u\to\infty} g(u)= \quad 0$$

Equation (3.111) is then decomposed into two different equations for ruin caused by claim and that caused by oscillation respectively. Firstly, we set σ_p=0 and α=0 to get the equation for ruin caused by claims, as given below

$$(\bar{r}u+c)q'''(u)+(2\bar{r}-\lambda+(\Theta_1+\Theta_2)(\bar{r}u+c))q''+(\lambda(\Theta\Theta_1+(1-\Theta)\Theta_2)+(\Theta_1+\Theta_2)(\bar{r}-\lambda)+\Theta_1\Theta_2(\bar{r}u+c))q'(u)=0 \tag{3.112}$$

with boundary conditions

$$g(u)= 1 \quad on\ u<0$$

$$g(0)= 1 \quad if\ \sigma_p^2>0$$

$$\lim_{u\to\infty} g(u)= 0$$

and for ruin due to oscillations, we set λ=0 and α=0 to get

$$\frac{1}{2}\sigma_p^2 q_\alpha''''(u)+(\bar{r}u+c+\frac{1}{2}(\Theta_1+\Theta_2)\sigma_p^2)q_\alpha'''(u)+(2\bar{r}+(\Theta_1+\Theta_2)(\bar{r}u+c)+\frac{1}{2}\Theta_1\Theta_2\sigma_p^2)q_\alpha''$$

$$+((\Theta_1+\Theta_2)\bar{r}+\Theta_1\Theta_2(\bar{r}u+c))q_\alpha'(u)=0 \tag{3.113}$$

with the boundary conditions

$$g(u)= \quad 1 \quad on\ u<0$$

$$g(0)= \quad 1 \quad if\ \sigma_p^2>0$$

$$\lim_{t\to\infty} g(u)= \quad 0$$

Equation (3.112) is similar to equation (2.18) of Paulsen and Gjessing (1997), thus following similar patterns to that of Paulsen and Gjessing (1997), the solution is obtained as

$$q(u)=C_1\int_u^\infty e^{-\Theta_1 u}(u+\frac{c}{\bar{r}})^{\lambda/\bar{r}-1}M((1-\Theta)\frac{\lambda}{\bar{r}},\frac{\lambda}{\bar{r}},(\Theta_1-\Theta_2)(y+\frac{c}{\bar{r}}))du$$

$$+C_2\int_u^\infty e^{-\Theta_1 u}(u+\frac{c}{\bar{r}})^{\lambda/\bar{r}-1}M((1-\Theta)\frac{\lambda}{\bar{r}},\frac{\lambda}{\bar{r}},(\Theta_1-\Theta_2)(y+\frac{c}{\bar{r}}))du \tag{3.114}$$

where

$$C_1=\frac{H_1(0)+(\lambda/\bar{r})H_1(1)}{(H_2(0)-H_2(1))(\bar{r}(c/\bar{r})^{\lambda/\bar{r}}H_1(0)+\lambda K_2(0))-(H_1(0)+\lambda H_1(1)/\bar{r})(\bar{r}(c/\bar{r})^{\lambda/\bar{r}}H_2(0)+\lambda K_1(0))}$$

$$C_2=\frac{H_2(0)-H_2(1)}{(H_2(0)-H_2(1))(\bar{r}(c/\bar{r})^{\lambda/\bar{r}}H_1(0)+\lambda K_2(0))-(H_1(0)+\lambda H_1(1)/\bar{r})(\bar{r}(c/\bar{r})^{\lambda/\bar{r}}H_2(0)+\lambda K_1(0))} \tag{3.115}$$

and $H_1(a)$, $H_2(a)$, $K_1(u)$ and $K_2(u)$ denote the following

$$H_1(x)=\mathrm{M}(a+\frac{\lambda\Theta}{\bar{r}},a+\frac{\lambda}{\bar{r}},\frac{c(\Theta_1-\Theta_2)}{\bar{r}})$$

$$H_2(x)=\mathrm{U}(a+\frac{\lambda\Theta}{\bar{r}},a+\frac{\lambda}{\bar{r}},\frac{c(\Theta_1-\Theta_2)}{\bar{r}})$$

$$K_1(u)=\int_u^{\infty} e^{-\Theta 1 u}(u+\frac{c}{\bar{r}})^{\lambda/\bar{r}-1}\mathrm{M}((1-\Theta)\frac{\lambda}{\bar{r}},\frac{\lambda}{r},(\Theta_1-\Theta_2)(y+\frac{c}{\bar{r}}))du$$

$$K_2(u)=\int_u^{\infty} e^{-\Theta 1 u}(u+\frac{c}{\bar{r}})^{\lambda/\bar{r}-1}\mathrm{U}((1-\Theta)\frac{\lambda}{\bar{r}},\frac{\lambda}{r},(\Theta_1-\Theta_2)(y+\frac{c}{\bar{r}}))du \quad (3.116)$$

3.4 Simulation Design

Monte Carlo simulation is carried out to investigate the behavior of the model and that of the probabilities of ruin for certain values of the parameters. The model in equation (3.35) is specifically examined since it gives the explicit representation of the reserve in real units. Computer program is written using MATLAB R2015a. The Brownian motion is generated by first generating $\varepsilon_i(i=1(1)N(t))\ N(0,1)$ and using the following recursive definitions (Sigman, 2007):

$$\begin{aligned} W(t_0) &= 0 \\ W(t_1) &= \varepsilon_1\sqrt{t_1-t_0}=\varepsilon_1\sqrt{t_1} \\ W(t_2) &= W(t_1)+\varepsilon_2\sqrt{t_2-t_1}=\varepsilon_1\sqrt{t_1-t_0}+\varepsilon_2\sqrt{t_2-t_1} \\ &\vdots \\ W(t_{N(t)}) &= \sum_{i=1}^{N(t)}\varepsilon_i\sqrt{t_i-t_{i-1}} \end{aligned} \quad (3.117)$$

where $0=t_0<t_1<\ldots<t_{N(t)}$ The simulation of the claim process is examined in two cases.

Since the time between successive arrivals are iid exponential, the arrival process is a Poisson process while the number of arrival $N(t)$ is Poisson distributed. The exponential inter-arrival time is simulated using

$$f(x)=-\frac{1}{\lambda}ln(u) \quad (3.118)$$

The claims $X_i(i=1(1)N(t))$ are generated from $U(0,1)$

Chapter 4

Results and Discussion

The simulation results for investigating the behavior of the risk reserve models and the ruin probabilities are presented and discussed in sections 4.1 and res:BRP. In both sections, the behavior of the models from both exponentially distributed claims and mixed exponential claims are investigated. Section contains the application to real life data.

In some situations, it is necessary to fix some parameters as to enable proper investigation of the parameter of interest. In these situations, the same parameter used by Paulsen et al. (2005) is adopted. That is, the parameters are set at c=1.1, λ=1, r=0.12, Θ=1, ϕ=0.6, σ_p=0.2, a=0.21, α=0 and σ_R=0.2.

4.1 Behavior of the Risk Reserve Processes

The behaviour of the reserve is investigated when the initial capital is set at u=25, in line with the current recapitalization of insurance companies in Nigeria. The safety loading "ϱ", interest "r", inflation "δ" and fraction invested in risky asset "ϕ" are varied as shown in table 4.1.

Table 4.1: Values of Parameters used in simulations

Parameter	Values										
ϱ	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
r	0.00	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20
δ	0.00	0.30	0.60	0.90	0.18	0.20	0.22	0.24	0.26	0.28	0.30
ϕ	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00

4.1.1 Behavior of The Risk Reserve Process with Exponential Claim When Interest r is Varied



Figure 4.1: The behavior of the reserve with respect to arrival epoch for ϱ=0.60, *a*=0.21, σ=0.20, δ=0.00, ϕ=0.00 and *r* is varied .

Comments:

Figure **Error! Reference source not found.** shows the behavior of the reserve process when *r* is varied, δ=0.00 and all the resources are invested in risk-less asset, that is ϕ=0.00. There is a

progressive increase in the reserve as the value of r is increased. When r=0 which implies there is no return from the risk-less investment, the value of the reserve decreases with increasing epoch. Thus, the income realized from premium is insufficient to payoff the claims and take care of administrative cost. Table E.1 on page 148 shows the values of the reserve for $r \leq 0.10$.



Figure 4.2: The behavior of the reserve with respect to arrival epoch for ϱ=0.60, *a*=0.21, σ=0.20, δ=0.18, ϕ=0.00 and *r* is varied .

Comments:

Figure **Error! Reference source not found.** shows the behavior of the reserve process when *r* is varied, δ=0.18 and all the resources are invested in risk-less asset, that is ϕ=0.00. The value of the

reserve decreases with increasing epoch for all values of $r \leq 0.18$. Though, there is an upward shift in the value of the reserve as r is increased. It can be deduced in this case that interest earned from fixed assets can only compensate for the effect of inflation and claims on the reserve, only when the rate $r > 0.18$. Table E.2 on page 149 shows the values of the reserve for $r \geq 0.10$.



Figure 4.3: The behavior of the reserve with respect to arrival epoch for ϱ=0.60, *a*=0.21, σ=0.20, δ=0.00, ϕ=0.20 and *r* is varied .

Comments:

Figure **Error! Reference source not found.** shows the behavior of the reserve process when *r* is varied, δ=0.00 and 20% of the reserve is continuously invested into risky assets, that is ϕ=0.20. The value of the reserve increases with increasing epoch for all values of *r*, though with a noticeable fluctuation. Also, as *r* is increased, there is an upward shift in the value of the reserve. Table E.3 on page 150 shows the values of the reserve for $r \leq 0.10$.

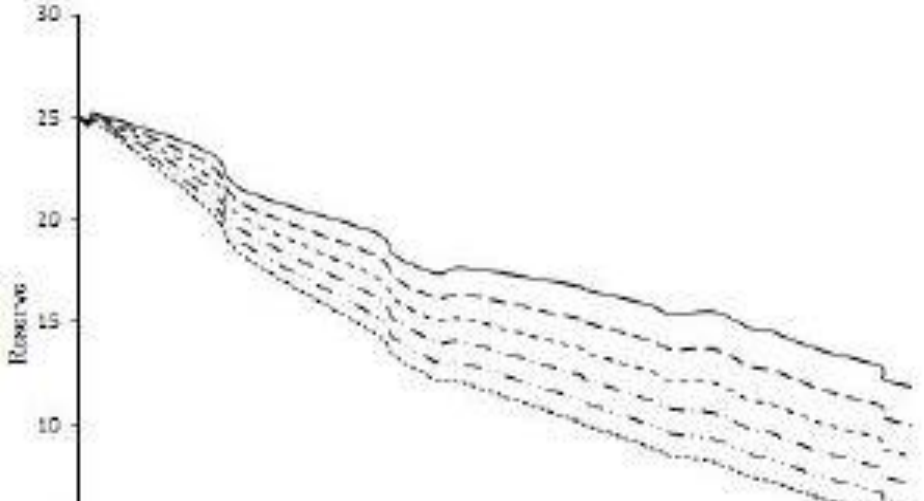

30
25
20
15
10

Figure 4.4: The behavior of the reserve with respect to arrival epoch for ϱ=0.60, *a*=0.21, σ=0.20, δ=0.18, ϕ=0.20 and *r* is varied .

Comments:

Figure **Error! Reference source not found.** shows the behavior of the reserve process when *r* is varied, δ=0.18 and 20% of the reserve is continuously invested into risky assets. The value of the reserve decreases with increasing epoch for all values of $r \leq 0.18$. Also, there is a noticeable fluctuation in the value of the reserve for all values of *r*. Table E.4 on page 151 shows the values of the reserve for $r \geq 0.10$.

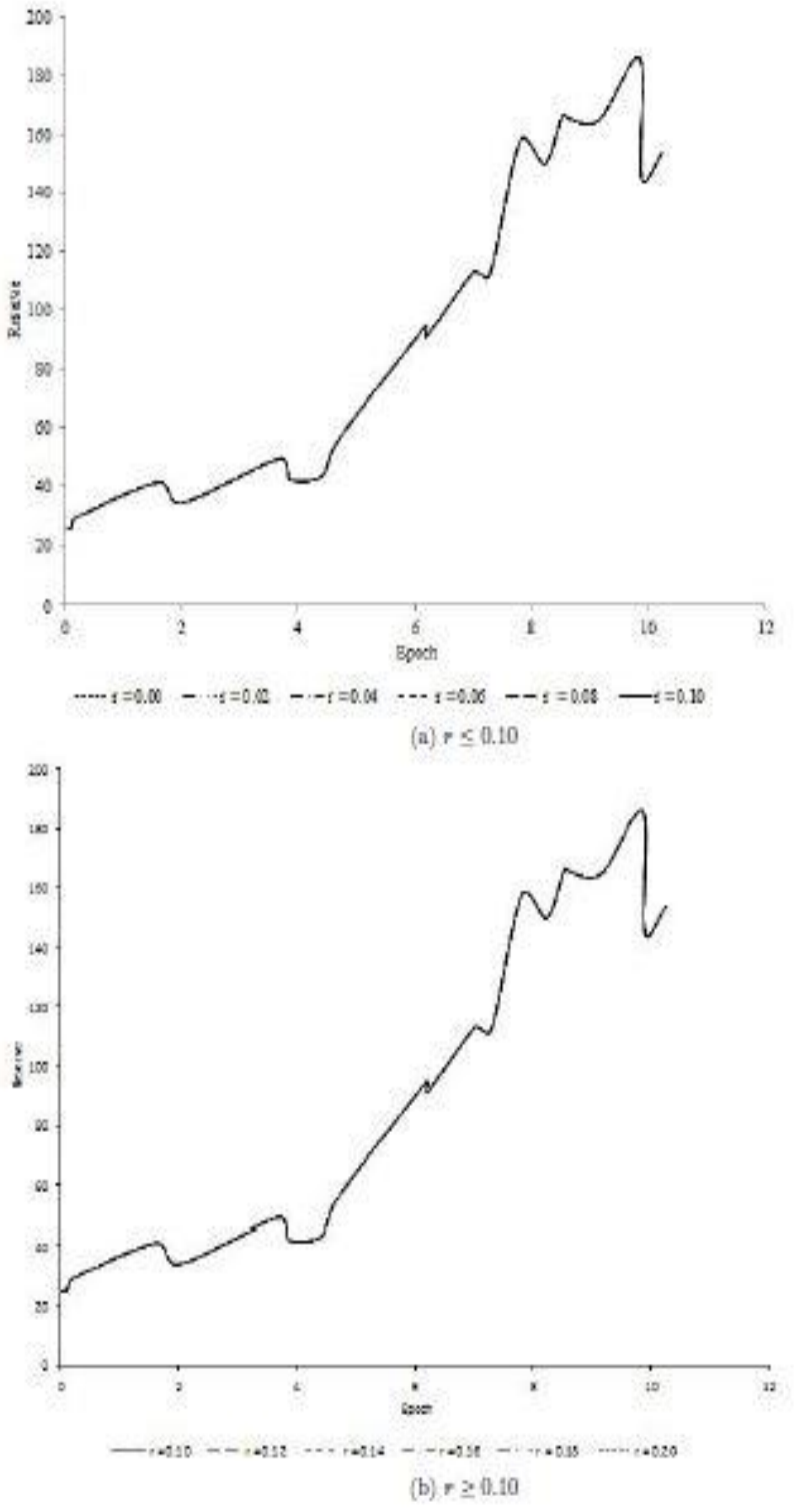


Figure 4.5: The behavior of the reserve with respect to arrival epoch for ϱ=0.60, *a*=0.21, σ=0.20, δ=0.00, ϕ=1.00 and *r* is varied .

Comments:

Figure **Error! Reference source not found.** shows the behavior of the reserve process when r is varied, δ=0.00 and 0% of the reserve is continuously invested into risky assets, that is ϕ=1.00. This implies all the reserve is invested into risky assets. Expectedly, variation of the interest rate has no effect on the reserve. Table E.5 on page 152 shows the value of the reserve for this case.

4.1.2 Discussion of Results Obtained When *r* is Varied

The results obtained when the interest rate is varied show that, for a model with an exponentially distributed claim sizes, the reserve increases as interest rate is increased. This is an expected behavior of the reserve. Also, when no investment is made into risky-less asset, the interest rate does not affect the value of the reserve.

When there is no inflation (that is δ=0.00) and investment is made into risk-less assets, it is not necessary to invest into risky assets to ensure the value of the reserve does not depreciate with increasing epoch. However, when inflation is greater than or equal to 18%, investing 20% and above into risky assets ensure that the value of the reserve does not depreciate with increasing epoch, assuming a safety loading of $\varrho \leq 0.6$.

4.1.3 Behavior of Risk Reserve Process with Exponential Claim When Safety Loading is Varied

Table 4.2: The behavior of the reserve with respect to arrival epoch for r =0.12, a=0.21, σ=0.20, δ=0.00, ϕ=0.00 and ϱ≤0.50 is varied .

Epoch	*ϱ=0*	*ϱ=0.1*	*ϱ=0.2*	*ϱ=0.3*	*ϱ=0.4*	*ϱ=0.5*
0.00000	25.00000	25.00000	25.00000	25.00000	25.00000	25.00000
0.11794	25.32731	25.32731	25.32731	25.32731	25.32731	25.32731
0.18794	25.45315	25.45315	25.45316	25.45316	25.45316	25.45316
1.61759	30.17262	30.17267	30.17273	30.17278	30.17284	30.17289
1.99044	31.49611	31.49623	31.49635	31.49647	31.49659	31.49671
3.69038	38.53647	38.53689	38.53731	38.53773	38.53815	38.53857
3.85862	39.09486	39.09559	39.09632	39.09706	39.09779	39.09852
4.42277	41.83451	41.83575	41.83699	41.83822	41.83946	41.84069
4.69567	43.09516	43.09698	43.09880	43.10063	43.10245	43.10428
6.18816	51.64828	51.65170	51.65513	51.65855	51.66197	51.66540
6.19570	51.69847	51.70268	51.70688	51.71108	51.71528	51.71948
7.00243	56.78928	56.79549	56.80169	56.80790	56.81411	56.82031
7.28650	58.77073	58.77862	58.78650	58.79439	58.80228	58.81017
7.79823	62.47074	62.48106	62.49137	62.50169	62.51201	62.52233
8.23207	65.86944	65.88240	65.89537	65.90833	65.92129	65.93425
8.52260	68.24608	68.26164	68.27719	68.29275	68.30831	68.32386
8.63321	69.07816	69.09609	69.11403	69.13196	69.14989	69.16782
9.15826	73.58022	73.60238	73.62453	73.64669	73.66885	73.69100
9.85818	80.07709	80.10508	80.13306	80.16105	80.18904	80.21702
9.88764	80.26857	80.29984	80.33110	80.36236	80.39363	80.42489
10.23884	83.77543	83.81204	83.84865	83.88526	83.92188	83.95849

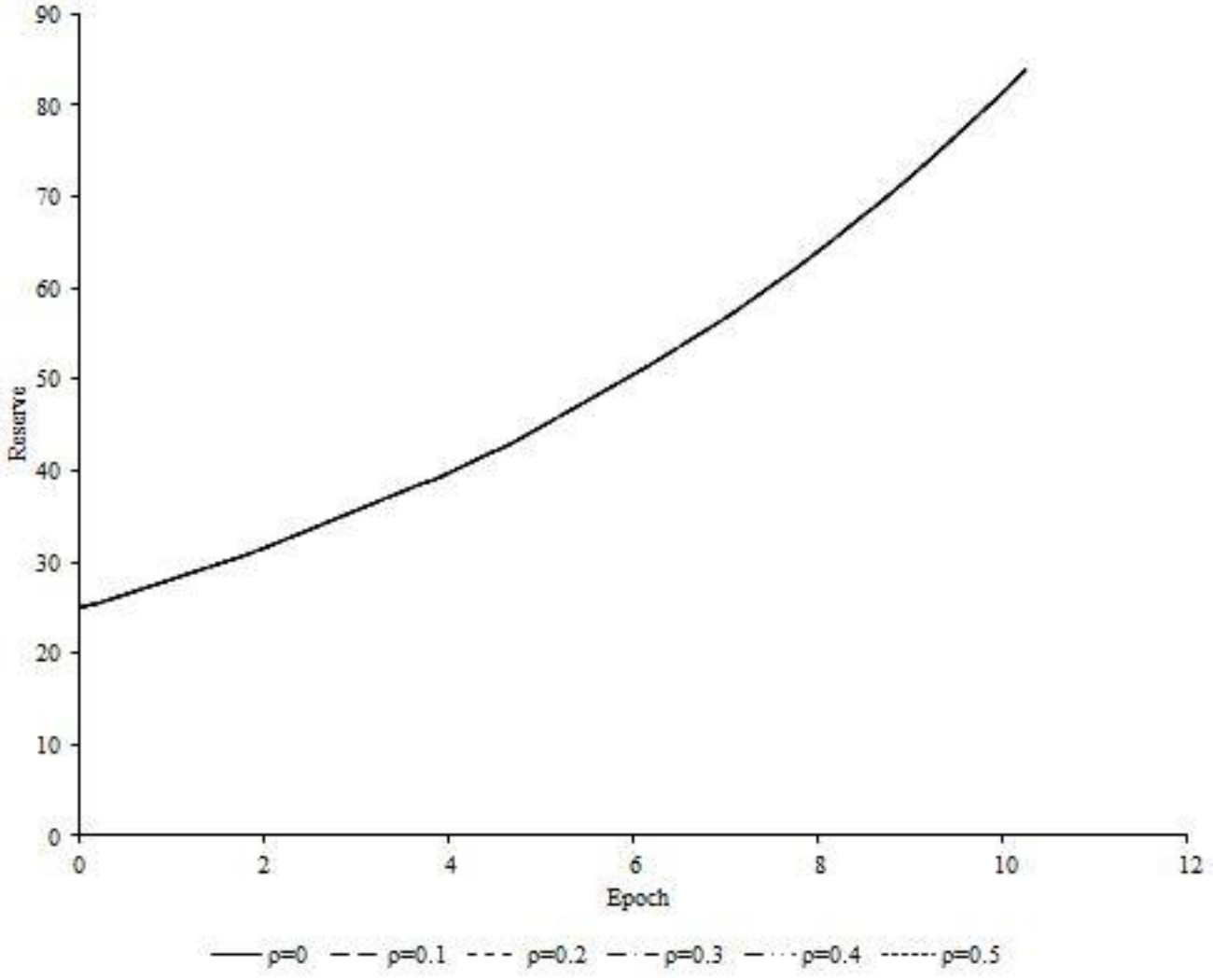


Figure 4.6: The behavior of the reserve with respect to arrival epoch for r =0.12, a=0.21, σ=0.20, δ=0.00, ϕ=0.00 and ϱ≤0.50 is varied .

Comments:

Table 4.2 and Figure 4.6 show the behavior of the reserve process when safety loading "ϱ" is varied, δ=0.00 and all the resources are invested in risk-less assets with rate *r*=0.12. From the figure, it is obvious that the reserve increases with increasing epoch for all values of ϱ. The changes in the reserve when ϱ is varied, though comparatively small, can be seen from table 4.2. These changes are quite small for the first few years and then increases subsequently.

Table 4.3: The behavior of the reserve with respect to arrival epoch for r=0.12, a=0.21, σ=0.20, δ=0.18, ϕ=0.00 and ϱ≤0.50 is varied .

Epoch	*ϱ=0*	*ϱ=0.1*	*ϱ=0.2*	*ϱ=0.3*	*ϱ=0.4*	*ϱ=0.5*
0.00000	25.00000	25.00000	25.00000	25.00000	25.00000	25.00000
0.11794	24.79528	24.79528	24.79528	24.79528	24.79528	24.79528
0.18794	24.60612	24.60612	24.60613	24.60613	24.60613	24.60613
1.61759	22.52456	22.52457	22.52458	22.52459	22.52460	22.52461
1.99044	21.97669	21.97671	21.97673	21.97676	21.97678	21.97680
3.69038	19.75115	19.75122	19.75128	19.75134	19.75140	19.75146
3.85862	19.43173	19.43182	19.43192	19.43202	19.43211	19.43221
4.42277	18.75154	18.75168	18.75183	18.75198	18.75212	18.75227
4.69567	18.37291	18.37311	18.37331	18.37351	18.37371	18.37391
6.18816	16.73513	16.73544	16.73576	16.73608	16.73639	16.73671
6.19570	16.72721	16.72759	16.72796	16.72834	16.72872	16.72910
7.00243	15.83841	15.83891	15.83940	15.83990	15.84039	15.84089
7.28650	15.55177	15.55236	15.55295	15.55354	15.55414	15.55473
7.79823	15.03765	15.03837	15.03908	15.03980	15.04051	15.04123
8.23207	14.63134	14.63218	14.63303	14.63387	14.63471	14.63555
8.52260	14.36439	14.36535	14.36631	14.36728	14.36824	14.36920
8.63321	14.24280	14.24388	14.24495	14.24603	14.24710	14.24817
9.15826	13.76243	13.76366	13.76490	13.76613	13.76736	13.76860
9.85818	13.15189	13.15331	13.15473	13.15615	13.15758	13.15900
9.88764	13.10886	13.11041	13.11196	13.11351	13.11505	13.11660
10.23884	12.81458	12.81630	12.81801	12.81973	12.82145	12.82316

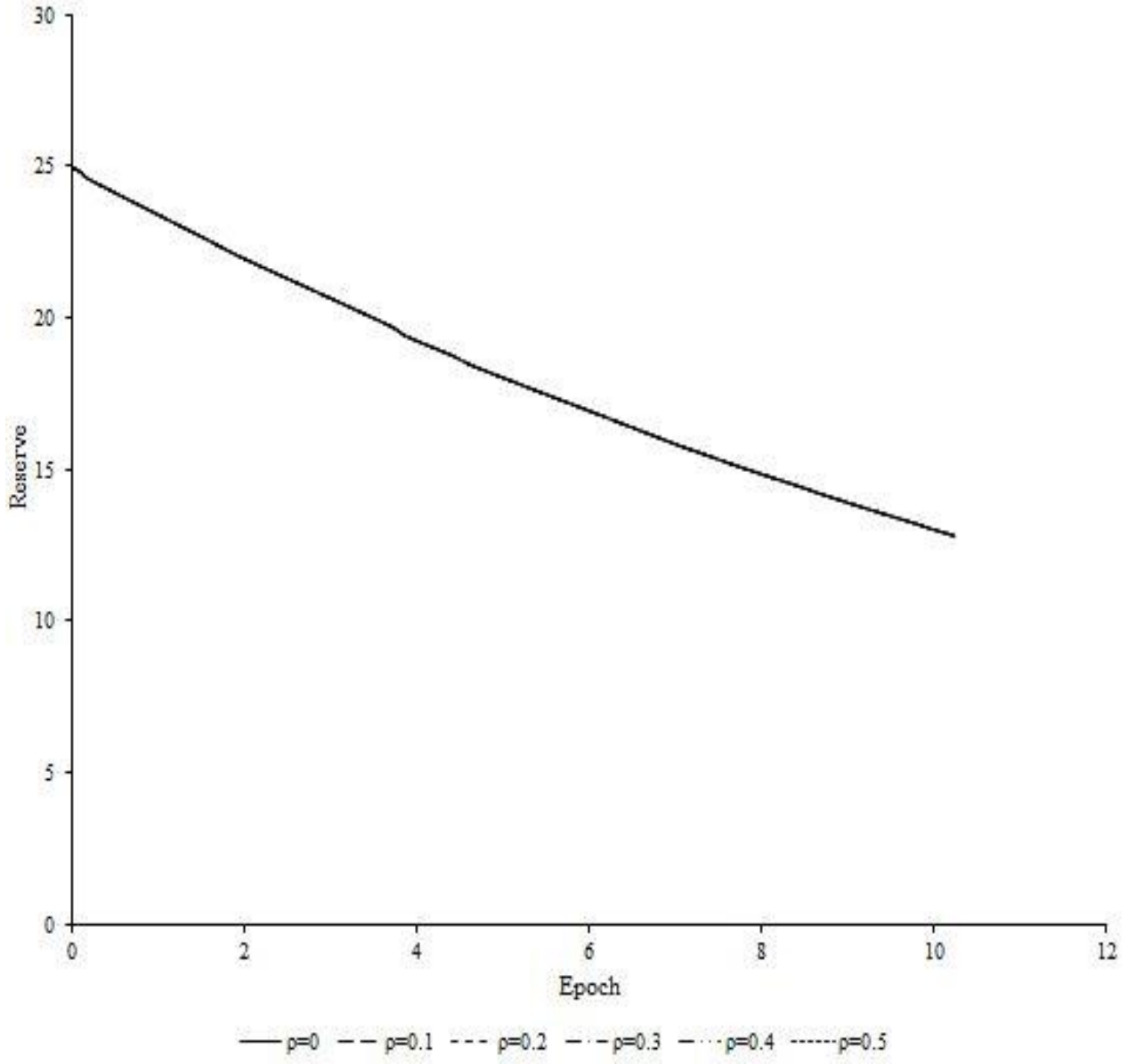


Figure 4.7: The behavior of the reserve with respect to arrival epoch for r=0.12, a=0.21, σ=0.2, δ=0.18, ϕ=0.0 and ϱ is varied .

Comments:

Table 4.3 and Figure 4.7 show the behavior of the reserve process when safety loading "ϱ" is varied, δ=0.18 and ϕ=0.0. The value of the reserve depreciates for all values of ϱ. Though the variation of ϱ increases the value of the reserve, especially at large epoch, but these increment is not enough to compensate for the depreciation due to the effect of inflation.

Table 4.4: The behavior of the reserve with respect to arrival epoch for r=0.12, a=0.21, σ=0.2, δ=0.00, ϕ=1.00 and ϱ is varied .

	ϱ=0	*ϱ=0.1*	*ϱ=0.2*	*ϱ=0.3*	*ϱ=0.4*	*ϱ=0.5*
0.00000	25.00000	25.00000	25.00000	25.00000	25.00000	25.00000
0.11794	25.65805	25.65805	25.65806	25.65806	25.65806	25.65806
0.18794	28.65615	28.65627	28.65638	28.65650	28.65662	28.65674
1.61759	40.43421	40.43486	40.43552	40.43617	40.43682	40.43747
1.99044	34.53933	34.53940	34.53947	34.53955	34.53962	34.53969
3.69038	49.76124	49.76331	49.76539	49.76746	49.76954	49.77162
3.85862	43.51890	43.52032	43.52175	43.52317	43.52460	43.52602
4.42277	45.40432	45.40471	45.40510	45.40550	45.40589	45.40628
4.69567	56.74838	56.76850	56.78863	56.80875	56.82887	56.84899
6.18816	93.45524	93.52016	93.58507	93.64999	93.71491	93.77982
6.19570	89.82084	89.85017	89.87950	89.90882	89.93815	89.96748
7.00243	111.13006	111.26056	111.39107	111.52158	111.65208	111.78259
7.28650	110.98897	111.03840	111.08782	111.13724	111.18666	111.23608
7.79823	150.98438	151.29039	151.59639	151.90239	152.20839	152.51440
8.23207	146.40020	146.43779	146.47538	146.51297	146.55056	146.58815
8.52260	161.12820	161.33894	161.54968	161.76042	161.97117	162.18191
8.63321	160.60662	160.77345	160.94029	161.10712	161.27396	161.44079
9.15826	162.72831	162.65520	162.58209	162.50897	162.43586	162.36274
9.85818	183.88686	184.11605	184.34523	184.57441	184.80360	185.03278
9.88764	148.16036	148.13313	148.10590	148.07868	148.05145	148.02422
10.23884	157.05667	157.16199	157.26730	157.37262	157.47793	157.58325

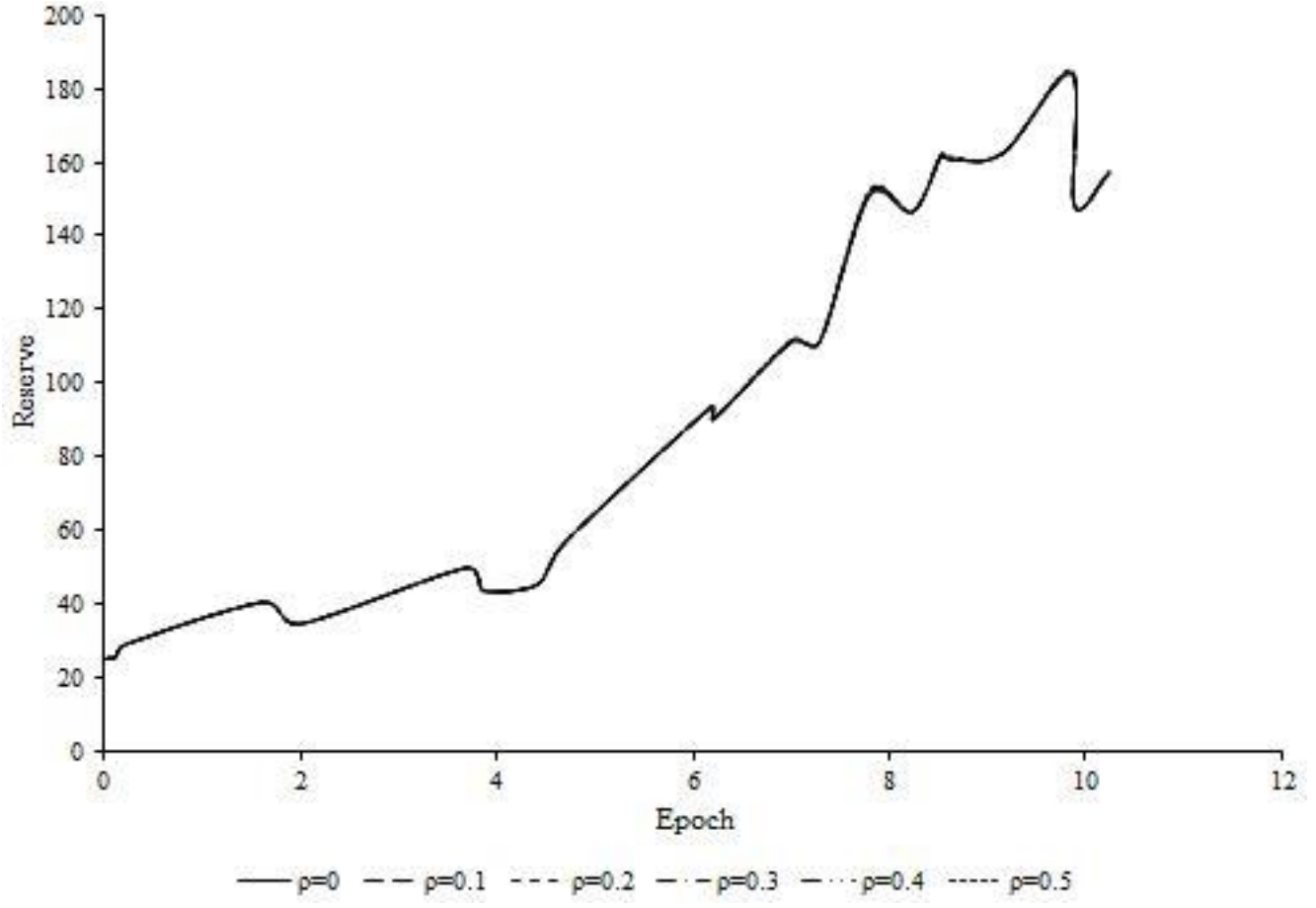


Figure 4.8: The behavior of the reserve with respect to arrival epoch for r=0.12, a=0.21, σ=0.2, δ=0.00, ϕ=1.00 and ϱ is varied .

Comments:

Table 4.4 and Figure 4.8 show the behavior of the reserve process when safety loading "ϱ" is varied, δ=0.00 and ϕ=1. The value of the reserve increases with increasing epoch though with obvious fluctuation. Like the previous cases, the changes in the values of the reserve is only noticeable at large epoch as shown in table 4.4.

Table 4.5: The behavior of the reserve with respect to arrival epoch for r=0.12, a=0.21, σ=0.2, δ=0.18, ϕ=1.00 and ϱ is varied .

	ϱ=0	*ϱ=0.1*	*ϱ=0.2*	*ϱ=0.3*	*ϱ=0.4*	*ϱ=0.5*
0.00000	25.00000	25.00000	25.00000	25.00000	25.00000	25.00000
0.11794	25.11907	25.11907	25.11907	25.11907	25.11907	25.11907
0.18794	27.70230	27.70235	27.70241	27.70246	27.70252	27.70257
1.61759	30.18957	30.18965	30.18973	30.18981	30.18989	30.18996
1.99044	24.09954	24.09956	24.09958	24.09960	24.09963	24.09965
3.69038	25.51183	25.51183	25.51184	25.51184	25.51184	25.51184
3.85862	21.62617	21.62628	21.62640	21.62652	21.62664	21.62676
4.42277	20.34728	20.34776	20.34824	20.34873	20.34921	20.34970
4.69567	24.17529	24.17523	24.17517	24.17511	24.17505	24.17499
6.18816	30.33194	30.33428	30.33661	30.33894	30.34128	30.34361
6.19570	29.14572	29.14584	29.14596	29.14608	29.14620	29.14632
7.00243	31.03993	31.04388	31.04783	31.05178	31.05573	31.05968
7.28650	29.50590	29.50570	29.50549	29.50528	29.50508	29.50487
7.79823	36.40770	36.41870	36.42969	36.44069	36.45168	36.46268
8.23207	32.80777	32.80502	32.80228	32.79953	32.79678	32.79404
8.52260	34.10757	34.11009	34.11261	34.11512	34.11764	34.12016
8.63321	33.34227	33.34234	33.34241	33.34248	33.34254	33.34261
9.15826	30.85961	30.85332	30.84703	30.84074	30.83445	30.82816
9.85818	30.45004	30.44963	30.44922	30.44880	30.44839	30.44798
9.88764	24.47344	24.47297	24.47251	24.47204	24.47158	24.47111
10.23884	24.21582	24.21567	24.21553	24.21539	24.21525	24.21510

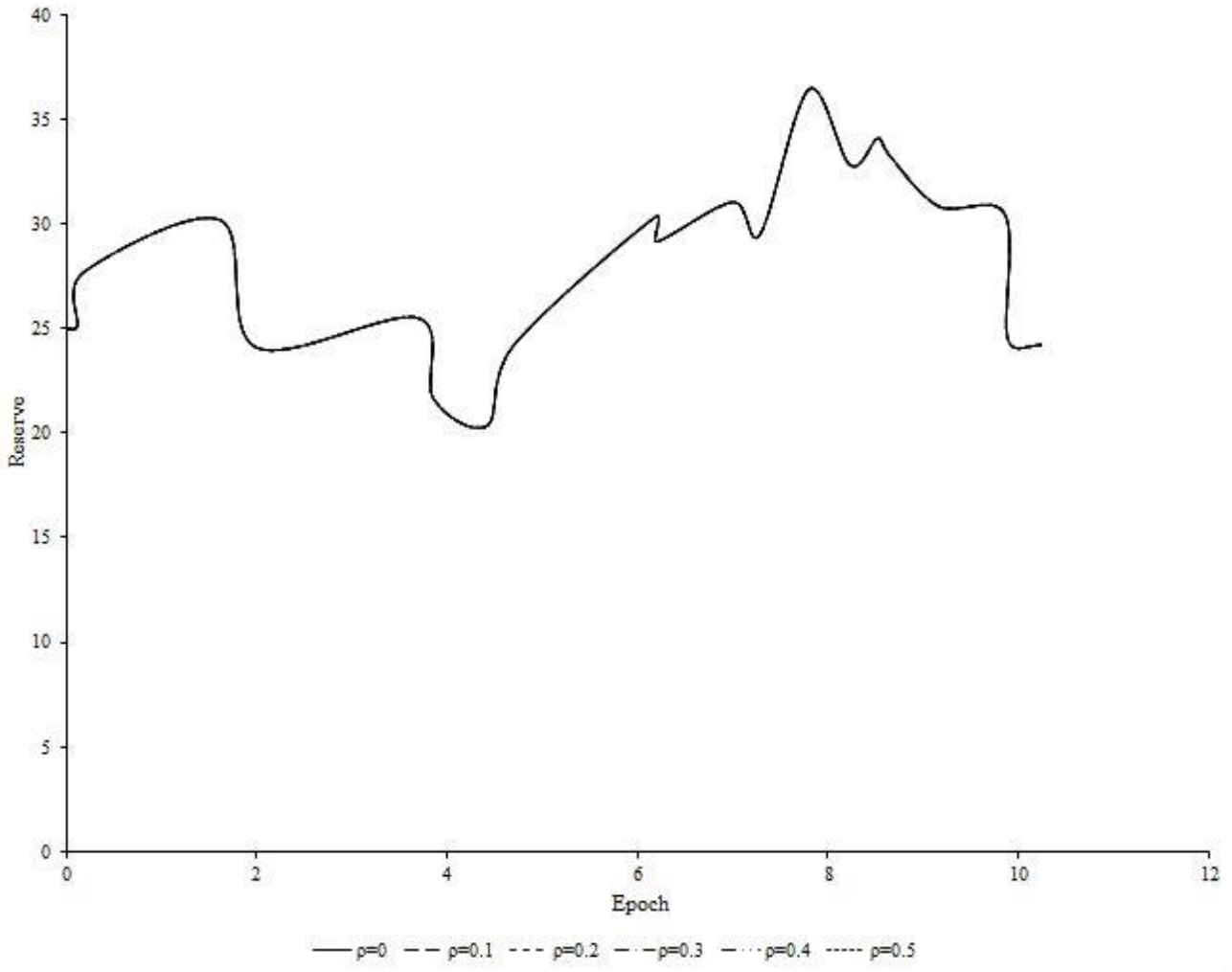


Figure 4.9: The behavior of the reserve with respect to arrival epoch for r=0.12, a=0.21, σ=0.2, δ=0.18, ϕ=1.00 and ϱ is varied .

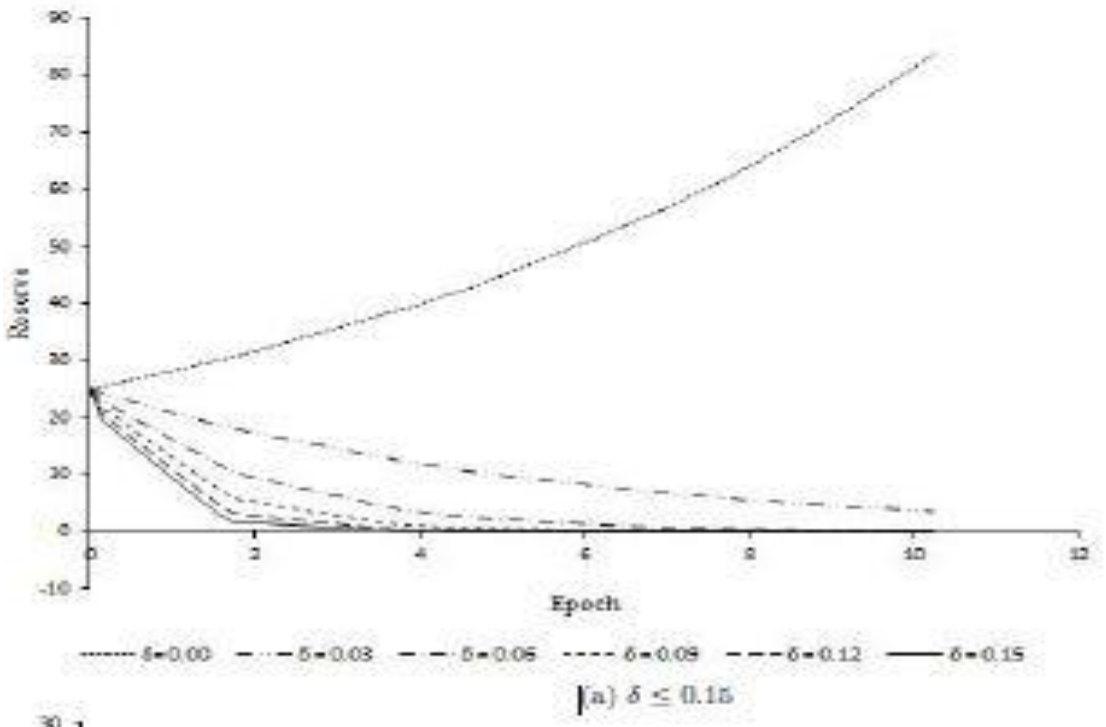
Comments:

Table 4.5 and Figure 4.9 show the behavior of the reserve process when safety loading "ϱ" is varied, δ=0.00 and ϕ=1. The value of the reserve fluctuates with increasing epoch. There is also a slight increase in the value of the reserve when ϱ is increased.

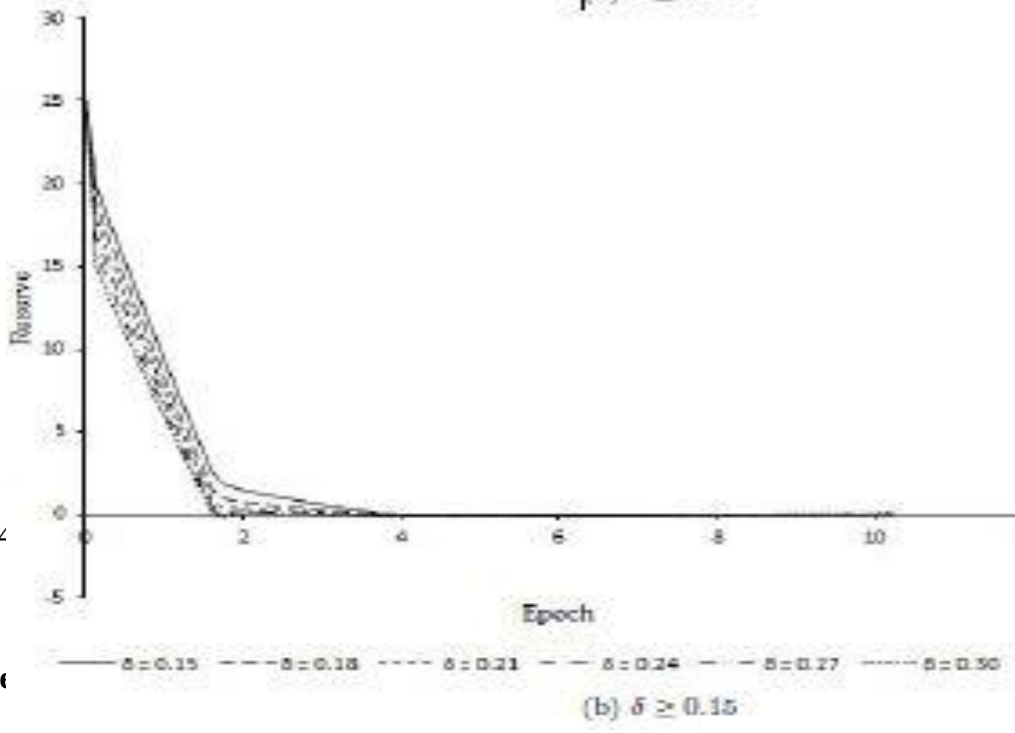
4.1.4 Discussion of Results Obtained When ϱ is Varied

It is worthy of note to mention that safety loading allows the inclusion of both administrative cost and the profit the company intends to make in the long run into the premium charged, through equation 1.3 on 19. However, when the claim size is exponentially distributed, the is only a slight increase in the value of the reserve when safety loading is increased. This slight increase is not sufficient in most case to compensate for the depreciating effect of inflation.

4.1.5 Behavior of Risk Reserve Process with Exponential Claim When drift ”δ” is Varied



(a) $\delta \leq 0.15$

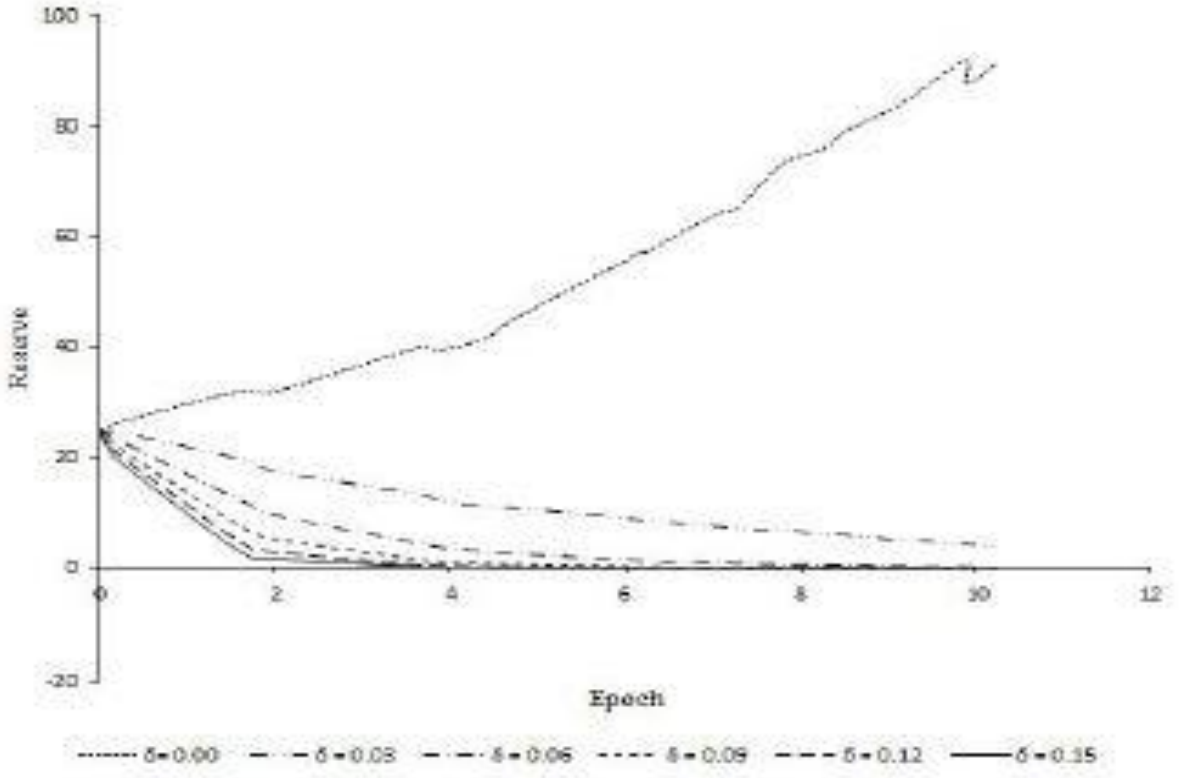


(b) $\delta \geq 0.15$

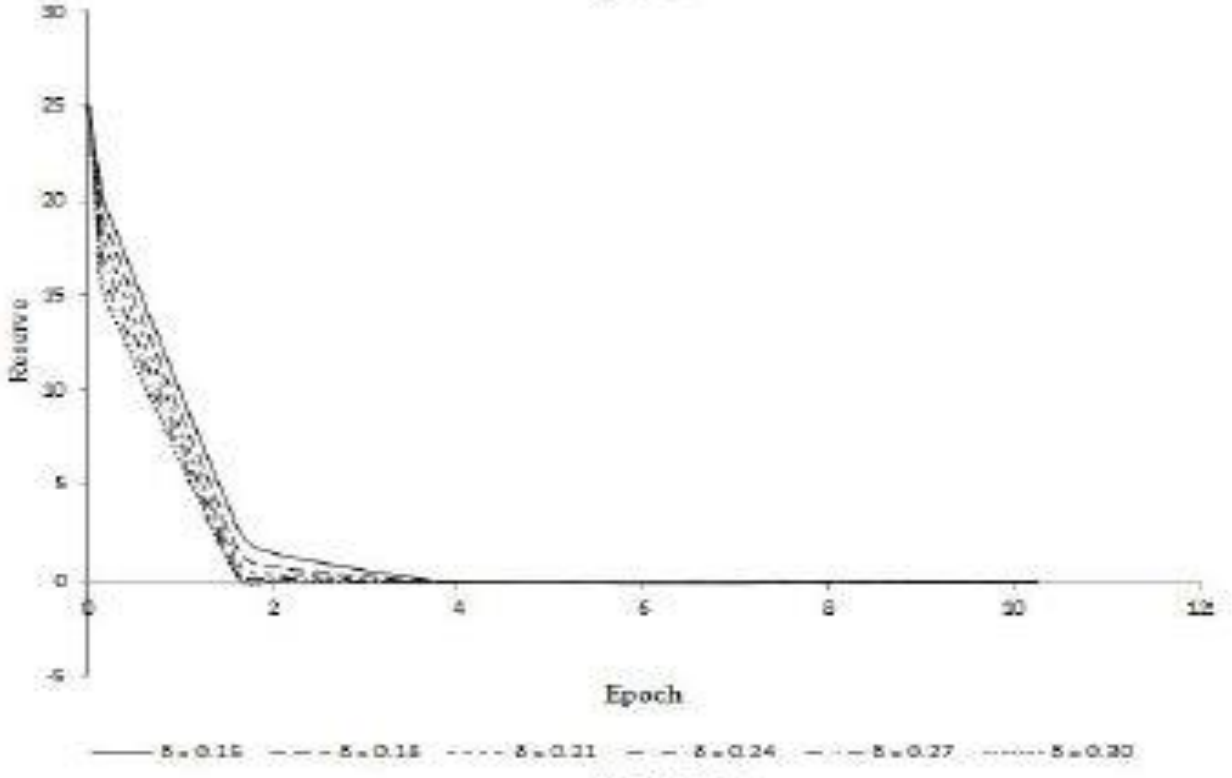
Figure 4 .20, ϕ=0.00

Comme

Figure **Error! Reference source not found.** show the behavior of the reserve when inflation is varied and all resources are invested into risk-less assets. The value of the reserve increases with increasing epoch when δ=0.00. This value decreases sharply for all other assumed values of inflation. There is no fluctuation in the reserve since no resource is invested into risky assets.



(a) $\delta \leq 0.15$

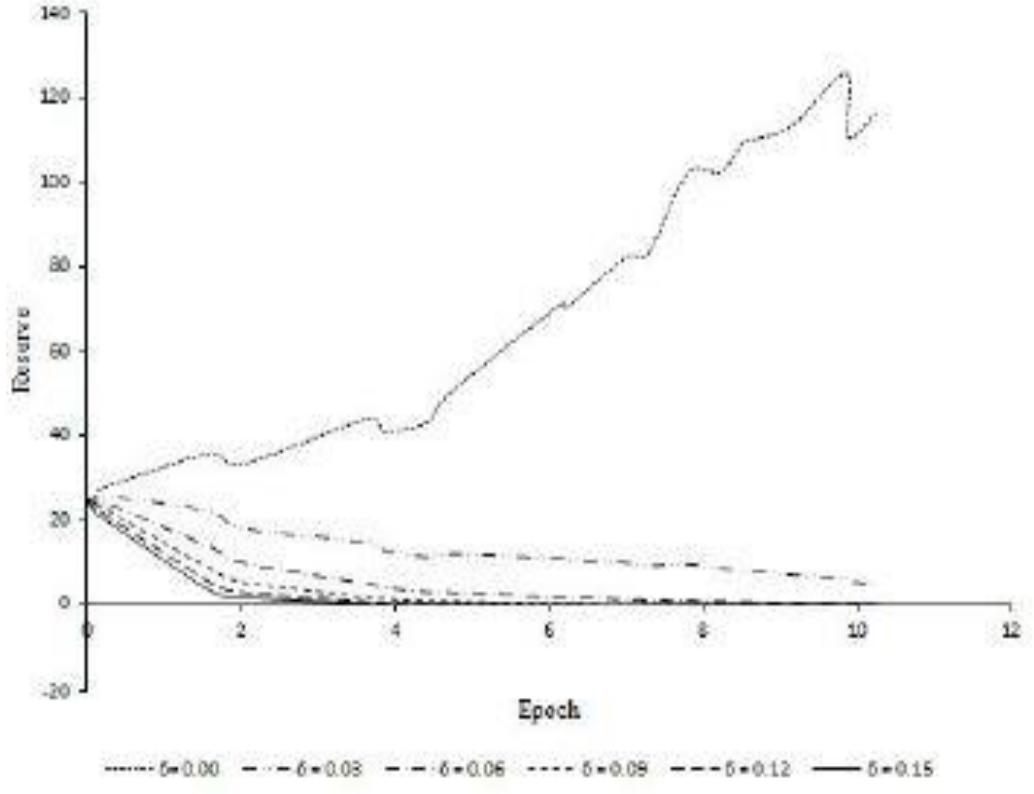


(b) $\delta \geq 0.15$

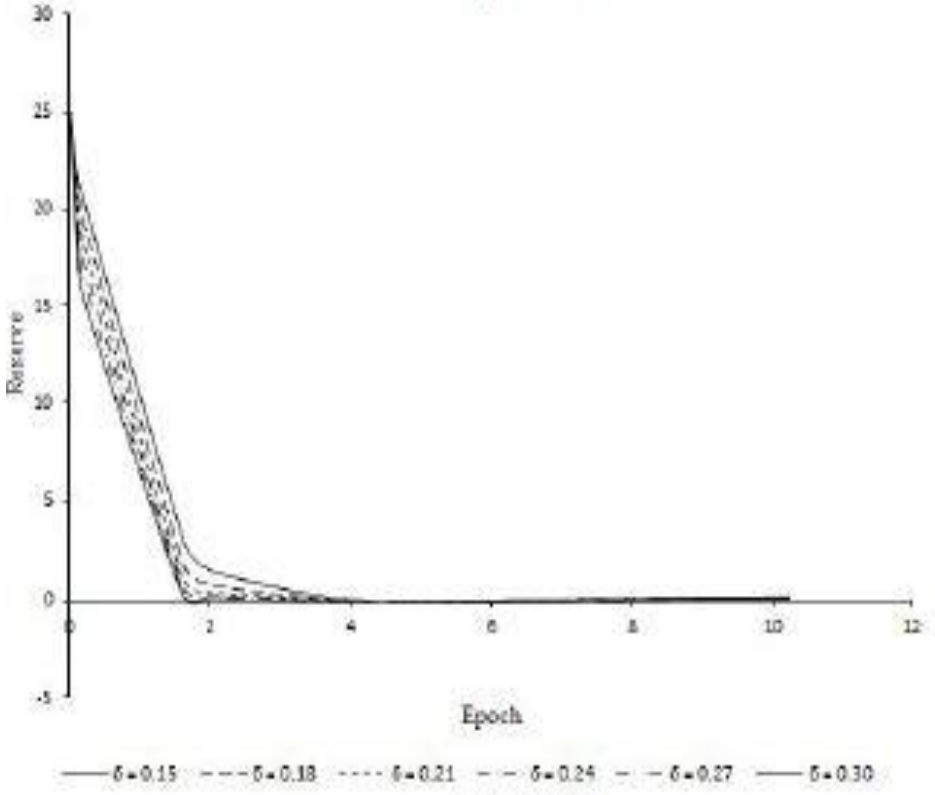
Figure 4.11: The behavior of the reserve with respect to arrival epoch for ϱ=0.60, *a*=0.21, σ=0.20, ϕ=0.20, *r*=0.12 and δ,is varied .

Comments:

Figure **Error! Reference source not found.** show the behavior of the reserve when inflation is varied and 20% of the resources is invested into risky assets. Similar to the behavior when all resources are invested into risky assets, the value of the reserve only increased assuming no inflation but decreases for all other values. The is also a noticeable fluctuation in the reserve as a result of investment into risky assets.



(a) $\delta \leq 0.15$



(b) $\delta \geq 0.15$

Figure 4.12: The behavior of the reserve with respect to arrival epoch for ϱ=0.60, *a*=0.21, σ=0.20, ϕ=0.60 *r*=0.12 and δ is varied .

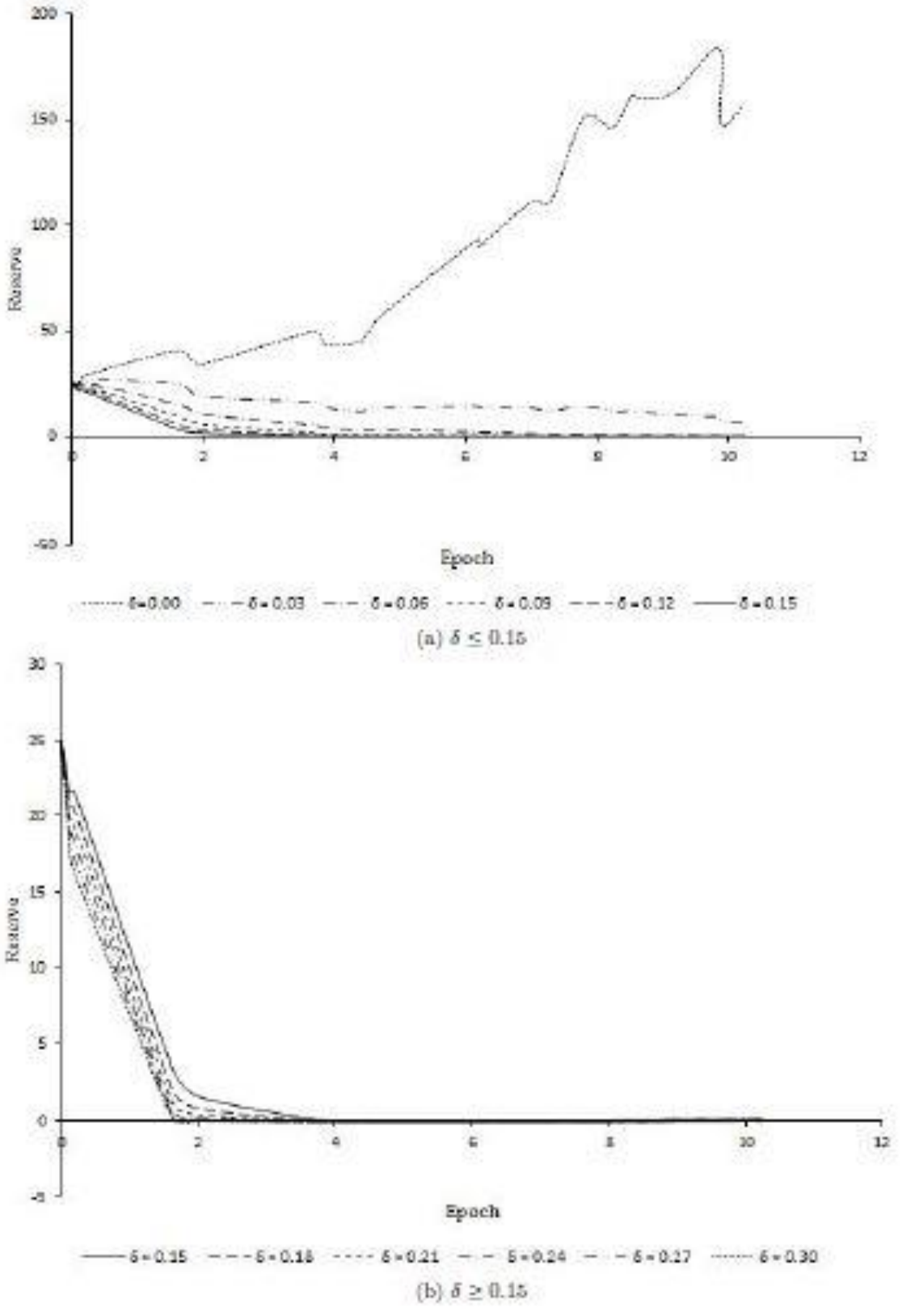


Figure 4.13: The behavior of the reserve with respect to arrival epoch for ϱ=0.60, *a*=0.21, σ=0.20, ϕ=0.20, *r*=0.12 and δ,is varied .

4.1.6 Discussion of Results Obtained When δ is Varied

The results obtained when inflation is varied show that there is a decrease in the value of the reserve resulting from inflation. As inflation is decreased, the value of the reserve progressively depreciates. The reserve increases only when inflation rate is assumed to be zero.

Fluctuation is also noticed in the values of the reserve when investment is made into risky assets, even when no inflation is assumed. In most cases, the effect of inflation is sharply felt before the second year and subsequently, the reserve begins to normalize.

4.1.7 Behavior of Risk Reserve Process with Exponential Claim When drift "ϕ" is Varied

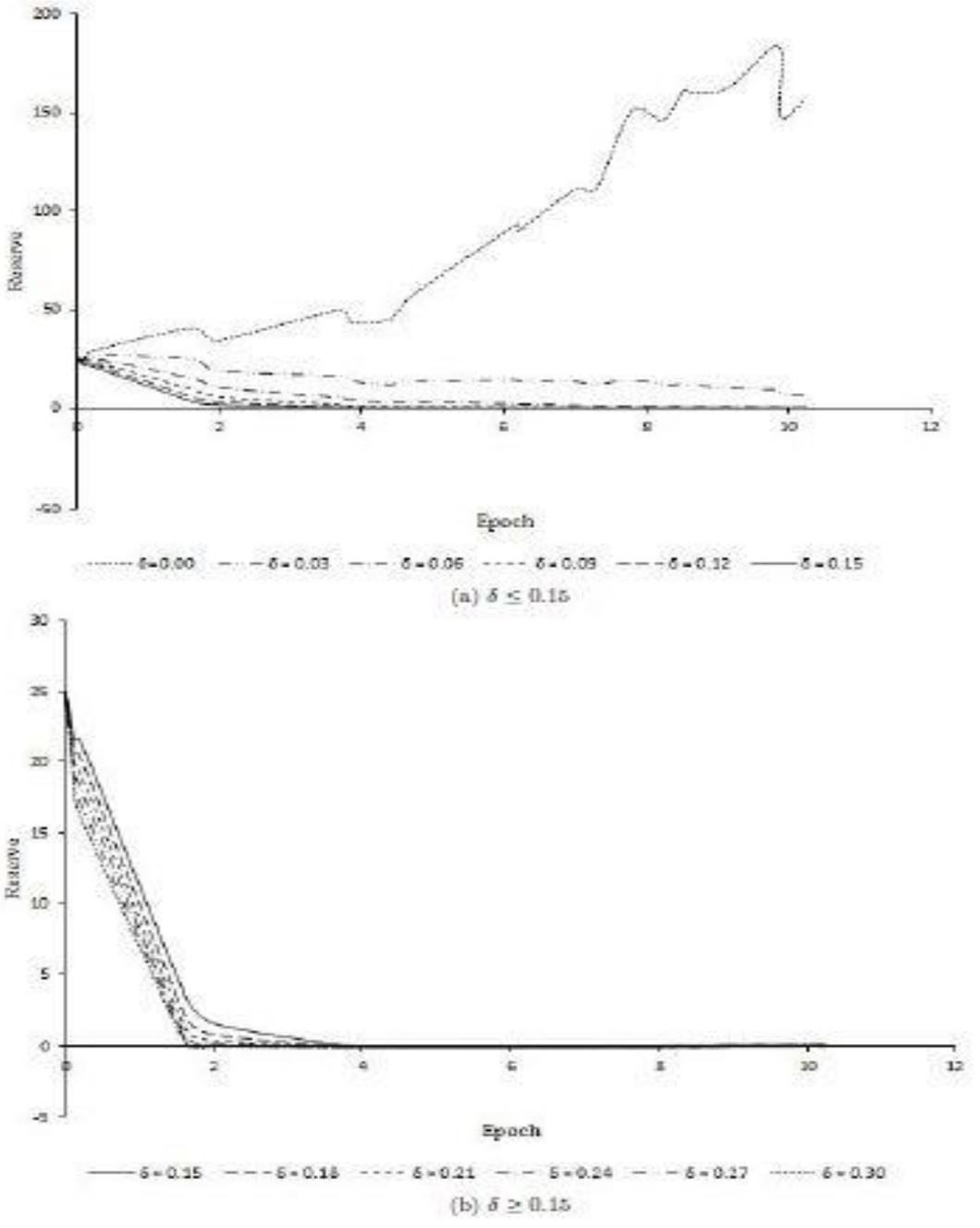


Figure 4.14: The behavior of the reserve with respect to arrival epoch for ϱ=0.60, *a*=0.21, σ=0.20, *r*=0.12, **δ=0.00** and ϕ is varied.

Comments: Figure **Error! Reference source not found.** show the behavior of the reserve when ϕ is varied and inflation is set at δ=0.00. When no investment is made into risky assets, that is ϕ=0.00, there is no fluctuation in the value of the reserve

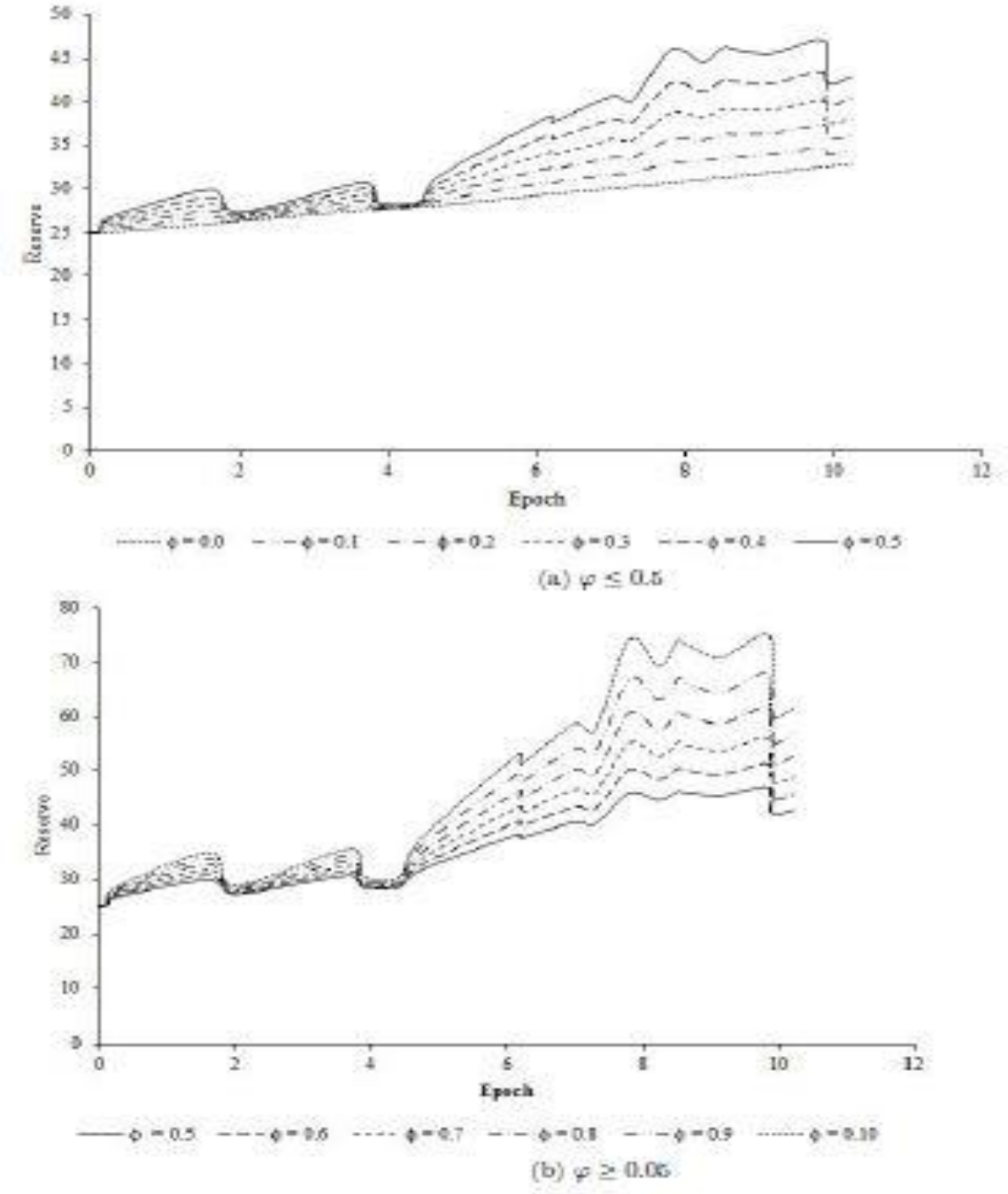


Figure 4.15: The behavior of the reserve with respect to arrival epoch for ϱ=0.60, *a*=0.21, σ=0.20, *r*=0.12, **δ=0.09** and ϕ is varied .

Comments: Figure **Error! Reference source not found.** show the behavior of the reserve when ϕ is varied and inflation is set at δ=0.09. When no investment is made into risky assets, there is no fluctuation in the value of the reserve but with investment into risky asset, fluctuation is observed. Also, the increase in that value of the reserve is noticed to be more pronounced as ϕ increases.

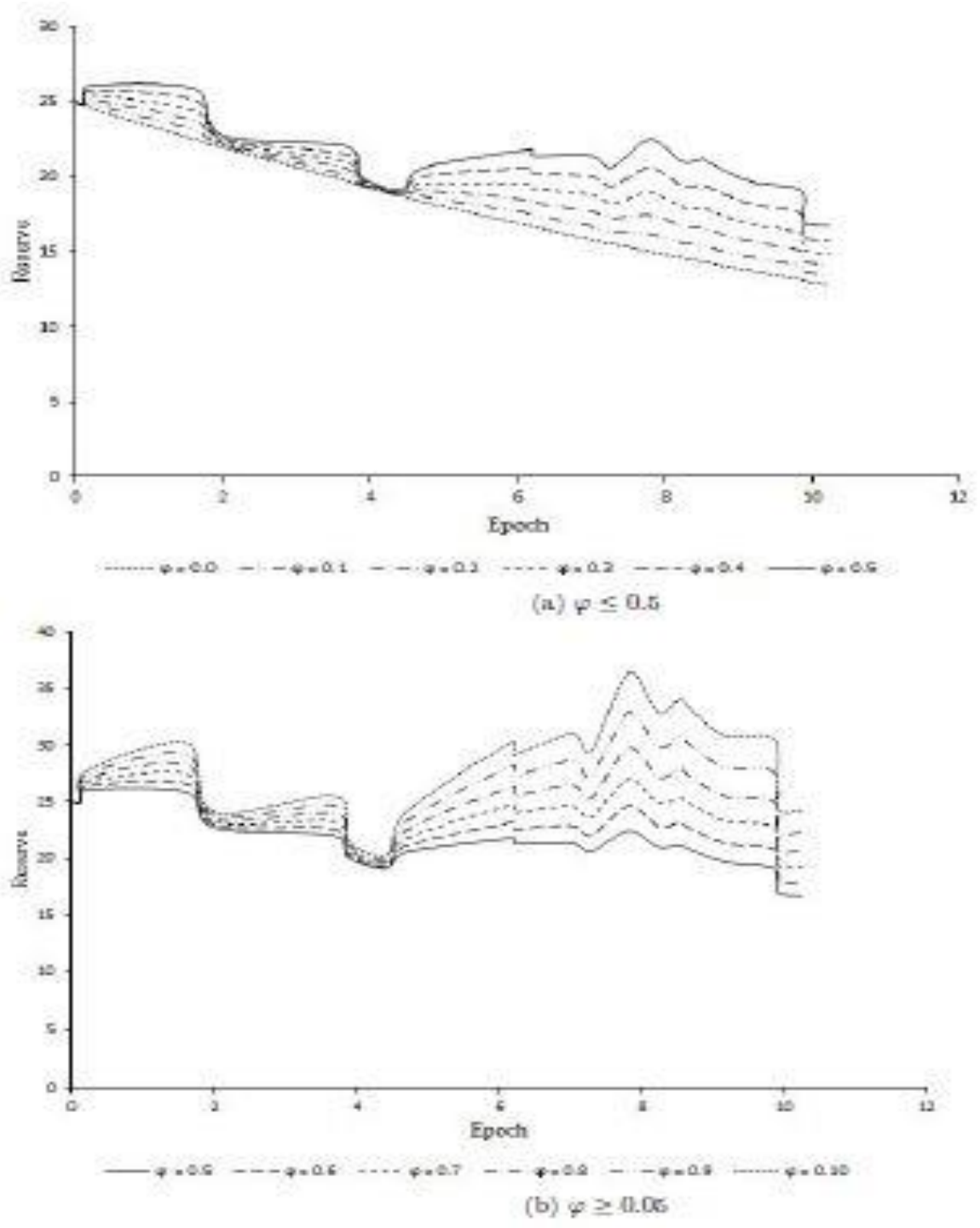


Figure 4.16: The behavior of the reserve with respect to arrival epoch for ϱ=0.60, *a*=0.21, σ=0.20, *r*=0.12, **δ=0.18** and ϕ is varied .

Comments: Figure **Error! Reference source not found.** show the behavior of the reserve when ϕ is varied and inflation is set at δ=0.18. When no investment is made into risky assets, there is no fluctuation in the value of the reserve but with investment into risky asset, fluctuation is observed. Also, the value of the reserve is noticed to decrease with increasing epoch especially when ϕ≤0.5.

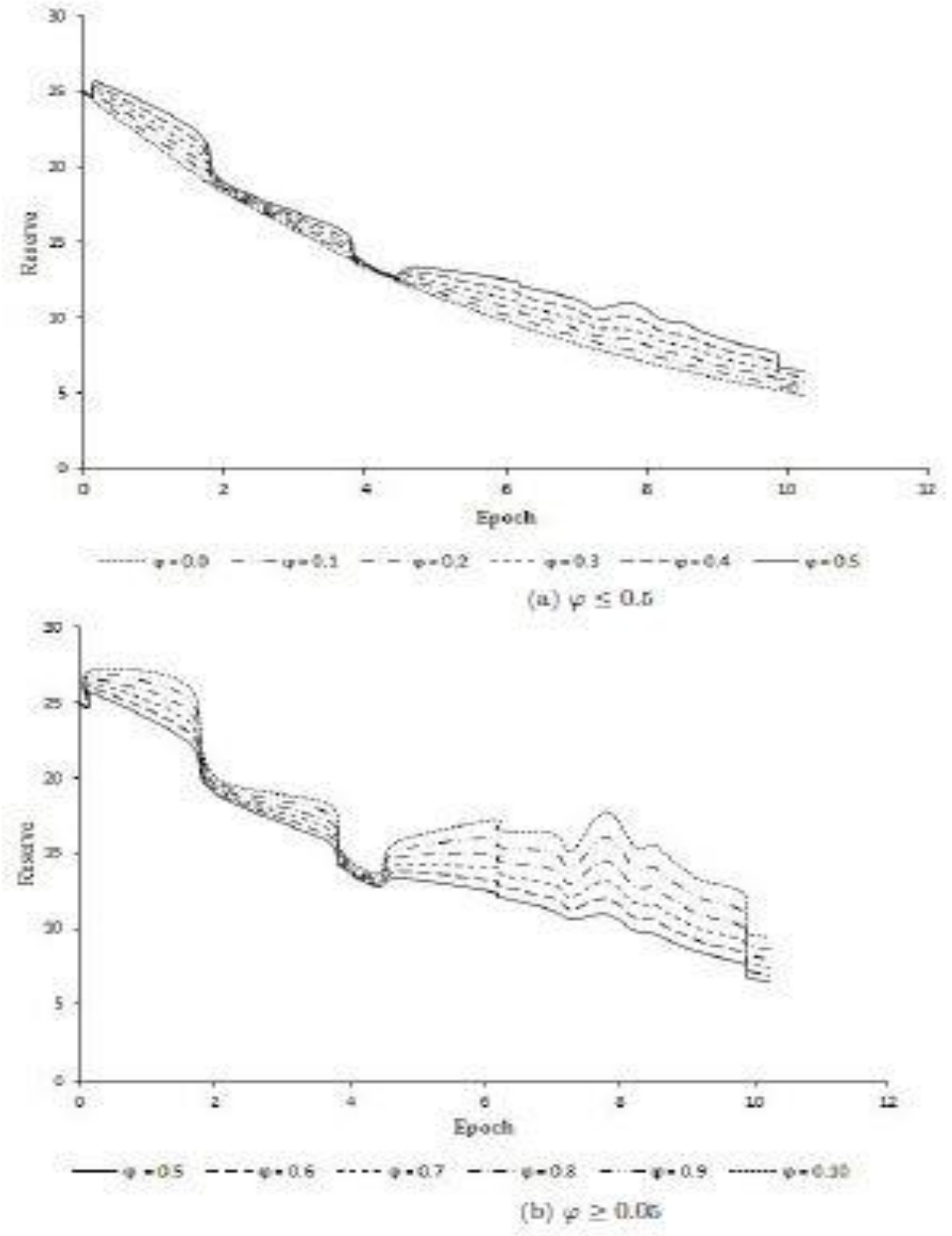


Figure 4.17: The behavior of the reserve with respect to arrival epoch for ϱ=0.60, *a*=0.21, σ=0.20, *r*=0.12, δ=0.27 and ϕ is varied .

Comments: Figure **Error! Reference source not found.** show the behavior of the reserve when ϕ is varied and inflation is set at δ=0.27. The fluctuation in the value of the reserve increases with increase in value of ϕ. No fluctuation is observed at ϕ=0.00. Also, the value of the reserve is noticed to decrease with increasing epoch for all values of ϕ.

4.1.8 Discussion of Results Obtained When ϕ is Varied

The results obtained when ϕ is varied show that there is no fluctuation in the value of the reserve when no investment is made into risky assets. However increasing the investments into risky assets increases the fluctuation. Also the value of the reserve is noticed to decrease with increasing epoch as inflation increases beyond δ=0.18, no matter the level of investment made into risky assets

4.2 Behaviour of the Ruin Probabilities

The behavior of ruin probability with respect to different parameters as defined in equation (3.80), where the focus is on ruin attributable mainly to the effect of claims, is examined below. All other parameters are as assumed above while the parameter of interest is varied over specific range of values. The estimates are presented in tables and graphs for better illustration and proper inference. In some cases were the curve of a particular situation has values bigger than others, the tables throw more lights on the results.

4.2.1 Ruin Probability Attributable to Claims When Claim Sizes are Exponential and Premium is Varied

Table 4.6: Ruin Probabilities Attributable to claim when premium rate *c* is varied, claim size is exponential and all other parameters kept are constant at **λ=0.50**, *r*=0.12, **Θ=0.50**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

u	c = 0.0001	c = 0.5	c = 1.0	c = 1.5	c = 2.0	c = 2.5
0	9.98851E-01	1.00374E-01	1.00750E-02	1.01127E-03	1.01505E-04	1.01885E-05
2	5.80246E-01	4.22415E-02	3.17482E-03	2.68403E-04	2.40376E-05	2.22456E-06
4	1.92536E-01	1.15687E-02	7.66954E-04	5.94451E-05	4.99296E-06	4.39807E-07
6	4.97425E-02	2.54172E-03	1.58599E-04	1.17187E-05	9.47279E-07	8.08716E-08
8	1.18289E-02	4.60053E-04	2.83840E-05	2.06025E-06	1.63650E-07	1.37561E-08
10	3.03232E-03	5.08255E-05	3.61247E-06	2.76726E-07	2.26159E-08	1.94117E-09
12	1.01932E-03	1.48738E-05	4.47062E-07	1.58780E-08	2.97263E-10	1.19653E-12
14	4.90668E-04	1.92001E-05	8.41442E-07	4.80024E-08	3.07032E-09	2.35948E-10
16	3.02431E-04	1.51792E-05	6.99722E-07	4.15924E-08	2.75207E-09	2.16646E-10
18	2.09797E-04	1.13608E-05	5.35475E-07	3.23582E-08	2.15579E-09	1.71771E-10
20	1.53992E-04	8.58783E-06	4.10806E-07	2.50731E-08	1.66849E-09	1.33847E-10

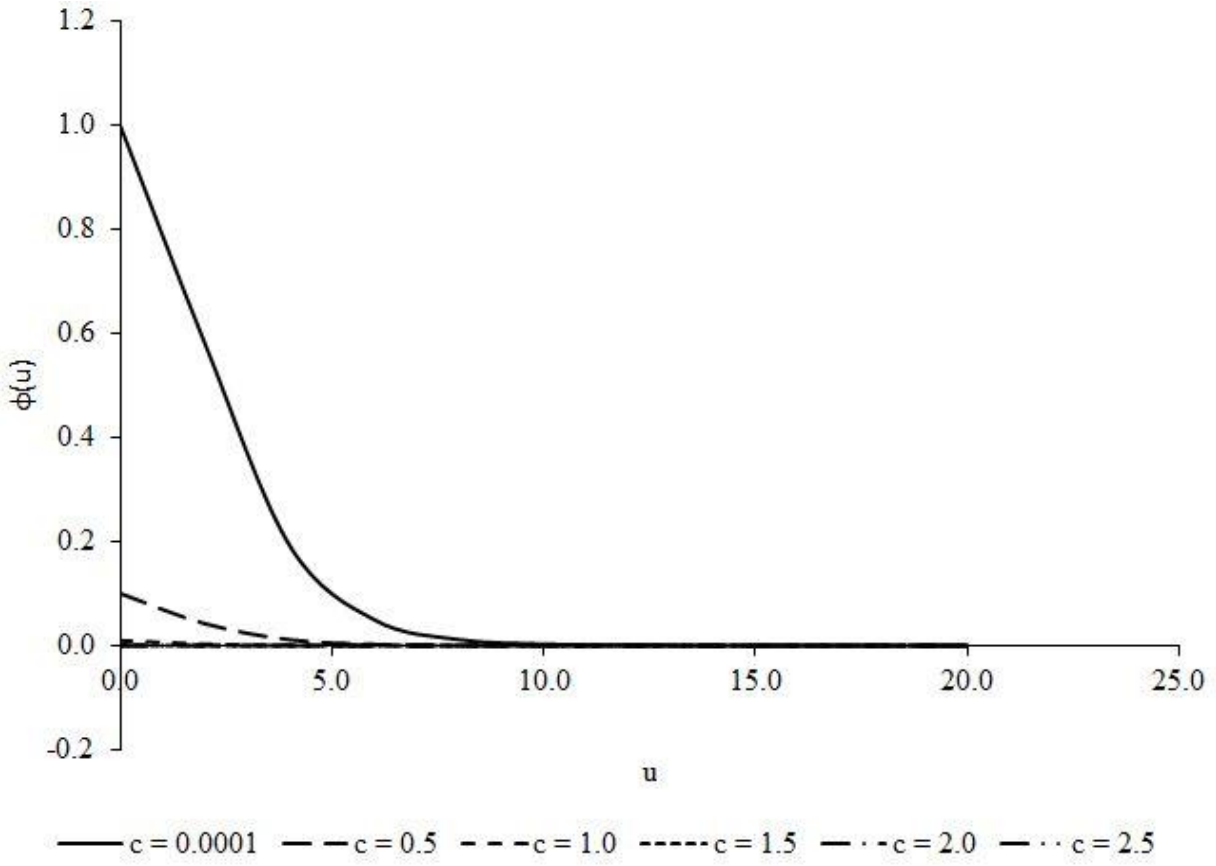


Figure 4.18: Ruin Probabilities Attributable to claim when premium rate *c* is varied, claim size is exponential and all other parameters kept are constant at **λ=0.50**, *r*=0.12, **Θ=0.50**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

Comments:

Figure 4.18 shows the behavior of the ruin probability attributable to claims when claim sizes is assumed to be exponential, premium is varied between c= 0.0001, 0.5, 1.0,1.5,2.0 and 2.5, λ=0.50, Θ=0.50 and other parameters are kept constant. From the figure, the ruin probability decreases as *c* is increased. There is a sharp decrease in ruin probability when $0 \leq u \leq 0.5$ for all values of the premium rate. Subsequently, the decrease becomes gradual. This is better illustrated by table 4.6.

Table 4.7: Ruin Probabilities Attributable to claim when premium rate *c* is varied, claim size is exponential and all other parameters kept are constant at **λ=0.50**, *r*=0.12, **Θ=1.00**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

u	c = 0.0001	c = 0.5	c = 1.0	c = 1.5	c = 2.0	c = 2.5
0	9.98851E-01	1.00374E-01	1.00750E-02	1.01127E-03	1.01505E-04	1.01885E-05
2	5.80246E-01	4.22415E-02	3.17482E-03	2.68403E-04	2.40376E-05	2.22456E-06
4	1.92536E-01	1.15687E-02	7.66954E-04	5.94451E-05	4.99296E-06	4.39807E-07
6	4.97425E-02	2.54172E-03	1.58599E-04	1.17187E-05	9.47279E-07	8.08716E-08
8	1.18289E-02	4.60053E-04	2.83840E-05	2.06025E-06	1.63650E-07	1.37561E-08
10	3.03232E-03	5.08255E-05	3.61247E-06	2.76726E-07	2.26159E-08	1.94117E-09
12	1.01932E-03	1.48738E-05	4.47062E-07	1.58780E-08	2.97263E-10	1.19653E-12
14	4.90668E-04	1.92001E-05	8.41442E-07	4.80024E-08	3.07032E-09	2.35948E-10
16	3.02431E-04	1.51792E-05	6.99722E-07	4.15924E-08	2.75207E-09	2.16646E-10
18	2.09797E-04	1.13608E-05	5.35475E-07	3.23582E-08	2.15579E-09	1.71771E-10
20	1.53992E-04	8.58783E-06	4.10806E-07	2.50731E-08	1.66849E-09	1.33847E-10

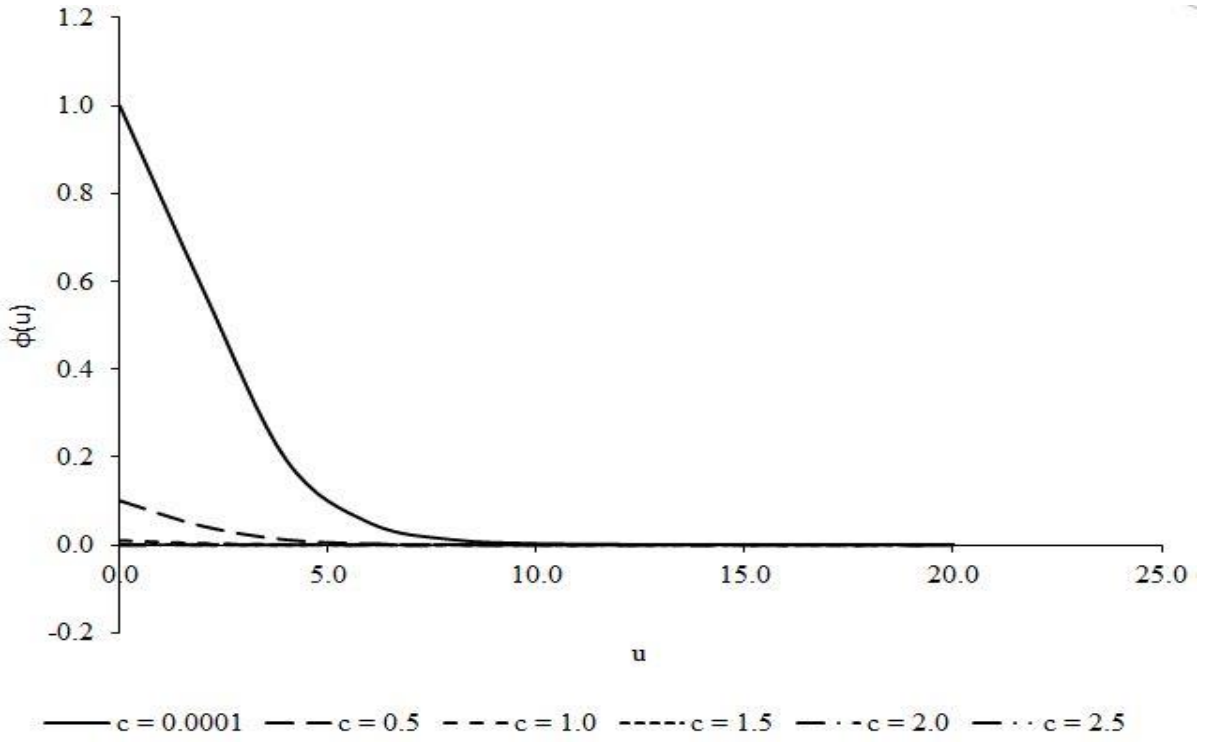


Figure 4.19: Ruin Probabilities Attributable to claim when premium rate *c* is varied, claim size is exponential and all other parameters kept are constant at **λ=0.50**, *r*=0.12, **Θ=1.00**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

Comments:

In Figure 4.19, λ and Θ are set at 0.50 and 1.00 respectively. The ruin probability decreases as the premium rate is increased. The decrease is sharp for $u<8$ and subsequently becomes gradual.

Table 4.7 further illustrates these estimates as more clarity.

Table 4.8: Ruin Probabilities Attributable to claim when premium rate *c* is varied, claim size is exponential and all other parameters kept are constant at **λ=0.5**, *r*=0.12, **Θ=1.5**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

u	c = 0.0001	c = 0.5	c = 1.0	c = 1.5	c = 2.0	c = 2.5
0.0	9.98277E-01	3.18004E-02	1.01127E-03	3.21587E-05	1.02266E-06	3.25210E-08
2	3.49797E-01	5.91893E-03	1.28133E-04	3.31296E-06	9.23236E-08	2.67887E-09
4	4.97163E-02	5.62129E-04	1.14848E-05	2.73174E-07	7.13276E-09	1.96675E-10
6	5.85626E-03	8.99705E-06	6.80313E-07	1.73119E-08	4.58096E-10	1.26638E-11
8	1.01706E-03	2.03511E-05	8.29095E-08	7.29411E-10	5.27251E-12	4.20200E-13
10	3.76068E-04	1.37762E-05	8.63860E-08	1.28046E-09	2.29190E-11	6.81369E-14
12	2.09233E-04	8.74885E-06	5.85557E-08	9.10449E-10	1.59559E-11	1.12275E-13
14	1.33382E-04	5.83433E-06	4.01639E-08	6.36321E-10	1.02614E-11	2.72223E-13
16	9.08498E-05	4.08360E-06	2.86062E-08	4.56239E-10	6.32342E-12	3.91982E-13
18	6.48014E-05	2.97180E-06	2.10433E-08	3.34278E-10	2.47714E-12	4.83997E-13
20	4.79132E-05	2.23143E-06	1.58862E-08	2.48545E-10	4.04358E-12	5.57152E-13

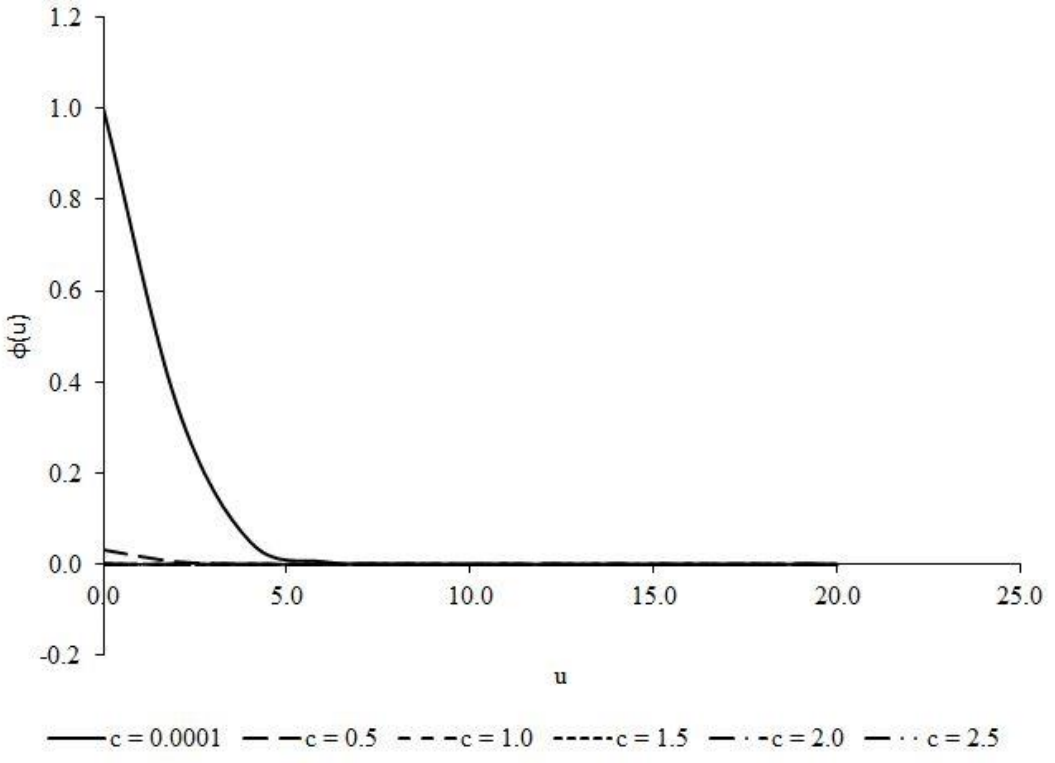


Figure 4.20: Ruin Probabilities Attributable to claim when premium rate *c* is varied, claim size is exponential and all other parameters kept are constant at **λ=0.5**, *r*=0.12, **Θ=1.5**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

Comments:

In Figure 4.20 and table 4.8 show the ruin probabilities when λ and Θ are set at 0.50 and 1.00 respectively. The ruin probability decreases as the premium rate is increased and there is a sharp decrease when u<5 and subsequently becomes gradual.

Table 4.9: Ruin Probabilities Attributable to claim when premium rate c is varied, claim size is exponential and all other parameters kept are constant at **λ=1.00**, r=0.12, **Θ=0.50**, ϕ=0.60, σ_p=0.20, a=0.21, α=0.00 and σ_R=0.20.

u	c = 0.0001	c = 0.5	c = 1.0	c = 1.5	c = 2.0	c = 2.5
0	9.99425E-01	3.16819E-01	1.00374E-01	3.18004E-02	1.00750E-02	3.19194E-03
2	4.26725E-01	3.14309E-01	9.75342E-02	2.98000E-02	9.00227E-03	2.70475E-03
4	1.83995E-01	2.97727E-01	8.83020E-02	2.56082E-02	7.32980E-03	2.08988E-03
6	7.98265E-02	2.59432E-01	7.29090E-02	2.00526E-02	5.45955E-03	1.48646E-03
8	3.47466E-02	2.05922E-01	5.49973E-02	1.44290E-02	3.76286E-03	9.85299E-04
10	1.51857E-02	1.49944E-01	3.83172E-02	9.65817E-03	2.42918E-03	6.15641E-04
12	6.71380E-03	1.01378E-01	2.49614E-02	6.08420E-03	1.48463E-03	3.66093E-04
14	3.05356E-03	6.44016E-02	1.53716E-02	3.64291E-03	8.66576E-04	2.08803E-04
16	1.46627E-03	3.88327E-02	9.02981E-03	2.08985E-03	4.86526E-04	1.14943E-04
18	7.64179E-04	2.24103E-02	5.09719E-03	1.15615E-03	2.64238E-04	6.13791E-05
20	4.38679E-04	1.24608E-02	2.78122E-03	6.20021E-04	1.39471E-04	3.19251E-05

Comments:

Figure 4.21 and table 4.9 show the ruin probabilities when λ and Θ are set at 1.00 and 1.50 respectively. When c=0.0001, the decrease in premium rate is sharp at u<0.50 and subsequently becomes gradual. However the behavior is different for other values of c as there is a gradual decrease. For c>0.0001, the ruin probability decreases as premium rate is increased.

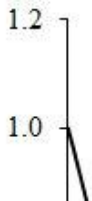


Figure 4.21: Ruin Probabilities Attributable to claim when premium rate *c* is varied, claim size is exponential and all other parameters kept are constant at **λ=1.00**, *r*=0.12, **Θ=0.50**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

Table 4.10: Ruin Probabilities Attributable to claim when premium rate *c* is varied, claim size is exponential and all other parameters kept are constant at **λ=1.00**, *r*=0.12, **Θ=1.00**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

u	c = 0.0001	c = 0.5	c = 1.0	c = 1.5	c = 2.0	c = 2.5
0	9.98851E-01	1.00374E-01	1.00750E-02	1.01127E-03	1.01505E-04	1.01885E-05
2	1.00000E+00	8.83105E-02	7.32991E-03	5.97408E-04	4.98020E-05	4.28414E-06
4	7.74794E-01	5.50036E-02	3.76293E-03	2.60720E-04	1.90309E-05	1.46865E-06
6	4.17462E-01	2.49644E-02	1.48466E-03	9.17506E-05	6.09624E-06	4.35098E-07
8	1.70391E-01	9.03089E-03	4.86536E-04	2.77029E-05	1.71854E-06	1.15718E-07
10	5.71879E-02	2.78155E-03	1.39473E-04	7.47350E-06	4.40175E-07	2.83425E-08
12	1.66805E-02	7.61634E-04	3.61835E-05	1.85062E-06	1.04741E-07	6.50147E-09
14	4.38348E-03	1.90785E-04	8.69327E-06	4.28928E-07	2.35316E-08	1.41511E-09
16	1.06274E-03	4.45941E-05	1.96686E-06	9.44357E-08	5.07885E-09	2.94788E-10
18	2.41594E-04	9.87321E-06	4.25429E-07	2.01566E-08	1.09335E-09	5.93686E-11
20	5.19963E-05	2.10346E-06	9.02835E-08	4.41352E-09	2.67318E-10	1.20908E-11

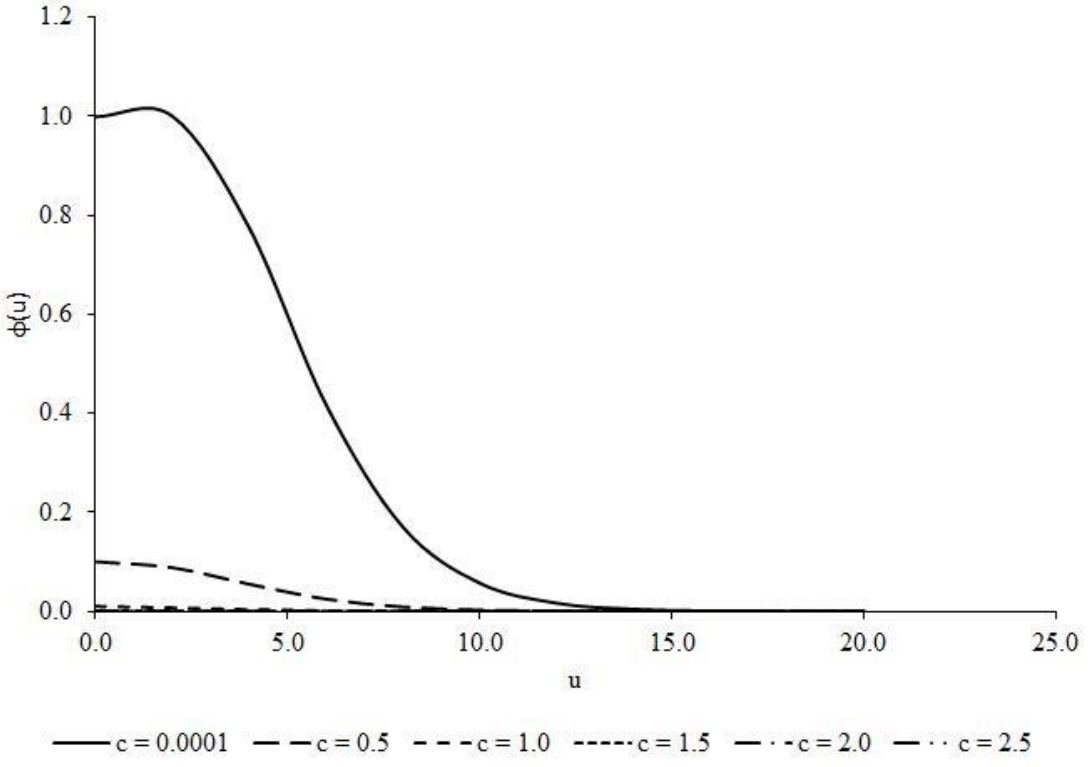


Figure 4.22: Ruin Probabilities Attributable to claim when premium rate *c* is varied, claim size is exponential and all other parameters kept are constant at **λ=1.00**, *r*=0.12, **Θ=1.00**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

Table 4.11: Ruin Probabilities Attributable to claim when premium rate *c* is varied, claim size is exponential and all other parameters kept are constant at **λ=1.00**, *r*=0.12, **Θ=1.50**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

u	c = 0.0001	c = 0.5	c = 1.0	c = 1.5	c = 2.0	c = 2.5
0	9.98277E-01	3.18004E-02	1.01127E-03	3.21587E-05	1.02266E-06	3.25210E-08
2	6.97792E-01	2.00550E-02	4.07439E-04	8.94996E-06	2.16039E-07	5.59399E-09
4	3.36279E-01	6.08500E-03	9.17509E-05	1.61520E-06	3.29676E-08	7.49231E-10
6	1.76850E-01	1.15630E-03	1.45647E-05	2.23149E-07	4.08092E-09	8.48425E-11
8	1.34957E-01	1.65100E-04	1.85062E-06	2.58019E-08	4.36857E-10	8.54473E-12
10	1.73174E-02	1.95408E-05	2.02354E-07	2.62734E-09	4.21958E-11	8.16168E-13
12	1.78714E-03	2.03350E-06	2.01559E-08	2.42147E-10	3.96711E-12	1.07665E-13
14	1.13997E-04	1.98973E-07	2.19456E-09	2.12456E-11	6.47775E-13	5.18127E-14
16	1.98146E-05	2.54050E-08	5.69767E-10	3.97549E-12	4.56820E-13	5.18213E-14
18	1.66853E-05	1.04374E-08	4.33955E-10	4.21450E-12	1.70707E-12	5.59036E-14
20	8.85145E-06	9.45032E-09	4.25772E-10	5.67536E-12	1.68310E-12	5.99051E-14

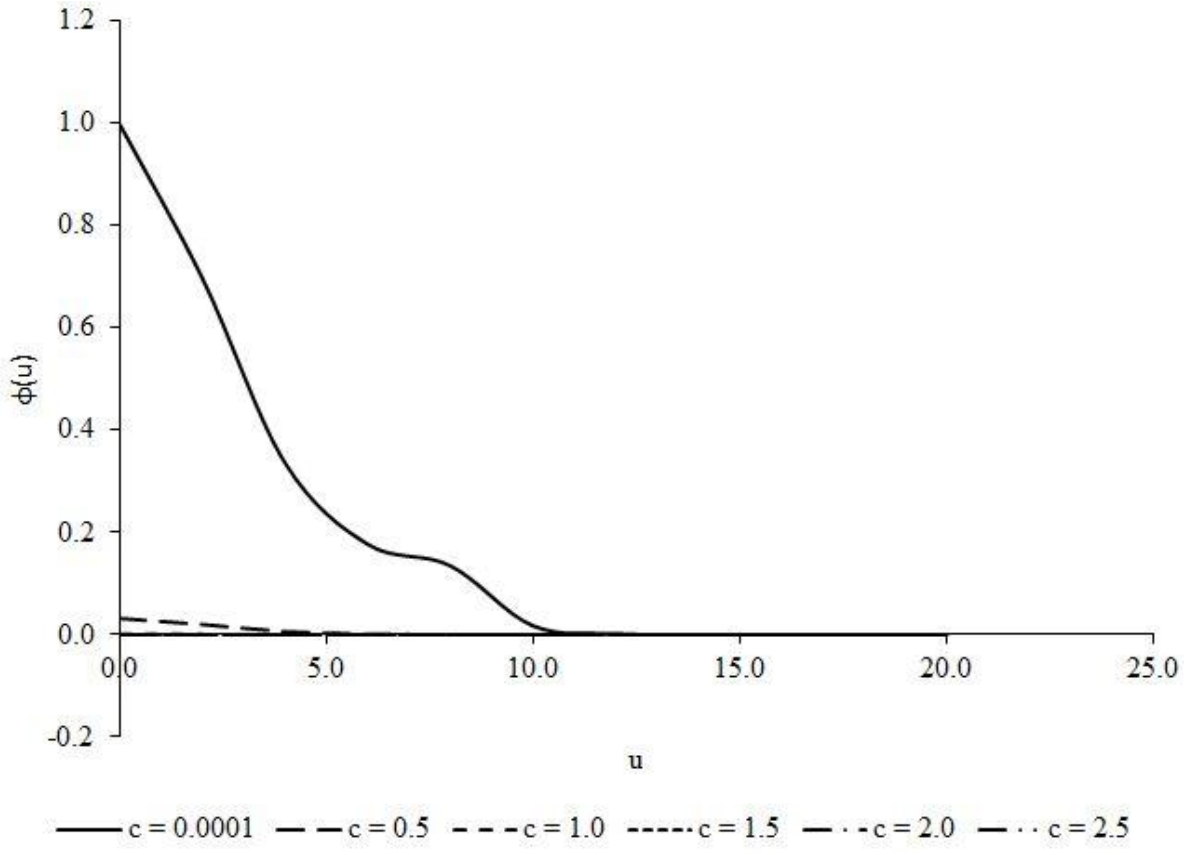


Figure 4.23: Ruin Probabilities Attributable to claim when premium rate *c* is varied, claim size is exponential and all other parameters kept are constant at **λ=1.00**, *r*=0.12, **Θ=1.50**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

Table 4.12: Ruin Probabilities Attributable to claim when premium rate *c* is varied, claim size is exponential and all other parameters kept are constant at **λ=1.50**, *r*=0.12, **Θ=0.50**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

u	c = 0.0001	c = 0.5	c = 1.0	c = 1.5	c = 2.0	c = 2.5
0	9.99425E-01	3.16819E-01	1.00374E-01	3.18004E-02	1.00750E-02	3.19194E-03
2	4.09820E-01	3.16794E-01	1.00305E-01	3.17010E-02	9.98117E-03	3.12514E-03
4	1.69752E-01	3.16092E-01	9.95789E-02	3.11517E-02	9.64758E-03	2.95281E-03
6	7.11326E-02	3.11622E-01	9.67251E-02	2.96247E-02	8.93309E-03	2.65136E-03
8	3.02074E-02	2.98156E-01	9.02263E-02	2.68218E-02	7.82751E-03	2.24521E-03
10	1.30259E-02	2.72142E-01	7.97505E-02	2.29173E-02	6.46160E-03	1.79136E-03
12	5.71622E-03	2.34607E-01	6.64258E-02	1.84459E-02	5.03027E-03	1.35057E-03
14	2.55896E-03	1.90509E-01	5.21561E-02	1.40214E-02	3.70735E-03	9.66703E-04
16	1.17157E-03	1.46019E-01	3.87435E-02	1.01111E-02	2.59968E-03	6.60292E-04
18	5.49934E-04	1.06095E-01	2.73617E-02	6.95245E-03	1.74333E-03	4.32516E-04
20	2.65289E-04	7.34483E-02	1.84656E-02	4.58132E-03	1.12338E-03	2.72935E-04

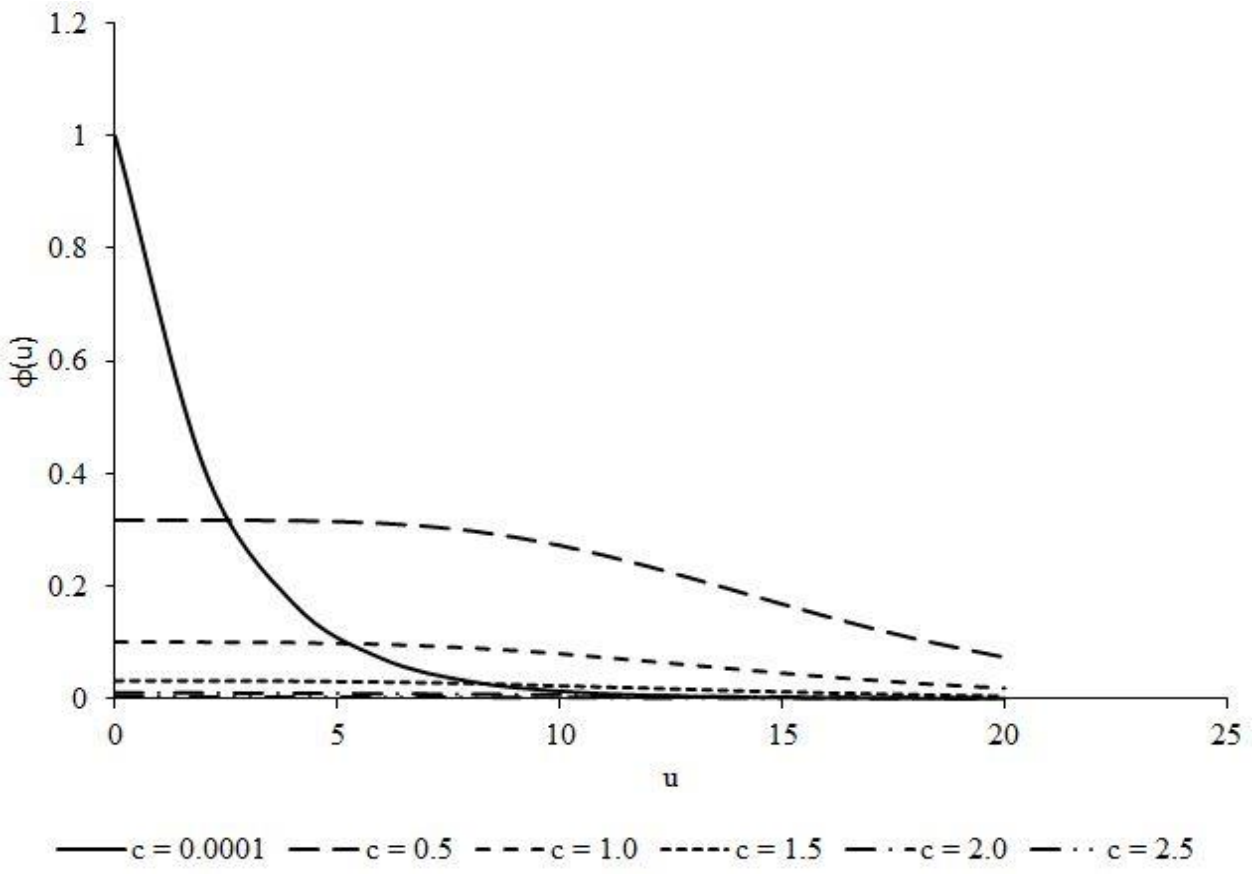


Figure 4.24: Ruin Probabilities Attributable to claim when premium rate *c* is varied, claim size is exponential and all other parameters kept are constant at **λ=1.50**, *r*=0.12, **Θ=0.50**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

Table 4.13: Ruin Probabilities Attributable to claim when premium rate *c* is varied, claim size is exponential and all other parameters kept are constant at **λ=1.50**, *r*=0.12, **Θ=1.00**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

u	c = 0.0001	c = 0.5	c = 1.0	c = 1.5	c = 2.0	c = 2.5
0	9.98851E-01	1.00374E-01	1.00750E-02	1.01127E-03	1.01505E-04	1.01885E-05
2	1.69655E-01	9.95790E-02	9.64758E-03	8.93382E-04	7.96628E-05	6.98773E-06
4	3.01904E-02	9.02263E-02	7.82751E-03	6.34864E-04	4.94509E-05	3.82105E-06
6	5.71307E-03	6.64258E-02	5.03027E-03	3.58388E-04	2.48189E-05	1.72793E-06
8	1.17094E-03	3.87435E-02	2.59968E-03	1.66309E-04	1.04762E-05	6.71227E-07
10	2.65150E-04	1.84656E-02	1.12338E-03	6.59458E-05	3.85228E-06	2.30969E-07
12	6.74828E-05	7.47954E-03	4.20877E-04	2.30686E-05	1.26843E-06	7.20641E-08
14	1.94821E-05	2.66134E-03	1.40678E-04	7.29537E-06	3.81833E-07	2.07513E-08
16	6.37036E-06	8.53281E-04	4.28701E-05	2.12456E-06	1.06763E-07	5.58727E-09
18	2.33467E-06	2.51310E-04	1.21081E-05	5.77844E-07	2.80673E-08	1.41871E-09
20	9.44854E-07	6.89890E-05	3.20925E-06	1.48370E-07	7.00379E-09	3.39632E-10

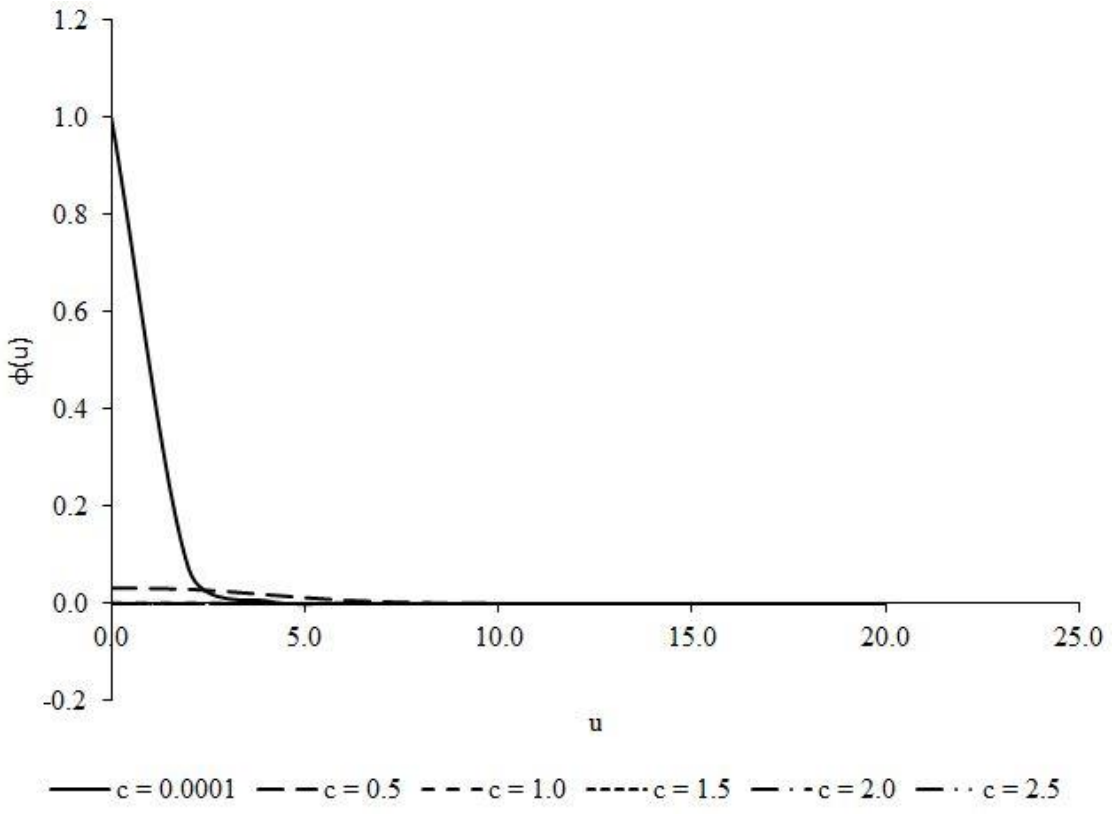


Figure 4.25: Ruin Probabilities Attributable to claim when premium rate *c* is varied, claim size is exponential and all other parameters kept are constant at **λ=1.50**, *r*=0.12, **Θ=1.00**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

Table 4.14: Ruin Probabilities Attributable to claim when premium rate *c* is varied, claim size is exponential and all other parameters kept are constant at **λ=1.50**, *r*=0.12, **Θ=1.50**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

u	c = 0.0001	c = 0.5	c = 1.0	c = 1.5	c = 2.0	c = 2.5
0	9.98277E-01	3.18004E-02	1.01127E-03	3.21587E-05	1.02266E-06	3.25210E-08
2	7.10523E-02	2.96247E-02	7.75960E-04	1.85148E-05	4.38680E-07	1.07765E-08
4	5.70992E-03	1.84459E-02	3.58388E-04	6.53343E-06	1.23518E-07	2.51978E-09
6	5.49350E-04	6.95245E-03	1.06571E-04	1.59776E-06	2.57164E-08	4.59057E-10
8	6.74486E-05	1.78242E-03	2.30686E-05	3.00660E-07	4.30464E-09	6.96015E-11
10	1.09684E-05	3.47017E-04	3.97385E-06	4.66963E-08	6.11990E-10	9.16179E-12
12	2.33361E-06	5.52914E-05	5.77844E-07	6.25890E-09	7.62504E-11	1.06077E-12
14	6.20608E-07	7.57404E-06	7.37615E-08	7.33301E-10	8.05243E-12	9.02320E-14
16	1.96423E-07	9.18750E-07	8.42221E-09	6.20309E-11	2.05949E-13	1.70455E-14
18	7.11666E-08	9.61806E-08	7.69115E-10	1.22962E-11	7.46439E-14	2.90485E-14
20	2.86967E-08	2.98587E-09	8.69352E-11	2.00096E-11	7.99320E-14	3.12455E-14

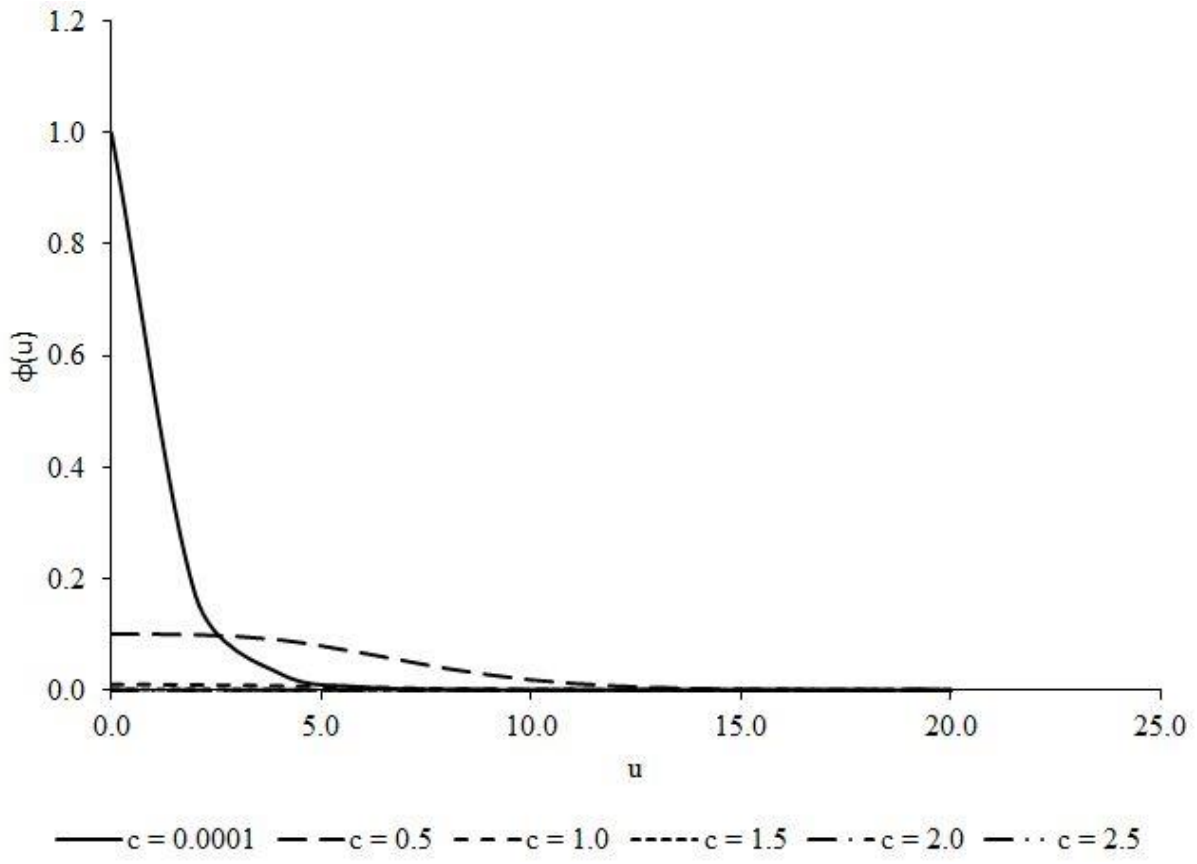


Figure 4.26: Ruin Probabilities Attributable to claim when premium rate *c* is varied, claim size is exponential and all other parameters kept are constant at **λ=1.50**, *r*=0.12, **Θ=1.50**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

4.2.2 Discussion of Results

The behavior of the probability of ruin attributable to claim when claim sizes are exponentially distributed shows that increasing premium rate causes significant reduction in the probability of ruin. This reduction is strongly felt when the initial capital is below 5.00 for all values of arrival rate λ. However, when the average claim size (1/Θ) is 2.00 and λ=1, the sharp decline in ruin probability is only felt when $c \leq 0.0001$. Increasing *u* causes decline in the ruin probability for all values of *c*.

4.2.3 Ruin Probability Attributable to Claims When Claim Sizes are Exponential and Average Claim Size is Varied

The behavior of ruin probability attributable to claims when claim sizes are exponentially distributed and average claim size is varied is examined below. In other to achieve this, the parameter Θ of the claim size distribution is varied (between 0.5−3.0), since the reciprocal of this parameter is the average claim size. This investigation is conducted for $0.10 \leq c \leq 2.00$, $0 \leq \lambda \leq 1.50$ and all other parameters kept constant.

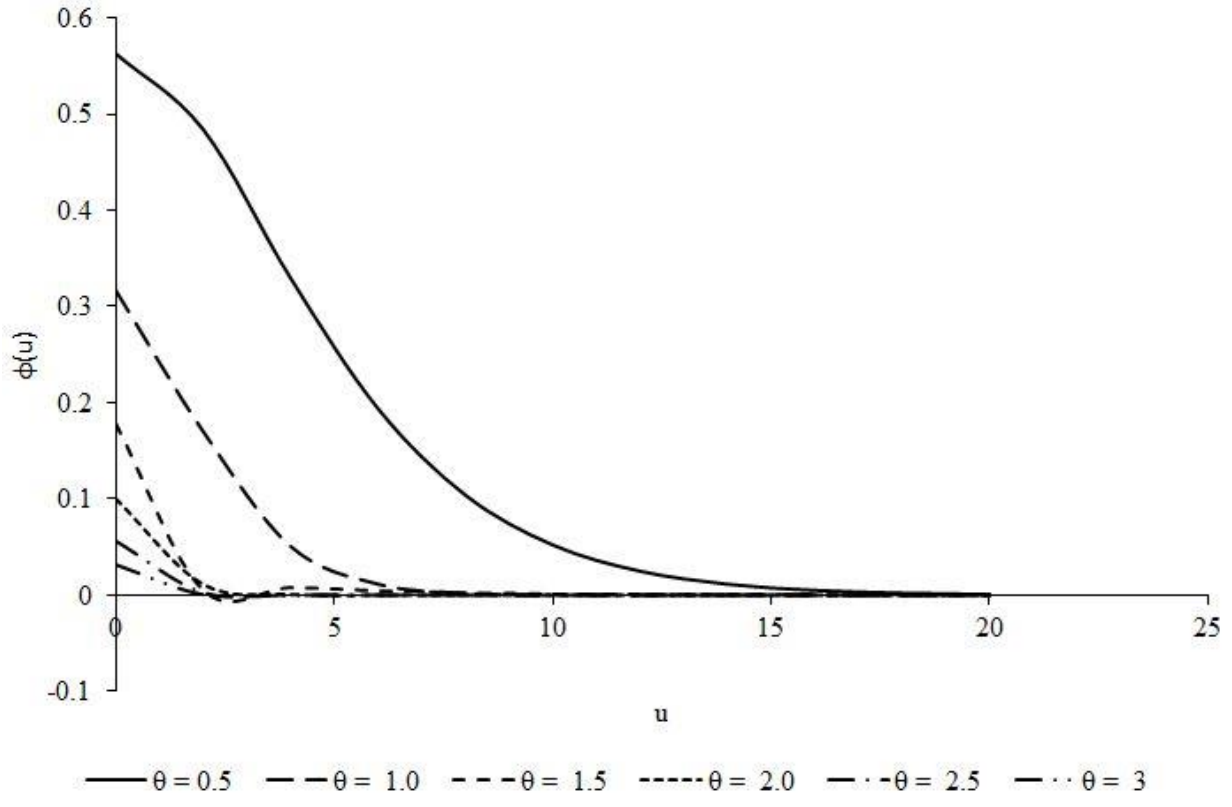


Figure 4.27: Ruin Probabilities Attributable to claim when Θ is varied, claim size is exponential and all other parameters kept constant at **c=0.10**, r=0.12, **λ=0.50**, ϕ=0.60, σ_p=0.20, a=0.21, α=0.00 and σ_R=0.20.

Comments:

Figure 4.27 shows the ruin probabilities when c=0.10 and λ=0.50. As Θ increases, ruin probability reduces for all values of Θ. This implies a decrease in the average claim sizes reduces the ruin probability. Also, the ruin probability for each of the values of Θ examined decrease as capital increases.

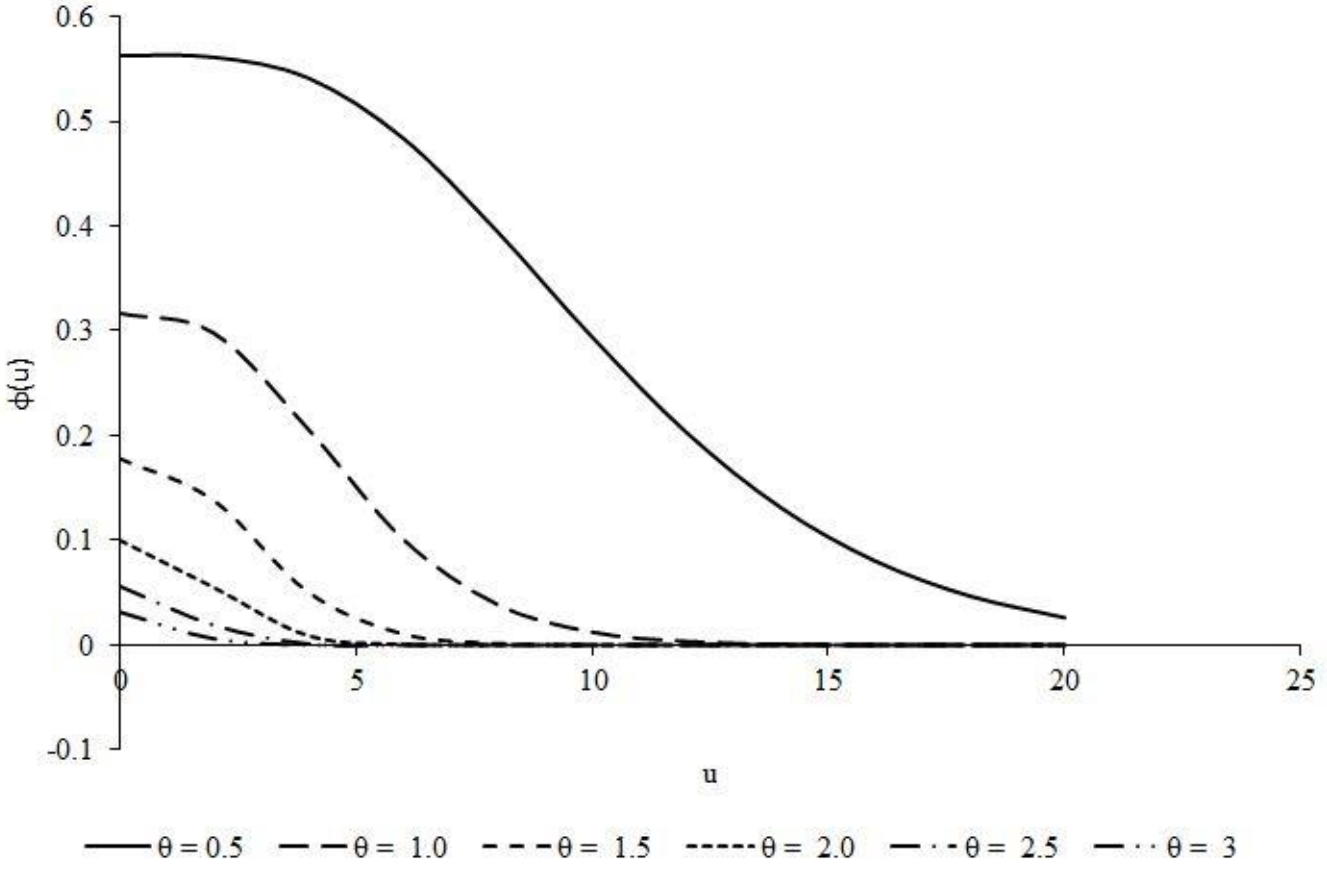


Figure 4.28: Ruin Probabilities Attributable to claim when Θ is varied, claim size is exponential and all other parameters kept constant at **c=0.10**, *r*=0.12, **λ=1.00**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

Comments:

Figure 4.27 shows the ruin probabilities when *c*=0.10 and λ=0.10. As Θ increases, ruin probability reduces for all values of Θ. Also as capital increases, ruin probability reduces.

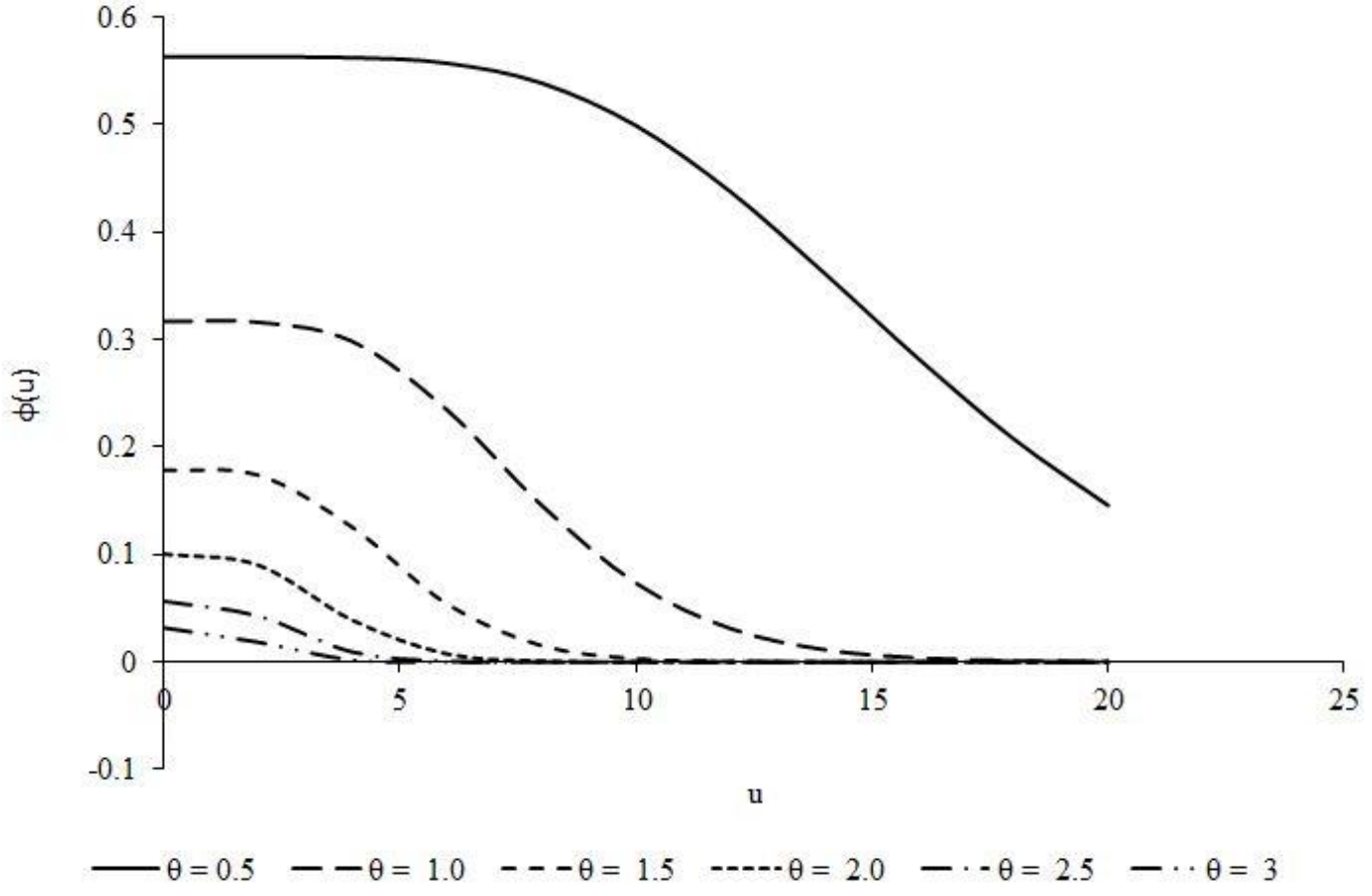


Figure 4.29: Ruin Probabilities Attributable to claim when Θ is varied, claim size is exponential and all other parameters kept constant at **c=0.10**, *r*=0.12, **λ=1.5**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

Comments:

The behavior of ruin probability in figure 4.29 is similar to the previous ones, as Θ increases, ruin probability reduces. Also ruin probability reduces with increase in capital.

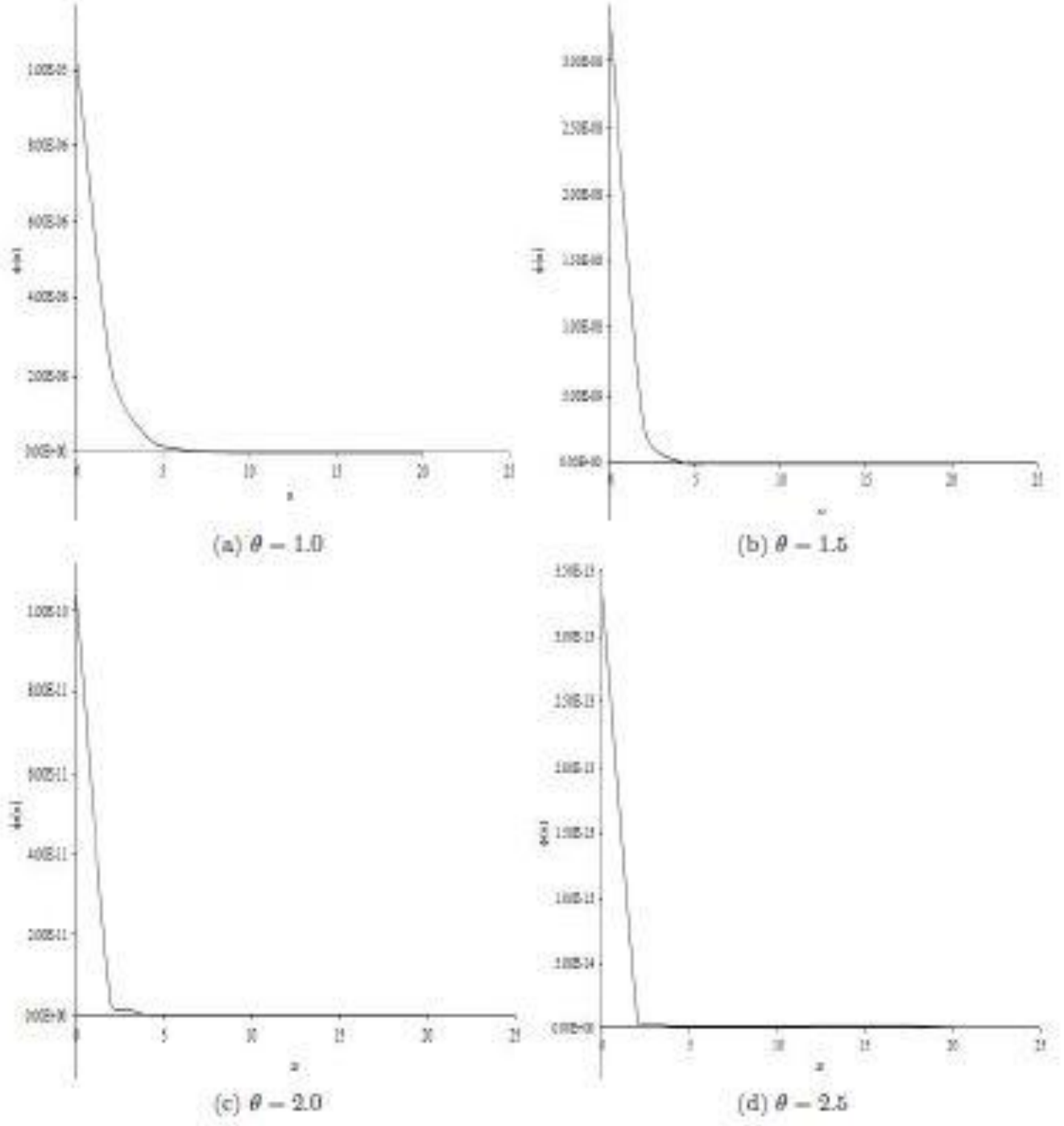


Figure 4.30: Subplots of ruin Probabilities Attributable to claim when claim size is exponential and all other parameters kept constant at **c=1.00**, r=0.12, **λ=0.50**, ϕ=0.60, σ_p=0.20, a=0.21, α=0.00 and σ_R=0.20.

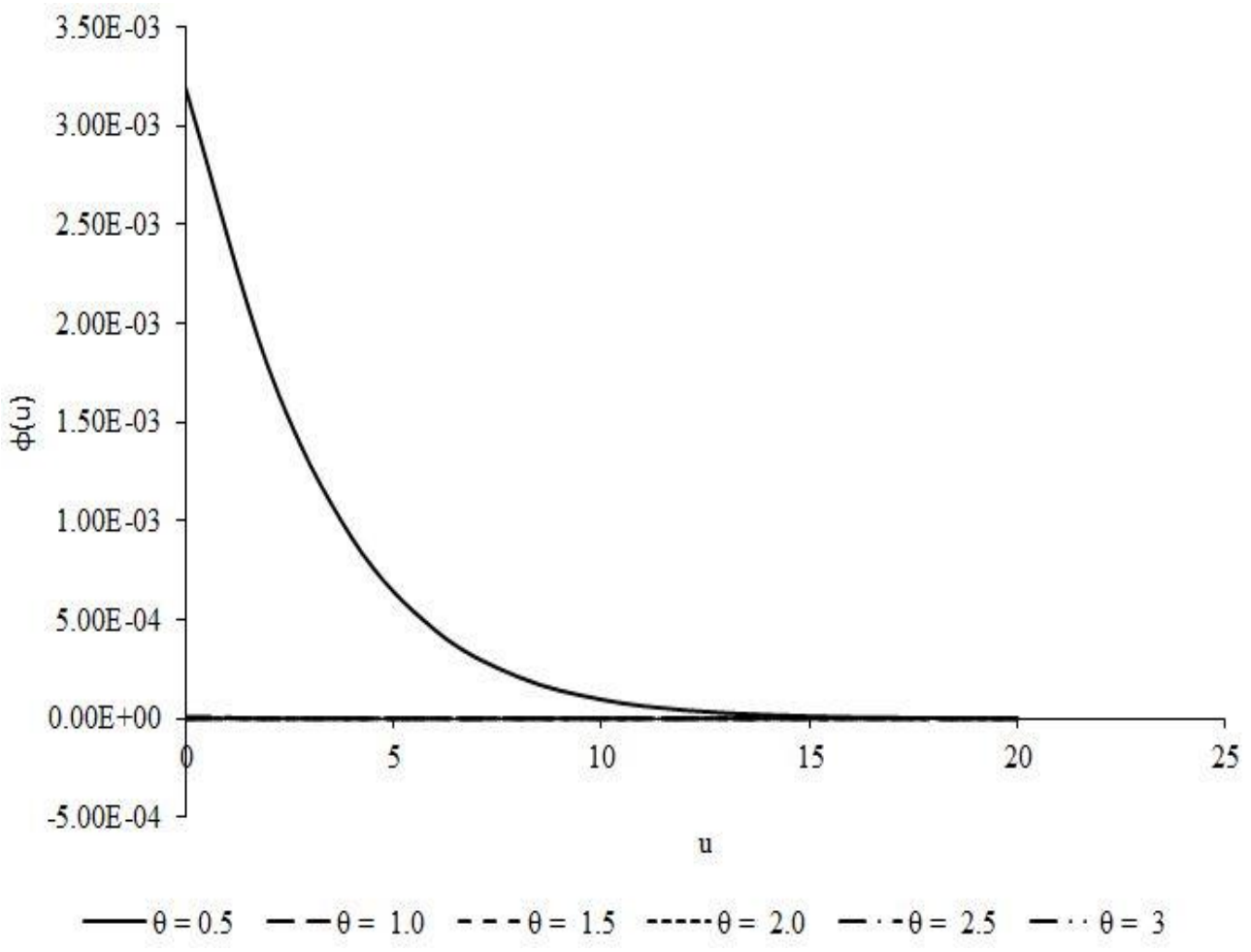


Figure 4.31: Ruin Probabilities Attributable to claim when Θ is varied, claim size is exponential and all other parameters kept constant at **c=1.00**, r=0.12, **λ=0.50**, ϕ=0.60, σ_p=0.20, a=0.21, α=0.00 and σ_R=0.20.

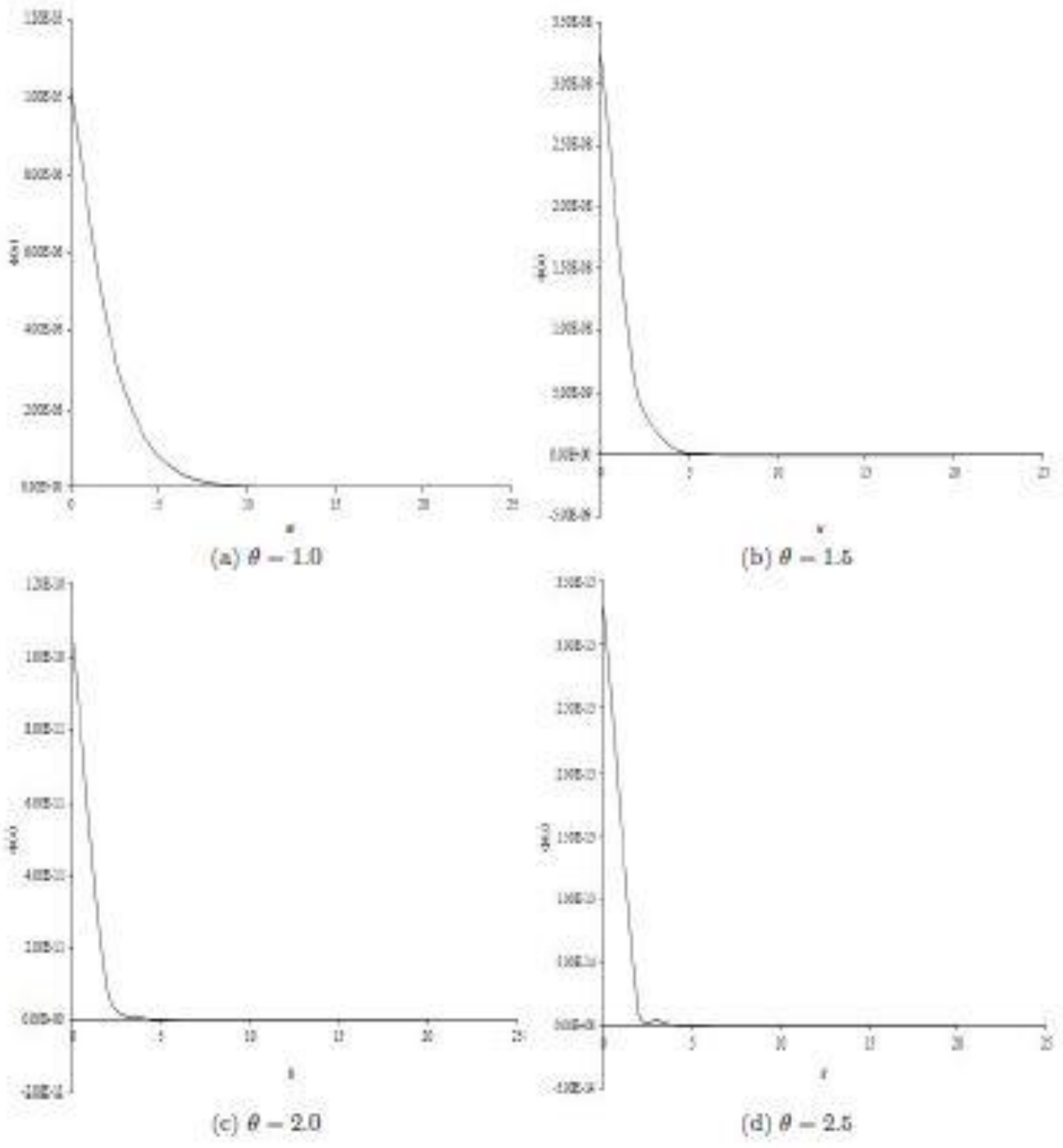


Figure 4.32: Subplots of ruin Probabilities Attributable to claim when claim size is exponential and all other parameters kept constant at **c=1.00**, r=0.12, **λ=1.00**, ϕ=0.60, σ_p=0.20, a=0.21, α=0.00 and σ_R=0.20.

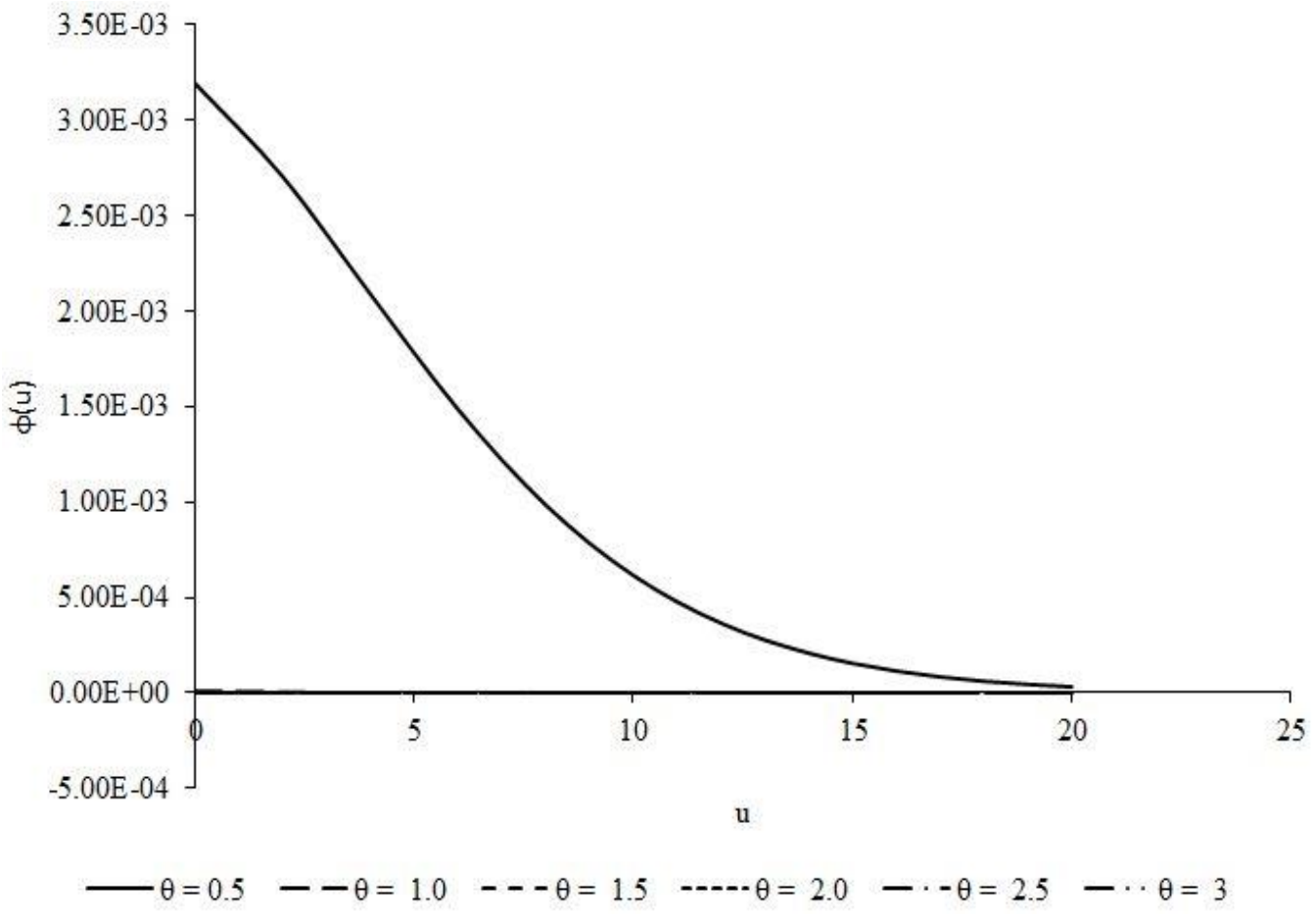


Figure 4.33: Ruin Probabilities Attributable to claim when Θ is varied, claim size is exponential and all other parameters kept constant at **c=1.00**, *r*=0.12, **λ=1.00**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

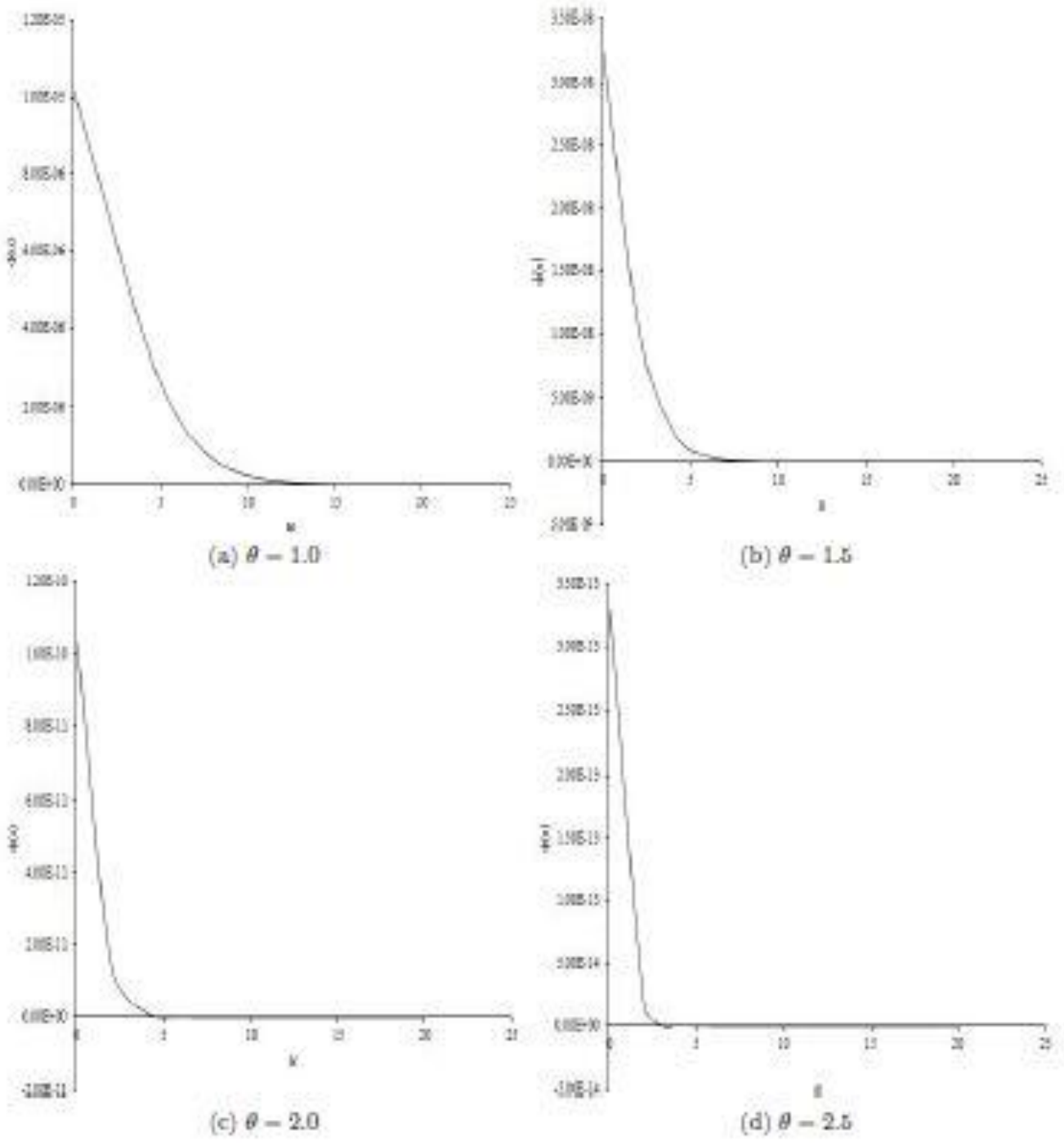


Figure 4.34: Subplots of ruin Probabilities Attributable to claim when claim size is exponential and all other parameters kept constant at **c=1.00**, r=0.12, **λ=1.50**, ϕ=0.60, σ_p=0.20, a=0.21, α=0.00 and σ_R=0.20.

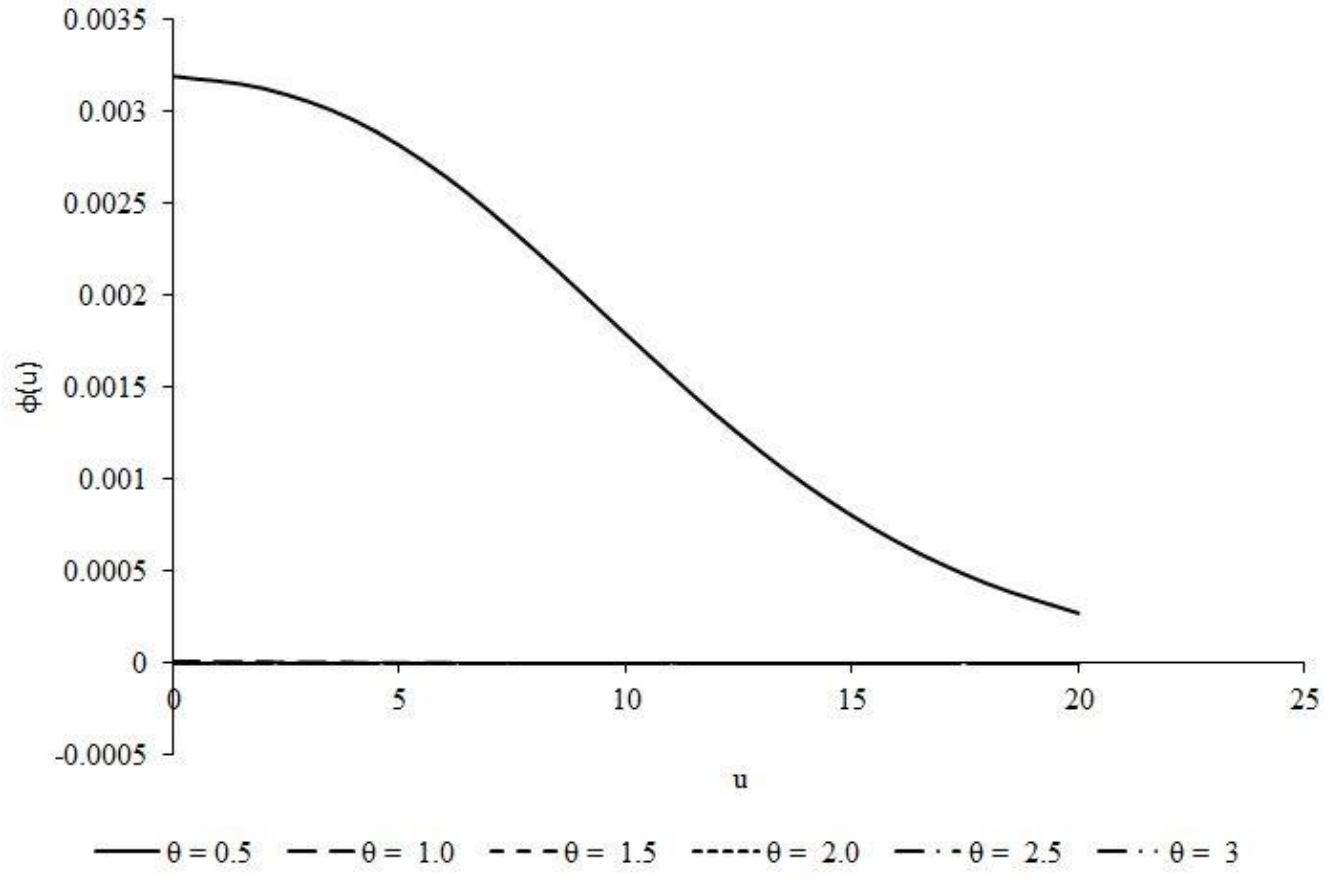


Figure 4.35: Ruin Probabilities Attributable to claim when Θ is varied, claim size is exponential and all other parameters kept constant at **c=1.00**, *r*=0.12, **λ=1.50**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

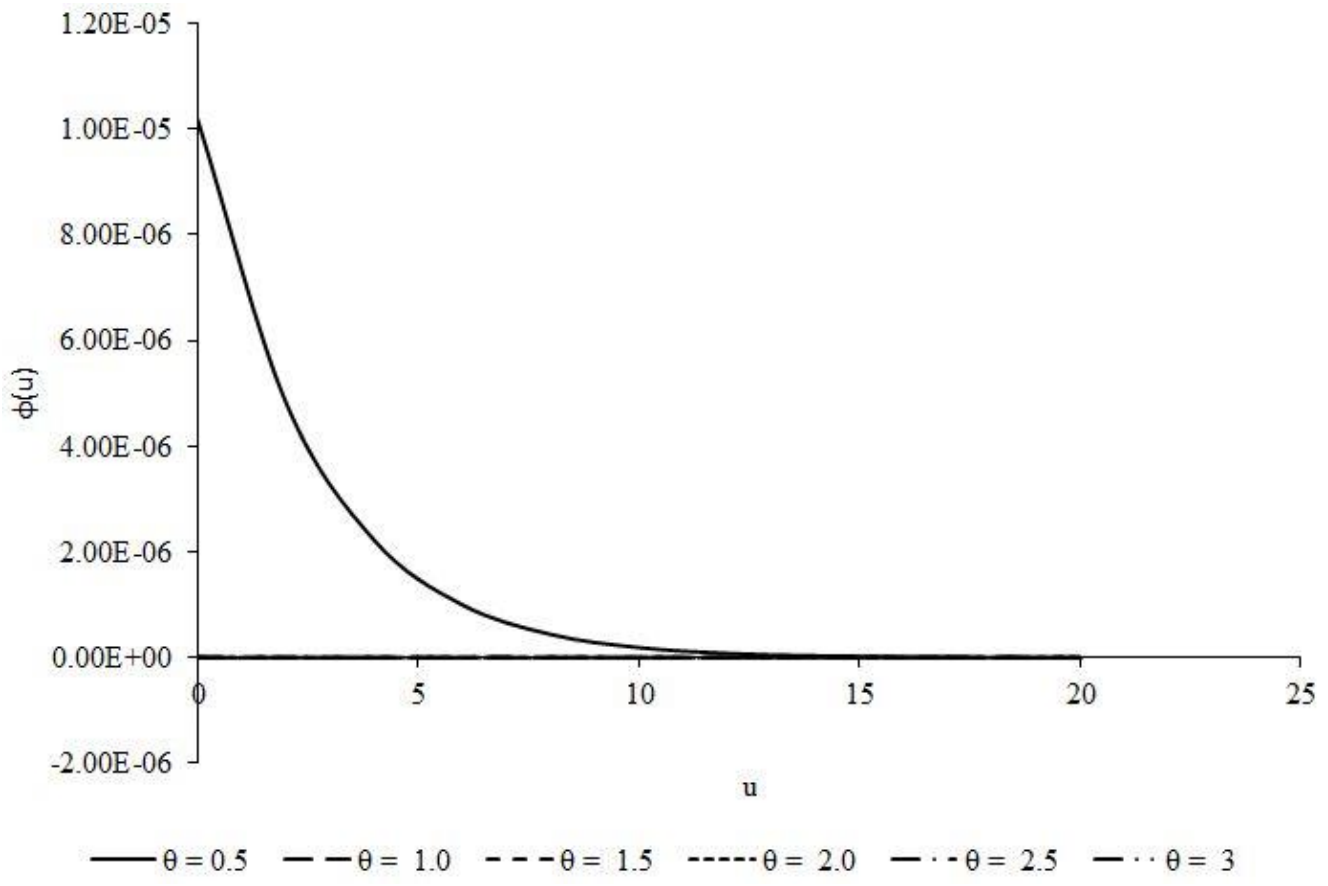


Figure 4.36: Ruin Probabilities Attributable to claim when Θ is varied, claim size is exponential and all other parameters kept constant at **c=2.00**, r=0.12, **λ=0.50**, ϕ=0.60, σ_p=0.20, a=0.21, α=0.00 and σ_R=0.20.

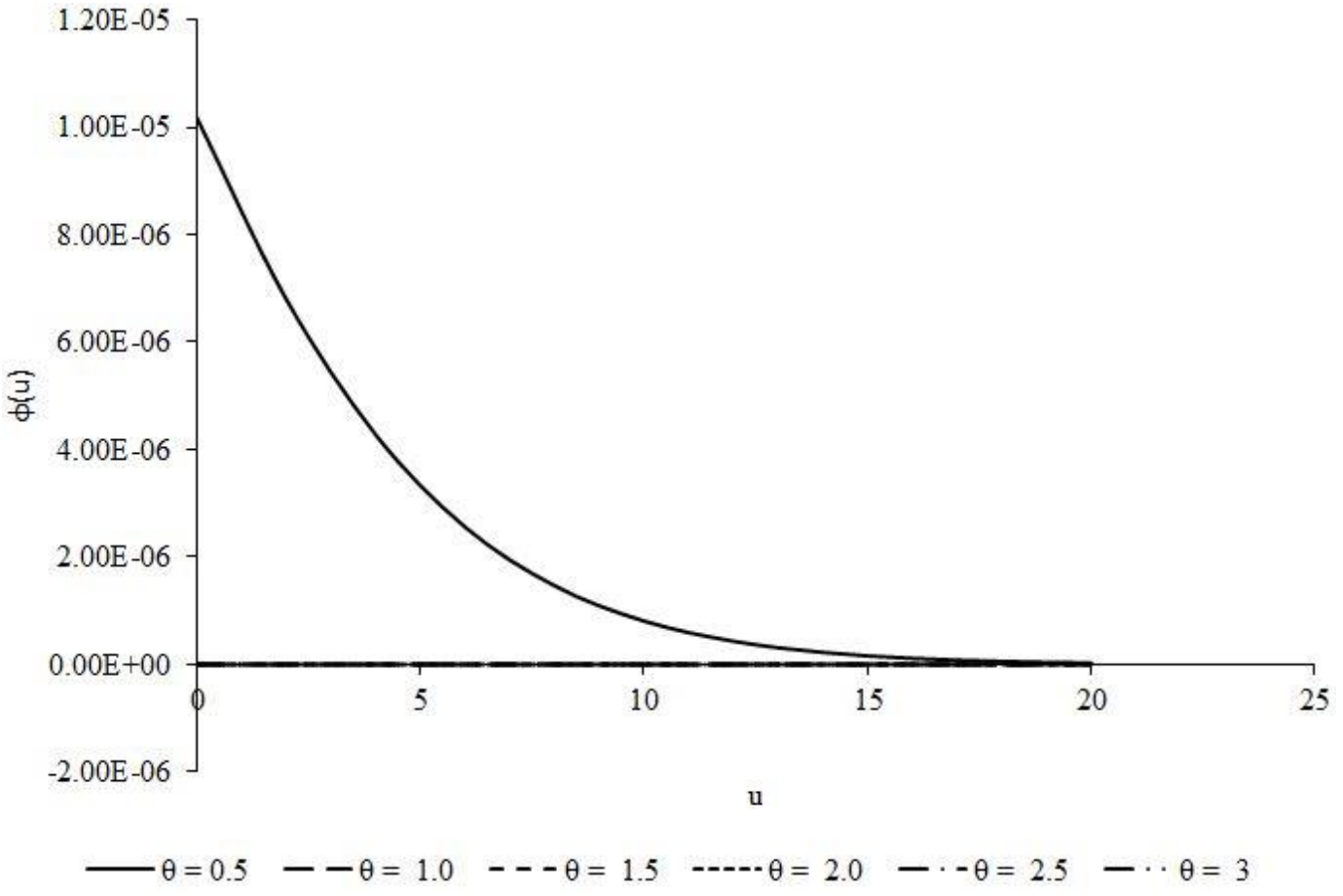


Figure 4.37: Ruin Probabilities Attributable to claim when Θ is varied, claim size is exponential and all other parameters kept constant at **c=2.00**, r=0.12, **λ=1.00**, ϕ=0.60, σ_p=0.20, a=0.21, α=0.00 and σ_R=0.20.

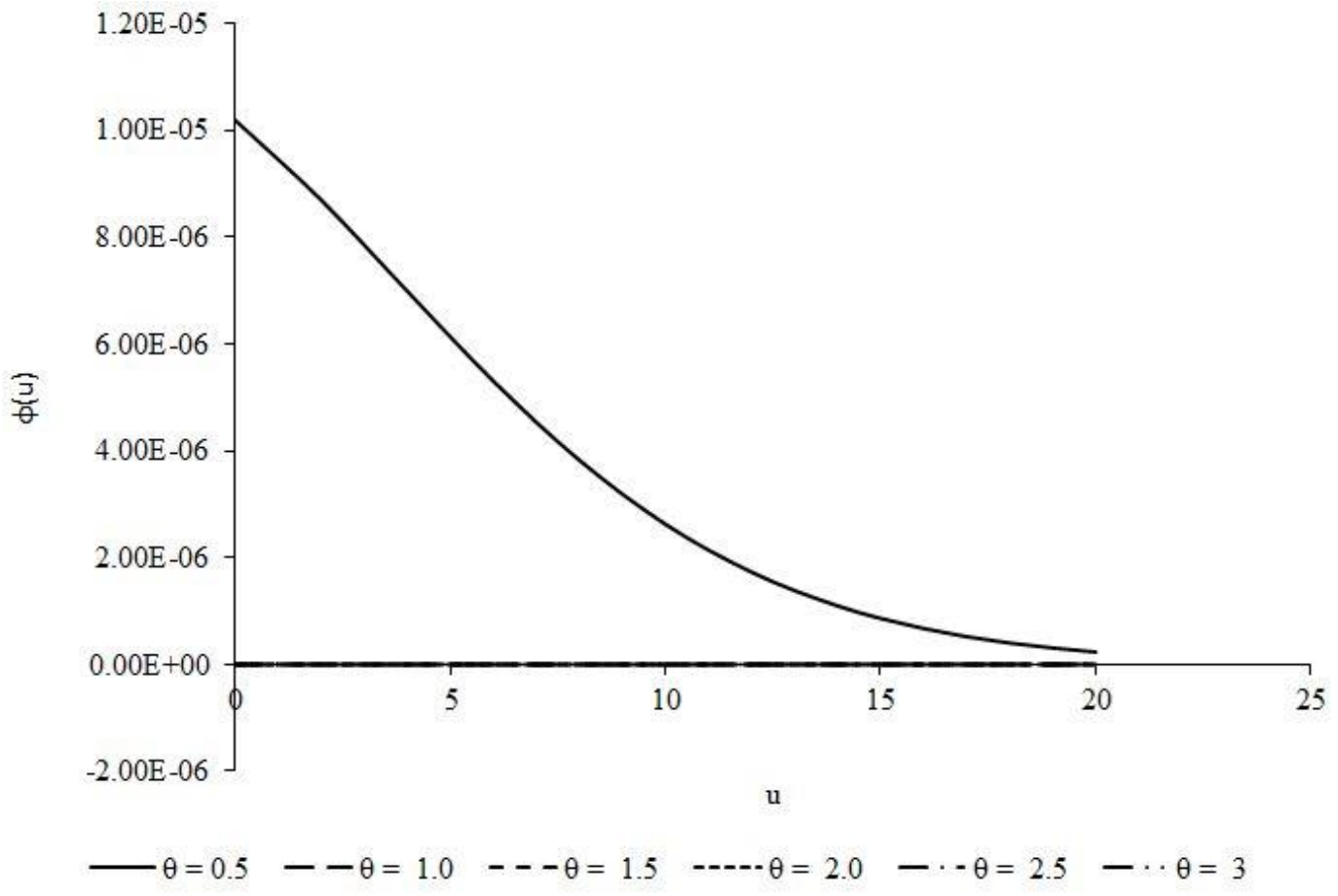


Figure 4.38: Ruin Probabilities Attributable to claim when Θ is varied, claim size is exponential and all other parameters kept constant at **c=2.00**, *r*=0.12, **λ=1.50**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

4.2.4 Discussion of Results

The behavior of ruin probability attributable to claim when claims sizes are exponential and parameter Θ is varied show that increase in Θ causes decrease in ruin probability. This implies that reduction in average claim sizes also reduces the ruin probability. However, the reduction in ruin probability is more rapid at lower values of λ.

Also, at different values of Θ, the ruin probability also decreases with increase in capital. At higher values of Θ, the reduction in ruin probability with increases in capital is less rapid.

4.2.5 Ruin Probability Attributable to Claims When Claim Sizes are Exponential and Claim Arrival Rate is Varied

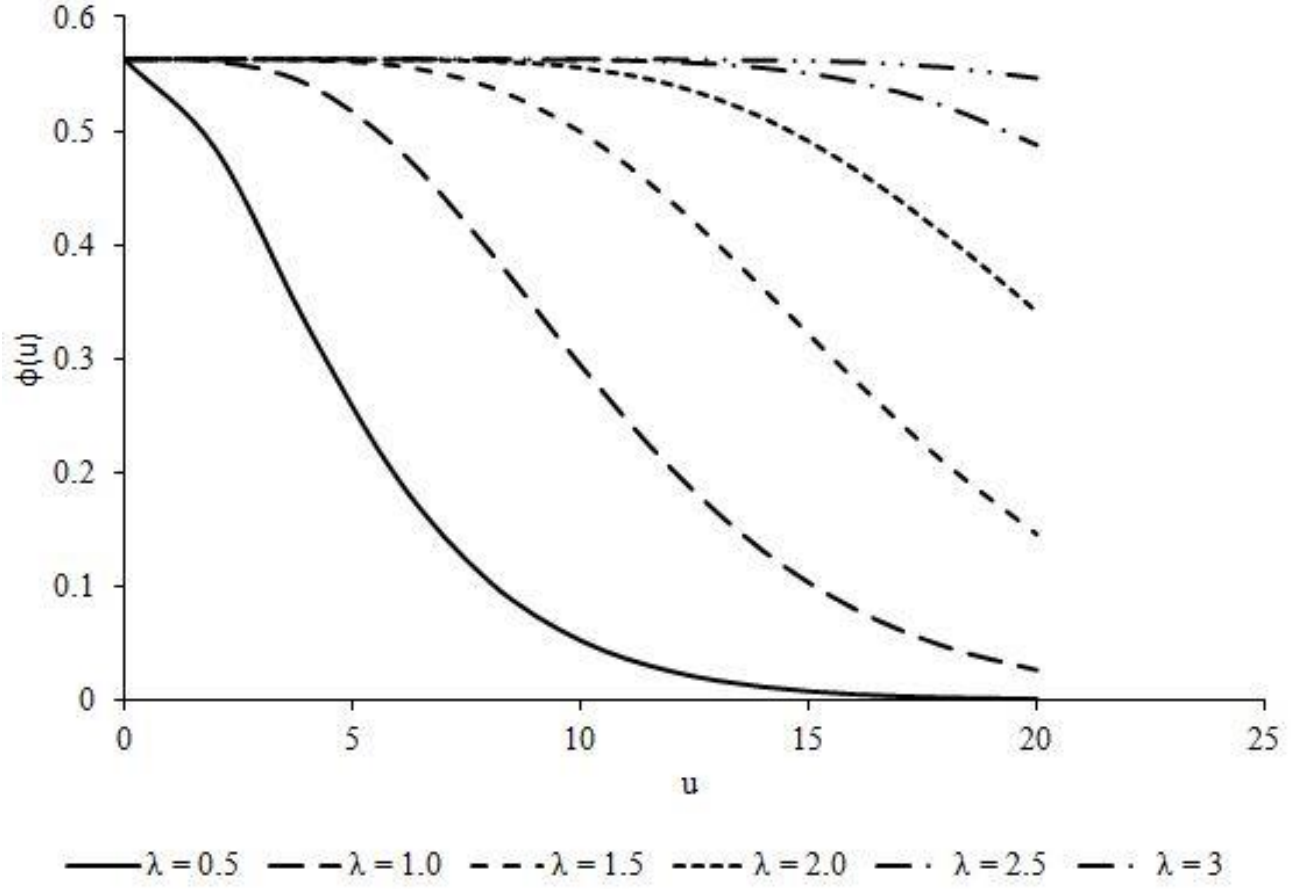


Figure 4.39: Ruin Probabilities Attributable to claim when claims arrival rate λ is varied, claim size is exponential and all other parameters kept constant at **c=0.10**, *r*=0.12, **Θ=0.50**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

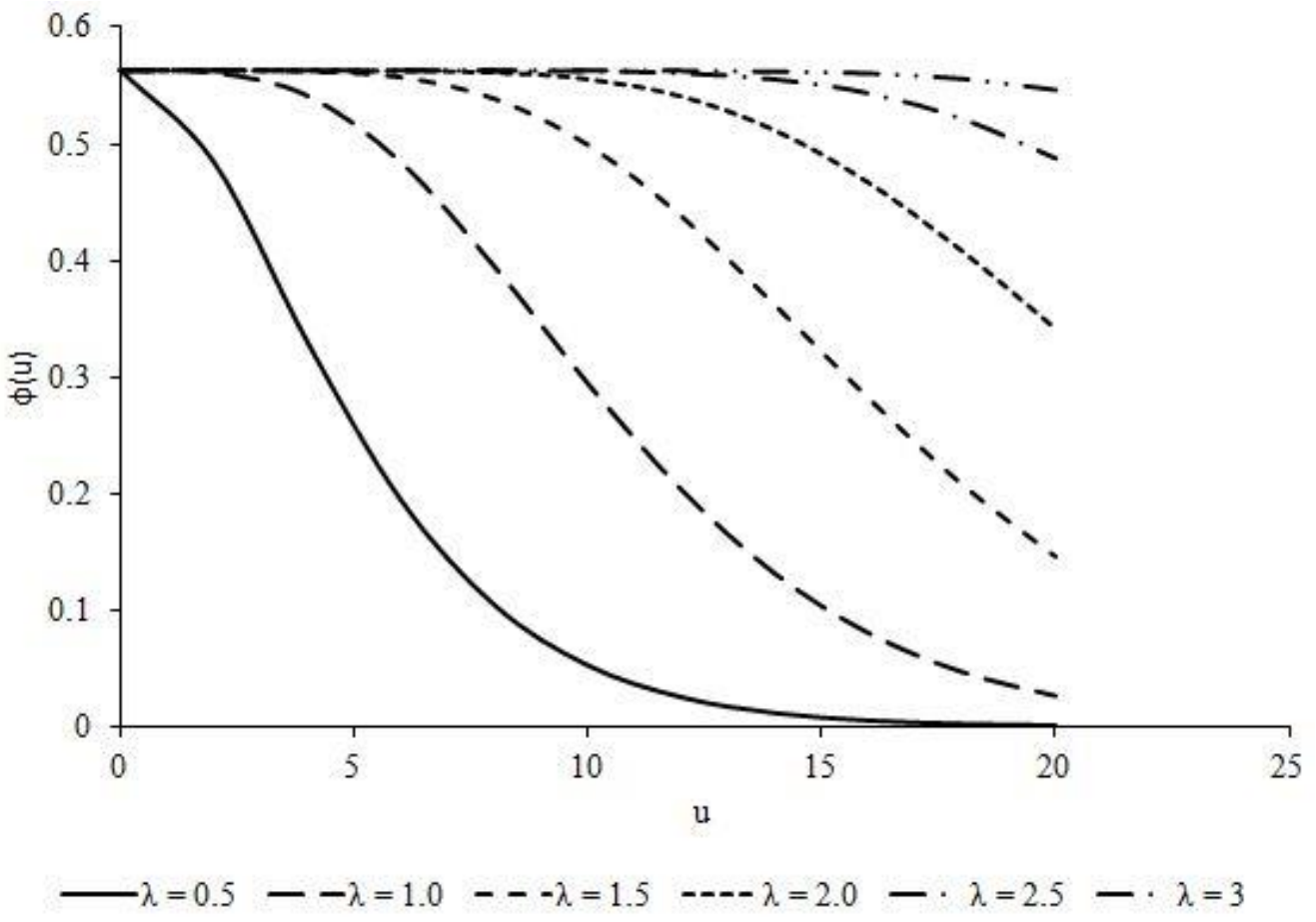


Figure 4.40: Ruin Probabilities Attributable to claim when claims arrival rate λ is varied, claim size is exponential and all other parameters kept constant at **c=0.10**, *r*=0.12, **Θ=1.00**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

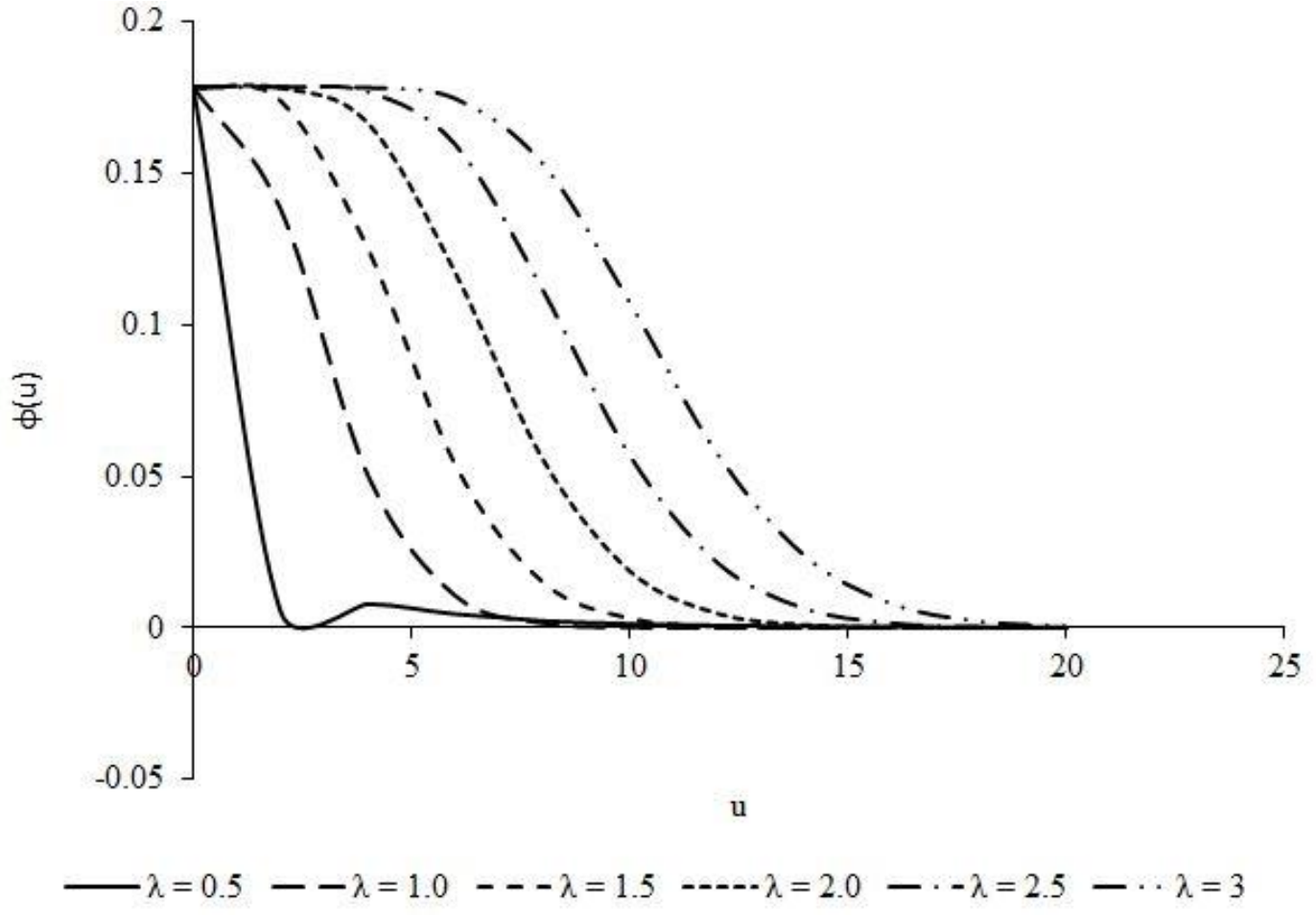


Figure 4.41: Ruin Probabilities Attributable to claim when claims arrival rate λ is varied, claim size is exponential and all other parameters kept constant at **c=0.10**, *r*=0.12, **Θ=1.5**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

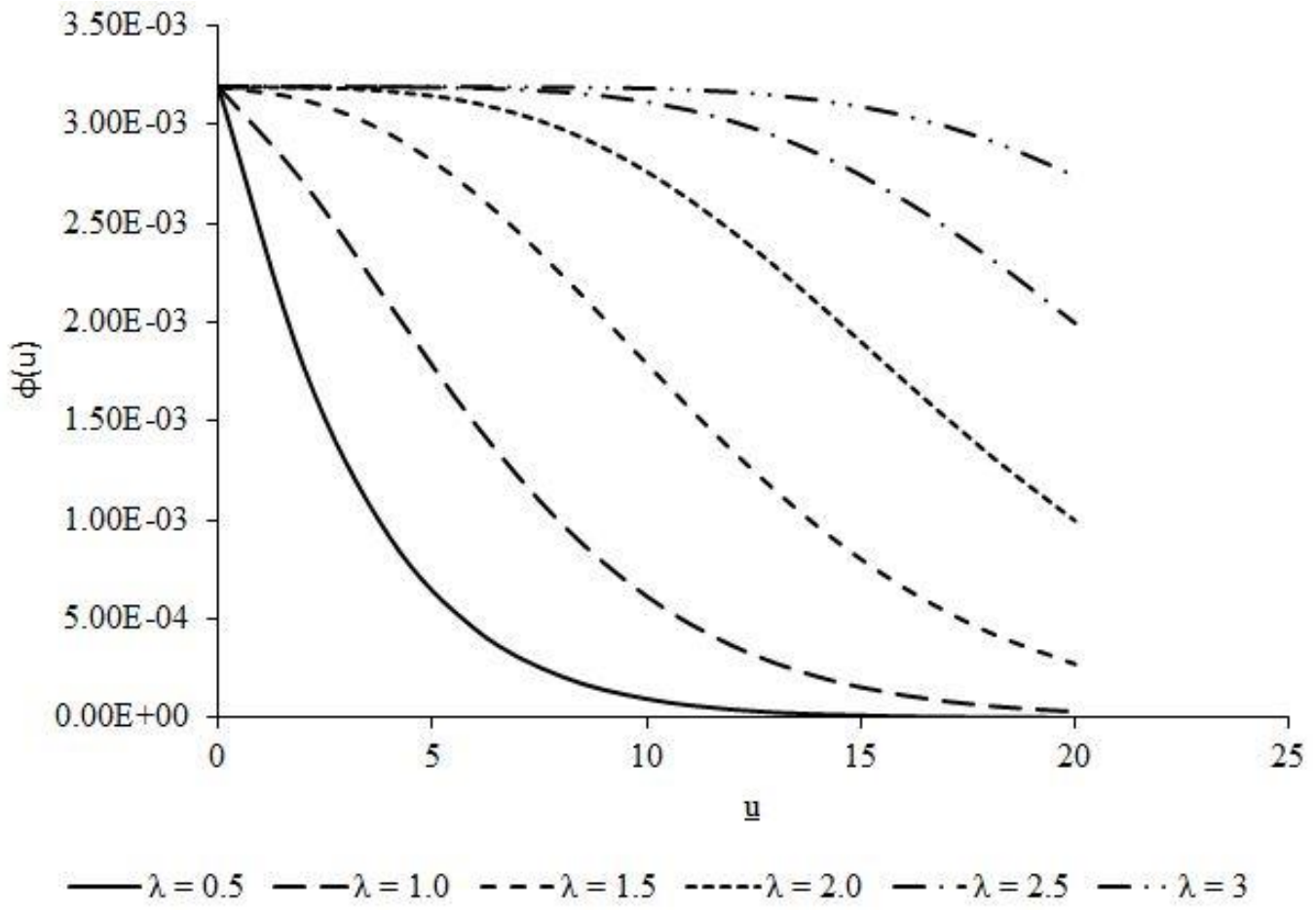


Figure 4.42: Ruin Probabilities Attributable to claim when claims arrival rate λ is varied, claim size is exponential and all other parameters kept constant at **c=1.00**, r=0.12, **Θ=0.50**, ϕ=0.60, σ_p=0.20, a=0.21, α=0.00 and σ_R=0.20.

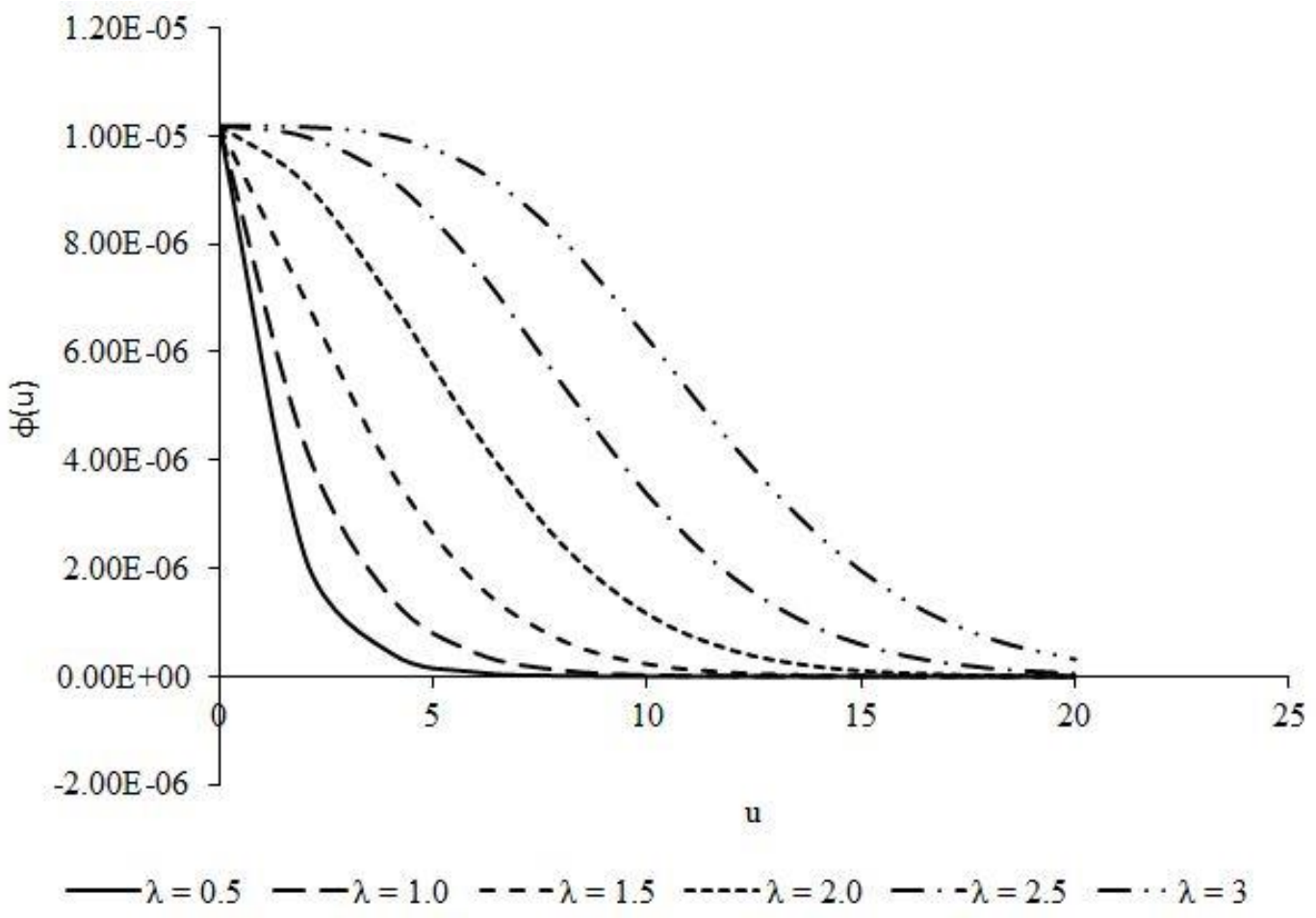


Figure 4.43: Ruin Probabilities Attributable to claim when claims arrival rate λ is varied, claim size is exponential and all other parameters kept constant at **c=1.00**, *r*=0.12, **Θ=1.00**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

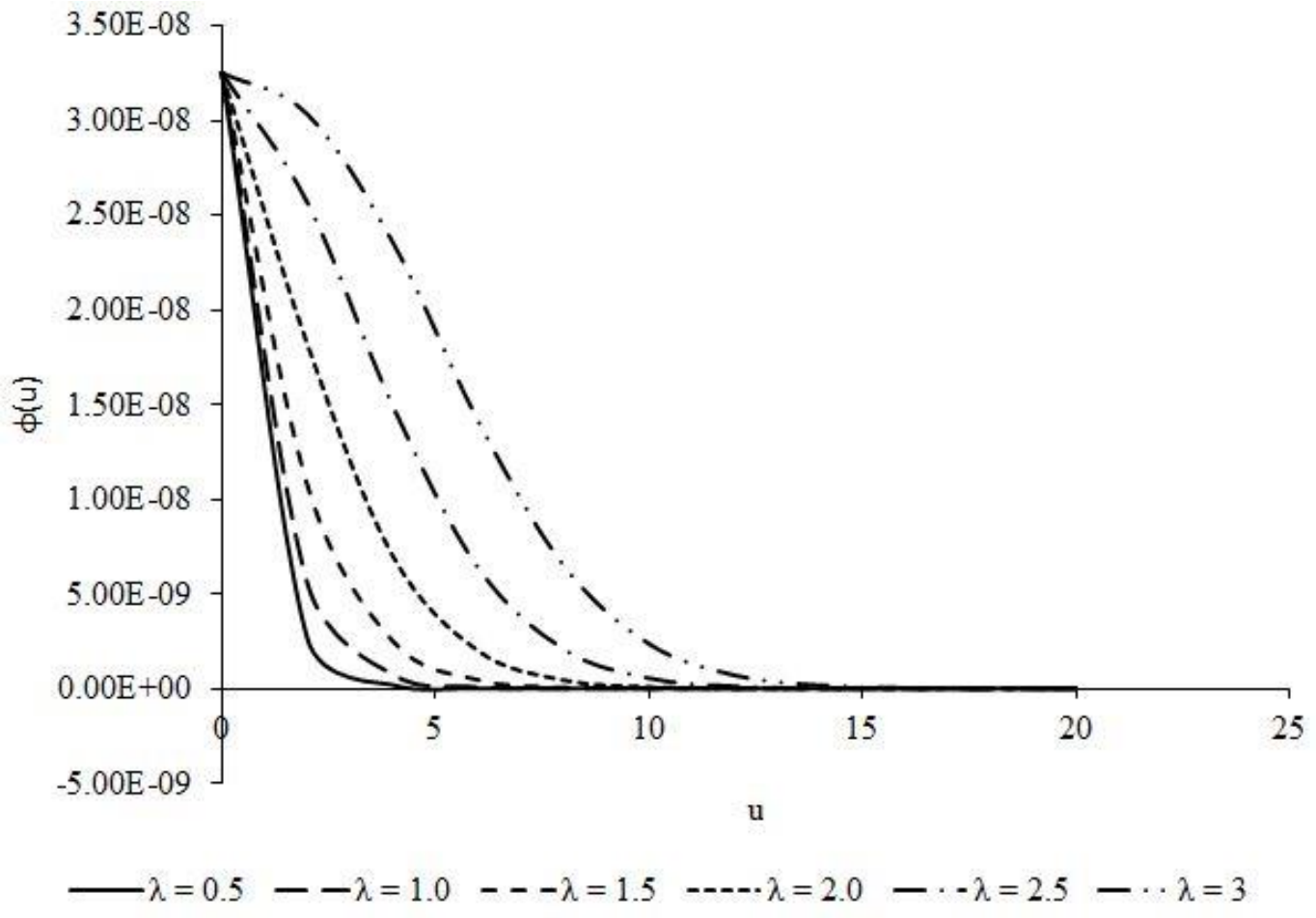


Figure 4.44: Ruin Probabilities Attributable to claim when claims arrival rate λ is varied, claim size is exponential and all other parameters kept constant at **c=1.00**, *r*=0.12, **Θ=1.50**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

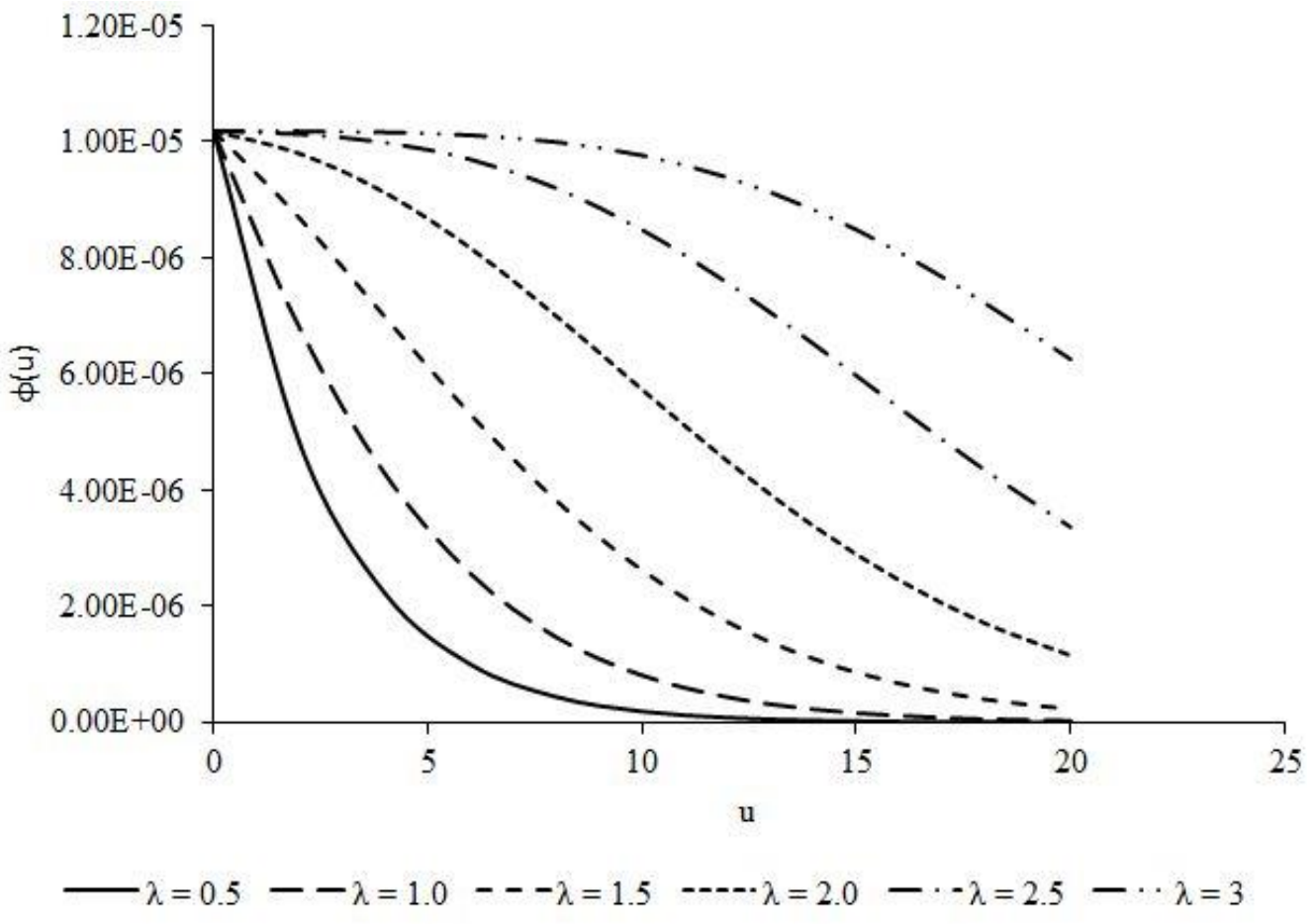


Figure 4.45: Ruin Probabilities Attributable to claim when claims arrival rate λ is varied, claim size is exponential and all other parameters kept constant at **c=2.00**, *r*=0.12, **Θ=0.50**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

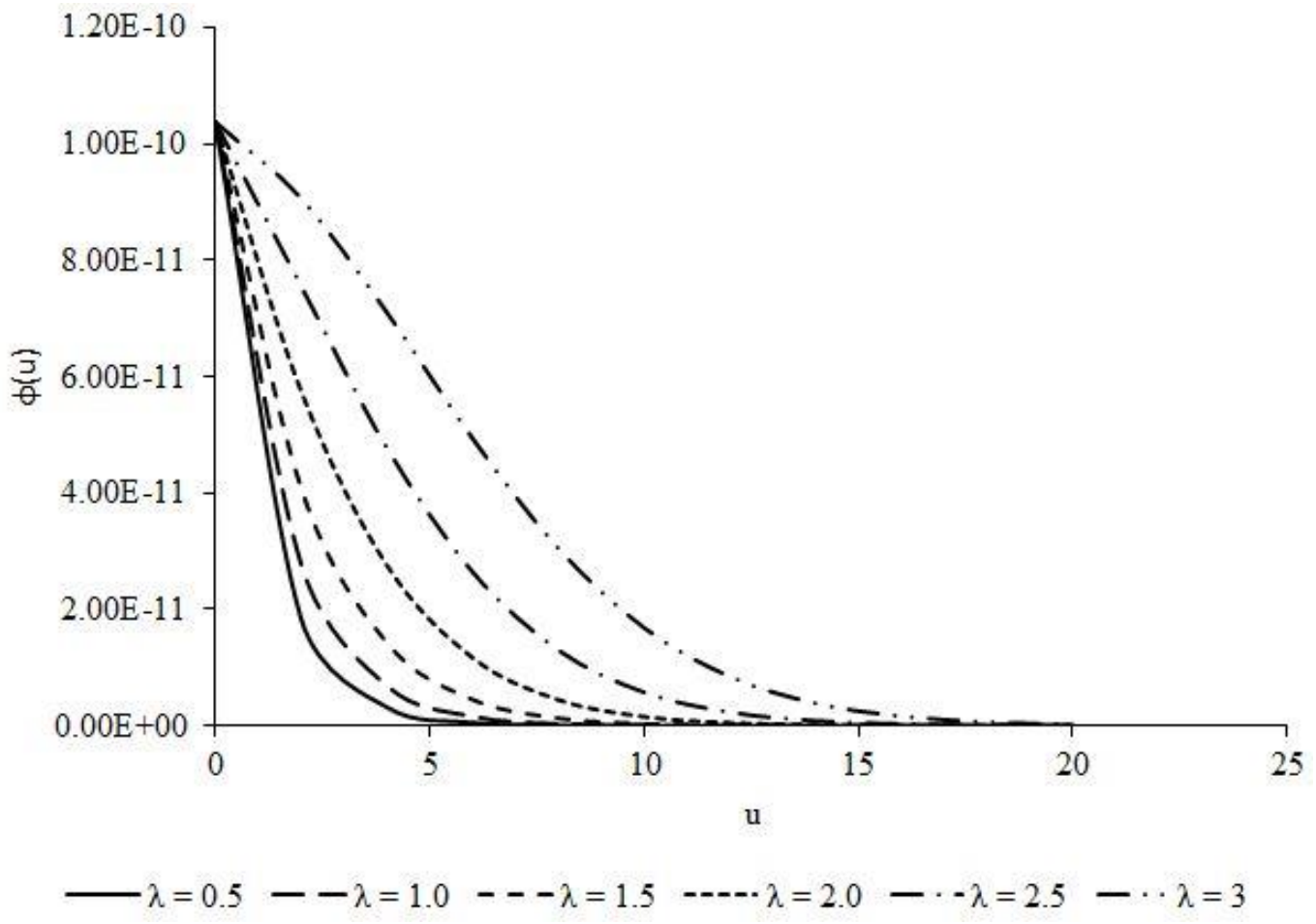


Figure 4.46: Ruin Probabilities Attributable to claim when claims arrival rate λ is varied, claim size is exponential and all other parameters kept constant at **c=2.00**, r=0.12, **Θ=1.00**, ϕ=0.60, σ_p=0.20, a=0.21, α=0.00 and σ_R=0.20.

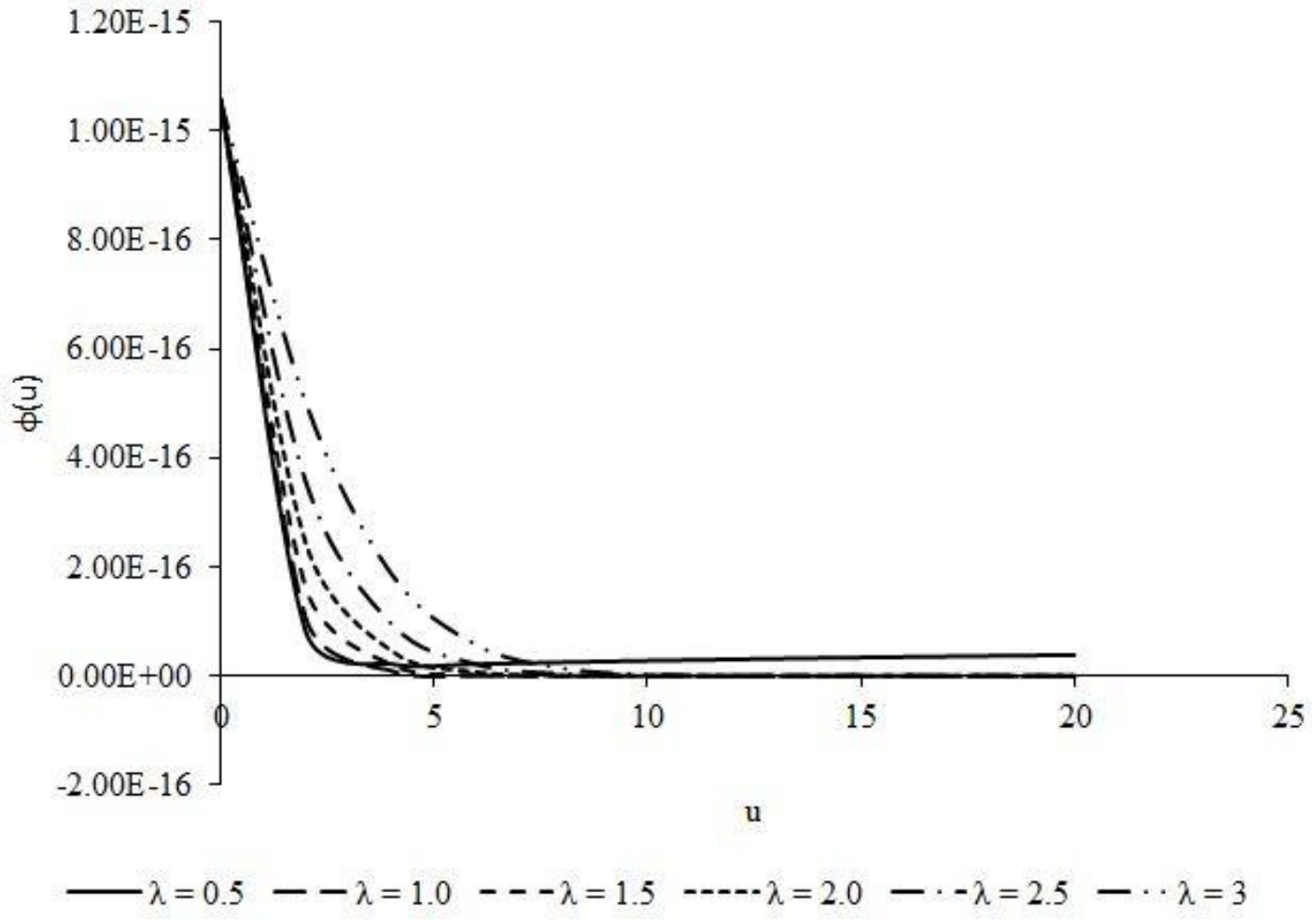


Figure 4.47: Ruin Probabilities Attributable to claim when claims arrival rate λ is varied, claim size is exponential and all other parameters kept constant at **c=2.00**, *r*=0.12, **Θ=1.50**, ϕ=0.60, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

4.2.6 Discussion of Results

The behavior of ruin probability when claim arrival rate is varied i shown in figures 4.39 to 4.47. From the figures, Increase in claims arrival rate λ increases the ruin probability provided the average claim sizes remains the same. If the average claim size is increased, the ruin probability also increases, irrespective of the claims arrival rate.

Also, the ruin probability decreases with increase in capital at for all values of λ.

4.2.7 Ruin Probability Resulting from Oscillation when Claim size are Exponential and Interest is varied

Table 4.15: Ruin Probabilities Resulting from Oscillation when Interest rate *r* is varied, claim size is exponential and all other parameters kept constant at *c*=1.1, λ=1.00, Θ=1.00, **ϕ=0.20**, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

u	r = 0.00	r = 0.06	r = 0.12	r = 0.18	r = 0.24
0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
1	4.363E-25	1.255E-25	3.585E-26	1.024E-26	2.927E-27
2	0.000E+00	1.763E-52	1.312E-54	9.776E-57	7.285E-59
3	0.000E+00	2.771E-81	4.904E-86	8.685E-91	1.539E-95
4	0.000E+00	4.872E-112	1.869E-120	7.178E-129	2.758E-137
5	0.000E+00	9.577E-145	7.263E-158	5.515E-171	4.190E-184
6	0.000E+00	0.000E+00	2.876E-198	3.938E-217	5.395E-236
7	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
8	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00

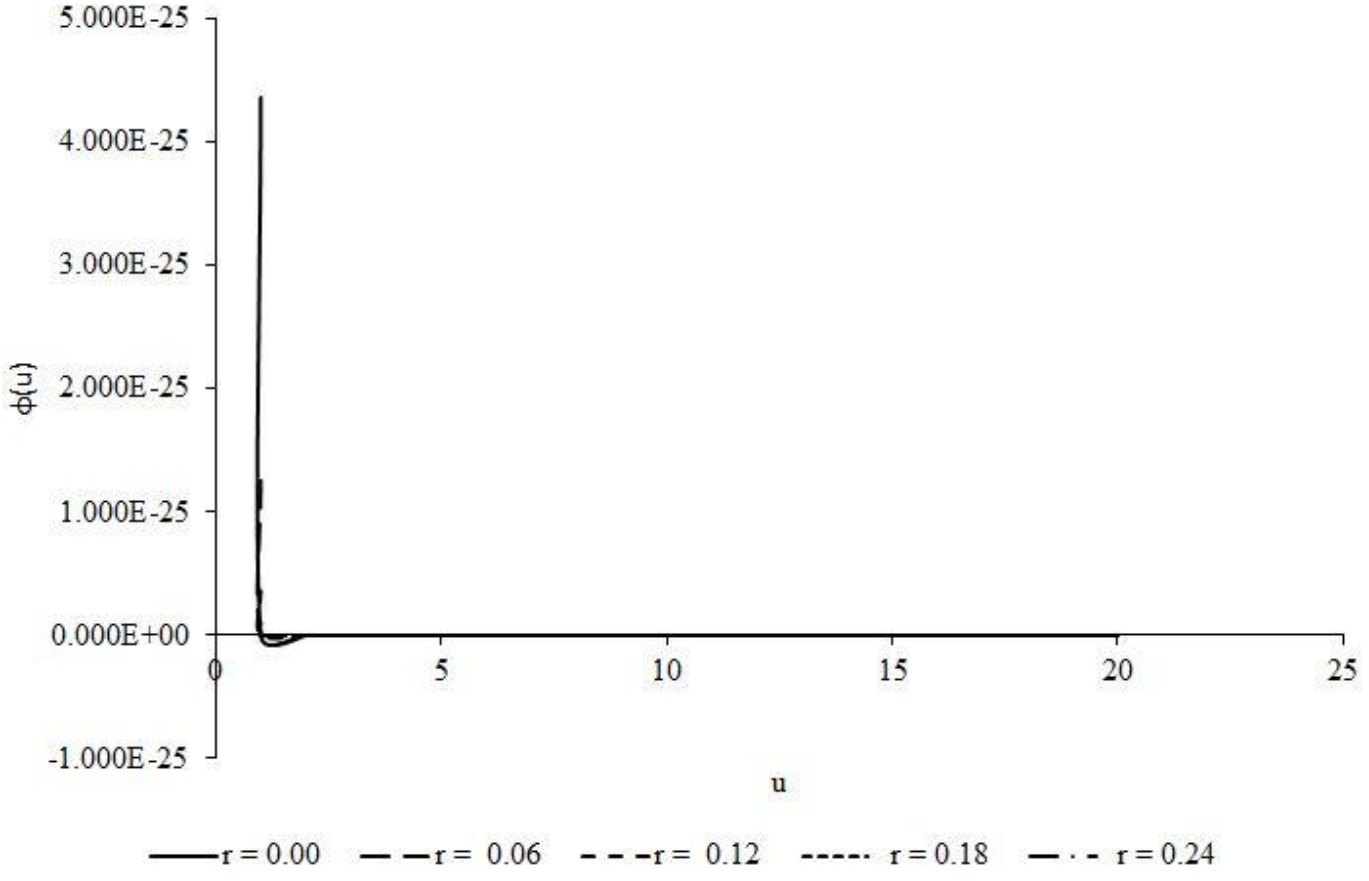


Figure 4.48: Ruin Probabilities Resulting from Oscillation when Interest rate *r* is varied, claim size is exponential and all other parameters kept constant at *c*=1.1, λ=1.00, Θ=1.00, **ϕ=0.20**, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

Table 4.16: Ruin Probabilities Resulting from Oscillation when Interest rate *r* is varied, claim size is exponential and all other parameters kept constant at *c*=1.1, λ=1.00, Θ=1.00, **ϕ=0.40**, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

u	r = 0.00	r = 0.06	r = 0.12	r = 0.18	r = 0.24
0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
1	1.257E-25	5.735E-26	2.241E-26	8.758E-27	3.423E-27
2	2.264E-52	8.243E-54	2.090E-55	5.299E-57	1.344E-58
3	5.790E-81	2.968E-84	8.102E-88	2.212E-91	6.042E-95
4	2.070E-111	2.676E-117	1.305E-123	6.369E-130	3.108E-136
5	1.003E-143	6.040E-153	8.735E-163	1.264E-172	1.829E-182
6	0.000E+00	3.410E-191	2.426E-205	1.727E-219	1.230E-233
7	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
8	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00

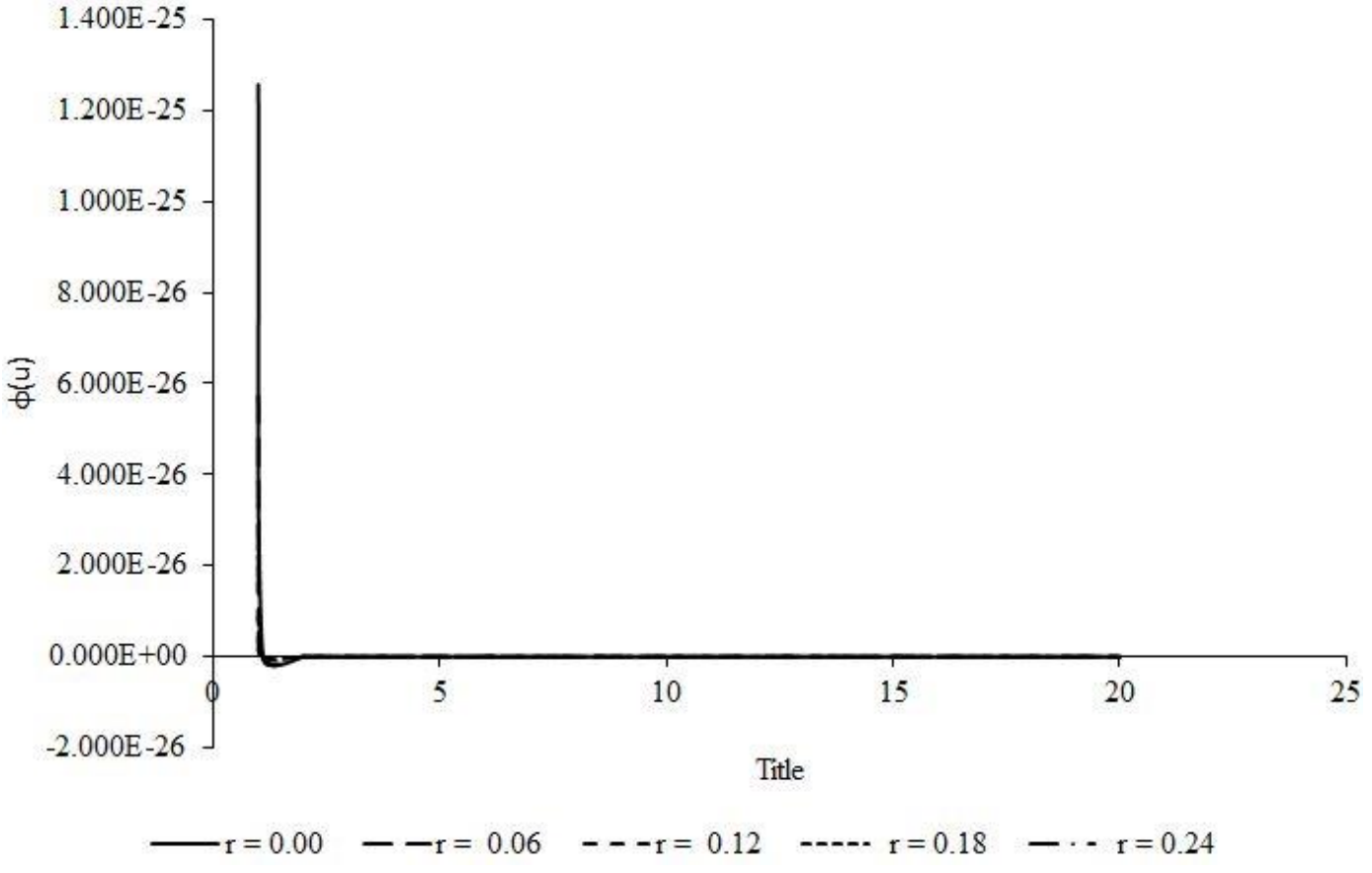


Figure 4.49: Ruin Probabilities Resulting from Oscillation when Interest rate *r* is varied, claim size is exponential and all other parameters kept constant at *c*=1.1, λ=1.00, Θ=1.00, **ϕ=0.40**, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

Ruin Probabilities Resulting from Oscillation when Interest rate *r* is varied, claim size is exponential and all other parameters kept constant at *c*=1.1, λ=1.00, Θ=1.00, **ϕ=0.60**, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

u	r = 0.00	r = 0.06	r = 0.12	r = 0.18	r = 0.24
0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
1	6.655E-26	2.621E-26	1.401E-26	7.489E-27	4.004E-27
2	7.475E-54	3.855E-55	3.328E-56	2.872E-57	2.479E-58
3	1.459E-84	3.181E-87	1.339E-89	5.635E-92	2.372E-94
4	5.023E-118	1.471E-122	9.117E-127	5.651E-131	3.503E-135
5	3.074E-154	3.812E-161	1.051E-167	2.896E-174	7.981E-181
6	3.362E-193	5.532E-203	2.047E-212	7.577E-222	2.804E-231
7	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
8	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00

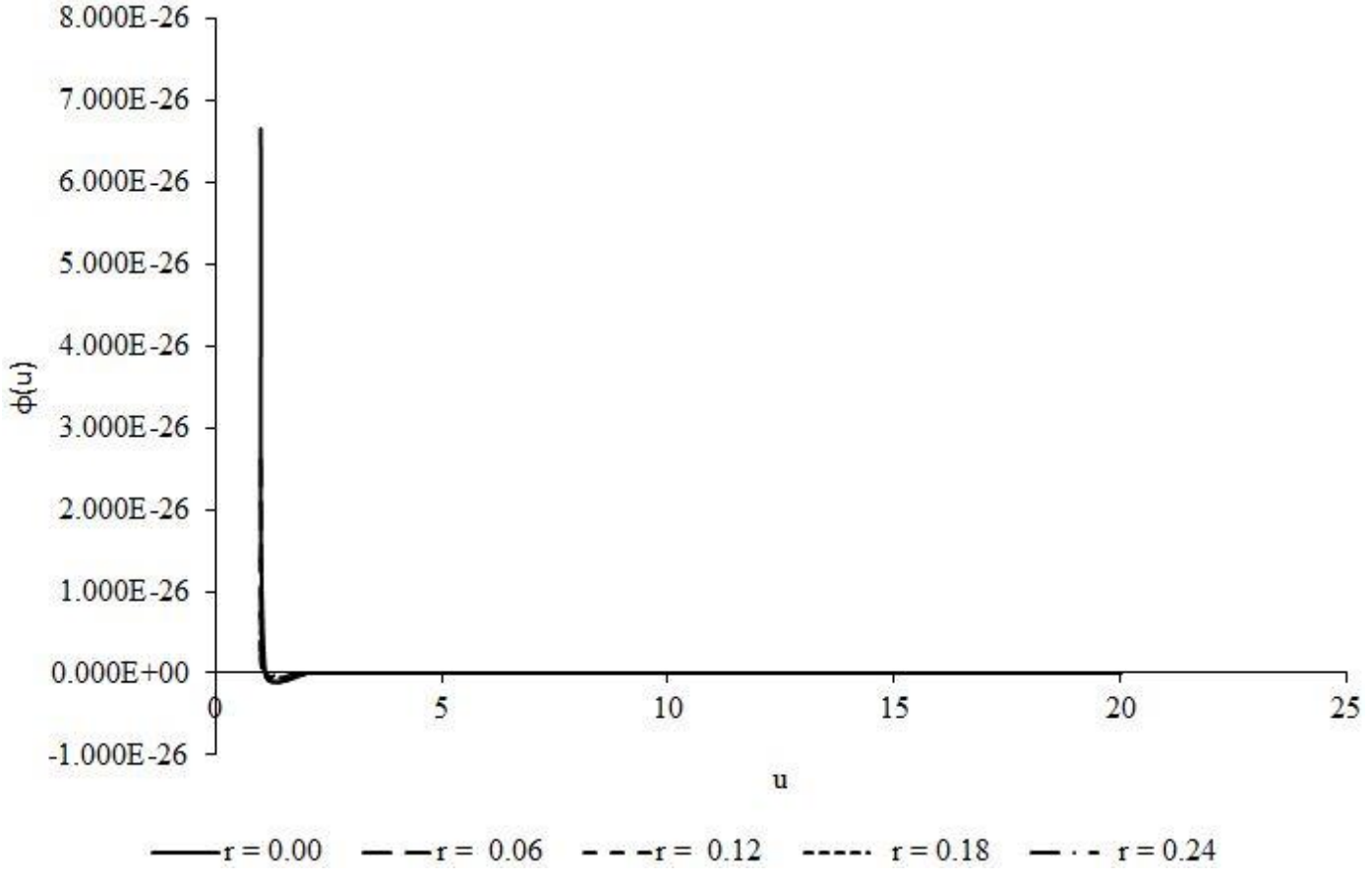


Figure 4.50: Ruin Probabilities Resulting from Oscillation when Interest rate *r* is varied, claim size is exponential and all other parameters kept constant at *c*=1.1, λ=1.00, Θ=1.00, **ϕ=0.60**, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

Table 4.17: Ruin Probabilities Resulting from Oscillation when Interest rate *r* is varied, claim size is exponential and all other parameters kept constant at *c*=1.1, λ=1.00, Θ=1.00, **ϕ=0.80**, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

u	r = 0.00	r = 0.06	r = 0.12	r = 0.18	r = 0.24
0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
1	2.194E-26	1.198E-26	8.758E-27	6.404E-27	4.682E-27
2	1.018E-55	1.804E-56	5.299E-57	1.557E-57	4.574E-58
3	1.024E-88	3.410E-90	2.212E-91	1.435E-92	9.312E-94
4	2.261E-125	8.090E-128	6.369E-130	5.014E-132	3.948E-134
5	1.104E-165	2.407E-169	1.264E-172	6.635E-176	3.483E-179
6	1.199E-209	8.979E-215	1.727E-219	3.323E-224	6.394E-229
7	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
8	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00

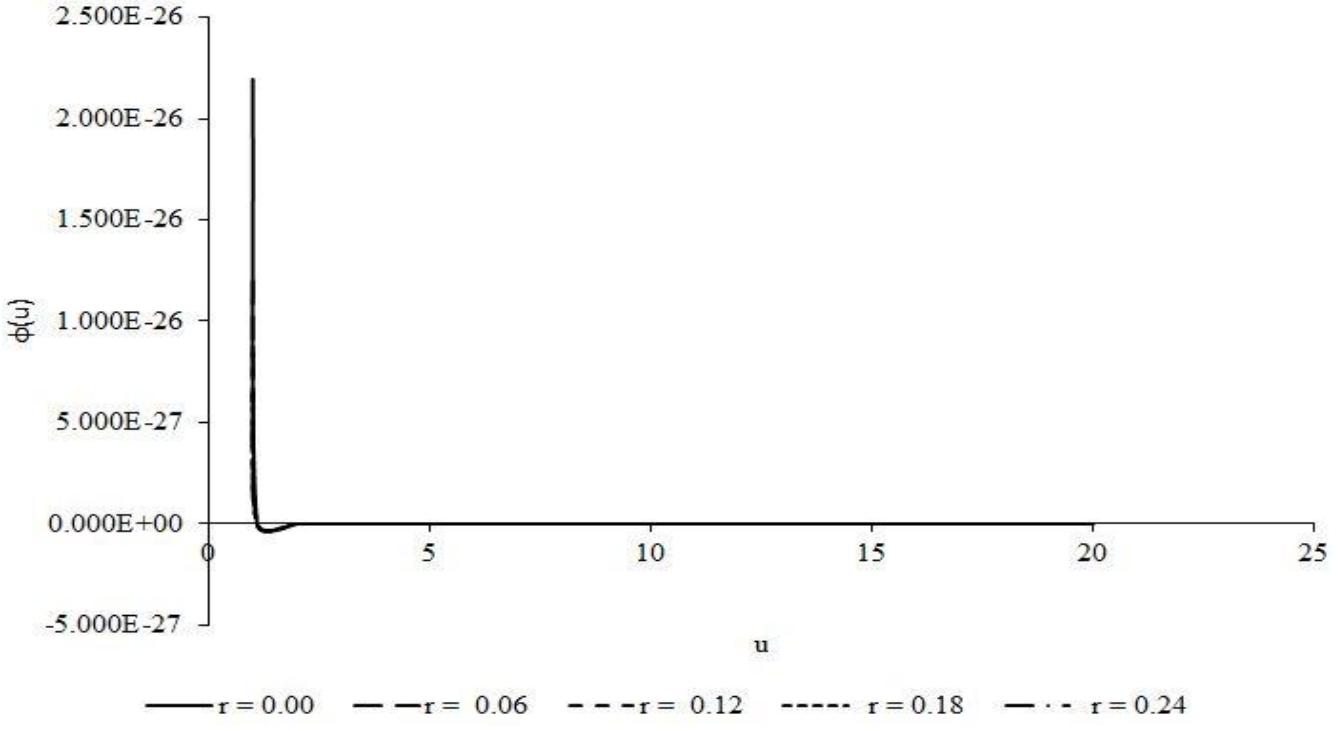


Figure 4.51: Ruin Probabilities Resulting from Oscillation when Interest rate *r* is varied, claim size is exponential and all other parameters kept constant at *c*=1.1, λ=1.00, Θ=1.00, **ϕ=0.80**, σ_p=0.20, *a*=0.21, α=0.00 and σ_R=0.20.

4.2.8 Discussion of Results

Figures 4.48 to 4.51 and 4.15 to 4.17 show the ruin probability due to oscillation when claim sizes are exponential and interest rate is varied. From the tables, reduction is noticed as interest rate is varied across a fixed capital at all ϕ. This reduction, though very minute, show that premium has effect on

ruin probability due to oscillation. The ruin probability due to oscillation becomes negligible as capital is increased.

4.2.9 Ruin Probability Resulting from Oscillation when Claim size are Exponential and Drift is varied

Table 4.18: Ruin Probabilities Resulting from Oscillation when Drift *a* is varied, claim size is exponential and all other parameters kept constant at *c*=1.1, λ=1.00, Θ=1.00, **ϕ=0.20**, σ_p=0.20, *r*=0.12, α=0.00 and σ_R=0.20.

u	*a*=0.5	*a*=1.0	*a*=1.5	*a*=2.0
1	7.890E-27	5.807E-28	4.275E-29	3.147E-30
2	3.523E-57	1.301E-61	4.806E-66	1.775E-70
3	8.889E-92	1.122E-101	1.416E-111	1.787E-121
4	1.267E-130	3.722E-148	1.094E-165	3.211E-183
5	1.019E-173	4.749E-201	2.212E-228	1.029E-255
6	4.629E-221	2.328E-260	0.000E+00	0.000E+00
7	0.000E+00	0.000E+00	0.000E+00	0.000E+00
8	0.000E+00	0.000E+00	0.000E+00	0.000E+00

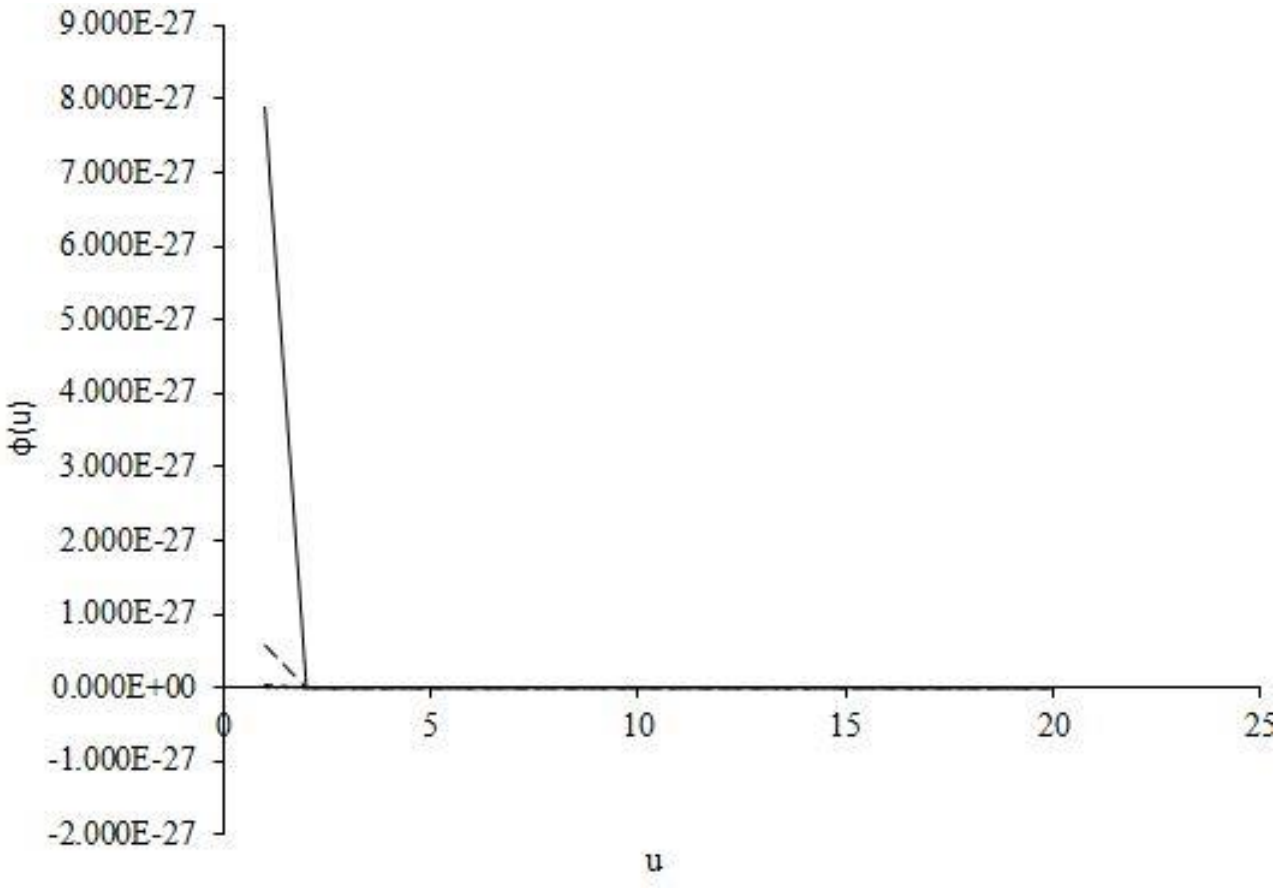


Figure 4.52: Ruin Probabilities Resulting from Oscillation when Drift *a* is varied, claim size is exponential and all other parameters kept constant at *c*=1.1, λ=1.00, Θ=1.00, **ϕ=0.20**, σ_p=0.20, *r*=0.12, α=0.00 and σ_R=0.20.

Table 4.19: Ruin Probabilities Resulting from Oscillation when Drift *q* is varied, claim size is exponential and all other parameters kept constant at *c*=1.1, λ=1.00, Θ=1.00, **ϕ=0.40**, σ_p=0.20, *r*=0.12, α=0.00 and σ_R=0.20.

u	*a*=0.5	*a*=1.0	*a*=1.5	*a*=2.0
1	1.086E-27	5.887E-30	3.023E-32	2.620E-35
2	1.507E-60	2.056E-69	2.509E-78	2.676E-87
3	2.665E-99	4.247E-119	5.689E-139	1.670E-158
4	6.004E-144	5.184E-179	3.523E-214	9.268E-249
5	1.723E-194	3.738E-249	0.000E+00	0.000E+00
6	6.290E-251	0.000E+00	0.000E+00	0.000E+00
7	0.000E+00	0.000E+00	0.000E+00	0.000E+00
8	0.000E+00	0.000E+00	0.000E+00	0.000E+00

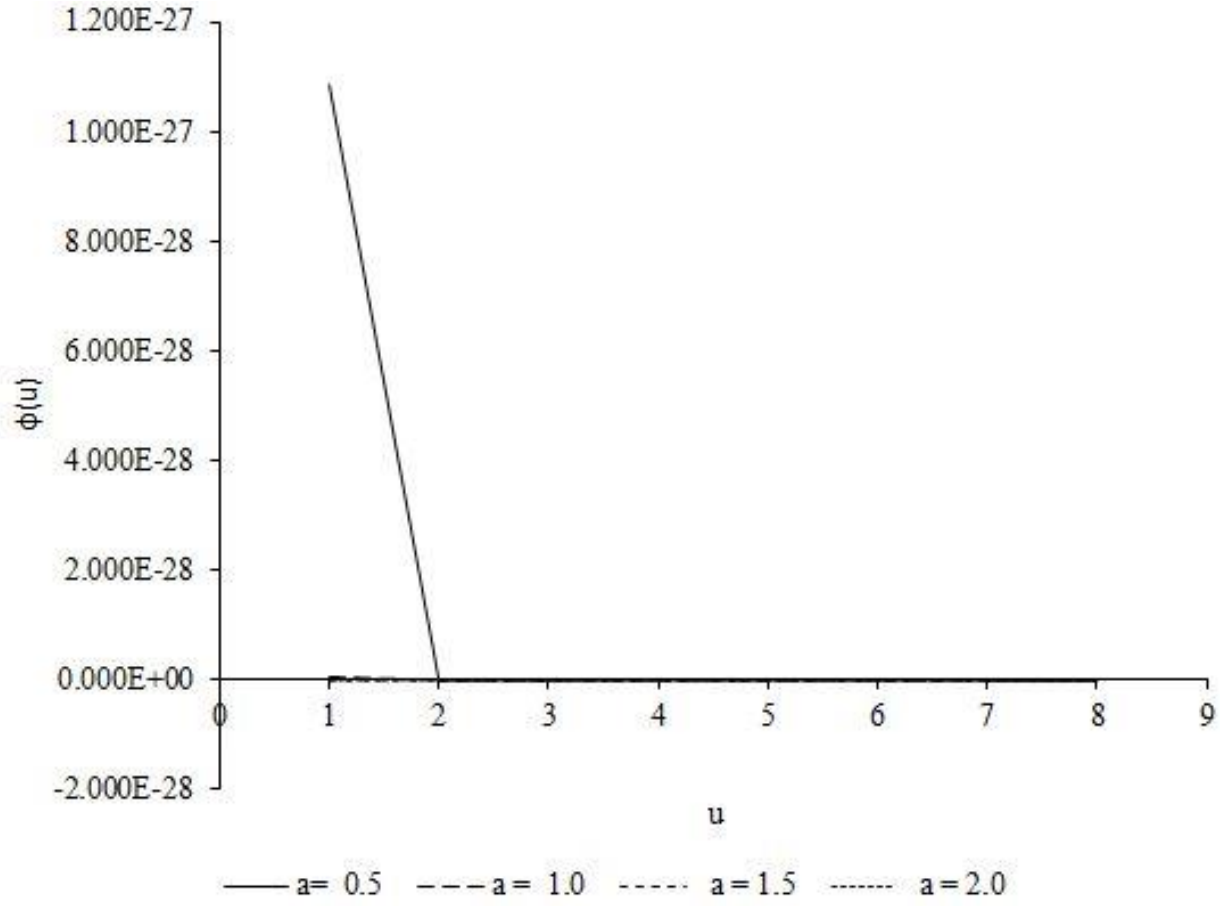


Figure 4.53: Ruin Probabilities Resulting from Oscillation when Drift *a* is varied, claim size is exponential and all other parameters kept constant at *c*=1.1, λ=1.00, Θ=1.00, **ϕ=0.40**, σ_p=0.20, *r*=0.12, α=0.00 and σ_R=0.20.

Table 4.20: Ruin Probabilities Resulting from Oscillation when Drift *a* is varied, claim size is exponential and all other parameters kept constant at *c*=1.1, λ=1.00, Θ=1.00, **ϕ=0.60**, σ_p=0.20, *r*=0.12, α=0.00 and σ_R=0.20.

u	*a*=0.5	*a*=1.0	*a*=1.5	*a*=2.0
1	1.495E-28	5.773E-32	2.902E-35	5.061E-38
2	6.446E-64	3.035E-77	5.633E-90	7.959E-103
3	7.991E-107	1.447E-136	1.830E-165	9.074E-195
4	2.846E-157	6.257E-210	1.442E-261	0.000E+00
5	2.911E-215	2.452E-297	0.000E+00	0.000E+00
6	8.549E-281	0.000E+00	0.000E+00	0.000E+00
7	0.000E+00	0.000E+00	0.000E+00	0.000E+00
8	0.000E+00	0.000E+00	0.000E+00	0.000E+00

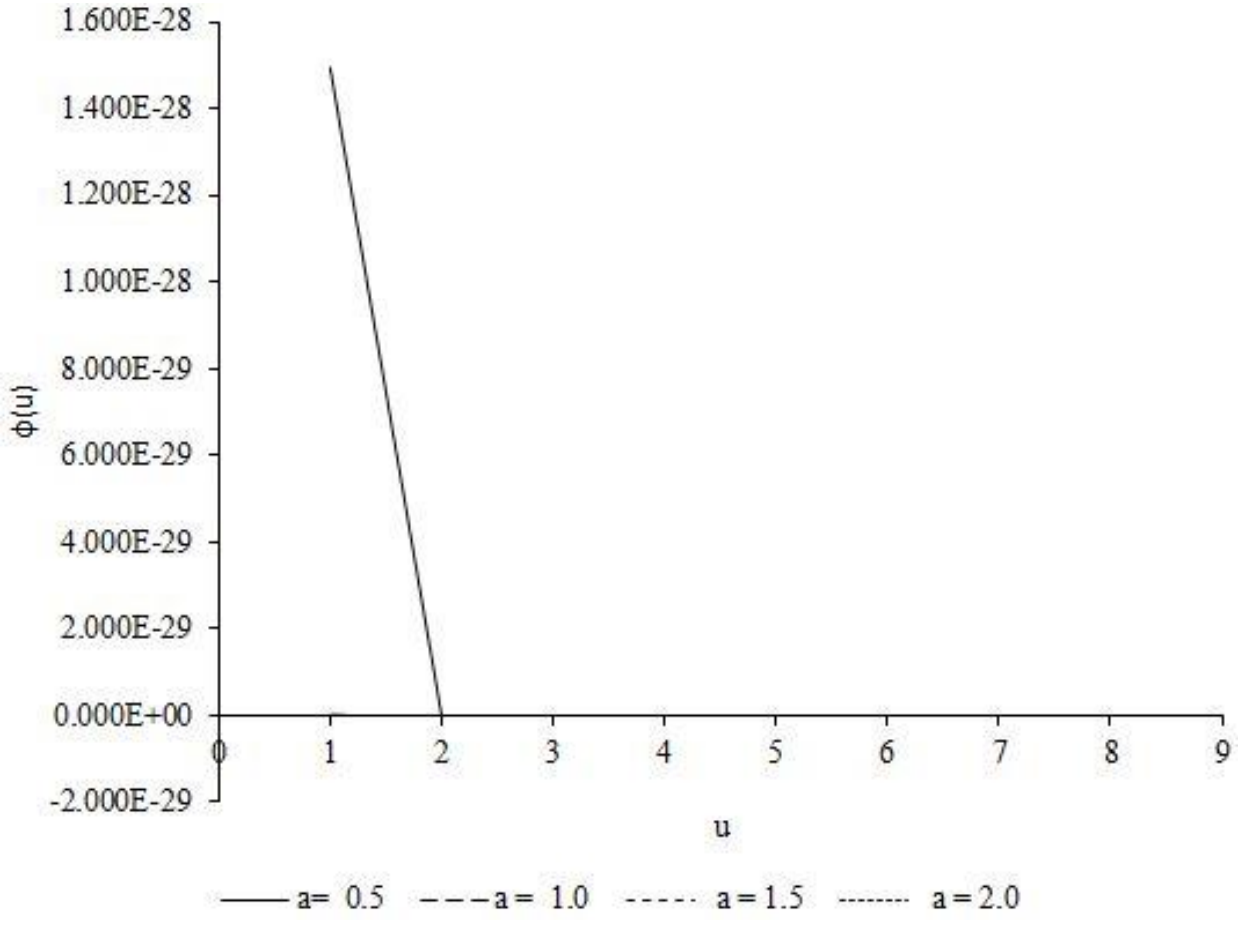


Figure 4.54: Ruin Probabilities Resulting from Oscillation when Drift *a* is varied, claim size is exponential and all other parameters kept constant at *c*=1.1, λ=1.00, Θ=1.00, **ϕ=0.60**, σ_p=0.20, *r*=0.12, α=0.00 and σ_R=0.20.

Table 4.21: Ruin Probabilities Resulting from Oscillation when Drift *a* is varied, claim size is exponential and all other parameters kept constant at *c*=1.1, λ=1.00, Θ=1.00, **ϕ=0.80**, σ_p=0.20, *a*=0.12, α=0.00 and σ_R=0.20.

u	*a*=0.5	*a*=1.0	*a*=1.5	*a*=2.0
1	2.059E-29	3.160E-34	1.001E-37	2.257E-42
2	2.759E-67	2.201E-86	9.809E-102	1.358E-119
3	2.396E-114	2.655E-154	2.298E-192	4.287E-232
4	1.349E-170	9.204E-241	0.000E+00	0.000E+00
5	4.920E-236	0.000E+00	0.000E+00	0.000E+00
6	0.000E+00	0.000E+00	0.000E+00	0.000E+00
7	0.000E+00	0.000E+00	0.000E+00	0.000E+00
8	0.000E+00	0.000E+00	0.000E+00	0.000E+00

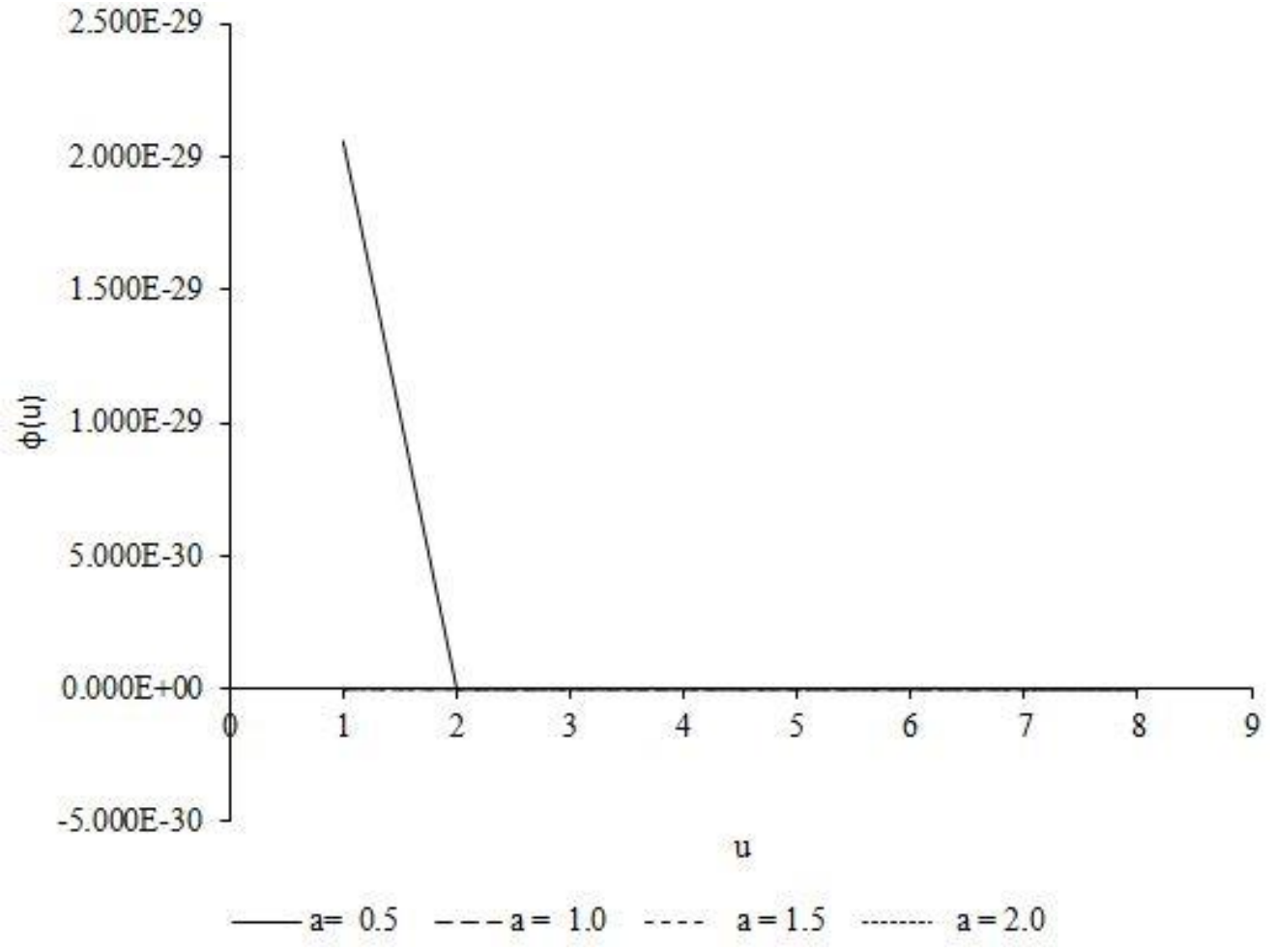


Figure 4.55: Ruin Probabilities Resulting from Oscillation when Drift *a* is varied, claim size is exponential and all other parameters kept constant at *c*=1.1, λ=1.00, Θ=1.00, **ϕ=0.80**, σ_p=0.20, *r*=0.12, α=0.00 and σ_R=0.20.

4.2.10 Discussion of Results

Figures 4.52 to 4.55 and 4.18 to 4.21 show the ruin probability due to oscillation when drift is varied at different ϕ. There is a reduction in ruin probability when drift is varied at a fixed capital and ϕ. This reduction though may be considered negligible as the value drift is increased. Also a rapid reduction in the ruin probability is observed at when the capital is increased.

The ruin probability due to oscillation is affected by drift though the effect could be considered negligible.

4.2.11 Ruin Probability Resulting from Oscillation when Claim size are Exponential and Volatility is varied

Table 4.22: Ruin Probabilities Resulting from Oscillation when Volatility σ_p is varied, claim size is exponential and all other parameters kept constant at c=1.1, λ=1.00, Θ=1.00, **ϕ=0.20**, a=0.21, r=0.12, α=0.00 and σ_R=0.20.

u	σ_p=0.5	σ_p=2.5	σ_p=4.5	σ_p=6.5
0	1.000E+00	1.000E+00	1.000E+00	1.000E+00
2	2.007E-09	2.539E-01	6.449E-01	7.882E-01
4	5.060E-20	5.003E-02	3.927E-01	6.063E-01
6	1.584E-32	7.610E-03	2.254E-01	4.549E-01
8	6.113E-47	8.904E-04	1.218E-01	3.326E-01
10	2.895E-63	7.991E-05	6.187E-02	2.368E-01
12	1.676E-81	5.490E-06	2.953E-02	1.641E-01
14	1.184E-101	2.882E-07	1.323E-02	1.105E-01
16	1.019E-123	1.155E-08	5.561E-03	7.214E-02
18	1.066E-147	3.530E-10	2.191E-03	4.559E-02
20	1.356E-173	8.220E-12	8.079E-04	2.774E-02

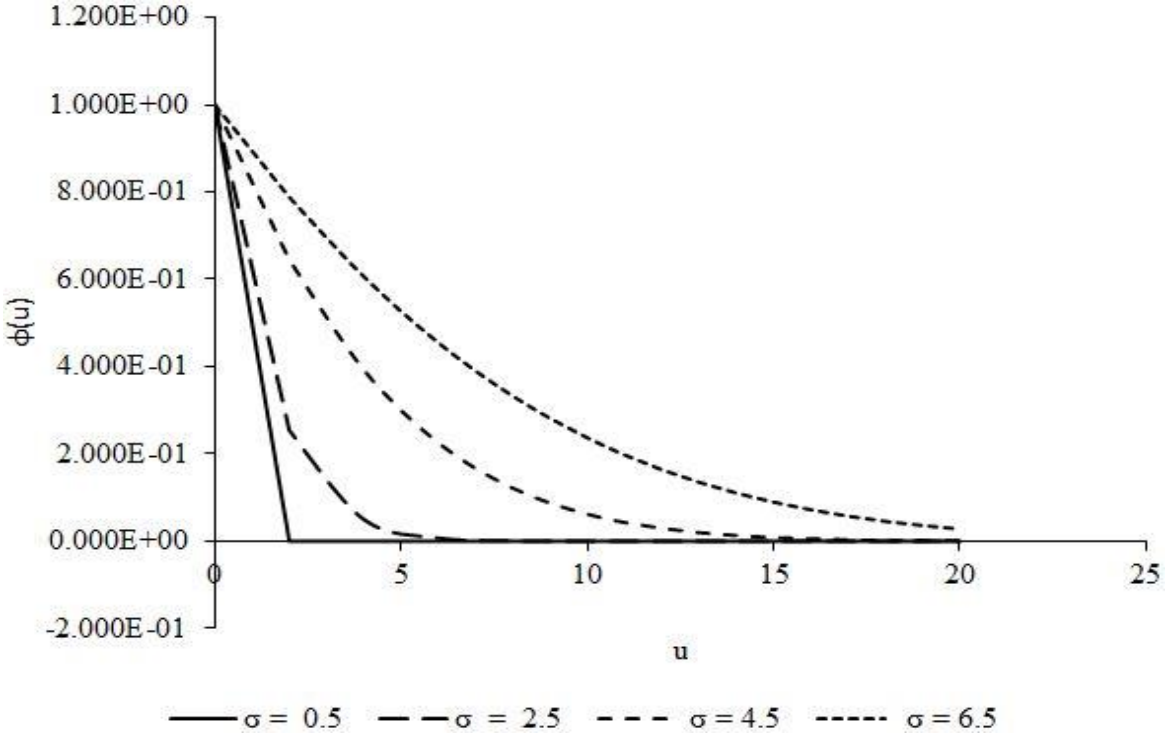


Figure 4.56: Ruin Probabilities Resulting from Oscillation when Volatility σ_p is varied, claim size is exponential and all other parameters kept constant at c=1.1, λ=1.00, Θ=1.00, **ϕ=0.20**, a=0.21, r=0.12, α=0.00 and σ_R=0.20.

Table 4.23: Ruin Probabilities Resulting from Oscillation when Volatility σ_p is varied, claim size is exponential and all other parameters kept constant at c=1.1, λ=1.00, Θ=1.00, **ϕ=0.40**, a=0.21, r=0.12, α=0.00 and σ_R=0.20.

u	σ_p=0.5	σ_p=2.5	σ_p=4.5	σ_p=6.5
0	1.000E+00	1.000E+00	1.000E+00	1.000E+00
2	1.468E-09	2.486E-01	6.350E-01	7.806E-01
4	1.535E-20	4.642E-02	3.780E-01	5.930E-01
6	1.126E-33	6.469E-03	2.105E-01	4.380E-01
8	5.737E-49	6.696E-04	1.095E-01	3.143E-01
10	2.020E-66	5.132E-05	5.315E-02	2.189E-01
12	4.898E-86	2.905E-06	2.403E-02	1.479E-01
14	8.149E-108	1.213E-07	1.012E-02	9.680E-02
16	9.290E-132	3.727E-09	3.961E-03	6.128E-02
18	7.245E-158	8.425E-11	1.441E-03	3.744E-02
20	3.863E-186	1.400E-12	4.870E-04	2.198E-02

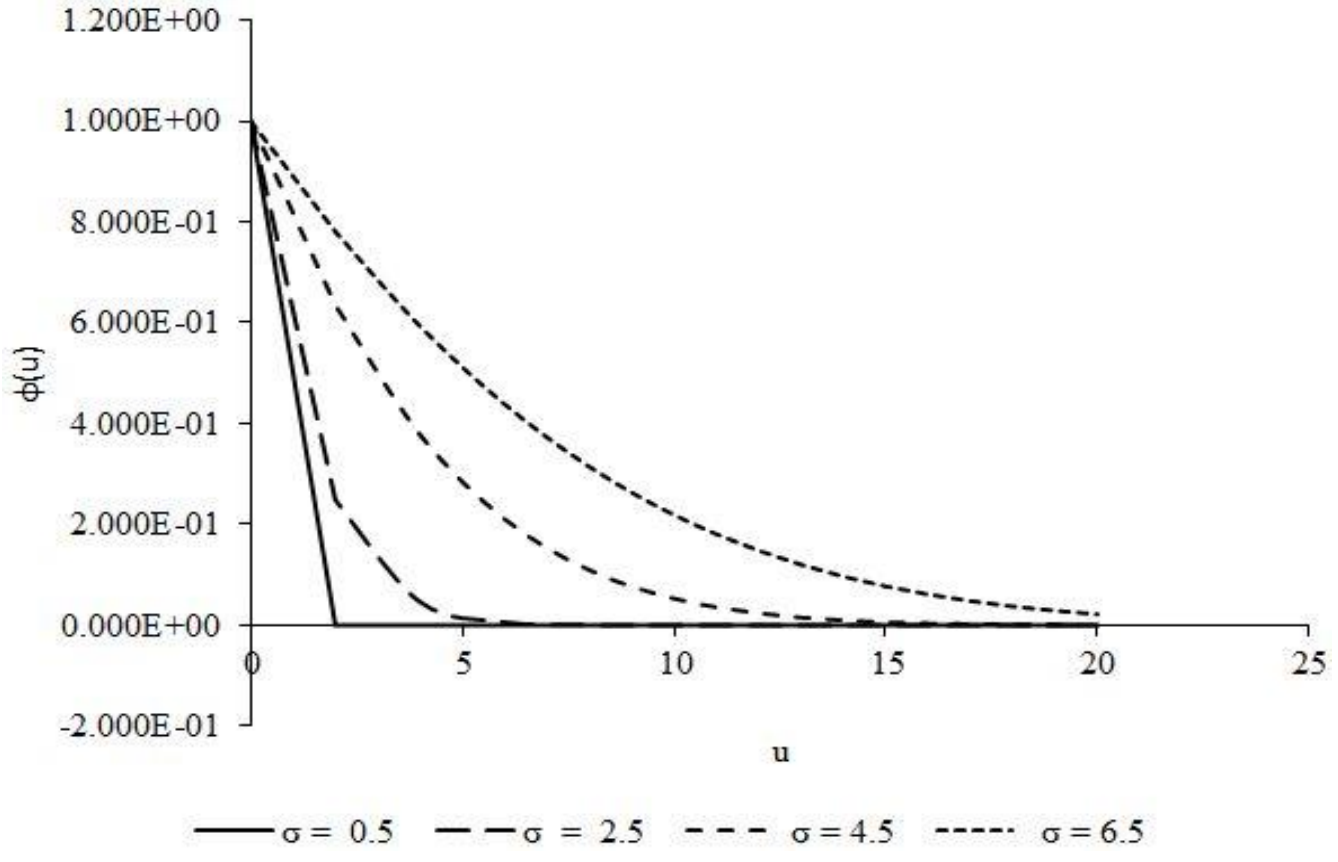


Figure 4.57: Ruin Probabilities Resulting from Oscillation when Volatility σ_p is varied, claim size is exponential and all other parameters kept constant at c=1.1, λ=1.00, Θ=1.00, **ϕ=0.40**, a=0.21, r=0.12, α=0.00 and σ_R=0.20.

Table 4.24: Ruin Probabilities Resulting from Oscillation when Volatility σ_p is varied, claim size is exponential and all other parameters kept constant at c=1.1, λ=1.00, Θ=1.00, **ϕ=0.60**, r=0.12, a=0.21, α=0.00 and σ_R=0.20.

u	σ_p=0.5	σ_p=2.5	σ_p=4.5	σ_p=6.5
0	1.00E+00	1.00E+00	1.00E+00	1.00E+00
2	1.07E-09	2.44E-01	6.26E-01	7.73E-01
4	4.66E-21	4.32E-02	3.64E-01	5.80E-01
6	8.02E-35	5.51E-03	1.97E-01	4.22E-01
8	5.41E-51	5.05E-04	9.87E-02	2.97E-01
10	1.42E-69	3.30E-05	4.58E-02	2.03E-01
12	1.44E-90	1.54E-06	1.96E-02	1.34E-01
14	5.65E-114	5.11E-08	7.76E-03	8.50E-02
16	8.53E-140	1.20E-09	2.83E-03	5.21E-02
18	4.96E-168	2.01E-11	9.53E-04	3.08E-02
20	1.11E-198	2.39E-13	2.95E-04	1.74E-02

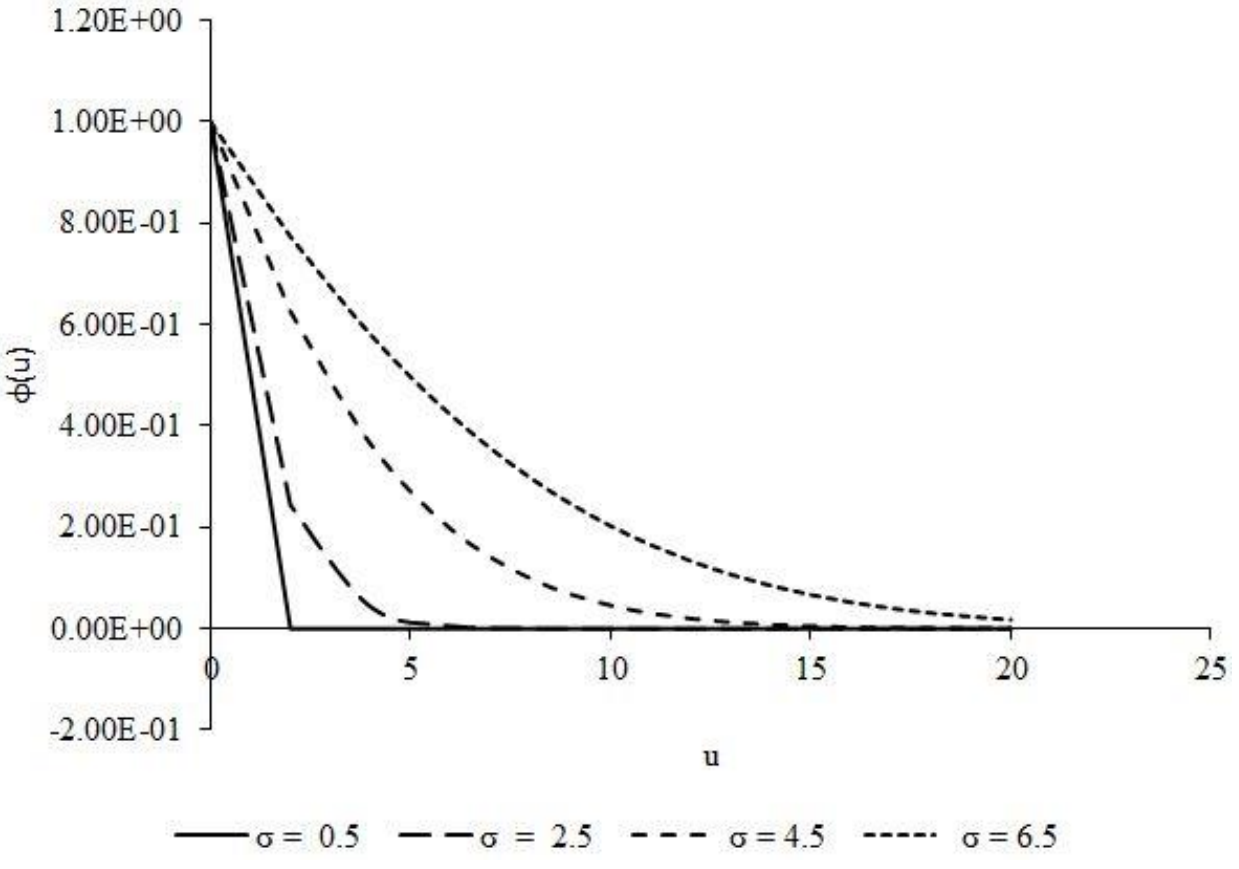


Figure 4.58: Ruin Probabilities Resulting from Oscillation when Volatility σ_p is varied, claim size is exponential and all other parameters kept constant at c=1.1, λ=1.00, Θ=1.00, **ϕ=0.60**, r=0.12, a=0.21, α=0.00 and σ_R=0.20.

Table 4.25: Ruin Probabilities Resulting from Oscillation when Volatility σ_p is varied, claim size is exponential and all other parameters kept constant at c=1.1, λ=1.00, Θ=1.00, **ϕ=0.80**, r=0.21, a=0.12, α=0.00 and σ_R=0.20.

u	σ_p=0.5	σ_p=2.5	σ_p=4.5	σ_p=6.5
0	1.000E+00	1.000E+00	1.000E+00	1.000E+00
2	7.878E-10	2.390E-01	6.168E-01	7.665E-01
4	1.422E-21	4.018E-02	3.516E-01	5.683E-01
6	5.747E-36	4.705E-03	1.847E-01	4.072E-01
8	5.128E-53	3.814E-04	8.921E-02	2.816E-01
10	1.002E-72	2.132E-05	3.955E-02	1.879E-01
12	4.272E-95	8.189E-07	1.607E-02	1.207E-01
14	3.954E-120	2.158E-08	5.978E-03	7.469E-02
16	7.932E-148	3.896E-10	2.034E-03	4.441E-02
18	3.442E-178	4.813E-12	6.321E-04	2.534E-02
20	3.229E-211	4.065E-14	1.793E-04	1.382E-02

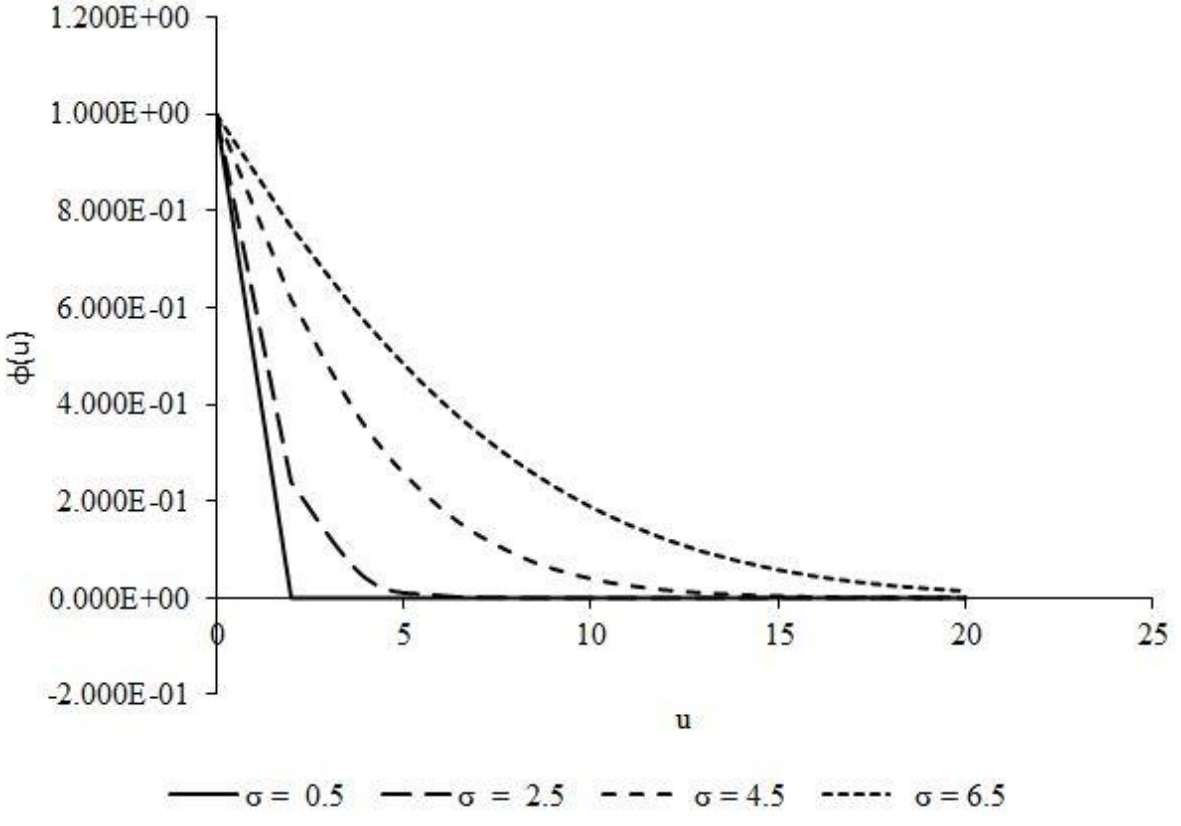


Figure 4.59: Ruin Probabilities Resulting from Oscillation when Volatility σ_p is varied, claim size is exponential and all other parameters kept constant at c=1.1, λ=1.00, Θ=1.00, **ϕ=0.80**, r=0.21, a=0.12, α=0.00 and σ_R=0.20.

4.2.12 Discussion of Results

Figures 4.56 to 4.59 and tables 4.22 to 4.59 show the ruin probability due to oscillation when volatility is varied at different values of ϕ. Increase in volatility is increases the ruin probability at fixed capital and ϕ. Also, increasing the fraction invested into risky assets increases the effect of volatility on ruin probability.

The effect of volatility on ruin probability wanes as capital invested into risky assets increases.

4.3 Application to Real life data

The model is applied to aggregated income and expenditure of insurance firms (Non-life) in Nigeria, extracted from the work of Ubom (2014), as shown in table 4.26. The data contain the premium and claim collected from 1990 - 2011. The investments captured in the data are government security, stock and bonds, real estate and mortgage, other loans, cash deposits and bill of exchange from 1996 - 2011. Investment before 1996 is not available since most insurance firms are dependent on premium only at this period. Also since the starting capital of the firms were not available on annual bases, the operating capital is considered as "*u*", thus the word "reserve" is used for *u* in place of "capital".

Table 4.26: Income and expenditure of Insurance firms in Nigeria between 1990-2011

	Income			Expenditure			
Year	Premium	Others	Total	Claims	Others	Total	Balance
1990	1.013674	0.034769	1.048443	0.306512	0.388611	0.695123	0.35332
1991	1.29624	0.037994	1.334234	0.386872	0.570902	0.957774	0.37646
1992	2.455691	0.07221	2.527901	0.612887	1.157628	1.770515	0.757386
1993	4.931918	0.969339	5.901257	2.684105	3.291629	5.975734	-0.07448
1994	14.51915	0.152526	14.67168	1.315294	2.483586	3.79888	10.8728
1995	13.52513	1.062524	14.58765	1.508882	3.856178	5.36506	9.222589
1996	11.09133	2.059232	13.15056	1.654069	4.26207	5.916139	7.234424
1997	10.94158	5.577439	16.51902	1.677282	4.822117	6.499399	10.01962
1998	11.68825	6.15822	17.84647	1.956214	5.218066	7.17428	10.67219
1999	14.59728	0.046578	14.64386	5.92318		5.92318	8.720678
2000	22.53146		22.53146	5.62952		5.62952	16.90194
2001	28.98129		28.98129	6.11052		6.11052	22.87077
2002	37.76589		37.76589	6.856145		6.856145	30.90975
2003	43.44181	0.502894	43.9447	9.4152		9.4152	34.5295
2004	50.10083	0.395046	50.49588	12.08408		12.08408	38.4118
2005	67.46556	0.280753	67.74631	12.4024		12.4024	55.34391
2006	81.58375	0.77814	82.36189	12.77447		12.77447	69.58742
2007	89.18489	1.065177	90.25007	25.13324		25.13324	65.11683
2008	107.2212	2.161268	109.3825	37.41255		37.41255	71.96992
2009	153.1271		153.1271	61.96915		61.96915	91.15797
2010	157.3368		157.3368	53.81535		53.81535	103.5215
2011	175.7568		175.7568	66.20476		66.20476	109.552

Source: Ubom (2014)

Bill of exchange, cash deposits and other loans are considered as investments with fixed returns, while Government security, Stock & Bonds, and Real estate & Mortgage are considered as investments with stochastic returns. Table 4.27 shows the investment portfolios of the firms.

Table 4.27: Investment portfolios of Insurance firms in Nigeria between 1990-2011

Year	Gov. Sec	Stock & Bond	Real Est. & Mortgage	Policy & Loans	Cash Deposits	Bill of Exchange	Total
1996	1.54616	4.04781	2.5232	0.79593	3.34706	0.1193	12.37946
1997	2.01201	4.09538	2.6835	0.84311	3.81591	0.16417	13.61308
1998	4.14588	3.63317	0.21195	2.30122	1.99319	3.37147	15.65688
1999	2.98721	4.17404	0.33265	4.12447	4.18416	5.78093	21.58346
2000	3.55895	4.99287	0.28234	5.21208	3.84437	7.30203	25.19264
2001	3.84271	6.88626	0.35933	6.7064	4.28455	10.17802	32.25727
2002	3.75208	8.35085	0.96031	7.90101	4.0954	11.88122	36.94087
2003	3.55895	4.99287	0.28234	5.21208	3.84437	7.30202	25.19263
2004	8.708	0	0.35184	8.35651	2.66858	2.59387	22.6788
2005	4.17806	61.80082	33.78815	5.5907	10.18535	6.30114	121.8442
2006	4.8581	121.8131	45.18677	7.88473	30.31417	6.30301	216.3599
2007	20.91481	222.2789	45.33191	12.94582	22.50809	5.26778	329.2473
2008	21.3749	227.1691	46.3292	13.2306	23.0039	5.3837	336.4914
2009	21.8452	232.1668	47.3485	13.5217	23.51	5.5021	343.8942
2010	22.3258	237.2744	48.3901	13.8192	24.0272	5.6232	351.4599
2011	22.8169	242.4945	49.4547	14.1232	24.5558	5.7469	359.1922

Source: Ubom (2014)

4.3.1 Distribution of claims and estimation of parameters of the model

The analysis of the data requires that the distribution of claims be determined and the parameters of the model be estimated from the data. Easyfit statistical software was used to determine the distribution of claim sizes. Hypothesis () was formulated for the goodness of fit test.

$$H_0: F_X(x) = F_0(x)$$

$$H_1: F_X(x) \neq F_0(x) \qquad for all x \qquad (4.1)$$

where $F_X(x)$ is the unknown distribution of the data and $F_0(x)$ is completely specified as exponential with parameter Θ. Figure 4.60 shows the fit of the distribution with the data.

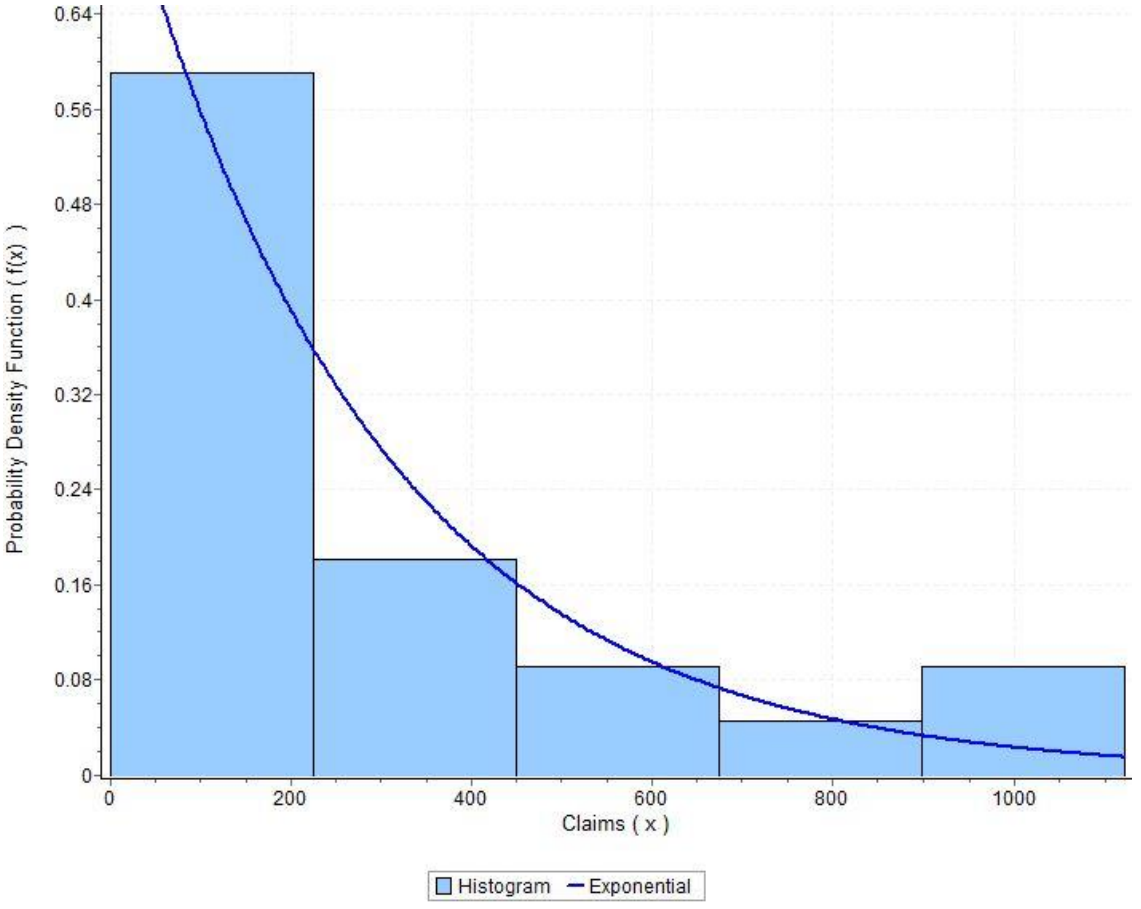


Figure 4.60: Distribution fit of the claim sizes in table 4.26

Figure 4.60 show the probability plot of the claim sizes and exponential distribution with parameter Θ=0.6711. It is obvious that the data is well approximated by exponential distribution. A Goodness of fit test on claim sizes using Kolmogorov-Smirnov test with level of significant α=0.05 is shown in table 4.28.

Table 4.28: Goodness of fit test on claim size

	Kolmogorov–Smirnov
Sample Size	22
Statistic	0.24426
P-Value	0.12176

Since p–value >0.05, H_0 is not rejected, thus the dataset is from exponential distribution with parameter Θ=0.6711.

Table 4.29: The parameters determined from the data

Year	u	c	λ	Θ	r	σ	a	ϕ
1990		0.02069	2.0670	0.6711				
1991	0.35332	0.02645	2.0941	0.6711				
1992	0.37646	0.05012	2.5042	0.6711				
1993	0.75739	0.10065	1.1484	0.6711				
1994	-0.07448	0.29631	6.8992	0.6711				
1995	10.87280	0.27602	5.6023	0.6711				
1996	9.22259	0.22635	4.1909	0.6711	0.1169	2.1602	0.5558	0.6557
1997	7.23442	0.22330	4.0771	0.6711	0.0480	2.4495	1.1546	0.6457
1998	10.01962	0.23854	3.7343	0.6711	0.0549	2.7386	1.6638	0.5104
1999	10.67219	0.29790	1.5403	0.6711	0.0533	3.0277	1.4269	0.3472
2000	8.72068	0.45983	2.5015	0.6711	0.0529	3.3166	1.2186	0.3507
2001	16.90194	0.59145	2.9643	0.6711	0.0549	3.6056	1.0202	0.3437
2002	22.87077	0.77073	3.4427	0.6711	0.0415	3.8944	0.8655	0.3536
2003	30.90975	0.88657	2.8838	0.6711	0.0411	4.1833	0.7915	0.3507
2004	34.52950	1.02247	2.5913	0.6711	0.0419	4.4721	0.7271	0.3995
2005	38.41180	1.37685	3.3998	0.6711	0.0383	4.7610	0.6463	0.8188
2006	55.34391	1.66497	3.9915	0.6711	0.0314	5.0498	0.5719	0.7943
2007	69.58742	1.82010	2.2178	0.6711	0.0355	5.3385	0.5191	0.8763
2008	65.11683	2.18819	1.7912	0.6711	0.0284	5.6273	0.5434	0.8763
2009	71.96992	3.12504	1.5444	0.6711	0.0268	5.9161	0.4593	0.8763
2010	91.15797	3.21096	1.8273	0.6711	0.0221	6.2048	0.3918	0.8763
2011	103.52146	3.58687	1.6592	0.6711	0.0143	6.4936	0.3451	0.8763

Table 4.29 show the parameters of the model as estimated from the data. The Probabilities of ruin attributable to claims, investments and total were determined for each of the years as shown in table 4.30.

4.3.2 Results and Discussion

Figures 4.61 to 4.64 show the behavior of the reserve and ruin probabilities over the years. From figure 4.61, there has been a progressive increase in the total reserve of insurance firms in Nigeria except for few occasions when declines have been witnessed. In 1994, the value of the reserve was noticed to have fallen below zero and subsequently rise above all previous values in the following year. Also the value of the reserve fell from 69.58742 billion in 2007 to 65.11683 billion in 2008.

In figure 4.62, ruin probability attributable to claim was noticed to have fluctuated. Increase in ruin probability was observed between 1995 and 1996 and between 2000 and 2001, while the probability decreased in all other years.

Table 4.30: Ruin probabilities for Insurance firms in Nigeria between 1991-2011

Year	u	φ_{claim}	φ_{invest}	φ_{total}
1991	0.35332	7.7990E-01		7.7990E-01
1992	0.37646	6.5849E-01		6.5849E-01
1993	0.75739	4.4444E-01		4.4444E-01
1994	-0.07448	5.2551E-02		5.2551E-02
1995	10.87280	5.2973E-02		5.2973E-02
1996	9.22259	4.2268E-01	1.8345E-04	4.2286E-01
1997	7.23442	3.3466E-01	6.6976E-04	3.3533E-01
1998	10.01962	7.7890E-02	3.7307E-06	7.7893E-02
1999	10.67219	8.1136E-03	4.7655E-04	8.5902E-03
2000	8.72068	7.3298E-02	1.2689E-02	8.5986E-02
2001	16.90194	7.6988E-03	2.1781E-05	7.7206E-03
2002	22.87077	1.5381E-03	4.4149E-07	1.5385E-03
2003	30.90975	1.1815E-05	1.8961E-09	1.1817E-05
2004	34.52950	2.9580E-07	1.1735E-08	3.0754E-07
2005	38.41180	1.2436E-08	6.8354E-12	1.2443E-08
2006	55.34391	6.5174E-10	7.6381E-10	1.4155E-09
2007	69.58742	1.5896E-09	4.6781E-09	6.2677E-09
2008	65.11683	6.5421E-10	8.4746E-09	9.1288E-09
2009	71.96992	1.0953E-12	6.6688E-08	6.6689E-08
2010	91.15797	2.4734E-12	9.8437E-07	9.8437E-07
2011	103.52146	1.1273E-13	4.6607E-06	4.6607E-06

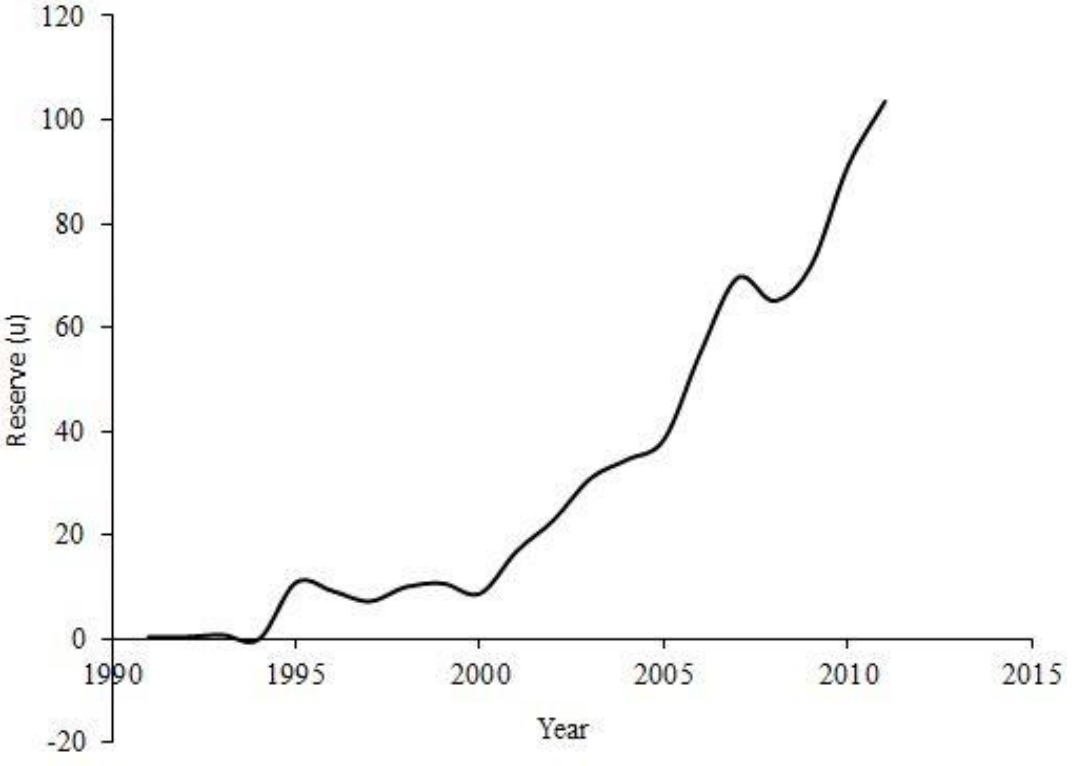


Figure 4.61: Reserve of the Insurance firms in Nigeria between 1991-2011

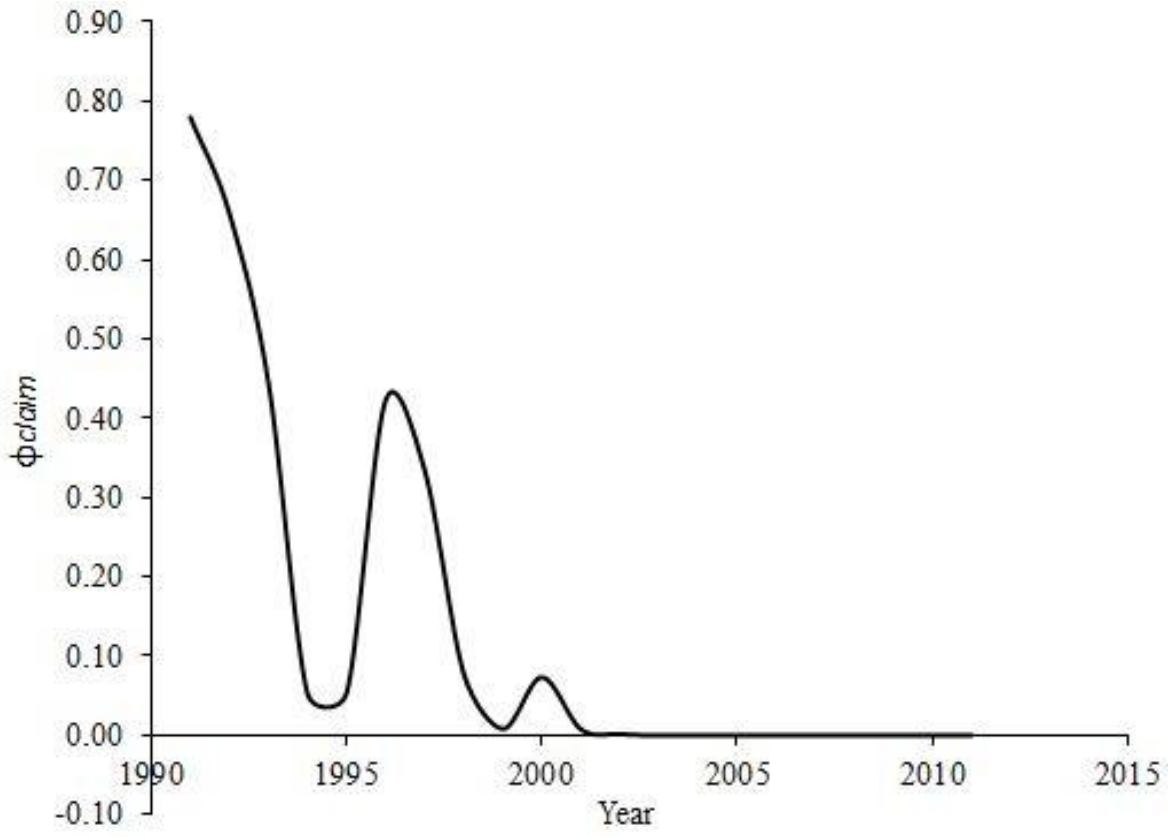


Figure 4.62: Ruin Probability due to claims for Insurance firm in Nigeria between 1991-2011

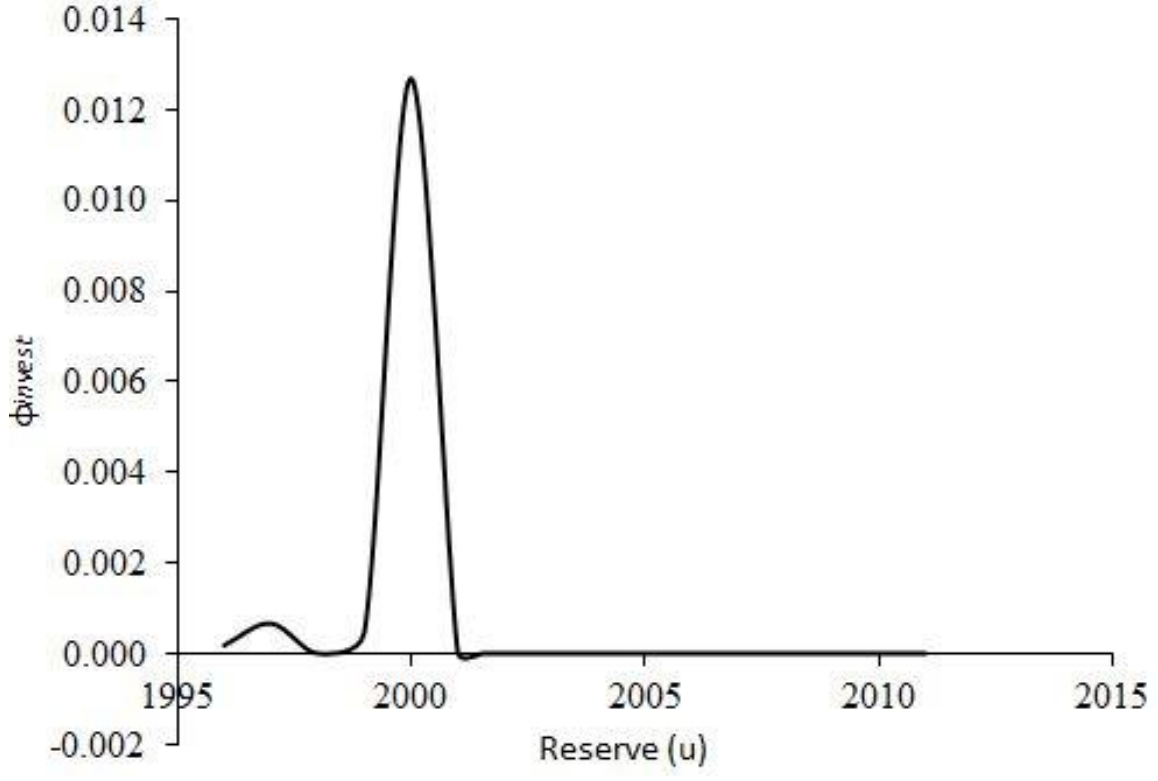


Figure 4.63: Ruin Probability due to Oscillation for Insurance firm in Nigeria between 1996-2011

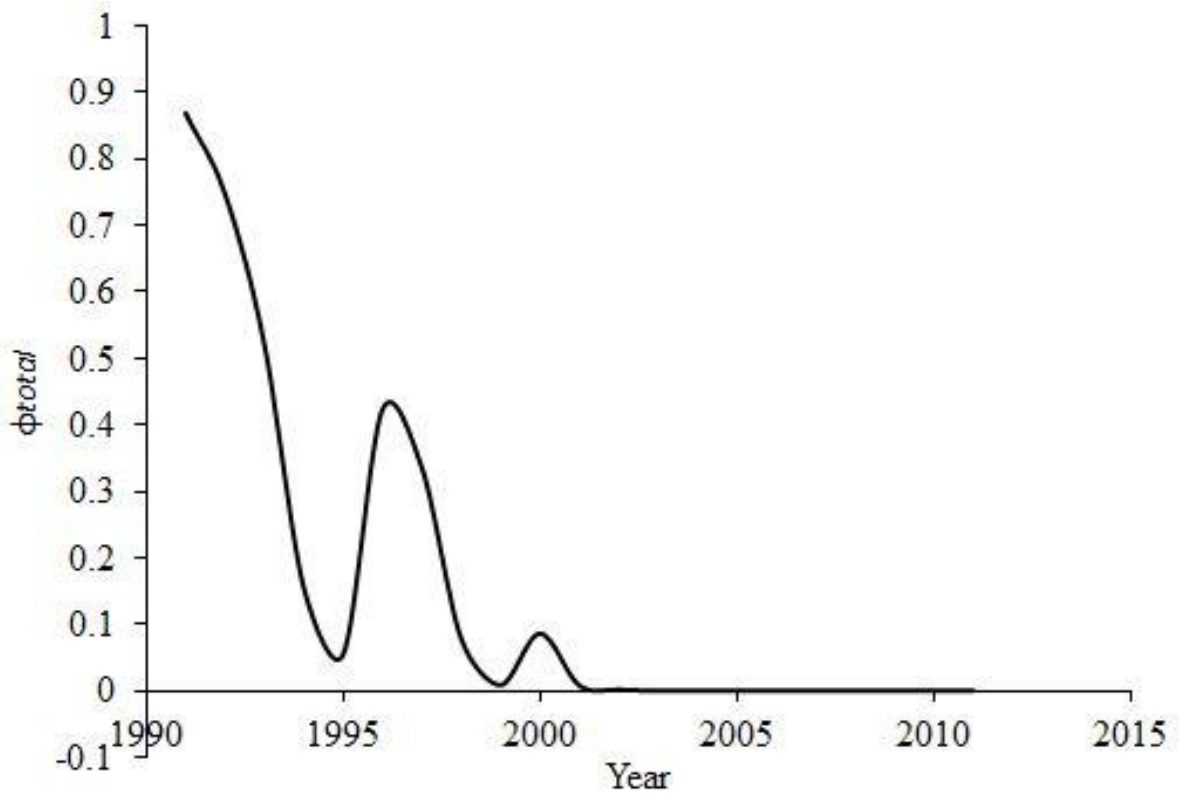


Figure 4.64: Ruin Probability for Insurance firm in Nigeria between 1991-2011

Figure 4.63 show the ruin probability due to investment only. From the figure, the companies were far from bankruptcy as a result of investments since the ruin probability is approximately zero in most cases. In figure 4.64, the sum of both ruin probabilities exhibits behavior similar to ruin due to claim.

It is noteworthy to mention that prior to 2005, companies were more liable to get ruined as a result of claims than investments. From 2008 and beyond, ruin probability due to investments dominated. This is clearly illustrated in table 4.30.

Chapter 5

Summary and Conclusion

5.0.1 Summary

In this work, a risk reserve model with investment and inflation is formulated and presented. The investment part of the model incorporated both risky and risk-less investments. The premium and risk-less investment are assumed to be a linear function while the risky investment is a geometric Brownian motion. Also, the claims arrival processes is assumed to be Markovian in nature. This model can be applied by any insurance firm irrespective of her investment strategy and claims sizes.

This thesis also illustrates how a Volterra integro-differential equation can be formulated for the ruin probability and converted into differential equations. For easier tractability, the differential equation was split into two equations representing ruin from claims and ruin from oscillations.

These two equations were solved analytically using theory of confluence hypergeometric function and results were obtained. The behavior of these results with varying parameters were further investigated. The model was applied to data obtained from the work of Ubom (2014).

5.0.2 Conclusion

Several authors have investigated several forms of ruin models while at the same time examined the ruin probabilities. Despite all these efforts and the huge success achieved so far, a ruin model considering stochastic and fixed investments whilst incorporating inflation which had not been considered remained the focus of this work. The ruin probabilities for this ruin model show some desirable results. It was observed that ruin can either be caused by claims or investments. The behavior of the ruin probability was investigated and it was observed that increase in premium, average claims size, claim arrival rate, interest and drift affects the probabilities attributable to claims. Also, ruin probability attributable to investment is affected by fraction of investment into risky assets, average claims size, volatility, interest and drift. The behavior of ruin attributable to investment when premium rate is varied does not follow any definite pattern. Hence, we cannot conclude that this ruin probability is affected by premuim rate. Another very important observation is that ruin is indpendent of Inflation.

5.0.3 Contribution to Knowledge

Despite several investigations and the successes that have been achieved by several authors, a ruin model with risky investment, risk-less investment and inflation all incorporated at the same time is

yet to be investigated. Thus, this work has been able to contribute the following to the existing body of knowledge;

1. A ruin model incorporating fixed investment, stochastic investments and inflation; which is a generalization of some of the existing models; has been formulated.

2. Ruin probabilities that can be attributed to claims and investments have been examined

The work has also been able to:

1. determine the parameters that affect these probabilities.

2. and show that inflation does not affect these probabilities of ruin.

5.0.4 Further Research

This research work can be further extended to include some events that were not considered. To this end, we suggest the following areas as objects of further studies:

1. a risk model with investments and inflation where dividend are paid when the reserve gets to a certain threshold could be studied.

2. the risk model could be considered with other possible claim distributions different from exponential and its mixtures.

3. the model could also be considered for situations where the claims inter-arrival time is Erlang distributed. This will allow for consideration of both true and false claims since insurance firms tends to investigate if a claim is actually genuine before payment is effected.

4. the premium could also be modeled as a stochastic process with similar properties to the claim process instead of a linear function considered here.

5. a dependence structure that allow the claim sizes to depend on inter-arrival time could also be considered. The dependency could be structured in such a way that inter-arrival time between several claims of similar sizes are small. This will capture situations were occurence of natural disaster causes several claims of similar sizes to arrive.

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Appendices

A Volterra Integro-differential Equation

An integral equation is an equation which contains the unknown function inside an integral sign. Generally, it can be written in the form;

$$u(x)=f(x)+\lambda \int_{g(x)}^{h(x)} K(x,t)u(t)dt \tag{A.1}$$

where $g(x)$ and $h(x)$ are the limits of integration, λ is a constant parameter, and $K(x,t)$ is a known function, of two variables x and t, called the *kernel function* of the integral equation. If the limits of integration are constants as given below;

$$u(x)=f(x)+\lambda \int_{a}^{b} K(x,t)u(t)dt \tag{A.2}$$

the equation is called *Fredholm integral equation*. If on the other hand one or both limits are variables as given below;

$$u(x)=f(x)+\lambda \int_{a}^{x} K(x,t)u(t)dt \tag{A.3}$$

the equation is called *Volterra integral equation*.

If the unknown function $u(x)$ appears only inside the integral sign, the equation is said to be an integral equation of the *first kind*. If on the other hand it appears both inside and outside the integral sign, the equation is said to be of the *second kind*. Also integral equations can be identified based on the function $f(x)$ in equations (A.1) – (A.3) above. If the function $f(x)$ is identically zero, the equation is said to be a *homogenous equation* otherwise it is called *inhomogenous*.

An equation containing both the integral and derivatives of the unknown function is called *integro-differential equation*. Thus the Volterra integro-differential equation is any equation of the form;

$$u^{(n)}(x)=f(x)+\lambda \int_{a}^{x} K(x,t)u(t)dt, \tag{A.4}$$

where the function $u^{(n)}(x)$ is the n^{th} derivative of the unknown function. The equation (A.4) may contain other derivatives of the unknown function.

B Hypergeometric Equations

The confluence hypergeometric equation is given as

$$xy''+(b-x)y'-ay=0 \tag{B.1}$$

One of the solutions is of the form (Slater, 1960; Georgiev and Georgieva-Grosse, 2005),

$$M(a,c;x)=\sum_{v=0}^{\infty}\frac{(a_v)}{(c)_v}\frac{x^v}{v!} \tag{B.2}$$

This solution is called the Kummer function and it is also known as the confluence hypergeometric function of the first kind. The symbol $(\alpha)_v$; where

$$(\alpha)_v=\alpha(\alpha+1)(\alpha+2)...(\alpha+v-1)=\Gamma(\alpha+v)/\Gamma(\alpha),$$

$$(1)_v=v! \tag{B.3}$$

α is any positive number (real or complex), v is any positive integer; is called the Pochhammer's symbol. Another solution of the equation which is called Tricomi's function or confluence hypergeometric function of the second kind is

$$U(a,c,x)=\frac{\Gamma(1-c)}{\Gamma(a+1-c)}M(a,c;z)+\frac{\Gamma(c-1)}{\Gamma(a)}x()^{1-c}M(a+1-c,2-c;z) \tag{B.4}$$

The Kummer's function is analytic, regular at zero and entire single-valued transcedental function of a,c,x (real or complex) except at $c\leq 0$ for which it has single poles (Georgiev and Georgieva-Grosse, 2005). Unlike the Kummer's function, the Tricomi's function can be extended to any integer c.

C Some Integral Formulae

Sometimes resolving multiple integral or differentiating under the integral could be very tricky, complex and time consuming. At this point, some useful mathematical tools could come handy. Among such useful tools are method for *reducing multiple integral to single integral* and *Leibnitz rule*

C.1 Reduction of Multiple Integral to Single Integral

Suppose it is desired to reduce the following double integral to single integral

$$\int_0^x\int_0^{x_1}F(y)dydx_1$$

it can be shown that

$$\int_0^x\int_0^{x_1}F(y)dydx_1=\int_0^x(x-y)F(y)dy \tag{C.1}$$

see Wazwaz (2011) pg.20 for the proof. Generally, multiple integral can be converted to a single integral using

$$\int_0^x\int_0^{x_1}\cdots\int_0^{x_{n-1}}F(x_n)dx_ndx_{n-1}\cdots dx_1=\frac{1}{(n-1)!}\int_0^x(x-y)^{n-1}F(y)dy \tag{C.2}$$

C.2 Leibnitz Rule

Let $f(x,t)$ be continuous and $\frac{\partial f}{\partial t}$ be continuous in a domain of the $x-t$ plane that includes the rectangle $a \leq x \leq b$, $t_0 \leq t \leq t1$, and let

$$F(x) = \int_{h(x)}^{g(x)} f(x,t)dt, \qquad (0.11)$$

then differentiation of the integral in (1.105) exists and is given by

$$F'(x) = \frac{dF}{dx} = f(x,h(x))\frac{h(x)}{dx} - f(x,g(x))\frac{dg(x)}{dx} + \int_{h(x)}^{g(x)}$$

frac∂f(x, t)∂dxdt, (0.12)

If $g(x)=a$ and $h(x)=b$ where a and b are constants, then the Leibnitz rule reduces to

$$F'(x) = \frac{dF}{dx} = \int_{b}^{a} \frac{\partial f(x,t)}{\partial dx}dt, \qquad (0.13)$$

which means that differentiation and integration can be interchanged Wazwaz (2011)

D Computer Program

D1 MATLAB Program

```
function f= realRisk(u,t,r,a,b,d,rt,Q,fr,sd)
% This program computes the real unit value of a risk process with
% deterministic inflation and variable stochastic investment
% where the investment a geometric brownian motion.
% u = initial capital, t = time, a = drift, d = inflation rate
% b = volatility, rt = arrival rate, Q = safety loading,
% fr = fraction invested, r = interest rate, sd = seed value,
% Wiener process W = e*sqrt(t) where e is white noise
% Simulating "arrival epoch" and "number of arrivals"
T = renPro(rt, t, sd); %T = poiPro(rt, t, sd);
N = length(T);
arlEpoch = zeros(N+1,1);
arlEpoch(1) = 0;
arlEpoch(2:N+1) = T;
resrv = zeros(N+1,1);
resrv(1) = u;
```

```
capital = zeros(N+1,1);

capital(1) = u;

% Simulating the Wiener Process (Brownian Motion process)

B = wiener(arlEpoch,sd);

w = B(:,3);

e = B(:,2);

% Simulating claim sizes

usd = sd+10;

x = claimSizes(N, usd);

claims = zeros(N+1,1);

income = zeros(N+1,1);

for n = 1:N

% computing the exponential

k = r-d + (a-r)*fr + (0.5*(fr*b)^2);

k1 = k*T(n) + fr*b*w(n);

% the 1st term of the Risk reserve due to initial capital

captl = u*exp(k1);

capital(n+1)= captl;

%simulating the claims

clm = 0;

for i = 1:n

h1 = k*( T(n)-T(i) );

h2 = fr*b*( w(n) - w(i) );

clm = clm + x(i) * exp(h1 + h2);

end

claims(n+1)= clm;

% the 2nd term of the Risk reserve due to income

k2 = ( k + b*e(n) )*( exp(k1)-1 );

miu = mean(claims);

c = ( 1 + Q )* miu * rt;

inc = c*k2;

income(n+1)= inc;

resrv(n+1) = captl + inc - clm;
```

```
%resrv(n+1) = k2;
end
%f = [arlEpoch premium];
f = [arlEpoch capital income claims resrv];
end
function f= realRisk(u,t,r,a,b,d,rt,Q,fr,sd)
% This program computes the real unit value of a risk process with
% deterministic inflation and variable stochastic investment
% where the investment a geometric brownian motion.
% u = initial capital, t = time, a = drift, d = inflation rate
% b = volatility, rt = arrival rate, Q = safety loading,
% fr = fraction invested, r = interest rate, sd = seed value,
% Wiener process W = e*sqrt(t) where e is white noise
% Simulating "arrival epoch" and "number of arrivals"
T = renPro(rt, t, sd); %T = poiPro(rt, t, sd);
N = length(T);
arlEpoch = zeros(N+1,1);
arlEpoch(1) = 0;
arlEpoch(2:N+1) = T;
resrv = zeros(N+1,1);
resrv(1) = u;
capital = zeros(N+1,1);
capital(1) = u;
% Simulating the Wiener Process (Brownian Motion process)
B = wiener(arlEpoch,sd);
w = B(:,3);
e = B(:,2);
% Simulating claim sizes
usd = sd+10;
x = claimSizes(N, usd);
claims = zeros(N+1,1);
income = zeros(N+1,1);
for n = 1:N

% computing the exponential
```

```
k = r-d + (a-r)*fr + (0.5*(fr*b)^2);

k1 = k*T(n) + fr*b*w(n);

% the 1st term of the Risk reserve due to initial capital

captl = u*exp(k1);

capital(n+1)= captl;

%simulating the claims

clm = 0;

for i = 1:n

h1 = k*( T(n)-T(i) );

h2 = fr*b*( w(n) - w(i) );

clm = clm + x(i) * exp(h1 + h2);

end

claims(n+1)= clm;

% the 2nd term of the Risk reserve due to income

k2 = ( k + b*e(n) )*( exp(k1)-1 );

miu = mean(claims);

c = ( 1 + Q )* miu * rt;

inc = c*k2;

income(n+1)= inc;

resrv(n+1) = captl + inc - clm;

%resrv(n+1) = k2;

end

%f = [arlEpoch premium];

f = [arlEpoch capital income claims resrv];

end

function f= wiener(t,sd)

% This program simulates Wiener process through the white noise

n = length(t);

w = zeros(n,1);

e = zeros(n,1);

w(1) = 0;

e(1) = 0;

e(2:n) = Whitenoise(n-1,sd);


for i = 2:n
```

```
w(i) = w(i-1) + e(i)*sqrt( t(i)-t(i-1) );

end

f = [t e w];

end

function f= Whitenoise(n,sd)

% This program simulates standard normal(gaussian)

% random variables with the following parameters

% Sample size = n; seed value = sd.

s = RandStream('swb2712','Seed',sd);

RandStream.setDefaultStream(s);

a = randn (n,1);

ave = mean(a);

stdv = std(a);

p = zeros(length(a),1);

for i=1:length(a)

p(i) = (a(i) - ave)/stdv;

end

f = p;

end

function f = unifrm(sd, n)

% This program simulates uniform(0,1) random variable with the

% seed value = sd

s = RandStream('swb2712','Seed',sd);

RandStream.setDefaultStream(s);

f = rand (n,1);
```

D.2 R Program

```
#exponential

#cl1=RuinrepEc(20,0.5);

RuinrepEc<-function(u,h){

m <- u/h+1 #h=0.5

x = 20

#hi= 0.75
```

```
co = 6#x/hi+2
v<-r <- matrix(nrow = m, ncol = co)
xh <-0
v[,1:2]<-RuinEc(u,h,xh)
#v[,1:2]<-RuinEc(u,h,0.0001)
xh = 0
for (k in 3:co){
xh=xh+ 0.5#hi
#xth=xth + 20
#xh =1/xth
v[,k]<-RuinEc(u,h,xh)[,2]
}
return(v)
}
RuinEc<-function(u,h,j){
m <- u/h+1
r <- matrix(nrow = m, ncol = 2)
r[1,1] <- 0;
r[1,2]<- RuinEclaim(r[1,1],j)
for ( k in 2:m){
r[k,1]<- r[k-1,1]+h;
r[k,2]<- RuinEclaim(r[k,1],j);
}
return(r)
}
Ruinreal<-function(reald){
m = length(u1)
r <- matrix(nrow = m, ncol = 2)
r[ ,1] <- u1;
#r[1,2]<- RuinEclaim(u1[1],reald[1,])
for ( k in 1:m){
```

```
r[k,2]<- RuinEclaim(u[k],k);

}

return(r)

}

RuinEclaim<-function(i,j){

u = i

tita = 1 #tita1[j]#1 ;

c = 1.1 #c1[j]#1.1 ;

r = j#0.12 #r1[j]#0.12

a = 0.21 #a1[j]#j#0.21

phi = 0.6 #phi1[j]#0.6

l <- 1;

alp = 0;

br <- r*(1-phi)+a*phi;

ctr = c*tita/br;

b1= (l+alp)/br

b2<- l/br

b3 <- alp/br

ex = exp(-ctr)

H11 <- -tita*ex*Re(kummerU(x = ctr, a= (1-b2),

b = 1- b1))-tita*(1-b1)*ex*Re(kummerU(x = ctr,

a= (2-b2), b = 2- b1))

H12t1<- tita*b1*ctr^(b1-1)*ex*Re(kummerM(x = ctr,

a= (1+b3), b = 1+ b1))

H12t2<- tita*ctr^b1*ex*Re(kummerM(x = ctr,

a= (1+b3), b = 1+ b1))

H12t3<- tita*ctr^b1 * ((b1+1)/(b3+1))*ex*Re(kummerM(x = ctr,

a= (2+b3), b = (2+ b1)))

H12 <- H12t1 - H12t2 + H12t3

H21t1 <- tita^2*ex*Re(kummerU(x = ctr,

a= (1-b2), b = 1- b1))
```

```
H21t2 <- 2*tita^2*(1-b2)*ex*Re(kummerU(x = ctr,
a= (2-b2), b = 2- b1))
H21t3 <- -tita^2*(1-b2)*(2-b2)*ex*Re(kummerU(x = ctr,
a= (3-b2), b = (3- b1)))
H21 <- H21t1 + H21t2 - H21t3
H22t1<- tita^2*b1*(b1-1)*ctr^(b1-2)*ex*Re(kummerM(x = ctr,
a= (1+b3), b = 1+ b1))
144
H22t2<- 2*tita^2*b1*ctr^(b1-1)*ex*Re(kummerM(x = ctr,
a= (1+b3), b = 1+ b1))
H22t3<- 2*tita*b1*ctr^(b1-1)*((b1+1)/(b3+1))*ex*Re(kummerM(x = ctr,
a= (2+b3), b = 2+ b1))
H22t4<- tita^2*ctr^b1*ex*Re(kummerM(x = ctr,
a= (1+b3), b = 1+ b1))
H22t5<- 2*tita^2*ctr^b1*ex*((b1+1)/(b3+1))*Re(kummerM(x = ctr,
a= (1+b3), b = 1+ b1))
H22t6<- tita^2*ctr^b1 * ((b1+1)/(b3+1))*((b1+2)/(b3+2))*ex*Re(kummerM(x = ctr,
a= (2+b3), b = (2+ b1)))
H22 <- H22t1 - H22t2 + H22t3 + H22t4 - H22t5 + H22t6
k1 <- c* H22 + (r- l - alp - c*tita)*H12
k2 <- c* H21 + (r- l - alp - c*tita)*H11
K1 <- k1 *Re(kummerU(x = ctr, a= (1-b2), b = 1- b1))
K2 <- k2*Re(kummerM(x = ctr, a= (1+b3), b = 1+ b1))
K <- K1-K2
C1num = k1*ex - alp*tita*Re(kummerM(x = ctr, a= (1+b3), b = 1+ b1))
C2num = alp*tita*Re(kummerU(x = ctr, a= (1-b2), b = 1- b1)) - k2*ex
C1 <- C1num/K
C2<- C2num/K
q1<- exp(-(u*tita + ctr))*Re(kummerU(x=(u*tita + ctr), a=1-b2, b=1-b1))
q2<- exp(-(u*tita + ctr))*Re(kummerM(x=(u*tita + ctr), a=1+b3, b=1+b1))
result<- abs(C1*q1 + C2*q2)
```

```
return(result)
}
zxcvgbhnjomkfevcdwghxbkfdcg vblk;wcemwnhdgbyxcufwnjk’xdrjcnlkfhd
#exponential Oscillation
RuinrepEs<-function(u,h){
m <- u/h+1
co = 7
v<-r <- matrix(nrow = m, ncol = 7)
xh <-0 #0.15
v[,1:2]<-RuinEs(u,h,xh)
for (k in 3:co){
xh = xh + 0.05
#xh=xh+hi
v[,k]<-RuinEs(u,h,xh)[,2]
}
return(v)
}
RuinEs<-function(u,h,j){
m <- u/h+1
r <- matrix(nrow = m, ncol = 2)
r[1,1] <- 0;
r[1,2]<- RuinEoscila(r[1,1],j)
for ( k in 2:m){
r[k,1]<- r[k-1,1]+h;
r[k,2]<- RuinEoscila(r[k,1],j);
}
return(r)
}
Ruinrealos<-function(reald){
m = length(reald$u1)
r <- matrix(nrow = m, ncol = 2)
```

```
r[ ,1] <- reald$u1;
#r[1,2]<- RuinEclaim(u1[1],reald[1,])
for ( k in 1:m){
r[k,2]<- RuinEoscila(reald[k],k);
}
return(r)
}
RuinEoscila<-function(i,j){
u = i #u1[j];
tita = 1 #tita1[j]#1 ;
c = 1.1 #c1[j]#1.1 ;
r = 0.12 #r1[j]#0.12
a = 0.21 #a1[j]#j#0.21
phi = j #phi1[j]#0.6
l <- 1;
alp = 0;
br <- r*(1-phi)+a*phi;
ctr = c*tita/br;
b1= (l+alp)/br;
b2<- l/br;
b3 <- alp/br;
ex = exp(-ctr);
sigp = 0.2 #i$sigma1[j]#j#0.2;
sigR = 0.2#i$sigma1[j]#0.2;
ba = (c/br) - (tita*sigp^2/(2*br));
intgrd1=function(x){(x+ba)*exp(-(tita*x+br*(x+ba)^2/sigp^2))
*Re(kummerM(x=(br*(x+ba)^2/sigp^2),0.5 ,1.5 ))};
intgrd2=function(x){exp(-(tita*x+br*(x+ba)^2/sigp^2))
*Re(kummerU(x=(br*(x+ba)^2/sigp^2),0 ,0.5 ))};
H10 = ba*exp(-r*ba^2/sigp^2)
*Re(kummerM(x=(r*ba^2/sigp^2), 0.5, 1.5));
```

```
H20 = exp(-r*ba^2/sigp^2)

*Re(kummerU(x=(r*ba^2/sigp^2), 0, 0.5));

H1p01 = (2*br*ba^2/sigp^2)*exp(-r*ba^2/sigp^2)

*Re(kummerM(x=(r*ba^2/sigp^2), 0.5, 1.5));

H2p01 = (2*br*ba^2/(3*sigp^2))*exp(-r*ba^2/sigp^2)

*Re(kummerM(x=(r*ba^2/sigp^2), 1.5, 2.5));

H1p0 = H10 - H1p01 + H2p01;

H2p0 = -(tita+2*br*ba)/sigp^2 * H20;

intgr10 = integrate(intgrd1,0,1.2*u )$val;

intgr20 = integrate(intgrd2,0,1.2*u )$val;

intgr1 = integrate(intgrd1,u,1.2*u )$val;

intgr2 = integrate(intgrd2,u,1.2*u )$val;

K = intgr20*(c*H1p0+(br-alp)*H10)-intgr10*(c*H2p0+(br-alp)*H20);

#K = intgrd2(0)*(c*H1p0+(br-alp)*H10)-intgrd1(0)*(c*H2p0+(br-alp)*H20);

C1num = c*H2p0 + (br-alp)*H20;

C2num = c*H1p0 + (br-alp)*H10;

C1 = C1num/K;

C2 = C2num/K;

#resltnum = intgr2*(c*H1p0 + (br-alp)*H10) - intgr1*(c*H2p0 + (br-alp)*H20);

#resltden = intgr20*(c*H1p0 + (br-alp)*H10) - intgr10*(c*H2p0 + (br-alp)*H20);

resltnum = intgrd2(u)*(c*H1p0 + (br-alp)*H10) - intgrd1(u)*(c*H2p0 + (br-alp)*H20);

resltden = intgrd2(0)*(c*H1p0 + (br-alp)*H10) - intgrd1(0)*(c*H2p0 + (br-alp)*H20);

result = resltnum/resltden;

return(result);

#return(K);

}
```

E Some Tables

E.1 Some simulated data for the reserve model with exponential claim size

Table E.1: The behavior of the reserve with respect to arrival epoch for $r \leq 0.10$, $\varrho=0.60$, $a=0.21$, $\sigma=0.20$, $\delta=0.00$, $\phi=0.00$.

Epoch	r=0.00	r=0.02	r=0.04	r=0.06	r=0.8	r=0.10
0.0000	25.0000	25.0000	25.0000	25.0000	25.0000	25.0000
0.1179	24.9714	25.0303	25.0895	25.1487	25.2081	25.2676
0.1879	24.8853	24.9790	25.0731	25.1676	25.2624	25.3576
1.6176	24.8306	25.6505	26.4973	27.3720	28.2755	29.2088
1.9904	24.7788	25.7898	26.8420	27.9369	29.0764	30.2623
3.6904	24.6859	26.5895	28.6393	30.8465	33.2231	35.7822
3.8586	24.5367	26.5191	28.6611	30.9754	33.4760	36.1778
4.4228	24.5102	26.7966	29.2955	32.0265	35.0112	38.2731
4.6957	24.4207	26.8479	29.5153	32.4468	35.6685	39.2093
6.1882	24.3877	27.6429	31.3297	35.5054	40.2348	45.5915
6.1957	24.3871	27.6465	31.3390	35.5219	40.2607	45.6291
7.0024	24.2732	27.9759	32.2396	37.1490	42.8022	49.3118
7.2865	24.2583	28.1228	32.5985	37.7822	43.7860	50.7399
7.7982	24.2160	28.3710	33.2336	38.9243	45.5843	53.3788
8.2321	24.2084	28.6170	33.8220	39.9675	47.2236	55.7913
8.5226	24.2019	28.7812	34.2200	40.6799	48.3526	57.4664
8.6332	24.1633	28.8012	34.3224	40.8953	48.7206	58.0377
9.1583	24.1299	29.0750	35.0251	42.1848	50.8001	61.1681
9.8582	24.0973	29.4590	36.0027	43.9891	53.7369	65.6354
9.8876	24.0625	29.4348	35.9960	44.0094	53.7973	65.7540
10.2388	24.0494	29.6338	36.5033	44.9538	55.3502	68.1424

Table E.2: The behavior of the reserve with respect to arrival epoch for ϱ=0.60, a=0.21, σ=0.20, δ=0.18, ϕ=0.00 and r≥0.10.

Epoch	*r = 0.10*	*r = 0.12*	*r = 0.14*	*r = 0.16*	*r = 0.18*	*r =0.20*
0.0000	25.0000	25.0000	25.0000	25.0000	25.0000	25.0000
0.1179	24.7369	24.7953	24.8538	24.9125	24.9714	25.0303
0.1879	24.5138	24.6061	24.6988	24.7919	24.8853	24.9790
1.6176	21.8044	22.5246	23.2685	24.0369	24.8306	25.6505
1.9904	21.1148	21.9768	22.8738	23.8073	24.7788	25.7898
3.6904	18.3353	19.7513	21.2761	22.9180	24.6859	26.5895
3.8586	17.9774	19.4320	21.0037	22.7019	24.5367	26.5191
4.4228	17.1492	18.7520	20.5036	22.4180	24.5102	26.7966
4.6957	16.7094	18.3735	20.2023	22.2120	24.4207	26.8479
6.1882	14.7582	16.7361	18.9763	21.5137	24.3877	27.6429

6.1957	14.7495	16.7283	18.9702	21.5099	24.3871	27.6465
7.0024	13.7340	15.8399	18.2650	21.0575	24.2732	27.9759
7.2865	13.4061	15.5535	18.0408	20.9217	24.2583	28.1228
7.7982	12.8251	15.0398	17.6319	20.6656	24.2160	28.3710
8.2321	12.3657	14.6339	17.3121	20.4746	24.2084	28.6170
8.5226	12.0666	14.3673	17.1002	20.3463	24.2019	28.7812
8.6332	11.9372	14.2460	16.9949	20.2674	24.1633	28.8012
9.1583	11.4076	13.7661	16.6045	20.0200	24.1299	29.0750
9.8582	10.7407	13.1562	16.1049	19.7042	24.0973	29.4590
9.8876	10.6997	13.1135	16.0622	19.6637	24.0625	29.4348
10.2388	10.3818	12.8197	15.8194	19.5097	24.0494	29.6338

Table E.3: The behavior of the reserve with respect to arrival epoch for ϱ=0.60, a=0.21, σ=0.20, δ=0.00, ϕ=0.20 and $r \leq 0.10$.

Epoch	*r = 0.00*	*r = 0.02*	*r = 0.04*	*r = 0.06*	*r = 0.08*	*r = 0.10*
0.0000	25.0000	25.0000	25.0000	25.0000	25.0000	25.0000
0.1179	25.0977	25.1452	25.1926	25.2402	25.2879	25.3357
0.1879	25.6681	25.7454	25.8230	25.9008	25.9789	26.0571
1.6176	27.3454	28.0652	28.8039	29.5620	30.3400	31.1384
1.9904	26.2530	27.1064	27.9875	28.8972	29.8364	30.8060
3.6904	27.9999	29.7137	31.5320	33.4611	35.5078	37.6793
3.8586	27.0045	28.7360	30.5781	32.5378	34.6227	36.8408
4.4228	27.0982	29.1018	31.2530	33.5626	36.0424	38.7047
4.6957	28.3854	30.6206	33.0311	35.6308	38.4346	41.4584

6.1882	31.3552	34.6572	38.3053	42.3355	46.7880	51.7071
6.1957	31.1103	34.3900	38.0138	42.0180	46.4423	51.3310
7.0024	32.2787	36.1564	40.4976	45.3576	50.7984	56.8897
7.2865	32.2063	36.2415	40.7797	45.8837	51.6240	58.0801
7.7982	34.3436	38.9748	44.2272	50.1845	56.9412	64.6052
8.2321	34.0105	38.8670	44.4133	50.7475	57.9816	66.2438
8.5226	34.6562	39.7972	45.6968	52.4671	60.2370	69.1545
8.6332	34.5733	39.7738	45.7525	52.6263	60.5296	69.6172
9.1583	34.5093	40.0392	46.4504	53.8837	62.5024	72.4963
9.8582	35.1808	41.2962	48.4682	56.8798	66.7458	78.3187
9.8876	33.4693	39.3057	46.1537	54.1892	63.6187	74.6852
10.2388	33.7659	39.8874	47.1119	55.6389	65.7040	77.5859

Table E.4: The behavior of the reserve with respect to arrival epoch for ϱ=0.60, a=0.21, σ=0.20, δ=0.18, ϕ=0.20 and $r \geq 0.10$.

Epoch	*r = 0.10*	*r = 0.12*	*r = 0.14*	*r = 0.16*	*r = 0.18*	*r = 0.20*
0.0000	25.0000	25.0000	25.0000	25.0000	25.0000	25.0000
0.1179	24.8035	24.8503	24.8973	24.9443	24.9914	25.0386
0.1879	25.1900	25.2659	25.3420	25.4183	25.4949	25.5717
1.6176	23.2457	23.8579	24.4861	25.1308	25.7925	26.4715
1.9904	21.4940	22.1931	22.9149	23.6601	24.4295	25.2238
3.6904	19.3087	20.4926	21.7487	23.0814	24.4953	25.9955
3.8586	18.3059	19.4817	20.7327	22.0635	23.4794	24.9857
4.4228	17.3416	18.6267	20.0065	21.4878	23.0783	24.7858
4.6957	17.6649	19.0591	20.5627	22.1842	23.9328	25.8187

6.1882	16.7464	18.5177	20.4748	22.6369	25.0255	27.6644
6.1957	16.6069	18.3648	20.3072	22.4534	24.8248	27.4451
7.0024	15.8537	17.7688	19.9128	22.3130	25.0001	28.0082
7.2865	15.3681	17.3046	19.4826	21.9321	24.6869	27.7851
7.7982	15.5382	17.6471	20.0389	22.7516	25.8280	29.3171
8.2321	14.7255	16.8430	19.2615	22.0235	25.1778	28.7800
8.5226	14.5535	16.7288	19.2251	22.0897	25.3769	29.1491
8.6332	14.3563	16.5318	19.0329	21.9083	25.2139	29.0140
9.1583	13.5800	15.7743	18.3185	21.2684	24.6883	28.6532
9.8582	12.8570	15.1154	17.7643	20.8710	24.5144	28.7873
9.8876	12.1962	14.3451	16.8667	19.8256	23.2974	27.3709
10.2388	11.8540	14.0275	16.5928	19.6206	23.1938	27.4109

Table E.5: The behavior of the reserve with respect to arrival epoch for ϱ=0.60, a=0.21, σ=0.20, δ=0.00, ϕ=1.00 and $r \leq 0.10$.

Epoch	*r = 0.00*	*r = 0.02*	*r = 0.04*	*r = 0.06*	*r = 0.08*	*r = 0.10*
0.0000	25.0000	25.0000	25.0000	25.0000	25.0000	25.0000
0.1179	25.6581	25.6581	25.6581	25.6581	25.6581	25.6581
0.1879	29.1414	29.1414	29.1414	29.1414	29.1414	29.1414
1.6176	41.2751	41.2751	41.2751	41.2751	41.2751	41.2751
1.9904	34.1527	34.1527	34.1527	34.1527	34.1527	34.1527
3.6904	49.1595	49.1595	49.1595	49.1595	49.1595	49.1595
3.8586	42.1514	42.1514	42.1514	42.1514	42.1514	42.1514
4.4228	43.4634	43.4634	43.4634	43.4634	43.4634	43.4634
4.6957	55.9205	55.9205	55.9205	55.9205	55.9205	55.9205
6.1882	94.5281	94.5281	94.5281	94.5281	94.5281	94.5281
6.1957	90.7790	90.7790	90.7790	90.7790	90.7790	90.7790

7.0024	112.9107	112.9107	112.9107	112.9107	112.9107	112.9107
7.2865	112.0351	112.0351	112.0351	112.0351	112.0351	112.0351
7.7982	157.4562	157.4562	157.4562	157.4562	157.4562	157.4562
8.2321	149.9559	149.9559	149.9559	149.9559	149.9559	149.9559
8.5226	166.3557	166.3557	166.3557	166.3557	166.3557	166.3557
8.6332	165.4960	165.4960	165.4960	165.4960	165.4960	165.4960
9.1583	165.0104	165.0104	165.0104	165.0104	165.0104	165.0104
9.8582	186.0887	186.0887	186.0887	186.0887	186.0887	186.0887
9.8876	145.3257	145.3257	145.3257	145.3257	145.3257	145.3257
10.2388	153.6580	153.6580	153.6580	153.6580	153.6580	153.6580

Table E.6: The behavior of the reserve with respect to arrival epoch for ϱ=0.60, a=0.21, σ=0.20, δ=0.18, ϕ=1.00 and $r \leq 0.10$.

Epoch	*r = 0.00*	*r = 0.02*	*r = 0.04*	*r = 0.06*	*r = 0.08*	*r = 0.10*
0.0000	25.0000	25.0000	25.0000	25.0000	25.0000	25.0000
0.1179	25.1191	25.1191	25.1191	25.1191	25.1191	25.1191
0.1879	28.1713	28.1713	28.1713	28.1713	28.1713	28.1713
1.6176	30.8173	30.8173	30.8173	30.8173	30.8173	30.8173
1.9904	23.8291	23.8291	23.8291	23.8291	23.8291	23.8291
3.6904	25.2008	25.2008	25.2008	25.2008	25.2008	25.2008
3.8586	20.9432	20.9432	20.9432	20.9432	20.9432	20.9432
4.4228	19.4737	19.4737	19.4737	19.4737	19.4737	19.4737
4.6957	23.8127	23.8127	23.8127	23.8127	23.8127	23.8127
6.1882	30.6678	30.6678	30.6678	30.6678	30.6678	30.6678
6.1957	29.4565	29.4565	29.4565	29.4565	29.4565	29.4565

7.0024	31.5151	31.5151	31.5151	31.5151	31.5151	31.5151
7.2865	29.7853	29.7853	29.7853	29.7853	29.7853	29.7853
7.7982	37.9286	37.9286	37.9286	37.9286	37.9286	37.9286
8.2321	33.6237	33.6237	33.6237	33.6237	33.6237	33.6237
8.5226	35.2031	35.2031	35.2031	35.2031	35.2031	35.2031
8.6332	34.3582	34.3582	34.3582	34.3582	34.3582	34.3582
9.1583	31.3429	31.3429	31.3429	31.3429	31.3429	31.3429
9.8582	30.8104	30.8104	30.8104	30.8104	30.8104	30.8104
9.8876	24.0373	24.0373	24.0373	24.0373	24.0373	24.0373
10.2388	23.6991	23.6991	23.6991	23.6991	23.6991	23.6991

Table E.7: The behavior of the reserve with respect to arrival epoch for $\varrho \leq 0.5$, r=0.12, a=0.21, σ=0.2, δ=0.00, ϕ=0.0.

Epoch	ϱ=0	ϱ=0.1	ϱ=0.2	ϱ=0.3	ϱ=0.4	ϱ=0.5
0.0000	25.0000	25.0000	25.0000	25.0000	25.0000	25.0000
0.1179	25.3273	25.6278	25.9319	26.2395	26.5509	26.8659
0.1879	25.4532	25.9363	26.4286	26.9303	27.4415	27.9625
1.6176	30.1726	35.4895	41.7408	49.0908	57.7327	67.8934
1.9904	31.4961	38.4611	46.9625	57.3394	70.0061	85.4684
3.6904	38.5365	55.8269	80.8509	117.0703	169.5013	245.4191
3.8586	39.0949	57.6098	84.8732	125.0316	184.2169	271.5220
4.4228	41.8345	65.2752	101.8236	158.8403	247.8707	387.1016
4.6957	43.0952	69.1347	110.8883	177.9001	285.6212	459.2381
6.1882	51.6483	96.3643	179.6921	335.1425	625.7569	1171.1080
6.1957	51.6985	96.5508	180.2484	336.6826	629.9444	1182.6352

7.0024	56.7893	115.0761	233.1045	472.7170	961.6806	1969.3653
7.2865	58.7707	122.5980	255.7247	534.4160	1122.2721	2380.7102
7.7982	62.4707	137.2749	301.7064	664.9810	1476.1226	3326.0257
8.2321	65.8694	151.2776	347.5984	801.7758	1866.9731	4434.8665
8.5226	68.2461	161.4569	382.3135	909.7669	2190.9157	5410.3678
8.6332	69.0782	165.3338	396.3039	956.0578	2342.0217	5925.7926
9.1583	73.5802	185.7729	469.9786	1198.7406	3118.1533	8457.6797
9.8582	80.0771	217.0596	589.9120	1620.0154	4563.2431	13578.8348
9.8876	80.2686	218.3694	596.2916	1650.1372	4718.2471	14485.5541
10.2388	83.7754	236.2758	669.5570	1928.7626	5785.2286	18941.6541

Table E.8: The behavior of the reserve with respect to arrival epoch for $\varrho \geq 0.5$, r=0.12, a=0.21, σ=0.2, δ=0.00, ϕ=0.0.

Epoch	ϱ=0.5	ϱ=0.6	ϱ=0.7	ϱ=0.8	ϱ=0.9	ϱ=1.0
0.0000	25.0000	25.0000	25.0000	25.0000	25.0000	25.0000
0.1179	26.8659	27.1846	27.5072	27.8336	28.1638	28.4980
0.1879	27.9625	28.4933	29.0343	29.5855	30.1473	30.7197
1.6176	67.8934	79.8404	93.8877	110.4052	129.8279	152.6677
1.9904	85.4684	104.3445	127.3899	155.5282	189.8893	231.8562
3.6904	245.4191	355.3891	514.7844	746.0384	1082.0276	1571.2374
3.8586	271.5220	400.4892	591.4186	875.0334	1298.4841	1935.5615
4.4228	387.1016	605.3666	948.8241	1492.4303	2360.4054	3764.3989
4.6957	459.2381	740.2491	1198.1430	1952.0459	3212.9409	5370.6009
6.1882	1171.1080	2201.0465	4166.8916	7984.0386	15597.8930	31406.1321

6.1957	1182.6352	2233.7267	4263.0253	8277.4869	16523.2668	34402.0797
7.0024	1969.3653	4084.3133	8670.6023	19179.4058	45367.2137	118184.3978
7.2865	2380.7102	5150.4824	11555.2769	27598.0420	72506.7423	215088.1527
7.7982	3326.0257	7717.8317	18904.0294	50627.2553	153501.8344	533807.4298
8.2321	4434.8665	10955.1858	29038.9039	85953.9486	292670.8923	1143411.7646
8.5226	5410.3678	14030.5286	39645.2275	127204.8027	473415.7962	2010717.8000
8.6332	5925.7926	15963.8001	47849.1866	166273.5177	675481.3624	3108010.8451
9.1583	8457.6797	24868.4124	83454.9822	331640.8800	1546194.9774	8076555.6395
9.8582	13578.8348	44878.9733	174720.1559	822583.5150	4538336.8759	27694980.3099
9.8876	14485.5541	50801.1949	216173.9304	1123348.4824	6757376.6274	44189010.4068
10.2388	18941.6541	72660.6775	344953.4621	1995900.0228	13155505.6226	92882783.0015

Table E.9: The behavior of the reserve with respect to arrival epoch for $\varrho \leq 0.5$, r=0.12, a=0.21, σ=0.2, δ=0.00, ϕ=1.

Epoch	ϱ=0.0	ϱ=0.1	ϱ=0.2	ϱ=0.3	ϱ=0.4	ϱ=0.5
0.0000	25.0000	25.0000	25.0000	25.0000	25.0000	25.0000
0.1179	25.6580	25.6580	25.6580	25.6580	25.6580	25.6580
0.1879	29.1411	29.1411	29.1411	29.1411	29.1411	29.1411
1.6176	41.2736	41.2736	41.2736	41.2736	41.2736	41.2736
1.9904	34.1527	34.1527	34.1527	34.1527	34.1527	34.1527
3.6904	49.1554	49.1554	49.1554	49.1554	49.1554	49.1554
3.8586	42.1491	42.1491	42.1491	42.1491	42.1491	42.1491
4.4228	43.4638	43.4638	43.4638	43.4638	43.4638	43.4638
4.6957	55.8776	55.8776	55.8776	55.8776	55.8776	55.8776
6.1882	94.3826	94.3826	94.3826	94.3826	94.3826	94.3826

6.1957	90.7191	90.7191	90.7191	90.7191	90.7191	90.7191
7.0024	112.6161	12.6166	112.6166	112.6166	112.6166	112.6166
7.2865	111.9371	11.9378	111.9378	111.9378	111.9378	111.9378
7.7982	156.7411	56.7416	156.7416	156.7416	156.7416	156.7416
8.2321	149.9051	49.9053	149.9053	149.9053	149.9053	149.9053
8.5226	165.8951	65.8955	165.8955	165.8955	165.8955	165.8955
8.6332	165.1501	65.1504	165.1504	165.1504	165.1504	165.1504
9.1583	165.2541	65.2541	165.2541	165.2541	165.2541	165.2541
9.8582	185.6311	85.6318	185.6318	185.6318	185.6318	185.6318
9.8876	145.4651	45.4650	145.4650	145.4650	145.4650	145.4650
10.2388	153.5071	53.5071	153.5071	153.5071	153.5071	153.5071

Table E.10: The behavior of the reserve with respect to arrival epoch for $\varrho \leq 0.5$, r=0.12, a=0.21, σ=0.2, δ=0.18, ϕ=1.

Epoch	ϱ=0.0	ϱ=0.1	ϱ=0.2	ϱ=0.3	ϱ=0.4	ϱ=0.5
0	25	25.00000	25.00000	25.00000	25.00000	25.00000
0.11794	25.11906	25.11906	25.11906	25.11906	25.11906	25.11906
0.18794	28.17118	28.17118	28.17118	28.17118	28.17118	28.17118
1.61759	30.81709	30.81709	30.81709	30.81709	30.81709	30.81709
1.99044	23.82903	23.82903	23.82903	23.82903	23.82903	23.82903
3.69038	25.20081	25.20081	25.20081	25.20081	25.20081	25.20081
3.85862	20.94283	20.94283	20.94283	20.94283	20.94283	20.94283

4.42277	19.47235	19.47235	19.47235	19.47235	19.47235	19.47235
4.69567	23.81305	23.81305	23.81305	23.81305	23.81305	23.81305
6.18816	30.66203	30.66203	30.66203	30.66203	30.66203	30.66203
6.19570	29.45622	29.45622	29.45622	29.45622	29.45622	29.45622
7.00243	31.50534	31.50534	31.50534	31.50534	31.50534	31.50534
7.28650	29.78591	29.78591	29.78591	29.78591	29.78591	29.78591
7.79823	37.89989	37.89989	37.89989	37.89989	37.89989	37.89989
8.23207	33.63091	33.63091	33.63091	33.63091	33.63091	33.63091
8.52260	35.19676	35.19676	35.19676	35.19676	35.19676	35.19676
8.63321	34.35828	34.35828	34.35828	34.35828	34.35828	34.35828
9.15826	31.35893	31.35893	31.35893	31.35893	31.35893	31.35893
9.85818	30.81162	30.81162	30.81162	30.81162	30.81162	30.81162
9.88764	24.03744	24.03744	24.03744	24.03744	24.03744	24.03744
10.23884	23.69879	23.69879	23.69879	23.69879	23.69879	23.69879

Table E.11: The behavior of the reserve, asset and claim with respect to arrival epoch when all parameters are fixed.

Epoch	Asset	Claim	Reserve
0.00000	25.00000	0.00000	25.00000
0.11794	25.00944	0.02864	24.98079
0.18794	26.79393	0.12091	26.67303
1.61759	27.24317	0.18045	27.06272
1.99044	23.17031	0.22851	22.94180
3.69038	22.93793	0.31374	22.62419
3.85862	20.54216	0.43634	20.10582
4.42277	19.40185	0.45692	18.94493
4.69567	21.71601	0.53467	21.18133
6.18816	24.22135	0.56657	23.65478
6.19570	23.65764	0.56715	23.09049
7.00243	24.14261	0.67720	23.46541
7.28650	23.20351	0.69097	22.51254

7.79823	26.36999	0.73562	25.63437
8.23207	24.32664	0.74297	23.58368
8.52260	24.78273	0.74946	24.03327
8.63321	24.39039	0.78709	23.60329
9.15826	22.83964	0.81765	22.02198
9.85818	22.18754	0.84652	21.34102
9.88764	19.24161	0.87325	18.36836
10.23884	18.90527	0.88316	18.02211
13.04243	20.43710	0.92381	19.51329
13.08495	23.96997	0.94792	23.02205
13.42828	23.70828	1.07531	22.63297
13.91949	23.13886	1.11011	22.02875
14.33607	23.08342	1.16765	21.91576
15.05060	24.26940	1.17892	23.09048
17.34246	22.33205	1.19000	21.14205
17.35771	25.00384	1.32926	23.67459
17.61945	24.94670	1.48701	23.45968
18.72042	25.69694	1.54627	24.15067

Table E.12: The behavior of the reserve with respect to arrival epoch when capital *u* is varied.

Epoch	*u = 25*	*u =30*	*u = 35*	*u = 40*
0.00000	25.00000	30.00000	35.00000	40.00000
0.11794	24.98079	29.98268	34.98457	39.98646
0.18794	26.67303	32.03179	37.39055	42.74931
1.61759	27.06272	32.51133	37.95995	43.40857
1.99044	22.94180	27.57580	32.20980	36.84379
3.69038	22.62419	27.21176	31.79932	36.38689
3.85862	20.10582	24.21402	28.32221	32.43041
4.42277	18.94493	22.82462	26.70431	30.58400
4.69567	21.18133	25.52510	29.86886	34.21263
6.18816	23.65478	28.49927	33.34376	38.18825
6.19570	23.09049	27.82198	32.55347	37.28497
7.00243	23.46541	28.29429	33.12317	37.95205

7.28650	22.51254	27.15306	31.79358	36.43411
7.79823	25.63437	30.90748	36.18059	41.45370
8.23207	23.58368	28.44877	33.31387	38.17897
8.52260	24.03327	28.98984	33.94641	38.90298
8.63321	23.60329	28.48131	33.35932	38.23734
9.15826	22.02198	26.58807	31.15416	35.72024
9.85818	21.34102	25.77802	30.21501	34.65200
9.88765	18.36836	22.21173	26.05511	29.89848
10.23884	18.02211	21.79966	25.57722	29.35477
13.04243	19.51329	23.60403	27.69478	31.78552
13.08495	23.02205	27.81669	32.61134	37.40598
13.42828	22.63297	27.37412	32.11527	36.85642
13.91949	22.02875	26.65595	31.28315	35.91035
14.33607	21.91576	26.53242	31.14907	35.76572
15.05060	23.09048	27.94485	32.79922	37.65358
17.34246	21.14205	25.60587	30.06969	34.53351
17.35771	23.67459	28.67535	33.67612	38.67688
17.61945	23.45968	28.44901	33.43834	38.42766
18.72042	24.15067	29.28963	34.42859	39.56755

Table E.13: The behavior of the reserve with respect to arrival epoch when Interest rate *r* is varied.

	r≤0.12					r>0.12			
Epoch	*r=0.04*	*r = 0.06*	*r=0.08*	*r=0.10*	*r=0.12*	*r=0.14*	*r=0.16*	*r = 0.18*	*r=0.20*
0.0000	25.0000	25.0000	25.0000	25.0000	25.0000	25.0000	25.0000	25.0000	25.0000
0.1179	24.8867	24.9102	24.9337	24.9572	24.9808	25.0044	25.0280	25.0516	25.0753
0.1879	26.5130	26.5529	26.5929	26.6329	26.6730	26.7132	26.7534	26.7937	26.8340
1.6176	25.6920	26.0280	26.3685	26.7133	27.0627	27.4166	27.7752	28.1384	28.5064
1.9904	21.5191	21.8663	22.2191	22.5776	22.9418	23.3119	23.6880	24.0701	24.4583
3.6904	20.0858	20.6925	21.3174	21.9611	22.6242	23.3072	24.0106	24.7353	25.4816
3.8586	17.7514	18.3130	18.8922	19.4896	20.1058	20.7414	21.3970	22.0732	22.7707
4.4228	16.4204	17.0183	17.6378	18.2798	18.9449	19.6342	20.3483	21.0884	21.8551
4.6957	18.1939	18.8990	19.6312	20.3916	21.1813	22.0015	22.8532	23.7377	24.6563
6.1882	19.3483	20.3458	21.3942	22.4963	23.6548	24.8724	26.1523	27.4977	28.9118
6.1957	18.8875	19.8610	20.8842	21.9598	23.0905	24.2790	25.5283	26.8415	28.2219

7.0024	18.6828	19.7791	20.9391	22.1666	23.4654	24.8397	26.2939	27.8327	29.4609
7.2865	17.7660	18.8502	20.0000	21.2194	22.5125	23.8839	25.3382	26.8805	28.5161
7.7982	19.8803	21.1858	22.5763	24.0572	25.6344	27.3142	29.1033	31.0087	33.0382
8.2321	18.0481	19.2974	20.6325	22.0591	23.5837	25.2128	26.9537	28.8141	30.8020
8.5226	18.2058	19.5159	20.9193	22.4227	24.0333	25.7586	27.6068	29.5868	31.7079
8.6332	17.8191	19.1176	20.5099	22.0027	23.6033	25.3194	27.1595	29.1324	31.2477
9.1583	16.3533	17.6176	18.9788	20.4442	22.0220	23.7206	25.5493	27.5181	29.6377
9.8582	15.4620	16.7609	18.1677	19.6911	21.3410	23.1278	25.0628	27.1583	29.4277
9.8876	13.3012	14.4205	15.6328	16.9460	18.3684	19.9090	21.5777	23.3851	25.3427
10.2388	12.8884	14.0169	15.2430	16.5750	18.0221	19.5942	21.3020	23.1574	25.1729
13.0424	12.6595	14.1108	15.7246	17.5187	19.5133	21.7308	24.1959	26.9362	29.9826
13.0850	14.9511	16.6597	18.5598	20.6726	23.0221	25.6345	28.5393	31.7693	35.3607
13.4283	14.5652	16.2661	18.1624	20.2764	22.6330	25.2600	28.1883	31.4525	35.0910
13.9195	13.9358	15.6307	17.5279	19.6516	22.0288	24.6895	27.6675	31.0006	34.7312
14.3361	13.6579	15.3772	17.3088	19.4785	21.9158	24.6534	27.7285	31.1824	35.0619
15.0506	14.0258	15.8944	18.0064	20.3931	23.0905	26.1387	29.5833	33.4760	37.8750
17.3425	11.9216	13.7655	15.8880	18.3308	21.1421	24.3771	28.0995	32.3826	37.3105
17.3577	13.2886	15.3633	17.7530	20.5051	23.6746	27.3244	31.5273	36.3669	41.9397
17.6195	13.0940	15.1584	17.5404	20.2888	23.4597	27.1177	31.3375	36.2053	41.8206
18.7204	12.9430	15.1400	17.6991	20.6796	24.1507	28.1928	32.8998	38.3810	44.7637