Optimal Resource Allocation in Supply Network with Competitive Market Concept

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Abstract

We propose a competitive market-based model which uses price mechanism in an economy to determine an optimal allocation of resources among a set of trading agents. A supply network is represented by a set of production agents, which make use of the resources available to them in the market to produce an output defined by their technologies and demand agents seeking to maximize returns based on manufacturing budget. We take into consideration, changes in quantity of primary production factors – capital and labour – at facility locations. The resource allocation table obtained from simulation is a measure of value of market resources and facility capacities in the supply network.

1 INTRODUCTION

The supply networks of most manufacturing systems have taken up a decentralized nature with the autonomy of individual organizations in the network. In the recent past, performance monitoring and optimization problems in supply networks have been approached from various perspectives ranging from game theoretic approaches to cost accounting methods and normative models based on mathematical programming. The major draw back of these approaches is their problem specific nature making their application in dynamic environments less than pragmatic. Most recently, a new paradigm based on multi-agent system (MAS) architecture has been found to be very useful in simulating the behaviour of a supply network in a dynamic environment. This methodology is anchored on a number of social system theories to obtain optimal solutions to complex problems. In our case, we make use of theories from microeconomics. We seek to obtain a Pareto-optimal allocation of resources among a set of agents in the market subject to some assumptions delineated in general equilibrium theory of a perfect competition market [3,4].

The ultimate goal of this research work is to provide a kernel for a strategic decision support system looking through the lenses of production resources and technology requirements of production facilities. As compared with our work in [2], this model focuses on resource allocation in an intra-organization supply network operating in a make-to-stock mode. This means that management is the only source of investment in the network. More explanation on the Walrasian market model can also be found in [1].

2 COMPETITIVE MARKET MECHANISM

In this section, we briefly describe the competitive market mechanism on which our model is based. We describe the competitive market as a Walrasian market

$g_i \subseteq G \; \forall\, i = 1, 2, 3 \ldots n$ *where G is a set of goods in the market and* $|G| = n$

g_i *is associated with* $p_i \subseteq P \; \forall\, i = 1, 2, 3 \ldots n$ *given that P is a vector of prices*

$x_j \subset X \; \forall\, j = 1,2,3 \ldots m$: *X is a set of trading agents in the market*

$e_k^j \subset E \; \forall\, k = 1,2,3 \ldots n$ *and* e_k^j = *endowment of trader j on market resource k and* $|E| = m x n$.

The market operates under the assumptions of conservation of value and conservation of products which simply means that there is no excess in either income or expenditure in the market neither is there excess in demand or supply of any market resource. Other assumptions which guarantee the existence of a unique equilibrium include, the weak axioms of profit maximization and gross substitution of products. Our goal is to find a price vector p* Є Ꝑ for which

$$\sum_{i=1}^{m} d_i^g = \sum_{i=1}^{m} s_i^g \; for \;\; all \;\; g \subset G \tag{2.1}$$

2.1 Equililibrium Computation

A number of methods have been proposed for computation of general equilibrium for multicommodity markets including the Walrasian price tatonnement algorithm and the Scarf algorithm based on fixed point theorem. However, we make use of an method popularly referrred to as market-oriented programming (MOP) [4] because of its superior performance [2]. Computation based on the MOP algorithm is done by making use of a market mechanism in which updated prices of resouces are continuously posted on a blackboard and trading agents submit their bids for market resources based on the posted prices. A market clearing point is reached at which point no new prices are posted on the board because trading agents are not willing to change their bids. It is at this point in the search space that the equilibrium price vector is arrived at and a Pareto optimal solution to the market problem is found.

The trading agents in the market are grouped as consumers and producers and are defined using the constraint optimization problem which they solve. In our model, both the consumption and production functions are represented as the Cobb-Douglas function with constant returns to scale. Equations (2.2) – (2.8) formally describes the formulations for the trading agents.

2.1.1 Consumer Agents

$$\max U^c = R_c \prod_{i=1}^{k} g_i^{\alpha_i^c} \text{ for } \sum_{i=1}^{k} \alpha_i^c = 1 \text{ for all } 1 \le c \le n \tag{2.2}$$

Subject to

$$\sum_{i=1}^{k} p_i e_i^c \le B^c \tag{2.3}$$

and $p_i > 0 \;\; for \;\; all \;\; p \subset P$ (2.4)

Where R_c = consumption scale of consumer c

α_i^c = preference index of consumer c of good i

e_i^c = endowment of consumer c of good i

g_i = bid of good i by consumer c

An optimal bid formulation is derived from a langrangean multiplier approach

$$g_i(p_i) = \frac{\alpha_i^c * B^c}{p_i} \tag{2.5}$$

2.1.1 Producer Agents

$$\max \pi^s(P) = p_s g_s - \sum_{\substack{i=1 \\ i \ne s}}^{k} p_i g_i - \sum_{j=1}^{2} p_j f_j \tag{2.6}$$

Subject to

$$g_s(P) = R_s \prod_{\substack{i=1 \\ i \ne s}}^{k} g_i^{\beta_i^s} . \prod_{j=1}^{2} f_j^{\gamma_j} \quad for \quad \sum_{i=1}^{k} \beta_i^s + \gamma_1 + \gamma_2 = 1 \tag{2.7}$$

The bidding function for the producer agent is given as:

$$g_i(p_i) = \left(\frac{p_i}{R_s \beta_i^s p_s} \right)^{\frac{1}{\beta_i^s - 1}} \tag{2.8}$$

Where β_i^s = technology index of good i for agent s

γ_j = technology index of primary factor j (capital and labour)

f_j = primary production factor j

3 EXPERIMENTS

3.1 Supply Network Model

The hypothetical supply network is made up of facilities spatially distributed over three regions R1, R2 and R3 with the initial allocated budget for each region represented as B1, B2 and B3 respectively. The problem is to optimally allocate resources among these facilities given the technology constraint of each facility and the initial budgetary allocation to each facility from each regions budget.

This supply network is modelled as a competitive market such that each of the units in the regions will make up the producers in the market and the consumers in the market are instantiated with an initial budget corresponding to the budgetary allocation of each facility. We represent the parameters of the trading agents in the form of input matrices as shown in figures 3.2 and 3.3.

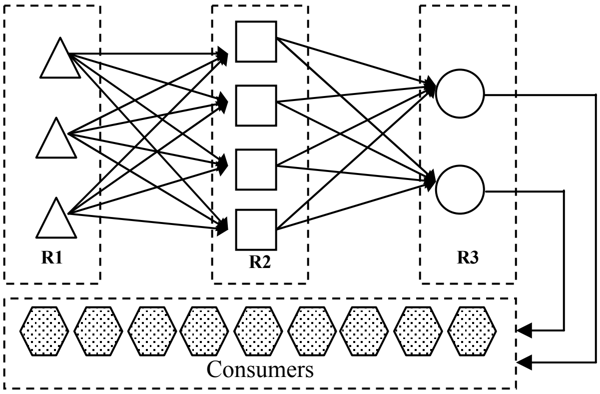


Figure 3.1. Supply Network Model with Regional Facilities

Figure 3.1 represents an intraorganization supply network with facilities distributed over three regions. Each of the regions has facilities which make up the producer agents in the market. For each of these producers, there is a corresponding consumer, which has initial capital and labour budgets; these budgets stand for the proposed investment of management on the facility. The consumers also have utility indices on the final products of the supply network representing the ratio of returns the firm is expecting from the supply network for investing in that facility. Therefore, R1 has 3 consumer agents, R2 has 4 and R3 has 2 consumer agents. It is these consumers that rent out there resources to the producers in return for final resources of the market. All the other resources in the market apart from the primary factors and the final goods are referred to as intermediate factors and are manufactured by producers agents in the layers preceding the final layer. Thus, all together, we have a total of 11 resources in the supply network market. It is on these resources that trading takes place within the market in order to determine the Pareto-optimal allocation of resources among the different facilities.

3.1.1 Agent Input Matrices

The matrices shown in figures 3.2. and 3.3. represent the input parameters of trading agents in the market; first those of the producers and then the consumers. The first matrix in each figure of 3.2(a-c) is the technology indices matrix for the producers while that of figure 3.3(a-c). is the utility indices matrix for the consumers. The second matrix of figure 3.2(a-c) is resource utilililization scale matrix of a producer while that of figure 3.3(a-c) is the resource endowment matrix of the consumer. Rows are agents (superscript) and columns are market commodities (subscript).

$$(a)\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_1^{11} & \gamma_2^{11} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_1^{12} & \gamma_2^{12} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_1^{13} & \gamma_2^{13} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$(b)\begin{bmatrix} \beta_1^{21} & \beta_2^{21} & \beta_3^{23} & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_1^{21} & \gamma_2^{21} \\ \beta_1^{22} & \beta_2^{22} & \beta_3^{23} & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_1^{22} & \gamma_2^{22} \\ \beta_1^{23} & \alpha_2^{23} & \beta_3^{3} & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_1^{23} & \gamma_2^{23} \\ \beta_1^{24} & \beta_2^{24} & \beta_3^{24} & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_1^{24} & \gamma_2^{24} \end{bmatrix} \begin{bmatrix} R_1^{21} & R_2^{21} & R_1^{23} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ R_1^{22} & R_2^{22} & R_2^{23} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ R_1^{23} & R_2^{23} & R_3^{23} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ R_1^{24} & R_2^{24} & R_4^{23} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$(c)\begin{bmatrix} 0 & 0 & 0 & \beta_4^{31} & \beta_5^{31} & \beta_6^{31} & \beta_7^{31} & 0 & 0 & \gamma_1^{31} & \gamma_2^{31} \\ 0 & 0 & 0 & \beta_4^{32} & \beta_5^{32} & \beta_6^{32} & \beta_7^{32} & 0 & 0 & \gamma_1^{32} & \gamma_2^{32} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & R_4^{31} & R_5^{31} & R_6^{31} & R_7^{31} & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & R_4^{32} & R_5^{32} & R_6^{32} & R_7^{32} & 0 & 0 & 1 & 1 \end{bmatrix}$$

Figure 3.2. Technology Matrices for Producers in Regions (a) R1 (b) R2 and (c) R3

$$(a)\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_8^{11} & \alpha_9^{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_8^{12} & \alpha_9^{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_8^{13} & \alpha_9^{13} & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_8^{11} & e_9^{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_8^{12} & e_9^{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_8^{13} & e_9^{13} & 0 & 0 \end{bmatrix}$$

$$(b)\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_8^{21} & \alpha_9^{21} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_8^{22} & \alpha_9^{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_8^{23} & \alpha_9^{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_8^{24} & \alpha_9^{24} & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_8^{21} & e_9^{21} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_8^{22} & e_9^{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_8^{23} & e_9^{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_8^{24} & e_9^{24} & 0 & 0 \end{bmatrix}$$

$$(c)\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_8^{31} & \alpha_9^{31} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_8^{32} & \alpha_9^{32} & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_8^{31} & e_9^{31} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_8^{32} & e_9^{32} & 0 \end{bmatrix}$$

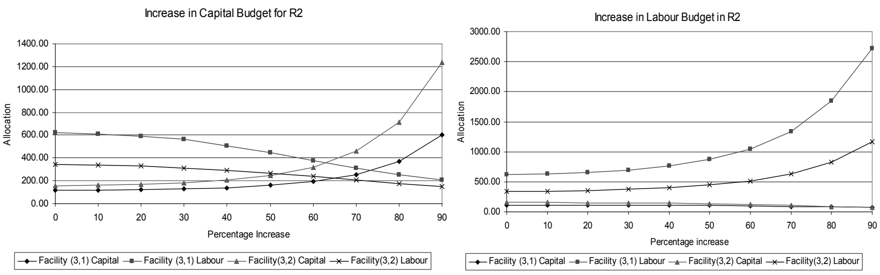
Figure 3.3. Utility and Endowment Matrices for Consumers in (a) R1 (b) R2 and (c) R3

3.1.2 Simulation

We ran simulations using hypothetical data for all the input parameters and the results we got show that we arrived at an approximate market clearing point. The results showed that the marginal rate of substation between any two market resources is constant for all the trading agents and the market clearing condition as given in equation (2.1) was satisfied. Sample results of simulations based on the MOP algorithm are discussed in [2].

4 DISCUSSION

An important application of the proposed model is in monitoring the effects budgetary variations have on the supply network in terms of changes in resource allocation and other indices like the final output of the supply network. We illustrate this effect by creating a linear variation in the budgets of agents in the different regions to see how resource allocation changes with the variations. Two graphs are presented in Figure 3.4 to show an example of what happens to the supply network when there is percentage increase in the budgets of capital or labour in a particular region of the network.



(a) (b)

Figure 3.4. (a)Variation of Capital Budget in R2 (b)Variation of labour Budget in R2

Figure 3.4(a) not only reveals that an increase in capital budget of region R2 of the network will cause a surge in the capital allocation of facilities in R3 and a reduction in their labour allocation but also evaluates these changes. Likewise, figure 3.4(b) depicts estimated primary factor allocation in R3 due to variation in labour budget in region R3. The same graph can be plotted for all the other market resources and facilities.

5 CONCLUSION

As stated earlier, the MOP algorithm implemented has been compared with the Walrasian tatonnement algorithm and Scarf algorithm and it was found to be more computationally efficient. We also showed how the model can be extended to accommodate variations in budgets due to mobility of labour and capital in the economy and changes in the relative prices of market commodities. The shape of the curves in Figure 3.4 depends on the distribution of initial budgets and the technology indices of the facilities.

As an extension to this work, we are interested in seeing how results derived from this model can be of use at the operational level of a supply network.

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