

**UNSTEADY HEAT AND MASS TRANSFER IN CONVECTIVE ROTATORY
RIVLIN-ERICKSEN FLOW PAST A POROUS VERTICAL PLATE**

BY

AGUNBIADE, Samson Ademola

Matric No: 95/040983

**A THESIS SUBMITTED TO THE DEPARTMENT OF MATHEMATICS FACULTY OF PHYSICAL
SCIENCES, UNIVERSITY OF ILORIN, ILORIN, NIGERIA,
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF THE DEGREE OF
DOCTOR OF PHILOSOPHY(Ph.D.) IN MATHEMATICS.**

JULY, 2018.

Certification

This is to certify that the research reported in this thesis was carried out by AGUNBIADE Samson Ademola with Matriculation Number 95/040983 in the Department of Mathematics, Faculty of Physical Sciences, University of Ilorin, Ilorin, Nigeria, for the award of the degree of Doctor of Philosophy in Mathematics.

.....
Dr. M. S. Dada
Supervisor

.....
Date

.....
Dr. Olubunmi A. Fadipe-Joseph
Departmental Postgraduate
Coordinator

.....
Date

.....
Prof. R. B. Adeniyi
Head of Department

.....
Date

.....
Internal Examiner from
Related Department

.....
Date

.....
External Examiner

.....
Date

DEDICATION

This thesis is dedicated to the Omniscience God who has been my helper all through the research work and to my wife Shola Agunbiade and our children Oluwabunmi, Oluwaseyi and Temitope Agunbiade.

ACKNOWLEDGEMENTS

I wish to return all the glory to the God who is Omniscience, Omnipresent and Omnipotent. He has been my all in all and without him, I am nothing.

My special appreciation goes to my uncommon, loving and humble supervisor, who is also my mentor. His sacrifice in term of time and experience, has been the bases for the successful completion of this thesis. I pray that God will reward him with blessings beyond his imaginations.

Furthermore, I wish to appreciate the entire staff of Mathematics Department, University of Ilorin, Ilorin for their supports in various forms to ensure the successful completion of the research work. These include: Prof. R. B. Adeniyi (the Head of Department), Dr. A. O. Fadipe-Joseph (the Departmental Postgraduate Coordinatior), Prof. J. A. Gbadeyan, Prof. T. O. Opoola, Prof. M. O. Bamigbola, Prof. M. O. Ibrahim, Prof. O. A. Taiwo, Dr. S. O. Makanjuola, Dr. K. Rauf, Dr. A. S. Idowu, Dr. E. O. Titiloye, Dr. K. O. Babalola, Dr. (Mrs.) Y. O. Aderinto, Dr. B. M. Yisa, Dr. J. U. Abubakar, Dr. K. A. Bello, Dr. (Mrs.) G. N. Bakare, Dr. O. T. Olotu, Dr. (Mrs.) C. N. Ejieji, Dr.(Mrs.) H. O. Ibrahim, Mr. B. M. Ahmed, Mrs. I. F. Usamot, Mr. O. A. Uweheren, Mr. O. Odetunde and Mr. A. Ayinla.

My profound gratitude goes to Mrs. Abigeli Agunbiade (my mother), Mrs. Elizabeth Adeboye, Mrs. Alice Ogunwole, Mr. Bisi Agunbiade, Pastor Sunday Agunbiade, Mrs. Feyintola Akanmu and Mr. Femi Agunbiade for their prayers, advice and moral support in the course of the programme.

I heartily express my appreciation to all my friends and colleagues for the wonderful environment they created in the Department.

Finally, I appreciate my wife and children for their understanding during the period of study.

ABSTRACT

Modelling of non-Newtonian fluid flow gives rise to non-linear problems which involves much computation. A non-Newtonian flow of Rivlin-Ericksen type past a porous vertical plate has been the subject of investigation by researchers due to its applications in engineering, food, paper and petroleum industries. A vast number of these investigations ignored rotatory and viscous dissipation effects of the fluid for simplification of problems. However, fluid flow realistically includes rotatory and viscous dissipation. Therefore, the objectives of the study were to: (i) determine the contribution of viscous dissipation to the Rivlin-Ericksen fluid; (ii) evaluate rotatory effects on convective Rivlin-Ericksen flow in a porous vertical plate; (iii) examine the effects of chemical reaction and thermal-diffusion on concentration profiles; (iv) analyze the influence of thermal radiation and viscous dissipation on fluid temperature; and (v) determine the impact of diffusion-thermo on concentration, temperature and velocity profiles.

The non-dimensional momentum, energy and species governing equations of the study respectively, are:

$$\left. \begin{aligned} \frac{\partial \xi}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \xi}{\partial \eta} &= -\frac{\partial P}{\partial x} + \frac{\partial^2 \xi}{\partial \eta^2} + 2\Omega v - (M + \frac{1}{k_p}) \xi \\ &+ G_h \vartheta + G_m \zeta - V_R \left(\frac{\partial^3 \xi}{\partial t \partial \eta^2} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial^3 \xi}{\partial \eta^3} \right) \\ \frac{\partial v}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial v}{\partial \eta} &= -\frac{\partial P}{\partial y} + \frac{\partial^2 v}{\partial \eta^2} - 2\Omega \xi - (M + \frac{1}{k_p}) v \\ &- V_R \left(\frac{\partial^3 v}{\partial t \partial \eta^2} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial^3 v}{\partial \eta^3} \right) \end{aligned} \right\} \quad (1)$$

$$\begin{aligned} \frac{\partial \vartheta}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \vartheta}{\partial \eta} &= \left(\frac{1}{P_r} + \frac{4}{3EP_r} \right) \frac{\partial^2 \vartheta}{\partial \eta^2} - \phi \vartheta + D_p \frac{\partial^2 \zeta}{\partial \eta^2} \\ &+ E_c \left(\left(\frac{\partial \xi}{\partial \eta} \right)^2 + \left(\frac{\partial v}{\partial \eta} \right)^2 \right) \end{aligned} \quad (2)$$

and

$$\frac{\partial \zeta}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \zeta}{\partial \eta} = \frac{1}{S_c} \frac{\partial^2 \zeta}{\partial \eta^2} - K_r \zeta + S_p \frac{\partial^2 \vartheta}{\partial \eta^2} \quad (3)$$

where ξ , v and w are non-dimensional velocity components in the x , y , and η

directions respectively, and ϑ , ζ , t , Ω , M , G_h , G_m , V_R and ϕ are, respectively, temperature, concentration, time, rotation parameter, magnetic parameter, Grashof number for heat transfer, Grashof number for mass transfer, viscoelasticity parameter and heat absorption coefficient. E , P_r , S_c , K_r , A , ϖ , ε , D_p , S_p and E_c are radiation parameter, Prandtl number, Schmidt number, chemical reaction parameter, suction velocity parameter, oscillation parameter, scalar constant, Dufour number, Soret parameter and Dissipation parameter respectively. These equations were reduced to ordinary differential equations using perturbation technique and the coupled ordinary differential equations obtained with a set of corresponding boundary conditions were solved for velocity, temperature and concentration using Adomian decomposition method. The effects of various fluid parameters on velocity, temperature and concentration were presented in tabular and graphical forms.

The findings of the study were that:

- (i) resultant velocity profiles were enhanced with an increase in the value of dissipation parameter;
- (ii) an increase in rotational parameter speed up the resultant velocity;
- (iii) concentration distribution decreased with a rise in chemical reaction and accelerated with an increase in thermal diffusion;
- (iv) temperature profiles experienced decline with an increase in the value of radiation parameter, while it was enhanced with an increase in the value of viscous dissipation parameter; and
- (v) the presence of diffusion-thermo parameter enhanced the temperature distribution and reduced the velocity and concentration profiles.

The study concluded that effects of rotatory and viscous dissipation have considerable influence on the velocity, temperature and concentration of Rivlin-Ericksen flow. It is therefore recommended that combined effects of rotatory, viscous dissipation, radiation, thermal-diffusion and diffusion-thermo be included when modeling Rivlin-Ericksen fluid flow for practical purpose.

NOMENCLATURE

x^*	dimensional distance upward the plate
y^*	dimensional distance normal to the plate
z^*	dimensional distance perpendicular to the planes of the plates
u^*, v^*, w^*	dimensional velocity components in the x^*, y^*, z^* directions respectively
t^*	dimensional non-dimensional concentration
ρ^*	fluid density
ϖ^*	oscillation parameter
K_1^*	dimensional chemical reaction rate parameter
Ω	rotation parameter
ν^*	kinematic viscosity
C_p	specific heat at constant pressure
σ	fluid electrical conductivity(Stefan-Boltzman constant)
B_0	magnetic induction
k_p^*	dimensional permeability of the porous medium
ξ	non-dimensional velocity
ϑ	non-dimensional temperature
η	non-dimensional distance
ζ	non-dimensional concentration
T^*	dimensional temperature
C^*	dimensional concentration
S_p	Thermal-diffusion parameter
E_c	Dissipation parameter
R_v	resultant velocity
P^*	dimensional pressure
D	chemical molecular diffusivity
ϕ_o	dimensional heat absorption coefficient
α	thermal diffusivity
g	gravitational acceleration
β_T, β_C	thermal, concentration expansion coefficient
β_1	kinematic viscoelasticity
T_h^*	plate dimensional temperature
C_h^*	plate dimensional concentration
E	radiation parameter
k_p	non-dimensional permeability of the porous medium

K_r	chemical reaction parameter
M	magnetic parameter
Pr	Prandtl number
Sc	Schmidt number
G_h , Gm	Grashof number for heat and mass transfer
ϕ	dimensionless heat absorption coefficient
V_R	dimensionless viscoelasticity parameter
ε	scalar constant
A	suction velocity parameter.
k'	mean absorption coefficient
W_o	scale of suction velocity contain non-zero positive constant
T_m	mean fluid temperature
K_T	thermal diffusion ratio
C_s	concentration susceptibility
K	thermal conductivity
H	a constant
T_0	temperature at the left plate
C_0	concentration at the left plate
h	distance of the plate
D_p	Dufour number

Contents

TITLE PAGE	i
Certification.....	ii
Dedication	iii
Acknowledgements	iv
Abstract.....	vi
Nomenclature	ix
Table of contents.....	xiii
1 GENERAL INTRODUCTION.....	1
1.1 Introduction.....	1
1.2 Statement of the Problem and Justification for the Study	2
1.3 Aim and Objectives of the Study	3
1.4 Definition of Terms and Variables	3
1.5 Outline of the Thesis.....	5
2 REVIEW OF RELATED LITERATURE.....	7
2.1 Preamble	7
2.2 Review of Previous Studies	7
3 METHODOLOGY	16
3.1 Formulation of the Research Problems	16
3.1.1 Radiation and Chemical reaction effects on Convective Rotatory Rivlin-Ericksen flow past a Porous Vertical Plate.....	16
3.1.1.1 Introduction.....	16
3.1.1.2 The Dimensional Governing Equations.....	18
3.1.1.3 Non-dimensional Variables	20

3.1.1.4	Non-dimensional Governing Equations.....	20
3.1.2	Combined effects of Thermal-Diffusion and Diffusion-Thermo on Convective Rotatory Rivlin-Ericksen flow past a Porous Vertical Plate.....	22
3.1.2.1	Introduction	22
3.1.2.2	The Dimensional Governing Equations	23
3.1.2.3	Non-dimensional Variables.....	24
3.1.2.4	Non-dimensional Governing Equations.....	24
3.1.3	Effects of Viscous Dissipation on Convective Rotatory Chemically Reacting Rivlin-Ericksen flow past a Porous Vertical Plate.....	26
3.1.3.1	Introduction	26
3.1.3.2	The Dimensional Governing Equations	27
3.1.3.3	Non-dimensional Variables.....	28
3.1.3.4	Non-dimensional Governing Equations.....	28
3.2	Method of Solution	29
3.2.1	Numerical Computaion of Research Problem 1	29
3.2.1.1	Perturbation Method	29
3.2.1.2	Further Simplification of the third order differential equations	33
3.2.1.3	Skin-friction, Nusselt and Sherwood Number in Term of Amplitude	39
3.2.2	Numerical Computation of Research Problem 2	43
3.2.2.1	Further Simplification of the third order differential equations	44
3.2.2.2	Adomian Decomposition Method.....	45
3.2.2.3	Solution to the Problem	46
3.2.3	Numerical Computation of Research Problem 3	51
3.2.3.1	Further Simplification of equations (3.159)-(3.167)	52
3.2.3.2	Solution to the Problem	54
4	RESULTS AND DISCUSSION OF THE FINDINGS	59
4.1	Radiation and Chemical reaction effects on Convective Rotatory Rivlin-Ericksen flow past a Porous Vertical Plate.....	59
4.2	Combined effects of Thermal-Diffusion and Diffusion-Thermo on Convective Rotatory Rivlin-Ericksen flow past a Porous Vertical Plate.....	80
4.3	Effects of Dissipation on Convective Rotatory Chemically Reacting Rivlin-Ericksen flow past a Porous Vertical Plate.....	101
5	SUMMARY, CONCLUSIONS AND RECOMMENDATIONS.....	123
5.1	Summary	123
5.2	Conclusion.....	123
5.3	Contribution to knowledge.....	124
5.4	Recommendations	125
	References	126

Chapter 1

GENERAL INTRODUCTION

1.1 Introduction

Viscoelastic materials exhibit both viscous and elastic characteristics under deformation. A viscous fluid is a type of fluid that resists shear flow. When a load is applied, it strains linearly with time. In other words, it exhibits time-dependent strain. While, under a load, elastic material is stretched, strained and returned to its original form in the absence of the load. Rivlin-Ericksen fluid which is being categorized as viscoelastic fluid is a non-Newtonian model proposed theoretically by Rivlin-Ericksen (1955). Generally, the relationship between shear stress and the shear rate for non-Newtonian fluids is non-linear and mostly is time-dependent. Some of the materials that are non-Newtonian fluids includes: molten polymers, salt solutions and some common substances; such as toothpaste, custard, paint, shampoo, starch suspensions, honey and blood (Rao, 2007).

Consideration of rotating medium in fluid dynamic is of great importance due to its relevance to numerous natural phenomena and application in many technological situations relating to Coriolis force. Rotating fluids are also relevant in science such as; oceanography, meteorology, atmospheric science and astrophysics. Some of the applications of rotating flow, particularly in porous media in the field of engineering, to mention but a few, are food processing industry, rotating machinery and chemical processing industry.

Convective heat transfer, which is also known as convection, describes how heat is transferred due to the fluid motion from one place to another. Convection is the major heat transfer for both liquids and gases. Natural buoyancy forces are responsible for the fluid motion when the fluid is heated. The study of mass transfer of convective fluid flow past a porous plate has gained the attention of many researchers due to its relevance. Therefore, the study of Rivlin-Ericksen fluid has become highly importance due to it application in petroleum industry, food and paper industry and similar activities.

1.2 Statement of the Problem and Justification for the Study

The study of Rivlin-Ericksen fluid flow through porous medium both in vertical and semi-infinite plate has gained the attention of many researchers as a result of its industrial applications in petroleum industry, paper industry, food processing, biochemical operations and transport polymers. Even in the medical aspect; the study of the flow of blood through arteries and veins which are of great interest are some of its applications. Therefore, this study presents the combined effects of thermal radiation, chemical reaction, viscous dissipation, thermal-diffusion and diffusion-thermo on mixed convective rotatory Rivlin-Ericksen flow past a porous vertical plate.

1.3 Aim and Objectives of the Study

The aim of this study is to investigate the effects of chemical reaction, thermal-diffusion, viscous dissipation, thermal radiation and diffusion-thermo on mixed convective rotatory Rivlin-Ericksen flow of incompressible and electrically conducting fluids past a porous vertical plate with a periodic suction.

The following are the objectives of this study:

- (i) determine the contribution of viscous dissipation to the Rivlin-Ericksen fluid;
- (ii) evaluate rotatory effects on convective Rivlin-Ericksen flow in a porous vertical plate; (iii) examine the effects of chemical reaction and thermal-diffusion on concentration profiles;
- (iv) analyze the influence of thermal radiation and viscous dissipation on fluid temperature; and
- (v) determine the impact of diffusion-thermo on concentration, temperature and velocity profiles.

1.4 Definition of Terms and Variables

- **FLUID:** A fluid can be defined as a substance which when subjected to external shearing force deforms continually and has the capacity to flow.
- **SPECIFIC HEAT:** This is referred to as the required heat for a unit mass temperature to be raised by one degree.
- **SHEARING FORCES:** These are forces that act on the surface tangentially, and causes deformation upon the material.
- **BOUNDARY LAYER:** Boundary layer resulted from relative motion between the fluid and the boundary.
- **HEAT TRANSFER:** This is the study of energy, how it is distributed, carried and diffused by and on some materials.
- **VISCOUS DISSIPATION:** In the thermodynamic systems, viscous dissipation resulted from an irreversible process. It also refers to as the rate of an irreversible work by shear stresses on the fluid.
- **THERMAL RADIATION:** This is the emission of energy continually from the body surface of all matter with temperature greater than zero.
- **PRANDTL NUMBER (P_r):** It can be described as a measure of viscosity and relative importance of heat conduction of the fluid. Like thermal conductivity and the viscosity, the Prandtl number is a material property that varies from fluid to fluid. Normally, small thermal conductivity and large viscosity leads to high Prandtl number and vice versa.
- **ECKERT NUMBER:** In fluid dynamics, Eckert number is a dimensionless number used to express the relationship that exist between enthalpy and kinetic energy of a flow
- **SCHMIDT NUMBER (S_c):** This is a non-dimensional number that shows the relationship between momentum diffusivity and mass diffusivity. In a fluid flow that involves both mass and momentum diffusion, the flow of the fluid can be characterised by the Schmidt number.
- **GRASHOF NUMBER (G_h):** In natural convection problem, it expresses the importance of buoyancy force to viscous force.
- **UNSTEADY FLOW:** This is the kind of flow where pressure, velocity and at a point; the cross

sectional area changes along with time.

- **PERMEABILITY:** This refers to the rate of flow of a liquid or gas through a porous material.
- **SUCTION:** It can be defined as a process where force is exerted on a liquid, gaseous or solid body by reducing the air pressure on part of its surface.
- **CONVECTIVE HEAT TRANSFER:** It can simply be referred to as convection. This is defined as the heat transfer as a result of movement of the fluid from one place to another.
- **MAGNETO HYDRODYNAMIC (MHD):** This refers to dynamics study of electrically conducting fluids. Magneto-hydrodynamics is derived from the word magneto means magnetic field, hydro means liquid and dynamics means movement.
- **STRESS AND STRAIN:** Stress means application of a force on a body, while the result of the stress is strain.
- **VISCOSITY:** This is a measure of fluid resistance to gradual deformation as a result of shear stress.
- **NEWTONIAN FLUIDS:** These are fluids that are in line with Newton's viscosity equation.
- **NON-NEWTONIAN FLUIDS:** In these kinds of fluid, linear relationship does not exist between shear stress and rate of deformation.

1.5 Outline of the Thesis

This thesis is made up of five chapters.

Chapter one contains the introduction, significance of the study, aim and objectives of the study, Definition of terms/variables and Outline of the thesis.

Chapter two contains the review of literature of relevant studies.

Chapter three consists of the physical configuration of the model, formulation of the research problem for Radiation and Chemical reaction effects on convective rotatory Rivlin-Ericksen flow past a permeable vertical plate, formulation of the research problem for Combined effects of thermal-diffusion and diffusion-thermo on convective rotatory Rivlin-Ericksen flow past a porous vertical plate and formulation of the research problem for Effects of dissipation on convective rotatory chemically reacting Rivlin-Ericksen flow past a permeable vertical plate, method of solution and the numerical computation of the research problems

Chapter four contains the presentation of the results both in graphical and tabular forms, and the discussion of the results obtained.

Chapter five is made up of summary, conclusion and recommendation.

Chapter 2

REVIEW OF RELATED LITERATURE

2.1 Preamble

Rivlin-Ericksen fluid has numerous applications in the field of engineering, chemical processing industry, rotating machinery, paper and food processing industry, petroleum industry and other areas that involve viscoelastic fluids. Hence, this study considered the effects of chemical reaction, thermal-diffusion, diffusion-thermo, viscous dissipation, thermal radiation on convective rotatory Rivlin-Ericksen flow of incompressible and electrically conducting fluids past a porous vertical plate with a periodic suction.

2.2 Review of Previous Studies

The study of Rivlin-Ericksen fluid has received attention from researchers as a result of its several applications. The relevance of this kind of fluid has gained attention of researchers such as Ravikumar et al.(2014) studied impacts of MHD and heat absorption on convective Rivlin-Ericksen fluid flow through a vertical permeable semi-infinite plate with variable suction. The result shown that an increase in the heat absorption coefficient and viscoelastic parameter decreased velocity. Influence of chemical reaction and radiation were not put into consideration in the paper. Investigation on laminar Rivlin-Ericksen flow past a parallel porous plates with injection and suction, where the lower plate is stretched was considered by Das et al.(2005). Uwanta and Hussaini (2012) discussed the influences of mass transfer on convective Rivlin-Ericksen hydromagnetic flow in the presence of time-dependent suction past a porous medium. They made use of approximate optically thin gray gas in their study. The impacts of heat transfer and radiation on MHD unsteady non-Newtonian fluid flow via a permeable medium with slip condition was investigated by Gbadeyan and Dada (2013). Dharmendra and Varshney (2012) considered MHD effect on stratified free convective Rivlin-Ericksen fluid flow through a permeable plate with mass and heat transfer. It was discovered and reported that increasing visco-elastic parameter leads to speed up velocity up to $y = 2$, after which a different trend was observed.

An investigation on the rotatory effects on MHD free convective Rivlin-Ericksen flow past a permeable medium in the presence of mass flux and constant heat was carried out by Varshney et al.(2013). It was reported that with an increase in rotation parameter, primary velocity decreases and secondary velocity increases. Lakshmi and Gomathi (2012) examined a flow of Rivlin-Ericksen fluid through a porous vertical wall. Constant suction was used in this study. It was discovered that as suction parameter increases, velocity decreases. Sreekanth et al.(2011) considered unsteady hydromagnetic Hele-Shaw flow of Rivlin-Ericksen fluid past a porous media and the results showed that with increase in viscoelastic parameter, velocity decreases. The above studies ignored the effects of some useful fluid parameters on the flow such as: heat

absorption, pressure gradient, radiation and chemical reaction. Reddy et al.(2013) studied MHD viscoelastic fluid flow with mass and heat transfer via an infinite vertical plate. The result shows that skin- friction coefficient is slowdown by increasing chemical reaction parameter. Nayak and Dash(2014) examined chemical reaction effect on viscoelastic MHD flow past a porous medium, but the effects of radiation and heat absorption were ignored. Dissipation and chemical reaction effects on free convective MHD Walter's memory fluid flow with heat sink and constant suction was carried out by Sekhar and Reddy (2012).

In all the studies discussed above, the effects of radiation on convective Rivlin-Ericksen flow was neglected. The effects of radiation as far as industrial applications is concerned, can not be over emphasized. Radiation effects is relevant in furnace design, glass production, space technology applications and spacecraft operating at higher temperatures, to mention but a few.

Consideration of rotating medium in fluid dynamic is of great importance due to its relevance to numerous natural phenomena and application in many technological situations relating to Coriolis force. Rotating fluids have its relevance in science such as; oceanography, meteorology, atmospheric science and astrophysics. Some of the applications of rotating flow, particularly in porous media in the field of engineering, to mention but a few are; food processing industry, rotating machinery and chemical processing industry. Recently, study of rotating flow has gained interest of many researchers due to its importance. Reddy et al. (2016a) analyzed the effects of rotation and thermal diffusion on viscoelastic fluid past a porous channel. Mutua et al. (2013) considered convective MHD Newtonian fluid flow with variable suction through porous plate and discovered that a decrease in rotation parameter resulted in increasing Skin friction along both x and y axes.

Singh (2013) studied thermal radiation effects on viscoelastic MHD fluid in a rotatory vertical plate. It was realised from the result that an increase in Ω , decreases the fluid velocity. Oldroyd-B MHD rotating fluid flow via a permeable vertical channel in the presence of radiation was carried out by Garg et al.(2014b). Guria and Jana (2013) examined rotatory viscoelastic fluid via a permeable plate under a uniform suction. They discovered that viscoelastic parameter made heat transfer at the plate to increase. Abdulmaleque (2017) investigated the effects of temperature-dependent injection/suction on non-Newtonian casson fluid flow with thermal radiation and viscous dissipation. Also, Garg et al. (2014a) presented oscillatory viscoelastic fluid flow through a porous rotating vertical channel with an assumption of an optically thin radiation and constant suction. The result showed that as the rotation parameter increased, the velocity decreased. The aforementioned studies have convincingly contributed to the studies of fluid flow but the effects of chemical reaction were neglected.

Hall current and rotation effects of MHD couette flow with magnetically inclined field was studied by Seth et al (2012). The result showed that an increase in angle of inclination enhanced both primary and secondary velocities profiles. Chakraborty and Panja (2009) discussed viscoelastic fluid of a couette oscillatory hydromagnetic flow. Hussaini et al (2015) studied Rivlin-Ericksen magnetohydrodynamics flow past a porous medium with slip condition. Jeffrey fluid of couette flow through a rotating channel was analyzed by Sreenadh and

Reddappa (2016). Jana et al. (2012) studied unsteady rotatory couette flow past a porous medium. These studies were carried out without considerations to the variability of suction, radiation and chemical reaction. Thermal radiation effects on rotatory viscoelastic MHD flow via a vertical plate was studied by Singh et al.(2011). It was reported that rotation parameter enhanced velocity profiles. Effects of chemical reaction and magnetic field was put into consideration on Oldroyd fluid.

Impacts of rotation on MHD Rivlin-Ericksen flow past a porous medium was analyzed by Sharma et al.(2013). Grag and Singh (2015) investigated radiating, rotating and chemically reacting viscoelastic fluid past a permeable vertical channel. Rita and Das (2013) studied convective viscoelastic MHD influence on oscillating fluid flow through a permeable medium. Likewise, Ravikumar et al.(2014) considered combined impacts of MHD and heat absorption on convective Rivlin-Ericksen fluid flow via a semi infinite vertical permeable plate in the presence of variable temperature and suction. result showed that an increase in Rivlin-Ericksen parameter diminished velocity. But, the above investigations overlooked the effects of thermal radiation.

In addition, simultaneous effects of heat and mass transfer on non-Newtonian fluid is significantly important to this study due to it application in engineering and various branches of science. Therefore, researchers such as ; Gaur and Jha (2016) investigated viscoelastic fluid flow with mass and heat transfer effects past a rotating permeable medium. An optically thin radiation and constant suction was considered in the study. The result showed that at high values of rotation parameter (Ω), velocity decreased. The effect of variable suction on oscillatory Jeffrey fluid with mass and heat transfer was analyzed by Joseph et al.(2016). It was reported that an increase in radiation parameter N , enhanced temperature profiles. Dada and Agunbiade (2016) examined the effects of chemical reaction and thermal radiation on convective Rivlin-Ericksen flow through a permeable vertical plate. It was discovered that temperature and velocity decreased as radiation parameter increased. Reddy et al.(2016c) presented non-Newtonian fluid with variable suction of a Non-Homogeneous permeable medium through a porous vertical plate. The resulted equations were solved using regular perturbation scheme. The result showed that an increase in G_m and G_h increased the velocity of the fluid. Rahman et al.(2016) considered a Newtonian fluid flow in a rotating system through a vertical permeable plate with heat transfer. The equations of the flow field were solved numerically.

A careful examination of all the above studies shown that combined impacts of chemical reaction, variable suction, pressure gradient and heat absorption on heat and mass transfer of convective Rivlin-Ericksen fluid flow in a rotating system have received little or no attention. Considering various phenomena, rotation has significantly numerous importance and applications such as, in meteorology, gaseous and nuclear reactors, cosmical fluid dynamics and various engineering applications.

Furthermore, Soret effect has its utilization in isotope separation and also relevant in areas such as petrology, hydrology, geosciences, etc. Therefore, Dulal and Babulal(2012) considered

the effect of Soret on MHD oscillatory flow in a permeable vertical medium along with radiation and chemical reaction. It was found that thermal-diffusion enhances concentration. Gbadeyan et al.(2011) examined the influence of thermal-diffusion and diffusion-thermo with mass and heat transfer on mixed convective viscoelastic fluid flow past a permeable medium. The result shown that both concentration and temperature profiles increase with the increase in thermal-diffusion parameter. Ibrahim and Suneetha (2015) investigated thermal-diffusion and chemical reaction effects on MHD viscoelastic fluid through an infinite vertical plate together with heat source. The result revealed that both velocity and temperature profiles increase with the increase in E_c .

Kurshinshiki fluid with the influence of Soret through a vertical porous plate with mass and heat transfer in the presence of variable suction was considered by Jimoh et al.(2014). It was discovered that at the boundary layer, increasing the heat sources parameter improved both velocity and temperature profiles. Deepthi and Prasada (2017) studied the combined effect of thermal radiation and thermal-diffusion on mass and heat transfer of mixed convective flow through a vertical channel. It was discovered that velocity and temperature increased as Soret parameter increased, while concentration reduced. However, the reverse was the case at higher values. Vijayakumar and Reddy (2017) investigated the effect of thermal radiation and Soret on MHD viscoelastic fluid flow through a porous plate. The result showed that an increase in Eckert number and radiation parameters increased and decreased the temperature profile respectively. Babu et al.(2017) considered the combined effects of diffusion-thermo and thermal-diffusion on MHD Jeffrey fluid flow in a stretching sheet. It was noticed that both Prandtl and Soret had a decline effect on temperature and a different trend was observed in concentration profile. Tasawar et al.(2017) investigated Dufour and Soret effects on peristaltic transport with MHD of convective Jeffrey fluid through a curved channel. It was acknowledged that temperature was enhanced with the increase in thermal-diffusion and diffusion-thermo.

Furthermore, Reddy et al.(2016b) studied the effects of MHD, diffusion-thermo and thermal-diffusion effects on convective Rivlin-Ericksen fluid flow via a vertical semi-infinite plate. Finite difference method was used and the result revealed that temperature rises and falls with increasing values of Dufour and Soret respectively. Also, species concentration profiles was accelerated with the increase in Soret parameter but a different trend was seen in the effect of Dufour. Chandra et al. (2015) considered the effect of solutal and thermal buoyancy on boundary layer MHD visco-elastic fluid flow through a porous plate with Soret and variable suction. It was reported that the presence of thermal-diffusion increases the fluid velocity, whereas the influence of the magnetic field reduces it. The effects of Soret on chemically reacting MHD Casson fluid flow via a vertical plate was examined by Charankumar et al.(2016). It was acknowledged that increase in thermal Grashof number led to increase in velocity.

Moreover, the effects of dissipation function on flow distribution can not be overemphasized due to its significance in controlling the rate of cooling. Mabood and Ibrahim (2016) investigated non-uniform heat source and Soret effects on convective Non-Darcian MHD Micropolar fluid flow in the presence of thermal radiation. The effect of dissipation was

considered and the result showed that temperature increased with an increase in Ec . Unsteady free convective MHD flow of a visco-elastic fluid past an inclined infinite porous plate with viscous dissipation, ohmic heating and heat source was studied by Umamaheswar et al.(2013). The result revealed that an increase in G_h made the Skin friction to increase. Dada and Adefolaju (2012) considered the influence of radiation, dissipation and MHD on convective mass and heat transfer past Darcy-forcheimer permeable channel. From the result, it could be seen that the values of Ec enhances velocity profiles. Sarma and Govardhan (2016) considered effects of Dufour and Soret on mass and heat transfer past a permeable vertical surface in the presence of viscous dissipation and radiation. Newtonian fluid was studied and it was acknowledged that Ec enhanced velocity profiles. Pal and Samad (2015) examined radiation and viscous dissipation effects on Non-Newtonian fluid via a surface with heat flux and generation. The result showed that increasing Ec led to increase in both velocity and temperature fields.

A careful examination of all the above named studies shows that combined effects of chemical reaction, variable suction, pressure gradient and heat absorption on heat and mass transfer of convective Rivlin-Ericksen fluid flow in a rotating system with viscous dissipation, diffusion-thermo and thermal diffusion have not received attention of researchers to the best knowledge of the author. Considering various phenomena, combined effects of thermal-radiation, thermal-diffusion, diffusion-thermo, and viscous dissipation in convective Rivlin-Ericksen fluid flow in a rotating system with heat and mass transfer past a permeable vertical plate, have serious importance and applications in; the field of engineering, chemical processing industry, rotating machinery, paper and food processing industry, petroleum industry and other areas that involve viscoelastic fluids. Hence, this work focuses on viscous dissipation, diffusion-thermo, thermal-diffusion and radiation effects on convective Rivlin-Ericksen fluid in a rotating system with chemical reaction. The solutions of the non-dimensional governing equations are obtained using perturbation and Adomian decomposition method with MAPLE 18 and MATHEMATICA 11.0 software.

Chapter 3

METHODOLOGY

3.1 Formulation of the Research Problems

3.1.1 Radiation and Chemical reaction effects on Convective Rotatory Rivlin-Ericksen flow Past a Porous Vertical Plate

3.1.1.1 Introduction

Chemical reaction and radiation effects on the fluid has great impact in fluid dynamic. Chemical reactions have tremendous impacts in changing the rate of mass diffusion. In majority of cases, a chemical reaction depends on the concentration of the species itself. A reaction is said to be first order if the rate of reaction is directly proportional to the concentration itself. Chemical reactions usually accompany a large amount of exothermic and endothermic reaction. These characteristics can be easily seen in a lot of industrial processes. Likewise, the role of thermal radiation is of major importance in the design of many advanced energy convection systems operating at high temperature and knowledge of radiative heat transfer becomes very important in nuclear power plants, various propulsion devices for aircraft and space vehicles.

Consider a non-Newtonian, two-dimensional incompressible mixed convective Rivlin-Ericksen flow of an electrically conducting fluid through a rotating vertical channel with a periodic suction. Here, fluid is bounded by two infinite vertical plates with distance h apart. Along the center line of the channel is the x -axis which is upward oriented, while z -axis is perpendicular to the plane of the plate. The fluid and the plates rotate in z -axis with a constant angular velocity Ω . In formulating this problem, the following assumptions were made:

- (i) the pressure is taken to be constant in the flow field;
- (ii) transversely applied magnetic Reynolds number and magnetic field are considered to be very small hence Hall effects and induced magnetic field are negligible;
- (iii) all the physical variables are function of z and t only;
- (iv) the uniform magnetic field is applied in z -direction that is perpendicular to the plate; and
- (v) compared to other chemical species, diffusing species is considered to be very low; therefore, Soret and Dufour effects are neglected.

With the above assumptions, below is the physical configuration of the flow of vertical plate subjected to a uniform transverse magnetic field with porous medium.

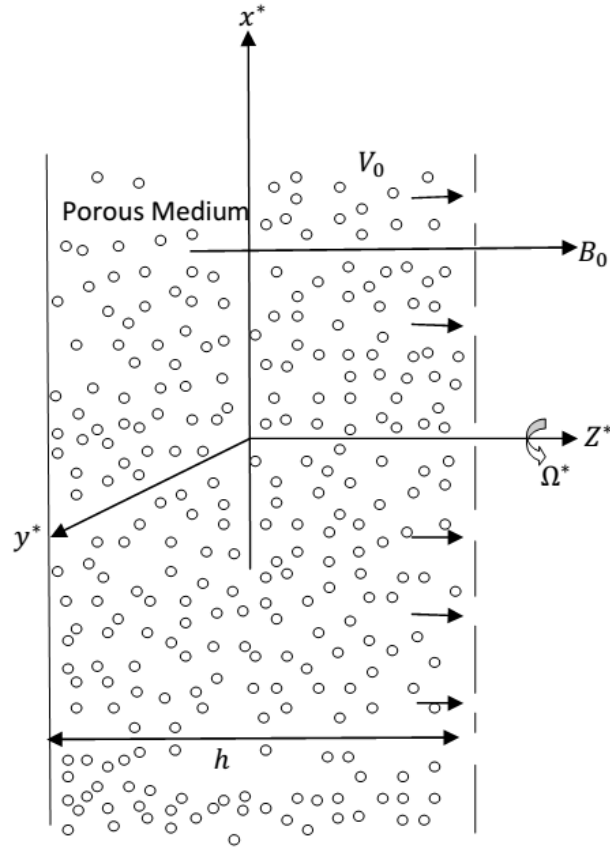


Figure 3.1 Physical Configuration of the Flow

3.1.1.2 The Dimensional Governing Equations

Based on the assumptions in the formulation of the research problem (Garg et al. 2014a), the dimensional governing equations of the unsteady, two-dimensional free convective Rivlin-Ericksen flow which comprising of continuity, momentum, energy and concentration equations are as follows:

Continuity equation:

$$\frac{\partial w^*}{\partial z^*} = 0 \quad (3.1)$$

Momentum equation:

$$\begin{aligned} \frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial z^*} = & -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + \nu \frac{\partial^2 u^*}{\partial z^{*2}} + 2\Omega^* v^* + g\beta_T(T^* - T_h^*) + \\ & g\beta_C(C^* - C_h^*) - \nu \frac{u^*}{k_p^*} - \frac{\sigma B_o^2 u^*}{\rho} - \beta_1 \left(\frac{\partial^3 u^*}{\partial t^* \partial z^{*2}} + v^* \frac{\partial^3 u^*}{\partial z^{*3}} \right) \end{aligned} \quad (3.2)$$

$$\frac{\partial v^*}{\partial t^*} + v^* \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} + \nu \frac{\partial^2 v^*}{\partial z^{*2}} - 2\Omega^* u^* - v^* \frac{\nu^*}{k_p^*} - \frac{\sigma B_o^2 v^*}{\rho} - \beta_1 \left(\frac{\partial^3 v^*}{\partial t^* \partial z^{*2}} + v^* \frac{\partial^3 v^*}{\partial z^{*3}} \right) \quad (3.3)$$

Energy equation:

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial z^*} = \alpha \frac{\partial^2 T^*}{\partial z^{*2}} - \frac{\phi_o}{\rho c_p} (T^* - T_h^*) - \frac{1}{\rho C_p} \frac{\partial q_R}{\partial z^*} \quad (3.4)$$

Concentration equation:

$$\frac{\partial C^*}{\partial t^*} + v^* \frac{\partial C^*}{\partial z^*} = D \frac{\partial^2 C^*}{\partial z^{*2}} - K_1^* (C^* - C_h^*) \quad (3.5)$$

The boundary conditions for the problem are:

$$\left. \begin{aligned} z^* = 0; u^* = v^* = 0, T^* = T_h + \varepsilon (T_0 - T_h) \cos \varpi^* t^*, C^* = C_h + \varepsilon (C_0 - C_h) \cos \varpi^* t^* \\ z^* = h; u^* = W_o (1 + \varepsilon \cos \varpi^* t^*), v^* = 0, T^* = T_h, C^* = C_h \end{aligned} \right\} \quad (3.6)$$

The suction velocity is expressed in exponential form as:

$v^* = -W_o (1 + \varepsilon A e^{i\varpi^* t^*})$ (Das et al(2011)), ε and εA are small values less than unity. q_R is the radiative heat flux and is defined, based on Rosseland approximation (Brewster (1972)), as:

$$q_R = -\frac{4\sigma}{3k'} \frac{\partial T^{*4}}{\partial z^*} \quad (3.7)$$

This present analysis is limited to optically thick fluid, hence Rosseland approximation is used. Considering the temperature differences within the flow to be sufficiently small, T^{*4} (quartic temperature function) can be expanded using Taylor series expansion and neglecting higher order terms (from second order) gives;

$$T^{*4} \approx 4T_h^{*3} T^* - 3T_h^{*4} \quad (3.8)$$

This is substituted into radiative heat flux term that was used in equation (3.4).

The pressure gradient for the fluid is considered in the form;

$$-\frac{1}{\rho} \frac{\partial P^*}{\partial y^*} = 0 \quad \text{and} \quad -\frac{1}{\rho} \frac{\partial P^*}{\partial x^*} = H \varepsilon \cos \varpi t \quad (3.9)$$

where H is a constant and it oscillates only in x-axis direction.

3.1.1.3 Non-dimensional Variables

The following are the non-dimensional variables.

$$\left. \begin{aligned} \xi &= \frac{u^*}{W_o}, v = \frac{v^*}{W_o}, t = \frac{t^* W_o^2}{\nu}, \eta = \frac{W_o z^*}{\nu} = \frac{z^*}{h}, k_p = \frac{k_p^* W_o^2}{\nu^2}, \\ x &= \frac{W_o x^*}{\nu}, \theta = \frac{T^* - T_h^*}{T_o^* - T_h^*}, \varpi = \frac{\varpi^* \nu}{W_o^2}, P_r = \frac{\nu \rho C_p}{k_p} = \frac{\nu}{\alpha}, \\ M &= \frac{\sigma B_o^2 \nu}{\rho W_o^2}, G_h = \frac{\nu g \beta_T (T_o^* - T_h^*)}{W_o^3}, \phi = \frac{\nu \phi_o}{\rho C_p W_o^2}, V_R = \frac{\beta_1 W_o^2}{\nu^2}, \\ Gm &= \frac{\nu g \beta_c (C_o^* - C_h^*)}{W_o^3}, y = \frac{W_o y^*}{\nu}, P = \frac{P^*}{\rho W_o^2}, \zeta = \frac{C^* - C_h}{C_o^* - C_h^*}, \\ S_c &= \frac{\nu}{D}, E = \frac{k k_p}{4 \sigma T_h^{*3}}, K_r = \frac{K_1^* \nu}{W_o^2}, \Omega = \frac{\Omega^* \nu}{W_o^2} \end{aligned} \right\} \quad (3.10)$$

3.1.1.4 Non-dimensional Governing Equations

The non-dimensional governing equations are obtained by substituting non-dimensional variables equation (3.10) into equations (3.2)-(3.5);

Momentum Equation

$$\begin{aligned} \frac{\partial \xi}{\partial t} - (1 + \varepsilon A e^{i\varpi t}) \frac{\partial \xi}{\partial \eta} &= H \varepsilon \cos \varpi t + \frac{\partial^2 \xi}{\partial \eta^2} + 2\Omega v - (M + \frac{1}{k_p}) \xi + G_h \theta + \\ G_m \zeta - V_R \left(\frac{\partial^3 \xi}{\partial t \partial \eta^2} - (1 + \varepsilon A e^{i\varpi t}) \frac{\partial^3 \xi}{\partial \eta^3} \right) \end{aligned} \quad (3.11)$$

$$\frac{\partial v}{\partial t} - (1 + \varepsilon A e^{i\varpi t}) \frac{\partial v}{\partial \eta} = \frac{\partial^2 v}{\partial \eta^2} - 2\Omega \xi - (M + \frac{1}{k_p}) v - V_R \left(\frac{\partial^3 v}{\partial t \partial \eta^2} - (1 + \varepsilon A e^{i\varpi t}) \frac{\partial^3 v}{\partial \eta^3} \right) \quad (3.12)$$

Energy equation:

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{i\varpi t}) \frac{\partial \theta}{\partial \eta} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial \eta^2} - \phi \theta + \frac{4}{3EP_r} \frac{\partial^2 \theta}{\partial \eta^2} \quad (3.13)$$

Concentration equation:

$$\frac{\partial \zeta}{\partial t} - (1 + \varepsilon A e^{i\varpi t}) \frac{\partial \zeta}{\partial \eta} = \frac{1}{S_c} \frac{\partial^2 \zeta}{\partial \eta^2} - K_r \zeta \quad (3.14)$$

Taking $\psi = \xi + iv$ equations (3.11) and (3.12) become

$$\frac{\partial \psi}{\partial t} - (1 + \varepsilon A e^{i\varpi t}) \frac{\partial \psi}{\partial \eta} = H \varepsilon \cos \varpi t + \frac{\partial^2 \psi}{\partial \eta^2} - (2i\Omega + F) \psi + G_h \theta +$$

$$G_m \zeta - V_R \left(\frac{\partial^3 \psi}{\partial t \partial \eta^2} - (1 + \varepsilon A e^{i\varpi t}) \frac{\partial^3 \psi}{\partial \eta^3} \right) \quad (3.15)$$

where

$$F = M + \frac{1}{k_p}.$$

The corresponding dimensionless boundary conditions are

$$\psi = 0, \mathcal{G} = 1 + \frac{\varepsilon}{2} (e^{i\varpi t} + e^{-i\varpi t}), \zeta = 1 + \frac{\varepsilon}{2} (e^{i\varpi t} + e^{-i\varpi t}) \text{ at } \eta = 0 \quad (3.16)$$

$$\psi = 1 + \frac{\varepsilon}{2} (e^{i\varpi t} + e^{-i\varpi t}), \mathcal{G} = 0, \zeta = 0 \text{ at } \eta = 1 \quad (3.17)$$

3.1.2 Combined effects of Thermal-Diffusion and Diffusion-Thermo on Convective Rotatory Rivlin-Ericksen flow past a Porous Vertical Plate

3.1.2.1 Introduction

In fluid flow that involves both heat and mass transfer, driving potentials and the fluxes relation are significantly noticed. The energy flux that is generated due to concentration gradient is referred to as diffusion-thermo, while mass flux resulting from temperature gradients is thermal-diffusion. Mostly, the effects of diffusion-thermo and thermal-diffusion are often neglected in most studies on the bases that they are of low magnitude in relation to the rest chemical species. The effects of Dufour and Soret become significant phenomena in areas like petrology, hydrology, geoscience, etc. The effect of thermal-diffusion is relevant, for example, in the separation of isotope and mixture of gases that has light molecular weight.

The following assumptions were made in the formulation of this problem:

- (i) An unsteady, laminar flow is considered.
- (ii) Induced magnetic field and Hall effects are ignored due to the fact that magnetic Reynold's number and transversely applied magnetic field was considered to be very small.
- (iii) A magnetic field B_0 of uniform strength is perpendicularly applied to the plates.
- (iv) There is a rotation of the entire system through the perpendicular axis to the plates.
- (v) Thermal-diffusion and diffusion-thermo are assumed to be of substantial magnitude, hence, they are not negligible.
- (vi) The plates are considered to be infinite in x -direction, hence all physical quantities excluding pressure are functions of z^* and t^* .
- (vii) In the flow field, pressure is taken to be constant.
- (viii) Finitely conducting fluid with constant physical properties is observed.

3.1.2.2 The Dimensional Governing Equations

With the above assumptions, the governing equations of the flow in the rotating channel are:
Continuity equation:

$$\frac{\partial w^*}{\partial z^*} = 0 \quad (3.18)$$

Momentum equation:

$$\begin{aligned} \frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial z^*} = & -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + \nu \frac{\partial^2 u^*}{\partial z^{*2}} + 2\Omega^* v^* + g\beta_T(T^* - T_h^*) + \\ & g\beta_C(C^* - C_h^*) - \nu \frac{u^*}{k_p} - \frac{\sigma B_0^2 u^*}{\rho} - \beta_1 \left(\frac{\partial^3 u^*}{\partial t^* \partial z^{*2}} + v^* \frac{\partial^3 u^*}{\partial z^{*3}} \right) \end{aligned} \quad (3.19)$$

$$\frac{\partial v^*}{\partial t^*} + v^* \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} + \nu \frac{\partial^2 v^*}{\partial z^{*2}} - 2\Omega^* u^* - v^* \frac{\nu^*}{k_p} - \frac{\sigma B_0^2 v^*}{\rho} - \beta_1 \left(\frac{\partial^3 v^*}{\partial t^* \partial z^{*2}} + v^* \frac{\partial^3 v^*}{\partial z^{*3}} \right) \quad (3.20)$$

Energy equation:

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial z^*} = \alpha \frac{\partial^2 T^*}{\partial z^{*2}} - \frac{\phi_o}{\rho c_p} (T^* - T_h^*) - \frac{1}{\rho C_p} \frac{\partial q_R}{\partial z^*} + \frac{Dk_T \partial^2 C^*}{C_p C_s \partial z^{*2}} \quad (3.21)$$

Concentration equation:

$$\frac{\partial C^*}{\partial t^*} + v^* \frac{\partial C^*}{\partial z^*} = D \frac{\partial^2 C^*}{\partial z^{*2}} - K_1^* (C^* - C_h^*) + \frac{DK_T}{T_m} \frac{\partial^2 T^*}{\partial z^{*2}} \quad (3.22)$$

The boundary conditions for the problem are:

$$\left. \begin{aligned} z^* = 0; u^* = v^* = 0, T^* = T_h + \varepsilon(T_0 - T_h) \cos \varpi^* t^*, C^* = C_h + \varepsilon(C_0 - C_h) \cos \varpi^* t^* \\ z^* = h; u^* = W_o(1 + \varepsilon \cos \varpi^* t^*), v^* = 0, T^* = T_h, C^* = C_h \end{aligned} \right\} \quad (3.23)$$

3.1.2.3 Non-dimensional Variables

The following non-dimensional variables are used to reduce the governing equations to non-dimensional form.

$$\left. \begin{aligned} \xi = \frac{u^*}{W_o}, v = \frac{v^*}{W_o}, t = \frac{t^* W_o^2}{\nu}, \eta = \frac{W_o z^*}{\nu} = \frac{z^*}{h}, k_p = \frac{k_p^* W_o^2}{\nu^2}, \\ g = \frac{T^* - T_h^*}{T_0^* - T_h^*}, \varpi = \frac{\varpi^* \nu}{W_o^2}, P_r = \frac{\nu \rho C_p}{k_p} = \frac{\nu}{\alpha}, M = \frac{\sigma B_0^2 \nu}{\rho W_o^2}, x = \frac{W_o x^*}{\nu} \\ G_h = \frac{\nu g \beta_T (T_0^* - T_h^*)}{W_o^3}, \phi = \frac{\nu \phi_o}{\rho C_p W_o^2}, V_R = \frac{\beta_1 W_o^2}{\nu^2}, G_m = \frac{\nu g \beta_c (C_0^* - C_h^*)}{W_o^3}, \\ y = \frac{W_o y^*}{\nu}, P = \frac{P^*}{\rho W_o^2}, \zeta = \frac{C^* - C_h}{C_0^* - C_h^*}, S_C = \frac{\nu}{D}, E = \frac{k' k_p}{4 \sigma T_h^{*3}}, \\ \Omega = \frac{\Omega^* \nu}{W_o^2}, K_r = \frac{K_1 \nu}{W_o^2}, D_p = \frac{DK_T (C_0^* - C_h^*)}{C_s C_p \nu (T_0^* - T_h^*)}, S_p = \frac{DK_T (T_0^* - T_h^*)}{\nu T_m (C_0^* - C_h^*)} \end{aligned} \right\} \quad (3.24)$$

3.1.2.4 Non-dimensional Governing Equations

Using equation (3.24) in equations (3.19)-(3.22) together with the boundary conditions. The equations become

Momentum Equation:

$$\begin{aligned} \frac{\partial \xi}{\partial t} - (1 + \varepsilon A e^{i\varpi t}) \frac{\partial \xi}{\partial \eta} = H \varepsilon \cos \varpi t + \frac{\partial^2 \xi}{\partial \eta^2} + 2\Omega v - (M + \frac{1}{k_p}) \xi + G_h g + \\ G_m \zeta - V_R \left(\frac{\partial^3 \xi}{\partial t \partial \eta^2} - (1 + \varepsilon A e^{i\varpi t}) \frac{\partial^3 \xi}{\partial \eta^3} \right) \end{aligned} \quad (3.25)$$

$$\frac{\partial v}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial v}{\partial \eta} = \frac{\partial^2 v}{\partial \eta^2} - 2\Omega \xi - (M + \frac{1}{k_p})v - V_R \left(\frac{\partial^3 v}{\partial t \partial \eta^2} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial^3 v}{\partial \eta^3} \right) \quad (3.26)$$

Energy equation:

$$\frac{\partial \mathcal{G}}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \mathcal{G}}{\partial \eta} = \left(\frac{1}{P_r} + \frac{4}{3EP_r} \right) \frac{\partial^2 \mathcal{G}}{\partial \eta^2} - \phi \mathcal{G} + D_p \frac{\partial^2 \zeta}{\partial \eta^2} \quad (3.27)$$

Concentration equation:

$$\frac{\partial \zeta}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \zeta}{\partial \eta} = \frac{1}{S_c} \frac{\partial^2 \zeta}{\partial \eta^2} - K_r \zeta + S_p \frac{\partial^2 \mathcal{G}}{\partial \eta^2} \quad (3.28)$$

Taking $\psi = \xi + iv$ equations (3.25) and (3.26) become

$$\begin{aligned} \frac{\partial \psi}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \psi}{\partial \eta} &= H \varepsilon \cos \omega t + \frac{\partial^2 \psi}{\partial \eta^2} - (2i\Omega + F)\psi + G_h \mathcal{G} + \\ G_m \zeta - V_R \left(\frac{\partial^3 \psi}{\partial t \partial \eta^2} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial^3 \psi}{\partial \eta^3} \right) \end{aligned} \quad (3.29)$$

where

$$F = M + \frac{1}{k_p}$$

Likewise, the corresponding boundary conditions are:

$$\left. \begin{aligned} \psi &= 0, \mathcal{G} = 1 + \frac{\varepsilon}{2} (e^{i\omega t} + e^{-i\omega t}), \zeta = 1 + \frac{\varepsilon}{2} (e^{i\omega t} + e^{-i\omega t}) \\ \text{at } \eta &= 0 \\ \psi &= 1 + \frac{\varepsilon}{2} (e^{i\omega t} + e^{-i\omega t}), \mathcal{G} = 0, \zeta = 0 \quad \text{at } \eta = 1 \end{aligned} \right\} \quad (3.30)$$

3.1.3 Effects of Viscous Dissipation on Convective Rotatory Chemically Reacting Rivlin-Ericksen flow past a Porous Vertical Plate

3.1.3.1 Introduction

Dissipation resulted from an irreversible process that is evident in homogeneous thermodynamic system. The process of transformation of energy from some initial form to some final form where the capacity of the final form to perform mechanical work is less compare to initial form, is known as dissipative process. An example of dissipative process is heat transfer. That is, transfer of energy from hotter to colder body. The effect of dissipation on fluid flow is significant due to its application in controlling the rate of cooling. Thus, the effects of dissipation on Rivlin-Ericksen fluid flow is considered.

The following assumptions were made in the formulation of this problem:

- (i) An unsteady, laminar flow is considered.
- (ii) Induced magnetic field and Hall effects are ignored due to the fact that magnetic Reynolds number and transversely applied magnetic field was considered to be very small.
- (iii) Thermal-diffusion and diffusion-thermo are assumed to be of substantial magnitude, hence, they are not negligible.
- (iv) A magnetic field B_0 of uniform strength is perpendicularly applied to the plates.
- (v) There is a rotation of the entire system through the perpendicular axis to the plates.
- (vi) The plates are considered to be infinite in x -direction, hence all physical quantities excluding pressure are functions of z^* and t^* .
- (vii) In the flow field, pressure is taken to be constant.
- (viii) Finitely conducting fluid with constant physical properties is observed.

3.1.3.2 The Dimensional Governing Equations

Continuity equation:

$$\frac{\partial w^*}{\partial z^*} = 0 \quad (3.31)$$

Momentum equation:

$$\begin{aligned} \frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial z^*} = & -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + \nu \frac{\partial^2 u^*}{\partial z^{*2}} + 2\Omega^* v^* + g\beta_T(T^* - T_h^*) + g\beta_C(C^* - C_h^*) - \\ & \nu \frac{u^*}{k_p^*} - \frac{\sigma B_o^2 u^*}{\rho} - \beta_1 \left(\frac{\partial^3 u^*}{\partial t^* \partial z^{*2}} + v^* \frac{\partial^3 u^*}{\partial z^{*3}} \right) \end{aligned} \quad (3.32)$$

$$\frac{\partial v^*}{\partial t^*} + v^* \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} + \nu \frac{\partial^2 v^*}{\partial z^{*2}} - 2\Omega^* u^* - \nu \frac{v^*}{k_p^*} - \frac{\sigma B_o^2 v^*}{\rho} - \beta_1 \left(\frac{\partial^3 v^*}{\partial t^* \partial z^{*2}} + v^* \frac{\partial^3 v^*}{\partial z^{*3}} \right) \quad (3.33)$$

Energy equation:

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial z^*} = \alpha \frac{\partial^2 T^*}{\partial z^{*2}} - \frac{\phi_o}{\rho c_p} (T^* - T_h^*) - \frac{1}{\rho C_p} \frac{\partial q_R}{\partial z^*} + \frac{DK_T \partial^2 C^*}{C_p C_s \partial z^{*2}} + \frac{\nu}{c_p} \left(\left(\frac{\partial u^*}{\partial z^*} \right)^2 + \left(\frac{\partial v^*}{\partial z^*} \right)^2 \right) \quad (3.34)$$

Concentration equation:

$$\frac{\partial C^*}{\partial t^*} + v^* \frac{\partial C^*}{\partial z^*} = D \frac{\partial^2 C^*}{\partial z^{*2}} - K_1^* (C^* - C_h^*) + \frac{DK_T}{T_m} \frac{\partial^2 T^*}{\partial z^{*2}} \quad (3.35)$$

The boundary conditions for the problem are:

$$\left. \begin{aligned} z^* = 0; u^* = v^* = 0, T^* = T_h + \varepsilon (T_0 - T_h) \cos \varpi^* t^*, \\ C^* = C_h + \varepsilon (C_0 - C_h) \cos \varpi^* t^* \\ z^* = h; u^* = W_o (1 + \varepsilon \cos \varpi^* t^*), v^* = 0, T^* = T_h, C^* = C_h \end{aligned} \right\} \quad (3.36)$$

3.1.3.3 Non-dimensional Variables

The following non-dimensional variables are used to reduce the governing equations to non-dimensional form.

$$\left. \begin{aligned} \xi = \frac{u^*}{W_o}, v = \frac{v^*}{W_o}, t = \frac{t^* W_o^2}{\nu}, \eta = \frac{W_o z^*}{\nu} = \frac{z^*}{h}, k_p = \frac{k_p^* W_o^2}{\nu^2}, \\ g = \frac{T^* - T_h^*}{T_0^* - T_h^*}, \varpi = \frac{\varpi^* \nu}{W_o^2}, P_r = \frac{\nu \rho C_p}{k_p} = \frac{\nu}{\alpha}, M = \frac{\sigma B_0^2 \nu}{\rho W_o^2}, x = \frac{W_o x^*}{\nu}, \\ G_h = \frac{\nu g \beta_T (T_0^* - T_h^*)}{W_o^3}, \phi = \frac{\nu \phi_o}{\rho C_p W_o^2}, V_R = \frac{\beta_1 W_o^2}{\nu^2}, G_m = \frac{\nu g \beta_c (C_0^* - C_h^*)}{W_o^3}, \\ y = \frac{W_o y^*}{\nu}, P = \frac{P^*}{\rho W_o^2}, \zeta = \frac{C^* - C_h}{C_0^* - C_h^*}, S_c = \frac{\nu}{D}, E = \frac{k' k_p}{4 \sigma T_h^{*3}}, \\ \Omega = \frac{\Omega^* \nu}{W_o^2}, K_r = \frac{K_1 \nu}{W_o^2}, E_c = \frac{W_o^2}{C_p (T_0^* - T_h^*)}, S_p = \frac{DK_T (T_0^* - T_h^*)}{\nu T_m (C_0^* - C_h^*)}, \\ D_p = \frac{DK_T (C_0^* - C_h^*)}{C_s C_p \nu (T_0^* - T_h^*)} \end{aligned} \right\} \quad (3.37)$$

3.1.3.4 Non-dimensional Governing Equations

Using the non-dimensional variables (3.37) in equations (3.32)-(3.35) including the boundary conditions. The equations become

Momentum Equation:

$$\frac{\partial \xi}{\partial t} - \left(1 + \varepsilon A e^{i \varpi t} \right) \frac{\partial \xi}{\partial \eta} = H \varepsilon \cos \varpi t + \frac{\partial^2 \xi}{\partial \eta^2} + 2 \Omega v - \left(M + \frac{1}{k_p} \right) \xi + G_h g +$$

$$G_m \zeta - V_R \left(\frac{\partial^3 \xi}{\partial t \partial \eta^2} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial^3 \xi}{\partial \eta^3} \right) \quad (3.38)$$

$$\frac{\partial v}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial v}{\partial \eta} = \frac{\partial^2 v}{\partial \eta^2} - 2\Omega \xi - (M + \frac{1}{k_p})v - V_R \left(\frac{\partial^3 v}{\partial t \partial \eta^2} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial^3 v}{\partial \eta^3} \right) \quad (3.39)$$

Energy equation:

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \theta}{\partial \eta} = \left(\frac{1}{P_r} + \frac{4}{3EP_r} \right) \frac{\partial^2 \theta}{\partial \eta^2} - \phi \theta + D_p \frac{\partial^2 \zeta}{\partial \eta^2} + E_c \left(\left(\frac{\partial \xi}{\partial \eta} \right)^2 + \left(\frac{\partial v}{\partial \eta} \right)^2 \right) \quad (3.40)$$

Concentration equation:

$$\frac{\partial \zeta}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \zeta}{\partial \eta} = \frac{1}{S_c} \frac{\partial^2 \zeta}{\partial \eta^2} - K_r \zeta + S_p \frac{\partial^2 \theta}{\partial \eta^2} \quad (3.41)$$

Taking $\psi = \xi + iv$ equations (3.38) and (3.39) become

$$\begin{aligned} \frac{\partial \psi}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \psi}{\partial \eta} &= H \varepsilon \cos \omega t + \frac{\partial^2 \psi}{\partial \eta^2} - (2i\Omega + F)\psi + G_h \theta + \\ G_m \zeta - V_R \left(\frac{\partial^3 \psi}{\partial t \partial \eta^2} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial^3 \psi}{\partial \eta^3} \right) \end{aligned} \quad (3.42)$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \theta}{\partial \eta} = \left(\frac{1}{P_r} + \frac{4}{3EP_r} \right) \frac{\partial^2 \theta}{\partial \eta^2} - \phi \theta + D_p \frac{\partial^2 \zeta}{\partial \eta^2} + E_c \frac{\partial \psi}{\partial \eta} \frac{\partial \bar{\psi}}{\partial \eta} \quad (3.43)$$

where

$$F = M + \frac{1}{k_p}$$

The boundary conditions are:

$$\left. \begin{aligned} \psi &= 0, \theta = 1 + \frac{\varepsilon}{2} (e^{i\omega t} + e^{-i\omega t}), \zeta = 1 + \frac{\varepsilon}{2} (e^{i\omega t} + e^{-i\omega t}) \\ \text{at } \eta &= 0 \\ \psi &= 1 + \frac{\varepsilon}{2} (e^{i\omega t} + e^{-i\omega t}), \theta = 0, \zeta = 0 \text{ at } \eta = 1 \end{aligned} \right\} \quad (3.44)$$

3.2 Method of Solution

3.2.1 Numerical Computation of Research Problem 1

3.2.1.1 Perturbation Method

The partial differential equations (3.13)-(3.15) are reduced to ordinary differential equations by Perturbation technique and analytical solutions were obtained for the resulting equations. Based on the nature of the boundary conditions, the following set of assumed solutions were used. (Garg et al., 2014b)

$$\left. \begin{aligned} \psi(\eta, t) &= \psi_0(\eta) + \frac{\varepsilon}{2} (\psi_1(\eta)e^{i\omega t} + \psi_2(\eta)e^{-i\omega t}) \\ \vartheta(\eta, t) &= \vartheta_0(\eta) + \frac{\varepsilon}{2} (\vartheta_1(\eta)e^{i\omega t} + \vartheta_2(\eta)e^{-i\omega t}) \\ \zeta(\eta, t) &= \zeta_0(\eta) + \frac{\varepsilon}{2} (\zeta_1(\eta)e^{i\omega t} + \zeta_2(\eta)e^{-i\omega t}) \end{aligned} \right\} \quad (3.45)$$

Substituting equations (3.45) into equations (3.13)-(3.15), after expansion, gives:

$$\begin{aligned} i\omega \frac{\varepsilon}{2} \psi_1(\eta)e^{i\omega t} - i\omega \frac{\varepsilon}{2} \psi_2(\eta)e^{-i\omega t} - (1 + \varepsilon A e^{i\omega t})(\psi_0'(\eta) + \frac{\varepsilon}{2} \psi_1'(\eta)e^{i\omega t} + \frac{\varepsilon}{2} \psi_2'(\eta)e^{-i\omega t}) = \\ H \frac{\varepsilon}{2} (e^{i\omega t} + e^{-i\omega t}) + (\psi_0''(\eta) + \frac{\varepsilon}{2} \psi_1''(\eta)e^{i\omega t} + \frac{\varepsilon}{2} \psi_2''(\eta)e^{-i\omega t}) - \\ (2i\Omega + F)(\psi_0(\eta) + \frac{\varepsilon}{2} \psi_1(\eta)e^{i\omega t} + \frac{\varepsilon}{2} \psi_2(\eta)e^{-i\omega t}) + G_h(\vartheta_0(\eta) + \frac{\varepsilon}{2} \vartheta_1(\eta)e^{i\omega t} + \frac{\varepsilon}{2} \vartheta_2(\eta)e^{-i\omega t}) + \\ G_m(\zeta_0(\eta) + \frac{\varepsilon}{2} \zeta_1(\eta)e^{i\omega t} + \frac{\varepsilon}{2} \zeta_2(\eta)e^{-i\omega t}) - V_R(i\omega \frac{\varepsilon}{2} \psi_1''(\eta)e^{i\omega t} - \\ i\omega \frac{\varepsilon}{2} \psi_2''(\eta)e^{-i\omega t} - (1 + \varepsilon A e^{i\omega t})(\psi_0'''(\eta) + \frac{\varepsilon}{2} \psi_1'''(\eta)e^{i\omega t} + \frac{\varepsilon}{2} \psi_2'''(\eta)e^{-i\omega t})) \end{aligned} \quad (3.46)$$

$$\begin{aligned} i\omega \frac{\varepsilon}{2} \vartheta_1 e^{i\omega t} - i\omega \frac{\varepsilon}{2} \vartheta_2 e^{-i\omega t} - (1 + \varepsilon A e^{i\omega t})(\vartheta_0' + \frac{\varepsilon}{2} \vartheta_1' e^{i\omega t} + \frac{\varepsilon}{2} \vartheta_2' e^{-i\omega t}) = \\ B(\vartheta_0'' + \frac{\varepsilon}{2} \vartheta_1'' e^{i\omega t} + \frac{\varepsilon}{2} \vartheta_2'' e^{-i\omega t}) - \phi(\vartheta_0 + \frac{\varepsilon}{2} \vartheta_1 e^{i\omega t} + \frac{\varepsilon}{2} \vartheta_2 e^{-i\omega t}) \end{aligned} \quad (3.47)$$

$$\begin{aligned} i\omega \frac{\varepsilon}{2} \zeta_1 e^{i\omega t} - i\omega \frac{\varepsilon}{2} \zeta_2 e^{-i\omega t} - (1 + \varepsilon A e^{i\omega t})(\zeta_0' + \frac{\varepsilon}{2} \zeta_1' e^{i\omega t} + \frac{\varepsilon}{2} \zeta_2' e^{-i\omega t}) = \\ \frac{1}{S_c}(\zeta_0'' + \frac{\varepsilon}{2} \zeta_1'' e^{i\omega t} + \frac{\varepsilon}{2} \zeta_2'' e^{-i\omega t}) - K_r(\zeta_0 + \frac{\varepsilon}{2} \zeta_1 e^{i\omega t} + \frac{\varepsilon}{2} \zeta_2 e^{-i\omega t}) \end{aligned} \quad (3.48)$$

From equations (3.46), (3.47) and (3.48) the following set of equations were obtained.

$$V_R \psi_0''' + \psi_0'' + \psi_0' - (2i\Omega + F)\psi_0 = -G_h \mathcal{G}_0 - G_m \zeta_0 \quad (3.49)$$

$$V_R \psi_1''' + (1 - V_R i \varpi) \psi_1'' + \psi_1' - (i \varpi + 2i\Omega + F) \psi_1 = -H - 2A \psi_0' - 2V_R A \psi_0''' - G_h \mathcal{G}_1 - G_m \zeta_1 \quad (3.50)$$

$$V_R \psi_2''' + (1 + V_R i \varpi) \psi_2'' + \psi_2' - (2i\Omega + F - i \varpi) \psi_2 = -H - G_h \mathcal{G}_2 - G_m \zeta_2 \quad (3.51)$$

$$B \mathcal{G}_0'' + \mathcal{G}_0' - \phi \mathcal{G}_0 = 0 \quad (3.52)$$

$$B \mathcal{G}_1'' + \mathcal{G}_1' - (\phi + i \varpi) \mathcal{G}_1 = -2A \mathcal{G}_0' \quad (3.53)$$

$$B \mathcal{G}_2'' + \mathcal{G}_2' - (\phi - i \varpi) \mathcal{G}_2 = 0 \quad (3.54)$$

$$\zeta_0'' + S_c \zeta_0' - S_c K_r \zeta_0 = 0 \quad (3.55)$$

$$\zeta_1'' + S_c \zeta_1' - S_c (K_r + i \varpi) \zeta_1 = -2A S_c \zeta_0' \quad (3.56)$$

$$\zeta_2'' + S_c \zeta_2' - S_c (K_r - i \varpi) \zeta_2 = 0 \quad (3.57)$$

where

$$B = \frac{1}{P_r} + \frac{4}{3EP_r}$$

The corresponding boundary conditions are:

$$\left. \begin{aligned} \psi_0 = \psi_1 = \psi_2 = 0, \mathcal{G}_0 = \mathcal{G}_1 = \mathcal{G}_2 = 1, \zeta_0 = \zeta_1 = \zeta_2 = 1 \text{ at } \eta = 0 \\ \psi_0 = \psi_1 = \psi_2 = 1, \mathcal{G}_0 = \mathcal{G}_1 = \mathcal{G}_2 = 0, \zeta_0 = \zeta_1 = \zeta_2 = 0 \text{ at } \eta = 1 \end{aligned} \right\} \quad (3.58)$$

Equations (3.52)-(3.57) are solved analytically with the boundary conditions (3.58). The root of the auxiliary equation for equation (3.52) and the complementary function, is obtained by assuming

$$\mathcal{G}_0 = e^{b\eta} \quad (3.59)$$

The solution to equation (3.52) is

$$\mathcal{G}_0 = g_1 e^{b_1 \eta} + g_2 e^{b_2 \eta} \quad (3.60)$$

where

$$b_1 = \frac{-1 + \sqrt{1 + 4BQ}}{2B} \text{ and } b_2 = \frac{-1 - \sqrt{1 + 4BQ}}{2B}$$

g_1 and g_2 are constants to be determined. Using the boundary conditions (3.58),

$$g_1 = \frac{-e^{b_2 - b_1}}{1 - e^{b_2 - b_1}} \text{ and } g_2 = \frac{1}{1 - e^{b_2 - b_1}}$$

Therefore, equation (3.60) reduces to

$$\mathcal{G}_0 = \frac{e^{b_1+b_2\eta} - e^{b_2+b_1\eta}}{e^{b_1} - e^{b_2}} \quad (3.61)$$

Solution of the homogeneous part of equation (3.53) is

$$\mathcal{G}_1 = g_3 e^{b_3\eta} + g_4 e^{b_4\eta} \quad (3.62)$$

where

$$b_3 = \frac{-1 + \sqrt{1 + 4B(i\varpi + \phi)}}{2B} \text{ and } b_4 = \frac{-1 - \sqrt{1 + 4B(i\varpi + \phi)}}{2B}$$

The particular integral solution is

$$\mathcal{G}_1 = k_1 e^{b_2\eta} + k_2 e^{b_1\eta} \quad (3.63)$$

Substituting for $\mathcal{G}_1$, $\mathcal{G}_1'$ and $\mathcal{G}_1''$ in equation (3.53) and equating their coefficients, gives;

$$k_1 = \frac{-2Ab_2 e^{b_1}}{(e^{b_1} - e^{b_2})(Bb_2^2 + b_2 - (\phi + i\varpi))}, k_2 = \frac{-2Ab_1 e^{b_2}}{(e^{b_1} - e^{b_2})(Bb_1^2 + b_1 - (\phi + i\varpi))}$$

The complete solution of equation (3.53) is obtained by adding equations (3.62) and (3.63) to have

$$\mathcal{G}_1 = g_3 e^{b_3\eta} + g_4 e^{b_4\eta} + k_1 e^{b_2\eta} + k_2 e^{b_1\eta} \quad (3.64)$$

Applying the boundary conditions (3.58) gives the final solution

$$\mathcal{G}_1 = \frac{e^{b_3\eta} - e^{b_4\eta}}{e^{b_4} - e^{b_3}} \left(e^{b_3} - k_1(e^{b_3} - e^{b_2}) - k_2(e^{b_2} - e^{b_1}) \right) + e^{b_3\eta} - k_1(e^{b_3\eta} - e^{b_2\eta}) - k_2(e^{b_3\eta} - e^{b_1\eta}) \quad (3.65)$$

Similarly, solutions to equations (3.54)-(3.57) along with the boundary conditions are given in equations (3.66)- (3.69) below

$$\mathcal{G}_2 = \frac{e^{b_5+b_6\eta} - e^{b_6+b_5\eta}}{e^{b_5} - e^{b_6}} \quad (3.66)$$

$$\zeta_0 = \frac{e^{b_7+b_8\eta} - e^{b_8+b_7\eta}}{e^{b_7} - e^{b_8}} \quad (3.67)$$

$$\zeta_1 = \frac{e^{b_9\eta} - e^{b_{10}\eta}}{e^{b_{10}} - e^{b_9}} \left(e^{b_9} - k_3(e^{b_9} - e^{b_8}) - k_4(e^{b_9} - e^{b_7}) \right) + e^{b_9\eta} \quad (3.68)$$

$$-k_3(e^{b_9\eta} - e^{b_8\eta}) - k_4(e^{b_9\eta} - e^{b_7\eta})$$

$$\zeta_2 = \frac{e^{b_{11}+b_{12}\eta} - e^{b_{12}+b_{11}\eta}}{e^{b_{11}} - e^{b_{12}}} \quad (3.69)$$

where

$$b_5 = \frac{-1 + \sqrt{1 + 4B(\phi - i\varpi)}}{2B}, b_6 = \frac{-1 - \sqrt{1 + 4B(\phi - i\varpi)}}{2B},$$

$$b_7 = \frac{-S_c + \sqrt{S_c^2 + 4S_c K_r}}{2}, b_8 = \frac{-S_c - \sqrt{S_c^2 + 4S_c K_r}}{2},$$

$$b_9 = \frac{-S_c + \sqrt{S_c^2 + 4S_c(K_r + i\varpi)}}{2}, b_{10} = \frac{-S_c - \sqrt{S_c^2 + 4S_c(K_r + i\varpi)}}{2},$$

$$b_{11} = \frac{-S_c + \sqrt{S_c^2 + 4S_c(K_r - i\varpi)}}{2}, b_{12} = \frac{-S_c - \sqrt{S_c^2 + 4S_c(K_r - i\varpi)}}{2},$$

$$k_3 = \frac{-2AS_c b_8 e^{b_7}}{(e^{b_7} - e^{b_8})(b_8^2 + S_c b_8 - S_c(K_r + i\varpi))},$$

$$k_4 = \frac{2AS_c b_7 e^{b_8}}{(e^{b_7} - e^{b_8})(b_7^2 + S_c b_7 - S_c(K_r + i\varpi))}$$

3.2.1.2 Further Simplification of the third order differential equations

The solutions of equations (3.49)-(3.51) are not feasible in the present forms since they are third order differential equations with only two boundary conditions. In order to derive sufficient and necessary boundary conditions, according to Beard and Walters (1964) and Garg et al.(2014b), the solutions of equations (3.49)-(3.51) are expressed in the forms:

$$\left. \begin{aligned} \psi_0(\eta) &= \psi_{01}(\eta) + V_R \psi_{02}(\eta) + 0(V_R^2) \\ \psi_1(\eta) &= \psi_{11}(\eta) + V_R \psi_{12}(\eta) + 0(V_R^2) \\ \psi_2(\eta) &= \psi_{21}(\eta) + V_R \psi_{22}(\eta) + 0(V_R^2) \end{aligned} \right\} \quad (3.70)$$

Introducing equation (3.70) to equations (3.49)-(3.51), the equations become:

$$\psi_{01}'' + \psi_{01}' - (2i\Omega + F)\psi_{01} = -G_h \vartheta_0 - G_m \zeta_0 \quad (3.71)$$

$$\psi_{02}'' + \psi_{02}' - (2i\Omega + F)\psi_{02} = -\psi_{01}''' \quad (3.72)$$

$$\psi_{11}'' + \psi_{11}' - (i\varpi + 2i\Omega + F)\psi_{11} = -H - 2A\psi_{01}' - G_h \vartheta_1 - G_m \zeta_1 \quad (3.73)$$

$$\psi_{12}'' + \psi_{12}' - (i\varpi + 2i\Omega + F)\psi_{12} = i\varpi \psi_{11}'' - \psi_{11}''' - 2A\psi_{01}''' - 2A\psi_{02}' \quad (3.74)$$

$$\psi_{21}'' + \psi_{21}' - (2i\Omega + F - i\varpi)\psi_{21} = -H - G_h \vartheta_2 - G_m \zeta_2 \quad (3.75)$$

$$\psi_{22}'' + \psi_{22}' - (2i\Omega + F - i\varpi)\psi_{22} = -\psi_{21}''' - i\varpi \psi_{21}'' \quad (3.76)$$

with the following boundary conditions:

$$\left. \begin{aligned} \psi_{01} = \psi_{02} = \psi_{11} = \psi_{12} = \psi_{21} = \psi_{22} = 0 \text{ at } \eta = 0 \\ \psi_{01} = \psi_{11} = \psi_{21} = 1, \psi_{02} = \psi_{12} = \psi_{22} = 0 \text{ at } \eta = 1 \end{aligned} \right\} \quad (3.77)$$

Substituting $\mathcal{G}_0$ and ζ_0 into equation (3.71) gives

$$\psi_{01}'' + \psi_{01}' - 2i\Omega + F\psi_{01} = -G_h \left(\frac{e^{b_1+b_2\eta} - e^{b_2+b_1\eta}}{e^{b_1} - e^{b_2}} \right) - \quad (3.78)$$

$$G_m \left(\frac{e^{b_7+b_8\eta} - e^{b_8+b_7\eta}}{e^{b_7} - e^{b_8}} \right)$$

The complementary function of equation (3.78) is

$$\psi_{01} = g_{13}e^{b_{13}\eta} + g_{14}e^{b_{14}\eta} \quad (3.79)$$

where,

$$b_{13} = \frac{-1 + \sqrt{1 + 4(2i\Omega + F)}}{2}, b_{14} = \frac{-1 - \sqrt{1 + 4(2i\Omega + F)}}{2}$$

g_{13} and g_{14} are constants to be determined. Let the particular integral solution be,

$$\psi_{01} = k_5e^{b_1\eta} + k_6e^{b_2\eta} + k_7e^{b_7\eta} + k_8e^{b_8\eta} \quad (3.80)$$

Substituting for ψ_{01} , ψ_{01}' and ψ_{01}'' in equation (3.78) and equating their coefficients, resulted to

$$k_5 = \frac{-Gre^{b_1}}{(e^{b_1} - e^{b_2})(b_2^2 + b_2 - (2i\Omega + F))},$$

$$k_6 = \frac{Gre^{b_2}}{(e^{b_1} - e^{b_2})(b_1^2 + b_1 - (2i\Omega + F))}, k_7 = \frac{-Gme^{b_7}}{(e^{b_7} - e^{b_8})(b_8^2 + b_8 - (2i\Omega + F))},$$

and,

$$k_8 = \frac{Gme^{b_8}}{(e^{b_7} - e^{b_8})(b_7^2 + b_7 - (2i\Omega + F))}$$

Hence, complete solution of equation (3.71) is the sum of equations (3.79) and (3.80), which gives

$$\psi_{01} = g_{13}e^{b_{13}\eta} + g_{14}e^{b_{14}\eta} + k_5e^{b_1\eta} + k_6e^{b_2\eta} + k_7e^{b_7\eta} + k_8e^{b_8\eta} \quad (3.81)$$

Applying the boundary conditions equation (3.77) to equation (3.81) gives

$$g_{13} = -(g_{14} + k_5 + k_6 + k_7 + k_8) \text{ and } g_{14} = \frac{1}{e^{b_{14}} - e^{b_{13}}} (1 + k_5(e^{b_{13}} - e^{b_2}) +$$

$$k_6(e^{b_{13}} - e^{b_1}) + k_7(e^{b_{13}} - e^{b_8}) + k_8(e^{b_{13}} - e^{b_7}))$$

Similarly, solutions to equations (3.72)-(3.76) along with the boundary conditions (3.77) are as follow:

$$\psi_{02} = g_{15}e^{b_{15}\eta} + g_{16}e^{b_{16}\eta} + k_9e^{b_{13}\eta} + k_{10}e^{b_{14}\eta} + k_{11}e^{b_{12}\eta} + k_{12}e^{b_{11}\eta} + k_{13}e^{b_8\eta} + k_{14}e^{b_7\eta} \quad (3.82)$$

$$\begin{aligned} \psi_{11} = & g_{17}e^{b_{17}\eta} + g_{18}e^{b_{18}\eta} + k_{15} + k_{16}e^{b_{13}\eta} + k_{17}e^{b_{14}\eta} + k_{18}e^{b_{12}\eta} + k_{19}e^{b_{11}\eta} + k_{20}e^{b_8\eta} + \\ & k_{21}e^{b_7\eta} + k_{22}e^{b_3\eta} + k_{23}e^{b_4\eta} + k_{24}e^{b_9\eta} + k_{25}e^{b_{10}\eta} \end{aligned} \quad (3.83)$$

$$\begin{aligned} \psi_{12} = & g_{19}e^{b_{19}\eta} + g_{20}e^{b_{20}\eta} + k_{26}e^{b_{13}\eta} + k_{27}e^{b_{14}\eta} + k_{28}e^{b_{12}\eta} + k_{29}e^{b_{11}\eta} + \\ & k_{30}e^{b_8\eta} + k_{31}e^{b_7\eta} + k_{32}e^{b_3\eta} + k_{33}e^{b_4\eta} + k_{34}e^{b_9\eta} + k_{35}e^{b_{10}\eta} \end{aligned} \quad (3.84)$$

$$\psi_{21} = g_{21}e^{b_{21}\eta} + g_{22}e^{b_{22}\eta} + k_{36}e^{b_6\eta} + k_{37}e^{b_5\eta} + k_{38}e^{b_{12}\eta} + k_{39}e^{b_{11}\eta} + k_{40} \quad (3.85)$$

$$\psi_{22} = g_{23}e^{b_{23}\eta} + g_{24}e^{b_{24}\eta} + k_{41}e^{b_{21}\eta} + k_{42}e^{b_{22}\eta} + k_{43}e^{b_6\eta} + k_{44}e^{b_5\eta} + k_{45}e^{b_{12}\eta} + k_{46}e^{b_{11}\eta} \quad (3.86)$$

where,

$$b_{15} = b_{13}, b_{16} = b_{14}, b_{17} = \frac{-1 + \sqrt{1 + 4(i\varpi + 2i\Omega + F)}}{2},$$

$$b_{18} = \frac{-1 - \sqrt{1 + 4(i\varpi + 2i\Omega + F)}}{2}, b_{19} = b_{17}, b_{20} = b_{18},$$

$$b_{21} = \frac{-1 - \sqrt{1 + 4(2i\Omega + F - i\varpi)}}{2}, b_{22} = \frac{-1 - \sqrt{1 + 4(2i\Omega + F - i\varpi)}}{2},$$

$$b_{23} = b_{21}, b_{24} = b_{22}, k_9 = \frac{-g_{13}b_{13}^3}{b_{13}^2 + b_{13} - (2i\Omega + F)}, k_{10} = \frac{-g_{14}b_{14}^3}{b_{14}^2 + b_{14} - (2i\Omega + F)},$$

$$k_{11} = \frac{-k_5b_2^3}{b_2^2 + b_2 - (2i\Omega + F)}, k_{12} = \frac{-k_6b_1^3}{b_1^2 + b_1 - (2i\Omega + F)},$$

$$k_{13} = \frac{-k_7b_8^3}{b_8^2 + b_8 - (2i\Omega + F)}, k_{14} = \frac{-k_8b_7^3}{b_7^2 + b_7 - (2i\Omega + F)}, k_{15} = -H,$$

$$k_{16} = \frac{-2Ag_{13}b_{13}}{b_{13}^2 + b_{13} - (i\varpi + 2i\Omega + F)}, k_{17} = \frac{-2Ag_{14}b_{14}}{b_{14}^2 + b_{14} - (i\varpi + 2i\Omega + F)},$$

$$k_{18} = \frac{-(2Ak_6b_2 + G_rk_1)}{b_2^2 + b_2 - (i\varpi + 2i\Omega + F)}, k_{19} = \frac{-(2Ak_7b_1 + G_rk_2)}{b_1^2 + b_1 - (i\varpi + 2i\Omega + F)},$$

$$k_{20} = \frac{-(2Ak_8b_8 + G_mk_3)}{b_8^2 + b_8 - (i\varpi + 2i\Omega + F)}, k_{21} = \frac{-(2Ak_9b_7 + G_mk_4)}{b_7^2 + b_7 - (i\varpi + 2i\Omega + F)},$$

$$k_{22} = \frac{-G_rg_3}{b_3^2 + b_3 - (i\varpi + 2i\Omega + F)}, k_{23} = \frac{-G_rg_4}{b_4^2 + b_4 - (i\varpi + 2i\Omega + F)},$$

$$k_{24} = \frac{-G_mg_9}{b_9^2 + b_9 - (i\varpi + 2i\Omega + F)}, k_{25} = \frac{-G_mg_{10}}{b_{10}^2 + b_{10} - (i\varpi + 2i\Omega + F)},$$

$$k_{26} = \frac{-2Ag_{13}b_{13}^3 + i\varpi k_{16}b_{13}^2 - k_{16}b_{13}^3}{b_{13}^2 + b_{13} - (i\varpi + 2i\Omega + F)}, k_{27} = \frac{-2Ag_{14}b_{14}^3 + i\varpi k_{17}b_{14}^2 - k_{17}b_{14}^3}{b_{14}^2 + b_{14} - (i\varpi + 2i\Omega + F)},$$

$$k_{28} = \frac{-2Ak_6b_2^3 + i\varpi k_{18}b_2^2 - k_{18}b_2^3}{b_2^2 + b_2 - (i\varpi + 2i\Omega + F)}, k_{29} = \frac{-2Ak_7b_1^3 + i\varpi k_{19}b_1^2 - k_{19}b_1^3}{b_1^2 + b_1 - (i\varpi + 2i\Omega + F)},$$

$$k_{30} = \frac{-2Ak_8b_8^3 + i\varpi k_{20}b_8^2 - k_{20}b_8^3}{b_8^2 + b_8 - (i\varpi + 2i\Omega + F)}, k_{31} = \frac{-2Ak_9b_7^3 + i\varpi k_{21}b_7^2 - k_{21}b_7^3}{b_7^2 + b_7 - (i\varpi + 2i\Omega + F)},$$

$$k_{32} = \frac{i\varpi k_{22}b_3^2 - k_{22}b_3^3}{b_3^2 + b_3 - (i\varpi + 2i\Omega + F)}, k_{33} = \frac{i\varpi k_{23}b_4^2 - k_{23}b_4^3}{b_4^2 + b_4 - (i\varpi + 2i\Omega + F)},$$

$$k_{34} = \frac{i\varpi k_{24}b_9^2 - k_{24}b_9^3}{b_9^2 + b_9 - (i\varpi + 2i\Omega + F)}, k_{35} = \frac{i\varpi k_{25}b_{10}^2 - k_{25}b_{10}^3}{b_{10}^2 + b_{10} - (i\varpi + 2i\Omega + F)},$$

$$k_{36} = \frac{-G_re^{b_5}}{(e^{b_5} - e^{b_6})(b_6^2 + b_6 - (2i\Omega + F - i\varpi))},$$

$$k_{37} = \frac{G_re^{b_6}}{(e^{b_5} - e^{b_6})(b_5^2 + b_5 - (2i\Omega + F - i\varpi))},$$

$$k_{38} = \frac{-G_me^{b_{11}}}{(e^{b_{11}} - e^{b_{12}})(b_{12}^2 + b_{12} - (2i\Omega + F - i\varpi))},$$

$$k_{39} = \frac{G_me^{b_{12}}}{(e^{b_{11}} - e^{b_{12}})(b_{11}^2 + b_{11} - (2i\Omega + F - i\varpi))}, k_{40} = -H,$$

$$k_{41} = \frac{-g_{21}b_{21}^2(b_{21}+i\varpi)}{b_{21}^2+b_{21}-(2i\Omega+F-i\varpi)}, k_{42} = \frac{-g_{22}b_{22}^2(b_{22}+i\varpi)}{b_{22}^2+b_{22}-(2i\Omega+F-i\varpi)},$$

$$k_{43} = \frac{-k_{36}b_6^2(b_6+i\varpi)}{b_6^2+b_6-(2i\Omega+F-i\varpi)}, k_{44} = \frac{-k_{37}b_5^2(b_5+i\varpi)}{b_5^2+b_5-(2i\Omega+F-i\varpi)},$$

$$k_{45} = \frac{-k_{38}b_{12}^2(b_{12}+i\varpi)}{b_{12}^2+b_{12}-(2i\Omega+F-i\varpi)}, k_{46} = \frac{-k_{39}b_{11}^2(b_{11}+i\varpi)}{b_{11}^2+b_{11}-(2i\Omega+F-i\varpi)},$$

$$g_{15} = -(g_{16}+k_{10}+k_{11}+k_{12}+k_{13}+k_{14}+k_{15}),$$

$$g_{16} = \frac{1}{e^{b_{16}}-e^{b_{15}}}(k_{10}(e^{b_{15}}-e^{b_{13}})+k_{11}(e^{b_{15}}-e^{b_{14}})+k_{12}(e^{b_{15}}-e^{b_2})+)$$

$$k_{13}(e^{b_{15}}-e^{b_1})+k_{14}(e^{b_{15}}-e^{b_8})+k_{15}(e^{b_{15}}-e^{b_7})),$$

$$g_{17} = -(g_{18}+k_{15}+k_{16}+k_{17}+k_{18}+k_{19}+k_{20}+k_{21}+k_{22}+k_{23}+k_{24}+k_{25}),$$

$$g_{18} = \frac{1}{e^{b_{18}}-e^{b_{17}}}(1+k_{15}(e^{b_{17}}-1)+k_{16}(e^{b_{17}}-e^{b_{13}})+k_{17}(e^{b_{17}}-e^{b_{14}})+k_{18}(e^{b_{17}}-e^{b_2})+)$$

$$k_{19}(e^{b_{17}}-e^{b_1})+k_{20}(e^{b_{17}}-e^{b_8})+k_{21}(e^{b_{17}}-e^{b_7})+k_{22}(e^{b_{17}}-e^{b_3})+k_{23}(e^{b_{17}}-e^{b_4})+)$$

$$k_{24}(e^{b_{17}}-e^{b_9})+k_{25}(e^{b_{17}}-e^{b_{10}})),$$

$$g_{19} = -(g_{20}+k_{26}+k_{27}+k_{28}+k_{29}+k_{30}+k_{31}+k_{32}+k_{33}+k_{34}+k_{35}),$$

$$g_{20} = \frac{1}{e^{b_{20}}-e^{b_{19}}}(k_{26}(e^{b_{19}}-e^{b_{13}})+k_{27}(e^{b_{19}}-e^{b_{14}})+k_{28}(e^{b_{19}}-e^{b_2})+k_{29}(e^{b_{19}}-e^{b_1})$$

$$+k_{30}(e^{b_{19}}-e^{b_8})+k_{31}(e^{b_{19}}-e^{b_7})+k_{32}(e^{b_{19}}-e^{b_3})+k_{33}(e^{b_{19}}-e^{b_4})+k_{34}(e^{b_{19}}-e^{b_9})+)$$

$$k_{35}(e^{b_{19}}-e^{b_{10}})), g_{21} = -(g_{22}+k_{36}+k_{37}+k_{38}+k_{39}+k_{40}),$$

$$g_{22} = \frac{1}{e^{b_{22}}-e^{b_{21}}}(1+k_{36}(e^{b_{21}}-e^{b_6})+k_{37}(e^{b_{21}}-e^{b_5})+k_{38}(e^{b_{21}}-e^{b_{12}})+)$$

$$k_{39}(e^{b_{21}}-e^{b_{11}})+k_{40}(e^{b_{21}}-1)),$$

$$g_{23} = -(g_{24} + k_{41} + k_{42} + k_{43} + k_{44} + k_{45} + k_{46}) ,$$

$$g_{24} = \frac{1}{e^{b_{24}} - e^{b_{23}}} (k_{41}(e^{b_{23}} - e^{b_{21}}) + k_{42}(e^{b_{23}} - e^{b_{22}}) + k_{43}(e^{b_{23}} - e^{b_6}) + k_{44}(e^{b_{23}} - e^{b_5}) \\ + k_{45}(e^{b_{23}} - e^{b_{12}}) + k_{46}(e^{b_{23}} - e^{b_{11}}))$$

Therefore, using equations (3.45) and (3.70), the solutions of velocity, temperature and concentration are given as:

$$\begin{aligned} \psi(\eta, t) = & g_{13}e^{b_{13}\eta} + g_{14}e^{b_{14}\eta} + k_5e^{b_2\eta} + k_6e^{b_1\eta} + k_7e^{b_8\eta} + k_8e^{b_7\eta} + \\ & V_R(g_{15}e^{b_{15}\eta} + g_{16}e^{b_{16}\eta} + k_9e^{b_{13}\eta} + k_{10}e^{b_{14}\eta} + k_{11}e^{b_2\eta} + k_{12}e^{b_1\eta} + k_{13}e^{b_8\eta} + k_{14}e^{b_7\eta}) + \\ & \frac{\varepsilon}{2}((g_{17}e^{b_{17}\eta} + g_{18}e^{b_{18}\eta} + k_{15} + k_{16}e^{b_{13}\eta} + k_{17}e^{b_{14}\eta} + k_{18}e^{b_2\eta} + k_{19}e^{b_1\eta} + k_{20}e^{b_8\eta} + k_{21}e^{b_7\eta} + \\ & k_{22}e^{b_3\eta} + k_{23}e^{b_4\eta} + k_{24}e^{b_9\eta} + k_{25}e^{b_{10}\eta} + V_R(g_{19}e^{b_{19}\eta} + g_{20}e^{b_{20}\eta} + k_{26}e^{b_{13}\eta} + k_{27}e^{b_{14}\eta} + \\ & k_{28}e^{b_2\eta} + k_{29}e^{b_1\eta} + k_{30}e^{b_8\eta} + k_{31}e^{b_7\eta} + k_{32}e^{b_3\eta} + k_{33}e^{b_4\eta} + k_{34}e^{b_9\eta} + k_{35}e^{b_{10}\eta})) \\ & e^{i\pi\tau} + (g_{21}e^{b_{21}\eta} + g_{22}e^{b_{22}\eta} + k_{36}e^{b_6\eta} + k_{37}e^{b_5\eta} + k_{38}e^{b_{12}\eta} + k_{39}e^{b_{11}\eta} + k_{40} + V_R(g_{23}e^{b_{23}\eta} + \\ & g_{24}e^{b_{24}\eta} + k_{41}e^{b_{21}\eta} + k_{42}e^{b_{22}\eta} + k_{43}e^{b_6\eta} + k_{44}e^{b_5\eta} + k_{45}e^{b_{12}\eta} + k_{46}e^{b_{11}\eta}))e^{-i\pi\tau} \end{aligned} \quad (3.87)$$

$$\begin{aligned} \vartheta(\eta, t) = & \frac{e^{b_1+b_2\eta} - e^{b_2+b_1\eta}}{e^{b_1} - e^{b_2}} + \frac{\varepsilon}{2}((\frac{e^{b_3\eta} - e^{b_4\eta}}{e^{b_4} - e^{b_3}}(e^{b_3} - k_1(e^{b_3} - e^{b_2}) - k_2(e^{b_2} - e^{b_1})) + \\ & e^{b_3\eta} - k_1(e^{b_3\eta} - e^{b_2\eta}) - k_2(e^{b_3\eta} - e^{b_1\eta}))e^{i\pi\tau} + (\frac{e^{b_5+b_6\eta} - e^{b_6+b_5\eta}}{e^{b_5} - e^{b_6}})e^{-i\pi\tau} \end{aligned} \quad (3.88)$$

and,

$$\begin{aligned} \zeta(\eta, t) = & \frac{e^{b_7+b_8\eta} - e^{b_8+b_7\eta}}{e^{b_7} - e^{b_8}} + \frac{\varepsilon}{2}((\frac{e^{b_9\eta} - e^{b_{10}\eta}}{e^{b_{10}} - e^{b_9}}(e^{b_9} - k_3(e^{b_9} - e^{b_8}) - k_4(e^{b_8} - e^{b_7})) + \\ & e^{b_9\eta} - k_3(e^{b_9\eta} - e^{b_8\eta}) - k_4(e^{b_9\eta} - e^{b_7\eta}))e^{i\pi\tau} + (\frac{e^{b_{11}+b_{12}\eta} - e^{b_{12}+b_{11}\eta}}{e^{b_{11}} - e^{b_{12}}})e^{-i\pi\tau} \end{aligned} \quad (3.89)$$

3.2.1.3 Skin-friction, Nusselt and Sherwood Number in Term of Amplitude

With reference to the boundary conditions (3.44), the amplitude is defined in terms of primary

and secondary velocities for steady and unsteady flow. Therefore, total resultant velocity ($\psi(\eta, t)$) is written as;

$$R_v = \sqrt{a^2 + b^2} \quad (3.90)$$

and velocity is defined as

$$\psi(\eta, t) = a + ib \quad (3.91)$$

where a and b are the real and imaginary part of $\psi(\eta, t)$ respectively. The Skin-friction is given as;

$$\tau(\eta) = \left(\frac{\partial \psi}{\partial \eta} \right)_{\eta=0,1} = \tau_m + i \tau_n \quad (3.92)$$

$$\beta_1 = |\tau(\eta)| \quad (3.93)$$

where τ_m and τ_n are the real and imaginary part of Skin-friction respectively, and β_1 is the Skin-friction in term of Amplitude.

Hence,

$$\tau(0) = \left. \begin{aligned} &g_{13}b_{13} + g_{14}b_{14} + k_5b_2 + k_6b_1 + k_7b_8 + k_8b_7 + \\ &V_R(g_{15}b_{15} + g_{16}b_{16} + k_9b_{13} + k_{10}b_{14} + k_{11}b_2 + k_{12}b_1 + \\ &k_{13}b_8 + k_{14}b_7) + \frac{\varepsilon}{2}[(g_{17}b_{17} + g_{18}b_{18} + k_{16}b_{13} + k_{17}b_{14} + k_{18}b_2 + \\ &k_{19}b_1 + k_{20}b_8 + k_{21}b_7 + k_{22}b_3 + k_{23}b_4 + k_{24}b_9 + k_{25}b_{10} + \\ &V_R(g_{19}b_{19} + g_{20}b_{20} + k_{26}b_{13} + k_{27}b_{14} + k_{28}b_2 + k_{29}b_1 + \\ &k_{30}b_8 + k_{31}b_7 + k_{32}b_3 + k_{33}b_4 + k_{34}b_9 + k_{35}b_{10}))e^{i\omega t} + \\ &(g_{21}b_{21} + g_{22}b_{22} + k_{36}b_6 + k_{37}b_5 + k_{38}b_{12} + k_{39}b_{11} + \\ &V_R(g_{23}b_{23} + g_{24}b_{24} + k_{41}b_{21} + k_{42}b_{22} + k_{43}b_6 + k_{44}b_5 + \\ &k_{45}b_{12} + k_{46}b_{11}))e^{-i\omega t} \end{aligned} \right\} \quad (3.94)$$

$$\tau(1) = \left. \begin{aligned} &g_{13}b_{13}e^{b_{13}} + g_{14}b_{14}e^{b_{14}} + k_5b_2e^{b_2} + k_6b_1e^{b_1} + k_7b_8e^{b_8} + \\ &k_8b_7e^{b_7} + V_R(g_{15}b_{15}e^{b_{15}} + g_{16}b_{16}e^{b_{16}} + k_9b_{13}e^{b_{13}} + k_{10}b_{14}e^{b_{14}} + \\ &k_{11}b_2e^{b_2} + k_{12}b_1e^{b_1} + k_{13}b_8e^{b_8} + k_{14}b_7e^{b_7}) + \frac{\varepsilon}{2}[(g_{17}b_{17}e^{b_{17}} + \\ &g_{18}b_{18}e^{b_{18}} + k_{16}b_{13}e^{b_{13}} + k_{17}b_{14}e^{b_{14}} + k_{18}b_2e^{b_2} + k_{19}b_1e^{b_1} + \\ &k_{20}b_8e^{b_8} + k_{21}b_7e^{b_7} + k_{22}b_3e^{b_3} + k_{23}b_4e^{b_4} + k_{24}b_9e^{b_9} + k_{25}b_{10}e^{b_{10}} \\ &+ V_R(g_{19}b_{19}e^{b_{19}} + g_{20}b_{20}e^{b_{20}} + k_{26}b_{13}e^{b_{13}} + k_{27}b_{14}e^{b_{14}} + k_{28}b_2e^{b_2} \\ &+ k_{29}b_1e^{b_1} + k_{30}b_8e^{b_8} + k_{31}b_7e^{b_7} + k_{32}b_3e^{b_3} + k_{33}b_4e^{b_4} + k_{34}b_9e^{b_9} \\ &+ k_{35}b_{10}e^{b_{10}}))e^{i\omega t} + (g_{21}b_{21}e^{b_{21}} + g_{22}b_{22}e^{b_{22}} + k_{36}b_6e^{b_6} + \\ &k_{37}b_5e^{b_5} + k_{38}b_{12}e^{b_{12}} + k_{39}b_{11}e^{b_{11}} + V_R(g_{23}b_{23}e^{b_{23}} + g_{24}b_{24}e^{b_{24}} + \\ &k_{41}b_{21}e^{b_{21}} + k_{42}b_{22}e^{b_{22}} + k_{43}b_6e^{b_6} + k_{44}b_5e^{b_5} + \\ &k_{45}b_{12}e^{b_{12}} + k_{46}b_{11}e^{b_{11}}))e^{-i\omega t} \end{aligned} \right\} \quad (3.95)$$

Nusselt number (Heat transfer coefficient) is defined as;

$$Nu(\eta) = -\left(1 + \frac{4}{3E}\right) \left(\frac{\partial \mathcal{G}}{\partial \eta}\right)_{\eta=0,1} = R_m + iR_n \quad (3.96)$$

$$\beta_2 = |Nu(\eta)| \quad (3.97)$$

where R_m and R_n are the real and imaginary part of Nusselt number, respectively, and β_2 is the Heat transfer coefficient in term of Amplitude.

Therefore,

$$Nu(0) = -\left(1 + \frac{4}{3E}\right) \left[\frac{1}{e^{b_1} - e^{b_2}} (b_2 e^{b_1} - b_1 e^{b_2}) + \frac{\varepsilon}{2} \left[\left(\frac{b_3 - b_4}{e^{b_4} - e^{b_3}} (e^{b_3} - k_1(e^{b_3} - e^{b_2})) - \right. \right. \right. \quad (3.98)$$

$$\left. k_2(e^{b_2} - e^{b_1})) + b_3 - k_1(b_3 - b_2) - k_2(b_3 - b_1) \right] e^{i\pi\tau} + \left(\frac{b_6 e^{b_5} - b_5 e^{b_6}}{e^{b_5} - e^{b_6}} \right) e^{-i\pi\tau}]$$

and

$$Nu(1) = -\left(1 + \frac{4}{3E}\right) \left[\frac{1}{e^{b_1} - e^{b_2}} (b_2 e^{b_1+b_2} - b_1 e^{b_2+b_1}) + \frac{\varepsilon}{2} \left[\left(\frac{b_3 e^{b_3} - b_4 e^{b_4}}{e^{b_4} - e^{b_3}} (e^{b_3} - \right. \right. \right. \quad (3.99)$$

$$\left. k_1(e^{b_3} - e^{b_2}) - k_2(e^{b_2} - e^{b_1})) + b_3 e^{b_3} - k_1(b_3 e^{b_3} - b_2 e^{b_2}) - k_2(b_3 e^{b_3} - b_1 e^{b_1}) \right] e^{i\pi\tau} +$$

$$\left(\frac{b_6 e^{b_5+b_6} - b_5 e^{b_6+b_5}}{e^{b_5} - e^{b_6}} \right) e^{-i\pi\tau}]$$

Sherwood Number(Mass transfer coefficient) is defined as:

$$Sh(\eta) = \left(\frac{\partial \zeta}{\partial \eta}\right)_{\eta=0,1} = \lambda_m + i\lambda_n \quad (3.100)$$

$$\beta_3 = |Sh(\eta)| \quad (3.101)$$

where λ_m and λ_n are the real and imaginary part of Sherwood Number respectively, and β_3 is the Mass transfer coefficient in term of Amplitude.

Hence,

$$Sh(0) = \frac{1}{e^{b_7} - e^{b_8}} (b_8 e^{b_7} - b_7 e^{b_8}) + \frac{\varepsilon}{2} \left[\left(\frac{b_9 - b_{10}}{e^{b_{10}} - e^{b_9}} (e^{b_9} - k_3(e^{b_9} - e^{b_8})) - \right. \right.$$

$$\left. k_4(e^{b_9} - e^{b_7})) + b_9 - k_3(b_9 - b_8) - k_4(b_9 - b_7) \right] e^{i\pi\tau} + \left(\frac{b_{12} e^{b_{11}} - b_{11} e^{b_{12}}}{e^{b_{11}} - e^{b_{12}}} \right) e^{-i\pi\tau}]$$

$$(3.102)$$

and,

$$Sh(1) = \frac{1}{e^{b_7} - e^{b_8}} (b_8 e^{b_7+b_8} - b_7 e^{b_8+b_7}) + \frac{\varepsilon}{2} \left[\left(\frac{b_9 e^{b_9} - b_{10} e^{b_{10}}}{e^{b_{10}} - e^{b_9}} (e^{b_9} - \right. \right.$$

$$\begin{aligned}
& k_3(e^{b_9} - e^{b_8}) - k_4(e^{b_9} - e^{b_7})) + b_9 e^{b_9} - k_3(b_9 e^{b_9} - b_8 e^{b_8}) - k_4(b_9 e^{b_9} - b_7 e^{b_7})) e^{i\varpi t} + \\
& (\frac{b_{12} e^{b_{11} + b_{12}} - b_{11} e^{b_{12} + b_{11}}}{e^{b_{11}} - e^{b_{12}}}) e^{-i\varpi t}]
\end{aligned}
\tag{3.103}$$

3.2.2 Numerical Computation of Research Problem 2

Based on the nature of the boundary conditions, the following set of assumed solutions are used. (Garg et al, 2014b)

$$\psi(\eta, t) = \psi_0(\eta) + \frac{\varepsilon}{2} (\psi_1(\eta)e^{i\varpi t} + \psi_2(\eta)e^{-i\varpi t}) \quad (3.104)$$

$$\mathcal{G}(\eta, t) = \mathcal{G}_0(\eta) + \frac{\varepsilon}{2} (\mathcal{G}_1(\eta)e^{i\varpi t} + \mathcal{G}_2(\eta)e^{-i\varpi t}) \quad (3.105)$$

$$\zeta(\eta, t) = \zeta_0(\eta) + \frac{\varepsilon}{2} (\zeta_1(\eta)e^{i\varpi t} + \zeta_2(\eta)e^{-i\varpi t}) \quad (3.106)$$

Applying the assumed solutions (3.104)-(3.106) in equations (3.27)-(3.29), give the following set of ordinary differential equations:

$$V_R \psi_0''' + \psi_0'' + \psi_0' - (2i\Omega + F)\psi_0 = -G_h \mathcal{G}_0 - G_m \zeta_0 \quad (3.107)$$

$$V_R \psi_1''' + (1 - V_R i\varpi)\psi_1'' + \psi_1' - (i\varpi + 2i\Omega + F)\psi_1 = -H - 2A\psi_0' - G_h \mathcal{G}_1 - G_m \zeta_1 - 2V_R A\psi_0''' \quad (3.108)$$

$$V_R \psi_2''' + (1 + V_R i\varpi)\psi_2'' + \psi_2' - (2i\Omega + F - i\varpi)\psi_2 = -H - G_h \mathcal{G}_2 - G_m \zeta_2 \quad (3.109)$$

$$B\mathcal{G}_0'' + \mathcal{G}_0' - \phi\mathcal{G}_0 = -D_p \zeta_0'' \quad (3.110)$$

$$B\mathcal{G}_1'' + \mathcal{G}_1' - (\phi + i\varpi)\mathcal{G}_1 = -2A\mathcal{G}_0' - D_p \zeta_1'' \quad (3.111)$$

$$B\mathcal{G}_2'' + \mathcal{G}_2' - (\phi - i\varpi)\mathcal{G}_2 = -D_p \zeta_2'' \quad (3.112)$$

$$\zeta_0'' + S_c \zeta_0' - S_c K_r \zeta_0 = -S_c S_p \mathcal{G}_0'' \quad (3.113)$$

$$\zeta_1'' + S_c \zeta_1' - S_c (K_r + i\varpi)\zeta_1 = -2AS_c \zeta_0' - S_c S_p \mathcal{G}_1'' \quad (3.114)$$

$$\zeta_2'' + S_c \zeta_2' - S_c (K_r - i\varpi)\zeta_2 = -S_c S_p \mathcal{G}_2'' \quad (3.115)$$

where

$$B = \frac{1}{P_r} + \frac{4}{3EP_r}$$

The corresponding boundary conditions are:

$$\left. \begin{aligned} \psi_0 = \psi_1 = \psi_2 = 0, \mathcal{G}_0 = \mathcal{G}_1 = \mathcal{G}_2 = 1, \zeta_0 = \zeta_1 = \zeta_2 = 1 \text{ at } \eta = 0 \\ \psi_0 = \psi_1 = \psi_2 = 1, \mathcal{G}_0 = \mathcal{G}_1 = \mathcal{G}_2 = 0, \zeta_0 = \zeta_1 = \zeta_2 = 0 \text{ at } \eta = 1 \end{aligned} \right\} \quad (3.116)$$

3.2.2.1 Further Simplification of the third order differential equations

The solutions of equations (3.107)-(3.109) are not feasible in the present forms since they are third order differential equations with only two boundary conditions. In order to derive sufficient and necessary boundary conditions, according to Beard and walters (1964) and Garg et al. (2014b), the solutions are expressed in the forms:

$$\psi_0(\eta) = \psi_{01}(\eta) + V_R \psi_{02}(\eta) + O(V_R^2) \quad (3.117)$$

$$\psi_1(\eta) = \psi_{11}(\eta) + V_R \psi_{12}(\eta) + O(V_R^2) \quad (3.118)$$

$$\psi_2(\eta) = \psi_{21}(\eta) + V_R \psi_{22}(\eta) + O(V_R^2) \quad (3.119)$$

Introducing equations (3.117)-(3.119) to equations (3.107)-(3.109), the equations become:

$$\psi_{01}'' + \psi_{01}' - (2i\Omega + F)\psi_{01} = -G_h \vartheta_0 - G_m \zeta_0 \quad (3.120)$$

$$\psi_{02}'' + \psi_{02}' - (2i\Omega + F)\psi_{02} = -\psi_{01}''' \quad (3.121)$$

$$\psi_{11}'' + \psi_{11}' - (i\varpi + 2i\Omega + F)\psi_{11} = -H - 2A\psi_{01}' - G_h \vartheta_1 - G_m \zeta_1 \quad (3.122)$$

$$\psi_{12}'' + \psi_{12}' - (i\varpi + 2i\Omega + F)\psi_{12} = i\varpi \psi_{11}'' - \psi_{11}''' - 2A\psi_{01}''' - 2A\psi_{02}' \quad (3.123)$$

$$\psi_{21}'' + \psi_{21}' - (2i\Omega + F - i\varpi)\psi_{21} = -H - G_h \vartheta_2 - G_m \zeta_2 \quad (3.124)$$

$$\psi_{22}'' + \psi_{22}' - (2i\Omega + F - i\varpi)\psi_{22} = -\psi_{21}''' - i\varpi \psi_{21}'' \quad (3.125)$$

with the following boundary conditions:

$$\left. \begin{aligned} \psi_{01} = \psi_{02} = \psi_{11} = \psi_{12} = \psi_{21} = \psi_{22} = 0 \text{ at } \eta = 0 \\ \psi_{01} = \psi_{11} = \psi_{21} = 1, \psi_{02} = \psi_{12} = \psi_{22} = 0 \text{ at } \eta = 1 \end{aligned} \right\} \quad (3.126)$$

3.2.2.2 Adomian Decomposition Method

Adomian decomposition method can be used for solving both linear and non-linear differential equations based on the following steps. Differential equation can be written in a general form as;

$$F\psi(\eta) = b \quad (3.127)$$

where F represents an operator of nonlinear ordinary differential equation containing both linear and nonlinear terms. $L\psi$ represents the linear term, where the invertible linear operator is L. Taking the highest-ordered derivative as L, L^{-1} is n-fold integration operator from 0 to η for $L = \frac{d^n}{d\eta^n}$. For the linear operator L, the remainder is R and $N\psi$ is the nonlinear term. Hence,

$$L\psi + R\psi + N\psi = b \quad (3.128)$$

$$L\psi = b - R\psi - N\psi \quad (3.129)$$

Since L is invertible, thus

$$L^{-1}L\psi = L^{-1}b - L^{-1}R\psi - L^{-1}N\psi \quad (3.130)$$

Let us consider ordinary differential equation of order two,

$$L^{-1}L\psi = \int_0^\eta \int_0^s \psi''(\eta) d\eta ds \quad (3.131)$$

$$L^{-1}L\psi = \psi - \psi(0) - \eta\psi'(0) \quad (3.132)$$

substituting for $L^{-1}L\psi$ in equation (3.130), the equation becomes;

$$\psi = \psi(0) + \eta\psi'(0) + L^{-1}b - L^{-1}R\psi - L^{-1}N\psi \quad (3.133)$$

Hence,

$$\psi = \psi(0) + \eta\psi'(0) + b\frac{\eta^2}{2} - \int_0^\eta \int_0^s (R\psi + N\psi) d\eta ds \quad (3.134)$$

ψ can be written in series form as:

$$\psi = \sum_{n=0}^{\infty} \psi_n \quad (3.135)$$

also, the nonlinear term as:

$$N\psi = \sum_{n=0}^{\infty} A_n \quad (3.136)$$

where

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left(F \left(\sum_{i=0}^n \lambda^i \psi_i \right) \right)_{\lambda=0} \quad n = 0, 1, 2, 3, \dots \quad (3.137)$$

Substituting equations (3.135) and (3.136) into equation (3.134) gives;

$$\sum_{n=0}^{\infty} \psi_n = \psi(0) + \eta\psi'(0) + b\frac{\eta^2}{2} - \int_0^\eta \int_0^s \left(R \sum_{n=0}^{\infty} \psi_n + \sum_{n=0}^{\infty} A_n \right) d\eta ds \quad (3.138)$$

The first three terms are identified as ψ_0 which is the initial approximation, that is

$$\psi_0 = \psi(0) + \eta\psi'(0) + b\frac{\eta^2}{2} \quad (3.139)$$

and

$$\psi_{n+1} = - \int_0^\eta \int_0^s \left(R \sum_{n=0}^{\infty} \psi_n + \sum_{n=0}^{\infty} A_n \right) d\eta ds \quad (3.140)$$

is the recurrence relation. All the components can be determined since A_0 depends on ψ_0 only, A_1 depends on ψ_0 and ψ_1 and so on. The solution will then be the n-term approximation to ψ .

3.2.2.3 Solution to the Problem

Equations (3.110)-(3.115) and (3.120)-(3.125) are highly coupled, as a result, analytical solutions is not viable. Hence, Adomian Decomposition Method is applied for the solution of the equations. Therefore, the initial approximation and the recurrence relation equations for equations (3.110)-(3.115) and (3.120)-(3.125) with the boundary conditions (3.116) and (3.126) according to MATHEMATICA software program are as follows:

$$\left. \begin{aligned} \zeta_0[0] &= 1 + h_1 * \eta; \\ \mathcal{G}_0[0] &= 1 + h_2 * \eta; \\ \psi_{01}[0] &= h_3 * \eta; \\ \psi_{02}[0] &= h_4 * \eta; \\ \zeta_1[0] &= 1 + h_5 * \eta; \\ \mathcal{G}_1[0] &= 1 + h_6 * \eta; \\ \psi_{11}[0] &= h_7 * \eta + H * \frac{\eta^2}{2}; \\ \psi_{12}[0] &= h_8 * \eta; \\ \zeta_2[0] &= 1 + h_9 * \eta; \\ \mathcal{G}_2[0] &= 1 + h_{10} * \eta; \\ \psi_{21}[0] &= h_{11} * \eta + H * \frac{\eta^2}{2}; \\ \psi_{22}[0] &= h_{12} * \eta; \end{aligned} \right\} \quad (3.141)$$

$$\zeta_0[n+1] = \int_0^\eta \left(\int_0^s (-S_c * \partial_\eta \zeta_0[n] + S_c * K_r * \zeta_0[n] - S_c * S_p * \partial_\eta (\partial_\eta \mathcal{G}_0[n])) d\eta \right) ds; \quad (3.142)$$

$$\mathcal{G}_0[n+1] = \int_0^\eta \left(\int_0^s \left(-\frac{1}{B} * \partial_\eta \mathcal{G}_0[n] + \frac{\phi}{B} * \mathcal{G}_0[n] - D_p * \partial_\eta (\partial_\eta \zeta_0[n]) \right) d\eta \right) ds; \quad (3.143)$$

$$\psi_{01}[n+1] = \int_0^\eta \left(\int_0^s (-1 * \partial_\eta \psi_{01}[n] + (2 * i * \Omega + F) * \psi_{01}[n] - G_h * \mathcal{G}_0[n] - G_m * \zeta_0[n]) d\eta \right) ds; \quad (3.144)$$

$$\psi_{02}[n+1] = \int_0^\eta \left(\int_0^s (-1 * \partial_\eta \psi_{02}[n] + (2 * i * \Omega + F) * \psi_{02}[n] - \partial_\eta \partial_\eta (\partial_\eta \psi_{01}[n])) d\eta \right) ds; \quad (3.145)$$

$$\begin{aligned} \zeta_1[n+1] &= \int_0^\eta \left(\int_0^s (-S_c * \partial_\eta \zeta_1[n] + S_c * (K_r + i * \varpi) * \zeta_1[n] - \right. \\ &\quad \left. 2 * A * S_c * \partial_\eta \zeta_0[n] - S_c * S_p * \partial_\eta (\partial_\eta \mathcal{G}_1[n])) d\eta \right) ds; \end{aligned} \quad (3.146)$$

$$\mathcal{G}_1[n+1] = \int_0^\eta \left(\int_0^s \left(-\frac{1}{B} * \partial_\eta \mathcal{G}_1[n] + \frac{\phi + i * \varpi}{B} * \mathcal{G}_1[n] - \frac{2 * A}{B} * \partial_\eta \mathcal{G}_0[n] - D_p * \partial_\eta (\partial_\eta \zeta_1[n]) \right) d\eta \right) ds; \quad (3.147)$$

$$\begin{aligned} \psi_{11}[n+1] = & \int_0^\eta \left(\int_0^s (-1 * \partial_\eta \psi_{11}[n] + (i * \varpi + 2 * i * \Omega + F) * \psi_{11}[n] - \right. \\ & \left. 2 * A * \partial_\eta \psi_{01}[n] - G_h * \mathcal{G}_1[n] - G_m * \zeta_1[n]) d\eta \right) ds; \end{aligned} \quad (3.148)$$

$$\begin{aligned} \psi_{12}[n+1] = & \int_0^\eta \left(\int_0^s (-1 * \partial_\eta \psi_{12}[n] + (i * \varpi + 2 * i * \Omega + F) * \psi_{12}[n] - \right. \\ & \left. 2 * A * \partial_\eta \partial_\eta (\partial_\eta \psi_{01}[n]) + i * \varpi * \partial_\eta (\partial_\eta \psi_{11}[n]) - \partial_\eta \partial_\eta (\partial_\eta \psi_{11}[n]) - 2 * A * \partial_\eta \psi_{02}[n] \right) d\eta ds; \end{aligned} \quad (3.149)$$

$$\zeta_2[n+1] = \int_0^\eta \left(\int_0^s (-S_c * \partial_\eta \zeta_2[n] + S_c * (kr - i * \varpi) * \zeta_2[n] - S_c * S_p * \partial_\eta (\partial_\eta \mathcal{G}_2[n])) d\eta \right) ds; \quad (3.150)$$

$$\mathcal{G}_2[n+1] = \int_0^\eta \left(\int_0^s \left(-\frac{1}{B} * \partial_\eta \mathcal{G}_2[n] + \frac{\phi - i * \varpi}{B} * \mathcal{G}_2[n] - D_p * \partial_\eta (\partial_\eta \zeta_2[n]) \right) d\eta \right) ds; \quad (3.151)$$

$$\begin{aligned} \psi_{21}[n+1] = & \int_0^\eta \left(\int_0^s (-1 * \partial_\eta \psi_{21}[n] + (2 * i * \Omega + F - i * \varpi) * \psi_{21}[n] - G_h * \mathcal{G}_2[n] - \right. \\ & \left. G_m * \zeta_2[n]) d\eta \right) ds; \end{aligned} \quad (3.152)$$

$$\begin{aligned} \psi_{22}[n+1] = & \int_0^\eta \left(\int_0^s (-1 * \partial_\eta \psi_{22}[n] + (2 * i * \Omega + F - i * \varpi) * \psi_{22}[n] - \partial_{\eta, \eta, \eta} \psi_{21}[n] - \right. \\ & \left. i * \varpi * \partial_\eta (\partial_\eta \psi_{21}[n]) \right) d\eta ds; \end{aligned} \quad (3.153)$$

where $h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}, h_{11}$ and h_{12} are constants to be determined. The approximate solutions of the above equations can be written as:

$$\left. \begin{aligned} \zeta_0 &= \sum_{a=0}^5 \zeta_0[a], \mathcal{G}_0 = \sum_{a=0}^5 \mathcal{G}_0[a]; \psi_{01} = \sum_{a=0}^5 \psi_{01}[a], \\ \psi_{02} &= \sum_{a=0}^5 \psi_{02}[a], \\ \zeta_1 &= \sum_{a=0}^5 \zeta_1[a], \mathcal{G}_1 = \sum_{a=0}^5 \mathcal{G}_1[a]; \psi_{11} = \sum_{a=0}^5 \psi_{11}[a], \\ \psi_{12} &= \sum_{a=0}^5 \psi_{12}[a], \\ \zeta_2 &= \sum_{a=0}^5 \zeta_2[a], \mathcal{G}_2 = \sum_{a=0}^5 \mathcal{G}_2[a]; \psi_{21} = \sum_{a=0}^5 \psi_{21}[a], \\ \psi_{22} &= \sum_{a=0}^5 \psi_{22}[a] \end{aligned} \right\} \quad (3.154)$$

Substituting the series solutions (3.154) into equations (3.104)-(3.106) and (3.117)-(3.119) gives the final solution for velocity, temperature and concentration distributions

$$\begin{aligned} \psi(\eta, t) = & \sum_{a=0}^5 \psi_{01}[a](\eta) + V_R \sum_{a=0}^5 \psi_{02}[a](\eta) + \frac{\varepsilon}{2} \left(\sum_{a=0}^5 \psi_{11}[a](\eta) + \right. \\ & \left. V_R \sum_{a=0}^5 \psi_{12}[a](\eta) e^{i\varpi t} + \left(\sum_{a=0}^5 \psi_{21}[a](\eta) + V_R \sum_{a=0}^5 \psi_{22}[a](\eta) \right) e^{-i\varpi t} \right) \end{aligned} \quad (3.155)$$

$$\mathcal{G}(\eta, t) = \sum_{a=0}^5 \mathcal{G}_0[a](\eta) + \frac{\varepsilon}{2} \left(\sum_{a=0}^5 \mathcal{G}_1[a](\eta) e^{i\varpi t} + \sum_{a=0}^5 \mathcal{G}_2[a](\eta) e^{-i\varpi t} \right) \quad (3.156)$$

$$\zeta(\eta, t) = \sum_{a=0}^5 \zeta_0[a](\eta) + \frac{\varepsilon}{2} \left(\sum_{a=0}^5 \zeta_1[a](\eta) e^{i\varpi t} + \sum_{a=0}^5 \zeta_2[a](\eta) e^{-i\varpi t} \right) \quad (3.157)$$

3.2.3 Numerical Computation of Research Problem 3

The partial differential equations (3.41), (3.42) and (3.43) are reduce to ordinary differential equations by Perturbation technique. Due to the nature of the boundary conditions, the assumed solutions can be written as follows.(Garg et al, 2014)

$$\left. \begin{aligned} \psi(\eta, t) &= \psi_0(\eta) + \frac{\varepsilon}{2} (\psi_1(\eta)e^{i\varpi t} + \psi_2(\eta)e^{-i\varpi t}) \\ \vartheta(\eta, t) &= \vartheta_0(\eta) + \frac{\varepsilon}{2} (\vartheta_1(\eta)e^{i\varpi t} + \vartheta_2(\eta)e^{-i\varpi t}) \\ \zeta(\eta, t) &= \zeta_0(\eta) + \frac{\varepsilon}{2} (\zeta_1(\eta)e^{i\varpi t} + \zeta_2(\eta)e^{-i\varpi t}) \end{aligned} \right\} \quad (3.158)$$

Substituting equations (3.158) into equation (3.41), (3.42) and (3.43) gives:

$$V_R \psi_0''' + \psi_0'' + \psi_0' - (2i\Omega + F)\psi_0 = -G_h \vartheta_0 - G_m \zeta_0 \quad (3.159)$$

$$\begin{aligned} V_R \psi_1''' + (1 - V_R i\varpi)\psi_1'' + \psi_1' - (i\varpi + 2i\Omega + F)\psi_1 &= -H - 2A\psi_0' - \\ &G_h \vartheta_1 \varpi - G_m \zeta_1 - 2V_R A\psi_0''' \end{aligned} \quad (3.160)$$

$$V_R \psi_2''' + (1 + V_R i\varpi)\psi_2'' + \psi_2' - (2i\Omega + F - i\varpi)\psi_2 = -H - G_h \vartheta_2 - G_m \zeta_2 \quad (3.161)$$

$$B\vartheta_0'' + \vartheta_0' - \phi\vartheta_0 = -E_c \psi_0' \bar{\psi}_0' - D_p \zeta_0'' \quad (3.162)$$

$$B\vartheta_1'' + \vartheta_1' - (\phi + i\varpi)\vartheta_1 = -2A\vartheta_0' - E_c \psi_0' \bar{\psi}_1' - E_c \psi_1' \bar{\psi}_0' - D_p \zeta_1'' \quad (3.163)$$

$$B\vartheta_2'' + \vartheta_2' - (\phi - i\varpi)\vartheta_2 = -E_c \psi_0' \bar{\psi}_2' - E_c \psi_2' \bar{\psi}_0' - D_p \zeta_2'' \quad (3.164)$$

$$\zeta_0'' + S_c \zeta_0' - S_c K_r \zeta_0 = -S_c S_p \vartheta_0'' \quad (3.165)$$

$$\zeta_1'' + S_c \zeta_1' - S_c (K_r + i\varpi)\zeta_1 = -2AS_c \zeta_0' - S_c S_p \vartheta_1'' \quad (3.166)$$

$$\zeta_2'' + S_c \zeta_2' - S_c (K_r - i\varpi)\zeta_2 = -S_c S_p \vartheta_2'' \quad (3.167)$$

where

$$B = \frac{1}{P_r} + \frac{4}{3EP_r}$$

The corresponding boundary conditions are:

$$\psi_0 = \psi_1 = \psi_2 = 0, \quad \vartheta_0 = \vartheta_1 = \vartheta_2 = 1, \quad \zeta_0 = \zeta_1 = \zeta_2 = 1 \quad \text{at } \eta = 0 \quad (3.168)$$

$$\psi_0 = \psi_1 = \psi_2 = 1, \mathcal{G}_0 = \mathcal{G}_1 = \mathcal{G}_2 = 0, \zeta_0 = \zeta_1 = \zeta_2 = 0 \text{ at } \eta = 1 \quad (3.169)$$

3.2.3.1 Further Simplification of equations (3.159)-(3.167)

Equations (3.159), (3.160) and (3.161) are third order differential equations with only two boundary conditions for each problem. In order to obtain necessary and sufficient boundary conditions (Beard and walters (1964) and Garg et al.(2014b)), the solutions are expressed in the forms:

$$\left. \begin{aligned} \psi_0(\eta) &= \psi_{01}(\eta) + V_R \psi_{02}(\eta) + 0(V_R^2) \\ \psi_1(\eta) &= \psi_{11}(\eta) + V_R \psi_{12}(\eta) + 0(V_R^2) \\ \psi_2(\eta) &= \psi_{21}(\eta) + V_R \psi_{22}(\eta) + 0(V_R^2) \\ \mathcal{G}_0(\eta) &= \mathcal{G}_{01}(\eta) + V_R \mathcal{G}_{02}(\eta) + 0(V_R^2) \\ \mathcal{G}_1(\eta) &= \mathcal{G}_{11}(\eta) + V_R \mathcal{G}_{12}(\eta) + 0(V_R^2) \\ \mathcal{G}_2(\eta) &= \mathcal{G}_{21}(\eta) + V_R \mathcal{G}_{22}(\eta) + 0(V_R^2) \\ \zeta_0(\eta) &= \zeta_{01}(\eta) + V_R \zeta_{02}(\eta) + 0(V_R^2) \\ \zeta_1(\eta) &= \zeta_{11}(\eta) + V_R \zeta_{12}(\eta) + 0(V_R^2) \\ \zeta_2(\eta) &= \zeta_{21}(\eta) + V_R \zeta_{22}(\eta) + 0(V_R^2) \end{aligned} \right\} \quad (3.170)$$

Applying equation (3.170) to equations (3.159)-(3.167) gives:

$$\psi_{01}'' + \psi_{01}' - (2i\Omega + F)\psi_{01} = -G_h \mathcal{G}_{01} - G_m \zeta_{01} \quad (3.171)$$

$$\psi_{02}'' + \psi_{02}' - (2i\Omega + F)\psi_{02} = -\psi_{01}''' - G_h \mathcal{G}_{02} - G_m \zeta_{02} \quad (3.172)$$

$$\psi_{11}'' + \psi_{11}' - (i\varpi + 2i\Omega + F)\psi_{11} = -H - 2A\psi_{01}' - G_h \mathcal{G}_{11} - G_m \zeta_{11} \quad (3.173)$$

$$\psi_{12}'' + \psi_{12}' - (i\varpi + 2i\Omega + F)\psi_{12} = i\varpi\psi_{11}'' - \psi_{11}''' - 2A\psi_{02}' - 2A\psi_{01}''' - G_h \mathcal{G}_{12} - G_m \zeta_{12} \quad (3.174)$$

$$\psi_{21}'' + \psi_{21}' - (2i\Omega + F - i\varpi)\psi_{21} = -H - G_h \mathcal{G}_{21} - G_m \zeta_{21} \quad (3.175)$$

$$\psi_{22}'' + \psi_{22}' - (2i\Omega + F - i\varpi)\psi_{22} = -\psi_{21}''' - i\varpi\psi_{21}'' - G_h \mathcal{G}_{22} - G_m \zeta_{22} \quad (3.176)$$

$$B\mathcal{G}_{01}'' + \mathcal{G}_{01}' - \phi\mathcal{G}_{01} = -E_c\psi_{01}'\bar{\psi}_{01}' - D_p\zeta_{01}'' \quad (3.177)$$

$$B\mathcal{G}_{02}'' + \mathcal{G}_{02}' - \phi\mathcal{G}_{02} = -E_c\psi_{01}'\bar{\psi}_{02}' - E_c\psi_{02}'\bar{\psi}_{01}' - D_p\zeta_{02}'' \quad (3.178)$$

$$B\mathcal{G}_{11}'' + \mathcal{G}_{11}' - (\phi + i\varpi)\mathcal{G}_{11} = -2A\mathcal{G}_{01}' - E_c\psi_{01}'\bar{\psi}_{11}' - E_c\psi_{11}'\bar{\psi}_{01}' - D_p\zeta_{11}'' \quad (3.179)$$

$$B\mathcal{G}_{12}'' + \mathcal{G}_{12}' - (\phi + i\varpi)\mathcal{G}_{12} = -2A\mathcal{G}_{02}' - E_c\psi_{01}'\bar{\psi}_{12}' - E_c\psi_{02}'\bar{\psi}_{11}' - E_c\psi_{11}'\bar{\psi}_{02}' - E_c\psi_{12}'\bar{\psi}_{01}' - D_p\zeta_{12}'' \quad (3.180)$$

$$B\mathcal{G}_{21}'' + \mathcal{G}_{21}' - (\phi - i\varpi)\mathcal{G}_{21} = -E_c\psi_{01}'\bar{\psi}_{21}' - E_c\psi_{21}'\bar{\psi}_{01}' - D_p\zeta_{21}'' \quad (3.181)$$

$$B\mathcal{G}_{22}'' + \mathcal{G}_{22}' - (\phi - i\varpi)\mathcal{G}_{22} = -E_c\psi_{01}'\bar{\psi}_{22}' - E_c\psi_{02}'\bar{\psi}_{21}' - E_c\psi_{21}'\bar{\psi}_{02}' - E_c\psi_{22}'\bar{\psi}_{01}' - D_p\zeta_{22}'' \quad (3.182)$$

$$\zeta_{01}'' + S_c\zeta_{01}' - S_cK_r\zeta_{01} = -S_cS_p\mathcal{G}_{01}'' \quad (3.183)$$

$$\zeta_{02}'' + S_c\zeta_{02}' - S_cK_r\zeta_{02} = -S_cS_p\mathcal{G}_{02}'' \quad (3.184)$$

$$\zeta_{11}'' + S_c\zeta_{11}' - S_c(K_r + i\varpi)\zeta_{11} = -2AS_c\zeta_{01}' - S_cS_p\mathcal{G}_{11}'' \quad (3.185)$$

$$\zeta_{12}'' + S_c\zeta_{12}' - S_c(K_r + i\varpi)\zeta_{12} = -2AS_c\zeta_{02}' - S_cS_p\mathcal{G}_{12}'' \quad (3.186)$$

$$\zeta_{21}'' + S_c\zeta_{21}' - S_c(K_r - i\varpi)\zeta_{21} = -S_cS_p\mathcal{G}_{21}'' \quad (3.187)$$

$$\zeta_{22}'' + S_c\zeta_{22}' - S_c(K_r - i\varpi)\zeta_{22} = -S_cS_p\mathcal{G}_{22}'' \quad (3.188)$$

subject to the following boundary conditions:

$$\left. \begin{aligned} \psi_{01} = \psi_{02} = \psi_{11} = \psi_{12} = \psi_{21} = \psi_{22} = 0 \text{ at } \eta = 0 \\ \psi_{01} = \psi_{11} = \psi_{21} = 1, \psi_{02} = \psi_{12} = \psi_{22} = 0 \text{ at } \eta = 1 \\ \mathcal{G}_{01} = \mathcal{G}_{11} = \mathcal{G}_{21} = 1, \mathcal{G}_{02} = \mathcal{G}_{12} = \mathcal{G}_{22} = 0 \text{ at } \eta = 0 \\ \mathcal{G}_{01} = \mathcal{G}_{11} = \mathcal{G}_{21} = 0, \mathcal{G}_{02} = \mathcal{G}_{12} = \mathcal{G}_{22} = 0 \text{ at } \eta = 1 \\ \zeta_{01} = \zeta_{11} = \zeta_{21} = 1, \zeta_{02} = \zeta_{12} = \zeta_{22} = 0 \text{ at } \eta = 0 \\ \zeta_{01} = \zeta_{11} = \zeta_{21} = 0, \zeta_{02} = \zeta_{12} = \zeta_{22} = 0 \text{ at } \eta = 1 \end{aligned} \right\} \quad (3.189)$$

3.2.3.2 Solution to the Problem

Therefore, the initial approximation and the recurrence relation equations for equations (3.171) - (3.188) with the boundary conditions (3.189) according to MATHEMATICA software program are as follows:

$$\left. \begin{aligned}
& \zeta_{01}[0] = 1 + h_{13} * \eta; & \mathcal{G}_{01}[0] &= 1 + h_{14} * \eta; \\
& \psi_{01}[0] = h_{15} * \eta; & \zeta_{02}[0] &= h_{16} * \eta; \\
& \mathcal{G}_{02}[0] = h_{17} * \eta; & \psi_{02}[0] &= h_{18} * \eta; \\
& \zeta_{11}[0] = 1 + h_{19} * \eta; & \mathcal{G}_{11}[0] &= 1 + h_{20} * \eta; \\
& \psi_{11}[0] = h_{21} * \eta + H * \frac{\eta^2}{2}; & \zeta_{12}[0] &= h_{22} * \eta; \\
& \mathcal{G}_{12}[0] = h_{23} * \eta; & \psi_{12}[0] &= h_{24} * \eta; \\
& \zeta_{21}[0] = 1 + h_{25} * \eta; & \mathcal{G}_{21}[0] &= 1 + h_{26} * \eta; \\
& \psi_{21}[0] = h_{27} * \eta + H * \frac{\eta^2}{2}; & \zeta_{22}[0] &= h_{28} * \eta; \\
& \mathcal{G}_{22}[0] = h_{29} * \eta; & \psi_{22}[0] &= h_{30} * \eta;
\end{aligned} \right\} \quad (3.190)$$

$$\zeta_{01}[n+1] = \int_0^\eta \left(\int_0^s (-S_c * \partial_\eta \zeta_{01}[n] + S_c * K_r * \zeta_{01}[n] - S_c * S_p * \partial_\eta (\partial_\eta \mathcal{G}_{01}[n])) d\eta \right) ds; \quad (3.191)$$

$$\mathcal{G}_{01}[n+1] = \int_0^\eta \left(\int_0^s \left(-\frac{1}{B} * \partial_\eta \mathcal{G}_{01}[n] + \frac{\phi}{B} * \mathcal{G}_{01}[n] - \right. \right.$$

$$\left. \left. \frac{E_c}{B} * \sum_{k=0}^n (\partial_\eta \psi_{01}[k] * \partial_\eta \bar{\psi}_{01}[n-k]) - D_p * \partial_\eta (\partial_\eta \zeta_{01}[n]) \right) d\eta \right) ds; \quad (3.192)$$

$$\psi_{01}[n+1] = \int_0^\eta \left(\int_0^s (-1 * \partial_\eta \psi_{01}[n] + (2 * i * \Omega + F) * \psi_{01}[n] - G_h * \mathcal{G}_{01}[n] - G_m * \zeta_{01}[n]) d\eta \right) ds; \quad (3.193)$$

$$\zeta_{02}[n+1] = \int_0^\eta \left(\int_0^s (-S_c * \partial_\eta \zeta_{02}[n] + S_c * K_r * \zeta_{02}[n] - S_c * S_p * \partial_\eta (\partial_\eta \mathcal{G}_{02}[n])) d\eta \right) ds; \quad (3.194)$$

$$\mathcal{G}_{02}[n+1] = \int_0^\eta \left(\int_0^s \left(-\frac{1}{B} * \partial_\eta \mathcal{G}_{02}[n] + \frac{\phi}{B} * \mathcal{G}_{02}[n] - \right. \right.$$

$$\left. \left. \frac{E_c}{B} * \sum_{k=0}^n (\partial_\eta \psi_{01}[k] * \partial_\eta \bar{\psi}_{02}[n-k]) - \frac{E_c}{B} * \sum_{k=0}^n (\partial_\eta \psi_{02}[k] * \partial_\eta \bar{\psi}_{01}[n-k]) \right. \right. \\
\left. \left. - D_p * \partial_\eta (\partial_\eta \zeta_{02}[n]) \right) d\eta \right) ds; \quad (3.195)$$

$$\psi_{02}[n+1] = \int_0^\eta \left(\int_0^s (-1 * \partial_\eta \psi_{02}[n] + (2 * i * \Omega + F) * \psi_{02}[n] - \partial_\eta \partial_\eta (\partial_\eta \psi_{01}[n]) \right. \\
\left. - G_h * \mathcal{G}_{02}[n] - G_m * \zeta_{02}[n]) d\eta \right) ds; \quad (3.196)$$

$$\zeta_{11}[n+1] = \int_0^\eta \left(\int_0^s (-S_c * \partial_\eta \zeta_{11}[n] + S_c * (K_r + i * \varpi) * \zeta_{11}[n] - \right.$$

$$2 * A * S_c * \partial_\eta \zeta_{01}[n] - S_c * S_p * \partial_\eta (\partial_\eta \mathcal{G}_{11}[n])) d\eta) ds; \quad (3.197)$$

$$\begin{aligned} \mathcal{G}_{11}[n+1] = & \int_0^\eta \left(\int_0^s \left(-\frac{1}{B} * \partial_\eta \mathcal{G}_{11}[n] + \frac{\phi + i * \varpi}{B} * \mathcal{G}_{11}[n] - \frac{2 * A}{B} * \partial_\eta \mathcal{G}_{01}[n] - \right. \right. \\ & \frac{E_c}{B} * \sum_{k=0}^n (\partial_\eta \psi_{01}[k] * \partial_\eta \bar{\psi}_{11}[n-k]) - \frac{E_c}{B} * \sum_{k=0}^n (\partial_\eta \psi_{11}[k] * \partial_\eta \bar{\psi}_{01}[n-k]) \\ & \left. \left. - D_p * \partial_\eta (\partial_\eta \zeta_{11}[n])) d\eta \right) ds; \end{aligned} \quad (3.198)$$

$$\begin{aligned} \psi_{11}[n+1] = & \int_0^\eta \left(\int_0^s (-1 * \partial_\eta \psi_{11}[n] + (i * \varpi + 2 * i * \Omega + F) * \psi_{11}[n] - \right. \\ & \left. 2 * A * \partial_\eta \psi_{01}[n] - G_h * \mathcal{G}_{11}[n] - G_m * \zeta_{11}[n]) d\eta \right) ds; \end{aligned} \quad (3.199)$$

$$\begin{aligned} \zeta_{12}[n+1] = & \int_0^\eta \left(\int_0^s (-S_c * \partial_\eta \zeta_{12}[n] + S_c * (K_r + i * \varpi) * \zeta_{12}[n] - \right. \\ & \left. 2 * A * S_c * \partial_\eta \zeta_{02}[n] - S_c * S_p * \partial_\eta (\partial_\eta \mathcal{G}_{12}[n])) d\eta \right) ds; \end{aligned} \quad (3.200)$$

$$\begin{aligned} \mathcal{G}_{12}[n+1] = & \int_0^\eta \left(\int_0^s \left(-\frac{1}{B} * \partial_\eta \mathcal{G}_{12}[n] + \frac{\phi + i * \varpi}{B} * \mathcal{G}_{12}[n] - \frac{2 * A}{B} * \partial_\eta \mathcal{G}_{02}[n] - \right. \right. \\ & \frac{E_c}{B} * \sum_{k=0}^n (\partial_\eta \psi_{01}[k] * \partial_\eta \bar{\psi}_{12}[n-k]) - \frac{E_c}{B} * \sum_{k=0}^n (\partial_\eta \psi_{02}[k] * \partial_\eta \bar{\psi}_{11}[n-k]) - \\ & \frac{E_c}{B} * \sum_{k=0}^n (\partial_\eta \psi_{11}[k] * \partial_\eta \bar{\psi}_{02}[n-k]) - \frac{E_c}{B} * \sum_{k=0}^n (\partial_\eta \psi_{12}[k] * \partial_\eta \bar{\psi}_{01}[n-k]) \\ & \left. \left. - D_p * \partial_\eta (\partial_\eta \zeta_{12}[n])) d\eta \right) ds; \end{aligned} \quad (3.201)$$

$$\begin{aligned} \psi_{12}[n+1] = & \int_0^\eta \left(\int_0^s (-1 * \partial_\eta \psi_{12}[n] + (i * \varpi + 2 * i * \Omega + F) * \psi_{12}[n] - \right. \\ & 2 * A * \partial_\eta \partial_\eta (\partial_\eta \psi_{01}[n]) - 2 * A * \partial_\eta \psi_{02}[n] + i * \varpi * \partial_\eta (\partial_\eta \psi_{11}[n]) - \partial_\eta \partial_\eta (\partial_\eta \psi_{11}[n]) - \\ & \left. G_h * \mathcal{G}_{12}[n] - G_m * \zeta_{12}[n]) d\eta \right) ds; \end{aligned} \quad (3.202)$$

$$\zeta_{21}[n+1] = \int_0^\eta \left(\int_0^s (-S_c * \partial_\eta \zeta_{21}[n] + S_c * (kr - i * \varpi) * \zeta_{21}[n] - S_c * S_p * \partial_\eta (\partial_\eta \mathcal{G}_{21}[n])) d\eta \right) ds; \quad (3.203)$$

$$\begin{aligned}
\mathcal{G}_{21}[n+1] = & \int_0^\eta \left(\int_0^s \left(-\frac{1}{B} * \partial_\eta \mathcal{G}_{21}[n] + \frac{\phi - i * \varpi}{B} * \mathcal{G}_{21}[n] - \right. \right. \\
& \frac{E_c}{B} * \sum_{k=0}^n (\partial_\eta \psi_{01}[k] * \partial_\eta \bar{\psi}_{21}[n-k]) - \frac{E_c}{B} * \sum_{k=0}^n (\partial_\eta \psi_{21}[k] * \partial_\eta \bar{\psi}_{01}[n-k]) \\
& \left. \left. - D_p * \partial_\eta (\partial_\eta \zeta_{21}[n]) \right) d\eta \right) ds; \tag{3.204}
\end{aligned}$$

$$\begin{aligned}
\psi_{21}[n+1] = & \int_0^\eta \left(\int_0^s (-1 * \partial_\eta \psi_{21}[n] + (2 * i * \Omega + F - i * \varpi) * \psi_{21}[n] - G_h * \mathcal{G}_{21}[n] - \right. \\
& \left. G_m * \zeta_{21}[n]) d\eta \right) ds; \tag{3.205}
\end{aligned}$$

$$\zeta_{22}[n+1] = \int_0^\eta \left(\int_0^s (-S_c * \partial_\eta \zeta_{22}[n] + S_c * (K_r - i * \varpi) * \zeta_{22}[n] - S_c * S_p * \partial_\eta (\partial_\eta \mathcal{G}_{22}[n])) d\eta \right) ds; \tag{3.206}$$

$$\begin{aligned}
\mathcal{G}_{22}[n+1] = & \int_0^\eta \left(\int_0^s \left(-\frac{1}{B} * \partial_\eta \mathcal{G}_{22}[n] + \frac{\phi - i * \varpi}{B} * \mathcal{G}_{22}[n] - \right. \right. \\
& \frac{E_c}{B} * \sum_{k=0}^n (\partial_\eta \psi_{01}[k] * \partial_\eta \bar{\psi}_{22}[n-k]) - \frac{E_c}{B} * \sum_{k=0}^n (\partial_\eta \psi_{02}[k] * \partial_\eta \bar{\psi}_{21}[n-k]) - \\
& \frac{E_c}{B} * \sum_{k=0}^n (\partial_\eta \psi_{21}[k] * \partial_\eta \bar{\psi}_{02}[n-k]) - \frac{E_c}{B} * \sum_{k=0}^n (\partial_\eta \psi_{22}[k] * \partial_\eta \bar{\psi}_{01}[n-k]) \\
& \left. \left. - D_p * \partial_\eta (\partial_\eta \zeta_{22}[n]) \right) d\eta \right) ds; \tag{3.207}
\end{aligned}$$

$$\begin{aligned}
\psi_{22}[n+1] = & \int_0^\eta \left(\int_0^s (-1 * \partial_\eta \psi_{22}[n] + (2 * i * \Omega + F - i * \varpi) * \psi_{22}[n] - \partial_{\eta, \eta, \eta} \psi_{21}[n] - \right. \\
& \left. i * \varpi * \partial_\eta (\partial_\eta \psi_{21}[n]) - G_h * \mathcal{G}_{22}[n] - G_m * \zeta_{22}[n]) d\eta \right) ds; \tag{3.208}
\end{aligned}$$

where $h_{13}, h_{14}, h_{15}, h_{16}, h_{17}, h_{18}, h_{19}, h_{20}, h_{21}, h_{22}, h_{23}, h_{24}, h_{25}, h_{26}, h_{27}, h_{28}, h_{29}$, and h_{30} are constants to be determined. The approximate solutions of the above equations can be written as:

$$\left. \begin{aligned} \zeta_{01} &= \sum_{a=0}^5 \zeta_{01}[a], \vartheta_{01} = \sum_{a=0}^5 \vartheta_{01}[a]; \psi_{01} = \sum_{a=0}^5 \psi_{01}[a], \\ \zeta_{02} &= \sum_{a=0}^5 \zeta_{02}[a], \vartheta_{02} = \sum_{a=0}^5 \vartheta_{02}[a]; \psi_{02} = \sum_{a=0}^5 \psi_{02}[a], \\ \zeta_{11} &= \sum_{a=0}^5 \zeta_{11}[a], \vartheta_{11} = \sum_{a=0}^5 \vartheta_{11}[a]; \psi_{11} = \sum_{a=0}^5 \psi_{11}[a], \\ \zeta_{12} &= \sum_{a=0}^5 \zeta_{12}[a], \vartheta_{12} = \sum_{a=0}^5 \vartheta_{12}[a]; \psi_{12} = \sum_{a=0}^5 \psi_{12}[a], \\ \zeta_{21} &= \sum_{a=0}^5 \zeta_{21}[a], \vartheta_{21} = \sum_{a=0}^5 \vartheta_{21}[a]; \psi_{21} = \sum_{a=0}^5 \psi_{21}[a], \\ \zeta_{22} &= \sum_{a=0}^5 \zeta_{22}[a], \vartheta_{22} = \sum_{a=0}^5 \vartheta_{22}[a]; \psi_{22} = \sum_{a=0}^5 \psi_{22}[a] \end{aligned} \right\} \quad (3.209)$$

Substituting the series solutions (3.209) into equations (3.158) and (3.170) gives the final solution for Velocity, Temperature and Concentration distributions

$$\psi(\eta, t) = \sum_{a=0}^5 \psi_{01}[a](\eta) + V_R \sum_{a=0}^5 \psi_{02}[a](\eta) + \frac{\varepsilon}{2} \left(\sum_{a=0}^5 \psi_{11}[a](\eta) + \right. \quad (3.210)$$

$$\left. V_R \sum_{a=0}^5 \psi_{12}[a](\eta) \right) e^{i\varpi t} + \left(\sum_{a=0}^5 \psi_{21}[a](\eta) + V_R \sum_{a=0}^5 \psi_{22}[a](\eta) \right) e^{-i\varpi t}$$

$$\vartheta(\eta, t) = \sum_{a=0}^5 \vartheta_{01}[a](\eta) + V_R \sum_{a=0}^5 \vartheta_{02}[a](\eta) + \frac{\varepsilon}{2} \left(\sum_{a=0}^5 \vartheta_{11}[a](\eta) + \right. \quad (3.211)$$

$$\left. V_R \sum_{a=0}^5 \vartheta_{12}[a](\eta) \right) e^{i\varpi t} + \left(\sum_{a=0}^5 \vartheta_{21}[a](\eta) + V_R \sum_{a=0}^5 \vartheta_{22}[a](\eta) \right) e^{-i\varpi t}$$

$$\zeta(\eta, t) = \sum_{a=0}^5 \zeta_{01}[a](\eta) + V_R \sum_{a=0}^5 \zeta_{02}[a](\eta) + \frac{\varepsilon}{2} \left(\sum_{a=0}^5 \zeta_{11}[a](\eta) + \right. \quad (3.212)$$

$$\left. V_R \sum_{a=0}^5 \zeta_{12}[a](\eta) \right) e^{i\varpi t} + \left(\sum_{a=0}^5 \zeta_{21}[a](\eta) + V_R \sum_{a=0}^5 \zeta_{22}[a](\eta) \right) e^{-i\varpi t}$$

Chapter 4

RESULTS AND DISCUSSION OF THE FINDINGS

4.1 Radiation and Chemical reaction effects on Convective Rotatory Rivlin-Ericksen flow past a Porous Vertical Plate

The partial differential equations (3.13)-(3.15), are solved analytically using perturbation method along with the boundary condition equations (3.16) and (3.17). The effects of various imbedded parameters in the governing equations are presented in graphical and tabular forms. Some of the parameters considered are; permeability of the porous medium k_p , radiation parameter E , chemical reaction parameter K_r , Prandtl number P_r , magnetic parameter M , Schmidt number S_c , Grashof number for mass transfer G_m , Grashof number for heat transfer G_h , dimensionless viscoelasticity parameter of the Rivlin-Ericksen fluid V_R , suction velocity parameter A , scalar constant ε and dimensionless heat absorption coefficient ϕ . Unless otherwise stated, the values used for the computations are: $t = 1$, $G_h = G_m = M = 5$, $V_R = 0.05$, $\phi = 0.005$, $P_r = 0.71$, $E = 3$, $\varepsilon = 0.2$, $A = k_p = 0.5$, $K_r = 2$ and $S_c = 1.002$.

The effects of chemical reaction K_r on $R_v(\eta)$ and $\zeta(\eta)$ profiles is presented in Figures 4.1 and 4.2. It is noticed that increase in the K_r parameter resulted in a decrease in $R_v(\eta)$ and an increase in $\zeta(\eta)$ profiles. This is as a result of the destructive chemical that reduces the boundary thickness and consequently, increases the mass transfer. Figures 4.3 and 4.4 show variation of values of S_c on the $R_v(\eta)$ and $\zeta(\eta)$ profiles. From these figures, it can be observed that $R_v(\eta)$ and $\zeta(\eta)$ decrease as S_c increases. Variational effect of M on R_v profiles is presented in Figure 4.5. In this figure, it is discovered that increase in M accelerate the $R_v(\eta)$ distribution.

Moreover, the effect of the variation of different values of Ω is presented in Figure 4.6. $R_v(\eta)$ distribution is enhanced with the increase in the values of Ω . Figure 4.7 displayed the variation of dimensionless heat absorption coefficient ϕ on $\vartheta(\eta)$ distribution. Naturally, the application of heat absorption coefficient is capable of decreasing the fluid temperature. Consequently, there is net reduction in the velocity of the fluid as a result of decrease in the thermal buoyancy effects. It is obvious from Figure 4.7 that an increase in ϕ , decreases the $\vartheta(\eta)$ profiles up to $\eta = 0.3$, after which a reverse effect is noticed. Again, Figures 4.8 and 4.9 depict the effect of E on $R_v(\eta)$ and $\vartheta(\eta)$ profiles, respectively. It is detected that velocity and $\vartheta(\eta)$ decreased with increase in E . In Figure 4.10, the effect of k_p is shown on $R_v(\eta)$.

It can be observed that $R_v(\eta)$ distribution decreases due to increase in permeability parameter.

Furthermore, Figures 4.11 and 4.12 deal with the variation of G_h and G_m respectively. It is discovered from these figures, that $R_v(\eta)$ profiles is enhanced as a result of an increase in both G_h and G_m . It is noticed that, the cooling of the plate correspond to the positive values of G_h . It can also be observed that there is rapid increase of velocity near the wall of the plate with an increase in G_h . In Figure 4.13, the effects of V_R on velocity is shown. These effects depict that as V_R increases, velocity profiles also increases. Also, Figures 4.14 and 4.15 illustrate the effect of variation in the values of Prandtl number P_r on $R_v(\eta)$ and $\theta(\eta)$ distribution. Prandtl number is the ratio of momentum diffusivity to thermal diffusivity. It is noteworthy to know that lower thermal conductivity material results in high velocity and vice versa. From Figure 4.14, it is obvious that R_v decreases with an increase in P_r . Moreover, Figure 4.15 depicts that temperature distribution reduces with an increase in Pr . Result showed that an increase in P_r makes the thermal conductivity of the fluid to increase. Hence, as the values of P_r is becoming higher, heat diffuses away more rapidly from the heated surface.

Figures 4.16, 4.17 and 4.18 reveal the effects of the ε on $R_v(\eta)$, $\theta(\eta)$ and $\zeta(\eta)$ distribution for different values of ε . It is noticed that $\theta(\eta)$ and $\zeta(\eta)$ profiles accelerate as the value of ε increases. On the contrast, an increase in ε , decreases the R_v profiles. Furthermore, effect of A on R_v and $\zeta(\eta)$ profiles is presented in Figures 4.19 and 4.20. It is obvious that $\zeta(\eta)$ distributions decreases as A increases, while R_v profiles is enhanced.

Finally, effect of V_R on $\tau(\eta)$ is observed at $\eta = 0$ and $\eta = 1$ in Figures 4.21 and 4.22, respectively. Viscoelasticity parameter of the Rivlin-Ericksen fluid V_R accelerate the $\tau(\eta)$ both at $\eta = 0$ and $\eta = 1$. Likewise, Figures 4.23 and 4.24 depict the effect of P_r on $Nu(\eta)$ at $\eta = 0$ and $\eta = 1$, respectively. At $\eta = 0$, increase in P_r makes the heat transfer coefficient to increase, while the $Nu(\eta)$ is slow down as P_r accelerates. The effects of K_r on Mass transfer coefficient both at $\eta = 0$ and $\eta = 1$ is presented in Figures 4.25 and 4.26, respectively. It is observed that an increase in K_r enhances the $Sh(\eta)$ both at $\eta = 0$ and $\eta = 1$.

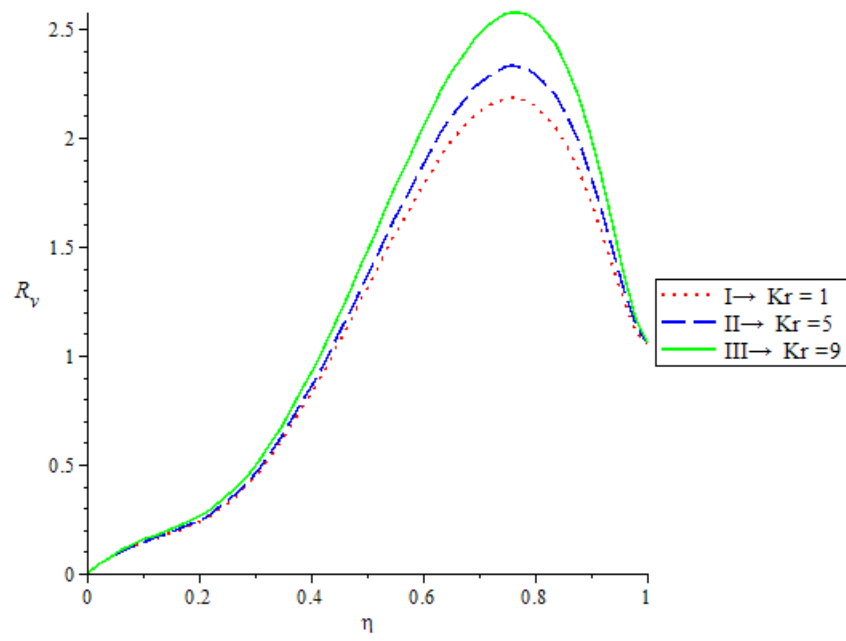


Figure 4.1. Variation of R_v with K_r

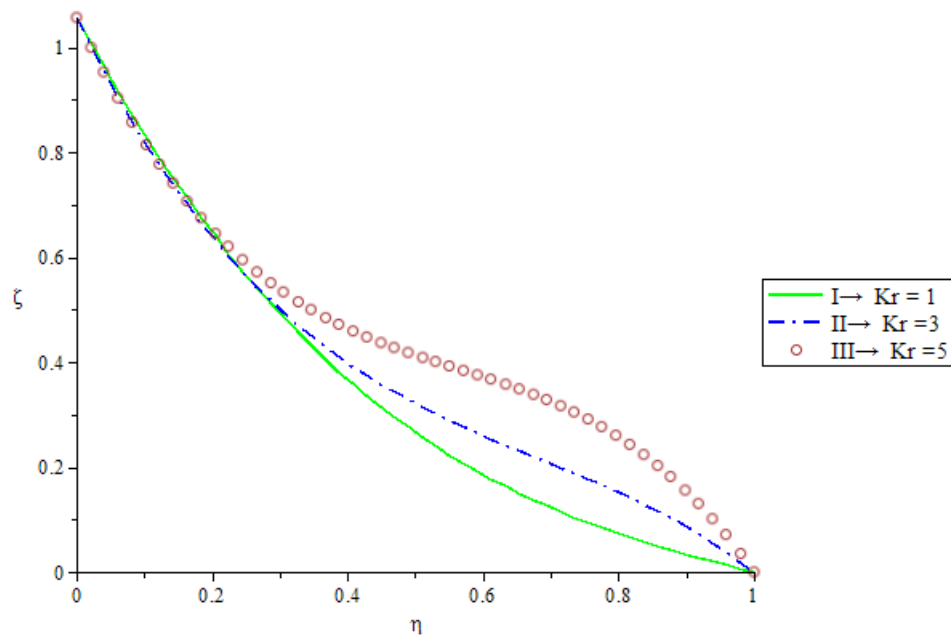


Figure 4.2. Variation of ζ with K_r

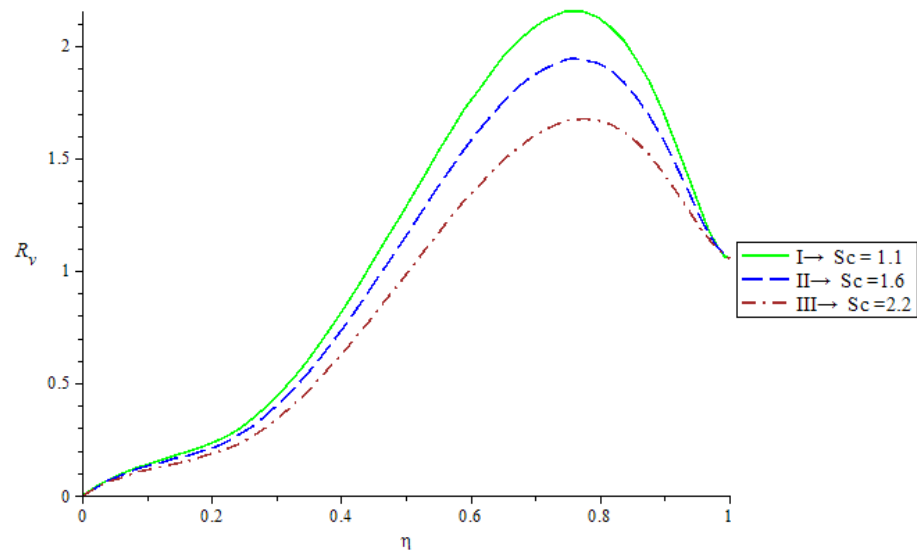


Figure 4.3. Variation of R_v with Sc

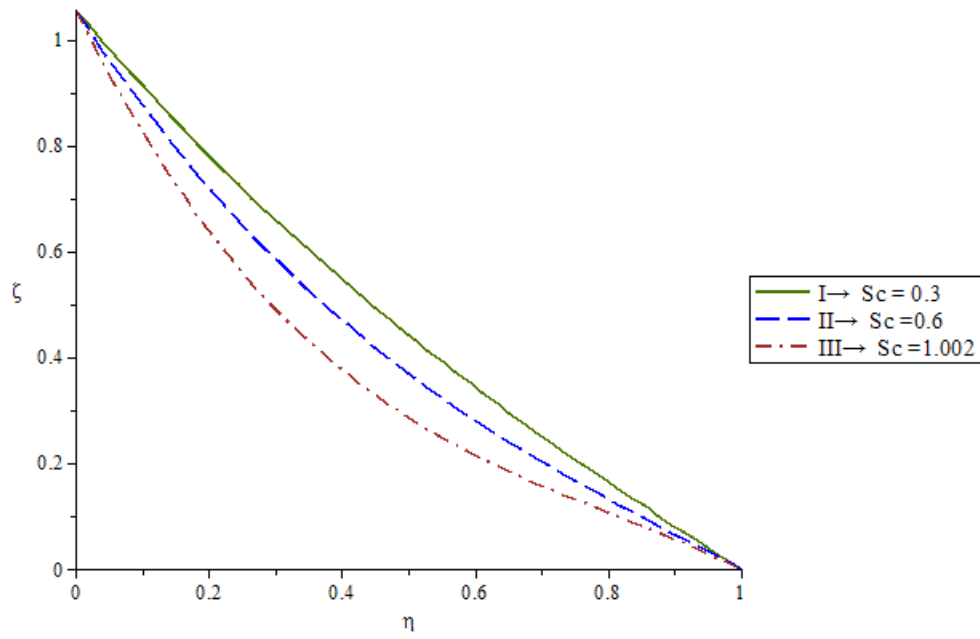


Figure 4.4. Variation of ζ with Sc

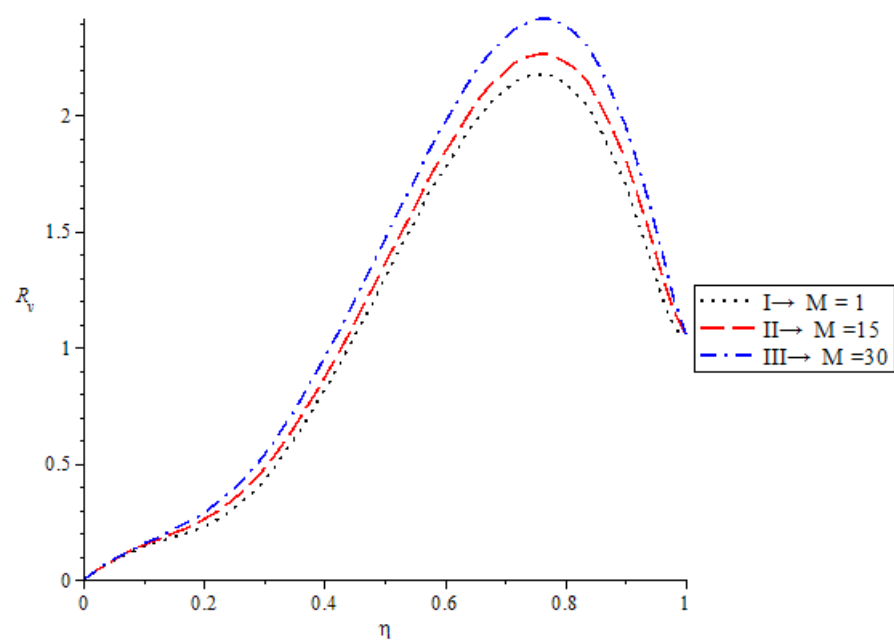


Figure 4.5. Variation of R_v with M

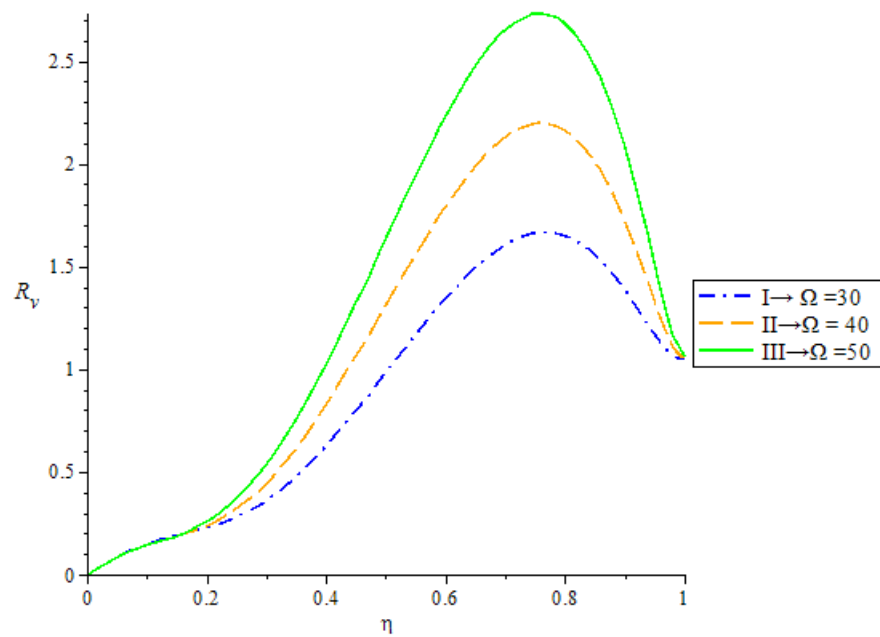


Figure 4.6. Variation of R_v with Ω

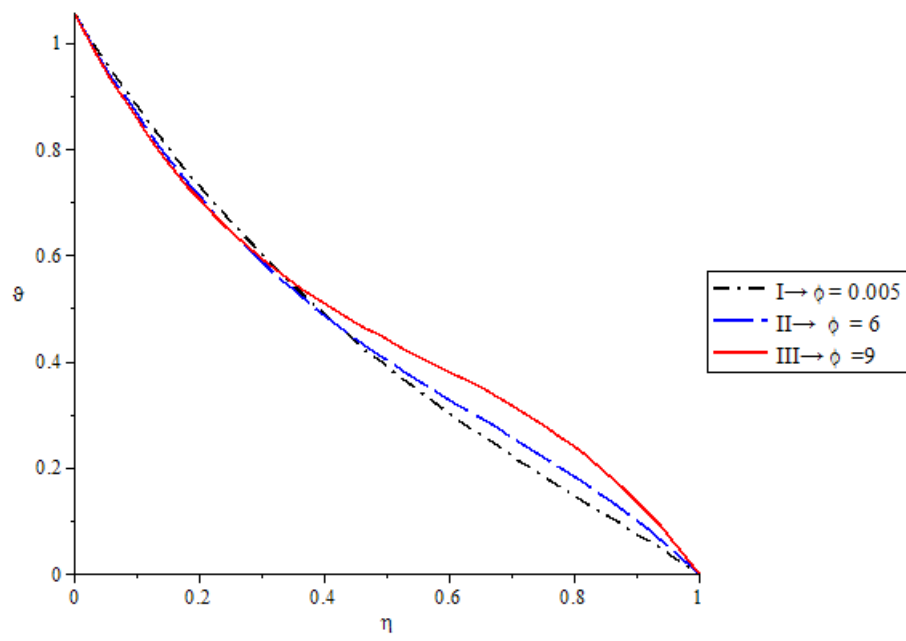


Figure 4.7. Variation of g with ϕ

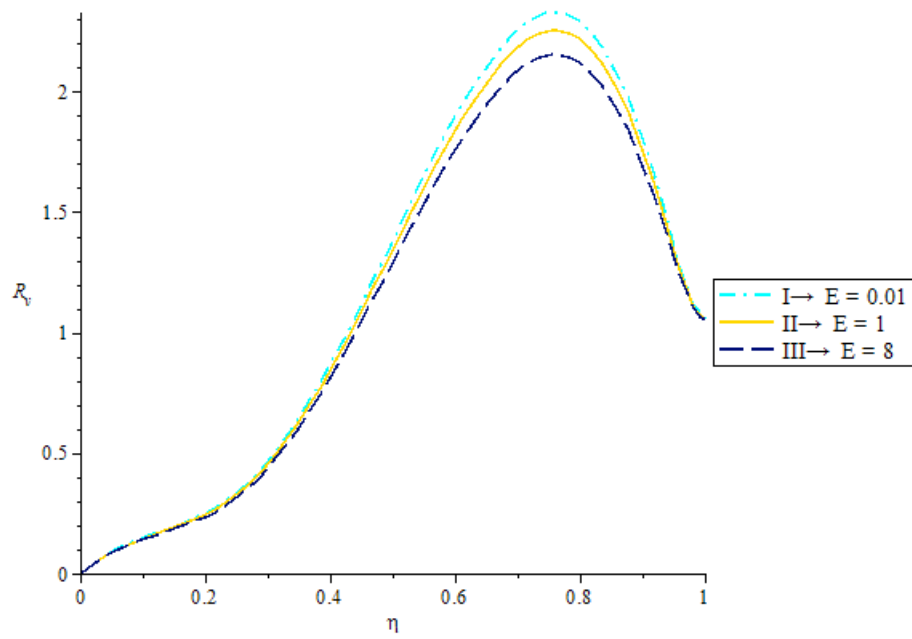


Figure 4.8. Variation of R_v with E

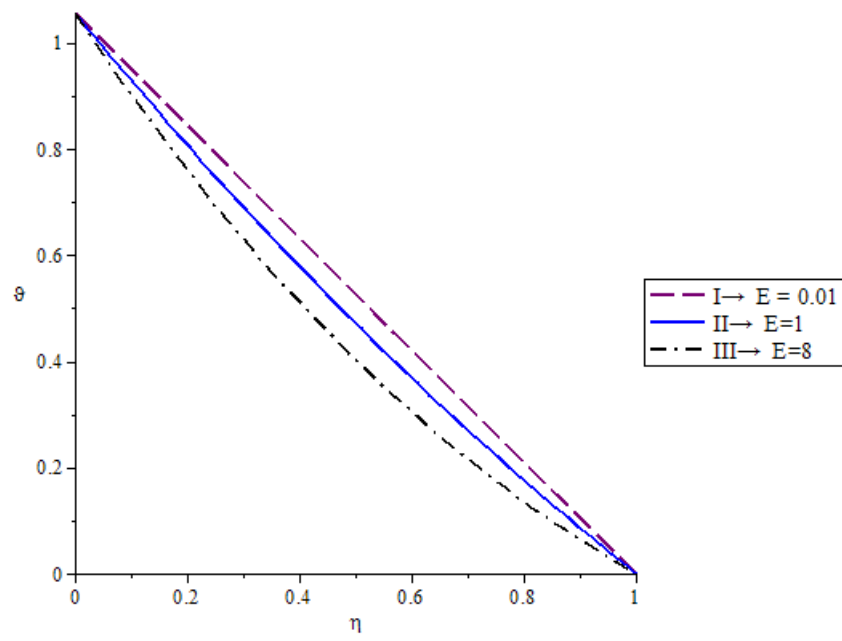


Figure 4.9. Variation of g with E

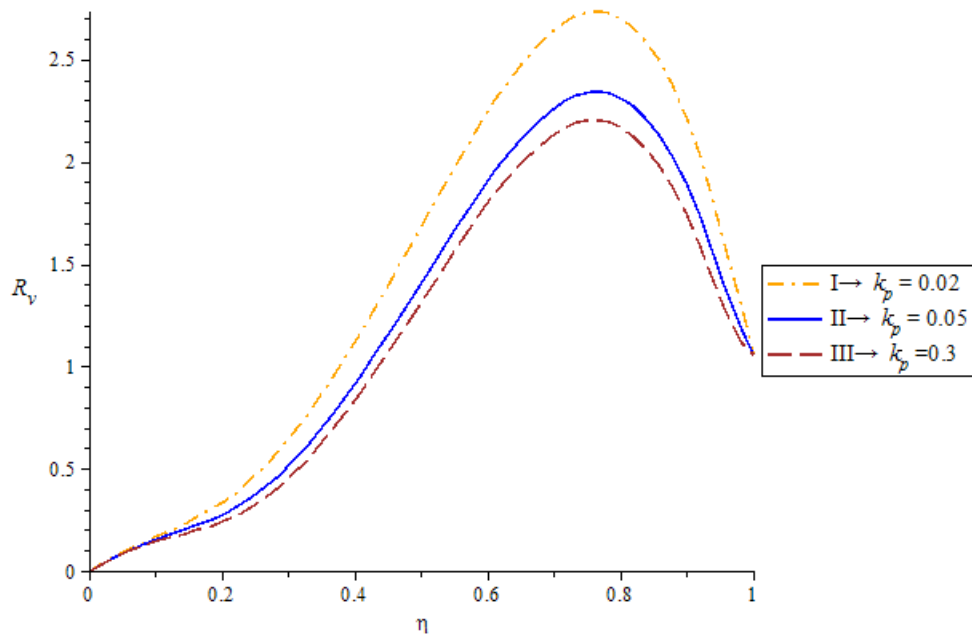


Figure 4.10. Variation of R_v with k_p

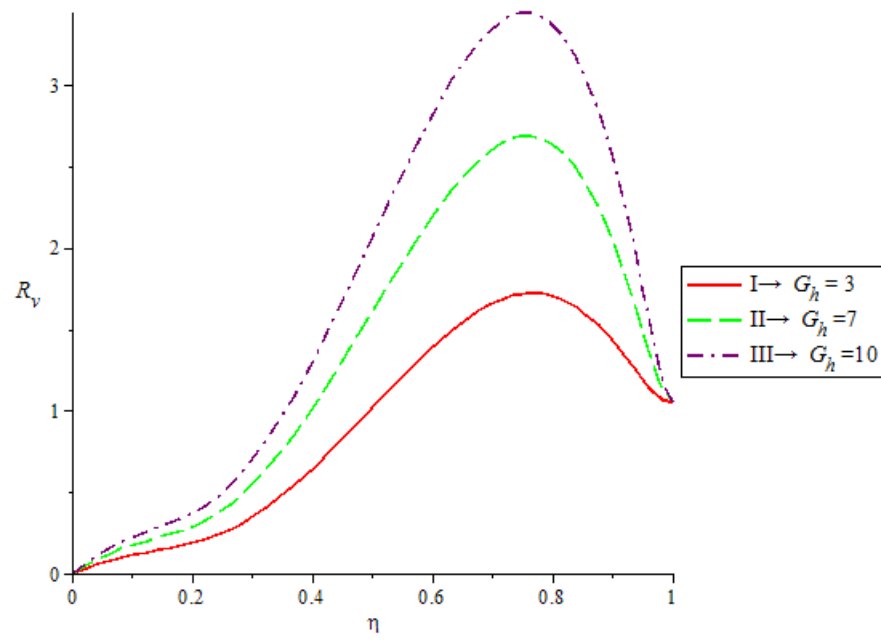


Figure 4.11. Variation of R_v with G_h

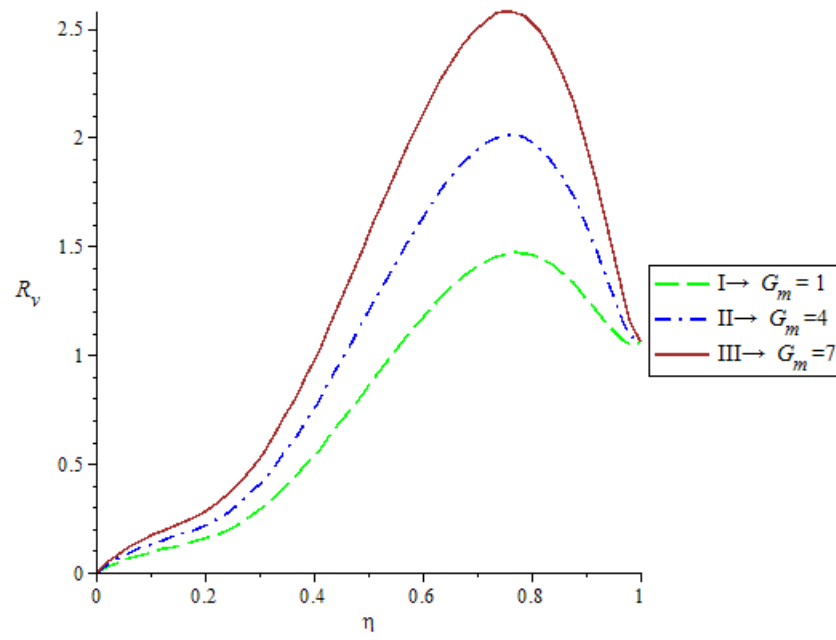


Figure 4.12. Variation of R_v with G_m

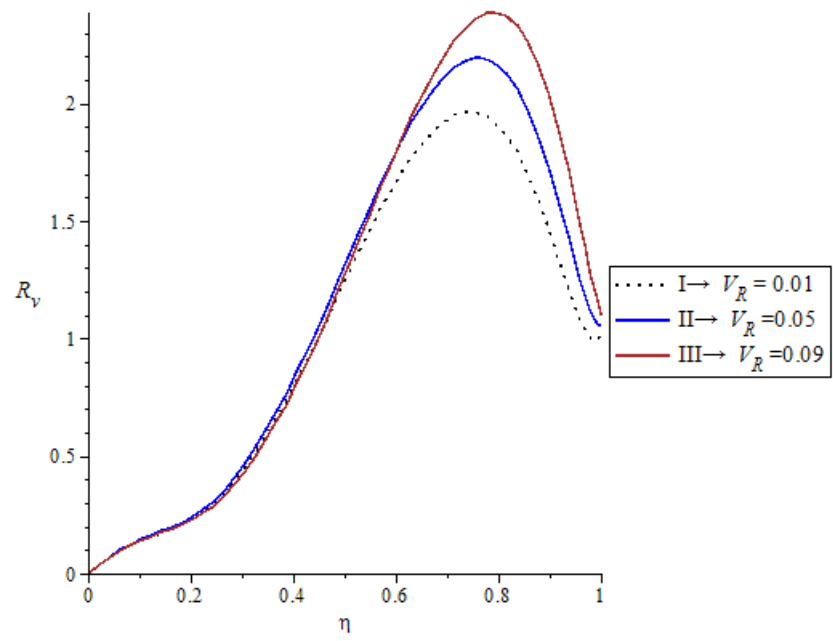


Figure 4.13. Variation of R_v with V_R

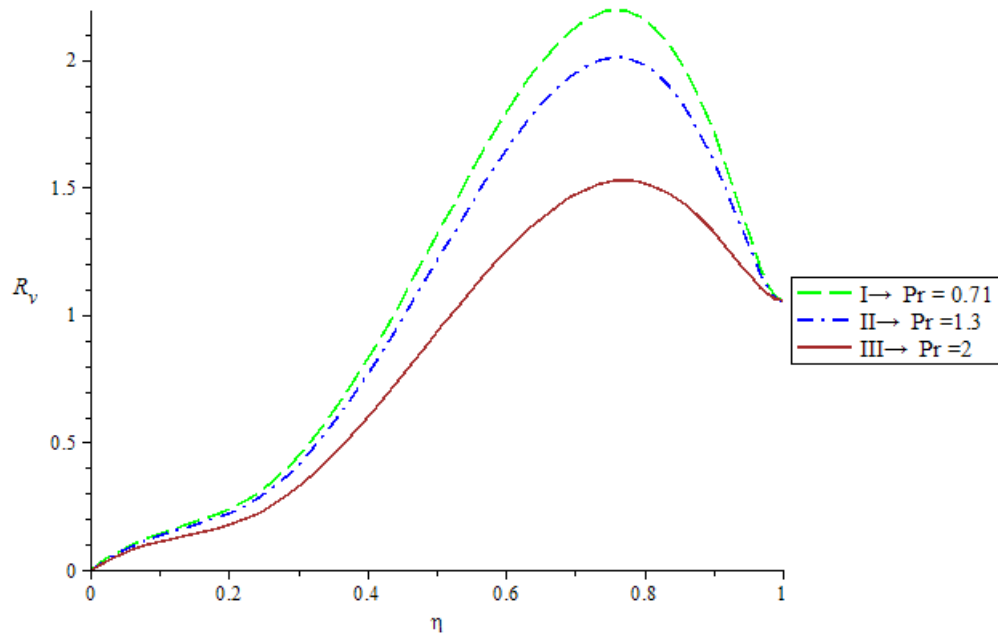


Figure 4.14. Variation of R_v with Pr

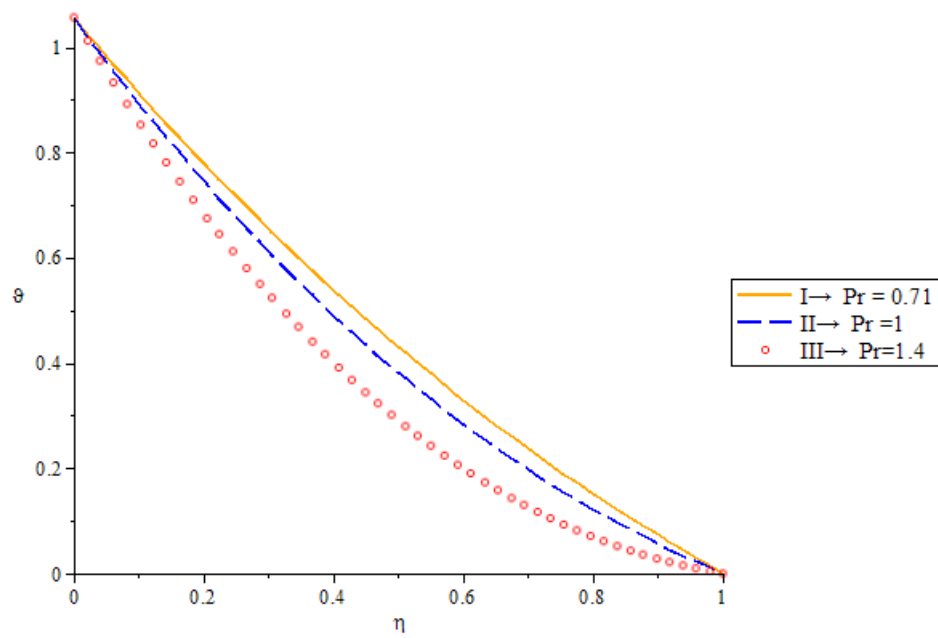


Figure 4.15. Variation of ϑ with Pr

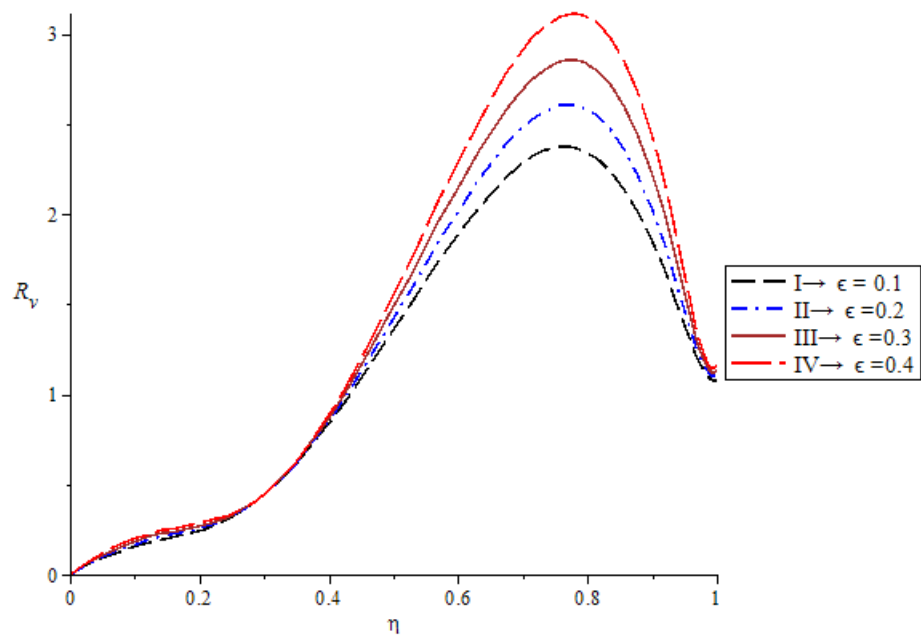


Figure 4.16. Variation of R_v with ϵ

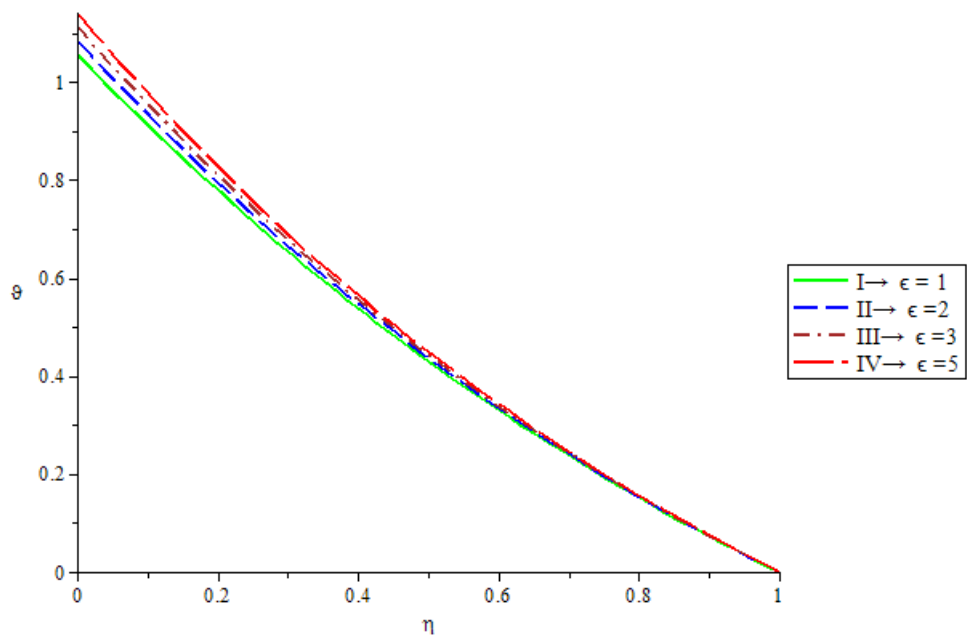


Figure 4.17. Variation of ϑ with ϵ

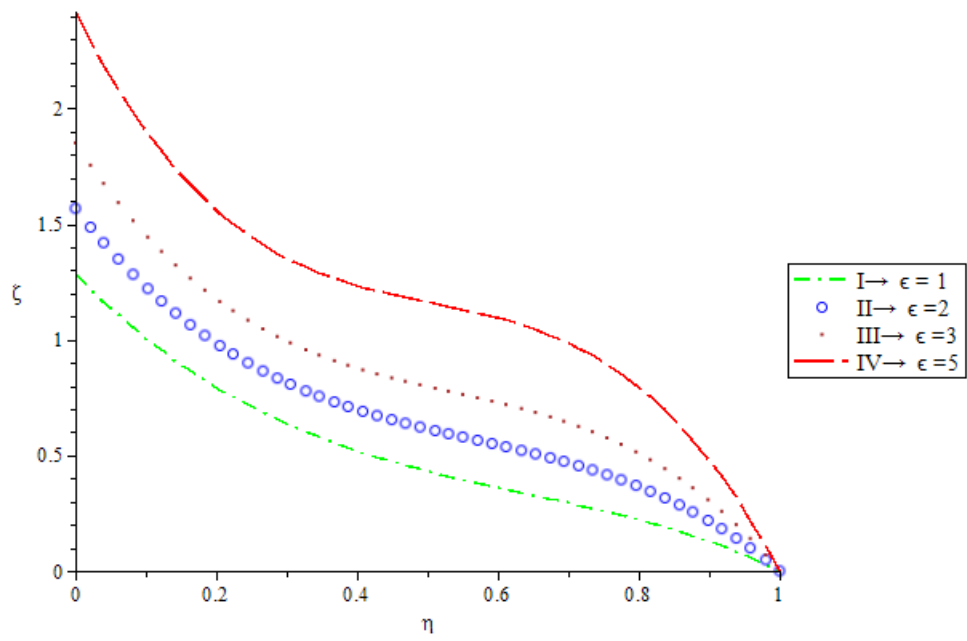


Figure 4.18. Variation of ζ with ϵ

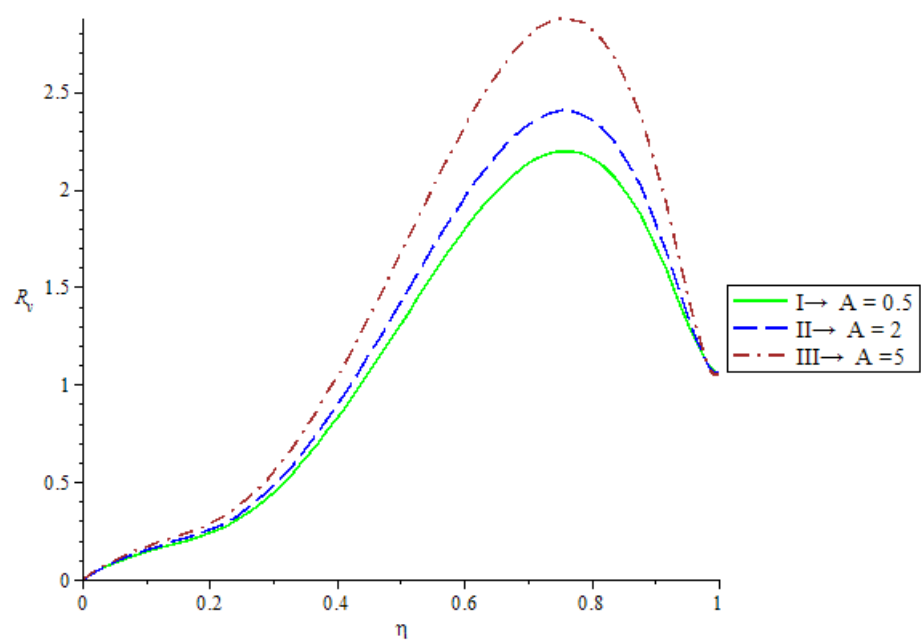


Figure 4.19. Variation of R_v with A

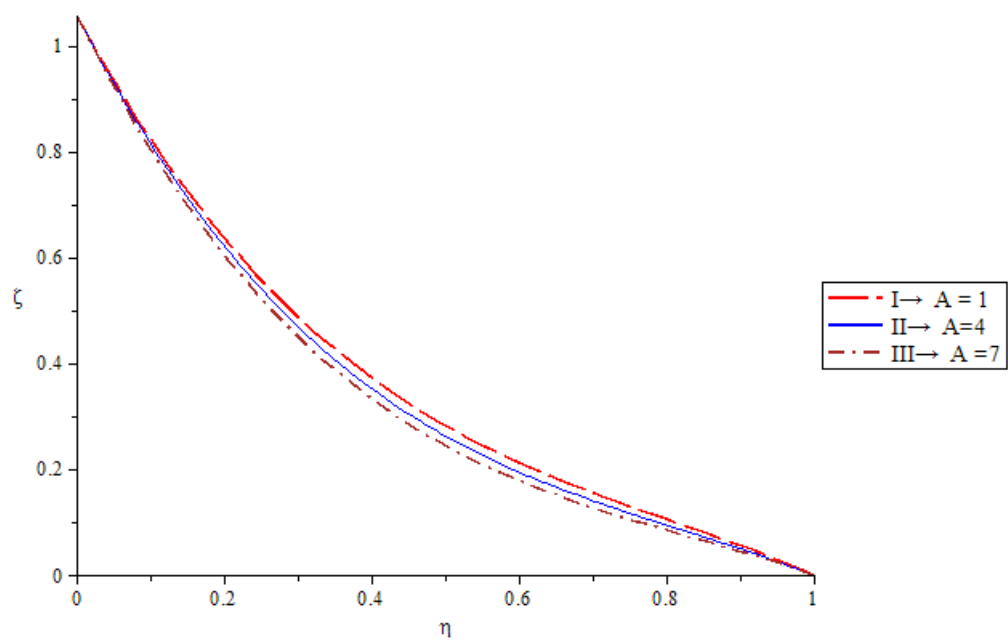


Figure 4.20. Variation of ζ with A

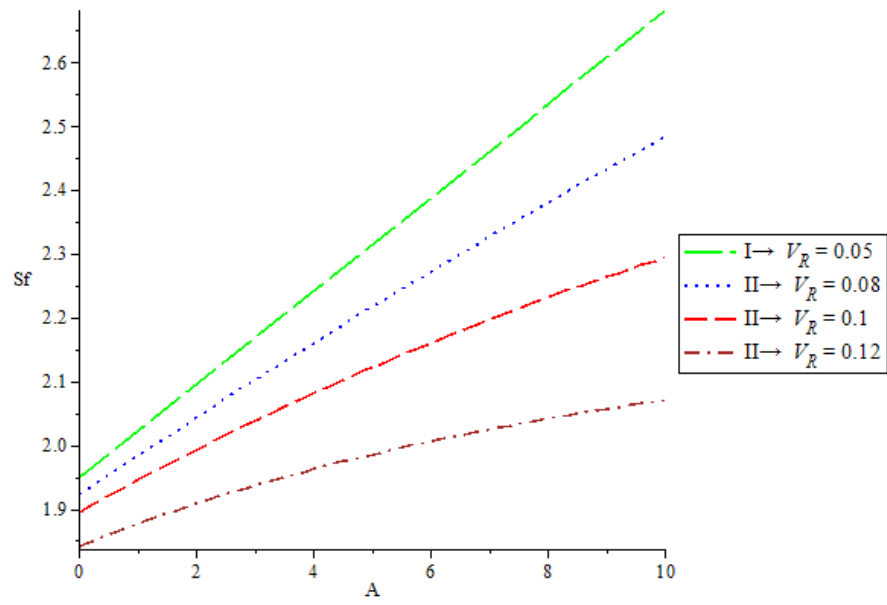


Figure 4.21. Skin-friction for different values of V_R at $\eta = 0$

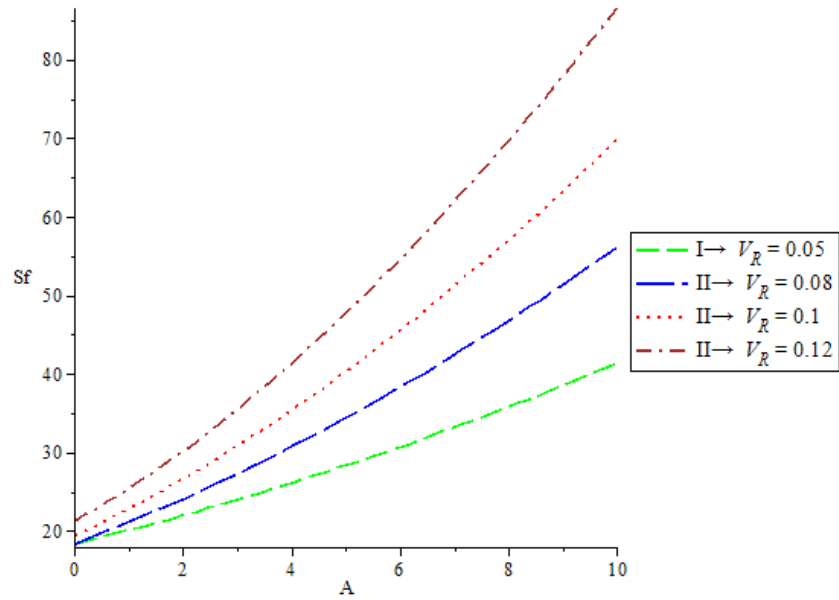


Figure 4.22. Skin-friction for different values of V_R at $\eta = 1$

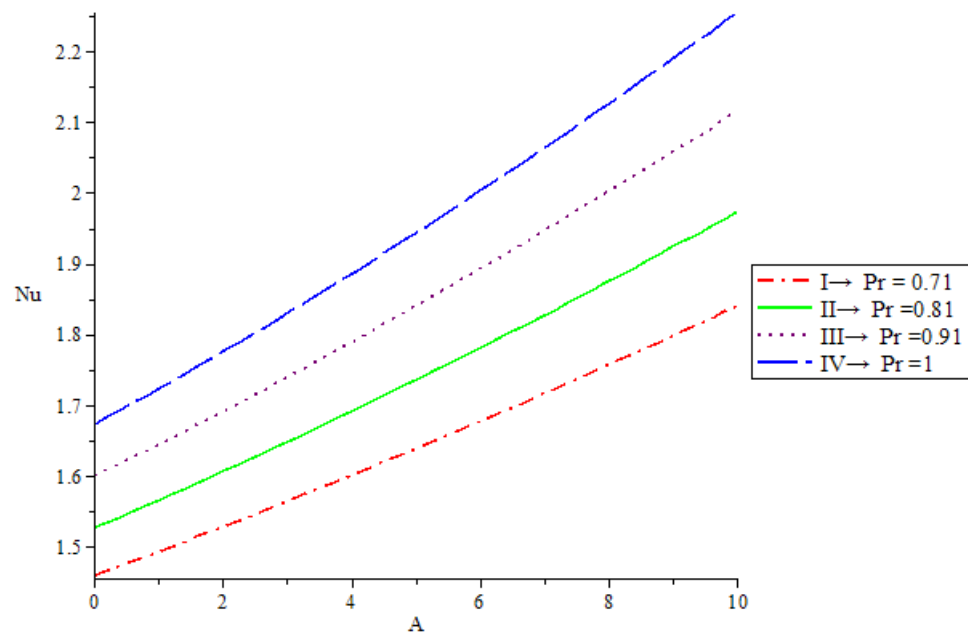


Figure 4.23. Nusselt number for different values of Pr at $\eta = 0$

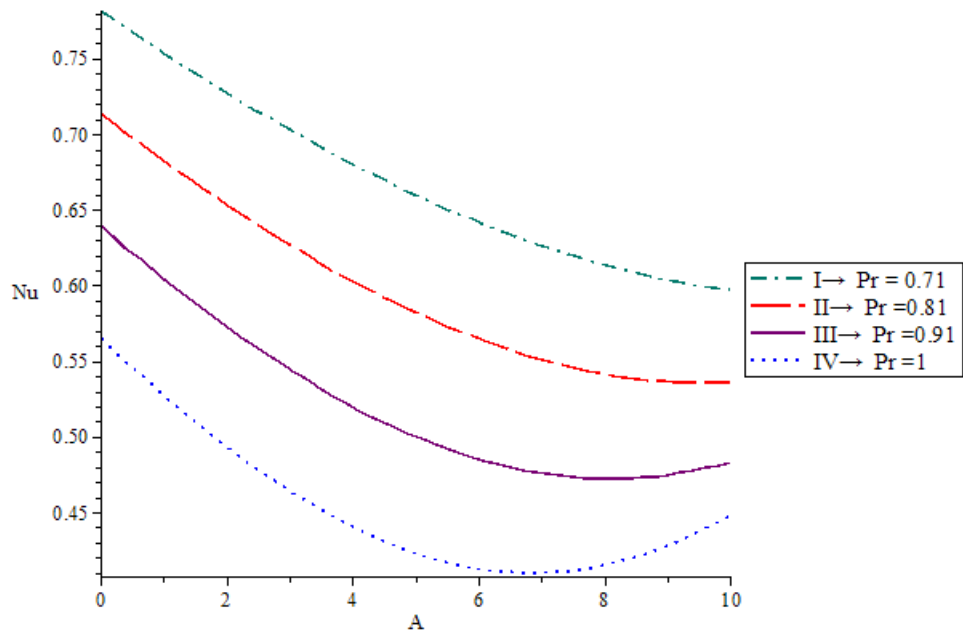


Figure 4.24. Nusselt number for different values of Pr at $\eta = 1$

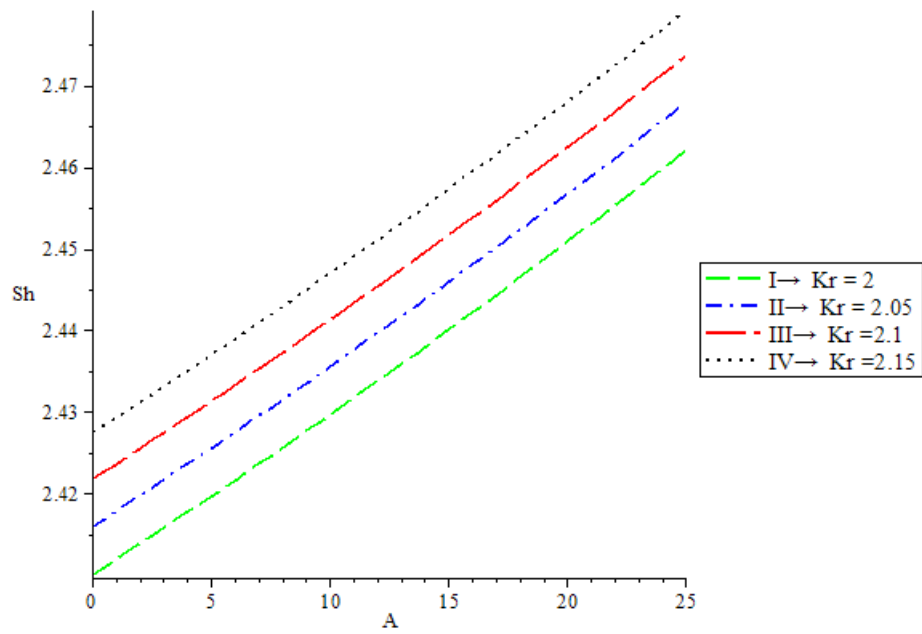


Figure 4.25. Sherwood number for different values of Kr at $\eta = 0$

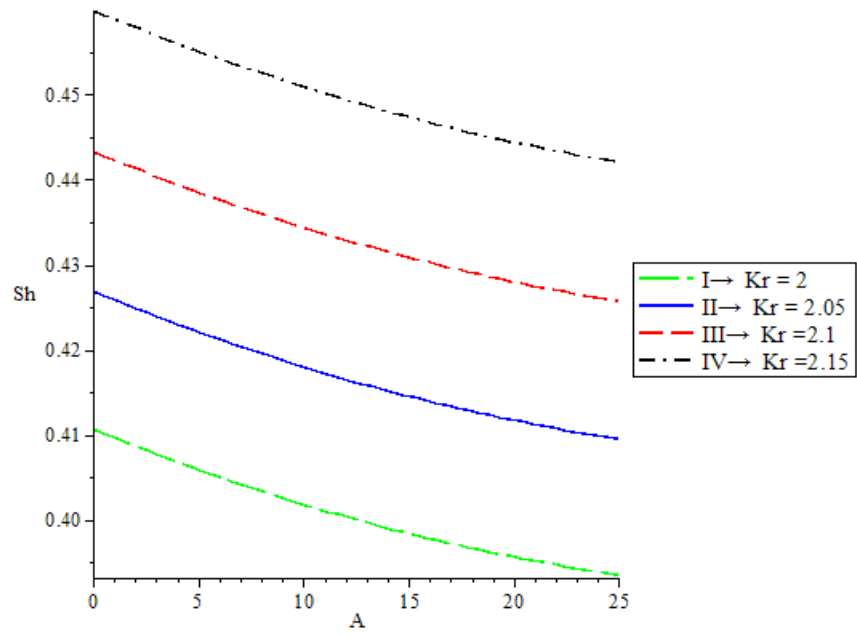


Figure 4.26. Sherwood number for different values of Kr at $\eta = 1$

Table 4.1 shows the variation of fluid parameters (K_r , S_c , M , ϕ , E and k_p) on $\tau(\eta)$, $Nu(\eta)$ and $Sh(\eta)$ at $\eta = 0$. Likewise, variation of fluid parameters (G_h , G_m , V_R , P_r , ε and A) on $\tau(\eta)$, $Nu(\eta)$ and $Sh(\eta)$ at $\eta = 0$ is presented in Table 4.2. While, Table 4.3 depicts the variation of fluid parameters (K_r , S_c , M , ϕ , E and k_p) on $\tau(\eta)$, $Nu(\eta)$ and $Sh(\eta)$ at $\eta = 1$. Also, variation of fluid parameters (G_h , G_m , V_R , P_r , ε and A) on $\tau(\eta)$, $Nu(\eta)$ and $Sh(\eta)$ at $\eta = 1$ is shown in table 4.4.

It is observed in Table 4.1 and Table 4.2 that an increase in S_c , M , ϕ , E and P_r makes the skin friction to decrease. While, K_r , K , G_h , G_m , V_R , ε and A enhance the value of the $\tau(\eta)$. $Nu(\eta)$ reduces with an increase in E . Increase in the values of ϕ , P_r , ε and A make $Nu(\eta)$ to increase. Likewise, $Sh(\eta)$ increases with an increase in K_r , S_c , ε and A .

Similarly, it is obvious in Table 4.3 and Table 4.4 that $\tau(\eta)$ is accelerated by an increase in K_r , M , G_h , V_R , ε and A . While, reverse effect is noticed for an increase in the values of S_c , ϕ , E , k_p , G_m and P_r . $Nu(\eta)$ is enhanced with an increase in ε but decelerated with increase in the values of ϕ , P_r , E and A . Likewise, increase in K_r and scalar constant accelerated $Sh(\eta)$ and it is decelerated by increasing the values of S_c and A .

Table 4.1: Variation of fluid parameters (K_r , S_c , M , ϕ , E and k_p) on Skin-friction, Nusselt number and Sherwood number at $\eta = 0$.

K_r	S_c	M	ϕ	E	k_p	τ	Nu	Sh
1	1.002	5	0.005	3	0.5	2.744404294	2.135941838	3.487979147
2	1.002	5	0.005	3	0.5	2.751522585	2.135941838	3.694249816
3	1.002	5	0.005	3	0.5	2.772263239	2.135941838	3.830172846
2	0.30	5	0.005	3	0.5	2.940620440	2.135941838	2.140523011
2	0.60	5	0.005	3	0.5	2.862071493	2.135941838	2.781200058
2	1.002	5	0.005	3	0.5	2.751522585	2.135941838	3.694249816
2	1.002	1	0.005	3	0.5	4.214488046	2.135941838	3.694249817
2	1.002	3	0.005	3	0.5	3.707778006	2.135941838	3.694249817
2	1.002	5	0.005	3	0.5	3.394266287	2.135941838	3.694249817
2	1.002	5	0.005	3	0.5	2.751522587	2.135941839	3.694249817
2	1.002	5	0.1	3	0.5	2.748753050	2.159435663	3.694249817
2	1.002	5	0.3	3	0.5	2.743247991	2.207383050	3.694249817
2	1.002	5	0.005	1	0.5	2.788944863	3.029919804	3.694249817
2	1.002	5	0.005	2	0.5	2.765532952	2.354841306	3.694249817
2	1.002	5	0.005	3	0.5	2.751522586	2.135941838	3.694249817
2	1.002	5	0.005	3	0.3	2.711333567	2.135941838	3.694249817
2	1.002	5	0.005	3	1	2.785445706	2.135941838	3.694249817
2	1.002	5	0.005	3	1.5	2.797574313	2.135941838	3.694249817

Table 4.2: Variation of fluid parameters (G_h , G_m , V_R , P_r , ϵ and A) on Skin-friction, Nusselt number and Sherwood number at $\eta = 0$.

G_h	G_m	V_R	P_r	ϵ	A	τ	Nu	Sh
4	5	0.05	0.71	0.2	0.5	2.484190776	2.135941838	3.694249817
5	5	0.05	0.71	0.2	0.5	2.751522587	2.135941838	3.694249817
6	5	0.05	0.71	0.2	0.5	3.021406392	2.135941838	3.694249817
5	1	0.05	0.71	0.2	0.5	2.359703321	2.135941838	3.694249817
5	2	0.05	0.71	0.2	0.5	2.613893913	2.135941838	3.694249817
5	3	0.05	0.71	0.2	0.5	2.871052090	2.135941838	3.694249817
5	5	0.01	0.71	0.2	0.5	2.446841450	2.135941838	3.694249817
5	5	0.03	0.71	0.2	0.5	2.596487361	2.135941838	3.694249817
5	5	0.04	0.71	0.2	0.5	2.673307448	2.135941838	3.694249817
5	5	0.05	0.71	0.2	0.5	2.751522586	2.135941840	3.694249817
5	5	0.05	1	0.2	0.5	2.700662241	2.459866611	3.694249817
5	5	0.05	1.5	0.2	0.5	2.566896985	3.247547291	3.694249817
5	5	0.05	0.71	0.2	0.5	3.270067856	2.135941838	3.694249817
5	5	0.05	0.71	0.3	0.5	3.560313707	2.245843917	3.807203141
5	5	0.05	0.71	0.4	0.5	3.851663736	2.355826685	3.921094250
5	5	0.05	0.71	0.2	0.4	2.740037542	2.133833744	3.689480411
5	5	0.05	0.71	0.2	1	2.809402892	2.146884336	3.719508720
5	5	0.05	0.71	0.2	2	2.927440694	2.170746160	3.776928437

Table 4.3: Variation of fluid parameters (K_r , S_c , M , ϕ , E and k_p) on Skin-friction, Nusselt number and Sherwood number at $\eta = 1$.

K_r	S_c	M	ϕ	E	k_p	τ	Nu	Sh
1	1.002	5	0.005	3	0.5	0.887860708	1.010935111	0.518536326
2	1.002	5	0.005	3	0.5	0.958124455	1.010935111	0.905939922
3	1.002	5	0.005	3	0.5	1.060510365	1.010935111	1.430316411
2	0.30	5	0.005	3	0.5	1.262843716	1.010935111	1.170554564
2	0.60	5	0.005	3	0.5	1.113988326	1.010935111	0.962208721
2	1.002	5	0.005	3	0.5	0.958124460	1.010935111	0.905939923
2	1.002	1	0.005	3	0.5	0.661726254	1.010935111	0.905939922
2	1.002	3	0.005	3	0.5	0.934071366	1.010935111	0.905939922
2	1.002	5	0.005	3	0.5	0.941516808	1.010935111	0.905939922
2	1.002	5	0.005	3	0.5	0.958124451	1.010935112	0.905939922
2	1.002	5	0.1	3	0.5	0.955152338	1.004521382	0.905939922
2	1.002	5	0.3	3	0.5	0.949585614	0.993636868	0.905939922
2	1.002	5	0.005	1	0.5	1.022304474	1.971385220	0.905939922
2	1.002	5	0.005	2	0.5	0.980843272	1.251516620	0.905939922
2	1.002	5	0.005	3	0.5	0.958124452	1.010935110	0.905939922
2	1.002	5	0.005	3	0.3	1.166401117	1.010935111	0.905939922
2	1.002	5	0.005	3	1	0.823955811	1.010935111	0.905939922
2	1.002	5	0.005	3	1.5	0.784887834	1.010935111	0.905939922

Table 4.4: Variation of fluid parameters (G_h , G_m , V_R , P_r , ϵ and A) on Skin-friction, Nusselt number and Sherwood number at $\eta = 1$.

G_h	G_m	V_R	P_r	ϵ	A	τ	Nu	Sh
4	5	0.05	0.71	0.2	0.5	0.660688019	1.010935111	0.905939922
5	5	0.05	0.71	0.2	0.5	0.958124445	1.010935111	0.905939922
6	5	0.05	0.71	0.2	0.5	1.325884834	1.010935111	0.905939922
5	1	0.05	0.71	0.2	0.5	1.374776128	1.010935111	0.905939922
5	2	0.05	0.71	0.2	0.5	1.257799569	1.010935111	0.905939922
5	3	0.05	0.71	0.2	0.5	1.143782227	1.010935111	0.905939922
5	5	0.01	0.71	0.2	0.5	0.269412789	1.010935111	0.905939922
5	5	0.03	0.71	0.2	0.5	0.630492288	1.010935111	0.905939922
5	5	0.04	0.71	0.2	0.5	0.799911946	1.010935111	0.905939922
5	5	0.05	0.71	0.2	0.5	0.958124444	1.010935113	0.905939922
5	5	0.05	1	0.2	0.5	0.888537986	0.783584931	0.905939922
5	5	0.05	1.5	0.2	0.5	0.806774865	0.360823048	0.905939922
5	5	0.05	0.71	0.2	0.5	2.318316012	1.010935111	0.905939922
5	5	0.05	0.71	0.3	0.5	3.232614245	1.033823030	1.078035355
5	5	0.05	0.71	0.4	0.5	4.169132023	1.059178157	1.252421782
5	5	0.05	0.71	0.2	0.4	0.937281455	1.013380615	0.910528321
5	5	0.05	0.71	0.2	1	1.066956246	0.999218937	0.884217841
5	5	0.05	0.71	0.2	2	1.303291665	0.978438077	0.847352658

4.2 Combined effects of Thermal-Diffusion and Diffusion-Thermo on Convective Rotatory Rivlin-Ericksen flow past a Porous Vertical Plate

The solutions for the partial differential equations (3.27)-(3.29) with the corresponding boundary conditions (3.30) are acquired by Adomian decomposition methods alongside with MATHEMATICA programming. The impacts of different parameters governing the flow field on velocity, temperature and species in the fluid are depicted in tabular and graphical forms. The parameters considered in this study includes: dimensionless viscoelasticity parameter of the Rivlin-Ericksen fluid (V_R), suction velocity parameter (A), rotation parameter (Ω), scalar constant (ε), chemical reaction parameter (K_r), radiation parameter (E), Prandtl number (P_r), Schmidt number (S_c), heat absorption coefficient (ϕ), Mass transfer Grashof number (G_m), Heat transfer Grashof number (G_h), permeability of the porous medium (k_p), Dufour parameter (D_p), Soret parameter (S_p) and magnetic parameter (M). Throughout the computations, the following are taken as default values: $t=1$, $G_h = G_m = M = 5$, $V_R = 0.05$, $\phi = 0.005$, $P_r = 0.71$, $E = 3$, $\varepsilon = 0.01$, $A = k_p = 0.5$, $K_r = 2$, $\Omega = 10$, $D_p = 0.1$, $S_p = 2$ and $S_c = 1.002$.

Figures 4.27 and 4.28 depict the effects of (A) on $\zeta(\eta)$ and ($R_v(\eta)$). It is obvious that as Suction velocity parameter increases, ($R_v(\eta)$) and $\zeta(\eta)$ increase. Variation of values of (ε) on $R_v(\eta)$, $\vartheta(\eta)$ and $\zeta(\eta)$ distribution is shown in Figures 4.29, 4.30 and 4.31. It is detected that increasing ε causes a corresponding increment on $R_v(\eta)$, $\vartheta(\eta)$ and $\zeta(\eta)$ profiles. The influence of G_m , G_r and k_p on velocity is illustrated in Figures 4.32, 4.33 and 4.34. From these Figures, $R_v(\eta)$ is enhanced by an increase in G_m , G_h and k_p . Figures 4.35 and 4.36 display the effect of the different values of K_r on $\zeta(\eta)$ and $R_v(\eta)$ profiles. It is observed that the more the value of K_r , the less the species and $R_v(\eta)$.

Figure 4.37 reveals the influence of M on the $R_v(\eta)$. It is clear from the figure that higher values of M decreases the flow velocity throughout the domain of the fluid. A drag force identified as Lorentz force is produced in electrically conducting fluid where magnetic field is applied. There is a decrease in the velocity of the fluid as a result of the effect of this drag force since fluid transport is resisted in the presence of the magnetic field. The contribution of radiation parameter is represented in Figures 4.38 and 4.39. From these figures, it is shown that $\vartheta(\eta)$ distribution is slowdown with a rise in E , However, $R_v(\eta)$ distribution is improved with a rise in E . This result revealed that higher values of radiation parameter are equivalent to increasing dominance conduction over E . Hence, there is reduction in buoyancy force and temperature in the thermal boundary layer.

The effect of Ω on $R_v(\eta)$ is seen in Figure 4.40. The result revealed that, higher values of

Ω enhanced $R_v(\eta)$ profiles, which showed an overwhelming effect of rotation. A diminishing in R_v due to a decrease in Ω is because of the presence of gravitational and Lorentz force rotating at very low speeds. This indicates that a friction factor is noticed, hence R_v decreases. The same trend is apparent in Figures 4.41 and 4.42, which represented R_v and $\vartheta(\eta)$ profiles for different values of P_r . P_r can be defined as the ratio of momentum diffusivity to thermal diffusivity. It is, therefore, obvious that lower thermal conductivity material leads to high velocity and a different trend is seen for higher thermal conductivity. Hence, in Figure 4.42, it is seen that an increase in P_r accelerates the $R_v(\eta)$ profiles. Likewise, in Figure 4.41, an increase in P_r reduces the thermal boundary layer thickness and average temperature within the boundary. This implies that, an increase in P_r makes the thermal conductivity of the fluid to increase, thus, resulted in rapid diffusivity of the heated surface.

Furthermore, effect of S_c on $\zeta(\eta)$ and $R_v(\eta)$ profiles is revealed in Figures 4.43 and 4.44. Here, it is observed that higher S_c leads to a decline in $\zeta(\eta)$ profiles, while the $R_v(\eta)$ is enhanced. Figure 4.45 depicts the variation of different values of (ϕ) . It is seen that with a rise in ϕ , temperature diminished. Thus, when heat is absorbed, the buoyancy force decreases the $\vartheta(\eta)$ profile. It is evident in Figures 4.46 that $R_v(\eta)$ is accelerated by an increase in V_R .

Figures 4.47, 4.48 and 4.49 detect the effects of variation of S_p on $R_v(\eta)$, $\vartheta(\eta)$ and $\zeta(\eta)$ profiles. A careful study of these figures shown that the presence of S_p enhances both R_v and $\zeta(\eta)$ profiles, while different trend is noticed in ϑ profiles. ϑ profiles decline with a rise in S_p . Effect of D_p on R_v , ϑ and $\zeta(\eta)$ profiles is presented in Figures 4.50, 4.51 and 4.52. R_v profiles diminishes as D_p is increased, while ϑ profiles increases. This is as a result of the generation of energy flux that enhances the temperature. A rise in D_p makes $\zeta(\eta)$ profiles to fall within $0 \leq \eta \leq 0.7$. and within $0.7 \leq \eta \leq 1$, a rise in $\zeta(\eta)$ profile is observed.

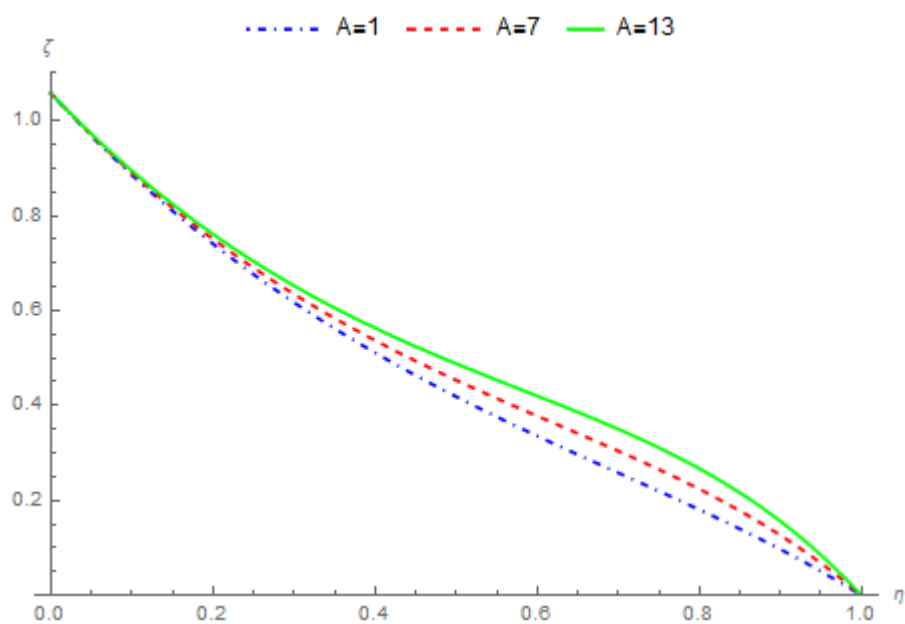


Figure 4.27. Variation of ζ with A

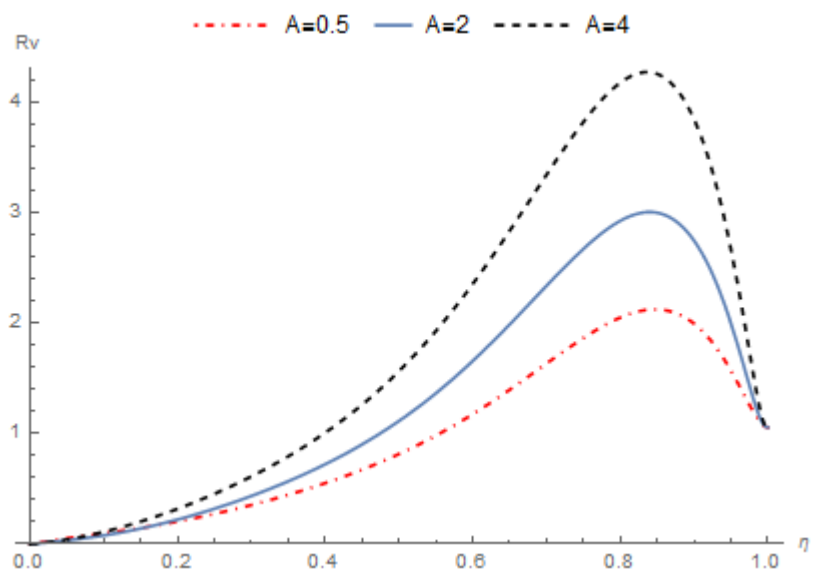


Figure 4.28. Variation of R_v with A

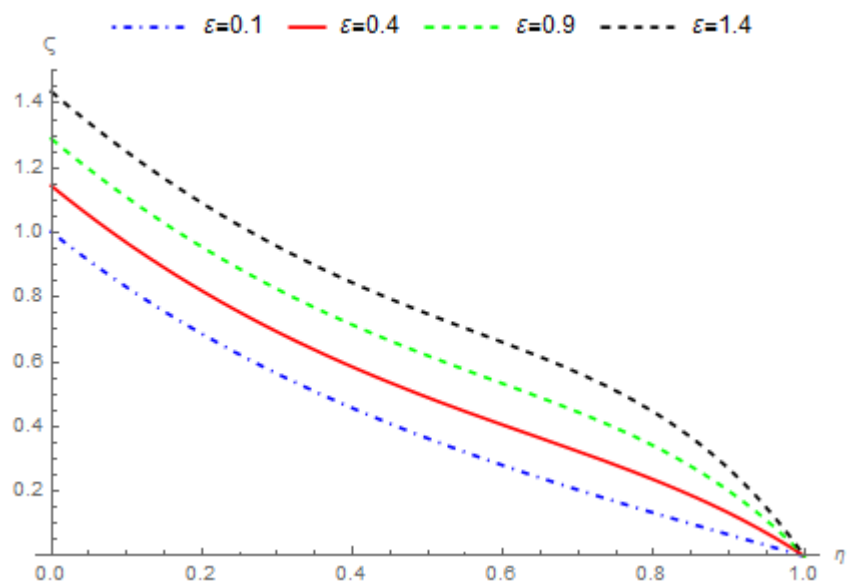


Figure 4.29. Variation of ζ with ε

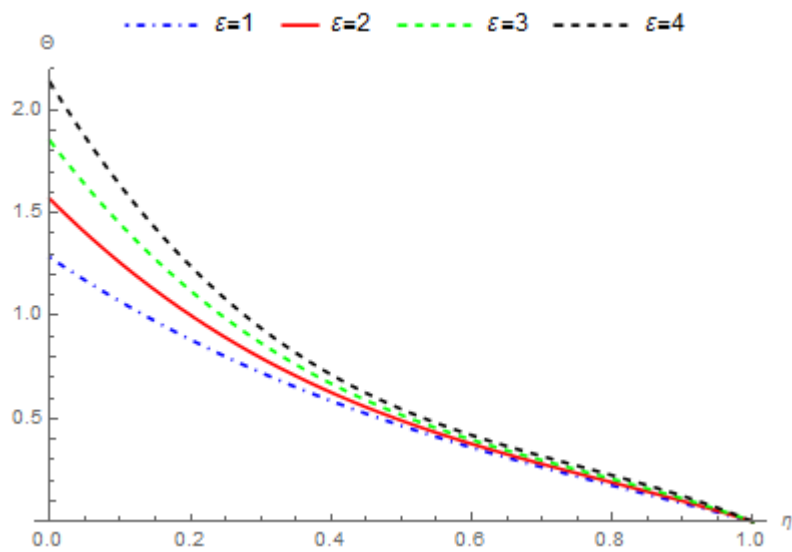


Figure 4.30. Variation of g with ε

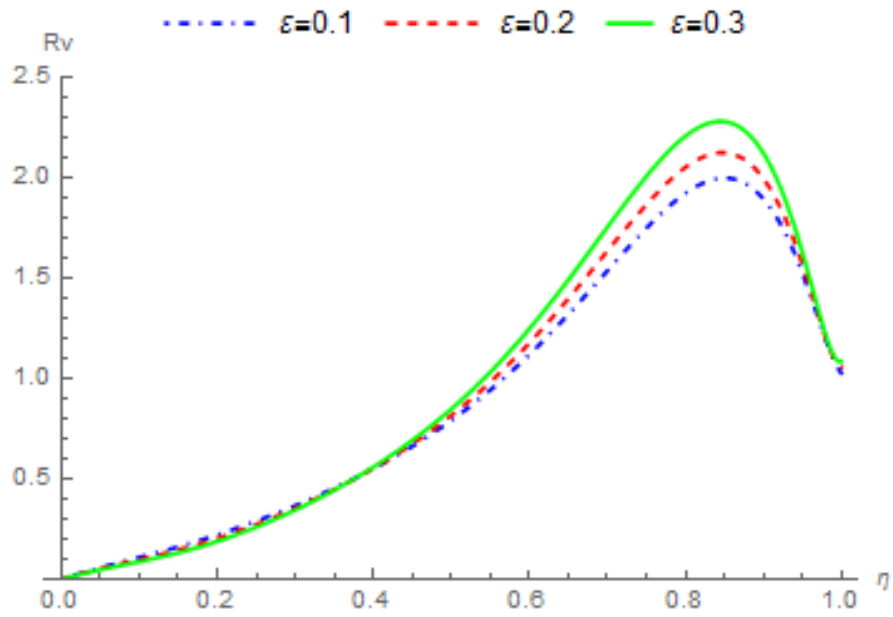


Figure 4.31. Variation of R_v with ε

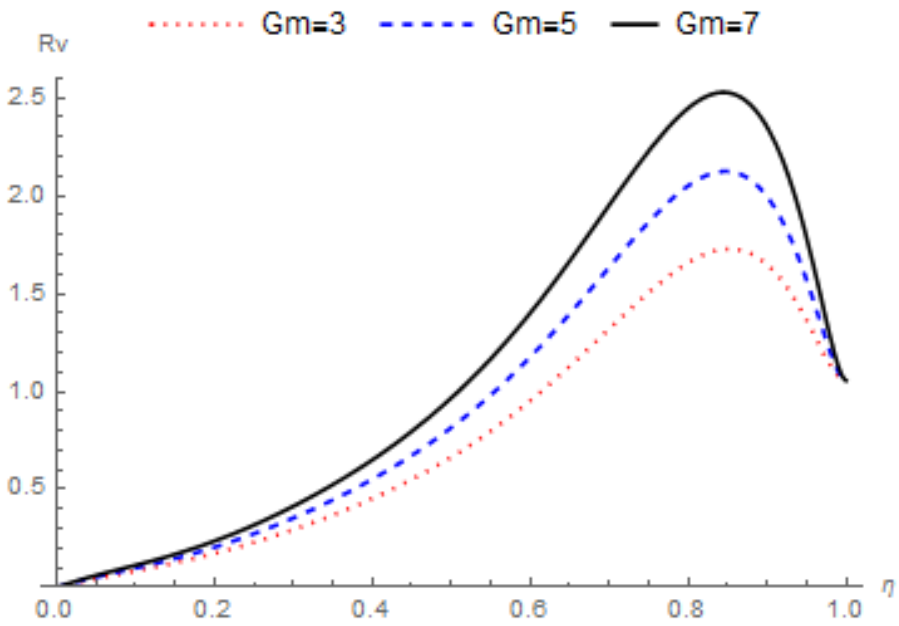


Figure 4.32. Variation of R_v with Gm

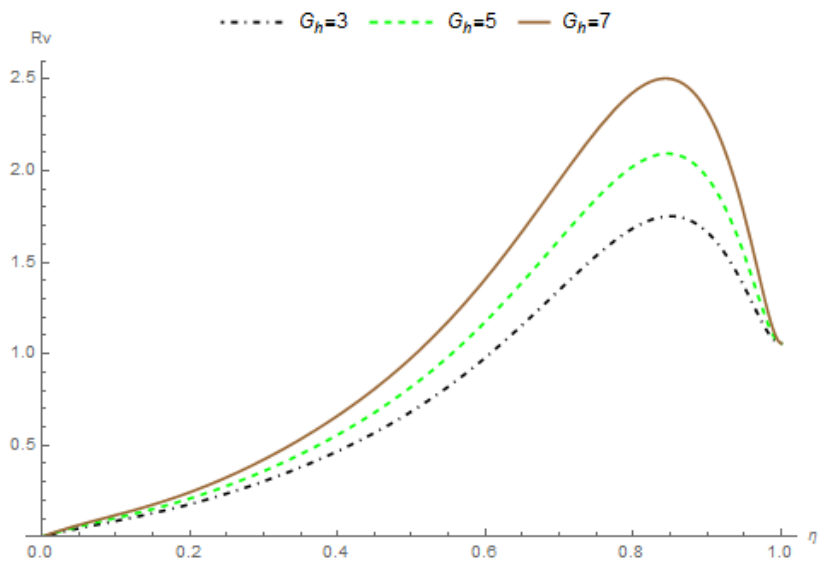


Figure 4.33. Variation of R_v with G_h

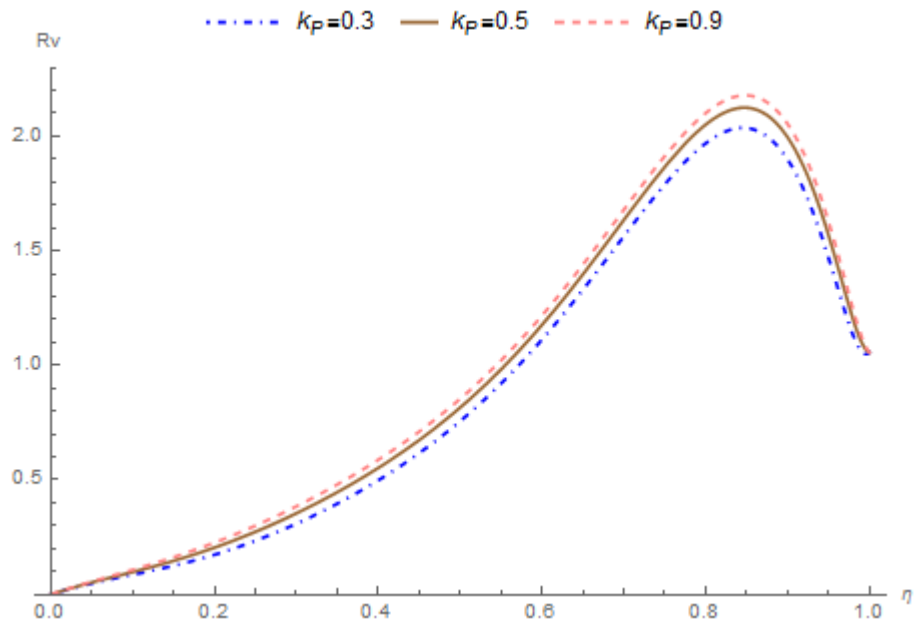


Figure 4.34. Variation of R_v with k_p

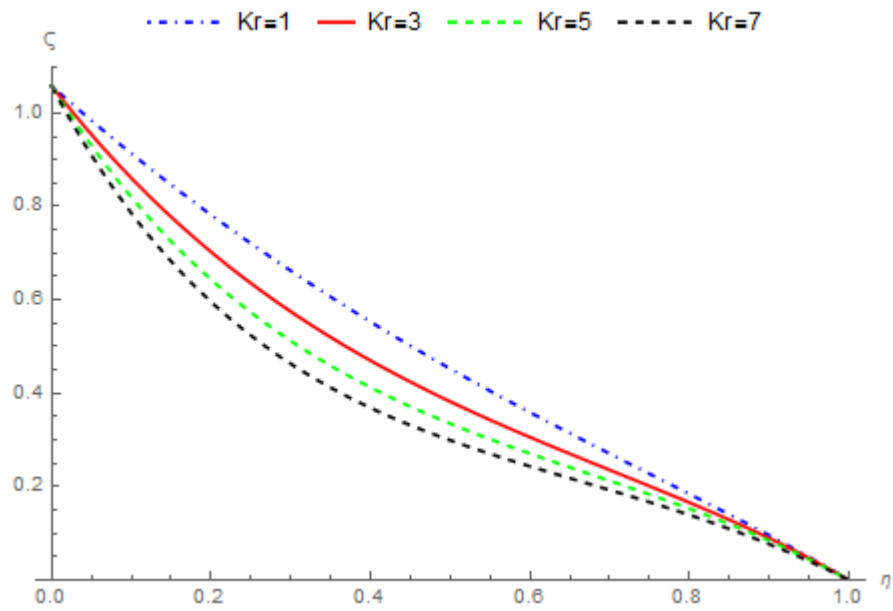


Figure 4.35. Variation of ζ with Kr

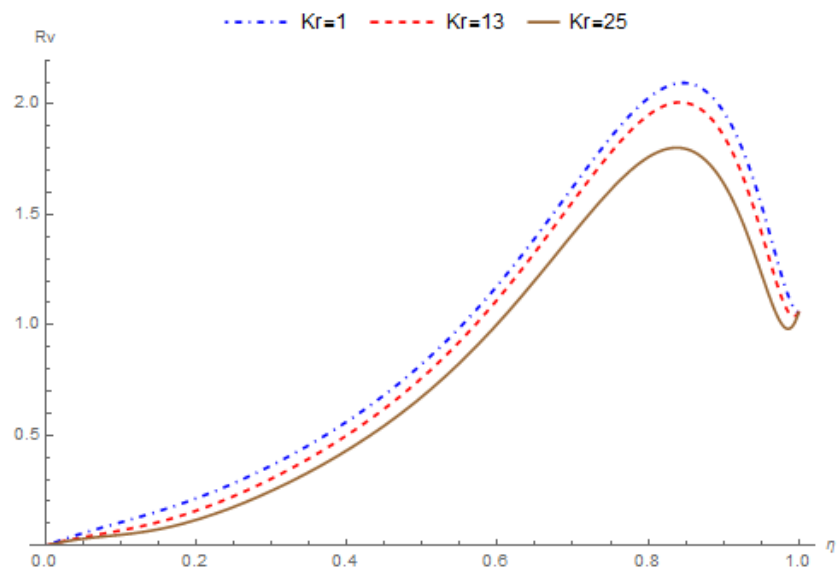


Figure 4.36. Variation of R_v with Kr

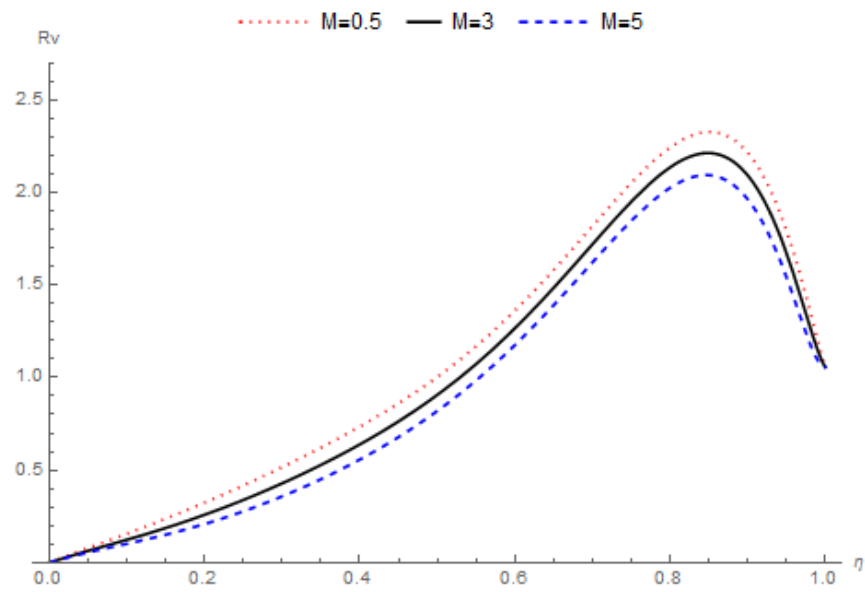


Figure 4.37. Variation of R_v with M

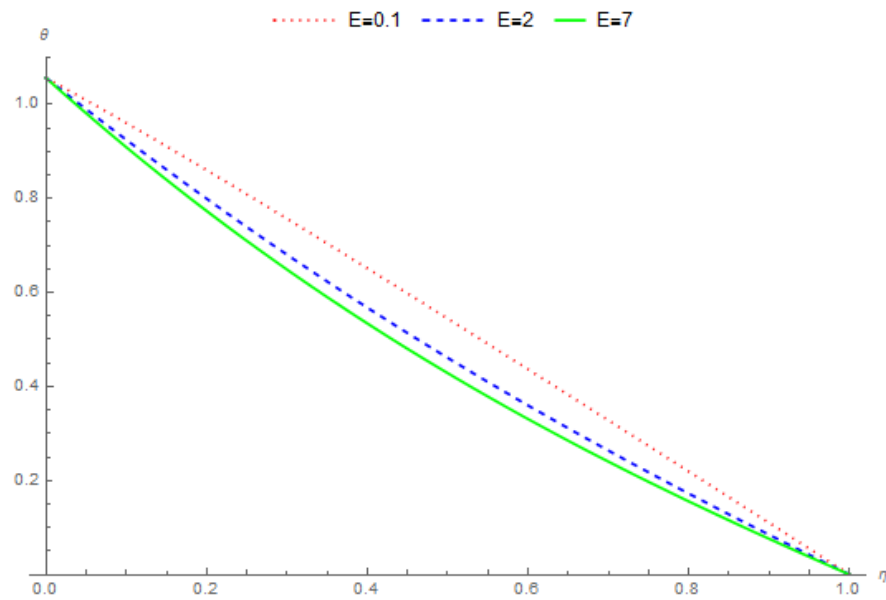


Figure 4.38. Variation of g with E

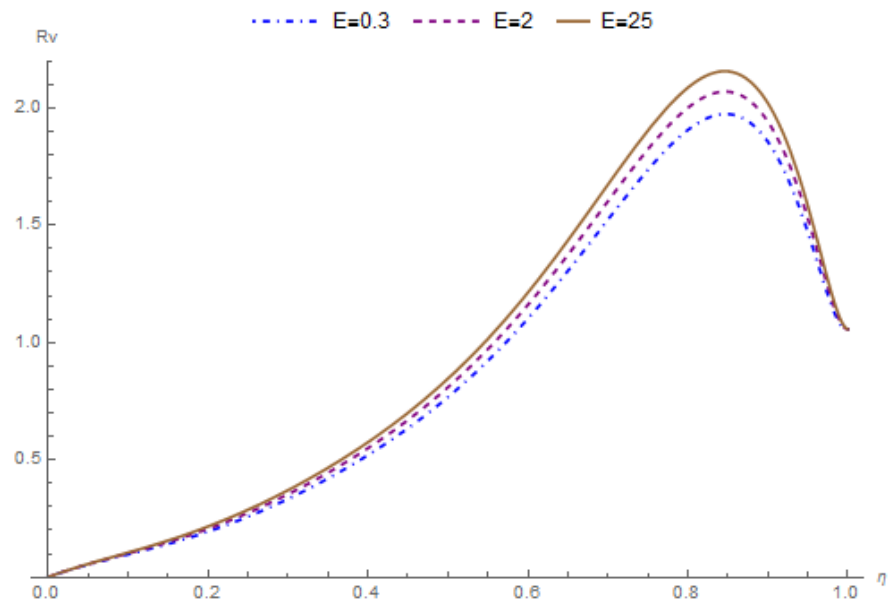


Figure 4.39. Variation of R_v with E

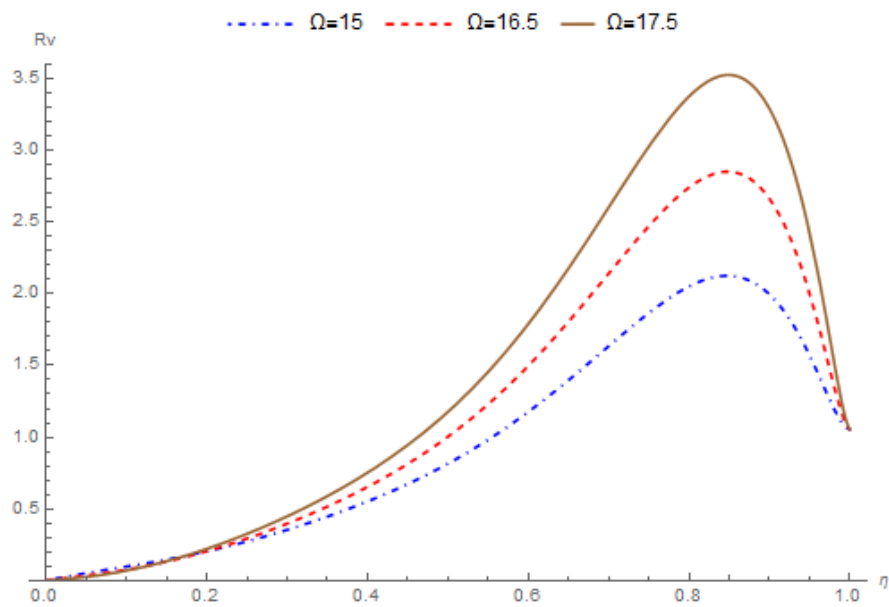


Figure 4.40. Variation of R_v with Ω

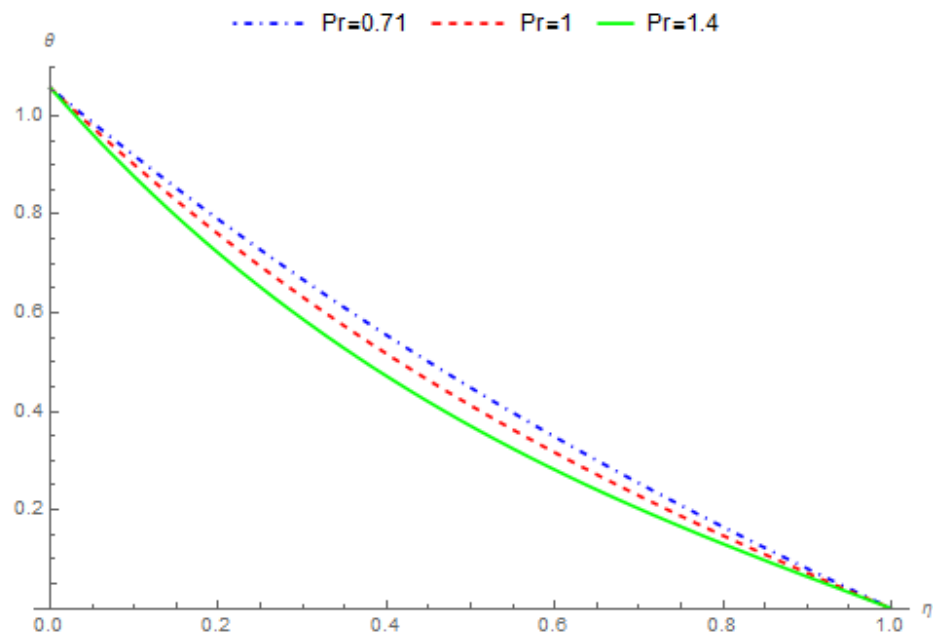


Figure 4.41. Variation of θ with Pr

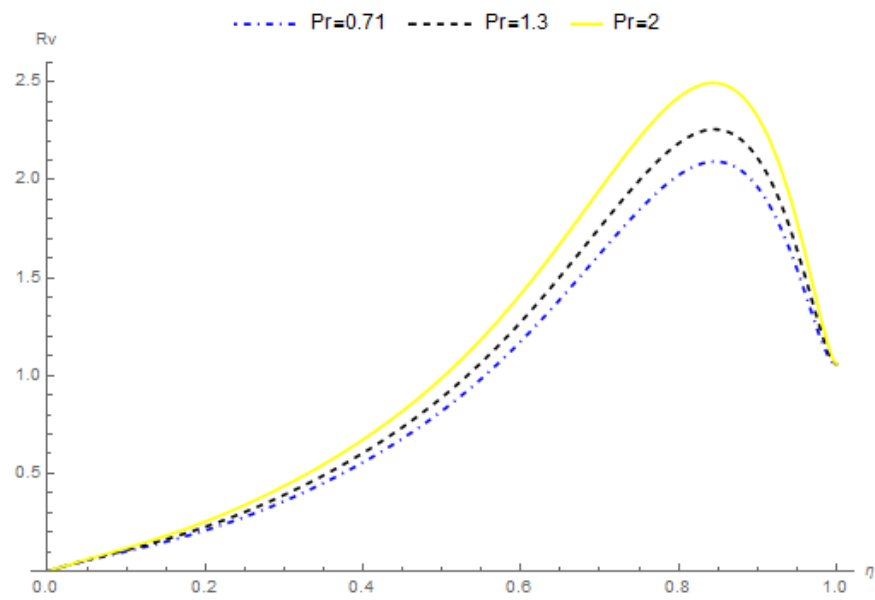


Figure 4.42. Variation of R_v with Pr

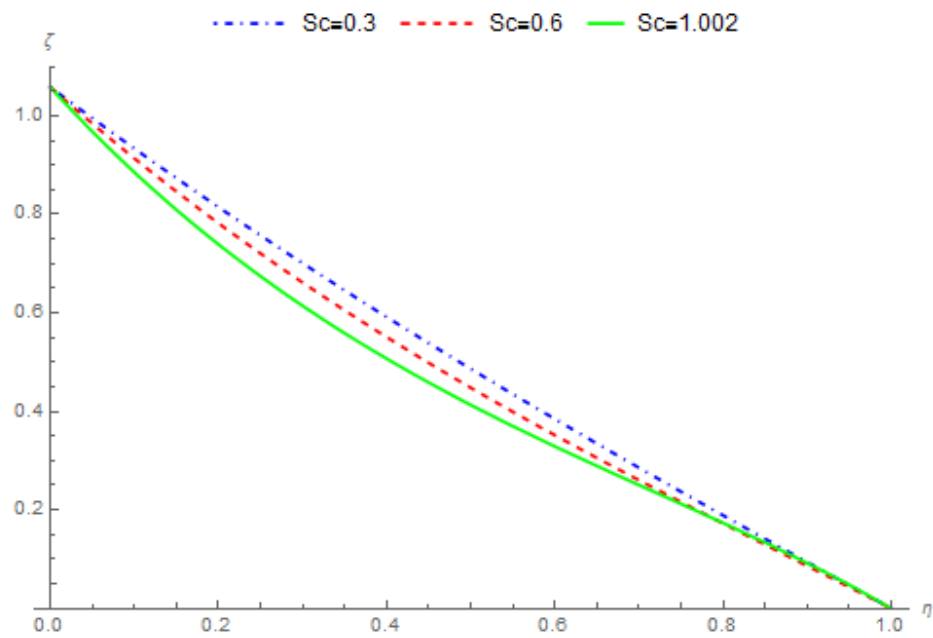


Figure 4.43. Variation of ζ with Sc

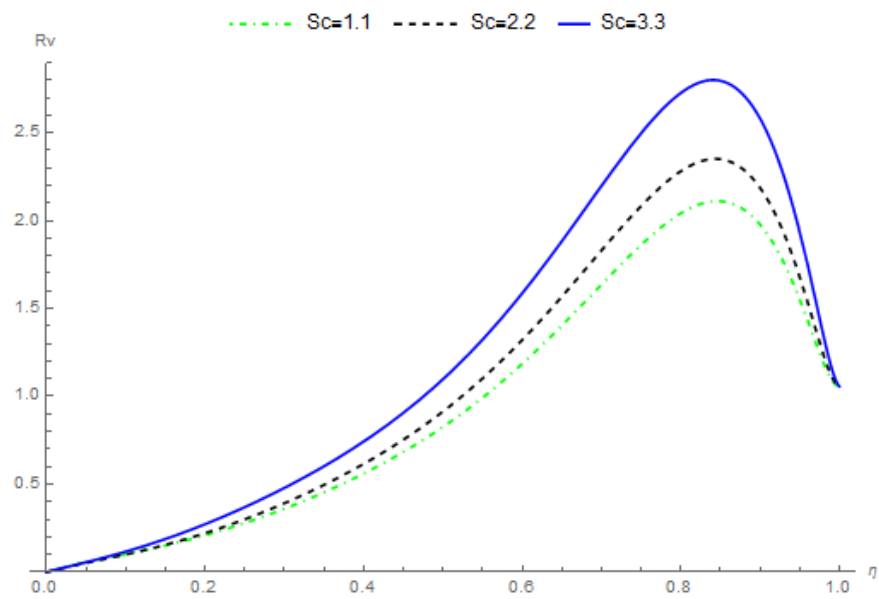


Figure 4.44. Variation of R_v with Sc

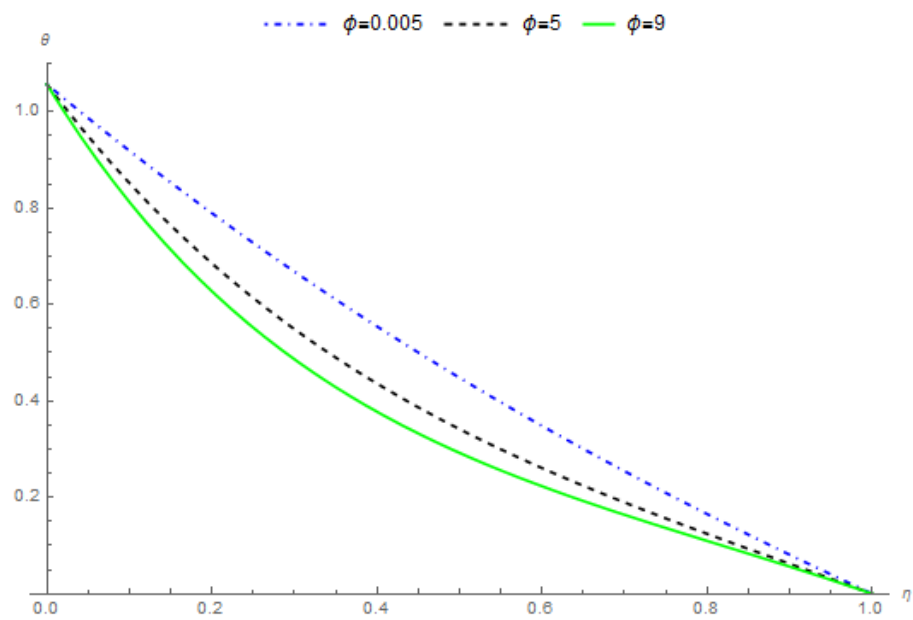


Figure 4.45. Variation of θ with ϕ

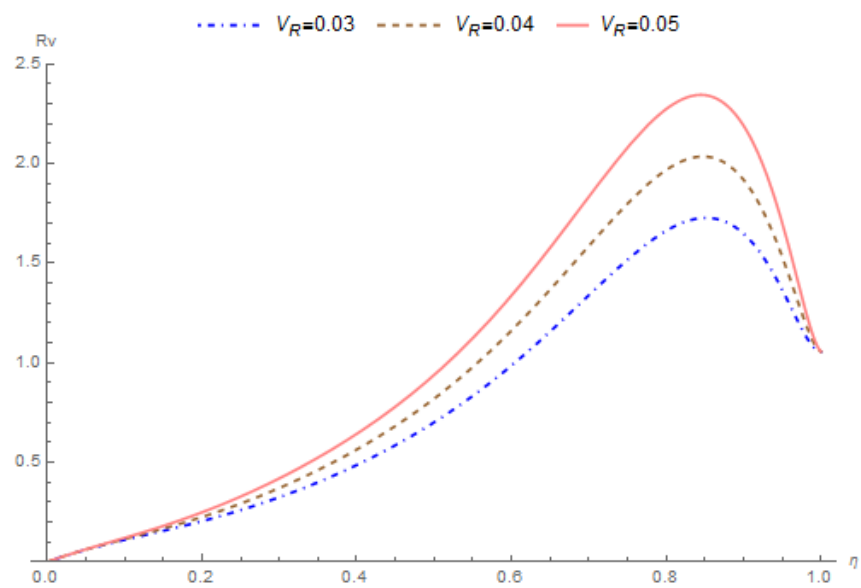


Figure 4.46. Variation of R_v with V_R

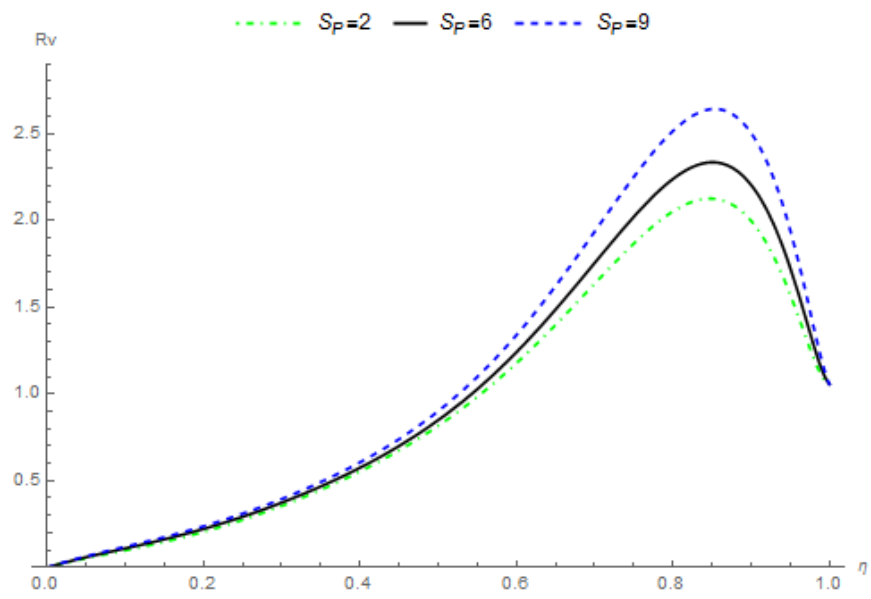


Figure 4.47. Variation of R_v with S_p

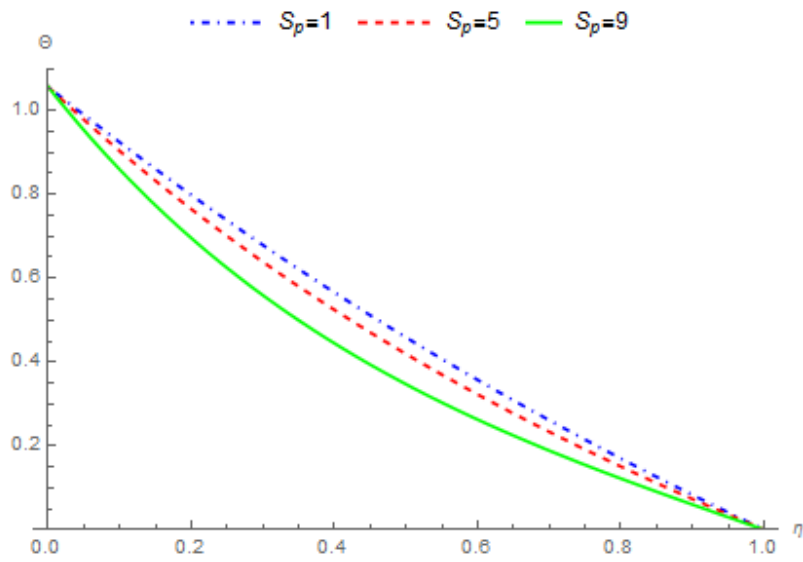


Figure 4.48. Variation of Q with S_p

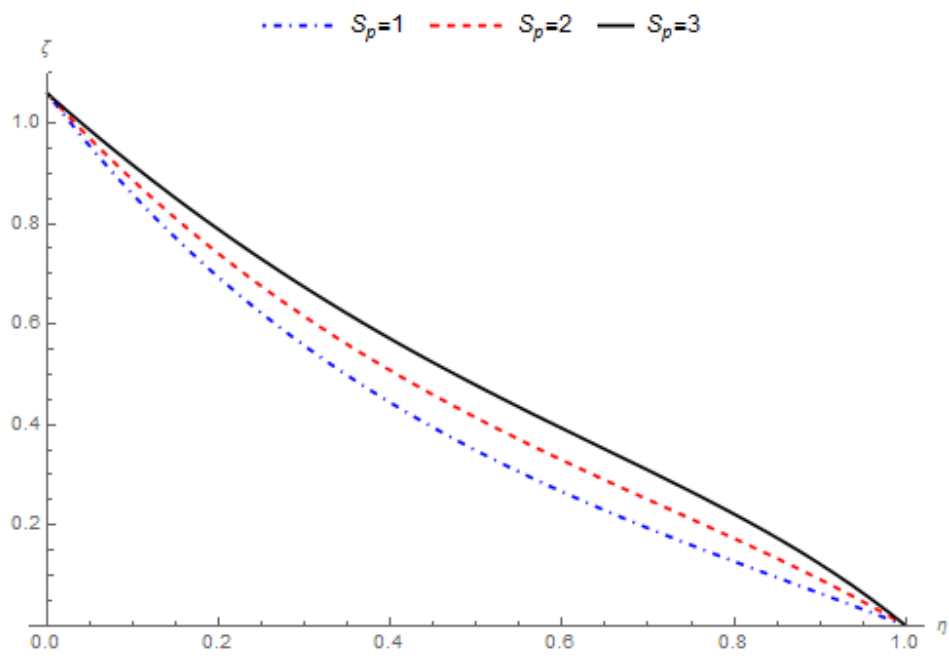


Figure 4.49. Variation of ζ with S_p

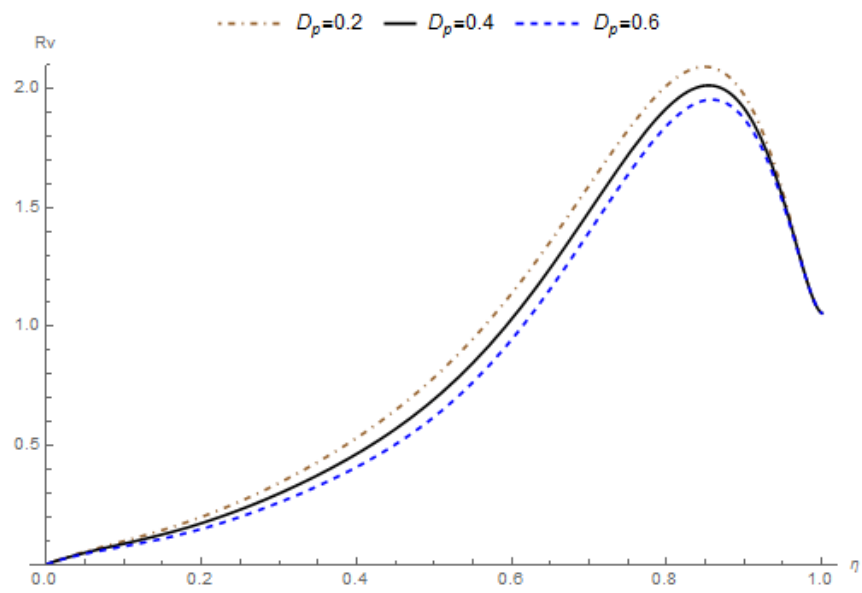


Figure 4.50. Variation of R_v with D_p

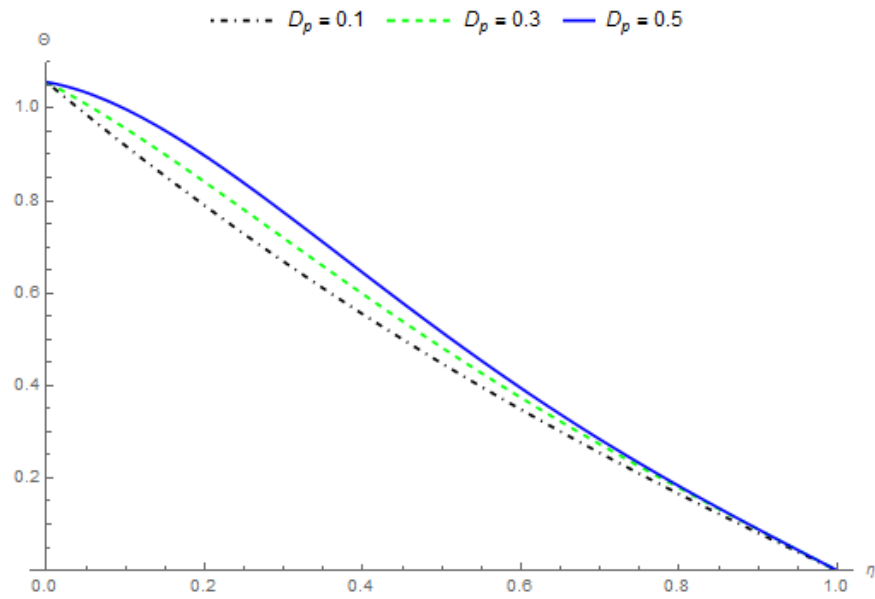


Figure 4.51. Variation of Θ with D_p

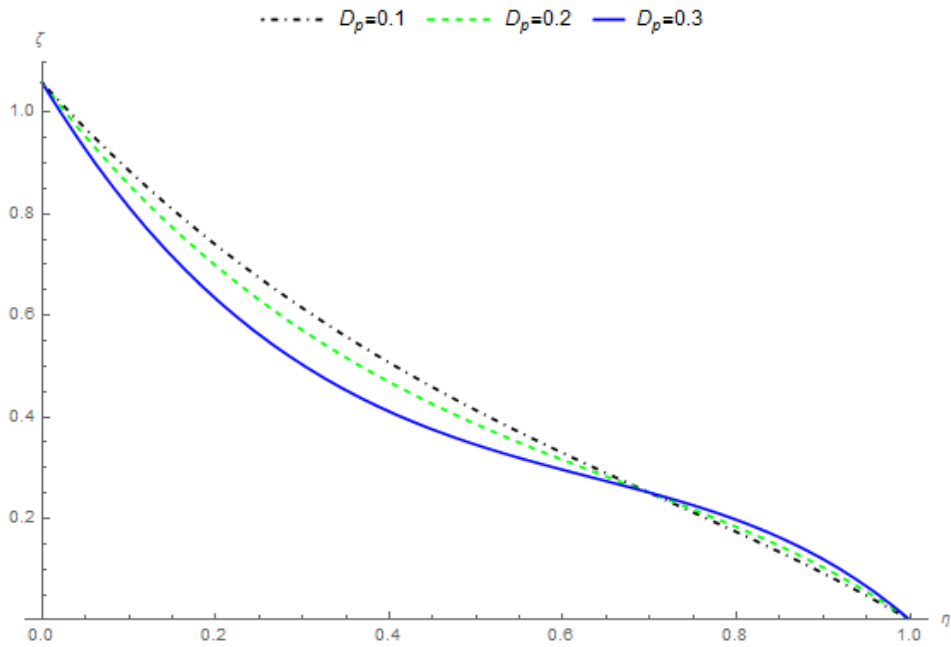


Figure 4.52. Variation of ζ with D_p

The computation in table 4.5 is done using Adomian Decomposition Method (ADM) and compared with fourth order Runge-Kutta method with shooting technique for skin friction at $\eta = 0$ and D is the difference between ADM and 4th Order R-K.

Table 4.5: Comparison of Skin-friction for various values of S_c , P_r , M , G_h , G_m , ϕ , K_r and E

S_c	P_r	M	G_h	G_m	Φ	K_r	E	ADM	$R-K$	$ D $
0.5	0.71	5	5	5	0.005	2	3	3.02608	3.02607	0.00001
1.0	0.71	5	5	5	0.005	2	3	2.90833	2.90847	0.00014
1.5	0.71	5	5	5	0.005	2	3	2.80761	2.80914	0.00153
1.002	0.5	5	5	5	0.005	2	3	2.92964	2.92977	0.00013
1.002	0.9	5	5	5	0.005	2	3	2.88813	2.88828	0.00015
1.002	1.3	5	5	5	0.005	2	3	2.84626	2.84656	0.0003
1.002	0.71	1.5	5	5	0.005	2	3	3.57819	3.57872	0.00053
1.002	0.71	2.5	5	5	0.005	2	3	3.35114	3.35153	0.00039
1.002	0.71	3.5	5	5	0.005	2	3	3.15537	3.15563	0.00026
1.002	0.71	5	2	5	0.005	2	3	2.16128	2.16142	0.00014
1.002	0.71	5	4	5	0.005	2	3	3.65903	3.65916	0.00013
1.002	0.71	5	6	5	0.005	2	3	3.15677	3.15691	0.00014
1.002	0.71	5	5	2	0.005	2	3	2.27213	2.27218	0.00005
1.002	0.71	5	5	4	0.005	2	3	2.69598	2.69608	0.0001
1.002	0.71	5	5	6	0.005	2	3	3.11983	3.12000	0.00017
1.002	0.71	5	5	5	0.003	2	3	2.90797	2.90811	0.00014
1.002	0.71	5	5	5	0.03	2	3	2.90702	2.90716	0.00014
1.002	0.71	5	5	5	0.3	2	3	2.89766	2.89779	0.00013
1.002	0.71	5	5	5	0.005	0.5	3	2.98536	2.98566	0.0003
1.002	0.71	5	5	5	0.005	1.5	3	2.93171	2.93191	0.0002
1.002	0.71	5	5	5	0.005	2.5	3	2.88578	2.88586	0.00008
1.002	0.71	5	5	5	0.005	2	2	2.91772	2.91785	0.00013
1.002	0.71	5	5	5	0.005	2	4	2.90175	2.90189	0.00014
1.002	0.71	5	5	5	0.005	2	6	2.89448	2.89462	0.00014

Table 4.6 - 4.7 display the variation of fluid parameters (K_r , E , Ω , S_p , D_p and V_R) on $\tau(\eta)$, $Nu(\eta)$ and $Sh(\eta)$ at $\eta = 0$ and $\eta = 1$. It is seen in Table 4.6 that the $\tau(\eta)$ is diminished with the presence of K_r , Ω , D_p and V_R , while it is strengthened by E and S_p . $Nu(\eta)$ is reduced with an increase in K_r , N , and D_p . On the other hand, increasing the values of S_p enhances the $Nu(\eta)$. In like manner, $Sh(\eta)$ increases with an increase in K_r and D_p . The mass transfer coefficient value is reduced with an increase in E and S_p .

Consequently, Table 4.7 shows that $\tau(\eta)$ is quickened by an increase in K_r , E , Ω , S_p and V_R , while higher values of D_p decreases the $\tau(\eta)$. $Nu(\eta)$ is risen with an increase in K_r and D_p but diminishes with increment in the values of E and S_p . Increasing E , S_p and D_p make $Sh(\eta)$ to rise and it decelerates by increasing the values of K_r .

Table 4.6: Values of Skin-friction, Nusselt number and Sherwood number for different parameters at $\eta = 0$

K_r	E	Ω	S_p	D_p	V_R	τ	Nu	Sh
1	3	15	2	0.1	0.05	1.26547	2.11822	1.51979
2	3	15	2	0.1	0.05	1.23144	2.05517	1.88701
3	3	15	2	0.1	0.05	1.20171	1.99740	2.21109
2	2	15	2	0.1	0.05	1.22856	2.24886	2.02036
2	3	15	2	0.1	0.05	1.23144	2.05517	1.88701
2	4	15	2	0.1	0.05	1.23326	1.95820	1.80325
2	3	10	2	0.1	0.05	2.30656	2.05517	1.88701
2	3	12	2	0.1	0.05	1.91408	2.05517	1.88701
2	3	14	2	0.1	0.05	1.47608	2.05517	1.88701
2	3	15	0.5	0.1	0.05	1.15877	1.96696	2.39242
2	3	15	1	0.1	0.05	1.18374	1.99150	2.23485
2	3	15	1.5	0.1	0.05	1.20808	2.02105	2.06581
2	3	15	2	0.2	0.05	1.23902	1.91244	2.24897
2	3	15	2	0.3	0.05	1.22160	1.68476	2.84090
2	3	15	2	0.4	0.05	1.17973	1.28395	3.77168
2	3	15	2	0.1	0.03	1.88677	2.05517	1.27000
2	3	15	2	0.1	0.04	1.24755	2.05517	1.88689
2	3	15	2	0.1	0.05	1.23144	2.05517	1.88701

Table 4.7: Values of Skin-friction, Nusselt number and Sherwood number for different parameters at $\eta = 1$

K_r	E	Ω	S_p	D_p	V_R	τ	Nu	Sh
1	3	15	2	0.1	0.05	31.07010	1.11707	1.01404
2	3	15	2	0.1	0.05	31.18150	1.15218	1.00215
3	3	15	2	0.1	0.05	31.30150	1.18574	0.99255
2	2	15	2	0.1	0.05	30.93190	1.39006	0.92341
2	3	15	2	0.1	0.05	31.18150	1.15218	1.00215
2	4	15	2	0.1	0.05	31.34460	1.03570	1.05271
2	3	10	2	0.1	0.05	10.07210	1.15218	1.00215
2	3	12	2	0.1	0.05	15.89760	1.15218	1.00215
2	3	14	2	0.1	0.05	31.18150	1.15218	1.00215
2	3	15	0.5	0.1	0.05	30.56020	1.17756	0.51807
2	3	15	1	0.1	0.05	30.73410	1.17484	0.66103
2	3	15	1.5	0.1	0.05	30.94190	1.16610	0.82349
2	3	15	2	0.2	0.05	30.65410	1.20466	1.20452
2	3	15	2	0.3	0.05	30.07210	1.25950	1.47638
2	3	15	2	0.4	0.05	29.37250	1.29107	1.92832
2	3	15	2	0.1	0.03	22.58470	1.15218	0.99932
2	3	15	2	0.1	0.04	26.85330	1.15218	1.00074
2	3	15	2	0.1	0.05	31.18150	1.15218	1.00215

4.3 Effects of Dissipation on Convective Rotatory Chemically Reacting Rivlin-Ericksen flow past a Porous Vertical Plate

The partial differential equations (3.41), (3.42) and (3.43) with the boundary condition equation (3.44) are solved using Adomian decomposition methods with the aid of MATHEMATICA software. The computations for various parameters in the governing equations are examined for velocity, temperature and concentration profiles and the effects are presented graphically. Some of the parameters considered are: Suction velocity parameter A , dimensionless viscoelasticity parameter of the Rivlin-Ericksen fluid V_R , magnetic parameter M , radiation parameter E , Prandtl number P_r , Schmidt number S_c , dimensionless heat absorption coefficient ϕ , Grashof number for heat transfer G_h , Grashof number for mass transfer G_m , scalar constant ε , permeability of the porous medium k_p , chemical reaction parameter K_r , rotation parameter Ω , dissipation parameter E_c , Dufour parameter D_p and Soret parameter S_p . The default values for the computations are: $G_h = G_m = M = 5, t = 1, V_R = 0.05, P_r = 0.71, \phi = 0.005, E = 3$, $A = k_p = 0.5, kr = 2, n = 0.9, S_c = 1.002, \varepsilon = 0.01, \Omega = 10, S_p = 2, D_p = 0.1$ and $E_c = 0.002$

Figures 4.53 and 4.54 display the effects of A on $\zeta(\eta)$ and $R_v(\eta)$ distributions. It is observed that increase in suction velocity leads to an increase in $R_v(\eta)$ and decrease in $\zeta(\eta)$ of the fluid. The variation of E_c on the $\vartheta(\eta)$, $R_v(\eta)$ and $\zeta(\eta)$ profile is shown in Figures 4.55, 4.56 and 4.57. E_c is the ratio of kinetic energy of the fluid flow to enthalpy. Thus, energy increases due to dissipation effect on the flow leading to higher fluid temperature and increase in buoyancy force. This effectually enhances the temperature profile. Therefore, it can be seen from these Figures that increase in the values of E_c enhances $\vartheta(\eta)$, $R_v(\eta)$ and $\zeta(\eta)$ profiles.

Figures 4.58, 4.59 and 4.60 exhibit the $R_v(\eta)$, $\vartheta(\eta)$ and $\zeta(\eta)$ profiles for different values of the (ε). It is found that an increase in ε enhances $R_v(\eta)$, $\vartheta(\eta)$ and $\zeta(\eta)$ distributions. The effects of G_m and G_h on R_v are displayed in Figures 4.61 and 4.62. It is evident in these figures that an increase in both G_h and G_m produces a significant increase in R_v profile. Variation of k_p on R_v is shown in Figure 4.63. It depicts that for higher values of k_p , velocity of the fluid is accelerated. Figure 4.64 and 4.65 reveal the variation of K_r on $\zeta(\eta)$ and $R_v(\eta)$. It is noticed that increasing values of K_r tends to reduce both $R_v(\eta)$ and $\zeta(\eta)$ distributions

The effect of M on $R_v(\eta)$ is shown in Figure 4.66. It is noticed that increase in the M resulted in a significant decrease in the flow velocity throughout the domain of the fluid. A kind of force known as drag force called Lorentz force is produced in electrically conducting fluid

where magnetic field is applied. There is a reduction in the velocity of the fluid due to this force, because fluid transport is opposed as a result of magnetic field. Figure 4.67 depicts the effect of Ω on $R_v(\eta)$. $R_v(\eta)$ profiles is enhanced with the increasing values of Ω .

Furthermore, the effects of P_r on $\theta(\eta)$ and $R_v(\eta)$ is observed in Figures 4.68 and 4.69 respectively. An increase in P_r results in an increase in $R_v(\eta)$ and decline of the temperature distribution profiles. At $\eta = 0.77$, a rise in fluid temperature is noticed. This is based on the fact that fluids with high P_r is of low thermal diffusivity resulted to low heat penetration, hence the thermal boundary layer is reduced. The variation values of heat absorption coefficient ϕ on temperature profiles is shown in Figure 4.70. It is noticed that increasing the values of ϕ produces a significant decrease in $\theta(\eta)$ distribution. Effects of viscoelastic parameter V_R and radiation parameter E on resultant velocity is illustrated in Figures 4.71 and 4.73 respectively. It is observed that $R_v(\eta)$ is enhanced by increase in V_R and E , while the reverse effect of E on temperature is shown in Figure 4.72. This implies that an increase in E makes temperature profile to decrease.

Variation of S_c on $\zeta(\eta)$ and $R_v(\eta)$ profiles is depicted in Figures 4.74 and 4.75 respectively. The S_c characterizes momentum diffusivity and mass diffusivity. Thus, species diffusivity rate transcend momentum diffusivity for higher values of S_c which decreases the concentration distribution. Therefore, species concentration for low values of S_c is higher. And for high values of S_c , opposite phenomenon is observed while $R_v(\eta)$ is enhanced with the increase in the values of S_c . Effect of Soret on $\zeta(\eta)$, $R_v(\eta)$ and $\theta(\eta)$ profiles are shown in Figures 4.76, 4.77 and 4.78 respectively. It is noted that increase in the values of S_p tend to increase both $\zeta(\eta)$ and $R_v(\eta)$ of the fluid. The fluid temperature reduce with a rise in S_p . Effect of D_p on $R_v(\eta)$, $\theta(\eta)$ and $\zeta(\eta)$ profiles is presented in Figures 4.79, 4.80 and 4.81 respectively. $R_v(\eta)$ profiles decelerated with an increase in D_p , while $\theta(\eta)$ profiles is improved. This is due to the fact that generation of energy flux increased the fluid temperature. A rise in D_p resulted in the decrease in the concentration profiles; a different trend is noticed at $\eta = 0.73$.

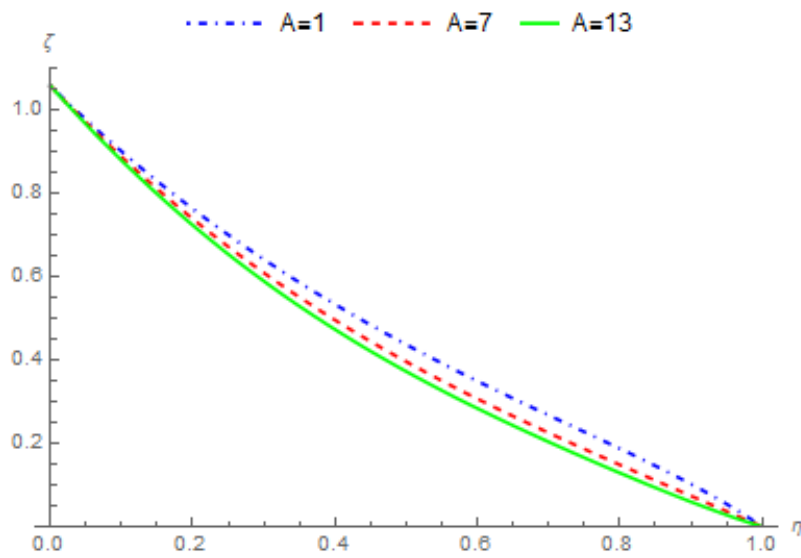


Figure 4.53. Variation of ζ with A

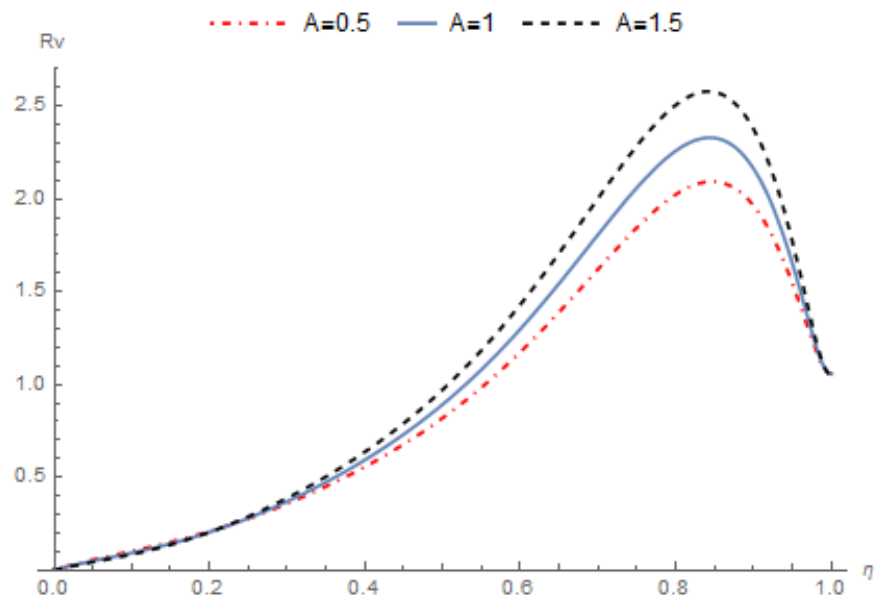


Figure 4.54. Variation of R_v with A

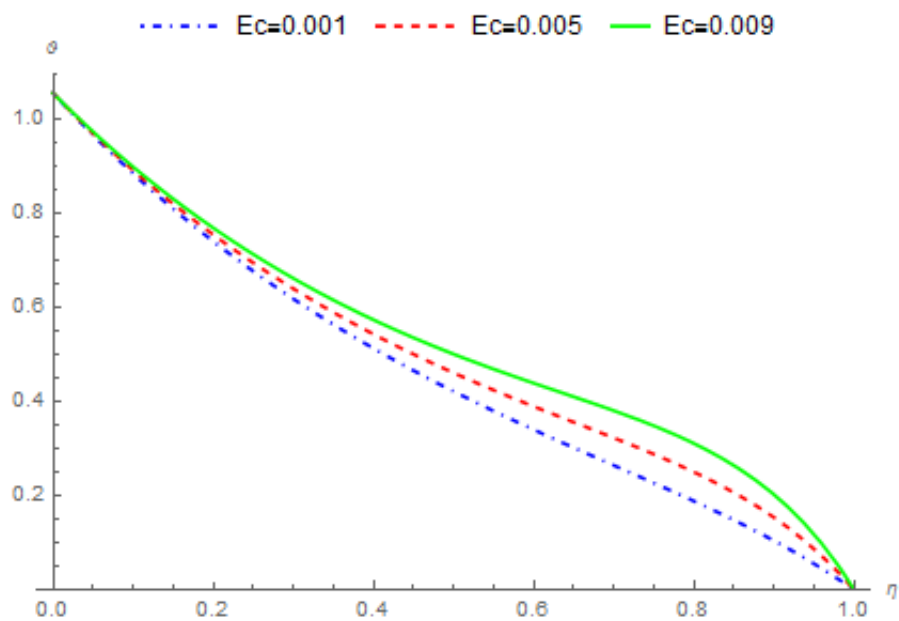


Figure 4.55. Variation of ϑ with Ec

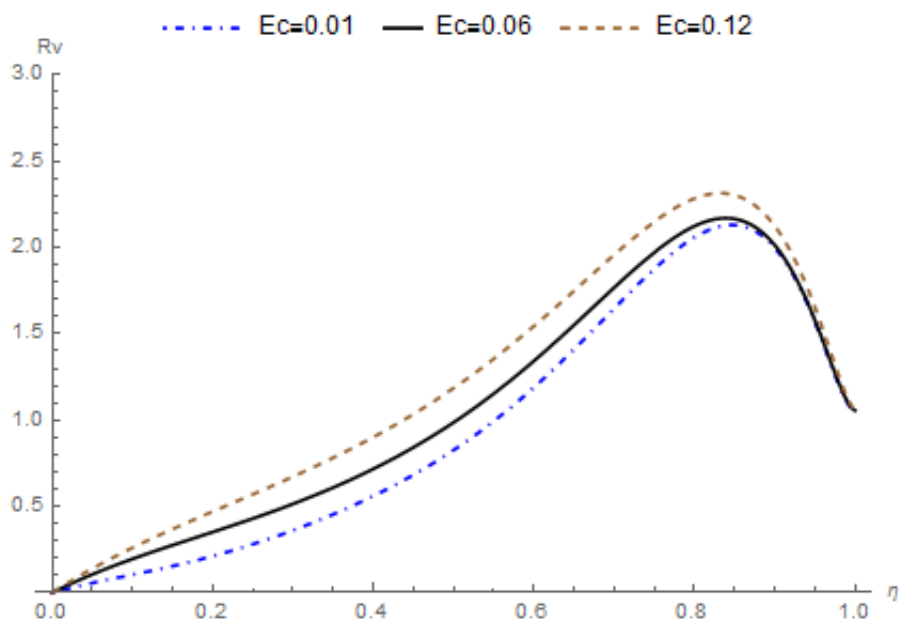


Figure 4.56. Variation of R_v with Ec

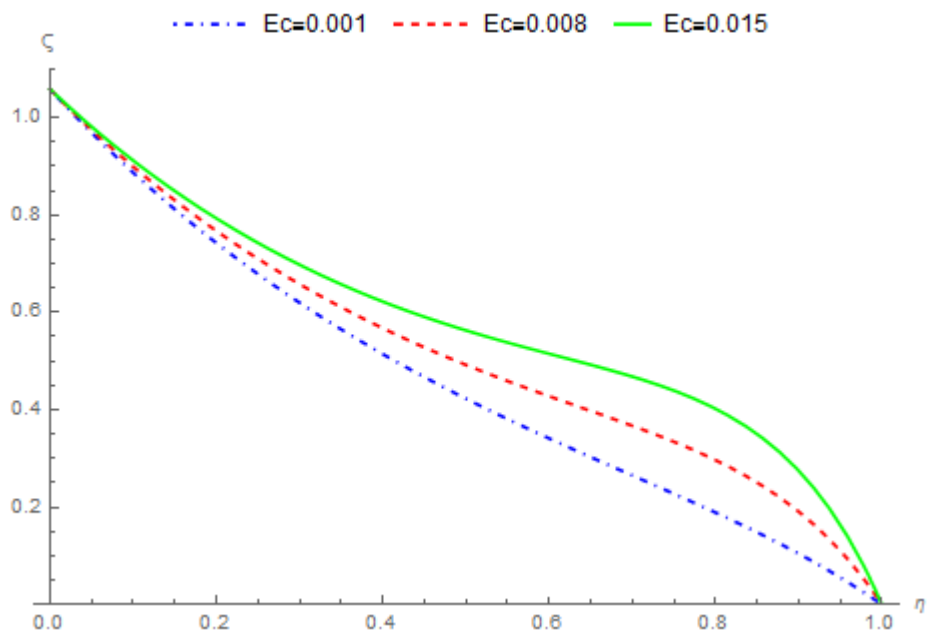


Figure 4.57. Variation of ζ with Ec

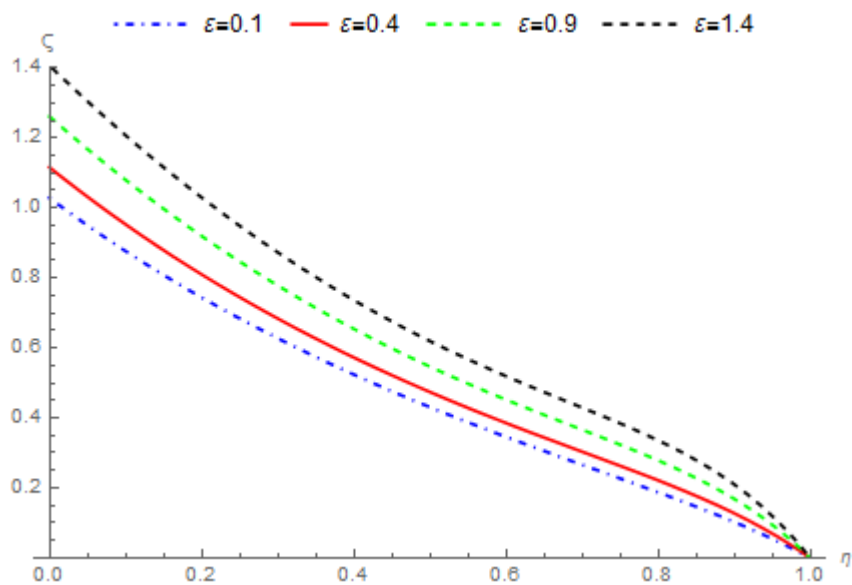


Figure 4.58. Variation of ζ with ε

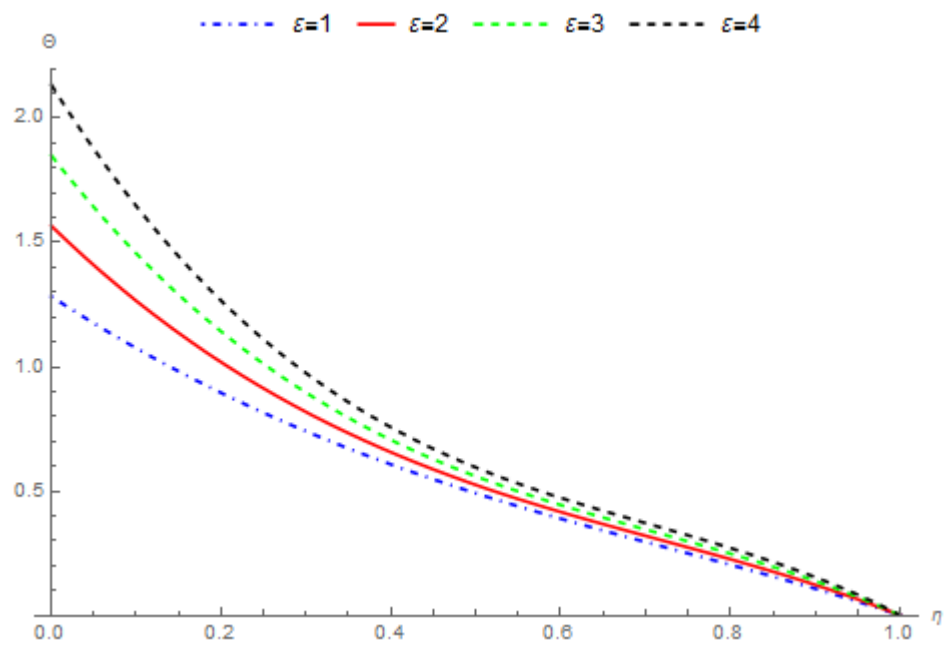


Figure 4.59. Variation of Θ with ε

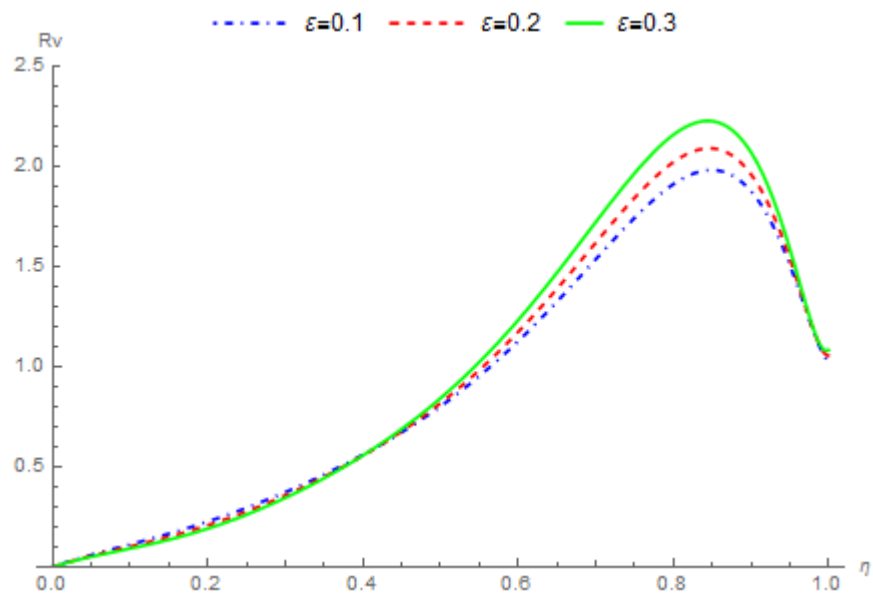


Figure 4.60. Variation of R_v with ε

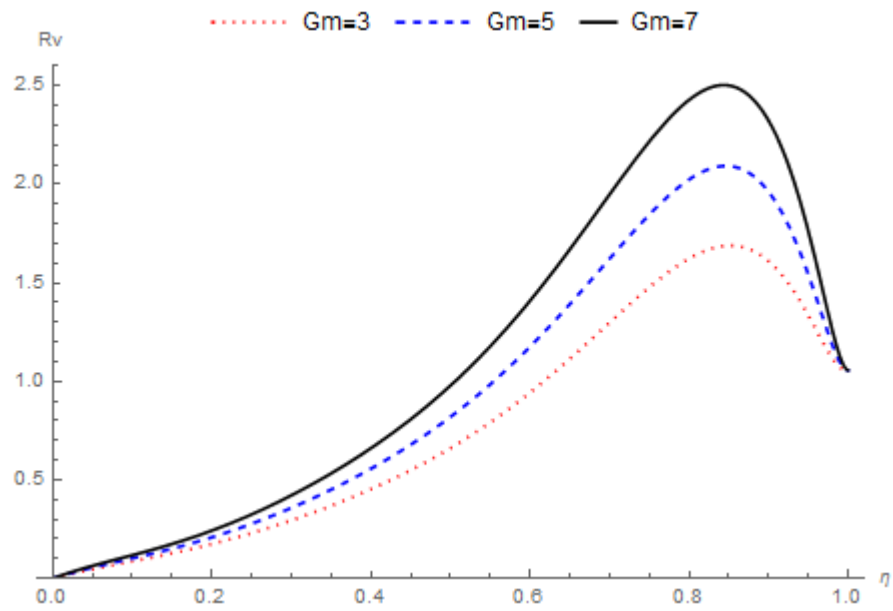


Figure 4.61. Variation of R_v with G_m

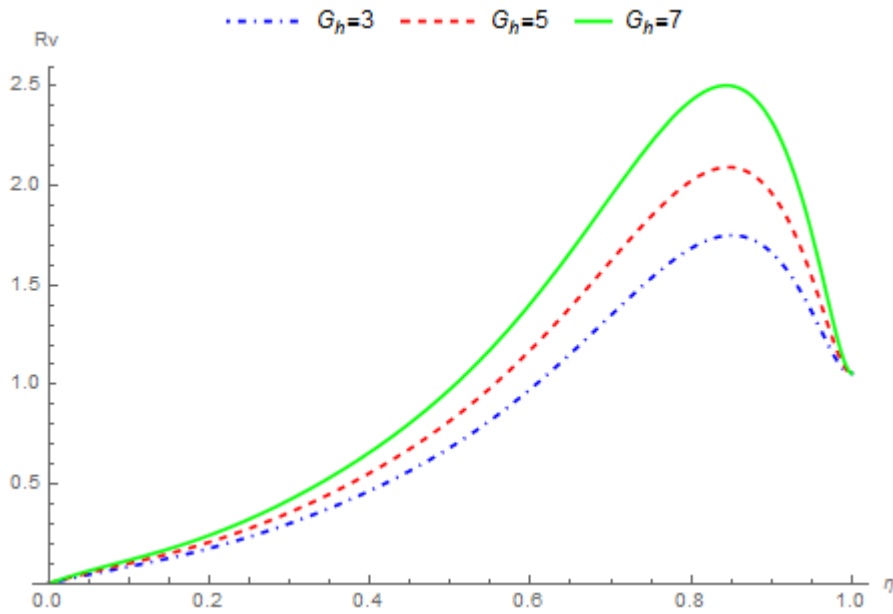


Figure 4.62. Variation of R_v with G_h

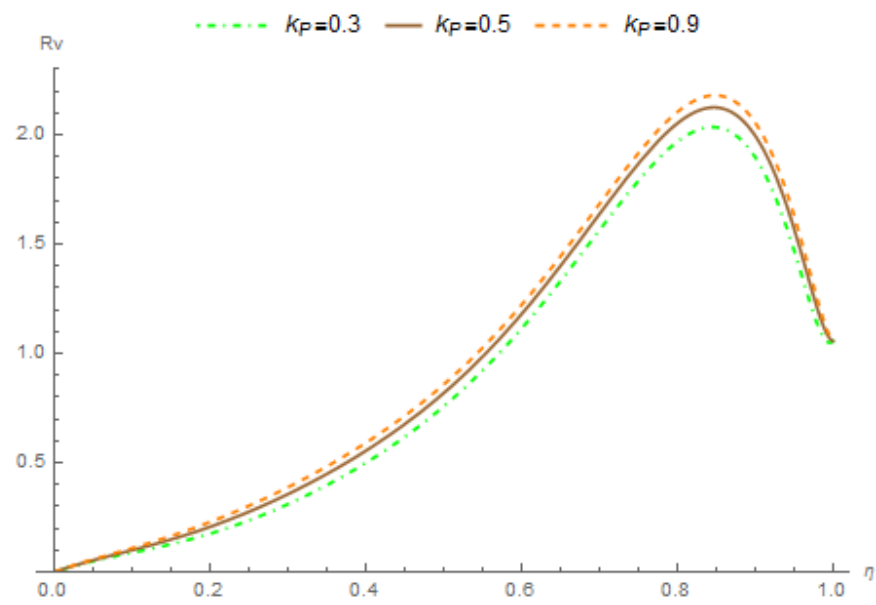


Figure 4.63. Variation of R_v with k_p

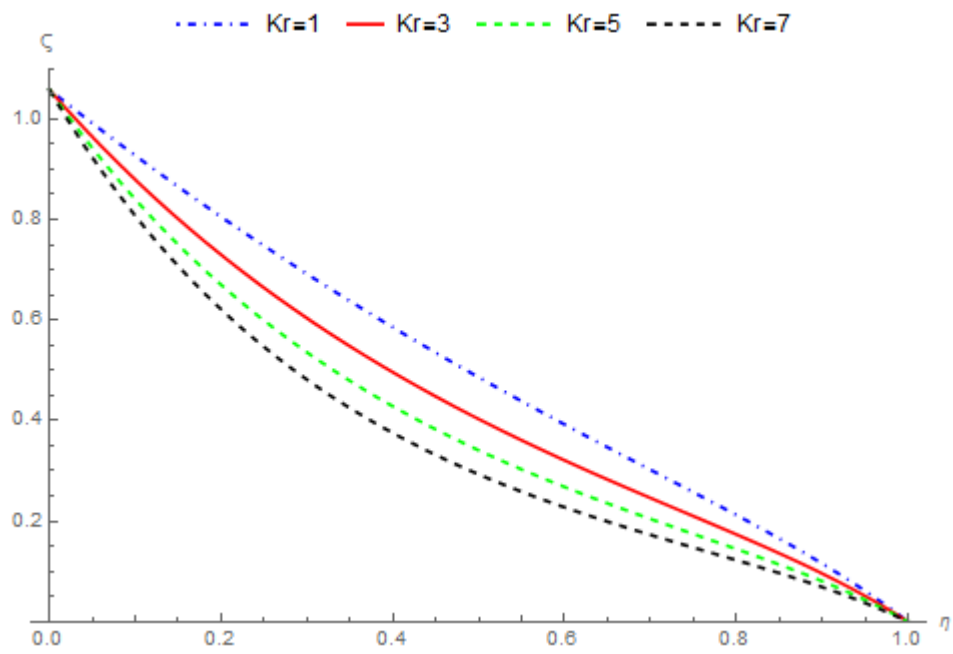


Figure 4.64. Variation of ζ with K_r

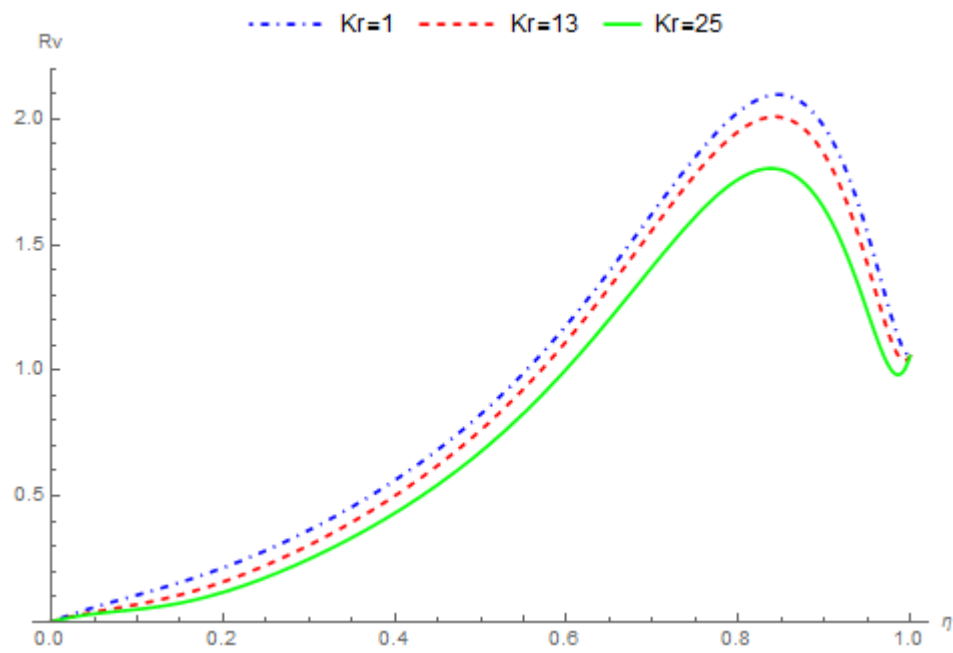


Figure 4.65. Variation of R_v with K_r

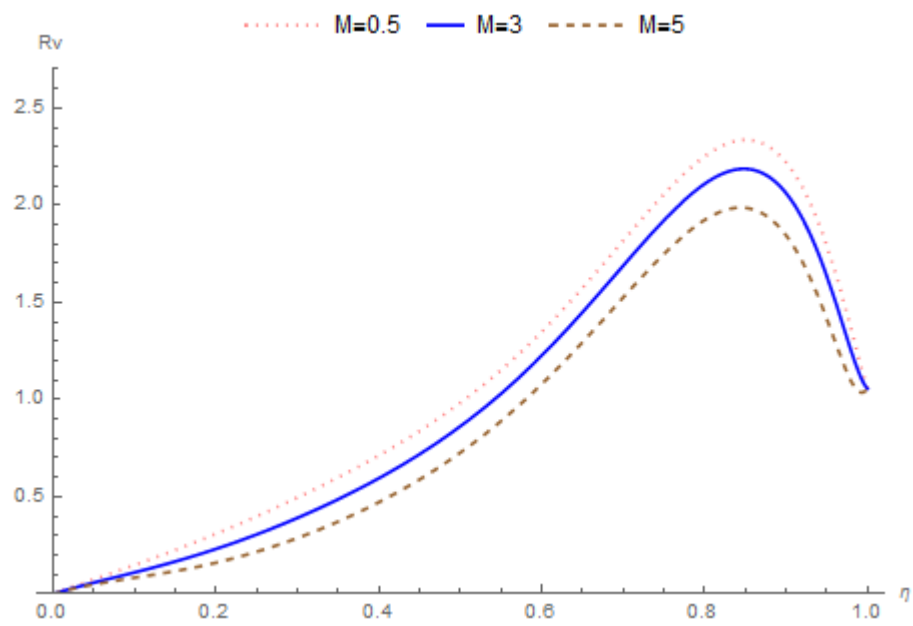


Figure 4.66. Variation of R_v with M

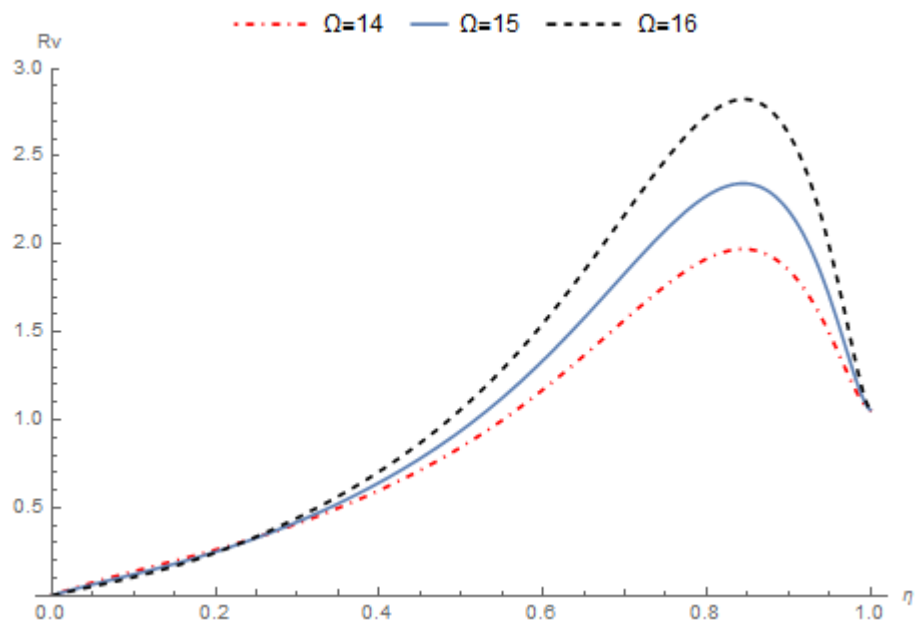


Figure 4.67. Variation of R_v with Ω

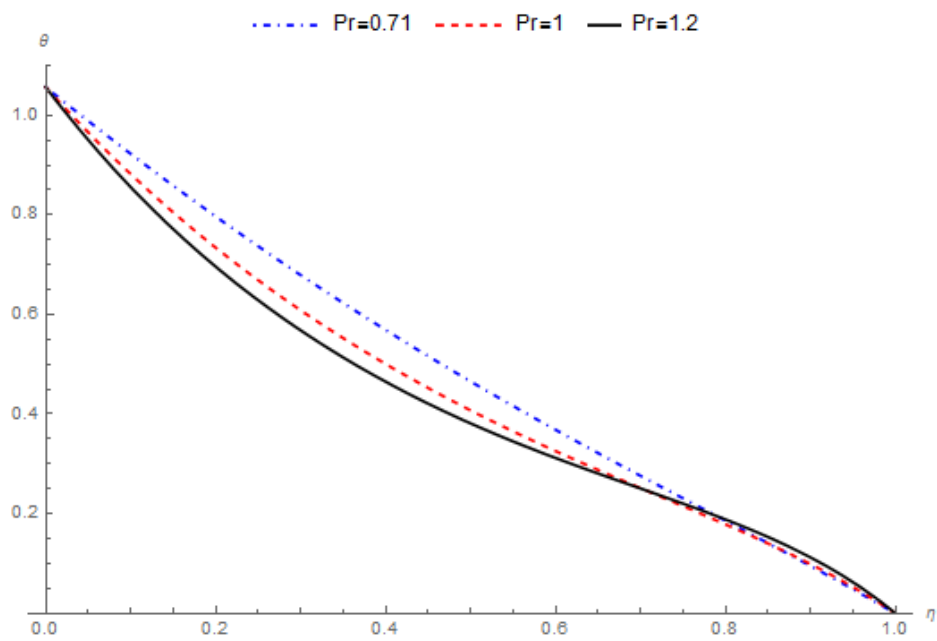


Figure 4.68. Variation of θ with Pr

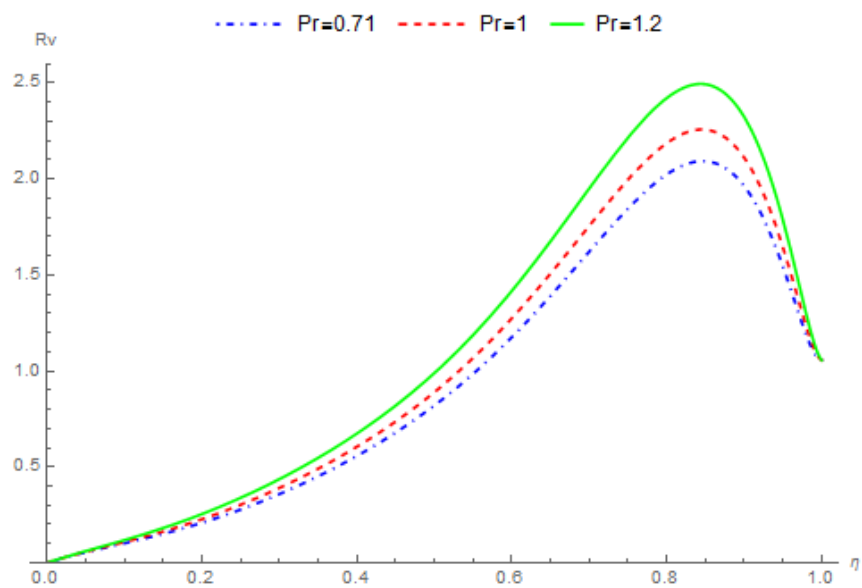


Figure 4.69. Variation of R_v with Pr

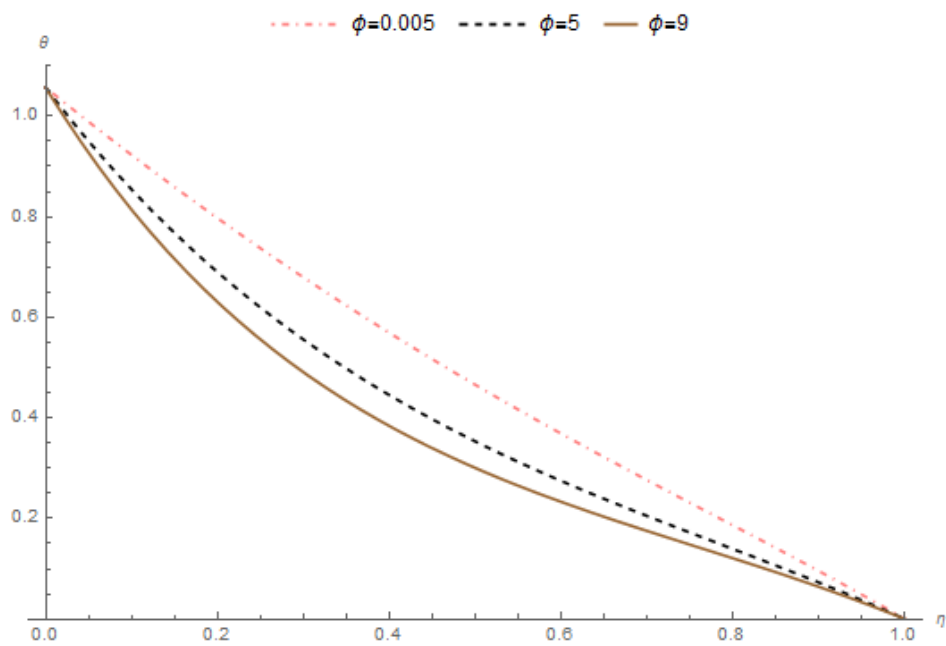


Figure 4.70. Variation of θ with ϕ

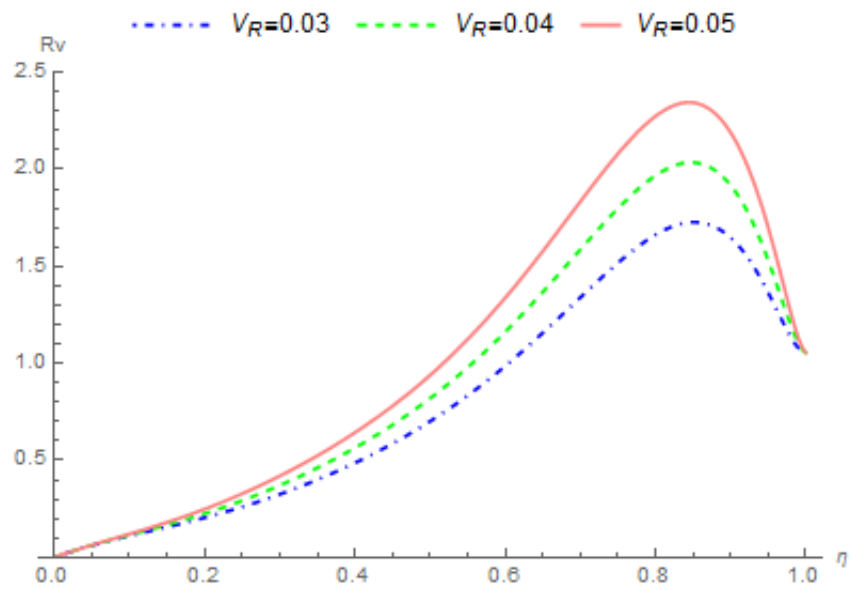


Figure 4.71. Variation of R_v with V_R

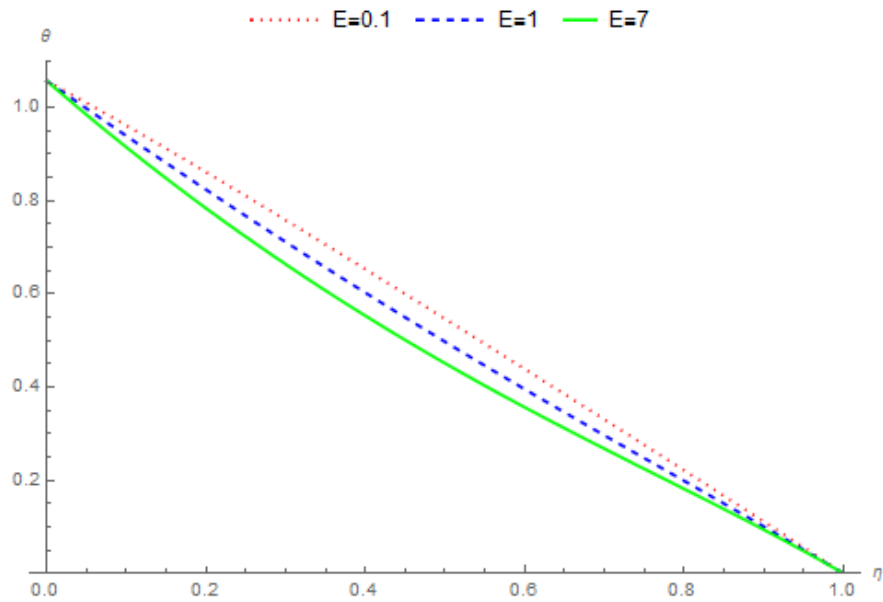


Figure 4.72. Variation of θ with E

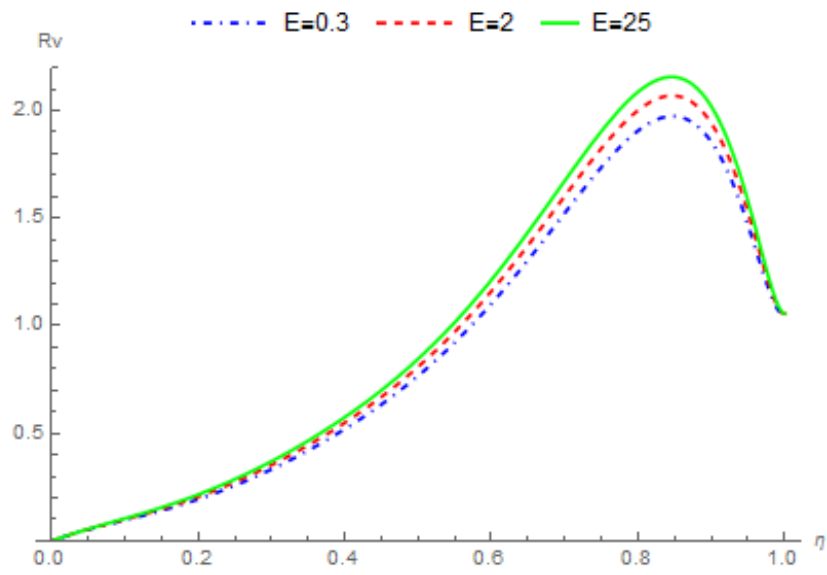


Figure 4.73. Variation of R_v with E

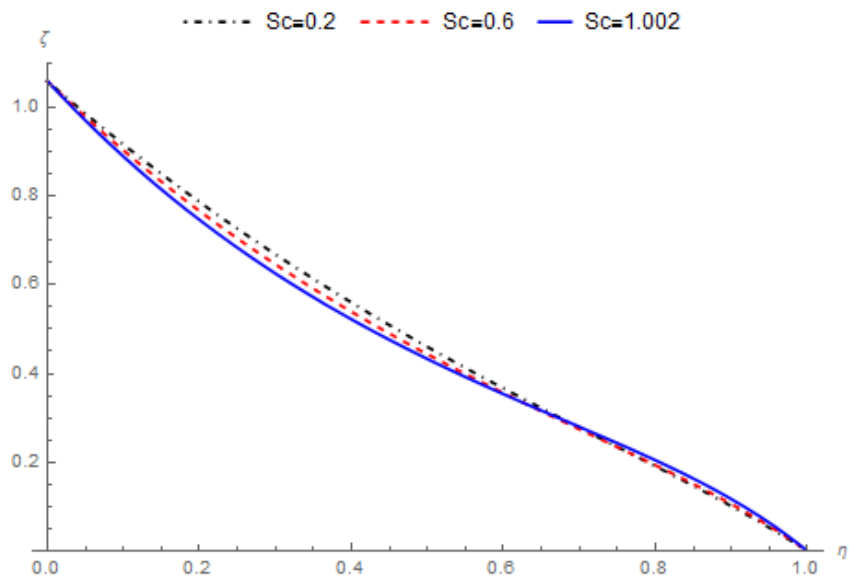


Figure 4.74. Variation of ζ with Sc

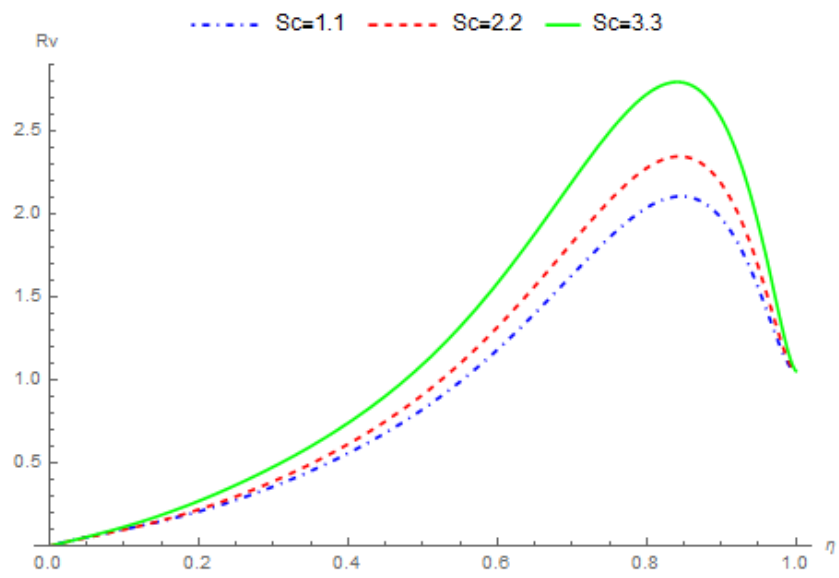


Figure 4.75. Variation of R_v with Sc

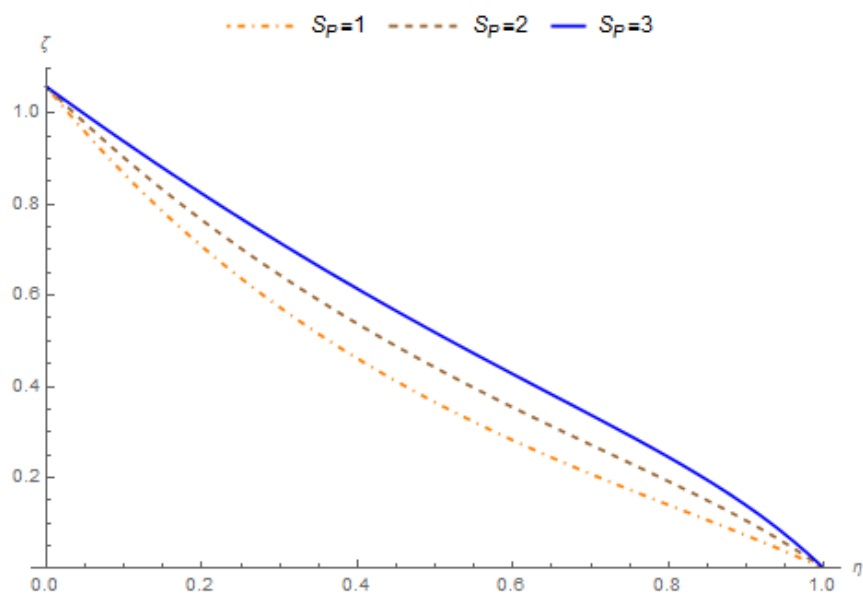


Figure 4.76. Variation of ζ with S_p

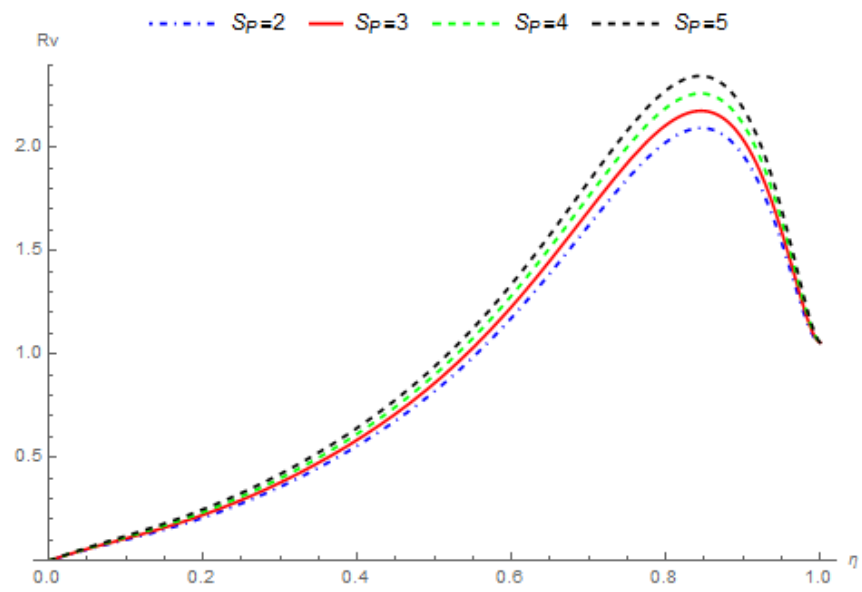


Figure 4.77. Variation of R_v with S_p

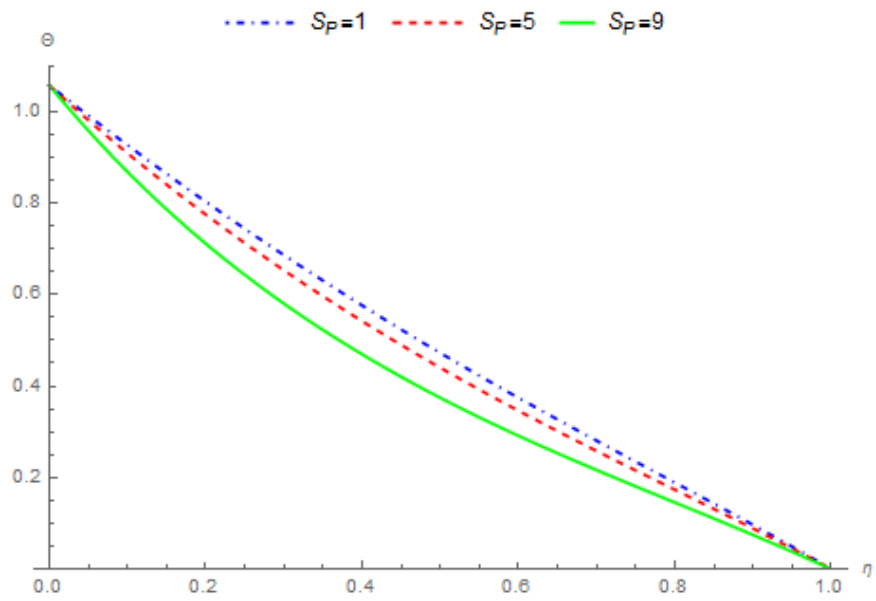


Figure 4.78. Variation of Q with S_p

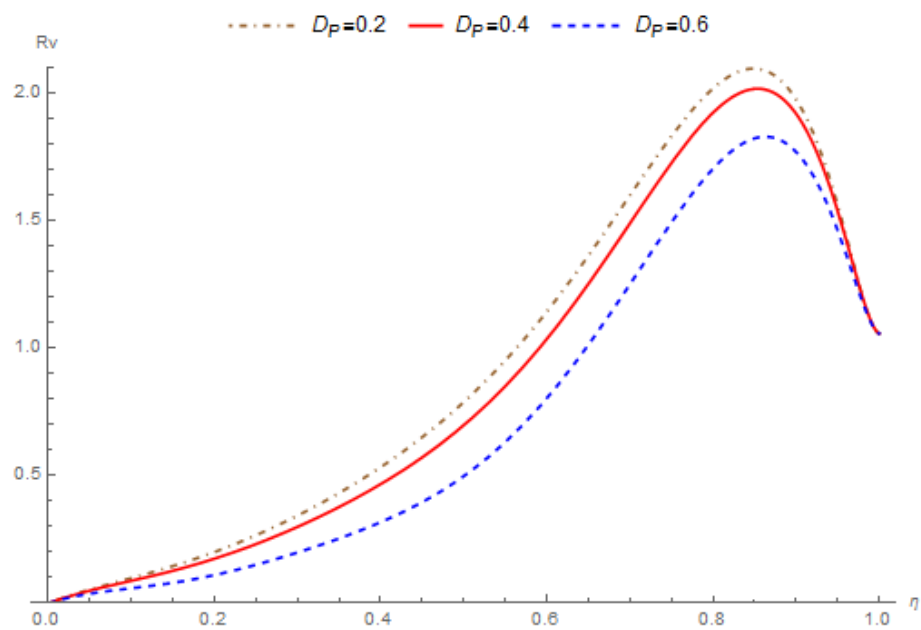


Figure 4.79. Variation of R_v with D_p

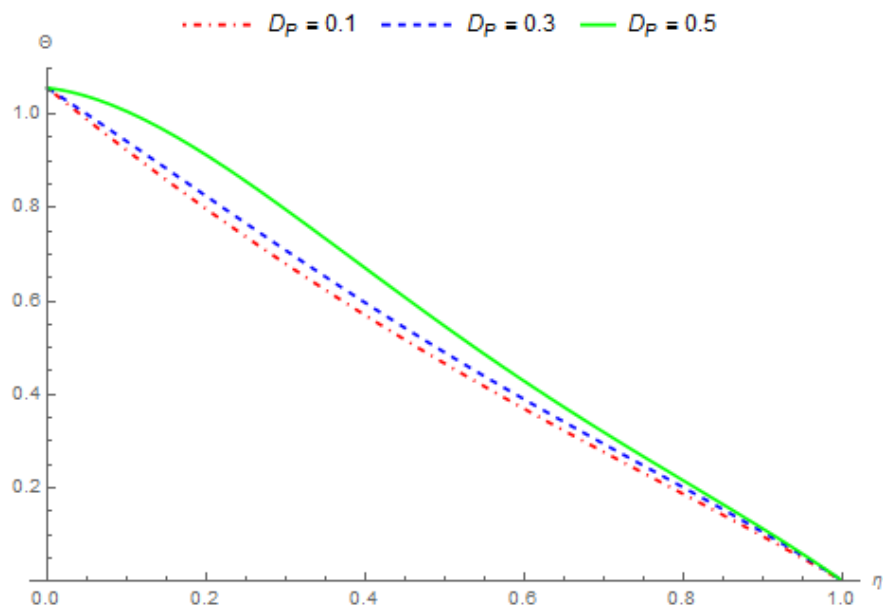


Figure 4.80. Variation of Θ with D_p

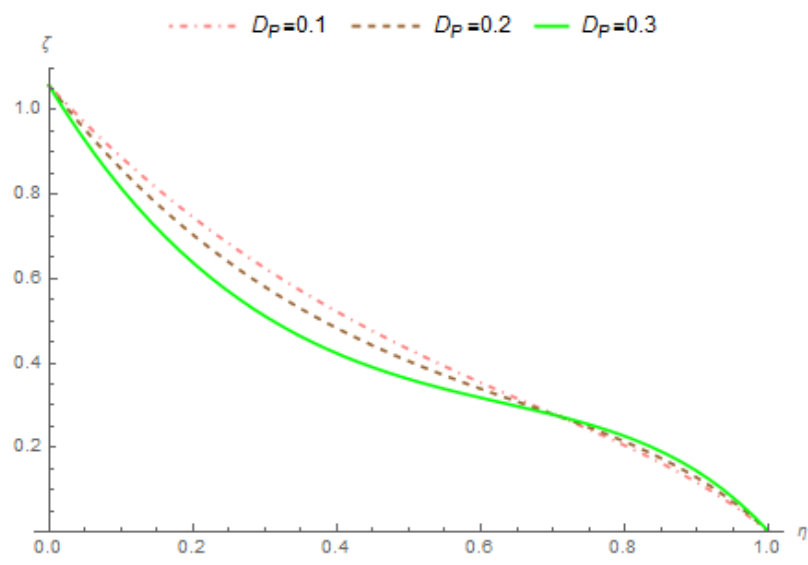


Figure 4.81. Variation of ζ with D_p

Table 4.8 is the table for Figure 4.54; it presented resultant velocity profile for different values of A . While, Table 4.9 is the table for Figure 4.71; it displayed resultant velocity profile for different values of V_R .

Table 4.8: Values of resultant velocity for different values of A

η	0	0.1	0.2	0.3	0.4	0.5
$A = 0.5$	0.00000	0.09999	0.20658	0.35601	0.55408	0.81782
$A = 1$	0.00000	0.08933	0.20438	0.37478	0.60301	0.90970
$A = 1.5$	0.00000	0.08206	0.20961	0.40155	0.65934	1.00866
η	0.6	0.7	0.8	0.9	1.0	
$A = 0.5$	1.17908	1.63712	2.05246	1.99707	1.05673	
$A = 1$	1.33106	1.86174	2.33253	2.23474	1.05673	
$A = 1.5$	1.49081	2.09585	2.62425	2.48542	1.05673	

Table 4.9: Values of resultant velocity for different values of V_R

η	0	0.1	0.2	0.3	0.4	0.5
$V_R = 0.03$	0.00000	0.09505	0.17216	0.27664	0.41972	0.61029
$V_R = 0.04$	0.00000	0.09682	0.18810	0.31534	0.48636	0.71382
$V_R = 0.05$	0.00000	0.09999	0.20658	0.35601	0.55408	0.81782
η	0.6	0.7	0.8	0.9	1.0	
$V_R = 0.03$	0.86748	1.19308	1.49900	1.50864	1.05673	
$V_R = 0.04$	1.02316	1.41495	1.77531	1.75133	1.05673	
$V_R = 0.05$	1.17908	1.63712	2.05246	1.99707	1.05673	

The variation of fluid parameters (K_r , E , Ω , S_p , D_p , V_R and E_c) on $\tau(\eta)$, $Nu(\eta)$ and $Sh(\eta)$ at $\eta = 0$ is presented in Table 4.10. Also, variation of fluid parameters (K_r , E , Ω , S_p , D_p , V_R and E_c) on $\tau(\eta)$, $Nu(\eta)$ and $Sh(\eta)$ at $\eta = 1$ is shown in Table 4.11. It is observed in Table 4.10 that an increase in K_r , Ω , D_p and V_R resulted in decrease in the $\tau(\eta)$. While, E , S_p and E_c enhance the value of the $\tau(\eta)$. $Nu(\eta)$ decelerated with an increase in K_r , E , Ω , D_p and E_c . While increase in the values of S_p and V_R make $Nu(\eta)$ to increase. $Sh(\eta)$ increases with an increase in K_r and D_p . On the other hand, increase in E , Ω , S_p , V_R and E_c decrease the $Sh(\eta)$.

Similarly, in Table 4.11, it is revealed that $\tau(\eta)$ is reduced by an increase in D_p and increases with an increase in K_r , E , Ω , S_p , V_R and E_c . $Nu(\eta)$ is enhanced with an increase in K_r , Ω , E_c and D_p but decreases with an increase in the values of E , S_p and V_R . Consequently, increase in E , S_p , E_c and Ω accelerate $Sh(\eta)$, while it is reduced by the values of K_r and V_R .

Table 4.10: Values of Skin-friction, Nusselt number and Sherwood number for different parameters at $\eta = 0$

K_r	E	Ω	S_p	D_p	E_c	V_R	τ	Nu	Sh
1	3	15	2	0.1	0.002	0.05	1.28158	2.06081	1.48037
2	3	15	2	0.1	0.002	0.05	1.24665	1.99652	1.85324
3	3	15	2	0.1	0.002	0.05	1.21614	1.93783	2.18230
2	2	15	2	0.1	0.002	0.05	1.24178	2.19274	1.98868
2	3	15	2	0.1	0.002	0.05	1.24665	1.99652	1.85324
2	4	15	2	0.1	0.002	0.05	1.24970	1.89784	1.76868
2	3	10	2	0.1	0.002	0.05	2.32384	2.01221	1.86317
2	3	12	2	0.1	0.002	0.05	1.93356	1.99753	1.85481
2	3	14	2	0.1	0.002	0.05	1.49398	1.99173	1.85117
2	3	15	0.5	0.1	0.002	0.05	1.16736	1.91194	2.38356
2	3	15	1	0.1	0.002	0.05	1.19455	1.93549	2.21733
2	3	15	1.5	0.1	0.002	0.05	1.22109	1.96384	2.03993
2	3	15	2	0.2	0.002	0.05	1.25449	1.84645	2.21936
2	3	15	2	0.3	0.002	0.05	1.23724	1.60984	2.81811
2	3	15	2	0.4	0.002	0.05	1.19540	1.19349	3.76191
2	3	15	2	0.1	0.001	0.05	1.23903	2.02584	1.87006
2	3	15	2	0.1	0.002	0.05	1.24665	1.99652	1.85324
2	3	15	2	0.1	0.003	0.05	1.25430	1.96722	1.83656
2	3	15	2	0.1	0.002	0.03	1.28689	1.99421	1.85354
2	3	15	2	0.1	0.002	0.04	1.26364	1.99536	1.85338
2	3	15	2	0.1	0.002	0.05	1.24665	1.99652	1.85324

Table 4.11: Values of Skin-friction, Nusselt number and Sherwood number for different parameters at $\eta = 1$

K_r	E	Ω	S_p	D_p	E_c	V_R	τ	Nu	Sh
1	3	15	2	0.1	0.002	0.05	31.08480	1.39350	1.43100
2	3	15	2	0.1	0.002	0.05	31.19390	1.42954	1.40590
3	3	15	2	0.1	0.002	0.05	31.31120	1.46313	1.38315
2	2	15	2	0.1	0.002	0.05	30.94580	1.66172	1.27095
2	3	15	2	0.1	0.002	0.05	31.19390	1.42954	1.40590
2	4	15	2	0.1	0.002	0.05	31.35560	1.31711	1.49187
2	3	10	2	0.1	0.002	0.05	10.02990	1.45777	1.29769
2	3	12	2	0.1	0.002	0.05	15.87190	1.50358	1.38090
2	3	14	2	0.1	0.002	0.05	24.89770	1.53651	1.41584
2	3	15	0.5	0.1	0.002	0.05	30.57180	1.44613	0.61153
2	3	15	1	0.1	0.002	0.05	30.74620	1.44597	0.85306
2	3	15	1.5	0.1	0.002	0.05	30.95440	1.44017	1.11887
2	3	15	2	0.2	0.002	0.05	30.66590	1.51392	1.60648
2	3	15	2	0.3	0.002	0.05	30.08070	1.60264	1.60648
2	3	15	2	0.4	0.002	0.05	29.37610	1.67770	2.32727
2	3	15	2	0.1	0.001	0.05	31.18790	1.47918	1.20417
2	3	15	2	0.1	0.002	0.05	31.19390	1.67770	1.40590
2	3	15	2	0.1	0.003	0.05	31.19950	1.88337	1.60709
2	3	15	2	0.1	0.002	0.03	22.57930	1.68641	1.42355
2	3	15	2	0.1	0.002	0.04	26.85770	1.68201	1.41390
2	3	15	2	0.1	0.002	0.05	31.19390	1.67770	1.40590

Chapter 5

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Summary

An analysis of the combined effects of chemical reaction, thermal radiation, rotatory, thermal-diffusion (Soret), diffusion-thermo (Dufour) and viscous dissipation on mixed convective Rivlin-Ericksen flow of an incompressible and electrically conducting fluids past a porous vertical plate with a periodic suction is studied. The dimensional governing equations are non-dimensionalised and the solutions were obtained using perturbation technique and Adomian decomposition method with MAPLE 18 and MATHEMATICA 11.0 software. The impacts of different parameters on velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number were illustrated both graphically and in tabular form. The results revealed that Sherwood number is enhanced with an increase in chemical reaction parameter, while both Skin friction and Nusselt number declined at $\eta = 0$.

5.2 Conclusion

Various fluid parameters involved in this study have significant influence on the fluid behaviour. The following are some of the findings:

- (i) The R_v increases with an increase in S_p , E_c , E , Ω , V_R , while it decreases with an increase in K_r and D_p .
- (ii) The temperature decreases with an increase in E , while it is enhanced with an increase in E_c and D_p .
- (iii) The concentration declines with an increase in K_r and D_p , an increase in concentration profiles is observed when there is an increase in S_p .
- (iv) Skin friction is enhanced with an increase in E , S_p , and E_c while, an increase in Ω , K_r , D_p and V_R for $\eta = 0$ slow down the skin friction. Consequently, at $\eta = 1$, skin friction increases with an increase in K_r , E , Ω , S_p , V_R and E_c ; it decreases with an increase in D_p .
- (v) Nusselt number at $\eta = 0$, is weakened with an increase in E , Ω , K_r , D_p , and E_c while, it is strengthened with an increase in S_p and V_R . At the other plate, for $\eta = 1$, Nusselt number increases with increase in Ω , K_r , D_p and E_c , and decreases when there is an increase in the E , S_p and V_R .
- (vi) Sherwood number accelerates with an increase in K_r and decelerates when there is an increase in E , Ω , S_p , E_c and V_R for $\eta = 0$. While at $\eta = 1$, Sherwood number

decreases with an increase in K_r and V_R , and is enhanced with an increase in E , Ω , S_p and E_c .

5.3 Contribution to knowledge

The following are the contributions to knowledge:

(i) Rotatory parameter reduced all the fluid properties at $\eta = 0$, but reverse effects is noticed at $\eta = 1$.

(ii) Increase in radiation parameter slow down temperature profiles, while it is enhanced by Dufour and dissipation parameter.

(iii) The resultant velocity distribution accelerates with a rise in dissipation, radiation, Soret and rotatory parameter. Decline in resultant velocity profiles is noticed for increase in Dufour and chemical reaction parameter.

(iv) Diffusion-thermo and chemical reaction weaken the concentration profiles while it is strengthen by an increase in thermal-diffusion.

Thus, combined effects of rotatory, viscous dissipation, radiation, thermal-diffusion and diffusion-thermo can be included in analysing Rivlin-Ericksen fluid flow for realistic practical applications.

5.4 Recommendations

The combined effects of chemical reaction, thermal radiation, rotatory, diffusion-thermo(Dufour), thermal-diffusion(Soret) and viscous dissipation on mixed convective Rivlin-Ericksen flow of an incompressible and electrically conducting fluids past a porous vertical plate with a periodic suction is considered. For further study, the effects of chemical reaction, thermal radiation, thermal-diffusion, viscous dissipation on rotatory Rivlin-Ericksen fluid flow should be looked into, for

(i) channel with slip condition

(ii) with Rivlin-Ericksen fluid of higher order

(iii) Hall current effects

References

- Abdulmaleque, K. L. (2017). Temperature-dependent suction/injection and variable properties on non-Newtonian Casson mixed convective MHD laminar fluid flow with viscous dissipation and thermal radiation. *American Journal of Heat and Mass Transfer*, **4**(2): 104-120.
- Babu, H. D. Venkateswarlu, B. and Narayana, S. P. V. (2017). Soret and Dufour effects on MHD radiative heat and mass transfer flow a stretching sheet. *Frontiers in Heat and Mass Transfer(FHMT)*, **8**(5): 1-9.
- Batcher, G. K. (2000). *An Introduction to Fluid Dynamics*. Cambridge Mathematical Library series, Cambridge University Press, 87-112.
- Beard, D. M. and Walters, K. (1964). Elastico-viscous boundary-layer flows- I: two-dimensional flow near the stagnation point. *Proc. Cambr. Phil. Soc.* (60): 667-674.
- Brewster, M. Q.(1972). *Thermal Radiation Transfer Properties*, John Wiley and sons, Chichester, 502-510.
- Chakraborty, G. and Panja, S. (2009). Oscillatory hydromagnetic couette flow of a viscoelastic Rivlin-Ericksen fluid. *J. Mech. Cont. and Math. Sci.*, **4** (1) : 403-409.
- Chandra, R. P., Raju, M. C. and Raju, G. S. S. (2015). Thermal and Solutal buoyancy effect on MHD boundary layer flow of a visco-elastic fluid past a porous plate with varying suction and heat source in the presence of Thermal Diffusion. *Applied and Computational Mathematics* , **4** (5) 1-7.
- Charankumar, G., Dharmaiah, G., Balamurugan, K. S. and Vedavathi, N. (2016). Chemical reaction and Soret effects on casson MHD fluid flow over a vertical plate. *Int. J. Chem. Sci.*, **14** (1): 213-221.
- Chen, W. and Lu, Z. (2004). An algorithm for Adomian decomposition method. *Applied Mathematics and Computation*, **159** : 221-235.
- Dada, M. S. and Agunbiade, S. A. (2016). Radiation and chemical reaction effects on convective Rivlin-Ericksen flow past a porous vertical plate. *Ife Journal of Science*, **18** (3) : 655-667.
- Dada, M. S. and Adefolaju, F. H. (2012). Dissipation, MHD and radiation effects on an unsteady convective heat and mass transfer in a Darcy-Forcheimer porous medium. *Journal of Mathematics Research*, **4** (2) : 110-127.
- Das, S. S. , Sahoo, S. K. , Dash, G. C. and Panda, J. P. (2005). Laminar flow of

elastico-viscous Rivlin-Ericksen fluid through porous parallel plates with suction and injection, the lower plate being stretched, *AMSE J. Mod.Meas.Cont.B*, **74**(8):1-22.

Das, S. S. , Maity, M. and Das, J. K. (2011). Effect of heat source and variable suction on unsteady viscous stratified flow past a vertical porous flat moving plate in the slip flow regime. *International Journal of Energy and Environment* **2**(2):375-382.

Deepthi, J. and Prasada, D. R. V. (2017). Mixed convective heat and mass transfer flow in a vertical channel with Soret effect and radiation. *International Journal of Emerging Trends in Engineering and Development*, **2** (7):1-11.

Dharmendra, K. and Varshney, N. K. (2012). Stratified Rivlin-Ericksen fluid effect on MHD free convection flow with heat and mass transfer past a vertical porous plate, *Ultra Scientist*, **24** (3): 527-534.

Dulal, P. and Babulal, T. (2012). Influence of Soret effect on MHD mixed convection oscillatory flow over a vertical surface in a porous medium with chemical reaction and thermal radiation. *International Journal of Nonlinear Science*, **14** (1): 66-78.

Garg, B. P., Singh, K. D. and Bansal, A. K. (2014a). An oscillatory MHD convective flow of viscoelastic fluid through porous medium filled in a rotating vertical porous channel with heat radiation. *International Journal of Engineering and innovative Technolgy* , **3** (12): 273-281.

Garg, B. P., Singh, K. D. and Bansal, A. K. (2014b). Rotating MHD convective flow of Oldroyd-B fluid through a porous medium in a vertical porous channel with thermal Radiation. *International Journal of Innovations in Engineering and Technology* , **4** (1) : 251-267.

Gaur, P. K. and Jha, A. K. (2016). Heat and mass transfer in viscoelastic fluid through rotating porous channel with Hall effect. *Open Journal of fluid Dynamics*, **6**,11-29.

Gbadeyan, J. A. , Idowu, A. S., Ogunsola, A. W., Agbola, O. O. and OLarewaju, P. O. (2011). Heat and mass transfer for Soret and Dufour's effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a viscoelastic fluid in the presence of magnetic field. *Global Journal of Science Frontier Research*, **11** (8): 96-115.

Gbadeyan, J. A. and Dada, M. S. (2013). On the influence of radiation and heat transfer on an unsteady MHD non-Newtonian fluid flow with slip in a porous medium. *Journal of Mathematics Research*, **5** (3): 40-50.

Grag, B. P. and Singh, K. D. (2015). Chemically reacting, radiating and rotating MHD Convective flow of viscoelastic fluid through porous medium in vertical channel.

International Journal of latest Trends in Engineering and Technology , **5** (2) : 314-326.

Guria, M. and Jana, R. N. (2013). Flow of a viscoelastic fluid past a porous plate in a rotating system. *Int. J. of Applied Mechanics and Engineering*, **18** (1) : 27-41.

Hussaini, S. A., Ramana, M. V. and Rafiuddin (2015). Magnetohydrodynamics Rivlin-Ericksen flow through porous medium in slip flow Regime. *International Journal of Science, Engineering and Technology Research* , **4** (7) : 2568-2572.

Ibrahim, S. M. and Suneetha, K. (2015). Chemical reaction and Soret effects on unsteady MHD flow of a viscoelastic fluid past an impulsively started infinite vertical plate with heat source/Sink. *International journal of Mathematics and Computational Science*, **1** (1) : 5-14.

Jana, M., Das, S. and Jana, R. N. (2012). Unsteady couette flow through a porous medium in a rotating system. *Open Journal of Fluid Dynamics*, **2**, 149-158.

Jimoh, A., Idowu, A. S. and Titiloye, E. O. (2014). Influence of Soret on unsteady MHD of Kuvshinshiki fluid flow with heat and mass Transfer past a vertical porous plate with vertical suction. *International journal of Recent and Innovation Trends in Computing and Communication* , **2** (9) : 2599-2611.

Joseph, K. M., Magaji, A. S., Peter, A. and Tijani, N. Z. (2016). Effect of variable suction on unsteady MHD oscillatory flow of Jeffrey fluid in a horizontal channel with heat and mass transfer. *Journal of Scientific and Engineering Research*, **3** (3): 599-610.

Lakshmi, R. and Gomathi, S. (2012). Study on flow of Rivlin-Ericksen fluid past a porous vertical wall with constant suction. *International Journal of Scientific and Research Publications*, **2** (9) : 1-16.

Mahood, F. and Ibrahim, S. M. (2016). Effects of Soret and Non-uniform heat source on MHD Non-darcian convective flow over a stretching sheet in a dissipative micropolar fluid with radiation. *Journal of Applied Fluid Mechanics*, **9** (5) : 2503-2513.

Mutua, N. M., Musyoki, N. M., Kinyanjui, M. N. and Kwanza, J. K. (2013). Magnetohydrodynamics free convection flow of a heat generating fluid past a semi-infinite vertical porous plate with variable suction. *International Journal of Applied Mathematics Research*, **2** (3) : 356-371.

Nayak, M. K. and Dash, G. C. (2014). Effect of chemical reaction on MHD flow of a visco-elastic fluid through porous medium. *Journal of Applied Analysis and computation*, **4** (4): 367-381.

Pal, D. R. and Samad, M. A. (2015). Combined effects of viscous dissipation and thermal

radiation on non-Newtonian fluid along a surface with heat generation and uniform surface heat flux. *Dhaka Univ. J. Sci.* **63** (2): 97-104.

Rahman, M. D., Asaduzzaman, M. D., Anisur Rahman, M. D. and Aktar, S. (2016). Heat transfer of a heat generation MHD fluid flow over a vertical porous flat plate in a rotating system. *Scholars Journal of Engineering and Technology* , **4** (8) : 344-358.

Rao, M. A. (2007). *Rheology of fluid and semisolid foods; Principles and Applications*. Springer, New York, (2nd ed) 18-22.

Ravikumar, V., Raju, M. C. and Raju, G. S. S. (2014). Combined effects of heat absorption and MHD on convective Rivlin- Ericksen flow past a semi-infinite vertical porous plate with variable temperature and suction. *Aim Shams Engineering Journal* 5, 867-875.

Reddy, S. K., Kesavaiah, D. C. and Shekar, M. N. R. (2013). MHD heat and mass transfer flow of a viscoelastic fluid past an impulsively started infinite vertical plate with chemical reaction. *International Journal of Innovative Research in Science Engineering and Technology*. **2** (4): 973-981.

Reddy, L. R., Raju, M. C., Raju, G. S. S. and Reddy, N. A. (2016a). Thermal diffusion and rotational effects on magneto hydrodynamic mixed convection flow of heat absorbing/generating viscoelastic fluid through a porous channel. *Frontiers in Heat and Mass Transfer* , **7** (20) : 1-12.

Reddy, P. C., Raju, M. C., and Raju, G. S. S. (2016b). Soret and Dufour effects on MHD free convective flow of Rivlin-Ericksen fluid past a semi-infinite vertical plate. *Advances and Applications in Fluid Mechanics*, **19** (2) : 401-414.

Reddy, S. H., Raju, M. C. and Reddy, E. K. (2016c). Magneto convective flow of a non-Newtonian fluid through non-homogeneous porous medium past a vertical porous plate with variable suction. *Journal of Applied Mathematics and Physics*, **4**, 233-248.

Rita, C. and Das, B. (2013). Viscoelastic effects on MHD free convective flow past an oscillating porous plate through porous medium with heat source. *International Journal of innovative Research in Science, Engineering and Technology*, **2** (12) : 7287-7295.

Rivlin, R. S. and Ericksen, J. I. (1955). Stress deformation relations for isotropic materials, *J. Rational Mech. Anal.* 4:323-329

Sarma, G. S. and Govardhan, K. (2016). Thermo-diffusion and Diffusion-thermo effects on free convective heat and mass transfer from vertical surface in a porous medium with viscous dissipation in the presence of Thermal radiation. *Archives of Current Research International*, **3** (1) : 1-11.

Sekhar, D. V. and Reddy, G. V. (2012). Chemical reaction effects on MHD unsteady free convective Walter's memory flow with constant suction and heat sink. *Advances in Applied Science Research*, **3**(4): 2141-2150.

Seth, G. S., Nandkeolyar, R. and Ansari, M. S. (2012). Effects of Hall current rotation unsteady MHD couette flow in the presence of an inclined Magnetic field. *Journal of Applied fluid Mechanics*, **5** (2): 67-74.

Sharma, A. K. Dubey, G. K. and Varshney, N. K. (2013). Effect of rotating on MHD flow of viscoelastic (Rivlin-Ericksen) fluid through porous medium with transpiration. *Ultra Scientist*, **25** (2) : 245-252.

Singh, K. D. (2013). Visco-Elastic MHD convective periodic flow through porous medium in a rotating vertical channel with thermal radiation. *Journal of Global Research in Mathematics Archives* **1** (4) : 8-20.

Singh, V. K., Gupta, P. C., Singh, J. and Varshney, N. K. (2011). Effect of rotatory system on MHD free convection and mass transfer flow of Oldroyd fluid through porous medium with constant heat and mass flux across moving plate. *International Journal of Mathematics Archive*, **2** (9) : 1716-1722.

Sreekanth, S., Saravana, R., Venkataramana, S. and Heemadri, R. R. (2011). Hydromagnetic Unsteady Hele-shaw flow of a Rivlin-Ericksen fluid Through porous media. *Advances in Applied Science Research*, **2** (3): 246-264.

Sreenadh, S. and Reddappa, B. (2016). Couette flow of a Jeffrey fluid in a rotating channel. *International Journal of Engineering Sciences and Research Technology*, **5** (5) : 218-227.

Tasawar, H. , Zahir, H., Tanveer, A. and Alsaedi, A. (2017). Soret and Dufour effects on MHD peristaltic transport of jeffrey fluid in a curved channel with convective boundary conditions. *PLoS Journal Pone* , **12** (2) : 1-50.

Umamaheswar, M., Varma, S. V. K. and Raju, M. C. (2013). Unsteady MHD free convective visco-elastic fluid flow bounded by an infinite inclined porous plate in the presence of heat source, viscous dissipation and Ohmic heating. *International Journal of Advanced Science and Technology* , **61** : 39-52.

Uwanta, I. J. and Hussaini, A. (2012). Effect of mass transfer on Hydromagnetic free convective Rivlin-Ericksen flow through a porous medium with time Dependent suction. *International Journal Engineering and Science*, **1** (4): 21-30.

Varshney, N. K., Satyabhan, S. and Singh, J. (2013). Effect of Rotatory Rivlin-Ericksen fluid on MHD free convection and mass transfer flow through porous medium with constant

heat and mass flux across moving plate. *IOSR Journal of Engineering* (IOSRJEN) **1** (1) : 010-017.

Vijayakumar, K. and Reddy, E. K. (2017). MHD boundary layer flow of a visco-elastic fluid past a porous plate with varying suction and heat source/sink in the presence of Thermal radiation and diffusion. *Global Journal of Pure and Applied Mathematics*, **13** (6): 2717-2733.