

SENSITIVITY OF PARAMETERS IN AN OPTIMAL CONTROL MODEL OF THE ELECTRIC POWER GENERATING SYSTEM

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ABSTRACT

The electric power generating systems which is expressed mathematically with an optimal control model relates two or more parameters that can be used to measure the condition or state of electric power generating systems. These parameters enable us to know the condition and the capacity of the generator, how to use, and how long to use, in order to maximize the general output and minimize the cost of generation. The sensitivity of the parameters is an approach given to a model so as to define the relative importance of the factors related to the model, because the whole parameter space is fully described. In this work, the sensitivity of parameters in an optimal control model of the electric power generation system proposed by Bamigbola and Aderinto (2009) is investigated so as to determine the relative importance of each parameter on the model results.

KEYWORDS: *Optimal control, electric power generating system, sensitivity, parameters, modelling.*

1. INTRODUCTION

Real life problems are often modelled as mathematical expressions which sometimes include parameters. Examples abound in electrical engineering, finance, economics and medical sciences, to name a few.

A model can be defined as a simplified representation of certain aspects of real life system. A mathematical model is a model created using

mathematical concepts such as functions and equations. When we create mathematical models, we move from real world into the theoretical world of mathematical concepts, where the model is built. We then manipulate the model using mathematical or computer aided techniques. Finally we re-enter the real world, which is then translated into useful solution to the real problem which implies that the start and end are in the real world, Edward and Hamson (1989). Thus, several researchers have worked on the applications of mathematical modelling into solving real life problems such as in: Mathematical modelling, a useful tool for STI control policy, Ashleigh et al (2013), Mathematical modelling application to corrosion in a petroleum industry, Oyelami and Asere (2009), Dynamic modelling of a wind turbine with doubly fed induction generator, Poller and Achilles (2003), A mathematical theory to model complex socio-economical systems by functional subsystems representation, Giulia et al (2008) and a host of others. As such, much emphasis has been on the operational aspect rather than the economical aspect of mathematical modelling.

However, several researchers have also worked on the sensitivity analysis of the modelling parameters used in various types of models. Fordor et al. (2010) worked on parameter sensitivity analysis of a synchronuous generator, Zivarian (2002) also worked on sensitivity analysis of a nonlinear lumped paramter model of HIV infection dynamics, Kathirgamanathan and Neitzart (2008) worked on optimal control parameter estimation in Aluminium extrusion for given product characteristics, Guo et al. (2016) worked on the performance evaluation and parameter sensitivity of energy-harvesting shock absorbers on different vehicles, Christopher and Fathalla (1999) worked on sensitivity anaylsis of parameters in modelling with delay-differential equations, Burns et al. (2007) worked on sensitivity analysis and parameter estimation for a model Chlamydia Trachomatis infection, Bastidas et al (2016) worked on parameter sensitivity and uncertain analysisfor a storm surge and wave model to metion but a few.

The main concern in mathematical modelling is to establish relationships between factors but this also almost invariably involves parameters.

Parameters are quantified factors that have constant values for a particular problem but can change from problem to problem, Edward and Hamson (1989). In general, a model can be of some use in predicting general behaviour in a descriptive fashion. To use the model in a practical way, we must solve for the numerical values of the parameters from given data. The use of data to obtain parameter values relevant to a particular application of the model is frequently called *calibrating* or sometimes *tuning* the model. The common methods of obtaining the parameter values are: graphical, statistical (usually involving least-squares estimation) and mathematical (usually requiring the solution of linear or nonlinear equations) methods. Parameters are used to identify characteristics, features, measurable factors that can help in defining a specific system. They are essential elements to take into consideration for the evaluation of an event, a project or any situation.

However in most practical problems, we are interested not only in the solution of the problem, but also in how the solution changes when the parameters of the problem change. Parameter sensitivity analysis is thus used to determine how sensitive the results of propagation of a parameter by varying the value of the parameter in a model. A mathematical model comprises of parameters, in which the more the parameters in the model, the higher the dimension of the model and the more complex to solve. Thus, the sensitivity of parameters makes it easier in determining the relative importance of each parameter, thus modifying the model by reducing the model's dimension which reduces the complexity.

Sensitivity of parameters helps in classifying modelling parameters into different types with respect to the relative importance of each parameter related to a model. The classification is as follows:

- Insensitive parameter: its influence on the model's accuracy is not critical. It usually ranging between 0-19%.
- Sensitive parameter: its influence on the model accuracy is significant and causes a change of between 20-60% in the model.

- Highly sensitive parameter: its influence on the model accuracy is very critical that is a little change will lead to a large change in the result of the model ranging from 60% and above.

In this study, we seek to assess the importance of some parameters included in mathematical models using an optimal control model of the electric power generating system as a case study. The parameter sensitivity analysis is hereby applied to the cost function of the model to investigate the relative importance of each parameter with a view of reducing the model's complexity.

2 MATHEMATICAL DESCRIPTION OF THE ELECTRIC POWER GENERATING SYSTEM

The electric power generating system can be described as follows: Considering the i^{th} generator with the rate of change in generation at time (t) depending on invested capital $C_i(t)$ and the generator output $G_i(t)$ in turn depending on the power input, generator capacity, running cost and transmission losses. Also, the investment on capital $C_i(t)$ at time t is known to be dependent on labour cost s_i , maintenance cost y_i , fuel cost rate r_i , actual capacity rate x_i , and the rate of cost of transmission to the grid centre γ_i . Because of the physical law that governs power generation and the operating limitation on the equipment, since the operation limitation on the equipment cannot be neglected because of its lifespan, we choose the generator capacity as our control $u_i(t)$ for which the upper bound is chosen to be 1 to represent the total capability of the generator.

In the above setting, an important objective is to minimize the total operating cost incurred in the process of generating the required quantity of electric power G at time t , and the components of the total operating cost are $C(t)$ and $u(t)$.

The electric power generating system described above has been modelled by Bamigbola and Aderinto (2009), as an optimal control problem of the form: Find the control u that minimizes the cost function

$$J(u) = \int_{t_0}^T [\delta^T C(t) + \eta u^T u] dt, (2.1)$$

subject to:

$$\frac{d}{dt} G_i(t) = \alpha_i + q_i C_i(t) G_i(t) - k_i G_i(t) (2.2)$$

$$\frac{d}{dt} C_i(t) = (s_i + y_i) + r_i C_i(t) G_i(t) + x_i u_i(t) C_i(t) + \gamma_i C_i(t) (2.3)$$

and

$$G_i(t_0) = G_0, \quad C_i(t_0) = C_0, \quad a_i \leq u \leq b_i, \quad i = 1, 2, \dots, m (2.4)$$

where $G_i(t)$ represents the amount of power generated by the i^{th} generator at the time t , $C_i(t)$ the capital investment on the i^{th} generator at time t , s_i labour cost, y_i maintenance cost, x_i actual capacity rate, r_i fuel cost rate, γ rate of cost of transmitting from generating station and $u(t)$ generator capacity while α_i , q_i and k_i are respectively the actual mechanical / electrical energy from the high pressure turbine and low pressure turbine (capacity of generator i), the corresponding running cost and the transmission loss rate, which depends on the distance from the grid centre, $\delta^T = (\delta_1, \delta_2, \delta_3, \dots, \delta_m)$ is the unit expenditure on the generators, η is a parameter to balance the size of the control.

The parameters in the above optimal control model, for which sensitivity analysis is to be conducted, are δ and η . Parameter δ is the unit expenditure on the generators, η represents the number of hours for which the machine can be on.

With these, problem (2.1)-(2.4) can be classified as being five-dimensional, having as its independent variables t , u , C , δ and η

3. COMPUTATIONAL CONSIDERATION

3.1 Algorithm for the parameters sensitivity

The following algorithm is constructed for sensitizing the parameters in the optimal control problem:

1. Select the parameters to be examined.
2. Assume values for each parameter.
3. Run the model using the selected series and calculate the objective function for each value of the generator capacity.
4. Evaluate parametric sensitivity, that is, define the sensitive and insensitive parameters using graphical and tabular representation.
5. Determine the relative importance of each parameter.

3.2 Computational details

To drive the application of the electric power generating system requires the supply of relevant data which requirement is met with the provision of data, tabulated in Table 5.3, obtained from the National Control Centre, Osogbo, Nigeria.

Table 1: Data for Electric power generating system

Data	One-generator	Three-generator
α	800MW	100 MW, 100 MW, 80 MW
q	0.3217 per unit	0.3217, 0.3112, 0.312 per unit
r	0.347 per unit	0.347 per unit
x	0.606	0.6.6, 0.50 2, 0.402

k	0.002MW' per unit	0.002MW'
s	200 per h (assumed)	50 per h (assumed)
y	100 per h (assumed)	50 per h (assumed)
γ	0.3421 per unit	0.3421 per unit
u	$a \leq u \leq b, 0 \leq u \leq 1,$	$a \leq u \leq b, 0.2 \leq u \leq 0.9,$
	$a = 0, b = 0$	$a = 0, b = 0$
δ	1-2 (estimated)	1-3 (estimated)
η	2-6 (estimated)	2-6 (estimated)

Source: National Grid Centre, Osogbo, Nigeria.

3.3 Numerical consideration

Thus, the numerical solution to (3.1)-(3.4) can be obtained by the application of finite difference method precisely by using trapezoidal rule for objective functional and fourth order Runge-Kutta scheme due to its capacity to yield very accurate results than other methods, for the constraints, Poppe and Kautz (1998).

The trapezoidal rule is given as

$$\int_a^b f(x)dx = \frac{h}{2}(f_0 + 2f_1 + 2f_2 + \dots + 2f_{n-1} + f_n)$$

$$= h\left(\frac{f_0 + f_n}{2} + \sum_{i=1}^{n-1} f_i\right), (3.1)$$

where $f(x_k) \equiv f_k$

The fourth order Runge-Kutta method for

$$\frac{dy}{dx} = f(x, y(x)), y(x_0) = y_0$$

is given by

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4), (3.2)$$

where

$$k_1 = f(x_n, y_n),$$

$$k_2 = f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_1), (3.3)$$

$$k_3 = f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_2),$$

$$k_4 = f(x_n + h, y_n + hk_3),$$

Utilizing (3.1)-(3.3) in (2.1)-(2.4) yields the following computational solution system as:

Table 2: Numerical solution for one-generator electric power model

$$h = 0.05, u = 0.1, 0.2, 0.3 \dots 1.0. a \leq t \leq b, a = 0, b = 1, G = 448.37443$$

U	C	J, $\delta = 1,$ $\eta = 2$	J, $\delta = 1,$ $\eta = 6$	J, $\delta = 2,$ $\eta = 2$	J, $\delta = 2,$ $\eta = 6$
0.1	172.95456	172.97456	173.01456	345.92912	345.96912
0.2	173.43002	173.51002	173.67002	346.94004	347.10004
0.3	173.94167	174.12167	174.48167	348.06334	348.42334
0.4	174.38331	174.70331	175.34331	349.08662	349.72662
0.5	174.87423	175.37324	176.37324	350.24648	351.24648
0.6	175.27690	175.99690	177.43690	351.27380	352.71380

0.7	175.82223	176.80223	178.76223	352.62446	354.58446
0.8	176.30126	177.58126	180.14126	353.88252	356.44252
0.9	176.78750	178.40750	181.64750	355.19500	358.43500
1.0	177.12043	179.12043	183.12043	356.24086	360.24086

Table 3: Numerical solution for three-generators electric power model

$h=0.05, u=0.1, 0.2, 0.3, \dots, 1.0, a \leq t \leq b, a=0, b=1, G_1=49.8663, G_2=49.8413, G_3=38.8947$

u	C_1	C_2	C_3	$J, \hat{\delta} = 3, \eta = 2$	$J, \hat{\delta} = 3, \eta = 6$	$J, \hat{\delta} = 4, \eta = 2$	$\hat{\delta} = 4, \eta = 6$	$J, \hat{\delta} = 5, \eta = 2$	$J, \hat{\delta} = 5, \eta = 6$
0.2	84.16832	84.02192	83.88128	756.29456	756.45456	1008.36608	1008.52608	1260.43760	1260.59760
0.3	84.56881	84.41291	83.88255	758.77281	759.13281	1011.63708	1011.99708	1264.50135	1264.86135
0.4	84.93559	84.64148	84.35942	762.12970	762.76977	1016.06636	1016.70636	1270.00295	1270.64295
0.5	85.32667	85.11172	84.60510	765.63047	766.63047	1020.67396	1021.67396	1275.71745	1276.71745

0.6	85.61276	85.23229	84.80787	767.67876	769.11876	1023.33168	1024.77168	1278.98460	1280.42460
0.7	86.09327	85.57560	85.07947	771.22502	773.18502	1027.97336	1029.93336	1284.72170	1286.68170
0.8	86.13012	85.88667	85.31879	773.28674	775.84674	1030.62232	1033.18232	1287.95790	1290.51790
0.9	86.62461	86.20128	85.56140	776.78187	780.02187	1035.16192	1038.40916	1293.55645	1296.79645

Converting the vector δ of three electric power generating systems to scalar $\hat{\delta}$, we took

$$\hat{\delta} = \|\delta\| = \sum_{j=1}^m |\delta_j|, \quad j = 1, 2, \dots, m$$

A computer program - Maple 16 software is used to implement Algorithm 3.1, in order to generate numerical results which are presented in Figure 1-6 below, since a good way of visualizing is by graphical representation, Edward and Hamson (1989). In addition, the time execution in implementing Algorithm 3.1 for different parameter systems are recorded in Table 4 below

Table 4: Execution time for Implementing Algorithm 3.1

System	Tw	One
One-generator	0.32	0.280s
Three-generators	0.43	0.372s

4. DISCUSSION OF RESULTS

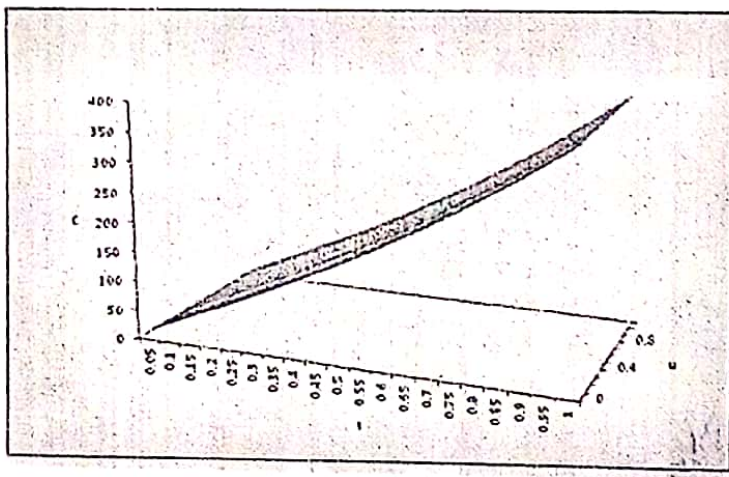


Figure 1: Graph of Cost of generation C versus time t and control u

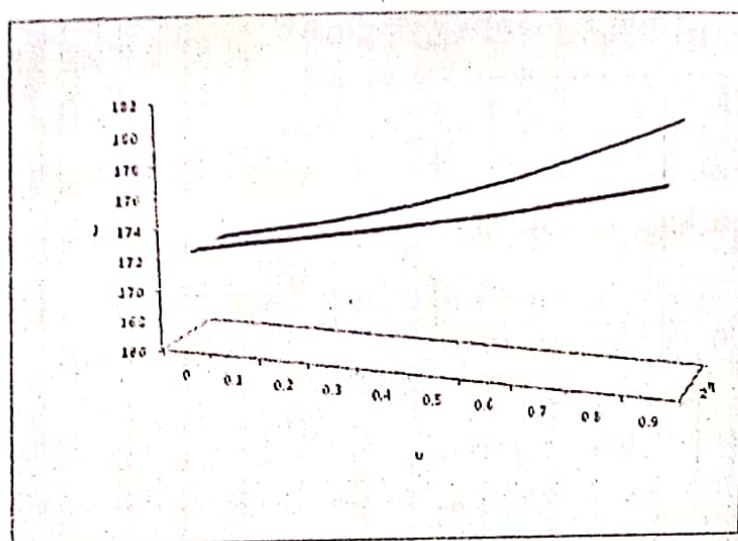


Figure 2: Graph of Objective function J versus control and parameter η for $\delta = 1$

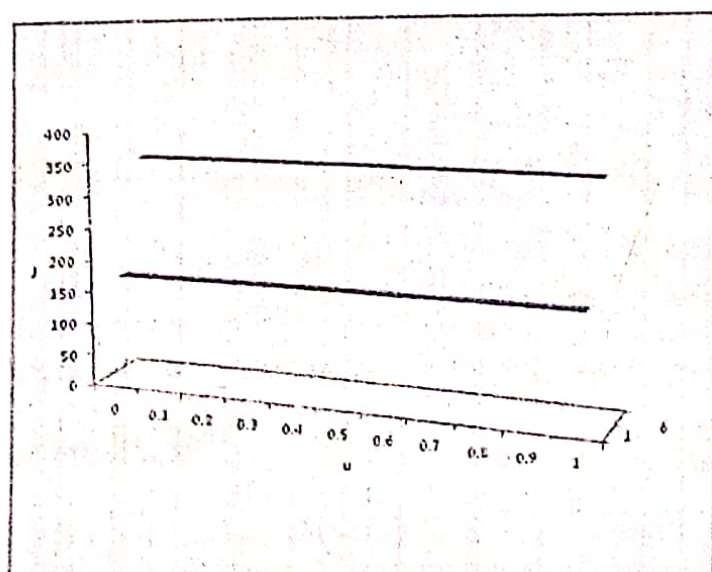


Figure 3: Graph of Objective function J versus control u and parameter δ for $\eta = 2$

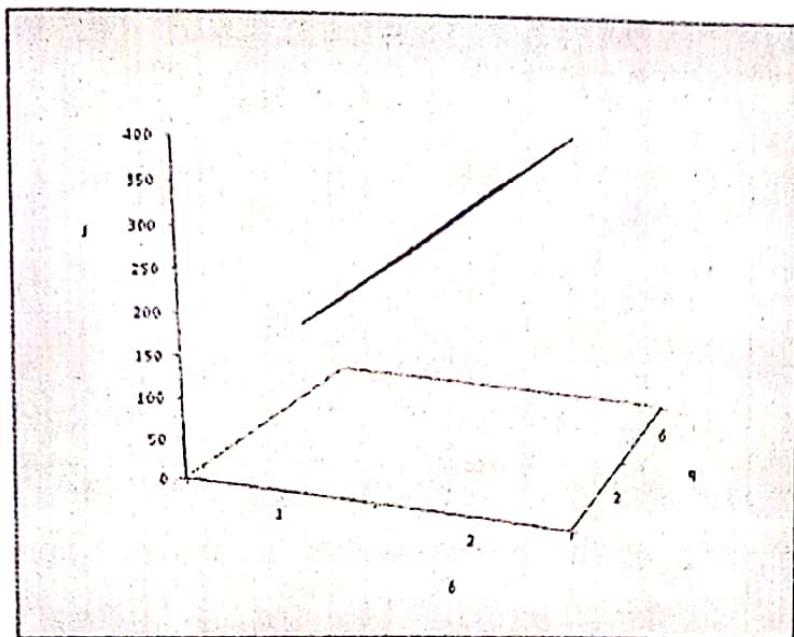


Figure 4: Graph of Objective function J versus parameter δ and η for $u = 1.0$

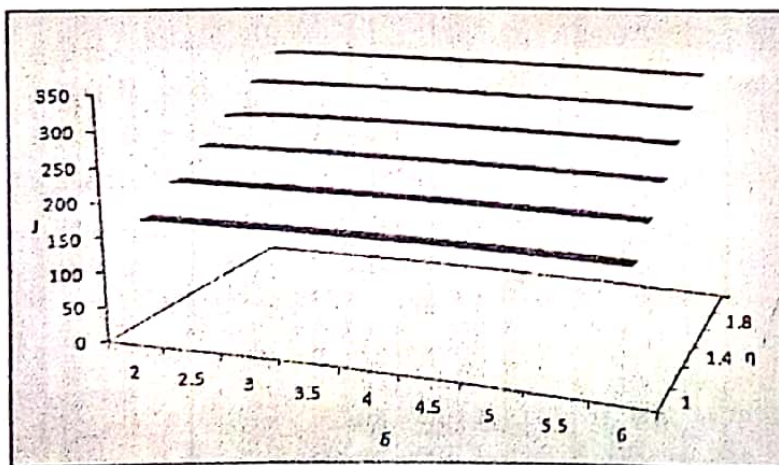


Figure 5: Graph of objective function J versus control u and parameters δ and η for one-generator system.

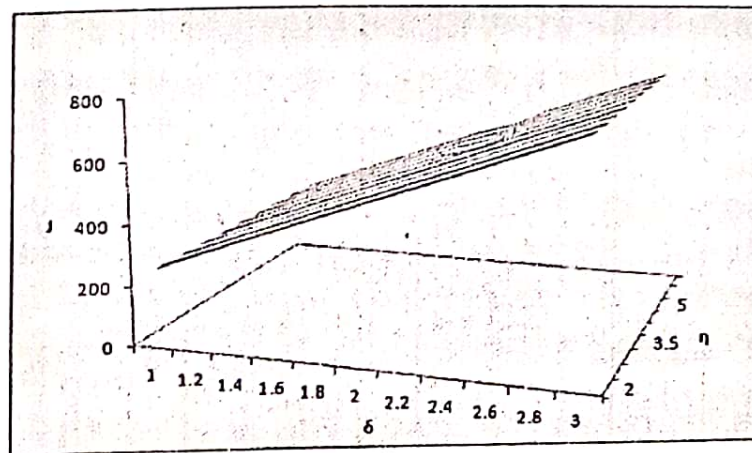


Figure 6: Graph of objective function J versus control u and parameters δ and η for three-generator system.

Figure 1 shows that the higher the control with time, the higher the cost of electric power generation.

Figure 2 shows that a change in the value of parameter η shows little difference in the output of the objective function when parameter δ is fixed.

Figure 3 shows that a little change in the value of parameter δ shows a significance difference in the output of the objective function when parameter η is fixed.

Figure 4 shows that the value of the objective function increases as the values of parameters η and δ increases at a given control value u . However, it is observed that change in the parameter η makes little or no significant difference in the value of the objection fuction meanwhile a little change in parameter δ shows a significant difference in the value of the objective function.

Figure 5 shows that a 3-dimensional graph of objective functional versus control u and parameters η and δ for a one-generator system. This shows a little significant difference in the valueof the objective function when

parameter η is varied while a huge significant difference is observed in the value of objective function when the parameter δ is varied.

Figure 6 shows that a 3-dimensional graph of objective functional versus control u and parameters η and δ for a three-generator system. This shows a little or no significant difference in the value of the objective function when parameter η is varied while a huge significant difference is observed in the value of objective function when the parameter δ is varied.

However, from the figures above, it can be established that the model gave the maximum generator output so far, and that the more we generate, the more the cost of generation. This suggests that for efficiency and effective functioning of the generators, monitoring the control is very essential.

Furthermore, it is clear from the graphical representation that the cost of generation J to a large extent depends on the parameter δ (unit of power expenditure on the power generating system) because there is a significant change in the value of J with relatively small change in δ . On the other hand, a sizable change in the value of η produces an insignificant change in the value of J . Based on sensitivity classification presented earlier, δ is adjudged to be highly sensitive while η is classified as an insensitive parameter. Removal of the insensitive parameter results in the modified model as

$$J(u) = \int_0^T [\delta^T C(t)] dt \quad (5.1)$$

subject to:

$$\frac{d}{dt} G_i(t) = \alpha_i + q_i C_i(t) G_i(t) - k_i G_i(t),$$

$$\frac{d}{dt} C_i(t) = (s_i + y_i) + r_i C_i(t) G_i(t) + x_i u_i(t) C_i(t) + \gamma_i C_i(t)$$

and

$$G_i(t_0) = G_0, C_i(t_0) = C_0, \quad a_i \leq u \leq b_i, i = 1, 2, \dots, m.$$

Effect of this modification on the computational process, as shown in Table 4.3, is a reduction of 15% in execution time.

5. CONCLUSION

Electric power generation as a real life endeavor needs to be studied as a multidisciplinary subject making use of contributions from the relevant fields. As shown in this work, mathematics has a lot to contribute in resolving practical problems in electric power generation as well as improving on its advancement. Analysis of sensitivity of parameters included in models obtained for real life problems is herein shown to be a worthwhile task of refining the model so as to maximize the accuracy of the solution, at the same time minimize the computational efforts and the time required.

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Appendix

The Maple16 software was used to solve the model