MODELLING OF ENUGU STATE MONTHLY RAINFALL USING BOX AND JENKINS METHODOLOGY

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ABSTRACT: The paper examined the rainfall distribution of Enugu state in Nigeria. Box-Jenkins methodology was used to build ARIMA model to analyze data and forecast for the period of 15 years, from January, 2002 to December, 2016 and to predict for the future. We observed that the average annual rainfall of Enugu state ranges from 124mm to 179mm. The irregularity in annual rainfall of Enugu State one and half decades ago is a bit large, indicating that climate stability is high in the state. Different time series models were diagnostically checked, and tested for Enugu state and at last an SARIMA (0, 0, 0)(1, 0, 1)$_{12}$ model is chosen as the proposed best model. The proposed model was used to forecast two years' monthly rainfall value for the state. The results indicated that relatively there is a tendency of increasing in trend of future rainfall values in the state.
KEYWORDS: Modelling; Box and Jenkins; ARIMA; Rainfall; SARIMA; Forecasting; Enugu State.

1. INTRODUCTION

Enugu State is one of the states in the southeastern part of Nigeria. It was created from the old Anambra state in 1991. It shares borders with Abia and Imo states, Ebonyi State, Benue state, Kogi state, and Anambra state to the south, east, northeast and west respectively.

The name "Enugu" (which was coined from *Enu Ugwu*) is synonymous to "the top of the hill" denoting the city's hilly topography ([***]). The most important cities in the state are Enugu (the state capital), Agbani, Awgu, Oji-River, Udi (site of the famous eastern coal mines) and Nsukka (home to the first university in Eastern Nigeria). The state has 17 administrative groupings, called Local Government Areas. A greater percentage of the population in the state is engaged in agriculture ([Lie71]), with a small proportion also engaged in white collar jobs.

Enugu is in the tropical rain forest zone with a derived savannah, with humidity highest between March and November ([Igw15]). Enugu state, being in the southern part of Nigeria, has the rainy season and dry season as the only weather conditions that occur yearly.

The topography of a region is an important component relating to variation in the climatic condition in various parts of a country. The climatic conditions of semi-arid zones exhibit extreme fluctuation both yearly and seasonally. Semi arid regions receive very small, irregular, and unreliable rainfall, while tropical regions essential receive rainfall all year round. Readers are referred to ([Ade10, Tak12]) for greater details.

In Nigeria, many regions experience rainfall throughout the year, but some regions experience seasonal and low rainfall thus necessitating irrigation ([AE09]). The pattern of rainfall usually exhibit spatial and temporal variability, which has effects on agricultural production, transportation, water supply, environment and urban planning ([AE09]). It has been noted that one may not be able to completely avoid damages due to extremes of rainfall but a forewarning could be of great use ([Nic80, MYM12]). Various approaches have been deployed to predict rainfall patterns ([Yev72, DK78, Tsa98, Cha91]). In practice, assessing the variability of rainfall is useful in decision making, management of risk and optimum usage of water resources of countries. In this study however, we used the univariate Box-Jenkins methodology to build ARIMA model in order to assess the rainfall pattern in Enugu State based on the data collected from Nigerian Meteorological Agency.

2. METHODOLOGY

An ARIMA model is an algebraic statement showing how a time-series variable is related to its own past values ([Pan83]). Box and Jenkins proposed a practical three-stage procedure for finding a good model time series model namely: identification,

parameter estimation and diagnostic checking ([Pan83]). This is what is known as the Box-Jenkins methodology.
The Box- Jenkins ARIMA (p, d, q) model include the autoregressive process (AR), the integrated process (I), and the moving average process (MA).
An Autoregressive (AR(p)) Process Model is defined as

$$X_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \Lambda + \phi_p x_{t-p} + w_t \qquad (1)$$

The model in lag operators takes the following form:

$$(1 - \phi_1 B - \phi_2 B^2 - \Lambda - \phi_p B^p) x_t = w_t \qquad (2)$$

The autoregressive operator $\phi(B)$ is defined to be

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \Lambda - \phi_p B^p \qquad (3)$$

The values of ϕ which make the process stationary are such that the roots of $\phi(B) = 0$ lie outside the unit ball in the complex plane [Cha91].
A Moving Average (MA(q)) Process Model is defined as

$$X_t = w_t - \theta_1 w_{t-1} - \theta_2 w_{t-2} - \ldots - \theta_q w_{t-q} \qquad (4)$$

In order to preserve its unique representation, usually the requirement is imposed that all roots of

$$\theta(B) = 1 + \theta_1 B + \theta_2 B^2 + \Lambda + \theta_q B^q = 0 \qquad (5)$$
$$|\,\theta_i\,| > 1.$$

Autoregressive Moving Average (ARMA(p,q)) model can be given as:

$$X_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \ldots + \phi_p x_{t-p} + w_t - \theta_1 w_{t-1} - \theta_2 w_{t-2} - \ldots - \theta_q w_{t-q} \qquad (6)$$

This can be simplified by a backward shift operator B to obtain

$$\phi(B) x_t = \theta(B) w_t \qquad (7)$$

Most time series in their raw form are non stationary. If the time series exhibits a trend, then this can be elimated through differencing. The sample autocorrelation function (ACF) and the sample partial autocorrelation function (PACF) are some of the common tools used to analyze univariate time series data.
The letter "I" in the acronym ARIMA corresponds to the number of times (d) the original series has been differenced; if a series has been differenced d times, it must subsequently be integrated d times to return it to its original overall level ([Pan83]).
Once the process has been transformed into stationarity (that is, it should have a constant mean, variance and correlation through time), we can proceed with the analysis. The Box-Jenkins ARIMA method is appropriate only for a time series that is stationary ([Pan83]).
A process (x_t) is said to be an ARIMA (p, 1, q) if it can be written as:

$$x_t - x_{t-1} = \phi_1 (x_{t-1} - x_{t-2}) + \ldots + \phi_p (x_{t-p} - x_{t-p-1}) + w_t - \theta_1 w_{t-1} - \ldots - \theta q w_{t-q} \qquad (8)$$

$$y_t = \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p} + w_t - \theta_1 w_{t-1} - \ldots - \theta q w_{t-q} \qquad (9)$$

Where:
$y_t = \nabla x_t = (x_t - x_{t-1}) \quad y_{t-1} = \nabla x_{t-1} = (x_{t-1} - x_{t-2}), \ldots$
$and \quad y_{t-p} = \nabla x_{t-p} = (x_{t-p} - x_{t-p-1})$

The Seasonal ARIMA (SARIMA) is used when the time series exhibits a seasonal variation, so as to properly capture the dynamics of the process. The modification made to the ARIMA model to account for seasonal behaviour. Due to the fact that several natural phenomena exhibit seasonal variations, it is necessary to incorporate autoregressive and moving average polynomials that include seasonal lags into the basic ARIMA model. In general, the seasonal and the non-seasonal operators could be combined in a multiplicative manner to produce a multiplicative seasonal autoregressive moving average model, denoted by ARMA (*p*, *q*) × (*P*, *Q*) *s*. A seasonal autoregressive notation (*P*) and a seasonal moving average notation (*Q*) will form the multiplicative Seasonal Autoregressive Integrated Moving Average model, denoted by ARIMA (*p*, *d*, *q*)*(*P*, *D*, *Q*) *s*, of ([BJ76]) and is given by:

$$(1 - \phi_p B)(1 - \Phi_P B^s)(1 - B)(1 - B^s) x_t = (1 + \theta_q B)(1 + \Theta_Q B^s) w_t \qquad (10)$$

where w_t is the Gaussian white noise process with zero mean and constant variance.
The first and the second parts of each compartment in equation (10) is the non-seasonal and seasonal aspect of AR(p), differencing (d=1) and MA(q), respectively at period s. The seasonal part of the model consists of terms that are very similar to the non-seasonal components of the model, but they involve backshifts of the seasonal period.

The Unit Root test can be used to test for stationarity. The null hypothesis asserts that the series has a unit root (that is, it is non-stationary). The alternative hypothesis is that the series do not have a unit root (that is, it is stationary). To detect whether a given series has a unit root, it can be assumed that the relationship between the current value (in time t) and last value (in time t-1) in the series is ([End95]):

$$x_t = \phi x_{t-1} + w_t \quad (11)$$

where x_t is an observation value at time *t*, w_t is assumed to be a normally distributed with mean zero and constant variance. This model is a first order autoregressive process. The time series x_t converges, as $t \to \infty$, to a stationary time series if $|\phi| < 1$. If $|\phi| \geq 1$, the series x_t is not stationary and the variance of x_t is time dependent ([DKN06, Tak12]). In other words, the series has a unit root.

The Unit Root test subsequently tests the following one-sided hypothesis:

H_0: $\phi = 1$ (has a unit root)

H_1: $\phi < 1$ (has root outside the unit circle)

If x_{t-1} is subtracted from both sides of equation (11), and introduce the difference operator, then we obtain the first order difference equation:

$$\nabla x_t = (\phi - 1)x_{t-1} + w_t \quad (12)$$

If ϕ is assumed to be 1, the effect of unit root can be eliminated from the actual series that has non stationarity via the first differencing. In addition to the assumption that $\{w_t\}$ is a Gaussian white noise process, it is further assumed not to be autocorrelated. If there is autocorrelation, the true magnitude of the test would be higher than the nominal size used ([Tak12]).

The Augmented Dickey-Fuller (ADF) and Kwiatkowski Phillips Schmidt Shin (KPSS) are among the important tests used to ascertain stationarity of time series data and were used in this study.

For the Augmented Dickey-Fuller test, the various cases of the test equation as are follows:

In a case where the time series does not have a trend component and potentially slow-turning around zero, the following test equation is to be used ([Tak12]):

$$\nabla x_t = \phi x_{t-1} + \theta_1 \nabla x_{t-1} + \Lambda + \theta_p \nabla x_{t-p} + w_t \quad (13)$$

In a case where the time series is flat and potentially slow-turning around a non-zero value, the following test equation is to be used ([Tak12]):

$$\nabla x_t = \theta_o + \phi x_{t-1} + \theta_1 \nabla x_{t-1} + \Lambda + \theta_p \nabla x_{t-p} + w_t \quad (14)$$

In a case where the time series has a trend in it (either up or down) and is potentially slow-turning around a trend line you would draw through the data, the following test equation is appropriate ([Tak12]):

$$\nabla x_t = \theta_o + \beta_t + \phi x_{t-1} + \theta_1 \nabla x_{t-1} + \Lambda + \theta_p \nabla x_{t-p} + w_t \quad (15)$$

Where: ∇x_t is the first differenced value of series (x_t) and w_t is the error term,

x_{t-1} is the first lagged value of the series (x_t)

∇x_{t-j} is the jth lagged first differenced of values, while $\theta_o, \beta_t, \phi, \theta_1, \theta_2, \Lambda\ \theta_p$ are parameters to be estimated.

According to ([GN86]), the problem of determining the optimal number of lags of the response variable arises, though several ways of choosing p have been proposed, but the following two simple rules of thumb are suggested: the frequency of the data or through an appropriate information criterion. So, the number of lags that minimizes the value of the Akaike information criterion (AIC) would be chosen ([GN86]).

The **Dickey-Fuller test-statistic** is associated with the ordinary least squares estimate of Φ. The Dickey-Fuller-test estimates π = ϕ+1 obtained from an ordinary regression and checks for Φ = 0 by computing the test statistic ([Tak12]).

It is noted that the purpose of the identification stage in the Box-Jenkins methodology is to determine the differencing required for achieving stationarity and the order of both the seasonal and the non-seasonal AR and MA operators for the residual series ([GN86]). The Autocorrelation Function (ACF) and the Partial Autocorrelation Function (PACF) are the two most useful tools in any attempt at univariate time series model identification ([GN86]).

The sample ACF (r_k) measures the amount of linear dependence between observations in a time series that are separated by a lag k ([Tak12]). To use the ACF in model identification, estimate r_k from the data and then plot r_k series against lag k up to a maximum lag of about five times the seasonality interval and this should be less than to one fourth of the series under study ([HML77]). To identify the number of non-seasonal and seasonal AR and MA parameters, the sample ACF is examined with what should be expected from the theoretical ACF ([HML77, Tak12]):

Table 1: Behaviour of the ACF and PACF for ARMA Models

	AR(p)	MA(q)	ARMA(p,q)
ACF	Tails off	Cuts off after lag q	Tails off
PACF	Cuts off after lag p	Tails off	Tails off

The partial autocorrelation function (PACF), can also be used for determining the possible order of the seasonal and non-seasonal AR and MA terms that should be incorporated in the model via the ACF and PACF. When the process is a pure ARIMA (*p, d, q*) model, r_k cuts off and is not significantly different from zero after lag p+sp ([Tak12]). If r_k damps out at lags that are multiples of s, this suggests the incorporation of a seasonal MA component into the model ([Tak12]). The failure of the partial autocorrelation function to truncate at other lags may imply that a non-seasonal MA term is required ([HML77]). The partial autocorrelations (ρ_{kk}) at lag *k* are estimated through successive autoregressive estimation. The first step is to model the x_t series by finite autoregressive models of order k ([Tak12]).

The selected model's parameters were estimated using the method of maximum likelihood. In the method of maximum likelihood, the likelihood function is maximized to obtain the parameter estimates. The likelihood function or joint density is the probability of obtaining the data, given its probability distribution.

One of the commonly used criteria for model comparison in time series analysis is the Akaike Information Criterion (AIC). The idea is to balance the risks of under fitting and over fitting. Akaike ([Aka78]) introduced the AIC in situations where there are competing models to select from, such that the model with the lowest AIC is chosen as the best model. It is defined as ([SS10]):

$$AIC = -2\log likehood + 2k \tag{16}$$

where k is the number of seasonal and non-seasonal autoregressive and moving average parameters to be estimated in the model ([Wei90]).

The optimal order of the model is determined by the value of *k*, which is a function of *p* and *q*, so that the value of k yielding the lowest AIC specifies the best model. Parsimony is a guiding principle in arriving at the best model ([Pan83]).

In testing the adequacy of the fitted model, the residuals could be extracted and examined whether they are independent random shocks consistent with a Gaussian white noise process ([Pan83]). At the diagnostic-checking stage, the residuals are used to test hypotheses about the independence of the random shocks ([Pan83]).

The basic analytical tool at the diagnostic-checking stage is the residual ACF ([Pan83]). A residual ACF is basically the same as any other estimated ACF, the only difference being that the residuals (w_t) from the estimated model are used instead of the observations in a realization (x_t) to calculate the autocorrelation coefficients ([Pan83]). The residuals w_t are given as:

$$w_t = \frac{\left(x_t - x_t^{t-1}\right)}{\sqrt{\rho_t^{t-1}}} \tag{17}$$

where $x_t - x_t^{t-1}$ is the one-step-ahead prediction of (x_t) based on the fitted model and $\sqrt{\rho_t^{t-1}}$ is the estimated one-step-ahead error variance ([Tak12]). If visual inspections of the residuals reveal that they are randomly distributed over time, then there is an indication that the proposed model is adequate ([Tak12, Pan83]).

Several statistical tests exist for diagnostic checking of randomness ([Tak12]). The Ljung-Box Q statistic, turning point and runs tests can be used for the diagnostic checking of residuals for independence ([Tak12]).

The Ljung-Box Q or Q(r) statistic can be used to check independence of residual instead of visual inspection of the sample autocorrelations ([Pan83]). A test of hypothesis can be conducted for the adequacy of the model using the chi-squared statistic ([Pan83]).

Another useful test is the portmanteau lack of fit test ([Tak12]). This test statistic is the modified *Q* -statistic originally proposed by ([BJ76]). Under the null hypothesis of model adequacy, the *Q*-statistic approximately follows the chi-squared distribution ([BJ76]). If a model is specified correctly, residuals should be uncorrelated and Q(r) should be small (p-value should be large) ([BJ76]).

The ultimate application of the Box-Jenkins methodology is to forecast future values of a time series ([Pan83]).

4. RESULTS AND DISCUSSION

Table 2: Data on monthly rainfall (in mm) of Enugu State from (January 2002 to December 2016)

Year	JAN	FEB	MAR	…	DEC
2002	0	6.1	25.8	----	0
2003	18.4	15.7	30.0	----	0
2004	32.4	0	32.3	----	0
2005	0	28.0	72.5	----	0
2006	0	46.5	10.4	----	0
2007	0	0	2.9	----	0
2008	0	6.4	4.8	----	33.1
2009	0	26.9	20.8	----	0
2010	43.9	4.6	78.9	----	0
2011	0	9.4	70.0	----	0
2012	1.2	0	56.7	----	30.2
2013	50.1	0	11.1	----	0
2014	1.0	0	2.2	----	0
2015	0	47.3	118.4	----	0
2016	39.0	35.7	13.0	----	0

The plot of the data displays a pronounced seasonal pattern in the series and as such truly describes the Rainfall data.

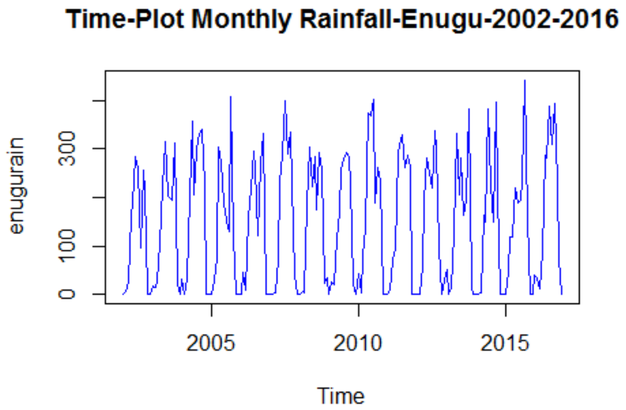
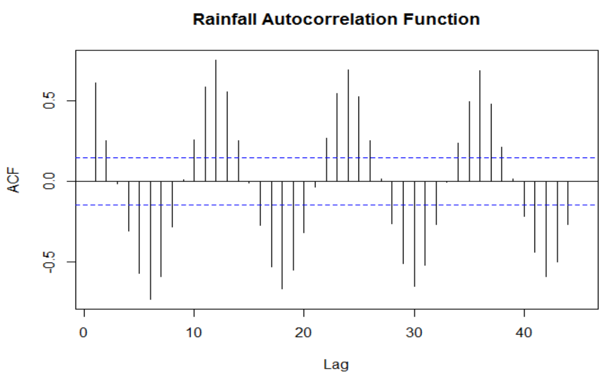
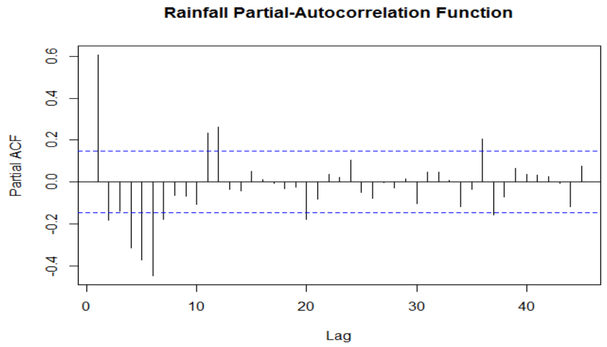


Figure 1: Time plot for Enugu monthly rainfall



(a)



(b)

Figure 2: Plot of autocorrelation and partial autocorrelation function

The data were subjected to a unit root test to confirm stationarity or otherwise. The following stationarity tests were applied on the series:

The KPSS test

H_0: The series is stationary

H_1: The series is not stationary

The ADF test

H_0 : The series is not stationary

H_1 : The series is stationary

Table 3: Test for Stationarity

Summary of Test statistics			
Test type	Test statistics	Lag order	P-value
KPSS	0.0296	3	0.1
ADF	-14.339	5	0.01

If the probability value (p-value) is greater than the pre-specified level of significance, the null hypothesis cannot be rejected and simple differencing is needed to render the series stationary. Since the p-value =0.01<0.05, the null hypothesis is rejected. It is concluded that the series is stationary under ADF. For the KPSS test, since the p-value=0.1, the null hypothesis cannot be rejected at the 0.05 level of significance and it is concluded that the series is stationary.

Having established stationarity of the time series data, the next step is the identification of the ARIMA model via the ACF and PACF plots. Tentative models were chosen based on the plots and the model with the smallest Akaike Information Criteria (AIC) and the Corrected Akaike Information Criteria (AICc) would be selected as the best fit.

Table 4: Overfitting for the Enugu rainfall data

ARIMA model	AICc
ARIMA(1,0,0)	2186.15
ARIMA(0,0,1)	2195.72
ARIMA(2,0,0)	2182.22
ARIMA(0,0,2)	2187.31
ARIMA(1,0,1)	2183.72
ARIMA(2,0,1)	2134.81
ARIMA(1,0,2)	2185.49
ARIMA(0,0,0)(1,0,0)[12]	2095.61
ARIMA(0,0,0)(0,0,1)[12]	2177.92
ARIMA(0,0,0)(2,0,0)[12]	2069.99
ARIMA(0,0,0)(0,0,2)[12]	2157.12
ARIMA(0,0,0)(1,0,1)[12]	2016.45 *
ARIMA(0,0,0)(2,0,1)[12]	2017.71
ARIMA(0,0,0)(1,0,2)[12]	2017.1
ARIMA(1,0,0)(1,0,0)[12]	2095.09
ARIMA(2,0,0)(1,0,0)[12]	2097.16
ARIMA(1,0,0)(2,0,0)[12]	2070.55
ARIMA(1,0,0)(0,0,1)[12]	2140.63
ARIMA(2,0,0)(0,0,1)[12]	2142.59
ARIMA(1,0,0)(0,0,2)[12]	2133.35
ARIMA(0,0,1)(1,0,0)[12]	2095.07
ARIMA(0,0,2)(1,0,0)[12]	2097.17
ARIMA(0,0,1)(2,0,0)[12]	2070.26
ARIMA(0,0,1)(0,0,1)[12]	2147.97
ARIMA(0,0,2)(0,0,1)[12]	2143.58
ARIMA(0,0,1)(0,0,2)[12]	2137.58

The identified model is SARIMA $(0,0,0)(1,0,1)_{12}$ which has the least AIC from Table 4 and the estimated parameters of the model are presented in Table 5.

Table 5: Estimation of parameter for SARIMA(0,0,0)(1,0,1)$_{12}$

Model Fit Statistics		
AIC	AICc	BIC
2016.22	2016.45	2028.99

Coefficients	Estimate	STD Error	t-value
Sar1	0.4867	0.0677	7.1891
Sma1	0.3938	0.0701	5.6177
Intercept	149.164	28.6029	5.215

The model is given as:

$$y_t = \mu + \Phi y_{t-12} - \Theta \varepsilon_{t-12} + \varepsilon_t$$

$$y_t = 149.1640 + 0.4867 y_{t-12} - 0.3938 \varepsilon_{t-12} + \varepsilon_t$$

The parameter coefficients of the model are found to be statistically significant given that the t-value are greater than 1.96 or 2 in absolute value and are within the bounds of -1 and 1.

MODEL CHECKING

The adequacy of the model is checked using the residual plots and the Box L-jung test in Figure 3 and Table 6.

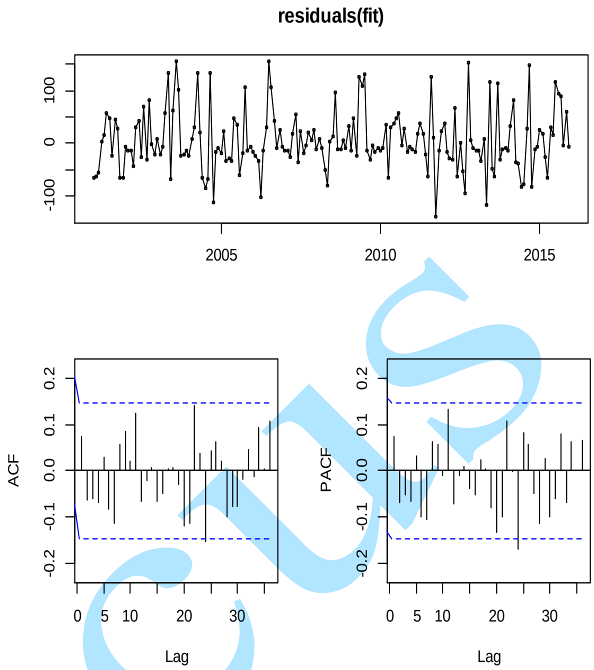


Figure 3: Plot of diagnostic check for Enugu rainfall data

In Table 6, the Box Ljung null hypothesis of uncorrelated residual against the alternative of correlated residuals. Since the p-value is greater than the pre-chosen 0.05 level of significance, the null hypothesis is not rejected.

Table 6: Summary of Test Statistics

Test type	Chi-squared	df	P-value
Ljung-Box	25.1373	20	0.1962

The Plot of the Fitted Model Values Superimposed on the Original Series:
The plot displayed in Figure 4 shows that the fitted values of the model fairly fits the original series.

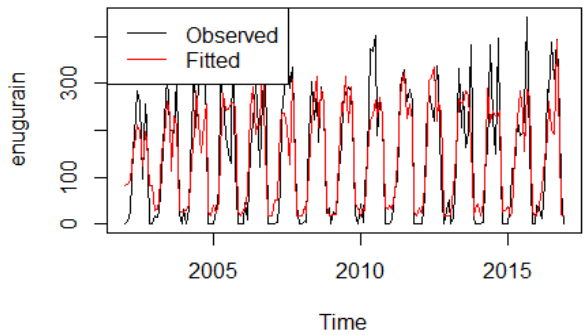


Figure 4: Plot for fitted model values superimposed on the original series

FORECAST: The fitted model is used to forecast the rainfall pattern for the next two years (January 2017-December 2018).

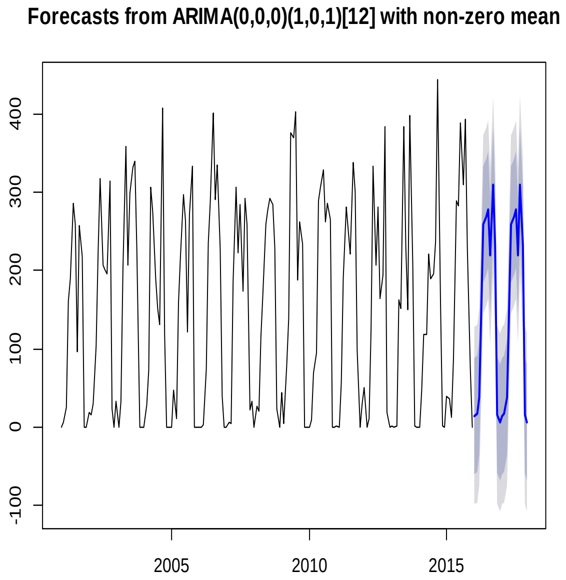


Figure 5: Plot of forcasted values using the fitted model

Table 7: Forecasted rainfall values (in mm) for the next two years (January 2017 to December 2018) and the 95% confidence interval for the forecasts

	Point Forecast	Lo 95	Hi 95
Jan 2017	14.58	-98.72	127.87
Feb 2017	17.22	-96.08	130.51
Mar 2017	38.50	-74.80	151.80
Apr 2017	147.97	34.67	261.27
May 2017	259.10	145.80	372.40
Jun 2017	268.20	154.91	381.50
Jul 2017	277.55	164.25	390.84
Aug 2017	218.58	105.28	331.88
Sep 2017	308.83	195.53	422.12
Oct 2017	231.39	118.09	344.68
Nov 2017	15.94	-97.35	129.24
Dec 2017	6.41	-106.88	119.71
Jan 2018	14.59	-98.74	127.92
Feb 2018	17.23	-96.10	130.56
Mar 2018	38.51	-74.82	151.84
Apr 2018	147.97	34.64	261.30
May 2018	259.09	145.76	372.43
Jun 2018	268.20	154.86	381.53
Jul 2018	277.54	164.20	390.87
Aug 2018	218.58	105.24	331.91
Sep 2018	308.82	195.48	422.15
Oct 2018	231.38	118.05	344.72
Nov 2018	15.96	-97.38	129.29
Dec 2018	6.43	-106.91	119.76

5. CONCLUSION

Based on the outcome of the result of the analysis, the time series model Seasonal ARIMA $(0,0,0)(1,0,1)_{12}$ for the monthly rainfall series of Enugu State was established as the best model having passed the diagnostics checking test and was used to forecast the monthly rainfall values for the next two years.
Rainfall pattern of Enugu State is found to have a steady pattern. The results show that there is a tendency of relatively increasing pattern of monthly rainfall over the forecast period from January 2017 to December 2018. The 95% confidence bounds were presented for the for the monthly rainfall forecast for the next two years (2017-2018).

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