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Optimal Control Applied to Electric Power Generating Systems Model.

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Abstract

Electric power is a fundamental part of the infrastructure of contemporary society since most of today's daily activity based on the assumption that the desired electric power is readily available. In this article, the characterization of the proposed mathematical model of electric power generating system by Bamigbola and Aderinto (2009) was analyzed via optimal control approach. Existence and uniqueness of the solution was established. Application to real life data was carried out for better understanding of the system.

Keywords: Optimal control, Electric power output, Generator control, Cost of generation.

1. Introduction

Electric power and its availability play on essential role in the advancement of modern society, because it serves as a fundamental resource for heating, cooling, as well as powering of computers and machineries that underpin the manufacturing process, communication and transportation sectors, to mention a few. In real life situations, continuous optimal control problems arise mostly in every aspect of human endeavour. Among these are the electric power systems, mainly the generation, transmission and distribution of electric energy.

Optimal flow of electric power systems as an optimization method for an energy management centre was developed in the 1960's Dommel and Tinney (1968). Since then a great number of research has been done and various optimization techniques have been used in order to find efficient solutions to these problems. Adejumobi (2005) looked at the effectiveness and efficiency of electrical power distribution system in Nigeria using power system security approach. Zhang and Tolbert (2005) studied the survey of reactive power planning model. Dmytro *et al.* (2007) looked at the model of electric power supply with fuel supply via variational inequalities.

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Bouktir and Sliman (2005) worked on optimal power flow of electrical network, Aderinto and Bamigbola (2012) qualitatively study electric power generating system model. Salaudeen and Aderinto (2014) looked at the iterative method for solving load flow analysis in electric power systems.

In the same vein, some researchers have worked on the applications of optimal control in different areas, Bhunu *et al.* (2008) analyzed the tuberculosis transmission model via optimal control. Naevadal (2003) looked at the solution of time optimal control problem via spread sheet approach. Adams et al. (2004) studied multidrug therapies for HIV using optimal control approach. Burden et al. (2003) looked at the application of optimal control to immunotherapy using cancer model as a case study. Fister *et al.* (1998) used optimal control approach to characterize an HIV model. Stengel *et al.* (2002) looked at the application of optimal control to immune response, Agusto (2008) applied optimal control to oxygen absorption in aquatic system. But little or no attention has been paid to the application of optimal control to electric power system.

The present effort is to use the optimal control approach to characterize a mathematical model of an electric power generation system for better understanding of the system in an attempt to generate at minimum cost and minimum losses.

2. Material and Methods

2.1. Establishment of the Model

Consider the mathematical model

$$\frac{dG(t)}{dt} = (P_{HP} + P_{LP}) - u_1 kG(t) + qC(t)G(t) - T_L G(t), \tag{2.1}$$

$$\frac{dC(t)}{dt} = (s + y) - u_2 xC(t) + rC(t)G(t) + T_c C(t) \tag{2.2}$$

with $G(0) = G_0, C(0) = C_0, (P_{HP} + P_{LP}) = \alpha, 0 \le a_i \le u_i \le b_i < 1$ Bamigbola and Aderinto (2009).

The problem to study is to find the controls u_1, u_2 that minimizes the cost functional:

$$J(u_1, u_2) = \int_0^T \left[\delta C(t) + \eta_1 u_1^2 + \eta_2 u_2^2\right] dt \tag{2.3}$$

subject to (2.1) and (2.2), where $G(t)$ is the amount of power generated by the ith generator at time t and $C(t)$ is the cost of production / generation at a particular time. a is the actual mechanical / electrical energy from the turbine, k is the rate of generation, q is the total running cost, T_L is the rate of energy loss during transmission, S is the labour cost at a particular time, y is the cost of maintenance, x is the capacity rate of generator, r is the fuel cost rate, T_c is the total cost of transmission. The control u_i is the load shedding rate, u_2 is the generator actual capacity rate. δ is the unit of power generating station. η_1 and η_2 are to balance the size of the control.

3. The Optimality System

We obtain the optimality systems (3.1) – (3.6) by applying the Pontryagin Minimum / Maximum Principle, Barbu (1994) to equation (2.1) and (2.2) and define the Langrangian which is the Hamiltonian augmented with penalty terms for the constraints.

$$\frac{dG(t)}{dt}=\left(P_{HP}+P_{Lp}\right)-\min\left\{\max\left\{a_2,\frac{1}{2\eta_1}\lambda_1 kG^*(t)\right\},b_1\right\}kG(t)+qC(t)G(t)-T_L C(t), \tag{3.1}$$

$$\frac{dG(t)}{dt}=(s+y)-\min\left\{\max\left\{a_1,\frac{1}{2\eta_2}\lambda_2 xC^*(t)\right\},b_2\right\}xC(t)+rC(t)+T_L C(t), \tag{3.2}$$

$$\lambda_1'=-\lambda_1\left[-\min\left\{\max\left\{a_1,\frac{1}{2\eta_1}\lambda_1 kG^*(t)\right\},b_1\right\}k+qC(t)-T_L\right]-\lambda_2\left[rC(t)\right], \tag{3.3}$$

$$\lambda_2'=-\delta-\lambda_1 qG(t)-\lambda_2\left[-\min\left\{\max\left\{a_2,\frac{1}{2\eta_2}\lambda_2 xC^*(t)\right\},b_2\right\}x+T_c+rG(t)\right], \tag{3.4}$$

where

$$\lambda_1(T)=\lambda_2(T)=0,\ G(0)=G_0, C(0)=C_0,\quad t\in[0,T],$$

$$u_1^*(t) = \min\left\{\max\left\{a_1, \frac{1}{2\eta_1} kG^*(t)\right\}, b_1\right\} \tag{3.5}$$

$$u_2^*(t) = \min\left\{\max\left\{a_2, \frac{1}{2\eta_2} \lambda_2 x C^*(t)\right\}, b_2\right\} \tag{3.6}$$

3.1 Uniqueness of the Optimality System

To successively discuss the uniqueness of the optimality system (3.1 – 3.6) we use the bounds for the state equations which implies that the adjoint system also has bounded coefficient and is linear in each adjoint variable. Thus the solutions of the adjoint system are bounded. Hence, there exist a D > 0 such that

$$|\lambda_i(t)| < D \quad for \ i = 1,2 \ on \ [0,T]$$

The following assumption in form of a lemma will be needed in proving the next theorem.

Lemma 3.1: The function $u^*(s) = \min(\max(s,a),b)$ is Lipschitz continuous in S, where $a < b$ are fixed positive constants.

Proof: Angelo (2005), Fister *et al.* (2000).

A real valued function f is called a Lipschitzian on a set $M \subset \Re^n$, if there is a constant $L = L(f,m) > 0$. Such that $|f(x) - f(y)| \le L\|x - y\|, \forall x, y \in M$.

Consider s_1, s_2 as real numbers and a, b as fixed positive constants, show that the Lipschitz continuity holds in all possible cases for $\max(s,a)$. Similar argument holds for $\min(\max(s,a),b)$ as well.

Case (i): Let $s_1 \ge a, s_2 \ge a$

$$|\max(s_1,a) - \max(s_2,a)| = |s_1 - s_2|$$

(ii) $s_1 \ge a, s_2 \le a$

$$|\max(s_1,a) - \max(s_2,a)| = |s_1 - a| \le |s_1 - s_2|$$

(iii) $s_1 \le a, s_2 \ge a$

$$\left|\max(s_1,a)-\max(s_2,a)\right|=|a-s_2|\le|s_1-s_2|$$

(iv) $s_1\le a, s_2\le a$

$$\left|\max(s_1,a)-\max(s_2,a)\right|=|a-a|=0\le|s_1-s_2|$$

Hence, $\left|\max(s_1,a)-\max(s_2,a)\right|\le|s_1-s_2|$ and u is Lipschitz continuous in S.

Theorem 3.1:

For T sufficiently small the solution to the optimality system (3.1) – (3.6) is unique.

Proof: suppose that $(G,C,\lambda_1,\lambda_2)$ *and* $\left(\bar{G},\bar{C},\bar{\lambda}_1,\bar{\lambda}_2\right)$ are two distinct solutions to the optimality system equation (3.1) –(3.6)

Let m > 0 be chosen such that $G=e^{mt}h$, $\quad C=e^{mt}j$, $\quad \lambda_1=e^{-mt}\omega$, $\quad \lambda_2=e^{-mt}z$,

$\bar{G}=e^{mt}\bar{h}$, $\quad \bar{C}=e^{mt}\bar{j}$, $\quad \bar{\lambda}_1=e^{-mt}\bar{\omega}$., $\quad \bar{\lambda}_2=e^{-mt}\bar{z}$

In addition, let

$$u_1^{*}(t)=\min\left\{\max\left\{a_1,\frac{1}{2\eta_1}k(\omega h)\right\},b_1\right\}$$

$$u_2^{*}(t)=\min\left\{\max\left\{a_2,\frac{1}{2\eta_2}\lambda_2x(zj)\right\},b_2\right\}.$$

and

$$\bar{u}_1^{*}(t)=\min\left\{\max\left\{a_1,\frac{1}{2\eta_1}k\left(\bar{\omega}\bar{h}\right)\right\},b_1\right\}$$

$$\bar{u}_2^{*}(t)=\min\left\{\max\left\{a_2,\frac{1}{2\eta_2}\lambda_2x\left(\bar{z}\bar{j}\right)\right\},b_2\right\}.$$

Now substitute $G=e^{mt}h$ into the first ODE of the optimality system (3.1) and obtain

$$h'+mh=(P_{HP}+P_{LP})e^{-mt}+qe^{mt}hj-T_Lh-\min\left(\max\left(a_1,\frac{1}{2\eta_1}k\left(e^{-mt}\omega h\right)\right),b_1\right)kh \tag{3.7}$$

Similarly, for $C = e^{mt} j,\quad \lambda_1 = e^{-mt}\omega \quad and \quad \lambda_2 = e^{-mt} z,$

we obtain

$$j' + mj = (s+y)e^{-mt} j + re^{mt} jh + T_c j - \min\left(\max\left(a_2, \frac{1}{2\eta_2} x(zj)\right), b_2\right) xj \tag{3.8}$$

$$-\omega' + m\omega = \omega\left[qe^{mt} j - T_L - \min\left\{\max\left\{a_1, \frac{1}{2\eta_1} k(\omega h)\right\} b_1\right\} k\right] + re^{mt} zj \tag{3.9}$$

$$-z' + mz = \delta e^{mt} + e^{mt} q\omega h + z\left[T_c + re^{mt} h - \min\left\{\max\left\{a_2, \frac{1}{2\eta_2} x(zj)\right\}, b_2\right\} x\right]. \tag{3.10}$$

Similarly for $\bar{G}, \bar{C}, \bar{\lambda}_1,$ *and* $\overline{\lambda_2}$, we have

$$\bar{h}' + m\bar{h} = (P_{HP} + P_{LP})e^{-mt} + qe^{mt}\,\overline{hj} - T_L \bar{h} - \min\left(\max\left(a_1, \frac{1}{2\eta_1} k(\overline{\omega h})\right), b_1\right) k\bar{h}, \tag{3.11}$$

$$\bar{j}' + m\bar{j} = (s+y)e^{-mt} + re^{mt}\,\overline{jh} + T_c \bar{j} - \min\left\{\max\left\{a_2, \frac{1}{2\eta_2} x(\bar{z}\bar{j})\right\}, b_2\right\} x\bar{j} \tag{3.12}$$

$$-\bar{\omega}' + m\bar{\omega} = \bar{\omega}\left[qe^{mt}\bar{j} - T_L - \min\left\{\max\left\{a_1, \frac{1}{2\eta_1} k(\overline{\omega h})\right\} b_1\right\} k\right] + re^{mt}\,\bar{z}\bar{j} \tag{3.13}$$

and

$$-\bar{z}' + m\bar{z} = \delta e^{mt} + e^{mt} q\overline{\omega h} + \bar{z}\left[T_c + re^{mt} h - \min\left\{\max\left\{a_2, \frac{1}{2\eta_2} x(\bar{z}\bar{j})\right\}, b_2\right\} x\right] \tag{3.14}$$

The next step is to subtract the equation for G *and* $\overline{G}$, V *and* $\overline{V}$, λ *and* $\overline{\lambda}_1$, λ_2 *and* $\overline{\lambda}_2$.

Multiply each new equation by an appropriate difference of functions and integrate from 0 *to* T. Using Lemma 3.1, we obtain:

$$\left|u_1^*(t)-\bar{u}_1^*(t)\right| \le \frac{k}{2\eta_1}\left|\omega h-\bar{\omega}\bar{h}\right|$$

and

$$\left|u_2^*(t)-\bar{u}_2^*(t)\right| \le \frac{x}{2\eta_2}\left|zh-\bar{z}\bar{h}\right|$$

Hence, we obtain

$$\frac{1}{2}\left(h+\bar{h}\right)^2(T)+m\int_0^T\left(h+\bar{h}\right)^2 dt \le q\int_0^T e^{mt}\left|hj-\bar{h}\bar{j}\right|\left|h-\bar{h}\right|dt+T_L\int_0^T\left|h-\bar{h}\right|^2 dt+k\int_0^T\left|u_1^* h-\bar{u}_1^*\bar{h}\right|\left|h-\bar{h}\right|dt$$
$$\le C_1\int_0^T\left[\left|h-\bar{h}\right|^2+\left|j-\bar{j}\right|^2+\left|\omega-\bar{\omega}\right|^2\right]dt+C_2e^{mT}\int_0^T\left[\left|h-\bar{h}\right|^2+\left|j-\bar{j}\right|^2\right]dt \quad (3.15)$$

$$\frac{1}{2}\left(j-\bar{j}\right)^2(T)+m\int_0^T\left(j-\bar{j}\right)^2 dt \le e^{-mt}(s+y)\int_0^T\left|j-j\right|^2 dt+T_c\int_0^T\left|j-j\right|^2 dt+r\int_0^T e^{mt}\left|jh-\bar{j}\bar{h}\right|\left|j-j\right|dt$$
$$+x\int_0^T\left|u_2^*-\bar{u}_2^*\right|\left|j-j\right|\left|j-\bar{j}\right|dt \le C_3\int_0^T\left[\left|j-j\right|^2+\left|z-\bar{z}\right|^2\right]dt+C_4e^{mT}\int_0^T\left[\left|h-\bar{h}\right|^2+\left|j-j\right|^2+\left|z-\bar{z}\right|^2\right]dt \quad (3.16)$$

$$\frac{1}{2}\left(\omega-\bar{\omega}\right)^2(0)+m\int_0^T\left(\omega-\bar{\omega}\right)^2 dt \le q\int_0^T e^{mt}\left|\omega j-\bar{\omega}\bar{j}\right|\left|\omega-\bar{\omega}\right|dt+T_L\int_0^T\left|\omega-\bar{\omega}\right|^2 dt+k\int_0^T\left|u_1^*\omega-\bar{u}_1^*\bar{\omega}\right|\left|\omega-\bar{\omega}\right|dt$$
$$+r\int_0^T e^{mt}\left|zj-\bar{z}\bar{j}\right|\left|\omega-\bar{\omega}\right|dt \le C_5\int_0^T\left[\left|h-h\right|^2+\left|\omega-\bar{\omega}\right|^2\right]dt+C_6e^{mT}\int_0^T\left[\left|j-\bar{j}\right|^2+\left|\omega-\bar{\omega}\right|^2+\left|z-z\right|^2\right]dt \quad (3.17)$$

and

$$\frac{1}{2}\left(z-\bar{z}\right)^2(0)+m\int_0^T\left(z-\bar{z}\right)^2 dt \le \delta\int_0^T e^{mt}\left|z-\bar{z}\right|dt+q\int_0^T e^{mt}\left|\omega h-\bar{\omega}\bar{h}\right|\left|z-\bar{z}\right|dt+T_c\int_0^T\left|z-\bar{z}\right|^2 dt$$
$$+r\int_0^T e^{mt}\left|zh-\bar{z}\bar{h}\right|\left|z-\bar{z}\right|dt+x\int_0^T\left|u_2^* z-\bar{u}_2^*\bar{z}\right|\left|z-\bar{z}\right|dt \le C_7\int_0^T\left[\left|j-\bar{j}\right|^2+\left|\omega-\bar{\omega}\right|^2+\left|z-\bar{z}\right|^2\right]dt$$
$$+C_8e^{3mT}\int_0^T\left[\left|h-h\right|^2+\left|j-\bar{j}\right|^2+\left|\omega-\bar{\omega}\right|^2+\left|z-\bar{z}\right|^2\right]dt. \quad (3.18)$$

To complete the proof for the uniqueness of the optimal control, the integral representations of $\left(h-\bar{h}\right),\left(j-\bar{j}\right),\left(\omega-\bar{\omega}\right),$ *and* $\left(z-\bar{z}\right)$ are combined, and the estimates gives the following:

$$\frac{1}{2}\left(h-\bar{h}\right)^2(T)+\frac{1}{2}\left(j-\bar{j}\right)^2(T)+\frac{1}{2}\left(\omega-\bar{\omega}\right)^2(0)+\frac{1}{2}\left(z-\bar{z}\right)^2(0)+m\int_0^T\left[\left(h-\bar{h}\right)^2+\left(j-\bar{j}\right)^2+\left(\omega-\bar{\omega}\right)^2+\left(z-\bar{z}\right)^2\right]dt$$
$$\le\left(C_9+C_{10}e^{3mT}\right)\int_0^T\left[\left(h-\bar{h}\right)^2+\left(j-\bar{j}\right)^2+\left(\omega-\bar{\omega}\right)^2+\left(z-\bar{z}\right)^2\right]dt. \quad (3.19)$$

Thus we have

$$\left(m-\left(C_9+C_{10}e^{3mT}\right)\right)\int_0^T\left[\left(h-\bar{h}\right)^2+\left(j-\bar{j}\right)^2+\left(\omega-\bar{\omega}\right)^2+\left(z-\bar{z}\right)^2\right]dt \le 0,$$

where C_9 *and* C_{10} depend on the coefficient and bound on h, j, ω, z.

If we choose m such that $m-\left(C_9+C_{10}e^{3mt}\right)>0$, then $\ln\left(\frac{m-C_9}{C_{10}}\right)>(3m)T$, since natural logarithm is an increasing functions.

Hence, if $m > C_9 + C_{10}$, it implies that:

$$T<\frac{1}{3m}\ln\left(\frac{m-C_9}{C_{10}}\right), \text{ then } h=\bar{h}, j=\bar{j}, \omega=\bar{\omega}, z=\bar{z}.$$

Hence, the solution to the optimality system is unique for small T. The uniqueness for a small time interval is usual for boundary value problems due to opposite time orientations, the state equations have initial conditions and the adjoint equations have time conditions. The optimal controls u_1^* *and* u_2^* are characterized in terms of the unique solution of the optimality system. See Barbu (1994), Fister *et al.* (1998) and Fister *et al.* (2000) for the proof of a similar uniqueness result.

4. Numerical Application

Mathematics has a vital role in solving practical problems in electric power generating systems as well as improving on its advancement. In this section, we obtained solution to the electric power generating system model using real life data via a numerical method. There are two types of systems of differential equations in the optimality system including one with control. An iterative method with fourth-order Runge-kutta scheme is used in solving the system, The controls are updated at the end of each iteration using the formula for optimal controls. Jain,(1983), Hosking *et al.*(1996), Pingping (2009).The following tabulated values were obtained from the National Control Centre, Nigeria

Tab. 4.1: Generator Parameters for a three-generator station. [National grid Centre, Nigeria]. Aderinto and Bamigbola (2012).

Parameter	Meaning	Value
α	actual mechanical / electrical energy available from the turbine	100MW,100MW,80MW $G_1 \le$ 100MW, $G_2 \le$ 100MW, $G_3 \le$ 80MW,
Q	total running cost	0.3217, #0.3112, 0.312Per unit
R	fuel cost rate	0.3478 each per unit
X	actual capacity rate	0.606, 0.502, 0.402
K	rate of energy loss during transmission	0.002 MW each per unit
S	labour cost	70 per h (assumed)
Y	maintenance cost	50 per h (assumed)
γ	Cost of transmitting from generating station	0.3421 per unit
U	generator actual capacity rate (control)	$a \le u \le b$, $0.3 \le u \le 0.9$, a = 0.0, b =1.0
δ	unit of power generating station	1 each
H	Parameters to balance the size of the control, .(Number of hours for which the machines can be on)	3

The optimal control problem under consideration is minimize

$$J(u)=\int_{t_o}^{t_f}\left[\delta^T C(t)+\eta\, u^T u\right]dt, \tag{4.1}$$

subject to

$$\frac{dG_i(t)}{dt}=\alpha_i+C_i(t)q_iG_i(t)-k_iG_i(t) \tag{4.2}$$

$$\frac{dC_i(t)}{dt}=(s_i+y_i)+C_i(t)r_iG_i(t)-u_i(t)x_iC_i(t)+\gamma_iC(t) \tag{4.3}$$

with $G_i(t_0)=G_0, \quad C_i(t_0)=C_0, \quad a_i \le u_i \le b_i$.

By the introduction of Langrangian operators, we have

$$L(G,C,u,\lambda_1,\lambda_2,M,N)=\left[\delta^T C_i + \eta u_i^T u_i\right]+\lambda_1\left[\alpha_i + C_i(t)q_i G_i(t) - k_i G_i(t)\right]$$

$$+\lambda_{2i}\left[(s_i+y_i)-x_i u_i C_i(t)+r_i C_i(t)G_i(t)+\gamma_i C(t)\right]$$

$$+\sum_{i=1}^{m} M_i(b_i-u_i)+N_i(u_i-a_i)$$

where $M_i,\ldots,M_m,N_i,\ldots,N_m \ge 0$ *are penalty multipliers satisfying* $M_i(b_i-u_i)=0, N_i(u_1-a_1)=0,$ *at* u_i^*

Thus, using the data in table 4.1, the computations for the solution are obtained as follows; Aderinto and Bamigbola (2012)

Tab. 4.2: Solution for a three-generators electric power model $h=0.05, u=0.2,0.3,0.4,\ldots,0.9,$ $\delta=1, \eta=3$, $i=1,2,3$

U	C_1	C_2	C_3	J at $t_f=1$	J at $t_f=6$
0.2	186.95889	186.63370	186.32130	560.27389	3361.64336
0.3	187.90813	187.56171	186.38327	562.66311	3375.97864
0.4	188.86003	188.20629	187.57888	566.08521	3396.51124
0.5	189.81461	189.33645	188.20943	569.63048	3417.78290
0.6	190.63340	189.78621	188.84115	572.50077	3435.00463
0.7	191.73181	190.57893	189.47405	576.19479	3457.16875
0.8	191.91594	191.37349	190.10813	579.15756	3474.94538
0.9	193.11363	192.16990	190.74339	583.23692	3499.42155
G1	99.900				
G2		99.850			
G3			77.920		

Tab. 4.3: Solution for a three-generators electric power model $h = 0.05, u = 0.3, 0.4, \ldots, 0.9$, $\delta = 1, \eta = 3$, $i = 1,2,3$, C_1, C_2, C_3 *in Table 4.2*

U	u_1	u_2	u_3	J at $t_f=1$	J at $t_f=6$
					3394.94311
0.2	0.3	0.4	0.5	565.82385	3402.22584
0.3	0.4	0.5	0.6	567.03764	3434.24923
0.4	0.5	0.6	0.7	572.37487	3454.74280
0.5	0.6	0.7	0.8	575.79047	3478.01215
0.6	0.7	0.8	0.9	579.66869	3435.67021
0.7	0.8	0.3	0.7	572.61170	3466.77933
0.8	0.9	0.8	0.5	577.79656	3460.38170
0.9	0.8	0.6	0.8	576.73028	3474.61780
	0.9	0.9	0.5	579.10297	3449.06581
	0.8	0.7	0.5	574.84430	

4.1. Discussion of Results

The results obtained shows that for efficiency and effective functioning of the generating machines in each station, monitoring of the control is very essential. Therefore, the choice of u_i is greatly dependent on the number of generating machines that are available and the number of hours or the duration in which the generation is to be carried out. Thus $u_1 = 0.8$, $u_2 = 0.7$, and $u_3 = 0.5$ is recommended for the three generators system above with $J = 574.84430$ *at* $t_f = 1$ and $J = 3449.06581$ at $t_f = 6$. Aderinto and Bamigbola (2012).

5. Conclusion

In this work, optimal control theory is applied to the mathematical model of electric power generating system in an attempt to characterize it. Optimality of the system is analyzed. Uniqueness of optimality solution to the system is established, and the optimality system was found to be unique for small T. Finally, numerical solution to the optimality system is reported using real life data. To a layman the result can be interpreted by saying that, the electric power generating systems can be expressed mathematically by using mathematical equations which relates two or more parameters that can be used to measure the condition or state of electric power generating systems. These parameters enable us to know the condition and the capacity of the generator, how to use, and how long to use, so as to maximize the generator output and minimize the cost of generation.

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