Effects of Eccentricity of the Ellipse and Type of Fluid flowing on Heat Transfer in Elliptic Pipes

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Abstract

The paper presents laminar forced convection of pseudo-plastic, Newtonian and dilatants fluids inside elliptic tubes with the intention of eliciting the effects of eccentricity of the geometry and the applicable heat exchanging fluids on flow pattern and heat transfer by the configuration. Power-Law for non-Newtonian fluids in conjunction with continuity, momentum and energy transport equations were adopted for the solution. A thermal boundary condition of uniform wall temperature was considered for the analysis. A computational algorithm was developed from the governing equations using a Finite Difference Method (FDM). A program was written in Quick Basic to generate the results for the influence of the variables considered to be of interest. The results obtained show that pseudo-plastic fluids flowing in elliptic duct transfer more heat and appear to be a better heat exchanging fluid than other fluids. For fluids with flow behavior index n=$\frac{1}{3}$, n=1 and n = 3, the mean Nusselt numbers are respectively 4.1734, 3.6906 and 3.2168. These results agree closely with the values obtained by Cotta and Ozisik [Nu =4.1753 for n= $\frac{1}{3}$ and Nu= 3.4090 for n=3]. It was noted that an eccentricity range of 0.866<e<1 is practically realistic for optimum heat transfer. It is also worth nothing that more heat is transferred from the end of minor axis than any other part of the ellipse's circumference. The results are applicable to the design and analysis of heat exchangers which are being incorporated in a wide variety of unique, compact and efficient cooling applications such as electronic cooling, nuclear reactors, medical and biomedical devices, and electric transformer.

Keywords: *Laminar, Convection, Non-Newtonian, Elliptic configuration*.

Nomenclature

a, b: Kozicki geometric constants
A, B: surface identifiers in Fig.1
D: Major diameter of the ellipse (m)
e: Eccentricity of the ellipse
f_d : Darcy friction factor
F_g: Body force (N)
F_W: Shear force along the wall (N/m^2)
g: Acceleration due to gravity (m/s^2)
K: Thermal conductivity of the fluid (W/m ^{o}C)
n: Flow behaviour index
Nu: Nusselt number
Nu_L: Local Peripheral Nusselt number
Nu_m: Mean Nusselt number
M: Total number of nodal points on the periphery of the elliptic surface
$\dot{m}$: Mass flow rate (kg/s)
P: Pressure (N/m^2)
P_A: Pressure at surface A of the control element in Fig. 1 (N/m^2)
P_B: Pressure at surface B of the control element in Fig. 1 (N/m^2)
r : radial coordinate
r^*: Dimensionless radial coordinate
R: Radial distance along the minor axis
Re_k: Power law Reynolds number
S_A, S_B: Surface area of the control element at sides A and B respectively (m^2)
T: Temperature (^{o}C)
T^*: Dimensionless Temperature
T^*_b: Dimensionless bulk temperature of the fluid
T_w^*: Dimensionless wall temperature
U_r : Velocity in radial direction(m/s)
U_ϑ : Velocity in azimuthal direction(m/s)
U_Z Velocity in axial direction (m/s)
U^*: Dimensionless Velocity
U_Z^*: Dimensionless axial Velocity
u_∞ : Free stream Velocity (m/s)
u^*_∞ : Dimensionless free stream Velocity
Z: Coordinate in the axial direction
Z_∞ : Axial distance at fully developed region
Z*: Dimensionless axial coordinate

Greek Symbol

α: Thermal diffusivity(m^2/s)

β: Momentum flux correction factor
ρ: Density(kg/m^3)
Γ: Radial distance along the circumference
ϑ: Coordinate in the azimuthal direction
μ: Absolute viscosity (N/

1.0 Introduction

To engineers and scientists non-Newtonian fluids are of great importance because of their industrial value. Peter (2001) stated in his work that it is usually much more difficult to study non-Newtonian fluids because of their complex nature and complex interactions with the flow fields. As a result, the majority of the information on non-Newtonian fluids is empirical and consists primarily of simple friction and heat transfer correlations. These correlations are mathematical expressions consisting of many constants and these constants are mostly determined by conducting experiments. According to Matthys (1995), it is pertinent for the engineers to be aware of the potential problems and have the ability to decide whether to use complex Newtonian fluid correlations or to use complex information available on non-Newtonian fluids. He also stated that the non-Newtonian power law equation can be converted and be used for Newtonian fluid by setting the fluid flow behaviour index as 1. He emphasized in his work that investigation into heat transfer problems involving heating and cooling of non-Newtonian fluids in heat exchangers can be of great economic benefits.

It is also important to understand the characteristics of the forced convection heat transfer mechanism in steady laminar non-Newtonian flow in elliptical ducts in order to exercise proper control over the performance of the heat exchanger and to economize the process. In most applications, it is the rate of heat transfer within process equipment which is of principal interest. However, with thermally sensitive working fluid, the temperature profile must be known and the maximum permissible temperature must not be exceeded (Chabra and Richardson, 1999).

Generally, the Nusselt number for Newtonian fluid is modified with a coefficient that involves a parameter reflecting the change in velocity profile - a power law exponent, n- and probably the aspect ratio if it is internal flow in a non circular duct (Matthys, 1995). In the present work eccentricity rather than aspect ratio was utilized. According to Uzun (2002), the most common heat exchangers employed in industries are circular and rectangular ducts. He carried out numerical investigation of heat transfer to Non-Newtonian fluid in arbitrary ducts and concluded that Non-Newtonian fluids with flow behaviour index less than one give a higher heat transfer coefficient than Newtonian fluids.

In 1986, Cotta and Ozisik investigated laminar forced convection of power-law non-Newtonian fluids in ducts. It was discovered by them that for any given Graetz number (Dimensionless axial distance), the Average Nusselt number for pseudo-plastic fluid (n<1) is found to be higher than dilatant fluids (n>1). In the research, they carried out numerical simulation for flow behavior index, $n = \frac{1}{3}$, n=1 and n=3.

Shah and London (1978) investigated the relationship between Nusselt number and aspect ratio of circular ducts and other cross section with uniform wall temperature in the fully developed flow region using Newtonian fluid as working fluid. They obtained a Nusselt number of 3.657 for circular geometry (eccentricity, e=0) and 2.976 for a square duct.

Mahjoob *et al* (2009) investigated numerically non-Newtonian fluid flow through rectangular micro channel. Kozicki et al (1967) presented a useful analysis for non-Newtonian fluid flow in elliptical ducts with different values of eccentricity and aspect ratio. They gave a generalized Reynolds number (power law Reynolds number) that can be used to estimate the hydrodynamic characteristics of non-Newtonian fluid in elliptical ducts. Adegun and Oladosu (2009) analyzed fluid flow and forced convective heat transfer in the entrance region of elliptical ducts. Though the work was limited to fluid of flow behaviour index n=1 (Newtonian fluid), they discovered a critical eccentricity e=0.866 for optimum heat transfer by convection. Adegun (2004), in his paper investigated the effects of different variables on fluid flow in elliptic configurations. Muzychka and Edge (2008) also studied laminar fluid flow and heat transfer in circular ducts and micro channels.

The present work investigates how elliptic tube could be utilized for high heat flux dissipation devices using a mathematical model that could take care of both Newtonian and non-Newtonian heat exchanging fluids.

2.0 Problem Formulation

2.1. Description of Physical Model and Basic Assumptions

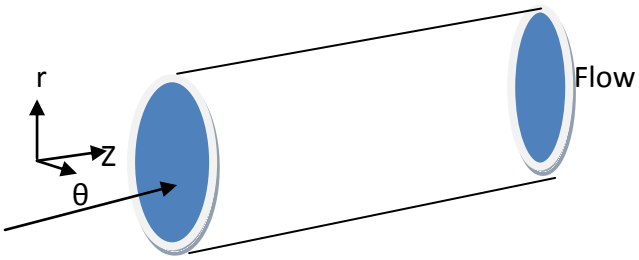


Fig.1: Physical model and Coordinate System

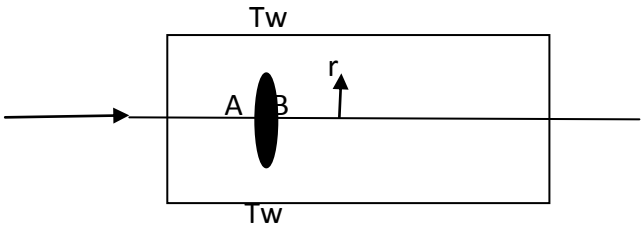


Fig. 2: Schematic for momentum balance and continuity

The model adopted for this work is an elliptic tube with modified cylindrical polar coordinate system (r,θ, z). It is assumed that the flow is fully developed and laminar, the shear stress applied is large enough to create a flow, velocity along the walls of the duct is zero (no slip boundary condition), the duct has a uniform wall temperature (UWT) and the flow is assumed incompressible.

2.2 Governing Equation

2.2.1 Continuity and Momentum Transport equations

For steady state incompressible fluid flow, the mathematical model for the conservation of mass is,

$$\frac{\partial U_r}{\partial r}+\frac{1}{r}\frac{\partial U_\theta}{\partial \theta}+\frac{\partial U_z}{\partial Z}+\frac{U_r}{r}=0 \qquad (1)$$

Since the flow is fully developed and the direction of flow is along the axis of the pipe and it is assumed that U_r and U_ϑ are insignificant compared to U_Z. Thus,

$$U_r = U_\theta = 0 \quad \text{and} \quad \frac{\partial U_z}{\partial Z} = 0 \tag{2}$$

Application of the conditions $U_r = U_\theta = 0$ to Navier-Stoke's equation yields a momentum equation (3) in cylindrical coordinate(r, θ, z):

$$\frac{\partial^2 U_z}{\partial r^2} + \frac{1}{r}\frac{\partial U_r}{\partial r} + \frac{1}{r^2}\frac{\partial^2 U_z}{\partial \theta^2} = \frac{1}{\mu}\left(\frac{dP}{dZ} - \rho g_z\right) \tag{3}$$

2.2.2 Momentum balance

From the Newton's second law of motion and following Frank (1999), the momentum balance equation is,

$$\dot{m}(\beta_A U_A - \beta_B U_B) = P_A S_A - P_B S_B - F_W - F_g \tag{4}$$

where,

F_g = Downward component of force (body force); F_w = Shear force along walls; S_A,S_B = Surface area of control element at point A and B respectively; P_A,P_B = Pressure at point *A* and *B* respectively; β_A, β_B = Dimensionless momentum flux correction factor.

2.2.3 Energy Transport equation

For a steady incompressible flow and with assumption that there are no chemical reactions, no heat sources within the fluid and negligible radiation effect, the energy equation is given as,

$$\frac{U_Z}{\alpha}\frac{\partial T}{\partial Z} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial Z^2} = 0 \tag{5}$$

2.2.4 Power law Equation for non-Newtonian fluid

According to Sestak *et al* (1974), the shear stress for power law non-Newtonian fluid is governed by

$$\tau = K\left(\frac{\partial U}{\partial r}\right)^n \tag{6}$$

2.3 Normalization Parameters

The following variables are used to non-dimensionalize the governing equations:

$$U_z^* = \frac{U_z}{U_\infty};\ Z^* = \frac{Z}{Z_\infty};\ r^* = \frac{r}{R};\ T^* = \frac{T - T_\infty}{T_w - T_\infty};\ Z^* = \frac{\alpha Z_\infty}{U_\infty D^2};\ \Gamma = \sqrt{\frac{\left(1 - e^2\right)}{\left(1 - e^2\cos\theta^2\right)}} \tag{7}$$

2.4 Normalized Governing Equations

Equations (1-6) are transformed into non-dimensional equations using normalization variables in equation (7).

2.4.1 Normalized Continuity and Momentum Transport Equations

The non-dimensional continuity equation is,

$$\frac{\partial U_z^*}{\partial r^*}=0 \tag{8}$$

while the momentum equation is,

$$\frac{\partial^2 U_z^*}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial U_z^*}{\partial r^*}+\frac{1}{r^{*2}}\frac{\partial^2 U_z^*}{\partial \theta^{*2}}=0 \tag{9}$$

It is assumed that the radius of the elliptic duct is far less than the length and thereof $\frac{r^2}{Z_\infty^2}<<1\approx 0$

2.4.2 Normalized Energy Transport Equation

The dimensionless form of energy transport is,

$$\frac{\partial^2 T^*}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial T^*}{\partial r^*}+\frac{1}{r^{*2}}\frac{\partial^2 T^*}{\partial \theta^{*2}}=\frac{U_z^*}{Z^*}\frac{\partial T^*}{\partial Z^*} \tag{10}$$

2.4.3 Normalized Power law for Non-Newtonian

The body force in the momentum balance equation (4) is assumed negligible, the flow across elemental areas S_A and S_B in Fig. 1 is also assumed to be the same. From equations (4 & 6), the non dimensional power law is derived as,

$$\frac{\partial U^*}{\partial r^*}=-\frac{1}{2}\left[-\frac{1}{8}f_d\,\mathrm{Re}_k\,8^{n-1}\left(b+\frac{a}{n}\right)^n\right]^{\frac{1}{n}}\left(\frac{r^*}{Z^*}\right) \tag{11}$$

where the Darcy friction factor,

$$f_d=\frac{2D\Delta P}{\frac{Z_\infty}{\rho U_\infty^2}}$$

and the Power Law Reynolds number is given as,

$$\mathrm{Re}_k = \frac{\rho U^{2-n} D^n}{8^{n-1} K\left(b + \frac{a}{n}\right)^n} \tag{12}$$

In equation (12), a and b are Kozicki geometric constant and the values of the geometric constants depend on the aspect ratio of the ellipse (Sestak *et al*,1974).

Table 1: Constant a and b used for Elliptical geometry as obtained from Frank (1999)

Aspect Ratio	Eccentricity (e)	a	B
0.00	1.0000	0.3084	0.9253
0.10	0.9949	0.3018	0.9053
0.20	0.9797	0.2907	0.8720
0.30	0.9539	0.2796	0.8389
0.40	0.9165	0.2702	0.8107
0.50	0.8660	0.2629	0.7886
0.60	0.8000	0.2575	0.7725
0.70	0.7141	0.2538	0.7614
0.80	0.6000	0.2515	0.7546
0.90	0.4359	0.2504	0.7510
1.00	0.0000	0.2500	0.7500

According to Chabra and Richardson (1999), at fully developed flow region, the product of the Darcy friction factor and Power law Reynolds number is,

$$f_d \,\mathrm{Re}_k = 64\left(b + \frac{a}{n}\right)^n \tag{13}$$

3.0 Finite Difference Analogue (FDA)

3.1 FDA of the momentum Transport

The FDA for the momentum transport equation (9) is,

$$U^*_{i,j,k} = \left(\frac{A_{i,j,k} + B_{i,j,k} + C_{i,j,k}}{D_{i,j,k}}\right) \tag{14}$$

where,

$$A_{i,j,k} = \left(U^*_{i+1,j,k} + U^*_{i-1,j,k}\right)\left(\Delta r^*\right)^2(\Delta\theta)^2 \;\; ; \;\; B_{i,j,k} = \left(U^*_{i,j+1,k} + U^*_{i,j-1,k}\right)\left(\Delta r^*\right)^2$$

$$\delta = \left[\left(\Delta r^*\right)^{2^*} r^*(\Delta\theta)^2\right] \;\; ; \;\; C_{i,j,k} = -\frac{1}{2}\left[-\frac{1}{8} f_d \,\mathrm{Re}_k \, 8^{n-1}\left(b + \frac{a}{n}\right)^n\right]^{\frac{1}{n}}\left(\frac{r^*}{Z^*}\right)\delta$$

$$D_{i,j,k} = 2\left[\left(\Delta r^*\right)^{2^*}(\Delta\theta)^2 + \left(\Delta r^*\right)^2\right]$$

3.2 FDA of the Energy Transport Equation

The FDA for the energy transport equation is obtained as,

$$T^*_{i,j,k} = \left(\frac{A_{i,j,k} + B_{i,j,k} - C_{i,j,k}}{\lambda}\right) \tag{15}$$

where,

$$A_{i,j,k} = \lambda(T^*_{i+1,j,k} + \eta T^*_{i-1,j,k}) \;\; ; \;\; B_{i,j,k} = \frac{T^*_{i,j+1,k}}{\left(\Delta r^*\right)^{2^*}\left(\Delta\theta^*\right)^2} \;\; ;$$

$$C_{i,j,k} = \frac{U^*_{i,j,k}}{4Z^*\Delta Z^*}T^*_{i,j,k} + T^*_{i,j,k-1} \; ; \; \lambda = \frac{1}{\left(\Delta r^*\right)^2} + \frac{1}{\left(2r^* + \Delta r^*\right)} \;\; ; \;\; \eta = \frac{1}{\left(\Delta r^*\right)^2} - \frac{1}{\left(2r^* + \Delta r^*\right)}$$

3.3 Applicable Boundary Conditions

The non-dimensional applicable boundary conditions are:

3.3.1 Cooling Process

The boundary conditions to be adopted if the heat exchanger is to be designed to cool the temperature of the working fluid in the pipe are:

i. At the wall surfaces: $r^* = \Gamma$, $U^* = 0$, $T^* = 0$

ii. At the core of the elliptic pipe: $r^* = 0$, $U^* = 1$, $T^* = 1$

3.3.2 Heating Process

If the heat exchanger is to be designed to increase the temperature of the fluid in the pipe, the following boundary conditions are to be used:

i. At the wall surfaces: $r^* = \Gamma$, $U^* = 0$, $T^* = 1$

ii. At the core of the elliptic pipe: $r^* = 0$, $U^* = 1$, $T^* = 0$

4.0 Peripheral Nusselt Number

The Nusselt number is a dimensionless quantity indicative of the rate of energy convection from the surface of the tube.

The local Nusselt number is estimated from equation (16) as.

$$Nu_L = \frac{D}{T_w - T_b}\frac{\partial T^*}{\partial r^*} \tag{16}$$

while the mean Nusselt number is the average of the local Nusselt numbers over the circumference of the ellipse and it is given as,

$$Nu_m = \frac{\sum_{1}^{M} Nu_L}{M} \tag{17}$$

where, M is the total number of nodal points on the wall surface. The bulk mean temperature is,

$$T_b^* = \frac{\int_0^{2\pi}\int_0^{\Gamma} T^*\left(r^*,\theta\right)U\left(r^*,\theta\right)r^* dr^* d\theta}{\int_0^{2\pi}\int_0^{\Gamma} U\left(r^*,\theta\right)r^* dr^* d\theta} \tag{18}$$

5.0 Results and Discussion

5.1 Velocity Profiles

Fig.3 shows the velocity distribution of non-Newtonian fluid along the major radius of elliptical tube of eccentricity e= 0.8660. The velocity distribution of $n = \frac{1}{3}$ and n=1 have the same value except for radial distance between 0.4 and 0.8 which is even insignificant when compared with the situation when the fluid index is 3 and 5.

Fig.4 shows the velocity distribution of dilatant fluid, n=3 along the major radius for different eccentricities. The graph reveals that for eccentricity of 0.866 and .9949 the numerical values of fluid flow velocity are almost the same but relatively higher for circular duct, e=0. This is an indication that the higher the eccentricity of the geometry the more the restriction to fluid flow in the pipe. This resistance to flow may arise from wall frictions.

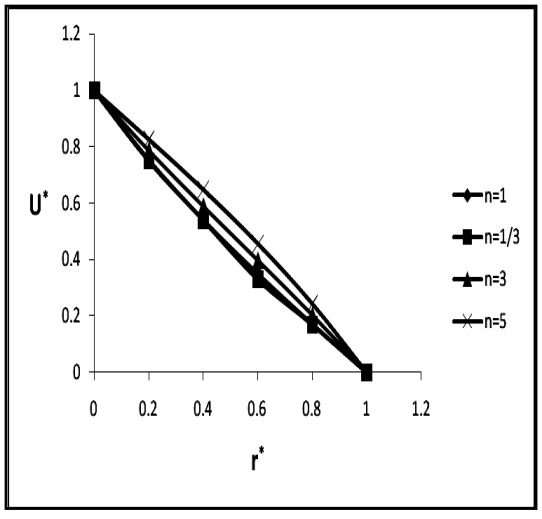


Fig.3: Velocity along major radius (θ = 0), e = 0.8660

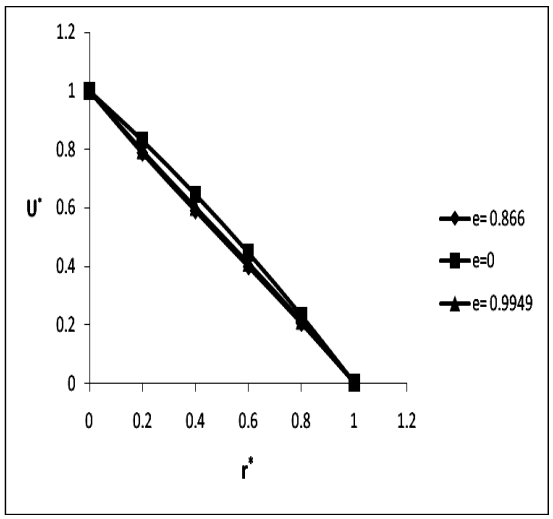


Fig.4: Velocity of dilatant (n=3) along major radius at different eccentricities

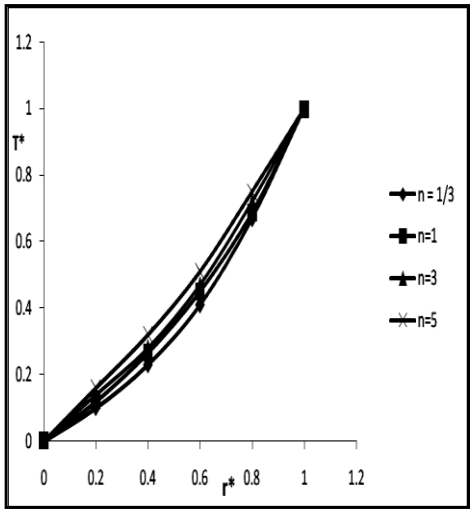


Fig.5: Temperature Profile (heating) at major radius (θ = 0), for e = 0.8660

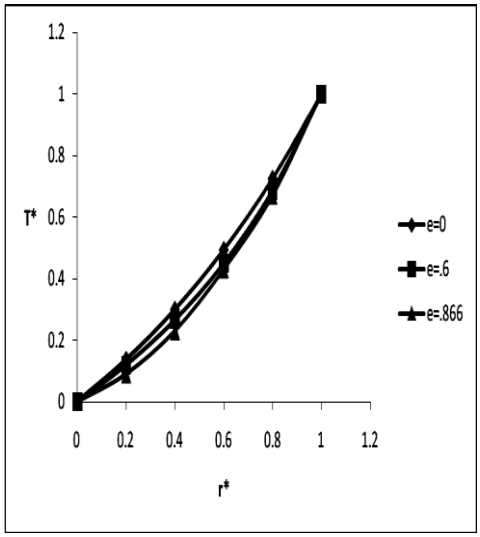


Fig.6: Temperature Distribution (heating) along major radius (θ = 0) for $n = \frac{1}{3}$

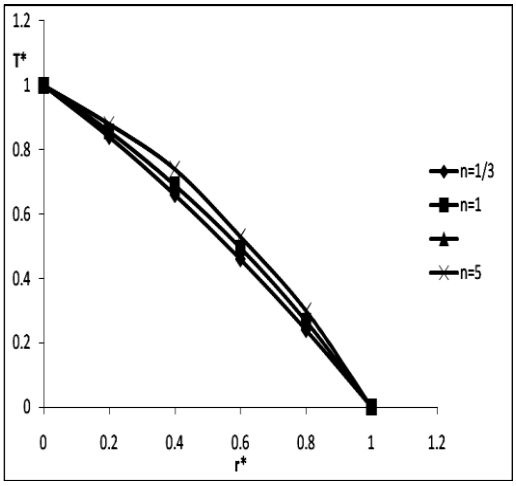


Fig.7: Temperature Distribution (cooling) along the radius for e = 0

5.2 Temperature Profiles

Fig.5 shows the temperature distribution (heating process) at the major radius for e= 0.8660. The plots depict a temperature increase with increasing flow behavior index n. Fig.6 is the temperature history of pseudo-plastic fluid of n= $\frac{1}{3}$ at θ = 0 for different eccentricities. It can be observed that the temperature increases as the eccentricity decreases. Fig.7 gives the temperature profile for cooling process along the radial distance for circular duct. The temperature decreases from the centre to the wall, with dilatant fluid having higher temperature.

5.3 Nusselt Number

Fig.8 shows the local Nusselt numbers around the peripheral walls of a circular tube for different flow behavior indices. For n=1 and n=3 the local Nusselt number dropped to a

minimum value at θ=90. But for pseudo-plastic fluid, n=1/3 the local Nusselt number rose to a maximum value at θ=90.

Fig.9 is the graph of local Nusselt number along the periphery of an elliptic duct of e=0.866 for flow behavior index, n=1/3, 1 and 3. For pseudo-plastic, Newtonian and dilatants fluids, the heat transfer is optimum at the wall of major axes and minimum at the minor axes. There is symmetry in the distribution of heat transfer on both segments of the cross section. The local Nusselt number increases as n reduces. It can be depicted from the figure that the local Nusselt number reduces from angle θ = 0 to 60 and remains fairly a constant value between 60 and 120. This heat transfer rate (Nusselt number) also increases from θ = 120 to 180. This shows that heat transfer is optimum along the minor axes.

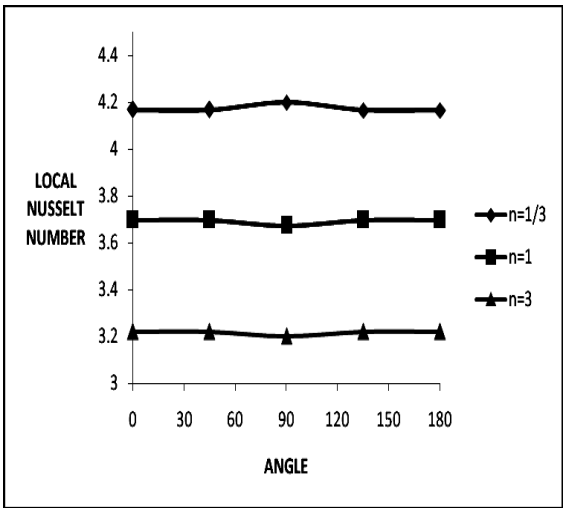


Fig.8: Local Nusselt number around the wall for different n and for e=0.

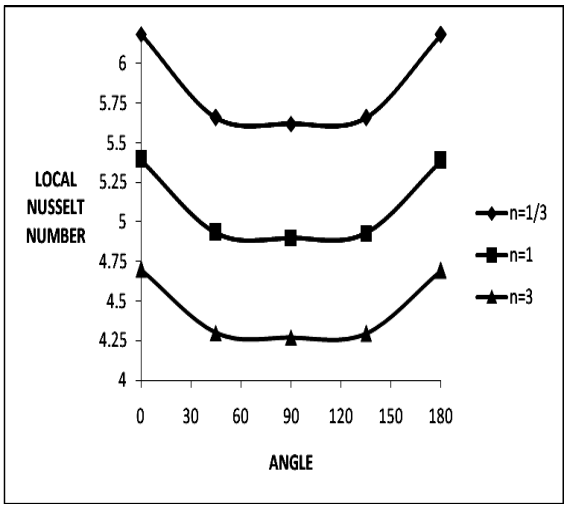


Fig.9: Local Nusselt number around the walls at different angles for e=0.866

Fig.10 shows the variation of local Nusselt number with flow behavior index for different eccentricities at θ = 90, that is along the major axis. For all the flow behavior index considered, the Nusselt number increases as the eccentricity increases but reduces with increasing flow behavior index, n. Fig.11 predicts the local Nusselt number at the end of minor radius for different eccentricities and for flow behavior index, n=1/3, 1 and 3. For all the eccentricities considered, the heat transfer by pseudo-plastic fluid is higher than that of dilatants while local Nusselt number increases with increasing eccentricity. For eccentricity range 0.8660<e< 1.0, there is a sharp increase in the local Nusselt number. This is an indication that the range is critical to heat enhancement.

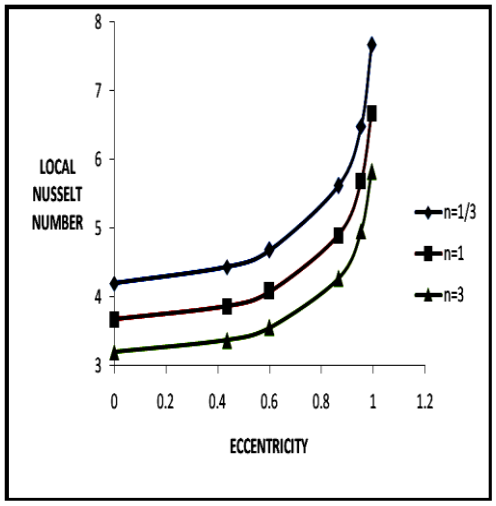


Fig. 10: Nusselt number with eccentricity for different flow behaviour index (θ = 90)

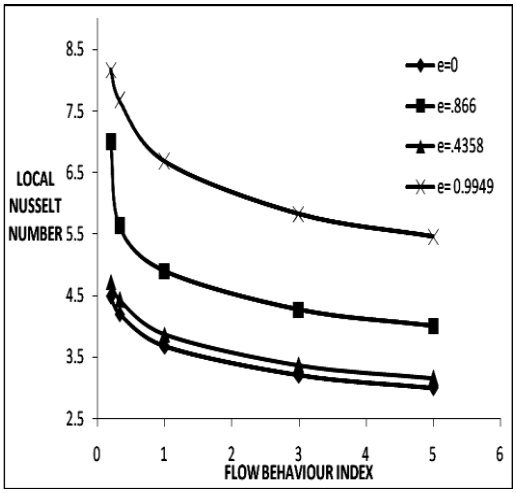


Fig.11: Local Nusselt number at minor radius (θ = 90)

Fig.12 is a relationship between the mean Nusselt number and the eccentricity at different values of flow behavior index. It can be observed from the plots that for all the values of eccentricity, the mean Nusselt number for pseudo-plastic is higher than dilatants fluids. Also, the mean Nusselt number increases with eccentricity. Fig. 13 is the plot of Mean Nusselt number versus flow behaviour index and for various eccentricities. It could be observed from the graph that the change in eccentricity, Δe has greater influence on heat transfer between e=0.866 and 0.9949 than between 0 and 0.7141.

Comparison of Figs.11, 12 and 13 shows that eccentricity ranges between 0.866 and 0.9 would be better for heat transfer enhancement in elliptic configuration.

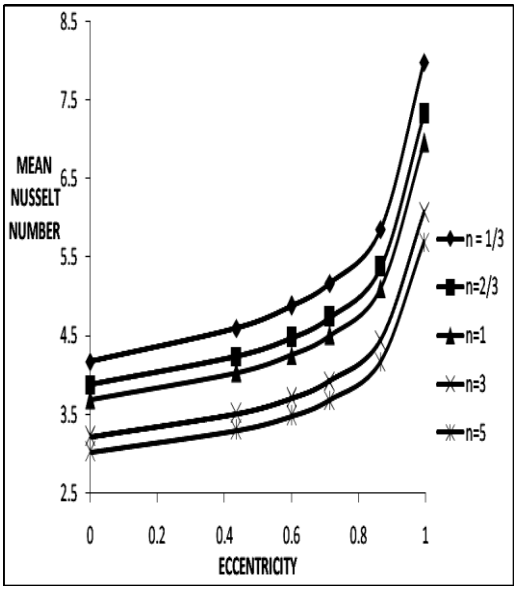


Fig.12: Mean Nusselt number with eccentricity

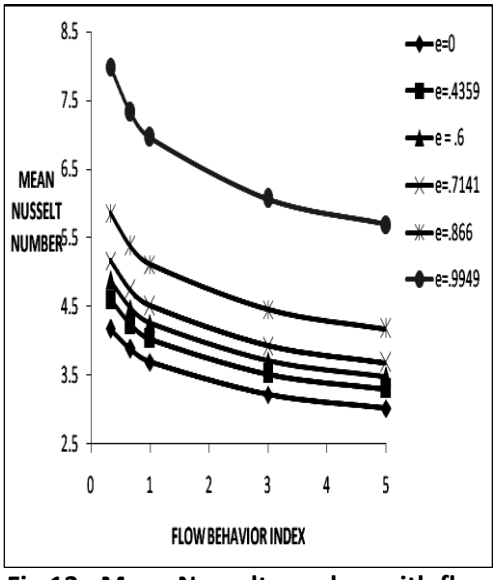


Fig.13: Mean Nusselt number with flow behaviour index and for various eccentricities

5.4 Validation

Table 4.1: Mean Nusselt number

n= 1/3	4.1753	4.1734
n= 2/3		3.8824
n = 1	3.6568	3.6906
n = 3	3.4090	3.2168
n= 5		3.0175

6.0 Conclusion

A numerical solution has been developed for laminar forced convection of power-law non-Newtonian fluids inside elliptical ducts using Finite Difference Method.

The Partial Differential Equations were transformed into modified cylindrical coordinate system and the computational domain of elliptic geometry was analyzed using QB-45 computer program.

Several values of eccentricity and flow behavior index were investigated. From the results obtained, the following are deduced:

Non-Newtonian fluids with flow behavior index $n < 1$ have higher heat transfer than those with $n > 1$ for all values of eccentricity. Therefore, pseudo-plastic fluids are better working fluid in heat exchangers irrespective of the values of eccentricity. This is in line with the work of Cotta and Ozisik [10].

For eccentricity $e \geq 0.8660$, the mean Nusselt number increases considerably. If dilatant fluid is cheaply available (may be in form of a waste product), a plant can be designed for a dilatant heat exchanger by using a duct of $e \geq 0.8660$ because at this value of eccentricity, heat is optimally augmented.

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