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An Empirical Study on the Run Length Properties of the Side-Sensitive Modified Group Runs Double Sampling (SSMGRDS) Control Chart for Monitoring the Process Mean Shifts

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Abstract: The side-sensitive modified group runs double sampling (SSMGRDS) control chart integrates the side-sensitive modified group runs (SSMGR) and double sampling (DS) charts concepts. The hybrid SSMGRDS chart which incorporates the sensitivity of its necessary control charts is an efficient procedure to enhance the mean shift detection speed. The average number of observations to signal (ANOS), percentiles of the number of observations to signal (PNOS) and median number of observations to signal (MNOS) as performance measures were employed to evaluate the performance of the SSMGRDS chart in this study. The results obtained indicates that the SSMGRDS chart is more important for large process detection mean shifts quickly as the shifts size increases. Besides, an extensive use of the MNOS and PNOS to evaluate the performance of the SSMGRDS chart leads to overall understanding of the performance of chart's due to the fact that the entire run length (RL) distribution has been considered. Generally, the findings herein unveil the RL properties of the SSMGRDS chart which will motivate professionals to use the chart for effective monitoring of their processes.

Keywords: Control chart, side-sensitive modified group runs double sampling (SSMGRDS), ANOS, MNOS, PNOS, process mean shifts

1. INTRODUCTION

Control charts have been commonly used in modern industries and other manufacturing sectors for process control to ensure the quality of the goods. The effort of organizations to gain a competitive advantage over their counterparts made them to ensure that they produce items of a high-quality standard to satisfy the needs of customers. One of the essential schemes which can help industries in achieving this vital goal is the Statistical Process Control (SPC) or Statistical Quality Control (SQC), which merely refers to the usage of statistical procedures to ensure that quality products are consistently produced. Control chart as an integral SPC tool is largely adopted in the surveillance of how a process change with time in industrial settings. The classical Shewhart $\bar{X}$ chart has been known for its capacity to discover large process mean shift rapidly. Although, the major disadvantage of the Shewhart $\bar{X}$ chart is that, it is not sensitive in detecting small and moderate shift of a process mean. Note that, a hybrid of control charting schemes is an extensive access to provide an augmented control chart that has a superior performance over the individual counterpart's performances.

To address this setback of the Shewhart $\bar{X}$ chart, several logical ideas have been used so as to increase the sensitivity of the Shewhart $\bar{X}$ chart, one of which is the use of double sampling (DS) $\bar{X}$ scheme. (Croasdale, 1974), was the first to introduce the DS $\bar{X}$ control chart, and later (Daudin, 1992) suggested the modified version of the DS $\bar{X}$ chart which considers an unmanageable

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condition at the first and combined sample stages concerning information acquired from both samples. (Tong & Lee, 2009) studied the performance of the DS $\bar{X}$ scheme under non-normality and discovered that the DS $\bar{X}$ control chart outperformed the Shewhart $\bar{X}$ chart at perceiving small mean shifts. Other literatures on the DS $\bar{X}$ and related control charts include (Khoo et al., 2013), (Teoh et al., 2014), (Teoh et al., 2016), (Umar & Nnamani, 2017), (Haq & Khoo, 2018), (Malela-Majika et al., 2019) and (Umar et al., 2019).

Group runs (GR) chart has been investigated by (Gadre & Rattihalli, 2004), the chart integrates the Shewhart $\bar{X}$ chart together with an extended version of the conforming run length (CRL) chart. The GR has remarkably smaller out-of-control average time to signal $\left(\mathrm{ATS}_1\right)$ when compared to the synthetic as well as Shewhart $\bar{X}$ control charts.

Additionally, (Gadre & Rattihalli, 2007) suggested the use of side sensitive group runs (SSGR) chart by joining the GR chart with the side sensitive procedure. The SSGR control chart surpass the Shewhart $\bar{X}$, synthetic and GR charts. Moreover, the modified version of the SSGR named SSMGR chart was proposed by (Gadre, Joshi & Rattihalli, 2010), which outperforms the existing GR control charts. Interested readers may found other related SSGR charts in (You et al., 2015), (Lim et al., 2015) and (Malela-Majika & Rapoo, 2019) for more details on the SSGR control charts.

Previous researches were carried out with the sole intention of developing new charts that are better and more efficient in detecting process shifts, and this eventually will result to more savings in cost. Thus, (Khoo et al., 2015) proposed the side sensitive group runs double sampling (SSGRDS) chart which combined the performance of both DS and SSGR control charts.

In addition, (Saha et al., 2017) initiated the modified side sensitive group runs double sampling (SSMGRDS) chart which incorporates the performance of both the side sensitive modified group runs (SSMGR) suggested by (Gadre & Rattihalli, 2006) and the DS charts.

Recently, an extensive study on the run length (RL) properties of the SSGRDS chart was suggested by (Chong et al., 2018), which examined the RL effect of various measures of performance on the SSGRDS chart.
In this study, the SSMGRDS control chart is extensively examined in terms of different performance measures such as, average number of observations to signal (ANOS), median number of observations to signal (MNOS) and percentiles of the number of observations to signal (PNOS).

The rest of this paper is organised as follows: In Section 2, the execution procedure of the SSMGRDS chart is presented. In Section 3, the three various performance measures (ANOS, MNOS, PNOS) employed in this study are discussed. In Section 4, the discussion of results and optimal design of the SSMGRDS chart are presented. Section 5 provides a discussion on the run length properties and finally some concluding remarks are drawn in Section 6.

2. IMPLEMENTATION OF THE SSMGRDS CHART

The SSMGRDS chart integrate the SSGR and DS control charting approach of (Gadre, Joshi & Rattihalli, 2010) and (Daudin, 1992), respectively. In addition, the SSMGRDS chart comprises the DS sub-chart and the CRL sub-chart extended version. CRL refers to the number of sampling stages between two successive nonconforming sampling stages. The first and second samples are denoted by n_1 and n_2 respectively. As for the DS sub-chart L and L_2 represents the control limits of the first and second sampling stages while L_1 stands for the warning limit of the first sampling stage.
Moreover, suppose the regions $I_1 = \left[-L_1, L_1\right]$, $I_2 = \left[-L, L_1\right) \cup \left(L_1, L\right]$, $I_3 = \left(-\infty, -L\right) \cup \left(L, +\infty\right)$, and $I_4 = \left[-L_2, L_2\right]$.Fig. 1 depict the graphical demonstration of the DS sub-chart, as it relates with the upcoming implementation steps.

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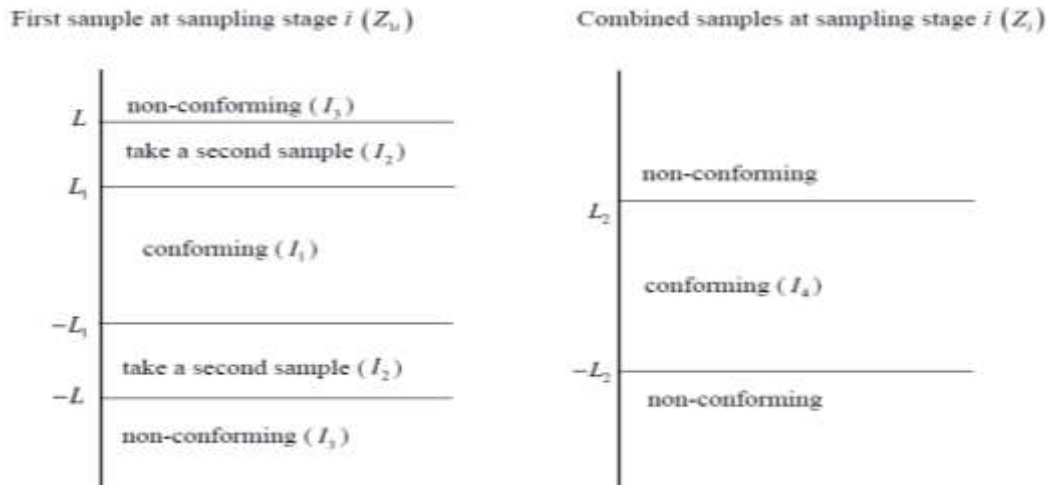


Figure1. A graphical view of the DS chart/DS sub-chart.

Note that, similar to the SSMGR chart, L_3 and L_4 denotes the warning and control limits of the CRL sub-chart, respectively. Similar to (Saha et al., 2017), the implementation of the SSMGRDS control chart are summarized as follows:

Step 1. Determine the optimal charting parameters, n_1, n_2, L_1, L_2, L_3, L_3 and L_4.

Step 2. A first sample having n_1 observations is drawn from a population, thereafter, the sample mean $\bar{X}_{1,i} = \frac{\sum_{j=1}^{n_1} X_{ij}}{n_1}$ is computed, where X_{1j}, for j=1,2,…, n_1, represents the observations in the first sample of sampling stage i. Then calculate $Z_{1,i} = \frac{\bar{X}_{1,i} - \mu_0}{\sigma/\sqrt{n_1}}$.

Step 3. If $Z_{1,i} \in I_1$, sampling stage i is conforming, then the control flow returns to Step 2.

Step 4. If $Z_{1,i} \in I_3$, sampling stage i is non-conforming, then move to Step 9.

Step 5. If $Z_{1,i} \in I_2$, a second sample having n_2 is drawn from the same population where the first sample is drawn, next is to calculate $\bar{X}_{2,i} = \frac{\sum_{j=1}^{n_2} X_{2j}}{n_2}$, where X_{2j}, for j=1,2,…, n_2, represents the observations in the second sample of sampling stage i.

Step 6. The overall sample mean, $\bar{X}_i = \frac{n_1\bar{X}_{1,i} + n_2\bar{X}_{2,i}}{n_1 + n_2}$, followed by $Z_i = \frac{\bar{X}_i - \mu_0}{\sigma/\sqrt{n_1 + n_2}}$, for sampling stage i.

Step 7. If $Z_i \in I_4$, then sampling stage i is conforming, then the control flow returns to Step 2.Otherwise, the process is declared as non-conforming and the control flow proceeds to Step 8.

Step 8. Calculate B_m value, where $m(=1, 2, \ldots)$ represents the number of non-conforming sampling stages.

Step 9. Declare the process as out-of-control if $B_1 \le L_4$ or for m> 1, $B_m \le L_3$ and $B_{m+1} \le L_4$,where B_m and B_{m+1} are the non-conforming sampling stages falls on the same side of the target mean value, μ_0, while L_3 and L_4 are the warning and control limits of the CRL sub-chart, respectively. Otherwise, proceeds to Step 10.

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Step 10. The control chart signals an out-of-control situation and counteractive steps are taken to examine the underlying procedure so that the out-of-control process is brought back into an in-control situation.

3. PERFORMANCE ASSESSMENT

The performance measure used in this study is the ANOS instead of the average run length (ARL). This is because the sample size of the SSMGRDS chart is not constant at each sampling phase (i.e. it can be either n_1 or n_1+n_2). Thus, the ARL which just measure the expected amount of sampling stages required to signal an out-of-control condition is less informative compared to the ANOS that measures the actual number of observations required by the chart to issue an out-of-control signal, for a specific shift size, δ. Additionally, the expected average number of observation (EANOS) which considers a random shifts in an overall range $(\delta_{\min}, \delta_{\max})$ is also useful to calculate the SSMGRDS chart. Besides this, the MNOS refers to the median number of observations taken from the start of monitoring process until the chart issue an out-of-control signal. On the other side, the PNOS focus on calculating the percentiles of observed numbers needed from the start of procedure monitoring until chart signals an out-of-control situation. In like manner, the expected median number of observation (EMNOS) and the expected median number of observation (EPNOS) which considers a random shifts in an overall range $(\delta_{\min}, \delta_{\max})$ are also useful methods to evaluate the SSMGRDS chart, respectively.

Suppose that the probability that the process is declared as in-control is (Daudin, 1992)

$$P_a = P_{a1} + P_{a2}, \quad (1)$$

where P_{a1} and P_{a2} denotes the probabilities that the process is declared as in-control at the first sample and the combined samples, respectively. In line with Eq. (1) (Daudin, 1992),

$$P_{a1} = \Pr(Z_1 \in I_1) = \Phi(L_1 + \delta\sqrt{n_1}) - \Phi(-L_1 + \delta\sqrt{n_1}) \quad (2)$$

where δ refers to the mean shift and $\Phi(\cdot)$ is the cumulative distribution function (cdf) of the standard normal random variable, respectively. Similarly,

$$P_{a2} = \Pr(Z \in I_4 \text{ and } Z_1 \in I_2) = \int_{z \in I_2^*} \Pr(Z \in I_4 | Z_1 = z)\phi(z)\,dz, \quad (3)$$

and

$$\Pr(Z \in I_4 | Z_1 = z) = \int_{z \in I_2^*} \left[\Phi(cL_2 + rc\delta - z\sqrt{n_1/n_2}) - \Phi(-cL_2 + rc\delta - z\sqrt{n_1/n_2})\right]\phi(z)\,dz, \quad (4)$$

where $\phi(\cdot)$ is the probability density function (pdf) of the standard normal variable.
Note that the standardized limit is,

$$I_2^* = \left[-L + \delta\sqrt{n_1}, -L_1 + \delta\sqrt{n_1}\right) \cup \left(L_1 + \delta\sqrt{n_1}, L + \delta\sqrt{n_1}\right]$$

$r = \sqrt{(n_1+n_2)}$ and $c = \sqrt{(n_1+n_2)/n_2}$.

The ANOS for a standardized mean shift, say $\delta = |\mu_1 - \mu_0|/\sigma$, where μ_1 denotes the out-of-control mean, computed as (Saha et al., 2017)

ANOS =

ANOS=

$$\frac{\left[1 + A_3 - A_4 - 2A_3A_4\alpha(1-\alpha)\right](n_1 + n_2 \Pr(Z_1 \in I_2))}{A_3A_4(1-P_a)\left[1-2\alpha(1-\alpha)\right]}, \quad (5)$$

where

$$\Pr(Z_1 \in I_2) = \Phi(L + \delta\sqrt{n_1}) - \Phi(L_1 + \delta\sqrt{n_1}) + \Phi(-L_1 + \delta\sqrt{n_1}) - \Phi(-L + \delta\sqrt{n_1}), \quad (6)$$

$$A_q = \Pr(B_m \le L_q) = 1 - (1-P_a)^{L_q}, \text{ for } q = 3 \text{ and } 4. \quad (7)$$

Also,

$$\alpha = \Pr(Z_1 > L \text{ or } \{Z > L_2 \text{ and } Z_1 \in (L_1, L)\} | \text{sampling stage is non-conforming})$$

$$= \frac{1 - \Phi(L_1 + \delta\sqrt{n_1}) - \int_{z \in I_2^{**}} \Phi(cL_2 + rc\delta - z\sqrt{n_1/n_2})\,\phi(z)\,dz}{1 - P_a}. \quad (8)$$

Here, $I_2^{**} = (L_1 + \delta\sqrt{n_1}, L + \delta\sqrt{n_1})$ and

$$P_a = \Phi(L_1 + \delta\sqrt{n_1}) - \Phi(-L_1 + \delta\sqrt{n_1}) + \int_{z \in I_2^*} \left[\Phi(cL_2 + rc\delta - z\sqrt{n_1/n_2}) - \Phi(-cL_2 + rc\delta - z\sqrt{n_1/n_2})\right]\phi(z)\,dz \quad (9)$$

The EANOS of the SSMGRDS control chart, for the shifts range $(\delta_{\min}, \delta_{\max})$ is computed as (Saha et al., 2017)

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$$\mathrm{EANOS}=\int_{\delta_{\min}}^{\delta_{\max}}\mathrm{ANOS}(\delta)\,\mathrm{g}(\delta)\,\mathrm{d}\delta, \quad (10)$$

where, ANOS (δ) is obtained from Eq. (5) and $\mathrm{g}(\delta)$ denotes the pdf of the shift size, δ. It is considered that each value of δ in the range, $\delta\in(\delta_{\min},\delta_{\max})$ occurs with the same probability. Therefore, the shift size δ follows a uniform, $U(\delta_{\min},\delta_{\max})$ distribution (Sparks, 2000). Thus, Eq. (10) becomes

$$\mathrm{EANOS}=\frac{1}{\delta_{\max}-\delta_{\min}}\int_{\delta_{\min}}^{\delta_{\max}}\mathrm{ANOS}(\delta)\,\mathrm{g}(\delta)\,\mathrm{d}\delta\cdot \quad (11)$$

4. RESEARCH FINDINGS AND OPTIMAL DESIGN

In this segment the result of the performance measures ANOS, MNOS and PNOS of the SSMGRDS control chart are presented. The in-control ANOS (ANOS_0), MNOS (MNOS_0) and PNOS (PNOS_0) were all fixed as 370 for a fair comparison. It is worthy to note that the 50th percentile (PNOS) is equivalent to the MNOS. Optimization programs incorporating the corresponding procedure are written in MATLAB software to compute the optimal parameters (n_1, n_2, L, L_1, L_2, L_3 and L_4) of the SSMGRDS chart that minimize the out-of-control ANOS (ANOS_1) when $\mathrm{ANOS}_0=370$. Besides that, the statistical analytical system (SAS) software was used to carry out a simulation studies in computing the out-of-control MNOS (MNOS_1) and PNOS (PNOS_1), respectively. The various combinations of the optimal parameters for the SSMGRDS control chart (n_1, n_2, L, L_1, L_2, L_3 and L_4) can be found in Table 1, where the in-control standard sample size $(\mathrm{ASS}_0)=5,7\text{ and }10$. The ANOS_1 and MNOS_1 values are presented in Table 2, while the PNOS_1 values are given in Table 3.

Table 1. Optimal charting parameters of the SSMGRDS chart by minimizing ANOS_1 when $\mathrm{ANOS}_0=370$ for $\mathrm{ASS}_0=5$ (first row), 7 (italicized) and 10 (third row)

δ	n_1	n_2	L	L_1	L_2	L_3	L_4
	3	10	4.8244	1.2815	1.6123	1	82
0.1	*4*	*14*	*4.8144*	*1.2419*	*1.4342*	*1*	*56*
	4	20	4.7675	1.0364	1.3110	1	36
	2	10	4.7675	1.0364	1.4309	1	36
0.2	*3*	*13*	*2.8299*	*1.0103*	*1.3073*	*1*	*25*
	6	12	2.2115	0.9147	1.3094	1	15
	3	7	2.4703	1.0381	1.3940	1	19
0.3	*5*	*6*	*2.3475*	*0.9303*	*1.3532*	*1*	*12*
	7	7	2.2115	0.7462	1.2634	1	8
	3	4	2.4703	0.6534	1.3911	1	8
0.5	*5*	*3*	*2.3475*	*0.4049*	*1.3252*	*1*	*5*
	6	5	2.2115	0.2186	1.2197	1	4
	3	3	2.4703	0.4122	1.3552	1	5
0.7	*4*	*4*	*2.3475*	*0.2938*	*1.1857*	*1*	*3*
	6	5	2.2115	0.2186	1.1411	1	3
	3	3	2.4703	0.4122	1.2204	1	3
1.0	*4*	*4*	*2.3475*	*0.2938*	*1.0759*	*1*	*2*
	5	6	2.2115	0.1760	1.0332	1	2
	3	3	2.4703	0.4122	1.1106	1	2
1.5	*3*	*5*	*2.3475*	*0.2290*	*1.0800*	*1*	*2*
	4	7	2.2115	0.1457	1.0403	1	2
	2	4	2.4703	0.3009	1.2192	1	3
2.0	*3*	*5*	*2.3475*	*0.2290*	*1.0800*	*1*	*2*
	3	8	2.2115	0.1231	1.1595	1	3

Table 2. ANOS_1 and MNOS_1 of the SSMGRDS chart when $\mathrm{ASS}_0=5,7\text{ and }10$.

δ	ANOS_1			MNOS_1		
	$\mathrm{ASS}_0=5$	$\mathrm{ASS}_0=7$	$\mathrm{ASS}_0=10$	$\mathrm{ASS}_0=5$	$\mathrm{ASS}_0=7$	$\mathrm{ASS}_0=10$
0.1	153.3552	145.495	139.9844	73	76	76
0.2	59.3342	58.2997	57.5748	36	38	36
0.3	32.2260	31.8796	31.7352	22	21	21
0.5	14.5358	14.8446	15.5226	10	8	11
0.7	8.9110	9.5824	10.7677	6	8	11
1.0	6.0221	6.8965	8.0481	6	8	5
1.5	4.3833	5.0091	5.5047	3	3	4
2.0	3.4381	3.6598	3.8404	2	3	3

Table 3. PNOS_1 of the SSMGRDS chart when $\mathrm{ASS}_0=5,7\text{ and }10$.

δ	PNOS_1				
	5th	25th	50th	75th	95th
	$\mathrm{ASS}_0=5$				
0.1	13	35	73	137	294
0.2	12	18	36	64	130
0.3	9	13	22	38	73
0.5	3	7	10	17	33
0.7	3	6	6	12	18

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1.0	3	6	6	6	12
1.5	3	3	3	6	6
2.0	2	2	2	6	6
$ASS_0 = 7$					
0.1	18	38	76	138	292
0.2	16	22	38	65	131
0.3	10	11	21	37	73
0.5	5	8	8	16	32
0.7	4	8	8	8	16
1.0	4	4	8	8	8
1.5	3	3	3	8	8
2.0	3	3	3	3	8
$ASS_0 = 10$					
0.1	24	40	76	140	284
0.2	12	18	36	66	126
0.3	7	14	21	35	70
0.5	6	11	11	17	33
0.7	6	6	11	11	22
1.0	5	5	5	11	11
1.5	4	4	4	4	11
2.0	3	3	3	3	11

Moreover, the graphical comparison of the out-of-control (OOC) $ANOS_1$, $MNOS_1$ and $PNOS_1$ (i.e. at 25th percentile) values when $ANOS_0 = MNOS_0 = PNOS_0 = 370$ and $ASS_0 = 7$ are shown in Fig. 2, respectively.

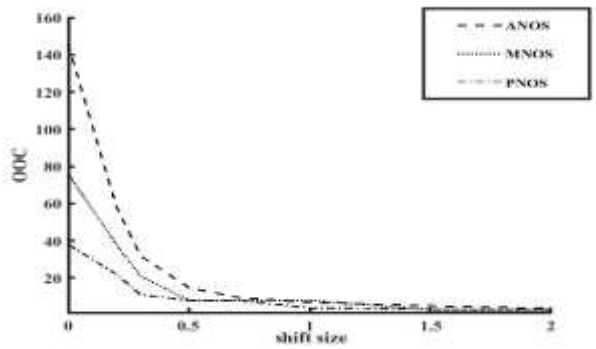


Figure 2. A graphical comparison of the OOC ($ANOS_1$, $MNOS_1$ and $PNOS_1$) values for the SSMGRDS chart when $ASS_0 = 7$ with different shift size

5. RUN LENGTH PROPERTIES EVALUATION

MNOS and PNOS are introduced in this analysis to assess the efficiency of the SSMGRDS control panel, in addition to ANOS. Considering both table 2, and Fig.2, it is apparent that the bulk of the values are smaller than the values, showing that the number of results reported is skewed to the right. Although, the level of skewness consistently reduces as δ increases. The interpretation of the performance of the SSMGRDS chart based on mainly ANOS could be misleading due to the skewness of the number of observations. It is obvious, that more information would be provided on the performance of the SSMGRDS chart via the use of MNOS and PNOS than ANOS because both the formers considered the overall RL distribution than the latter. Moreover, the impact of outliers on median is quite minimal in comparison to average, thus, creating it a more suitable and reliable value to be use in the interpretation of the chart's performance.

In Table 3, the percentile shows the assurance level for the out-of-control signal to occur. For example, when $ASS_0 = 7, \delta = 0.1$, we are 95% confident that the SSMGRDS chart at the 292th observations would issue an out-of-control signal. It can also be seen from Table 3 that the unrestrained condition signalled faster as δ increases (i.e. for moderate shift, δ = 0.5 and 0.7) while the unrestrained condition is issued at the same examination for all percents when δ is large (i.e. $\delta \geq 1.0$). It is obvious from Tables 2 and 3, that there is a decrease in the values of $ANOS_1$ $MNOS_1$ and $PNOS_1$ as δ increases, for $ASS_0 = 5, 7 \text{ and } 10$. For instance, when $ASS_0 = 7$, the $ANOS_1(MNOS_1)$ decrease from 145.50 (76) when δ = 0.1 to 3.66 (3) when δ = 2.0. In the like manner, for the 50th percentile when $ASS_0 = 7$, the $PNOS_1$ decreases from 76 when δ = 0.1 to 3 when δ = 2.0. Moreover, the detecting ability of the SSMGRDS chart based on the $ANOS_1$ is relatively decreasing as the ASS_0 increases.

A small values of $ANOS_1$ $MNOS_1$ and $PNOS_1$ are desirable for a swift detection of an out-of-control process mean shift. Thus, the SSMGRDS chart becomes further effective and susceptible in finding the process mean shift. Generally, control charts are likely to detect large process mean shifts faster compared to small shifts. This is because large shifts are more likely to lead a important loss of superiority and cost effective.

6. CONCLUSION

This study examines the performance of the SSMGRDS control chart using ANOS, MNOS and PNOS. Looking at

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the fact that the model size of the SSMGRDS chart is not consistent at all sampling phase, it is imperative to use the ANOS as performance measure rather than the standard ARL quality measurement. This is because, the ANOS provides more information about the process mean shifts as it estimates the real number of interpretation required to issue an OOC condition. In addition, the MNOS and PNOS are adopted in this study as performance measures so as to generally evaluate the performance of the SSMGRDS chart effectively. The results indicates that the dissemination of the number of considerations is altered to the right as majority of the $ANOS_1$ values are higher than the $MNOS_1$ and $PNOS_1$ values, respectively.

Thus, MNOS and PNOS provides the most suitable values of the middle tendency, and hence are better identifiers of the SSMGRDS control chart's performance for swift detection ability of the process mean shift. Moreover, it is obvious that the SSMGRDS chart performs well as the process shifts size increases, which is justified for swift detection of large shifts that lead to a substantial loss of superiority faster.

7. ACKNOWLEDGEMENTS

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