Nig. J. Pure & Appl. Sci. Vol. 27 (2014): 2562 - 2577
Printed in Nigeria

0794-0378
(C) 2014 Faculty of Physical Sciences and Faculty of Life Sciences, Univ. of Ilorin, Nigeria

THE UNSTEADY RADIATIVE AND MHD FREE CONVECTIVE TWO IMMISCIBLE FLUID FLOWS THROUGH IN A HORIZONTAL CHANNEL

M. S. Dada*, C. Onwubuoya, K. M. Joseph

Department of Mathematics, University of Ilorin – Nigeria
*Corresponding author's email address and telephone number: dadamsa@gmail.com, +2348033547911

ABSTRACT

In this present work, the unsteady radiative and MHD free convective two immiscible fluid flows through a horizontal channel have been investigated. The upper channel is assumed to be porous while the lower is non-porous. The partial differential equations governing the fluid flow are transformed to ordinary differential form by a regular perturbation method and the analytical solutions for each fluid flow are obtained and these solutions matched at the interface. The results are presented for various values of the fluid parameters such as, Grash of number, radiative parameter, and frequency parameter, Prandtl number, viscosity and conductivity ratio. It is found among others results that as the thermal radiation from the wall temperature decreases, the temperature profiles and thermal boundary layers increases.

Key words: Unsteady flow, MHD, thermal radiation, immiscible fluid,

INTRODUCTION

Several attempts has been made in the past years to study the problems of unsteady magneto-hydrodynamics (MHD) free convective flow through a porous medium in a horizontal parallel plates because of its various applications in engineering processes such as oil extraction, oil transport, thermal energy storage, MHD pumps, MHD flow meter and cooling of nuclear reactors etc. It finds its place also in physiology such as blood flow in the arteries. The immiscible fluid flow through a porous medium and heat transfer is significant in the problems of petroleum extraction and transport.

Examining the wide range of applications of such flow, some authors and scholars have made their contributions. Notable among them are Chamka (2000, 2003) who considered the flow of two-immiscible fluids in porous and non-porous channels, and the unsteady MHD convective heat and mass transfer past semi- infinite vertical permeable plate with absorption. Also, Israle *et al.*, (2004) investigated the influence of viscous dissipation and radiation on unsteady MHD free convention flow past an infinite heated vertical plate in the porous medium with time dependent suction. Umavathi *et al.*, (2004, 2008, 2010) reported on oscillatory flow

and heat transfer in two immiscible fluids, unsteady magnetohydrodynamic two fluid flow and heat transfer in a horizontal channel and unsteady flow and heat transfer through a porous media sandwiched between viscous fluids.

Vafai and Kim (1990) was one of the earlier studies that presented the exact solution for the fluid mechanics of the interface region between a porous medium and a fluid layer. Malashetty and Leela (1991) discussed the magnetohydrodynamic heat transfer in two fluid flows. Malashetty and Umavathi (1997) studied two phase MHD flow and heat transfer in an inclined channel. Reddy and N.B Reddy (2009) analyzed the radiation and mass transfer effect on an unsteady MHD free convention flow past a heated vertical porous plate with viscous dissipation. Kumar *et al.*, (2009) discussed the unsteady MHD and heat transfer of two viscous immiscible fluids through a porous medium in horizontal channel. Kumar *et al.*, (2010) studied the unsteady flow and heat transfer through a porous media sandwiched between viscous fluids. Simon (2013) studied the convective flow of two immiscible fluids and heat transfer with porous along an inclined channel with pressure gradient.

The effects of thermal radiation on fluid flow have led some researchers to carry out studies on the influence of thermal radiation on the single phase fluid flow. Prakash and Ogulu (2006) investigated the unsteady two dimensional flows of radiating and chemically reacting MHD fluid with time dependent suction. Dada (2012) discussed radiation, dissipation and MHD effects on fluid flow past an impulsively started vertical Newtonian heating plate.
In the above investigations, the effect of thermal radiation is neglected in most of the studies on multiple phase flows and due to the influence of thermal radiation not only on single phase but also on multiple phase fluid flows, this present work hereby investigates the effect of radiation on an unsteady MHD free convective two immiscible fluid flows in a horizontal channel.

FORMULATION OF THE PROBLEM

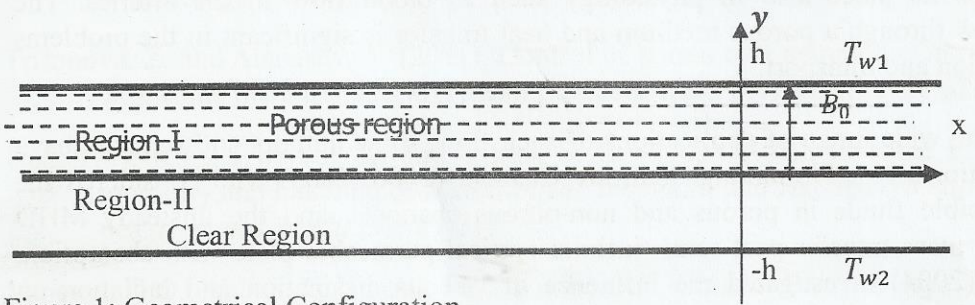


Figure 1: Geometrical Configuration

In the Figure 1 above, the upper channel is porous while the lower channel is non-porous denoted as region I and region II respectively. The fluid flowing through region 0≤y≤h (region I) having density ρ_1, dynamic viscosity μ_1 specific heat capacity c_p and thermal conductivity k_1. The fluid flowing through $-h \leq y \leq 0$ $(region\ II)$ having ρ_2, dynamic viscosity μ_2, specific heat capacity c_p, and thermal conductivity k_2. The magnetic Reynolds number is assumed very small. Further, the magnetic field is not strong enough to cause electrical dissipation. The equation governing the fluid flow are given by

$$\frac{\partial V_1'}{\partial y'} = 0 \tag{1}$$

$$\rho_1\left(\frac{\partial U_1'}{\partial t'} + V_1'\frac{\partial U_1'}{\partial y'}\right) = \mu_1\frac{\partial^2 U_1'}{\partial y'^2} - \frac{\partial \rho'}{\partial x'} - \sigma B_0^2 U_1' + \rho_1 g\beta f_1(T_1' - T_{w1}') \tag{2}$$

$$\rho_1 C_p\left(\frac{\partial T_1'}{\partial t'} + V_1'\frac{\partial T_1'}{\partial y'}\right) = K_1\frac{\partial^2 T_1'}{\partial y'^2} - \frac{\partial q_r}{\partial y} \tag{3}$$

Region II

$$\frac{\partial V_2'}{\partial y'} = 0 \tag{4}$$

$$\rho_2\left(\frac{\partial U_2'}{\partial t'} + V_2'\frac{\partial U_2'}{\partial y'}\right) = \mu_2\frac{\partial^2 U_2'}{\partial y'^2} - \frac{\partial \rho'}{\partial x'} - \sigma B_0^2 U_2' - \frac{\mu_2}{K'}U'_2 + \rho_2 g\beta f_2(T_2' - T_{w2}') \tag{5}$$

$$\rho_2 C_p\left(\frac{\partial T_2'}{\partial t'} + V_2'\frac{\partial T'_2}{\partial y'}\right) = K_2\frac{\partial^2 T'_2}{\partial y'^2} - \frac{\partial q_r}{\partial y} \tag{6}$$

where U' the x'-component fluid velocity is V' is the y component fluid velocity and T' is the fluid temperature.

The subscripts 1 and 2 corresponds to Region I and Region II respectively. The boundary conditions on velocity are no slip boundary conditions, which required that x'-component velocity must varnish at the wall. Therefore, the boundary conditions on temperature are isothermal conditions. We also assume the continuity of velocity, shear stress, temperature and heat flux at the interface between the two fluid layers at y=0

Therefore, the boundary and the interface conditions on the velocity are

$$U_1'(\mathrm{h}) = 0$$
$$U_2'(-\mathrm{h}) = 0$$
$$U_1'(0) = U_2' = 0 \tag{7}$$
$$\mu_1\frac{\partial U_1'}{\partial y'} = \mu_2\frac{\partial U_2'}{\partial y} \text{ at y' } = 0$$

The thermal boundary and interface conditions on temperature for both fluids are given by

$$T_1'(\mathrm{h}) = T_{w1}'$$
$$T_2'(-\mathrm{h}) = T_{w2}'$$
$$T_1'(0) = T_2' \quad (0) \tag{8}$$
$$K_1\frac{\partial T_1'}{\partial y'} = K_2\frac{\partial T'_2}{\partial y'} \text{ at y' } = 0$$

The continuity equations for both equations (1) and (4) implies that V_1' and V_2' are independent of y, they can be at most a function of time alone. Hence, we write (assuming $V_1' = V_2' = V'$)

$$V' = V_0(1 + \varepsilon A e^{i\omega t}) \tag{9}$$

where A is a real positive constant, ε is a very small positive quantity such that εA << 1. Here it is assumed that the transpiration velocity V' varies periodically with time about a non-zero constant mean V_0.

By the use of the following non-dimensional quantities

$$U_i = \frac{U_i}{U}, y = \frac{y'}{h}, t = \frac{t'\vartheta_1}{h^2}, V = \frac{h}{\vartheta_1}V' = \frac{v}{|V_0|}, P = -\frac{h^2}{\mu_1 u}\left(\frac{\partial p}{\partial x}\right)$$

$$\theta_i = \frac{T_i' - T_{w1}'}{T_{w2}' - T_{w1}'}, Pr = \frac{\mu_1 C_p}{k_1}, \alpha_1 = \frac{\mu_2}{\mu_1}, \beta_1 = \frac{k_2}{k_1}, \tau_1 = \frac{\rho_2}{\rho_1}, m_1 = \frac{\beta_{f2}}{\beta_{f1}} \quad (10)$$

$$Gr = \frac{\rho_1 g h \beta f_1 (T_{w2}' - T_{w1}')}{\mu_1 U}, M = \frac{h\sigma B_0^2}{\rho_1 U}, F = \frac{4I'h_1^2}{k_1}, \frac{\partial qr}{\partial y} = 4(T_i' - T_{w1}')I',$$

equations (2) - (6) become,

for region I

$$\frac{\partial U_1}{\partial t} + \left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial U_1}{\partial y} = \frac{\partial^2 U_1}{\partial y^2} + P - M^2 U_1 + Gr\theta_1 \quad (11)$$

$$\frac{\partial \theta_1}{\partial t} + \left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial \theta_1}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta_1}{\partial y^2} - \frac{F\theta_1}{Pr} \quad (12)$$

For region II

$$\frac{\partial U_2}{\partial t} + \left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial U_2}{\partial y} = \alpha_1\xi_1\frac{\partial^2 U_2}{\partial y^2} + \xi_1 P - M^2 U_2 - \alpha_1\xi_1 k^2 U_2 + Grm_1\theta_2 \quad (13)$$

$$\frac{\partial \theta_2}{\partial t} + \left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial \theta_2}{\partial y} = \frac{\beta_1\xi_1}{Pr}\frac{\partial^2 \theta_2}{\partial y^2} - \frac{\beta_1\xi_1}{Pr}F\theta_2 \quad (14)$$

where $\Pr = \frac{\mu_1 C_p}{k_1}$ is the Prandtl number, $\alpha_1 = \frac{\mu_2}{\mu_1}$ is the ratio of viscosities $\beta_1 = \frac{k_2}{k_1}$ is the ratio of thermal conductivities and $\xi_1 = \frac{\rho_2}{\rho_1}$ is the ratio of the densities.

The hydrodynamic, thermal boundary and interface conditions in non-dimensional form are as follows

$$U_1(1) = 0$$
$$U_2(-1) = 0$$
$$U_1(0) = U_2(0) \quad (15)$$
$$\frac{\partial U_1}{\partial y} = \alpha_1\frac{\partial U_2}{\partial y} \; at \; y = 0$$

$$\theta_1(1) = 1$$
$$\theta_2(-1) = 0$$
$$\theta_1(0) = \theta_2(0) \quad (16)$$
$$\frac{\partial \theta_1}{\partial y} = \beta_1\frac{\partial \theta_2}{\partial y} \; at \; y = 0$$

METHOD OF SOLUTION

In order to solve the governing equations (11) - (14) under the boundary and interface conditions (15) – (16), we separate $U_1(y,t), \theta_1(y,t), U_2(y,t), \theta_2(y,t)$ into steady and unsteady parts by assuming,

$$U_1(y,t) = U_{10}(y) + \varepsilon e^{i\omega t} U_{11}(y)$$
$$\theta_1(y,t) = \theta_{10}(y) + \varepsilon e^{i\omega t} \theta_{11}(y)$$
$$U_2(y,t) = U_{20}(y) + \varepsilon e^{i\omega t} U_{21}(y) \quad (17)$$
$$\theta_2(y,t) = \theta_{20}(y) + \varepsilon e^{i\omega t} \theta_{21}(y)$$

By substituting (17) into equations (11)-(14), equating the harmonic and non-harmonic terms, and neglecting the higher order of ε^2, we get the following sets of ordinary differential equations:

Region I

Harmonic coefficient

$$\frac{\partial^2 U_{10}}{\partial y^2} = \frac{\partial U_{10}}{\partial y} + M^2 U_{10}(y) = -P - Gr\theta_{10} \quad (18)$$

$$\frac{\partial^2 \theta_{10}}{\partial y^2} - Pr\frac{\partial \theta_{10}}{\partial y} - F\theta_{10}(y) = 0 \quad (19)$$

Non Harmonic coefficients

$$\frac{\partial^2 U_{11}}{\partial y^2} - \frac{\partial U_{11}}{\partial y} - (M^2 + i\omega)U_{11}(y) = \frac{\partial U_{10}}{\partial y} - Gr\theta_{11}(y) \quad (20)$$

$$\frac{\partial^2 \theta_{11}}{\partial y^2} - Pr\frac{\partial \theta_{11}}{\partial y} - (F + i\omega Pr)\theta_{11}(y) = Pr\frac{\partial \theta_{10}}{\partial y} \quad (21)$$

Region II

Harmonic coefficients

$$\frac{\partial^2 U_{20}}{\partial y^2} - \frac{1}{\alpha_1\xi_1}\frac{\partial U_{20}}{\partial y} - \left(\frac{\xi_1 M^2 + \alpha_1\xi_1 K^2}{\alpha_1\xi_1}\right) U_{20}(y) = -\frac{\xi_1 P}{\alpha_1\xi_1} - \frac{GrM_1}{\alpha_1\xi_1}\theta_{20}(y) \quad (22)$$

$$\frac{\partial^2 \theta_{20}}{\partial y^2} - \frac{\beta_1\xi_1}{Pr}\frac{\partial \theta_{20}}{\partial y} - \theta_{20}(y) \quad (23)$$

Non Harmonic coefficients

$$\frac{\partial^2 U_{21}}{\partial y^2} - \frac{1}{\alpha_1\xi_1}\frac{\partial U_{21}}{\partial y} - \left(\frac{\xi_1 M^2 + \alpha_1\xi_1 K^2 + i\omega}{\alpha_1\xi_1}\right) U_{21}(y) = \frac{1}{\alpha_1\xi_1}\frac{\partial U_{20}}{\partial y} - \frac{GrM_1}{\alpha_1\xi_1}\theta_{21}(y) \quad (24)$$

$$\frac{\partial^2 \theta_{21}}{\partial y^2} - \frac{\beta_1\xi_1}{Pr}\frac{\partial \theta_{21}}{\partial y} + \left(\frac{1 + \beta_1\xi_1 i\omega}{Pr}\right)\theta_{21} = \frac{\beta_1\xi_1}{Pr}\frac{\partial \theta_{20}}{\partial y} \quad (25)$$

The corresponding boundary and interface conditions become:

Harmonic terms

$$U_{10}(1) = 0$$
$$U_{20}(-1) = 0$$
$$U_{10}(0) = U_{20}(0) \quad (26)$$
$$\frac{\partial U_{10}}{\partial y} = \alpha_1 \frac{\partial U_{20}}{\partial y} \text{ at } y = 0$$

Non-Harmonic

$$U_{11}(1) = 0$$
$$U_{21}(-1) = 0$$
$$U_{11}(0) = U_{21}(0) \quad (27)$$
$$\frac{\partial U_{11}}{\partial y} = \alpha_1 \frac{\partial U_{21}}{\partial y} \; at \; y = 0$$
$$\theta_{10}(1) = 1$$
$$\theta_{20}(-1) = 0$$
$$\theta_{10}(0) = \theta_{20}(0) \quad (28)$$
$$\frac{\partial \theta_{10}}{\partial y} = \beta_1 \frac{\partial \theta_{20}}{\partial y} \; at \; y = 0$$
$$\theta_{11}(1) = 1$$
$$\theta_{21}(-1) = 0$$
$$\theta_{11}(0) = \theta_{21}(0) \quad (29)$$
$$\frac{\partial \theta_{11}}{\partial y} = \beta_1 \frac{\partial \theta_{21}}{\partial y} \; at \; y = 0$$

Now, the equations (18)-(25) are ordinary linear coupled differential equations with constant coefficients.

The analytical solutions of the differential equations (18) to (25) are known under the boundary conditions (26) to (29) and are given by:

$$U_{10}(y) = C_3 e^{m3y} + C_4 e^{m4y} + A_1 + A_2 e^{m1y} + A_3 e^{m2y} \quad (30)$$
$$U_{20}(y) = C_{11} e^{m11y} + C_{12} e^{m12y} + E_1 + E_2 e^{m9y} + E_3 e^{m10y} \quad (31)$$
$$\theta_{10}(y) = C_1 e^{m1y} + C_2 e^{m2y} \quad (32)$$
$$\theta_{20}(y) = C_9 e^{m9y} + C_{10} e^{m10y} \quad (33)$$
$$U_{11}(y) = C_7 e^{m7y} + C_8 e^{m8y} + D_1 e^{m3y} + D_2 e^{m4y} + D_3 e^{m1y} + D_4 e^{m2y} + D_5 e^{m5y} + D_6 e^{m6y} \quad (34)$$
$$U_{21}(y) = C_{15} e^{m15y} + C_{16} e^{m16y} + G_1 e^{m11y} + G_2 e^{m12y} + G_3 e^{m9y} + G_4 e^{m10y} + G_5 e^{m13y} + G_6 e^{m14y} \quad (35)$$
$$\theta_{11}(y) = C_5 e^{m5y} + C_6 e^{m6y} + B_1 e^{m1y} + B_2 e^{m2y} \quad (36)$$
$$\theta_{21}(y) = C_{13} e^{m13y} + C_{14} e^{m14y} + F_1 e^{m9y} + F_2 e^{m10y} \quad (37)$$

where

$$C_1 = \frac{L_6 C_{10} - L_3}{L_4}, C_2 = L_1 - C_1 e^{m1-m2}$$
$$C_3 = -C_4 e^{m4-m3} - Q_7, C_4 = \frac{Q_{16} - C_{12} Q_{15}}{Q_{10}}, C_5 = -C_6 e^{m6-m5} L_{15}$$
$$C_6 = \frac{L_{24} - L_{23} C_{14}}{L_{18}}, C_7 = -C_8 e^{m8-m7} - Q_{23}, \; C_8 = \frac{Q_{32} - C_{16} Q_{31}}{Q_{26}}$$
$$C_9 = -C_{10} e^{m9-m10}, C_{10} = \frac{L_2 L_3 - L_1 L_4}{L_2 L_6 - L_4 L_5}, C_{11} = -C_{12} e^{m11-m12} - Q_{12}$$
$$C_{12} = \frac{Q_{10} Q_{14} - Q_8 Q_{16}}{Q_{10} Q_{13} - Q_8 Q_{15}}, C_{13} = -C_{14} e^{m13-m14} - L_{20}$$

$$C_{14} = \frac{L_{22}L_{18} - L_{16}L_{24}}{L_{21}L_{18} - L_{16}L_{23}}, C_{15} = -C_{16}e^{m15-m16} - Q_{28}$$

$$C_{16} = \frac{Q_{26}Q_{30} - Q_{24}Q_{32}}{Q_{26}Q_{29} - Q_{24}Q_{31}}$$

$$L_1 = \frac{1}{e^{m2}}, L_2 = 1 - e^{(m1-m2)}, L_3 = \frac{m2}{e^{m2}}, L_4 = m1 - m2e^{m1-m2}$$

$L_5 = 1 - e^{m9-m10}, L_6 = \beta_{m_{10}} - \beta_{m_9}e^{m9-m10}, L_7 = B_1e^{m1} + B_2e^{m2},$
$L_8 = F_1e^{-m9} + F_2e^{-m10}, L_9 = B_1 + B_2, L_{10} = F_1 + F_2, L_{11} = L_{10} - L_9, L_{12} =$
$m_1B_1 + m_2B_2, L_{13} = \beta_{m_9F_1} + \beta_{m_{10}F_2}, L_{14} = L_{13} - L_{12}, L_{15} = \frac{L_7}{e^{m5}},$
$L_{20} = \frac{L_8}{e^{-m13}}, L_{21} = e^{m15-m14} - 1, L_{22} = L_{17} - L_{20}, L_{23} = \beta_{m_{13}}e^{m15-m14} - \beta_{m_{14}}, L_{24} = L_{19} -$
$\beta_{m_{13}}L_{20},$
$Q_1 = A_1 + A_2e^{m1} + A_3e^{m2}, Q_2 = E_1 + E_2e^{-m9} + E_3e^{-m10},$
$Q_3 = A_1 + A_2 + A_3, Q_4 = E_1 + E_2 + E_3, Q_5 = Q_4 - Q_3,$
$Q_6 = \alpha m9E_2 + \alpha m10E_3 - m1A_2 - m2A_3, Q_7 = \frac{Q_1}{e^{m3}}, Q_8 = 1 - e^{m4-m3},$
$Q_9 = Q_5 + Q_7, Q_{10} = m4 - m3\, e^{m4-m3}, Q_{11} = Q_6 + m3Q_7,$
$Q_{12} = \frac{Q_2}{e^{-m11}}, Q_{13} = e^{m11-m12} - 1, Q_{14} = Q_9 - Q_{12},$
$Q_{15} = \alpha m11e^{m11-m12} - \alpha m12, Q_{16} = Q_{11} - \alpha m11Q_{12},$
$Q_{17} = D_1e^{m3} + D_2e^{m4} + D_3e^{m1} + D_4e^{m2} + D_5e^{m5} + D_6e^{m6},$
$Q_{18} = G_1e^{-m11} + G_2e^{-m12} + G_3e^{-m9} + G_4e^{-m10} + G_5e^{-m13} + G_6e^{-m14},$
$Q_{19} = D_1 + D_2 + D_3 + D_4 + D_5 + D_6,$
$Q_{20} = G_1 + G_2 + G_3 + G_4 + G_5 + G_6,$
$Q_{21} = m3D_1 + m4D_2 + m1D_3 + m2D_4 + m5D_5 + m6D_6,$
$Q_{22} = \alpha m11G_1 + \alpha m12G_2 + \alpha m9G_3 + \alpha m10G_4 + \alpha m13G_5 + \alpha m14G_6,$
$Q_{23} = \frac{Q_{17}}{e^{m7}}, Q_{24} = 1 - e^{m8-m7}, Q_{25} = Q_{20} - Q_{19} + Q_{23}$
$Q_{26} = m8 - m7e^{m8-m7}, Q_{27} = Q_{22} - Q_{21} + m7Q_{23}, Q_{28} = \frac{Q_{18}}{e^{-m15}},$
$Q_{29} = e^{m15-m16} - 1, Q_{30} = Q_{25} - Q_{28}, Q_{31} = \alpha m15e^{m15-m116} - \alpha m16,$
$Q_{32} = Q_{27} - \alpha m15Q_{28},$
$A_1 = -\frac{P}{M^2}, A_2 = -\frac{GrC_1}{m_1^2 - m_1 + M^2}, A_3 = -\frac{GrC_1}{m_2^2 - m_2 + M^2},$
$E_1 = \frac{eP}{d}, E_2 = -\frac{fC_9}{m_9^2 - Cm_9 - d}, E_3 = -\frac{fC_{10}}{m_{10}^2 - Cm_{10} - d},$
$D_1 = \frac{C_3m_3}{m_3^2 - m_3 - a}, D_2 = \frac{C_4m_4}{m_4^2 - m_4 - a}, D_3 = \frac{A_2m_1 + B_1}{m_1^2 - m_1 - a}, D_4 = \frac{A_3m_2 + B_2}{m_2^2 - m_2 - a},$
$D_5 = \frac{C_5}{m_5^2 - m_5 - a}, D_6 = \frac{C_6}{m_6^2 - m_6 - a}, G_1 = \frac{C\,C_{11}m_{11}}{m_{11}^2 - Cm_{11} - h}, G_2 = \frac{C\,C_{12}m_{12}}{m_{12}^2 - Cm_{12} - h},$
$G_3 = \frac{C\,E_2m_9 - fF_1}{m_9^2 - Cm_9 - a}, G_4 = \frac{C\,E_3m_{10} - fF_2}{m_{10}^2 - Cm_{10} - h}, G_5 = \frac{-FC_{13}}{m_{13}^2 - Cm_{13} - h}, G_5 = \frac{-FC_{14}}{m_{14}^2 - Cm_{14} - h},$
$B_1 = \frac{PrC_1m_1}{m_1^2 - Prm_1 - b}, B_2 = \frac{PrC_2m_2}{m_2^2 - Prm_2 - b}, F_1 = \frac{gC_9}{m_9^2 - gm_9 + k}, F_2 = \frac{gC_{10}}{m_{10}^2 - gm_{10} + k},$

$$m_1 = \Pr + \frac{\sqrt{Pr^2+4f}}{2}, m_2 = \Pr - \frac{\sqrt{Pr^2+4f}}{2}, m_3 = \frac{1+\sqrt{1-4m^2}}{2},$$
$$m_4 = \frac{1-\sqrt{1-4m^2}}{2}, m_5 = \Pr + \frac{\sqrt{Pr^2+4b}}{2}, m_6 = \Pr - \frac{\sqrt{Pr^2+4b}}{2}, m_7 = \frac{1+\sqrt{1+4a}}{2},$$
$$m_8 = \frac{1-\sqrt{1+4a}}{2}, m_9 = \frac{g+\sqrt{g^2+4}}{2}, m_{10} = \frac{g-\sqrt{g^2+4}}{2}, m_{11} = \frac{C+\sqrt{C^2+4d}}{2}, m_{12} = \frac{C-\sqrt{C^2+4d}}{2},$$
$$m_{13} = \frac{g+\sqrt{g^2-4k}}{2}, m_{14} = \frac{g-\sqrt{g^2-4k}}{2}, m_{15} = \frac{C+\sqrt{C^2+4h}}{2}, m_{16} = \frac{C-\sqrt{C^2+4h}}{2}$$

The coefficient of the skin friction at the upper plate is given by

$$\tau_u = [(\frac{du_{10}}{dy})_{y=1} + \in e^{i\omega t}(\frac{du_{11}}{dy})_{y=1}] \quad (38)$$

while the coefficient of the skin friction at the lower plate is given by

$$\tau_l = [(\frac{du_{20}}{dy})_{y=-1} + \in e^{i\omega t}(\frac{du_{21}}{dy})_{y=-1}] \quad (39)$$

In addition, the rate of heat transfer in terms of Nusselt number at the upper plate is defined by

$$Nu_u = -[(\frac{d\theta_{10}}{dy})_{y=1} + \in e^{i\omega t}(\frac{d\theta_{11}}{dy})_{y=1}] \quad (40)$$

and the Nusselt number at the lower plate is given by

$$Nu_l = -[(\frac{d\theta_{20}}{dy})_{y=-1} + \in e^{i\omega t}(\frac{d\theta_{21}}{dy})_{y=-1}] \quad (41)$$

RESULTS AND DISCUSSION

In the present analysis, we study the effect of radiation on unsteady MHD free convective two immiscible fluid flows through a horizontal channel. The upper channel (Region I) is assumed to be porous while the lower (Region II) is clear. The partial differential equations governing the fluid flow are transformed into ordinary differential equations and solved analytically. The solutions are presented graphically for different values of the fluid parameters.

Figure 2 depicts the effect of Grashof number, Gr on velocity. It is observed that as Gr increases, velocity also increase, that is Grashof number and velocity are directly proportional to each other. Figure 3 shows the effect of Prandtl number on temperature. The prandtl defines the ratio of momentum diffusivity to the temperature diffusivity. It is noticed that as Pr decreases, the temperature props up. This is because, if Pr increases, the thermal diffusivity decreases and these

phenomena lead to decreasing of energy ability that reduces the thermal boundary layer. Figures 4 and 5 respectively show the effect of conductivity ratio on velocity and temperature. It is found that as the conductivity ratio decreases, both the velocity and the temperature increases. Figure 6 depicts the effect of viscosity ratio on velocity .it was observed that as viscosity ratio decreases, the velocity increases across the two regions.

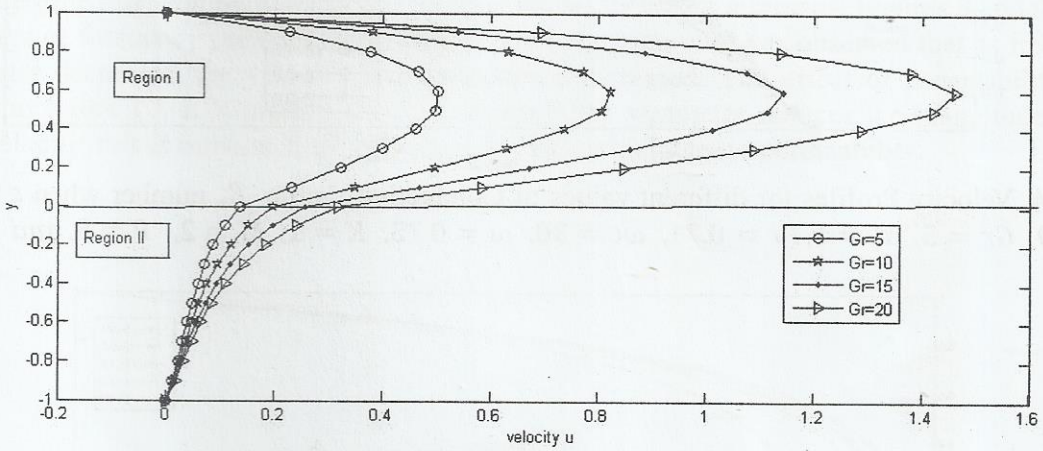


Figure 2. Velocity Profiles for different values of Grashof number when $\varepsilon = 0.1,\ A = 0.9,\ Pr = 0.71,\ \alpha_1 = 1,\ \beta_1 = 1,\ \omega t = 30,\ \omega = 0.75,\ K = 1,\ M = 2,\ P = 5, and\ F = 1$

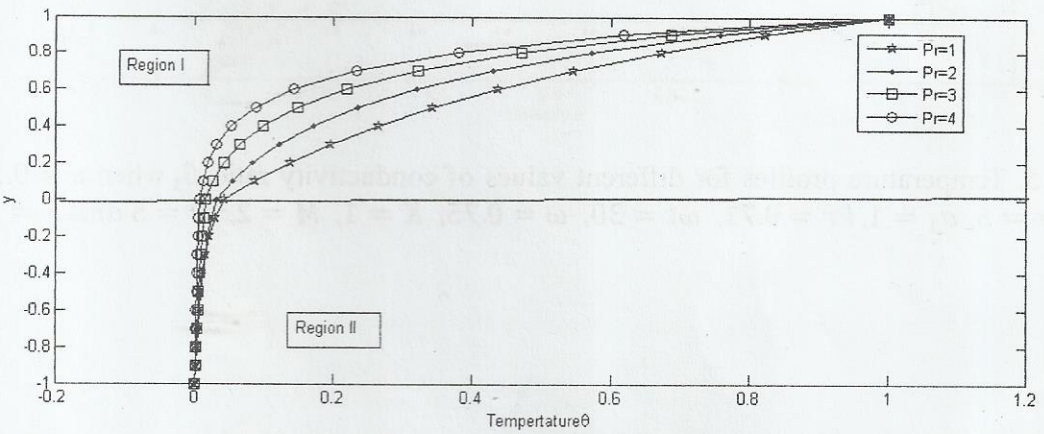


Figure 3. Temperature profiles for different values of Prandtl number when $\varepsilon = 0.1,\ A = 0.9,\ Gr = 5,\ \alpha_1 = 1,\ \beta_1 = 1,\ \omega t = 30,\ \omega = 0.75,\ K = 1,\ M = 2,\ P = 5, and\ F = 1$

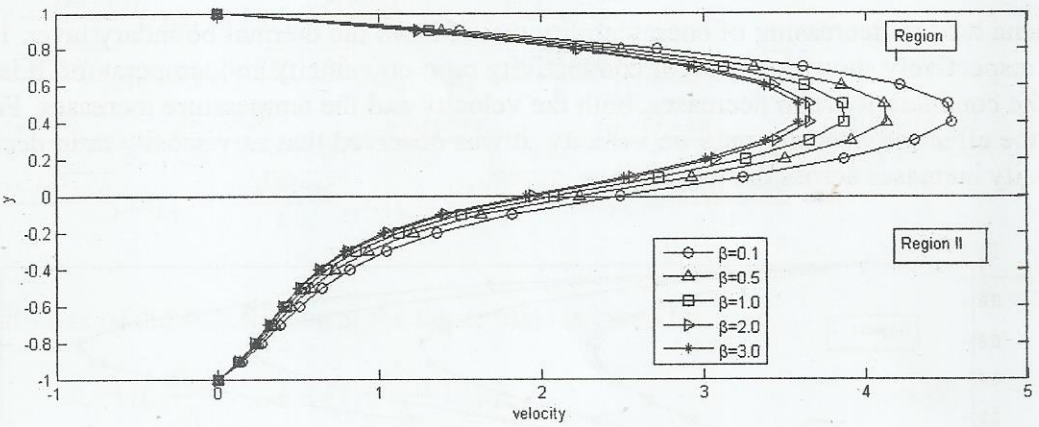


Figure 4. Velocity Profiles for different values of Conductivity ratio β_1 number when $\varepsilon = 0.1$, $A = 0.9$, $Gr = 5$, $\alpha_1 = 1, Pr = 0.71$, $\omega t = 30$, $\omega = 0.75$, $K = 1$, $M = 2$, $P = 5, and\ F = 1$

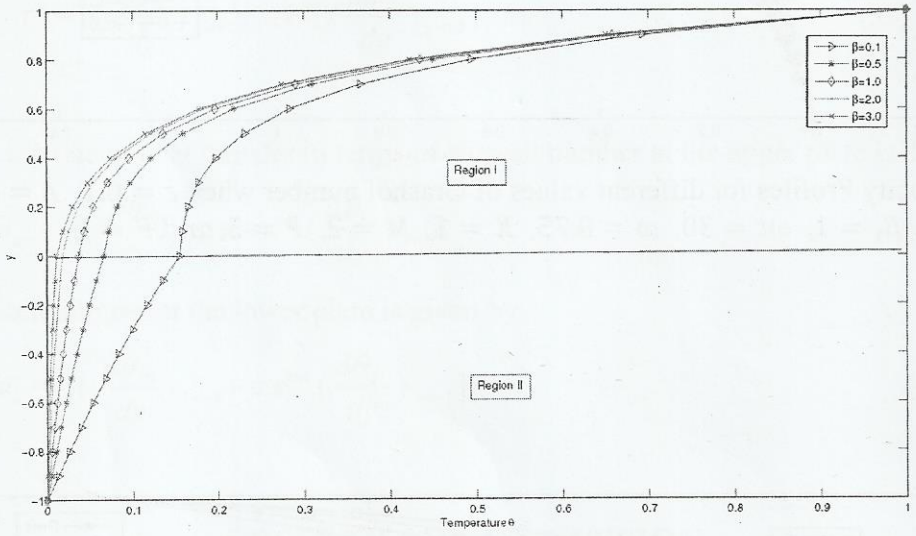
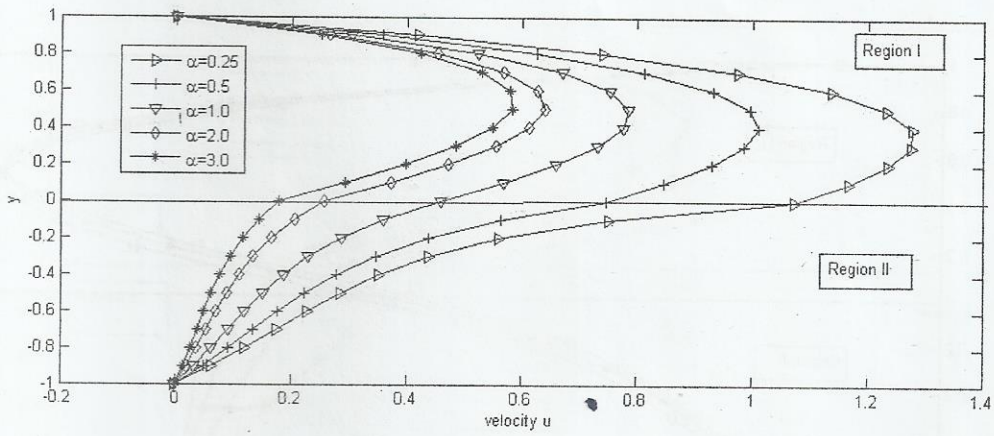


Figure 5. Temperature profiles for different values of conductivity ratio β_1 when $\varepsilon = 0.1$, $A = 0.9$, $Gr = 5$, $\alpha_1 = 1, Pr = 0.71$, $\omega t = 30$, $\omega = 0.75$, $K = 1$, $M = 2$, $P = 5\ and\ F = 1$

Figure 6. Velocity profiles for different values of viscosity ratio α_1 when $\varepsilon = 0.1,\ A = 0.9,\ Gr = 5,\ \beta_1 = 1, Pr = 0.71,\ \omega t = 30,\ \omega = 0.75,\ K = 1,\ M = 2,\ P = 5\ and\ F = 1$

The influence of the thermal radiation parameter F was shown in figure 7. It is clear that thermal radiation enhances the temperature in the boundary region. Thus, radiation should be minimized to enhance better cooling environment. The radiation parameter F defines the relative contribution of conduction heat transfer to thermal radiation transfer. It is found that as F decreases, the temperature profiles and thermal boundary layers increases. Figures 8 and 9 depict the effect of frequency parameter on velocity and temperature. It was observed that as frequency parameter decreases, the velocity and temperature increased. The effect of permeability K is shown in figure 10. It is noticed that the permeability parameter reduces the magnitude of the fluid velocity, this is because fluid velocity decreases with increase permeability.



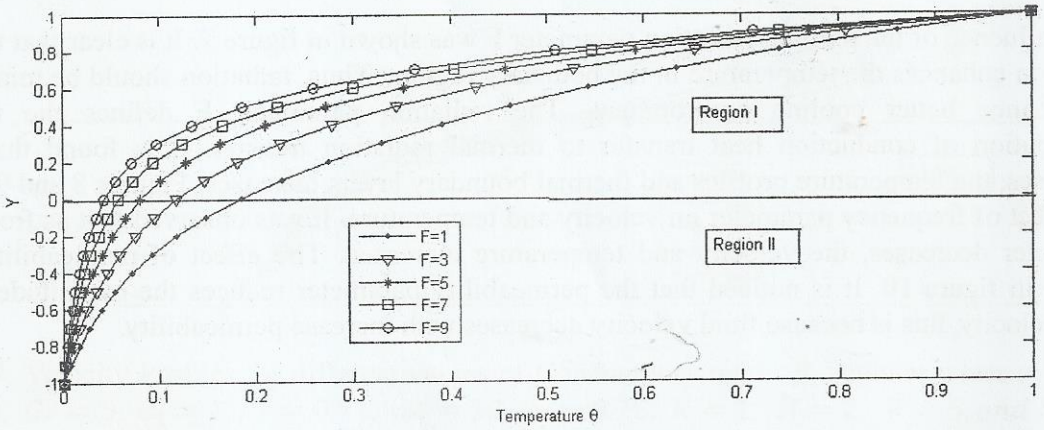


Figure 7. Temperature profiles for different values of thermal radiation parameters F when $\varepsilon = 0.1,\ A = 0.9,\ Gr = 5, \alpha_1 = 1,\ \beta_1 = 1, Pr = 0.71,\ \omega t = 30,\ \omega = 0.75,\ K = 1,\ M = 2,$ $and\ P = 5$

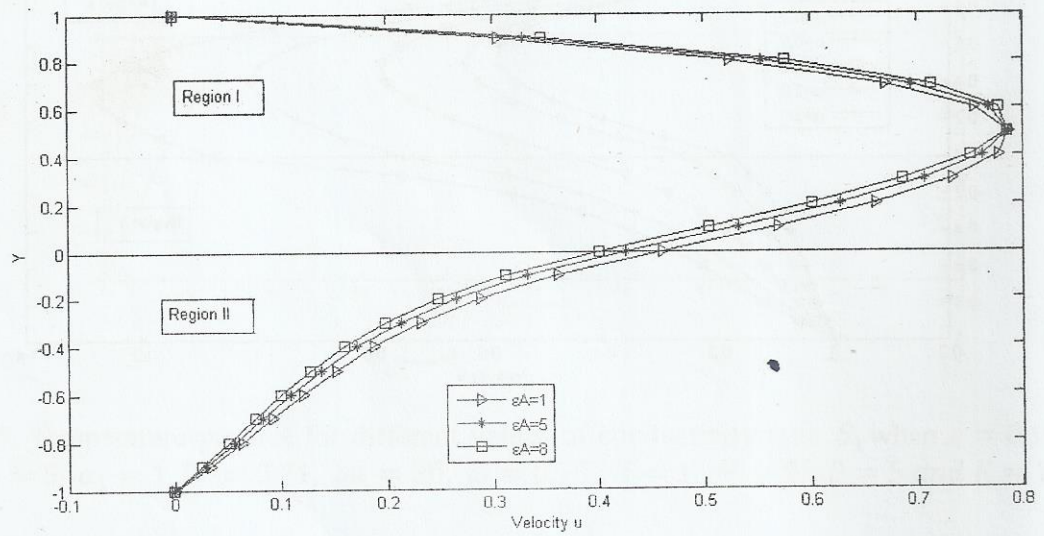


Figure 8. Velocity profiles for different values frequency parameters εA when $Gr = 5, \alpha_1 = 1,\ \beta_1 = 1, Pr = 0.71,\ \omega t = 30,\ \omega = 0.75,\ K = 1,\ M = 2,\ P = 5\ and\ F = 1$

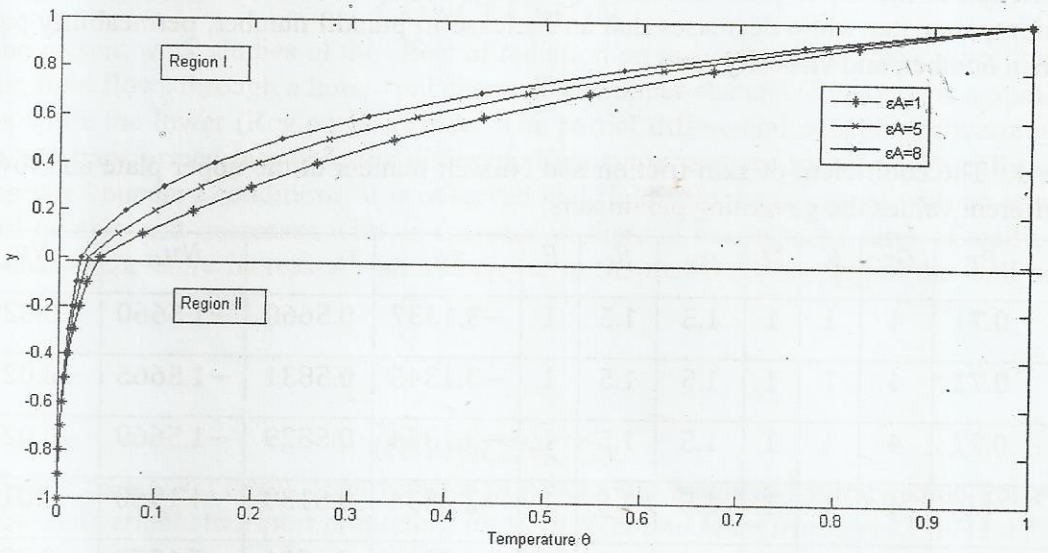


Figure 9. Temperature profiles for different values frequency parameters εA when $Gr = 5, \alpha_1 = 1,\ \beta_1 = 1, Pr = 0.71,\ \omega t = 30,\ \omega = 0.75,\ K = 1,\ M = 2,\ P = 5\ and\ F = 1$

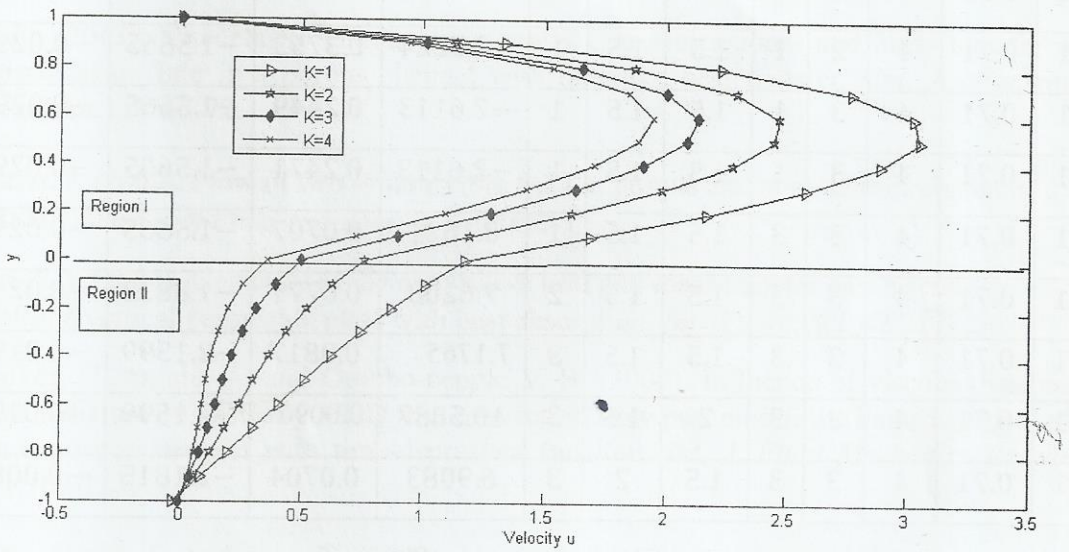


Figure 10. Velocity profiles for different values of porous permeability K when $\varepsilon = 0.1, A = 0.9, Gr = 5, \alpha_1 = 1, \beta_1 = 1, Pr = 0.71, \omega t = 30, \omega = 0.75,\ M = 2,\ P = 5\ and\ F = 1$

Table 1 shows that the coefficient of the skin friction at the upper plate increases with an increase in Prandtl number, permeability parameter, Hartman number, and viscosity ratio, but decreases with an increase in Grashof number, conductivity and radiation parameter. The coefficient of the

skin friction at the lower plate increases with an increase in Grashof number, conductivity and radiation parameter while decreases due an increase in prandtl number, permeability parameter, Hartman number, and viscosity ratio.

Table 1. The coefficient of skin friction and Nusselt number at the upper plate and lower plate for different values the governing parameters.

ε	Pr	Gr	K	M	α_1	β_1	F	τ_U	τ_L	Nu_U	Nu_L
0	0.71	4	1	1	1.5	1.5	1	−3.1337	0.5660	−1.5660	−0.0292
0.01	0.71	4	1	1	1.5	1.5	1	−3.1345	0.5831	−1.5665	−0.0292
0.02	0.71	4	1	1	1.5	1.5	1	−3.1354	0.5829	−1.5669	−0.0292
0.01	4.0	4	1	1	1.5	1.5	1	−2.2534	0.5139	−4.1980	−0.0180
0.01	7.0	4	1	1	1.5	1.5	1	−1.8016	0.4506	−7.1350	−0.0019
0.01	0.71	5	1	1	1.5	1.5	1	−3.6135	0.6212	−1.5665	−0.0292
0.01	0.71	6	1	1	1.5	1.5	1	−4.0925	0.6593	−1.5665	−0.0292
0.01	0.71	4	2	1	1.5	1.5	1	−2.8224	0.3753	−1.5665	−0.0292
0.01	0.71	4	3	1	1.5	1.5	1	−2.6113	0.2449	−1.5665	−0.0292
0.01	0.71	4	3	1	1.5	1.5	1	−2.6113	0.2471	−1.5665	−0.0292
0.01	0.71	4	3	3	1.5	1.5	1	8.1621	0.0707	−1.5665	−0.0292
0.01	0.71	4	3	3	1.5	1.5	2	7.6208	0.0771	−1.8832	−0.0238
0.01	0.71	4	3	3	1.5	1.5	3	7.1765	0.0817	−2.1599	−0.0197
0.01	0.71	4	3	3	2	1.5	3	10.5887	0.0096	−2.1599	−0.0197
0.01	0.71	4	3	3	1.5	2	3	6.9083	0.0704	−2.1815	−0.0084

Furthermore, the Nusselt number at the upper plate decreases with an increase in prandtl number, viscosity ratio, thermal conductivity ratio and radiation parameter but remained constant for an increase in Grashof number, permeability parameter and Hartman number, whereas at the lower plate, the Nusselt number increases with an increase in the prandtl number, viscosity ratio, thermal conductivity ratio and radiation parameter but remained constant for an increase in Grashof number, permeability parameter and Hartman number.

CONCLUSION

The present work studies of the effect of radiation on unsteady MHD free convective two immiscible fluid flows through a horizontal channel. The upper channel (Region I) is assumed to be porous while the lower (Region II) is clear. The partial differential equations governing the fluid flow are transformed into ordinary differential equations and are solved analytically under the appropriate boundary conditions. It is observed that fluid velocity increases with an increase in Grashof number and decreases with an increase in thermal conductivity ratio. Finally, it is observed that temperature increases with a decrease in thermal radiation within the wall of the channel.

REFERENCES

Alireza, S. and Sahai, V. (1990). Heat transfer in developing magneto hydrodynamic Poiseuille flow and variable transport properties. *Int. J. of Heat and Mass transfer*. 33: 1711-1720.

Dada, M. S. (2012). Radiation, Dissipation and MHD effects on fluid flow past an impulsively started vertical Newtonian heating plate. *International Journal of Engineering, Tome X – Fascicule*. 3:197-202.

Das, U. J. (2013). Viscoelastic effects on unsteady two dimensional and mass transfer of a viscoelastic fluid in a porous channel with radiative heat transfer. *Int. J. of scientific research*. (5): 67-72.

Chamkha, A. J. (2000). Flow of two –immiscible fluid in porous and non –porous channels, *ASME Journal of Fluid engineering*. 122:117-124.

Chamkha, A. J. (2003). Unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable plate with heat absorption. *Int. J. Eng. Sci.* 42:217-230.

Israle-cookey, C., Ogulu, A. and Omubo-pepple V. B. (2004). Influence of viscous dissipation and radiation on unsteady MHD free convection flow past on infinite heated vertical plate in a porous medium with time-dependent function. *Int. J. Fluid Mechanics Research*. 31:107-123.

Kumar, N., Gupta, S. and Jain, T. (2009). Unsteady MHD and heat transfer of two viscous immiscible fluids through a porous medium in a horizontal channel. *J. Damghan Universities of Basic Science.s* 2(1): 87-98.

Kumar, N., Jain, T. and Gupta, S. (2012). Unsteady MHD Free convective flow through porous medium sandwiched between vision fluids. *Int. J. Energy and Technology*. 4(27): 1-11.

Malashetty, M. S. and Leela, V. (1991). Magneto hydrodynamic heat transfer two fluid flows. Proceeding of ASME HTD. 159.

Malashetty, M. S. and Umavathi, J. C. (1997). Two phase magneto hydrodynamic flow and heat transfer in an inclined channel. *Int. J. Multiphase flow.* 23: 545-560.

Prakash, J. and Ogulu, A. (2006). Unsteady two dimensional flow of radiating and chemically reacting MHD fluid with time dependent suction. *Indian J. pure and applied physics.* 44:805-810

Reddy, M. G. and Reddy, N. B. (2009). Radiation and mass transfer effect on an unsteady MHD free convection flow past a heated vertical porous plate with viscous dissipation. *Int. J. of Appl. Math. and Mech.* 6 (6): 96-110.

Simon, D. and Shagaiya, Y. D. (2013). Convective flow of two immiscible fluids and heat transfer with porous along inclined channel with pressure gradient. *Int. J. of Engineering and Science.* 2(4):12-18.

Umavathi, J. C. Chamkha, A. J. and Abdul Mateen. (2004). Oscillatory flow and heat transfer in to immiscible fluids. *Int. J. Fluid Mechanics Research.* 31:107-123.

Umavathi, J. C., Chamkha, A. J., Abdul Mateen, and Prathap Kumar, J. (2008). Unsteady magneto-hydrodynamic two fluid flow and heat transfer in a horizontal channel. *J. of Heat and Technology.* 26(2): 121-133.

Umavathi, J.C., Liu, I. C., Prathap-kumar, J. and Shaik-meera D. (2010). Unsteady flow and heat transfer of porous media Sandwiched between vision fluids, *Applied Mathematics and mechanics.* 31(12): 1497-1516.

Vafai, K. and Kim, S. J. (1990). Fluid mechanics of the interface region between a porous medium and a fluid layer; an exact solution. *International journal of heat and fluid flow.* 2 (3): 254-256.