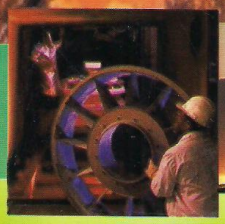
NSE TECHNICAL TRANSACTIONS

ISSN 11195363

A TECHNICAL PUBLICATION OF THE NIGERIAN SOCIETY OF ENGINEERS







APRIL - JUNE, 2011 VOL.46, NO.2

Supported by:

Education Trust Fund

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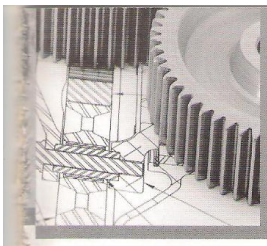
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Development and Assessment of a Quintic Polynomial Model for the Prediction of Maximum Daily Rainfall in Ilorin, Nigeria

O.O. Olofintoye and A.W. Salami

Department of Civil Engineering, University of Ilorin,

P.M.B 1515, Ilorin, Nigeria**geotoseen@yahoo.co.uk**

Abstract

The main purpose of this study is to compare parametric density functions with a nonparametric function to estimate maximum daily rainfall of Ilorin, Nigeria. The parametric density functions include, Normal, Lognormal, Pearson III, Log-Pearson III, Gumbel, and Log-Gumbel probability functions. The nonparametric approach is a Quintic regression polynomial. The probability distribution functions were fitted to the annual duration series of the maximum daily rainfall, then a Quintic regression polynomial was also fitted using the method of Least Squares. Results showed that the Quintic polynomial model predicts maximum daily precipitation much better than other parametric methods. Thus a Quintic regression polynomial can be used as an alternative approach for precipitation frequency analysis.

Keywords: *Parametric functions, Nonparametric function, Normal, Lognormal, Pearson III, Log-Pearson III, Gumbel, Log-Gumbel, Quintic regression polynomial, Least Squares.*

Introduction

Rainfall is a form of precipitation which descends in droplets, and is the commonest form of precipitation in the tropics. The existence of man and other living creatures on the earth is greatly dependent on rainfall occurrence, distribution, amount, frequency and its environmental impact. As a consequence of this, the future occurrence of rainfall deserves more attention than the present. Hence, it is of paramount importance to devise a way by which the future occurrences can be predicted within certain degree of freedom (Ogunlela and Oyewole, 2010).

The importance of rainfall to hydrology, civil engineering, agriculture, and many other facets of human endeavors cannot be over emphasized. Thus, much efforts have been made in the study and understanding of the phenomena behind the rainfall process, so as to be able to predict future rainfall events, and to guard against rainfall events that may have devastating effects on the environment. Extreme rainfall events have caused catastrophe in several parts of the world. High rainfall intensity over an extended period have caused flooding in some areas resulting in loss of life and property whereas extremely low rainfall in some other places have resulted in drought, and famine (Olofintoye, 2007). Thus, it is imperative to analyse previous rainfall data to generate basic information on the future occurrence and its impact on the environment (Ogunlela and Oyewole, 2010). An important problem in water resources engineering therefore, is the estimation of the maximum daily rainfall for a certain return period using frequency analysis.

In frequency analysis, an assumed probability distribution is fitted to the available data to estimate



the magnitude corresponding to return periods and the appropriate distribution models that represent the data is chosen. The choice of the probability distribution model is almost arbitrary as no physical basis is available to rationalize the use of any particular function and the search for the proper distribution function has been the subject of several studies (Wilson, 1969; Warren et al, 1972; Viessman et al, 1989; Anifowose and Salami 2008; Olofintoye et al, 2009). For instance Yusuf and Salami (2009) in a study to determine the effect of Jebba dam on hydro-Meteorological variables within the Niger River Basin, fitted various probability distribution models to the annual mean values of the hyro-meteorological variables (reservoir inflow and outflow, rainfall, humidity, evaporation, temperature, sunshine and wind speed) of the area, to determine the model that is most appropriate for the prediction of these variables. The analysis indicated that the Gumbel probability distribution models best fit Reservoir inflow, outflow and humidity at post dam stage. It was also deduced that the maximum and minimum temperatures, evaporation and sunshine during dam construction and post dam stages were best fitted by the Gumbel probability distribution model. However, the Log-Pearson distribution model was the best-fit model for rainfall and wind speed at both dam construction and post dam stages, while Log-Normal best fit humidity at dam construction stage. Mahdavi et al, (2010) carried out a study to select the best frequency distribution to estimate average annual precipitation and assess the effects of data length on the selection of suitable distribution using 65 stations data of Mazandaran and Golestan provinces in Iran. Relative residual mean square (RMS) was used to determine the best-fitted distribution to the annual series and precipitation was estimated for different return periods. Relative frequency of first classes of fitted distributions showed that Normal and Pearson distributions fitness decreased and Gumbel distribution had more fitness with data series by increasing statistical period length. The best-fitted distribution is Pearson with 15-year data; Log-Pearson for 20, 25 and 30-year periods. Based on Moment method and total given scores, two-parameter normal distribution has the best fitness in all statistical periods. Also, Ogunlela (2001) in a stochastic analysis of rainfall event in Ilorin used probability distribution functions. He concluded that the Log-Pearson type III distribution best described the maximum daily rainfall data while the Normal distribution best characterized the maximum monthly rainfall for Ilorin. Hence, he suggested that the maximum daily rainfall may not be exponentially and Log-normally distributed.

Hydrologic processes are believed to be so complex that they can be interpreted and explained only in probabilistic sense, and that rainfall analysis like many other hydrologic processes is also complex that it can be explained, interpreted, analysed and used in forecasting only in probability sense, since its occurrence and distribution are a function of nature (Ogunlela and Oyewole, 2010; Viessman et al, 1989). Thus probability distribution functions have hitherto been used to model various hydrologic processes. Recent researches has however revealed that nonparametric functions can be used as an alternative approach for precipitation frequency analysis. For instance, Jou et al, (2009) in a study compared parametric density functions (Normal, two and three parameter Log-normal, two parameter Gamma, Pearson, Log-Pearson type III, and Gumbel extreme value type I) with nonparametric Fourier series to estimate annual precipitation for five rain gauge stations in Iran. The result showed that the Fourier series predict annual precipitation much better than other parametric methods based on a Mean Relative Deviation (MRD) and Mean Square Relative Deviation (MSRD) goodness-of-fit test. They thus concluded that the Fourier series can be used as a better alternative approach for precipitation frequency analysis. Jou et al, (2009) noted that these parametric methods have been successfully applied in many cases, but, have some disadvantage because of not fitting to the observed data very well or diverting from the extreme tails. Also, some other problems in selection of these methods, are difficulties to chose the best distribution function and estimation of their parameters (particularly for skewed data). Jou et al, (2009) also reported that Guo in 1991 proposed a nonparametric variable kernel estimation model which provides an alternative way in flood quantile estimation when historical floods data are available, and was shown that the nonparametric kernel estimator fitted the real data points closer than its parametric counterparts.

In mathematics, a quintic equation is a polynomial equation of degree five (Wikipedia, 2010). Polynomials may be used to approximate functions and model processes, the basis for this being the Weiestrass approximation theorem which states that "If a function f(x) is continuous on a finite interval [a, b], there exists a polynomial $P_n(x)$ of degree n such that $|f(x) - P_n(x)| <$ ERROR, throughout the interval [a, b], for any given ERROR > 0 (Gerald and Wheatley, 2006). Polynomials have been used to model processes in science, engineering, and more importantly, hydrology. For example Zaw and Naing, (2009) applied Multivariable polynomial regression (MPR) to implement a precipitation forecast model over Myanmar. The model output

result is station wide monthly and annual rainfall amount during summer monsoon season. The proposed model results were compared with the result produced by multiple linear regression model (MLR). From the experimental results, it was observed that using MPR method achieves closer agreement between actual and estimated rainfall than using MLR.

This study is an assessment of a non parametric quintic polynomial regression function, fitted to the annual duration series of the maximum daily rainfall of Ilorin, Nigeria, using the method of Least Squares, against six parametric probability distribution functions (Normal, Lognormal, Pearson III, Log-Pearson III, Gumbel, and Log-Gumbel).

Materials and Method

Ilorin is the state capital of Kwara state, Nigeria, in the tropical hinterland climatic zone. It is located on latitude 829 N, and longitude 435 E, with a relief of about 305m above sea level. The average annual rainfall is about 1234.4mm. The southwest monsoon prevails during the wet season while the northeast harmattan wind prevails during the dry season. The mean annual temperature is around 27 C. Relative humidity is higher during the wet season.

prevails during the wet season while the northeast harmattan wind prevails during the dry season. The mean annual temperature is around 27 C. Relative humidity is higher during the wet season.

Data Collection

The daily rainfall data for Ilorin was collected from the Nigerian Meteorological Agency (NIMET), Oshodi Lagos, Nigeria. NIMET is the agency responsible for the measurement, control, and storage of rainfall and other meteorological data of major cities in Nigeria. The nature of data is rainfall depth (mm) recorded for everyday of the year. A total of 54 years (1952 – 2005) was used for the analysis.

Data Analysis

The annual maximum rainfall values were selected for each year to produce an annual duration series. Table 1 presents a summary of the statistics of the series.

Table 1: Summary of statistics for peak daily rainfall for Ilorin (1952 – 2005)

Mean value $\bar{x}$, (mm)	84.44
Standard deviation σ, (mm)	23.38
Skewness coefficient G	0.92
Median (mm)	79.90
Minimum (mm)	46.50
Maximum (mm)	160.5

The series was then ranked according to Weilbull's plotting position and the corresponding return periods were estimated. The series was evaluated with six methods of probability distribution functions (Gumbel (EVI type1), Log-Gumbel (LG), Normal (N), Log-Normal (LN), Pearson III, and Log-Pearson type III (LP_3) and then fitted with a quintic polynomial function using the method of Least Squares. Goodness-of-fit tests were adopted for the selection of the best-fit model.

Evaluation of Probability Distribution Models

The probability distribution analysis was carried out in accordance with standard procedure (Wilson, 1969; Warren et al, 1972; Viessman et al, 1989; Mustapha and Yusuf, 1999). The mathematical expressions obtained for the various probability distribution functions are presented in Table 2.

Development of a Quintic Polynomial Model

A polynomial equation of degree 5 (quintic) was fitted to the data using the method of least squares. A general procedure for fitting data with a polynomial of degree n using the method of least-squares follows (Gerald and Wheatley, 2006):

Let Y_i represent an experimental value, and let y_i be a value from the equation:

$$y_i = a_0 + a_1 x_i + a_2 x_i^2 + \cdots + a_n x_i^n \qquad 1$$

Where $a_0 \ldots a_n$ are coefficients of the polynomial and x_i is a particular value of the variable assumed to be free of error. We wish to determine the best values for $a_0 \ldots a_n$ so that the y's predict the function values that correspond to x-values. Define the error of prediction as

$$e_i = Y_i - y_i \qquad 2$$

The method of least-squares suggests that the estimates of the regression coefficients be chosen so that the sum of the squared differences (S) between observed sample values Y_i and the estimated value y_i, is minimized (Gerald and Wheatley, 2006; Soong, 2004). ie.

$$S = e_1^2 + e_2^2 + \cdots + e_N^2 = \sum_{i=1}^{N} e_i^2$$

$$= \sum_{i=1}^{N} (Y_i - a_0 - a_1 x_i - a_2 x_i^2 - \cdots - a_n x_i^n)^2 \qquad 3$$

Should be a minimum. N is the number of (x, Y)-pairs. At the minimum for S, all the partial derivatives $\frac{\partial S}{\partial a_0}, \frac{\partial S}{\partial a_1}, \cdots, \frac{\partial S}{\partial a_n}$ vanish, and we write the $(n + 1)$ equations as follows:

$$\frac{\partial S}{\partial a_0} = 0 = \sum_{i=1}^{N} 2(Y_i - a_0 - a_1 x_i - a_2 x_i^2 - \cdots - a_n x_i^n)(-1)$$

$$\frac{\partial S}{\partial a_1} = 0 = \sum_{i=1}^{N} 2(Y_i - a_0 - a_1 x_i - a_2 x_i^2 - \cdots - a_n x_i^n)(-x_i) \qquad 4$$

$$\vdots \qquad \vdots$$

$$\frac{\partial S}{\partial a_n} = 0 = \sum_{i=1}^{N} 2(Y_i - a_0 - a_1 x_i - a_2 x_i^2 - \cdots - a_n x_i^n)(-x_i^n)$$

Dividing each equation by 2 and rearranging gives the $(n+1)$ normal equations:

$$a_0 N + a_1 \sum x_i + a_2 \sum x_i^2 + \cdots + a_n \sum x_i^n = \sum Y_i$$

$$a_0 \sum x_i + a_1 \sum x_i^2 + a_2 \sum x_i^3 + \cdots + a_n \sum x_i^{n+1} = \sum x_i Y_i$$

$$a_0 \sum x_i^2 + a_1 \sum x_i^3 + a_2 \sum x_i^4 + \cdots + a_n \sum x_i^{n+2} = \sum x_i^2 Y_i \quad (5)$$

$$\vdots \qquad\qquad \vdots$$

$$a_0 \sum x_i^n + a_1 \sum x_i^{n+1} + a_2 \sum x_i^{n+2} + \cdots + a_n \sum x_i^{2n} = \sum x_i^n Y_i$$

These equations are solved simultaneously to obtain the coefficients of the polynomial $a_0 \ldots a_n$

Fitting regression polynomials by solving large sets of linear equations is a tedious task, fortunately, commercial softwares like MATLAB and Mathematica are available for curve fitting. Herein, the MATLAB 2007 software was used to fit the quintic regression polynomial and to obtain the required polynomial equation. This equation is presented in Table 2.

Comparison of the Precipitation Models

The mathematical expressions obtained for each function were used to predict rainfall for different return periods and were used in performing the statistical tests (goodness-of-fit tests) for the selection of the best fit model. In this study, for comparison of nonparametric and parametric methods, the Mean Relative Deviation (MRD) and Mean Square Relative Deviation (MSRD) statistics were used to measure the goodness-of-fit of the developed models. These statistical terms are defined as follows (Jou et al, 2009):

$$MRD = \frac{1}{n}\sum_{i=1}^{n} \frac{|x_i - \hat{x}_i|}{x_i} \times 100 \quad (6)$$

$$MSRD = \frac{1}{n}\sum_{i=1}^{n} \left(\frac{x_i - \hat{x}_i}{x_i} \times 100\right)^2 \quad (7)$$

Where x and are the observed and estimatedmaximum daily precipitation respectively and n is the sample size. In general, low values of MRD and MSRD indicate good fit (Jou et al, 2009). MRD and MSRD values were computed for each developed model, and the model with the minimum values of MRD and MSRD was chosen as the best-fit model. The computed values of MRD and MSRD are presented in Table 2. Graphical plots (Figures 1 to 7) were also made to provide insight as to how well the models fit the data.

Results and Discussion

Results

Table 2 presents the developed model equations and the respective values of MRD and MSRD.

Table 2: Developed Model equations

SN	Model	Model Equation	MRD	MSRD
1	Normal	$R_e = 84.44 + 23.38K$	5.1429	41.6133
2	Lognormal	$Log(R_e) = 1.91 + 0.12K$	2.1899	8.1068
3	Pearson III	$R_e = 84.44 + 23.38K'$	1.9507	6.6136
4	Log Pearson III	$Log(R_e) = 1.91 + 0.12K'$	1.9573	6.4051
5	Gumbel	$R_e = 84.44 + 23.38(0.78Y_T - 0.45)$	2.0903	7.1862
6	Log Gumbel	$Log(R_e) = 1.91 + 0.12(0.78Y_T - 0.45)$	3.5036	22.7369
7	Quintic	$R_e = -1826.9x^5 + 4834.4x^4 - 4856.6x^3 + 2328.4x^2 - 599.7x + 160$ *or* $R_e = ((((4834.4 - 1826.9x)x - 4856.6)x + 2328.4)x - 599.7)x + 160$*	1.7187	5.0753

R_e – Estimated rainfall using a model
K – Normal variate obtained from Normal distribution tables for a given return period
K′ - Variate obtained from Pearson distribution tables for a given return period and skew coefficient
Y_T – Reduced variate computed as $Y_T = -\ln[-\ln(1 - {}^1/_T)]$ for the given return period T
x – Exceedence probability corresponding to a given return period T; ($x = {}^1/_T$).
* - polynomial model expressed in nested form

Figures 1 to 7 shows the plots of the various models.

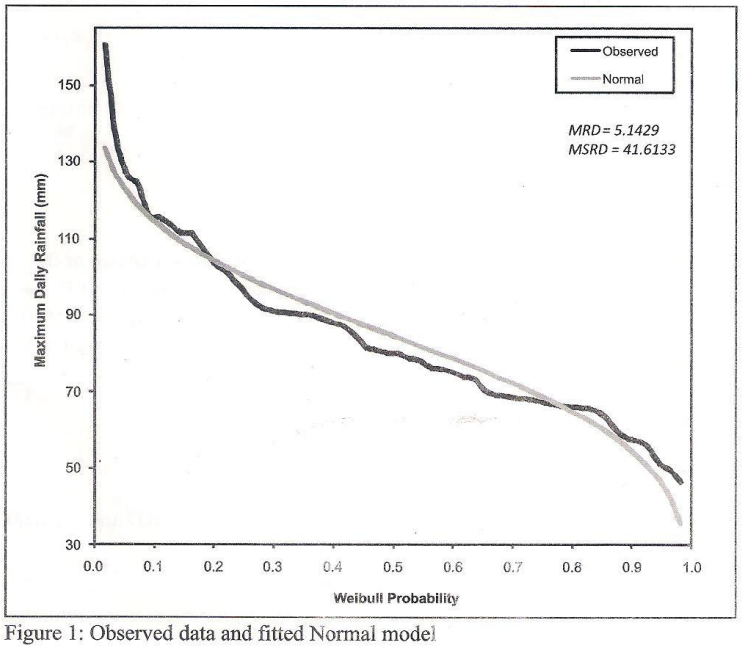


Figure 1: Observed data and fitted Normal model

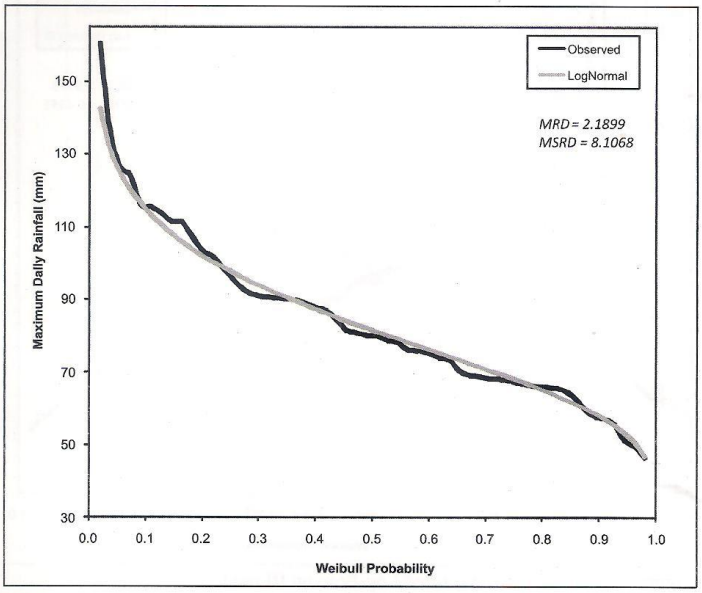


Figure 2: Observed data and fitted Lognormal model

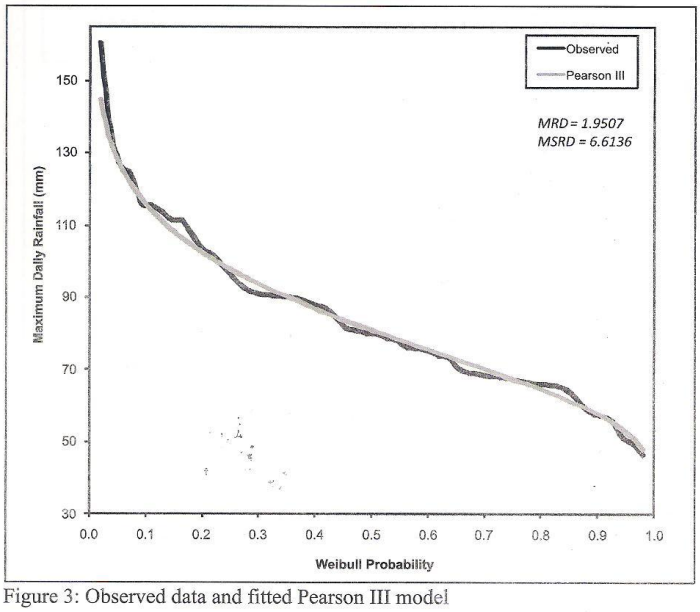


Figure 3: Observed data and fitted Pearson III model

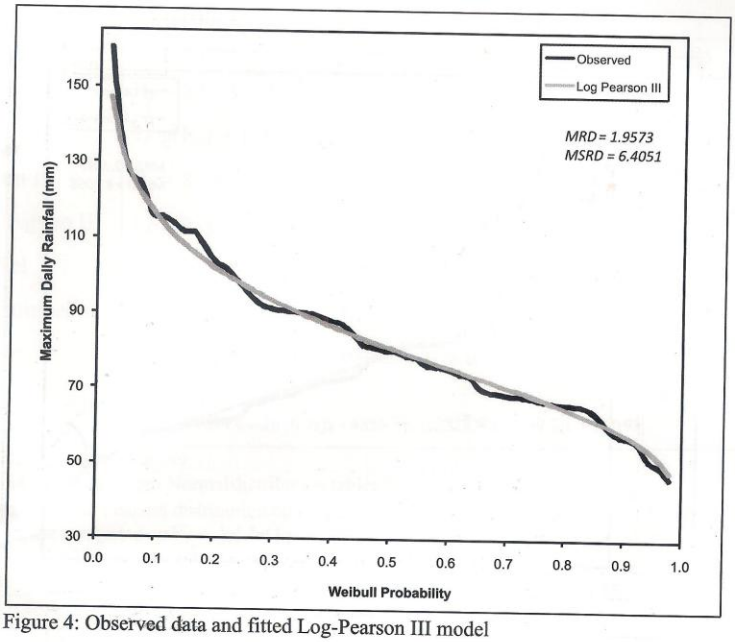


Figure 4: Observed data and fitted Log-Pearson III model

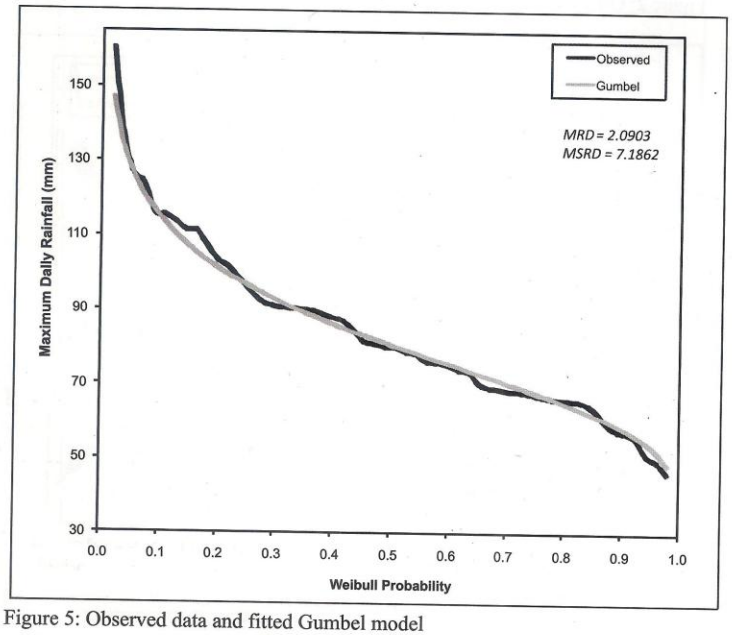


Figure 5: Observed data and fitted Gumbel model

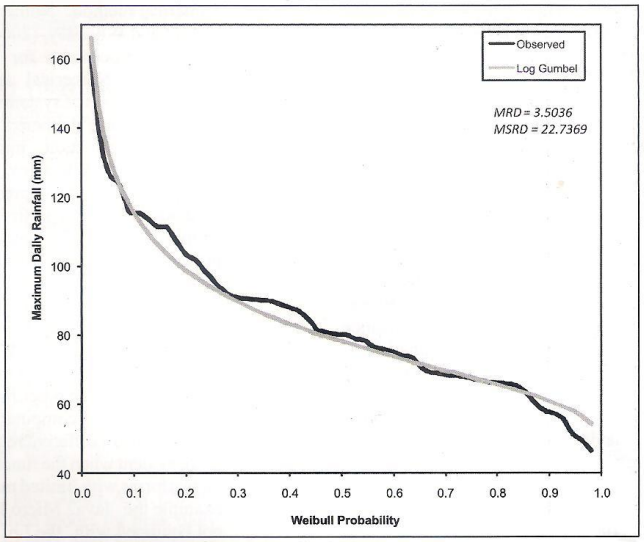


Figure 6: Observed data and fitted Log-Gumbel model

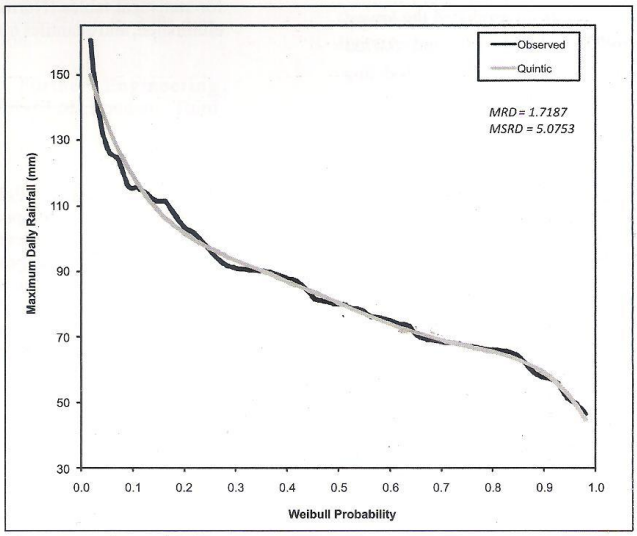


Figure 7: Observed data and fitted Quintic model

Discussion of results

In general, the developed models exhibit good fit to the data. The Normal distribution has been noted to provide a reasonable approximation in the central, but is inadequate at one or both tails of the distribution

(Viessman et al, 1989). This is in accord with the results obtained in this study as the Normal distribution exhibited the poorest fit as it produced the highest values of MRD and MSRD (5.1429 and 41.6133, respectively). Therefore the use of the Normal distribution for the prediction of maximum daily rainfall in Ilorin, Nigeria should be restricted.

Viessman et al, (1989) has noted that many hydrologic variables exhibit a marked right skewness partly due to the influence of natural phenomena. The computed skew coefficients from the observed data (0.92) revealed that the maximum daily rainfall distributions at Ilorin is positively skewed. Also, the 3-parameter Pearsonian distributions, which take into cognizance the use of the skew coefficient in the estimation of future rainfalls, were found to exhibit very good fits as they produced lower values of MRD and MSRD than the Gumbel and Normal functions. In view of this, it can be said that the maximum daily rainfall distribution of Ilorin is positively skewed and the Pearson III and Log-Pearson III distributions may be conveniently used for the prediction of future maximum daily rainfall events in Ilorin, Nigeria.

In this study, the Quintic polynomial function was chosen as the best-fit model as it produced the lowest values of MRD and MSRD (1.7187 and 5.0753 respectively), see Table 2 and Figure 7. Modeling processes using polynomials may be viewed as a numerical method. Numerical analysis as defined by Gerald and Wheatley, (2006), is the development and study of procedures for solving problems with a computer. Numerical analysis aims at reducing problems to a set of systematic procedure (algorithms) in which the only mathematical operations required are addition, subtraction, multiplication, and division (Gerald and Wheatley, 2006). Furthermore, polynomials can be expressed in nested form. The method of nesting is particularly suitable for use with a simple computer or calculator, as the calculation proceeds in a chain formation involving only the mathematical operations of addition, subtraction, and multiplication (Stroud, 1996). These simple operations are exactly the functions that a computer can perform (Gerald and Wheatley, 2006). Hence the quintic polynomial function developed here has a main advantage over the other functions as it can be readily adapted for use in computer programs for computation of quantiles for different return periods. This becomes more evident when the functions are to be programmed on platforms with limited mathematical functions. For example the Java2 Micro Edition(J2ME) platform is not equipped with the Ln, Log, and Power functions which are required in the computation of the Gumbel functions and the Log-transforms. Moreover, the need for statistical tables (Normal, and Pearson tables) is eliminated, and quantiles may be computed directly.

Conclusion

Studies of rainfall distribution cannot be overlooked in the design of reservoirs, artificial waterways, drainage networks, retention ponds, and other water-related structures. Probability distribution models were developed to predict the maximum daily rainfall of Ilorin, Nigeria. A Quintic regression polynomial was also fitted to model the distribution of maximum daily rainfall. It was found that the Quintic polynomial model was the best-fit model based on Mean Relative Deviation and Mean Squared Relative Deviation goodness-of-fit tests. Polynomials can be readily adapted for use in computer programs, hence it is suggested here that the developed Quintic polynomial function be used for the prediction of maximum daily rainfall of Ilorin. Also the range of length of data used (54 years) is statistically large thus giving a good approximation of the population parameters, hence the developed models are recommended for use in the prediction of maximum daily rainfall of Ilorin, Nigeria.

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