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Mathematical modeling of electric power flow and the minimization of power losses on transmission lines

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ABSTRACT

The importance of electric power in today's world cannot be overemphasized, for it is the key energy source for industrial, commercial and domestic activities. Its availability in the right quantity is essential to advancement of civilization. Electrical energy produced at power stations is transmitted to load centres from where it is distributed to the consumers through the use of transmission lines run from one place to another. As a result of the physical properties of the transmission medium, some of the transmitted power are lost to the surroundings. The power losses could take off a sizeable portion of the transmitted power since the transmission lines usually span a long distance, sometimes several hundred kilometers. The overall effect of power losses on the system is a reduction in the quantity of power available to the consumers. As such, adequate measures must be put in place to reduce power losses to the barest minimum. Thus, in this paper, we developed a mathematical model for determining power losses over typical transmission lines, as the resultant effect of ohmic and corona power losses, taking into cognizance the flow of current and voltage along the lines. Application of the classical optimization technique aided the formulation of an optimal strategy for minimization of power losses on transmission lines. With the aid of the new models it is possible to determine current and voltage along the transmission lines. In addition, we note that the analytical method does not involve any design or construction and so is less expensive than other models reported in the literature. Hence, the goal of this paper is to address a very well-known engineering problem – reducing the power losses on transmission lines to the barest minimum.

1. Introduction

Energy is a basic necessity for the development of any nation. Although, there are different forms of energy, the most important of them is electrical energy. A modern and civilized society is so much dependent on the use of electrical energy because it has been the most powerful vehicle for facilitating economic, industrial and social developments [21].

The ever increasing use of electrical energy for industrial, domestic and commercial purposes necessitated the bulk production of electrical energy. This bulk production is achieved with the help of suitable power production stations which

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are generally referred to as electric power generating stations or electric power plants. A generating station usually employs a prime mover coupled with an alternator to produce electric power [18].

Electrical energy produced at power stations is transmitted to load centres from where it is distributed to the consumers [10]. The transmission and distribution subsystems are very important to the electric power system, because without these subsystems the generated power cannot get to the load centres not to talk of getting finally to the consumers.

The connection between the power stations and the load centres is effected with the use of transmission lines, usually conductors run from one place to another and supported on towers. However, the arrangement of the power system places the transmission subsystem in a critical position since it is only the quantum of energy delivered to the distribution subsystem that will be said to be available for consumption in the system. For this reason, what happens in the transmission subsystem demand a careful examination. In designing extra high voltage single-circuit, Sakhavati et al. [26] highlight the importance of transmission lines.

The efficiency of the transmission component of the electric power system is known to be hampered by a number of problems, especially in third-world countries. The major problems identified in [14] include application of inappropriate technology, inadequacy of materials, equipment and man-power, among others. Even in developed countries, efficiency of the transmission subsystem has been shown to depend on the topology and type of the electric power grid adopted, see Albert et al. [3], Kinney et al. [15] and Han and Ding [12].

From the physics of electric power transmission, when a conductor is subjected to electric power (or voltage), electric current flows in the medium. Resistance to the flow produces heat (thermal energy) which is dissipated to the surroundings. This power loss is referred to as ohmic loss [28]. Furthermore, if the applied voltage exceeds a critical level, another type of power loss, called the corona effect [9] occurs. The power losses accumulate as the induced current flows and the corona effect propagate along the transmission lines. The power losses could take off a sizeable portion of the transmitted power since transmission lines usually span a long distance, sometimes several hundred kilometers [11]. The overall effect of power losses on the system is a reduction in the quantity of power available to the consumers. Thus, adequate measures must be put in place to reduce power losses to the barest minimum.

A lot of research works have been undertaken on minimization of electric power losses, out of which we cite a few. Ramesh et al. [23] looked at minimization of power losses in distribution networks by using feeder restructuring, implementation of distributed generation and capacitor placement method. Rugthaicharoencheep and Sirisumrannukul [25] employed the use of feeder reconfiguration for loss reduction in distribution system with distributed generators by Tabu Search. Numphetch et al. [19] worked on loss minimization using optimal power flow based on swarm intelligences while Abddullah et al. [1] studied transmission loss minimization and power installation cost using evolutionary computation for improvement of voltage stability. In addition to active power losses, series reactive power losses of transmission system were also considered as one of the multiple objectives, and Zakariya [30] made a comparison between the corona power loss associated with HVDC (High Voltage Direct Current) transmission lines and the ohmic power loss.

In most of the above research works, much emphasis has been on reduction of losses using feeder reconfiguration, implementation of distributed generation and capacitor placement which are capital intensive. A more general approach would be to blend the art of mathematical modeling with the mathematical precision of the classical optimization technique for minimization of power losses.

In a previous effort (see Oke and Bamigbola [20]), the classical optimization technique was applied for minimization of the power losses function without taking into consideration the fact that the voltage varies along the transmission lines. In this paper, we developed a mathematical model for determining power losses over typical transmission lines as the resultant effect of ohmic and corona losses but taking into cognizance the flow of current and voltage along the lines. Application of the classical optimization technique aided the formulation of an optimal strategy for minimizing power losses on transmission lines.

With the aid of the new models it is possible to determine current and voltage along transmission lines. In addition, the present approach is economical in terms of cost and effort as it does not involve any design or construction of electrical appliances.

2. Power flow on transmission lines

In this section, we derive the expressions which voltage and current must satisfy on uniform transmission lines. A real transmission line will have some series resistance associated with power losses in the conductor [4]. There may also be some shunt conductance if the insulating material holding two conductors has some leakage current. Therefore, resistance and conductance are responsible for power losses on transmission lines [7]. To this end, we formulate a model for a lossy transmission line where the effect of the series resistance (R) and shunt conductance (G) are taken care of.

2.1. Model formulation

Herein, we are interested in determining the extent to which current and voltage outputs differ from their input values over an elemental portion of the transmission line. As such, we consider an equivalent circuit of a transmission line of length Δx containing resistance $R\Delta x$, capacitance $C\Delta x$, inductance $L\Delta x$ and conductance $G\Delta x$ as shown in the circuit in Fig. 1 below. The circuit illustrates how power (both voltage and current) flow through the transmission medium [13].

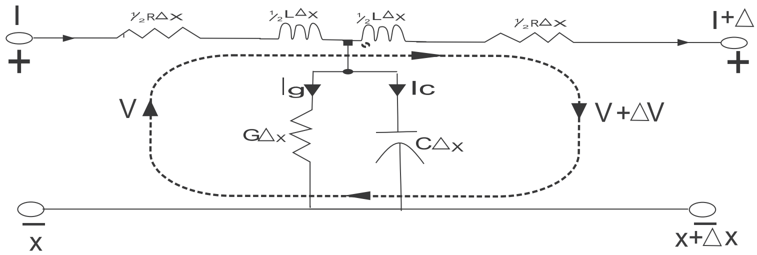

Fig. 1. An equivalent circuit of a high voltage transmission line.

Applying the Kirchoff's voltage law which states that the sum of all branch voltages in a loop equals zero (see Paul [22]), on the equivalent circuit of the transmission line, we have

$$V = \frac{1}{2}RI\Delta x + \frac{1}{2}L\frac{\partial I}{\partial t}\Delta x + \frac{1}{2}L\left[\frac{\partial I}{\partial t} + \frac{\partial \Delta I}{\partial t}\right]\Delta x + \frac{1}{2}R[I + \Delta I]\Delta x + V + \Delta V,$$

which on simplification, dividing through by Δx and taking limits as Δx tends to zero, simplifies to

$$\frac{\partial V}{\partial x} = -\left[RI + L\frac{\partial I}{\partial t}\right]. \tag{2.1}$$

Using the Kirchoff's current law which states that the sum of all branch current entering a node equals zero (see Paul [22]),

$$\frac{\partial I}{\partial x} = -\left[GV + \frac{G}{2}\frac{\partial V}{\partial x}\Delta x + C\frac{\partial V}{\partial t} + \frac{C}{2}\frac{\partial^2 V}{\partial t \partial x}\Delta x\right], \tag{2.2}$$

on the equivalent circuit of the transmission line and simplifying as above, we have

$$\frac{\partial I}{\partial x} = -\left[GV + C\frac{\partial V}{\partial t}\right]. \tag{2.3}$$

The partial differential equations in (2.1) and (2.3) describe the flow of current and voltage on a lossy transmission line.

Differentiating (2.1) with respect to x, gives

$$\frac{\partial^2 V}{\partial x^2} = -\left[L\frac{\partial^2 I}{\partial x \partial t} + R\frac{\partial I}{\partial x}\right], \tag{2.4}$$

and (2.3) with respect to t, yields

$$\frac{\partial^2 I}{\partial t \partial x} = -\left[C\frac{\partial^2 V}{\partial t^2} + G\frac{\partial V}{\partial t}\right]. \tag{2.5}$$

Substituting Eqs. (2.3) and (2.5) in Eq. (2.4), gives

$$\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2} + LG\frac{\partial V}{\partial t} + RGV + RC\frac{\partial V}{\partial t}. \tag{2.6}$$

Differentiating Eq. (2.1) with respect to t and Eq. (2.3) with respect to x and simplifying as above, we have

$$\frac{\partial^2 I}{\partial x^2} = LC\frac{\partial^2 I}{\partial t^2} + CR\frac{\partial I}{\partial t} + RGI + GL\frac{\partial I}{\partial t}. \tag{2.7}$$

Eqs. (2.6) and (2.7) are hyperbolic partial differential equations (pdes) for lossy transmission lines. These are the expressions depicting the flow of electric power along transmission lines and they govern the flow of current and voltage over the lines.

Dividing Eq. (2.6) by LC and substituting therein $\lambda = \frac{G}{C}$, $\beta = \frac{R}{L}$ and $\phi = \frac{1}{CL}$, yield

$$\frac{\partial^2 V}{\partial t^2} + (\lambda + \beta)\frac{\partial V}{\partial t} + (\lambda.\beta)V(x,t) = \phi\frac{\partial^2 V}{\partial x^2}. \tag{2.8}$$

which can be solved subject to the following boundary conditions:

$$V(x,0) = V_0$$

and

$$V(l,t) = 0, \quad l \to \infty$$

where V_0 denotes the initial voltage applied to the conductor.

2.2. Model solution

We now provide an analytical solution to problem (2.8) using a version of the eigenfunction method (see Courant and Hilbert [6]) by assuming a solution of the form

$$V(x,t) = X(x)T(t) \tag{2.9}$$

to give

$$\phi\frac{d^2X}{dx^2} - \alpha X = 0 \tag{2.10}$$

and

$$\frac{d^2T}{dt^2} + (\lambda+\beta)\frac{dT}{dt} + (\lambda\beta-\alpha)T = 0. \tag{2.11}$$

In transmitting electric power over the transmission lines, the elapsed time is very small. Reckoning the elapsed time as the unit time yields

$$\alpha = \lambda\beta. \tag{2.12}$$

We can then solve (2.10) subject to the boundary conditions $V(0,t) = V_0(t)$ and $V(l,t) = 0,\ l \to \infty$ where $V_0(t)$ denotes the voltage applied to the starting part of the conductor, to give

$$V(x,t) = V_0(t)e^{-x\sqrt{\frac{\alpha}{\phi}}}. \tag{2.13}$$

Employing the relation $V = IR$, we have

$$I(x,t) = I_0(t)e^{-x\sqrt{\frac{\alpha}{\phi}}}, \tag{2.14}$$

where $I_0(t) = \frac{V_0(t)}{R}$.

With the aid of the last two equations, i.e., (2.13) and (2.14), the quantity of current and voltage at any point on the transmission line can be discerned as illustrated with a transmission line with $\sqrt{\frac{\alpha}{\phi}} = 7.15 \times 10^{-5}$. The available electric current and voltage at different places on the transmission network are presented in Table 1 for a maximum length of about 300 km. It should be noted that the transmitted current and voltage are 19.17 A and 330 kV respectively. As such, Fig. 2 below depicts the transmitted and available currents and voltages.

3. Power losses on transmission lines

In this section, a mathematical model for determining power losses on high voltage transmission lines is formulated as a linear combination of ohmic and corona losses. The resulting model function was then minimized using the classical optimization technique.

3.1. Model for determination of power losses

The main reason for losses on transmission lines is the resistance of the conductor against the flow of current. As a result, heat is produced in the conductor and this increases the temperature of the conductor [16]. The rise in the conductor's temperature further increases the resistance of the conductor and this consequently increases the power losses. The value of the ohmic power loss [27] is given as

$$L_O = I^2R \text{ kW/km/phase}, \tag{3.1}$$

where I denotes current along the conductor and R represents resistance of the conductor.

Table 1
Available current and voltage along a 330 kV single circuit of a typical high voltage transmission network.

Length of line (km)	Current (A)	Voltage (kV)
10	19.14	329.5
20	19.09	329.1
50	19.00	327.6
100	18.87	325.3
200	18.60	320.7
300	18.33	316.1

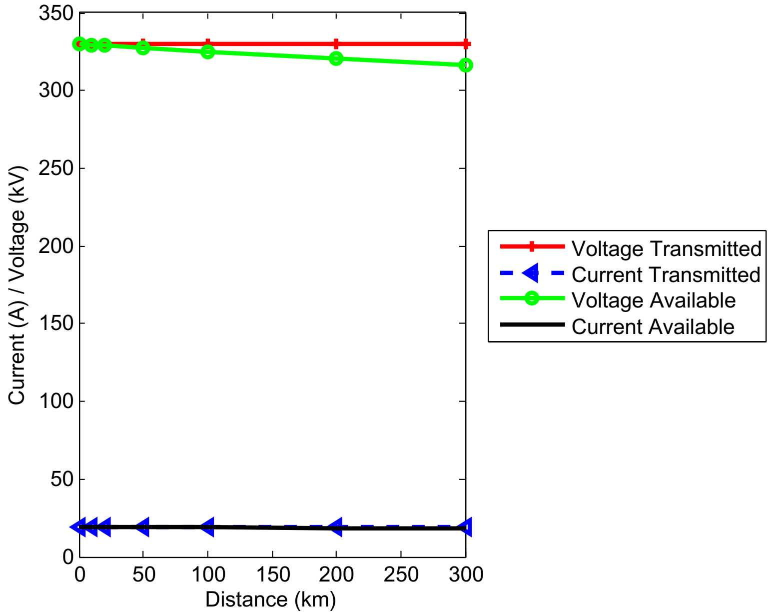

Fig. 2. Transmitted and available current and voltage of the transmission line.

The formation of corona on high voltage transmission lines is associated with a loss of power, which will have some effect on the efficiency of the transmission line [2]. The corona power loss for a fair weather condition as given in [17,29] has the value

$$L_C = 242\frac{(f+25)}{\delta}.\sqrt[4]{\frac{r}{d}}(V-V_c)^2.(10)^{-5}\ \text{kW/km/phase}, \tag{3.2}$$

where f represents the frequency of transmission, δ denotes the air density factor, r is radius of the conductor, d represents the space between the transmission lines, V is the operating voltage per phase and V_c denotes the disruptive critical voltage. High voltage gradients of above 18 kV/cm surrounding conductors is known to cause corona discharge, see Bayliss and Hardy [5, p. 645].

The total losses on a transmission line is then given as

$$T_L = L_O + L_C \tag{3.3}$$

i.e.,

$$T_L = I^2R + 242\frac{(f+25)}{\delta}.\sqrt[4]{\frac{r}{d}}(V-V_c)^2.(10)^{-5}. \tag{3.4}$$

With resistance R of the conductor given by

$$R = \frac{\rho L}{A}, \tag{3.5}$$

the power losses is therefore given by

$$T_L = I^2\frac{\rho L}{A} + 242\frac{(f+25)}{\delta}.\sqrt[4]{\frac{A}{\pi d^2}}(V-V_c)^2.(10)^{-5}\ \text{kW/km/phase}, \tag{3.6}$$

where ρ represents the resistivity of the conductor, L, the length of the conductor and A is the cross-sectional area of the conductor.

Eq. (3.6) is therefore the mathematical model for determining power losses over a transmission line. This model is dependent on a number of transmission related factors such as frequency, distance of transmission, operating voltage, etc. We now use the classical optimization technique, based on differential calculus, to minimize the model in order to get the optimum electric power losses during transmission.

3.2. Optimum power losses

The problem of finding the optimum power losses on transmission lines can be posed as

$$\text{Minimize}_{(I,V,d)}\ T_L = I^2\frac{\rho L}{A} + 242\frac{(f+25)}{\delta}.\sqrt[4]{\frac{A}{\pi d^2}}(V-V_c)^2.(10)^{-5}\ \text{kW/km/phase}. \tag{3.7}$$

Eq. (3.7) is therefore the corresponding nonlinear multi-variable unconstrained optimization problem for the transmission power losses function in (3.6) above. Assuming that the transmission related factors are continuous, then (3.7) can be solved using the classical method of optimization.

In order to determine the stationary points of (3.7) [8], we differentiate with respect to the selected variables to get

$$\frac{\partial T_L}{\partial I} = \frac{2I\rho L}{A}, \tag{3.8}$$

$$\frac{\partial T_L}{\partial V} = 484\frac{(f+25)}{\delta}.\sqrt[4]{\frac{A}{\pi d^2}}(V-V_0).(10)^{-5} \tag{3.9}$$

and

$$\frac{\partial T_L}{\partial d} = -121\frac{(f+25)}{\delta}.\sqrt[4]{\frac{A}{\pi}}(V-V_0)^2 d^{\frac{-3}{2}}(10)^{-5}. \tag{3.10}$$

Eqs. (3.8)–(3.10) give the extremum points as $I = 0,\ V = V_0$ and $d \longrightarrow \infty$.

In practical terms, the optimal value of d can be obtained from the following condition which is satisfied preceding the occurrence of corona discharge (see Bayliss and Hardy [5, pp. 645–649]):

$$V_g = \frac{U_p}{[r\log_e(d/r)]} \leqslant 18, \tag{3.11}$$

where V_g denotes voltage surface gradient; U_p, the phase voltage and r, the radius of the conductor. By solving (3.11), we have

$$d \geqslant re^{\frac{U_p}{18r}},$$

from which the optimum value of d is obtained as

$$d^* = r.e^{\frac{U_p}{18r}}. \tag{3.12}$$

The second order partial derivatives with respect to the variables are

$$\frac{\partial^2 T_L}{\partial I^2} = \frac{2\rho L}{A}, \tag{3.13}$$

$$\frac{\partial^2 T_L}{\partial I \partial V} = 0, \tag{3.14}$$

$$\frac{\partial^2 T_L}{\partial I \partial d} = 0, \tag{3.15}$$

$$\frac{\partial^2 T_L}{\partial V \partial I} = 0, \tag{3.16}$$

$$\frac{\partial^2 T_L}{\partial V^2} = 484\frac{(f+25)}{\delta}.\sqrt[4]{\frac{A}{\pi d^2}}.(10)^{-5}, \tag{3.17}$$

$$\frac{\partial^2 T_L}{\partial V \partial d} = -242\frac{(f+25)}{\delta}.\sqrt[4]{\frac{A}{\pi}}(V-V_0)d^{\frac{-3}{2}}(10)^{-5}, \tag{3.16}$$

$$\frac{\partial^2 T_L}{\partial d \partial I} = 0, \tag{3.18}$$

$$\frac{\partial^2 T_L}{\partial d \partial V} = -242\frac{(f+25)}{\delta}.\sqrt[4]{\frac{A}{\pi}}(V-V_0)d^{\frac{-3}{2}}(10)^{-5} \tag{3.19}$$

and

$$\frac{\partial^2 T_L}{\partial d^2} = \frac{363}{2}\frac{(f+25)}{\delta}.\sqrt[4]{\frac{A}{\pi}}(V-V_0)^2.d^{-\frac{5}{2}}(10)^{-5}. \tag{3.20}$$

The Hessian matrix (Rao [24]) is therefore given as

$$H = \begin{pmatrix} \frac{\partial^2 T_L}{\partial I^2} & \frac{\partial^2 T_L}{\partial I \partial V} & \frac{\partial^2 T_L}{\partial I \partial d} \\ \frac{\partial^2 T_L}{\partial V \partial I} & \frac{\partial^2 T_L}{\partial V^2} & \frac{\partial^2 T_L}{\partial V \partial d} \\ \frac{\partial^2 T_L}{\partial d \partial I} & \frac{\partial^2 T_L}{\partial d \partial V} & \frac{\partial^2 T_L}{\partial d^2} \end{pmatrix}$$

i.e.,

$$H = \begin{pmatrix} \frac{2\rho L}{A} & 0 & 0 \\ 0 & 484\frac{(f+25)}{\delta}\sqrt[4]{\frac{A}{\pi d^2}}(10)^{-5} & -242\frac{f+25}{\delta}\sqrt[4]{\frac{A}{\pi}}(V-V_0)d^{\frac{-3}{2}}(10)^{-5} \\ 0 & -242\frac{f+25}{\delta}.\sqrt[4]{\frac{A}{\pi}}(V-V_0)d^{\frac{-3}{2}}(10)^{-5} & \frac{363}{2}\frac{(f+25)}{\delta}.\sqrt[4]{\frac{A}{\pi}}(V-V_0)^2.d^{-\frac{5}{2}}(10)^{-5} \end{pmatrix}$$

for which

$$|H_1| = \left|\frac{2\rho L}{A}\right| > 0,$$

$$|H_2| = \begin{vmatrix} \frac{2\rho L}{A} & 0 \\ 0 & 484\frac{(f+25)}{\delta}.\sqrt[4]{\frac{A}{\pi d^2}}.(10)^{-5} \end{vmatrix} = \frac{2\rho L}{A}.484\frac{(f+25)}{\delta}.\sqrt[4]{\frac{A}{\pi d^2}}.(10)^{-5} > 0$$

and

$$\begin{aligned} |H| &= \begin{vmatrix} \frac{2\rho L}{A} & 0 & 0 \\ 0 & 484\frac{(f+25)}{\delta}.\sqrt[4]{\frac{A}{\pi d^2}}.(10)^{-5} & -242\frac{(f+25)}{\delta}.\sqrt[4]{\frac{A}{\pi}}(V-V_0)d^{\frac{-3}{2}}(10)^{-5} \\ 0 & -242\frac{(f+25)}{\delta}.\sqrt[4]{\frac{A}{\pi}}(V-V_0)d^{\frac{-3}{2}}(10)^{-5} & \frac{363}{2}\frac{(f+25)}{\delta}.\sqrt[4]{\frac{A}{\pi}}(V-V_0)^2.d^{-\frac{5}{2}}(10)^{-5} \end{vmatrix} \\ &= \frac{2\rho L}{A}\left[\left(484\frac{(f+25)}{\delta}.\sqrt[4]{\frac{A}{\pi d^2}}.(10)^{-5}\right)\left(\frac{363}{2}\frac{(f+25)}{\delta}.\sqrt[4]{\frac{A}{\pi}}(V-V_0)^2.d^{-\frac{5}{2}}(10)^{-5}\right) - \left(242\frac{(f+25)}{\delta}\sqrt[4]{\frac{A}{\pi}}(V-V_0)d^{\frac{-3}{2}}(10)^{-5}\right)^2\right] \\ &= \frac{3\rho L}{A}\left[242\frac{(f+25)}{\delta}\sqrt[4]{\frac{A}{\pi}}(V-V_0)d^{\frac{-3}{2}}\times 10^{-5}\right]^2 > 0 \end{aligned}$$

4. Discussion on results

Since $|H_1|$, $|H_2|$ and $|H|$ are all greater than zero, then it shows that the Hessian matrix of power losses over transmission lines is positive definite at the optimum point, i.e., $x^* = (0, V_c, d^*)^T$. Hence, the power losses is minimum at $I = 0$, $V = V_c$ and d^*.

Technically, this implies that the total power losses on transmission lines will only be minimum if

(i) power is transmitted at a very low current along transmission lines which will reduce the ohmic or line loss on the conductors. This conforms with the principle of electric power transmission.
(ii) the operating voltage is equal to the disruptive critical voltage. When this happens, there is no ionization of air around the conductor and hence no corona is formed. Therefore, there will be no corona loss, and
(iii) the spacing between the transmission lines should be larger than the value of $r.e^{\frac{U_p}{18r}}$. This is because an increase in the spacing between conductors reduces electro-static stresses and consequently the corona effect.

In summary, the optimal strategy is to operate the transmission system at the disruptive critical voltage and in such a way that the spacing between the conductors is large when compared to their diameter.

5. Conclusion

Loss minimization is a very important area of concern in electricity transmission and even in distribution. This therefore underscores the relevance of this research work. Herein, we first presented the relevant background on the problem including illustration with a circuit diagram. The application of Kirchoff's current and voltage laws aided in the derivation of mathematical expressions, in the form of partial differential equations with practical boundary conditions, that describe the flow of electric power along transmission lines. The power flow models obtained are unique as they have not been suggested previously in literature. We then use the solution to the obtained mathematical physics equations to predict network parameters such as current and voltage along a transmission line with very realistic results.

Finally, the classical optimization technique is employed for the minimization of power losses on transmission lines, thus giving a solution which on implementation is sure to yield the optimal operating strategy.

References

[1] N.R.H. Abddullah, M. Ismail, M.O. Mohammad, Transmission loss minimization and power installation cost using evolutionary computation for improvement of voltage stability, in: Proc. 14th Int. Middle East Power Syst. Conf., 2010.
[2] M. Abdel-Salam, Electrical Breakdown of Gases in High-Voltage Engineering: Theory and Practice, in: M.E. Khalifa (Ed.), Marcel Dekker, New York, 2000, pp. 93–121.
[3] R. Albert, I. Albert, G.L. Nakarado, Structural vulnerability of the North American power grid, Phys. Rev. E 69 (2004) 1–4.
[4] H. Ambriz-Pe'rez, C.R. Fuerte-Esquivel, High voltage direct current modeling in optimal power flows, Electr. Power Energ. Syst. 30 (2008) 157–168.
[5] C.R. Bayliss, B.J. Hardy, Transmission and Distribution Electrical Engineering, Elsevier, Amsterdam, 2007.
[6] R. Courant, D. Hilbert, Methods of Mathematical Physics, 1989 (ISBN 0-471-50447-5).
[7] T. Dhaene, D, De Zutter, Selection of lumped element models for coupled lossy transmission lines, IEEE Trans. Comput. Aided Des. 11 (1992) 805–815.
[8] J.H. Drake, L.K. Kirchmayer, R.B. Kayall, H. Wood, Optimum operation of a hydrothermal system, Trans. AIEE Power Apparatus Syst. 62 (1962) 242–250.
[9] R.D. Dunlop, R. Gutman, P.P. Marchenko, Analytical development of loadability characteristics for EHV and UHV lines, IEEE Trans. PAS 98 (2) (1979) 606–607.
[10] J.D. Glover, M.S. Sarma, T.J. Overbye, Power System and Design, fifth ed., Cengage Learning, Stamford, 2008.
[11] M.W. Gustafson, J.S. Baylor, Transmission loss evaluation for electric systems, IEEE Trans. Power Syst. 3 (3) (1988) 1026–1032.
[12] P. Han, M. Ding, Analysis of cascading failures in small-world power grid, Int. J. Energy Sci. 1 (2) (2011) 99–104.
[13] W.H. Hayt Jr., J.E. Kemmerly, Engineering Circuit Analysis, seventh ed., McGraw-Hill, New York, 2006.
[14] A.P. Kenneth, A.T. John, The benefit of distribution generation to the Nigerian electric utility system, J. Basic Appl. Sci. Res. 3 (2) (2013) 381–385.
[15] R. Kinney, P. Crucitti, R. Albert, V. Latora, Modeling cascading failures in the North American power grid, Eur. Phys. J. B 46 (2005) 101–107.
[16] V. Král, S. Rusek, L. Rudolf, Calculation and estimation of technical losses in transmission networks, Electr. Rev. 88 (8) (2012) 88–91.
[17] V.K. Mehta, R. Mehta, Principles of Power Systems, S. Chand Company Limited, New Delhi, 2008.
[18] A.V. Meier, Electric Power Systems: A Conceptual Introduction, Wiley, London, 2006.
[19] S. Numphetch, L. Uthen, K. Umaporn, U. Dusit, K. Thanatchai, Loss minimization using optimal power flow based on swarm intelligences, ECTI Trans. Electr. Electron. Commun. Eng. 9 (1) (2011) 212–222.
[20] M.O. Oke, O.M. Bamigbola, Minimization of losses on electric power transmission lines, J. Math. Theor. Model. 3 (7) (2013) 28–31.
[21] O.I. Okoro, E. Chikuni, Power sector reforms in Nigeria: opportunities and challenges, J. Energy South. Afr. 18 (3) (2007) 52–57.
[22] C.R. Paul, Fundamentals of Electric Circuit Analysis, John Wiley, New York, 2001.
[23] L. Ramesh, S.P. Chowdhury, S. Chowdhury, A.A. Natarajan, C.T. Gaunt, Minimization of power loss in distribution networks by different techniques, Int. J. Electr. Power Eng. 2 (1) (2009) 1–6.
[24] S.S. Rao, Optimization Theory and Applications, Wiley Eastern Limited, New Delhi, 1989.
[25] N. Rugthaicharoencheep, S. Sirisumrannukul, Feeder reconfiguration for loss reduction in distribution system with distributed generators by Tabu Search, GMSARN Int. J. 3 (2009) 47–54.
[26] A. Sakhavati, M. Yaltagiani, S.S. Ahari, S.S. Mahaei, 765 kV transmission line design (electrical section), Int. J. Electr. Comput. Eng. 2 (5) (2012) 698–707.
[27] I. Sarajcev, M. Majstrovic, I. Medic, Calculation of losses in electric power cables as the base for cable temperature analysis, J. Adv. Comput. Methods Heat Transfer 4 (2003) 529–537.
[28] T. Smed, G. Andersson, G.B. Sheble, L.L. Grigsby, A new approach to AC/DC power flow, IEEE Trans. Power Syst. 6 (3) (1991) 1238–1244.
[29] E.A. Yahaya, T. Jacob, M. Nwohu, A. Abubakar, Power loss due to corona on high voltage transmission lines, IOSR J. Electr. Electron. Eng.(IOSR-JEEE) 8 (3) (2013) 14–19.
[30] M.A. Zakariya, Corona power loss versus ohmic power loss in HV transmission lines, in: Proc. Power Conf., Dhahran, Saudi Arabia, 2010.