

Heat and Mass Transfer of Oscillatory Visco-elastic Radiating Hydromagnetic Flow through a Vertical Channel with Hall Effects

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Abstract

Heat and mass transfer of oscillatory hydrodynamics convective Visco-elastic incompressible fluid flow in a vertical channel with Hall effects is investigated. The equations governing the problem are simplified, non-dimensionalized and solved using Differential Transform Method. The results compared well with existing literature. The effects of the salient fluid parameters are discussed and presented in tabular and graphical forms.

Keywords: Heat and Mass Transfer, Hall Current, Visco-elastic, Hydromagnetic, porosity

1. Introduction

Investigation of heat and mass transfers are important in science and technology due to their applications in industries and manufacturing process. Some specific applications of heat and mass transfers are observed in drying of solid materials, distillation, extraction, absorption and in chemical processing industries such as food processing and polymer production. Additionally, the application of heat and mass transfer are also encountered in buoyancy induced motion in the atmospheres as well in bodies of water, quasi-solid bodies such as earth and others. Some authors have carried out some research works on the analysis of heat and mass transfer. Sharma and kalpana (2009) investigated effect of mass transfer on three

dimensional unsteady mixed convective flows past an infinite vertical moving porous plate with periodic suction. Reddy (2014) discussed radiation and viscous dissipation effects on unsteady MHD free convective mass transfer flow past an infinite vertical porous plate with Hall current in the presence of chemical reaction. In his results, it is observed that there is a decrease in the temperature boundary layer when the Prandtl number and radiation parameters are increased while an increase in the Eckert number increases the temperature. Also, an increase in the Schmidt number and chemical reaction parameter decreases the concentration boundary layer. The problem of simultaneous heat and mass transfer with entire range of buoyancy ratio for most practical chemical species

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in dilute and aqueous solutions was studied by Lin and WU (1995).

The effect of magnetic field cannot be neglected in liquid metals, electrolytes and ionized gases. MHD heat and mass transfer process are useful in power engineering, metallurgy, astrophysics and geophysics. Hence, Sharma *et al.* (2012) examined mass transfer with chemical reaction in MHD mixed convection flow along a vertical stretching sheet. Chen (2004) studied combined heat and mass transfer of an electrically conducting fluid in MHD free convection from a vertical surface with Ohmic heating and viscous dissipation. It was reported in the study that an increase in buoyancy ratio of mass and thermal diffusion enhances the velocity profile while it reduces the concentration distribution in the boundary layer and thus increases concentration gradient at the wall. The velocity profile also decreases by an increase in magnetic field. In the above studies, Hall current effect is assumed to be insignificant. However, if the magnetic field is very strong the effect of Hall current needs to be considered because the electrical conductivity is affected by magnetic field which causes the reduction of conductivity parallel to the electric field.

An appreciable discussion of Hall current has been presented by Cowling (1957). Soundalgekar (1979) analyzed the Hall and ion slip effects in MHD coquette flow with heat transfer. Hall effect with simultaneous thermal and

mass diffusion on unsteady hydromagnetic flow past a vertical plate was analyzed by Acharya *et al.* (1995). Constant suction/injection normal to the plate was assumed. The results were discussed with respect to the Schmidt number, hydromagnetic, Hall and suction parameters.

Radiation effects are put into consideration when heat and mass transfer flow occur in engineering process especially in the design of pertinent equipment. Examples of such engineering areas are nuclear power plants and various proportion devices for missiles, space vehicles and aircrafts. Soundalgekar and Takhan (1993) have examined radiation effects on the free convection flow of a gas past a semi-infinite plate using Cogley-Vincentine Giles equilibrium model. Radiation effect on mixed convection along an isothermal vertical plate using Rosseland approximation was analyzed by Hosain and Takhar (1996). More works on radiative heat transfer can be found in the work of Takhar *et al.* (1996), Abo-eldahab (2004) and Muthucumarswamy and Kumar (2004).

The above studies have not considered non Newtonian fluid flow. Non Newtonian fluid flow is very useful in industries and food manufacturing process such as molten plastics, polymer processing applications, food stuffs or slurries. Hayat *et al.* (2008) studied the hydrodynamic elastico-viscous fluid over a stretching surface. Olajuwon (2011) discussed the convection heat and

mass transfer in a hydromagnetic flow of a second grade fluid where thermal radiation and thermal diffusion are put into consideration. It was examined that an increase in second grade parameter resulted to a decrease in velocity profile of the fluid. MHD free convection flow of Visco-elastic fluid past a vertical porous plate was investigated by Chowdhury *et al.* (2000).

Hall current effect on Visco-elastic MHD oscillatory convective flow through a porous medium in a vertical channel with heat radiation was discussed by Singh *et al.* (2014). However, the effect of mass transfer is neglected in the studied. Hence, the present work deals with the effect of Hall current on Visco-elastic MHD oscillatory convective flow of heat and mass transfer through a porous medium in a vertical channel with heat radiation.

2. Mathematical Analysis

Consider a fully developed radiative MHD free convective flow of Visco-elastic electrically conducting incompressible fluid with heat and mass transfer through a porous medium in an infinitely long insulated vertical plates with distance 'd' apart. Cartesian coordinate system is introduced with X-axis taken vertically upward along the flow direction. The Z-axis is chosen perpendicularly to the plane. The fluid is subjected to a transversely applied uniform magnetic field of strength B_0 . There is a rise in Lorentz force in y direction which induces a cross flow in that direction due to presence of Hall current. Since the plates are infinitely long, all the physical quantities are functions of z and t alone. The equations governing the flow of the fluid are:

Continuity equation

$$\nabla \cdot \vec{V} = 0 \quad (1)$$

Momentum equation

$$\rho \left[\frac{\partial \vec{V}}{\partial t^*} + (\vec{V} \cdot \nabla) \vec{V} \right] = -\nabla P + J \times B_0 + \mu \nabla^2 \vec{V} - \mu \frac{\vec{V}}{K} + g\beta(T^* - T_0) + g\beta^*(C^* - C_0) + \nabla \cdot \Xi \quad (2)$$

Energy equation

$$\rho C_p \left[\frac{\partial T^*}{\partial t^*} + (\vec{V} \cdot \nabla) T^* \right] = K \nabla^2 T^* - \nabla q_r \quad (3)$$

Concentration equation

$$\frac{\partial C^*}{\partial t^*} = D \nabla^2 C^* - K_T (C^* - C_0) \quad (4)$$

where the generalized Ohm's law and Maxwell equations respectively are

$$\vec{J} = \sigma (\vec{E} + \vec{V} \times \vec{B}) - \frac{\sigma}{\rho n_e} (\vec{J} \times \vec{B} - \nabla P_e) \quad (5)$$

Maxwell Equations:

$$\nabla \cdot \vec{J} = 0, \quad \nabla \times \vec{E} = 0, \quad \nabla \cdot \vec{B} = 0 \quad (6)$$

$\vec{V}(u^*, v^*, w^*)$ is the velocity field along x^* , y^* and z^* directions respectively, g is the acceleration due to gravity, P is the pressure field, β is the volumetric coefficient of thermal expansion, β^* is the volumetric coefficient of expansion with concentration, ρ is the fluid density, D is the chemical molecular diffusivity, μ is the viscosity, K is the thermal conductivity, $\vec{J}(J_x^*, J_y^*, J_z^*)$ is the current density, $\vec{B}(B_x^*, B_y^*, B_z^*)$ is the magnetic induction, $\vec{E}$ is the electric field, q_r is the radiative heat flux. The electric field $\vec{E} = 0$ since no polarization voltage is imposed on the flow as a result of no energy is added or extracted from the fluid by the electric field. The induced magnetic field is small compare to applied magnetic field therefore the induced magnetic field is neglected so that $B_x^* = B_y^* = 0$ hence $\vec{B} = (0, 0, B_0)$. This assumption holds when magnetic Reynold number is very small Chaudhary and Jain (2006). Ξ is the Cauchy stress tensor and it is defined for an incompressible homogeneous fluid of second order, Coleman and Noll (1960), as

where ∇ represents the gradient operator and $\frac{d}{dt}$ is the material time derivative. Markovitz and Coleman (1994) examined that μ_1, μ_3 are positive and μ_2 is negative.

The equation of conservation $\nabla \cdot \vec{J} = 0$ implies J_z is a constant. Since the plates are insulated $J_z = 0$ at the plates hence equal to zero everywhere in the fluid. The generalized Ohm's law including Hall current in the absence of electric field is

$$\vec{J} + \frac{\omega_e \tau_e}{B_0} (\vec{J} \times \vec{B}) = \sigma \left(\vec{V} \times \vec{B} + \frac{\nabla P_e}{en_e} \right) \quad (10)$$

ω_e is the electron frequency and τ_e is the electron collision time, P_e is the electron pressure, e is the electric charge, n_e is the number density of electron and σ is the electrical conductivity. If the ionize gases weak the thermoelectric pressure and ion slip are negligible. equation (10) in its component form reduces to

$$j_x = \frac{\sigma B_0}{1 + m^2} (mu + v) \quad (11)$$

$$j_y = \frac{\sigma B_0}{1 + m^2} (mv + u) \quad (12)$$

where $m = \omega_e \tau_e$ is the Hall current. Since the plates are not porous, the integration of continuity equation (1) gives $w^* = 0$. The Boussinesq approximation equations governing the flow are

$$\frac{\partial u^*}{\partial t^*} = \frac{-1}{\rho} \frac{\partial P}{\partial x^*} + v_1 \frac{\partial^2 u^*}{\partial z^{*2}} + v_2 \frac{\partial^3 u^*}{\partial z^{*2} \partial t^*} + \frac{\sigma B_0^2}{\rho} \frac{(mv^* - u^*)}{(1 + m^2)} - v_1 \frac{u^*}{K^*} + g\beta(T^* - T_0) + g\beta^*(\phi^* - \phi_0) \quad (13)$$

$$\frac{\partial v^*}{\partial t^*} = \frac{-1}{\rho} \frac{\partial P}{\partial y^*} + v_1 \frac{\partial^2 v^*}{\partial z^{*2}} + v_2 \frac{\partial^3 v^*}{\partial z^{*2} \partial t^*} + \frac{\sigma B_0^2}{\rho} \frac{(mu^* - v^*)}{(1 + m^2)} - v_1 \frac{v^*}{K^*} \quad (14)$$

$$\rho C_p \frac{\partial T^*}{\partial t^*} = K \frac{\partial^2 T^*}{\partial z^{*2}} - \frac{\partial q}{\partial z^*} \quad (15)$$

$$\frac{\partial \phi^*}{\partial t^*} = D \frac{\partial^2 \phi^*}{\partial z^{*2}} - \frac{\partial q}{\partial z^*} - K_r(\phi^* - \phi_0) \quad (16)$$

together with the appropriate boundary conditions

$$u^* = v^* = 0, \quad T^* = T_0, \quad \phi^* = \phi_0 \quad \text{at} \quad z^* = -\frac{d}{2} \quad (17)$$

and

$$v^* = u^* = 0, \quad T^* = T_0 + (T_w - T_0) \quad \phi^* = \phi_0 + (\phi_w - \phi_0) \quad \text{at} \quad z^* = \frac{d}{2} \quad (18)$$

with dimensionless quantities

$$\eta = \frac{z^*}{d}, x = \frac{x^*}{d}, y = \frac{y^*}{d}, u = \frac{u^*}{U}, v = \frac{v^*}{U}, T = \frac{T^* - T_0}{T_w - T_0}, Re = \frac{Ud}{\nu_1},$$

$$\phi = \frac{\phi^* - \phi_0}{\phi_w - \phi_0}, t = \frac{t^* U}{d}, w = \frac{w^* d}{U}, p = \frac{p^*}{\rho U^2}, H^2 = B_0 \frac{\sigma d}{\nu_1}, K = \frac{K^*}{d^2},$$

$$Gr = \frac{g \beta d^2 (T_w - T_0)}{\nu_1 U}, Pe = \frac{\rho C_p d U}{k}, N = \frac{2 \alpha d}{\sqrt{k}}, \gamma = \frac{\nu_2 Re}{d^2}, Sc = \frac{\nu_1}{D},$$

$$Gr_c = \frac{g \beta^* d^2 (C_w - C_0)}{\nu_1 U}, \xi = \frac{K_r d^2}{\nu_1} \quad (19)$$

The radiative heat flux q_r in the energy equation follows Cogley et al, 1986 takes the form

$$\frac{\partial q_r}{\partial z^*} = 4\alpha^2 (T^* - T_0) \quad (20)$$

where

$$\alpha^2 = \int_0^\infty k_{\lambda n} \frac{\partial e_{b\lambda}}{\partial T} d\lambda$$

$k_{\lambda\eta}$ is the absorption coefficient at the walls and $e_{b\lambda}$ is the Plank's function.

Substituting dimensionless quantities in equation (19) into equations (13) – (16) with the conditions (17) – (18) we have

$$Re \frac{\partial u}{\partial t} = -Re \frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial \eta^2} + \gamma \frac{\partial^3 u}{\partial \eta^2 \partial t} + H^2 \frac{(mv - u)}{1 + m^2} - \frac{u}{K} + GrT + G_c \phi \quad (21)$$

$$Re \frac{\partial v}{\partial t} = -Re \frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial \eta^2} + \gamma \frac{\partial^3 v}{\partial \eta^2 \partial t} + H^2 \frac{(mu - v)}{1 + m^2} - \frac{v}{K} \quad (22)$$

$$Pe \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial \eta^2} - N^2 T \quad (23)$$

$$ReS_c \frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial \eta^2} + S_c \xi \phi \quad (24)$$

with transformed boundary conditions

$$u = v = 0, \quad T = 0, \quad \phi = 0 \quad \text{at } \eta = -\frac{1}{2} \quad (25)$$

$$u = v = 0, \quad T = \cos \omega t, \quad \phi = \cos \omega t \quad \text{at } \eta = \frac{1}{2} \quad (26)$$

where Re is Reynolds number, K is the porous medium permeability, H is Hartmann number, γ is the visco-elastic parameter, Gr is Grashof number, G_c is modified Grashof number Pe is pecllet number, N is the radiation parameter, S_c is Schmidt number and ξ is the reaction parameter.

Oscillatory internal flow in a non-dimensional pressure gradient oscillating only in the direction of x -axis, the pressure gradients take the form

$$-\frac{\partial P}{\partial x} = A \cos \omega t, \quad \text{and} \quad -\frac{\partial P}{\partial y} = 0 \quad (27)$$

where A is a constant.

Equation (21) and (22) are combined by introducing a complex function $F = u + iv, i = \sqrt{-1}$

$$Re \frac{\partial F}{\partial t} = -Re \frac{\partial p}{\partial x} + \frac{\partial^2 F}{\partial \eta^2} + \gamma \frac{\partial^3 F}{\partial \eta^2 \partial t} - H^2 \frac{(mi+1)}{(1+m^2)} F - \frac{F}{K} + G_r T + G_c \phi \quad (28)$$

with the complex form of the boundary conditions

$$F = 0, \quad T = 0, \quad \phi = 0 \quad \text{at} \quad \eta = -\frac{1}{2} \quad (29)$$

$$F = 0, \quad T = \cos \omega t, \quad \phi = \cos \omega t \quad \text{at} \quad \eta = \frac{1}{2} \quad (30)$$

Equations (23), (24) and (28) are solved by assuming a complex form solution

$$F(\eta, t) = F_0(\eta)e^{i\omega t}, \quad T(\eta, t) = \theta_0(\eta)e^{i\omega t}, \quad \phi(\eta, t) = \phi_0(\eta)e^{i\omega t} \quad \text{at} \quad \eta = \frac{1}{2} \quad (31)$$

with

$$-\frac{\partial P}{\partial x} = Ae^{i\omega t}$$

Substituting assumed solutions (31) into equations (23), (24) and (28) with their boundary conditions (29) and (30), the equations reduce to

$$(1+i\omega)F_0'' - \left[i\omega Re + H^2 \frac{(mi+1)}{(1+m^2)} + \frac{1}{K} \right] F_0 + G_r \theta_0 + G_c \phi_0 + ARe = 0 \quad (32)$$

$$\theta_0'' - (i\omega p_e + N^2)\theta_0 = 0 \quad (33)$$

$$\phi_0'' - Sc(\xi + i\omega Re)\phi_0 = 0 \quad (34)$$

with boundary conditions

$$F = 0, \quad \theta_0 = 0, \quad \phi_0 = 0 \quad \text{at} \quad \eta = -\frac{1}{2} \quad (35)$$

$$F = 0, \quad \theta_0 = 1, \quad \phi_0 = 1 \quad \text{at} \quad \eta = \frac{1}{2} \quad (36)$$

1. Analysis of Differential Transform Method (DTM)

DTM is an iterative procedure to obtain analytic Taylor series solutions of differential equations. The basic definitions and application procedures of DTM are introduced as follows. Consider a

function $F(x)$ which is analytic in a domain D and let $x = x_0$ represent any point in D . The function $F(x)$ is then represented by a power series whose center is located at x_0 . The differential transform of the function $F(x)$ is given by Kaya and Ozgumus (2007)

$$f(k) = \frac{1}{k!} \left(\frac{d^k F(x)}{dx^k} \right)_{x=x_0} \quad (37)$$

where $F(x)$ is the original function and $f(k)$ is the transformed function. The inverse transformation is defined as follows:

$$F(x) = \sum_{k=0}^{\infty} (x - x_0)^k f(k) \quad (38)$$

Equation (38) can be written in finite series as

$$F(x) = \sum_{k=0}^n (x - x_0)^k f(k) \quad (39)$$

Table 1: Differential Transform Theorems

$F(x) = G(x) \pm H(x)$	$f(k) = g(k) \pm h(k)$
$F(x) = \lambda G(x)$	$f(k) = \lambda g(k)$
$F(x) = G(x)H(x)$	$f(k) = \sum_{l=0}^k g(k-l)h(l)$
$F(x) = \frac{d^n G(x)}{dx^n}$	$f(k) = \frac{(n+1)!}{n!} g(k+n)$
$F(x) = x^n$	$f(k) = \delta(k-n) = \begin{cases} 0 & \text{if } k \neq n \\ 1 & \text{if } k = n \end{cases}$

3.1 DTM Method of Solution

Taking the differential transform of equations (32), (33) and (34) by using the theorem introduced in table 1 we arrived at recurrence relations

$$a(k+2) = \frac{1}{(1+i\omega\gamma)(k+1)(k+2)} \left(\left(ReI\omega + \frac{H^2}{(1+m^2)}(ml+1) + \frac{1}{K} \right) * a(k) - Gr * f(k) - Gc * g(k) - A * Re * \delta(k) \right) \quad (40)$$

$$f(k+2) = \frac{1}{(k+1)(k+2)} ((I\omega Pe + N^2) * f(k)) \quad (41)$$

$$g(k+2) = \frac{1}{(k+1)(k+2)} (Sc(\xi + I\omega Re) * g(k)) \quad (42)$$

where $a(k)$, $f(k)$ and $g(k)$ are differential transformed of $F_0(\eta)$, $\theta_0(\eta)$ and $\phi_0(\eta)$ respectively. Assuming $a(0)$, $a(1)$, $f(0)$, $f(1)$, $g(0)$, $g(1) = a_0$, a_1 , f_0 , f_1 , g_0 , g_1 , where $k = 0..n$ using equation (39) and recurrence relations in equations (40), (41) and (42) the following series solutions are obtained.

$$F_0(\eta) = a_0 + a_1 + \frac{\eta^2}{2+2i\omega\gamma} \left(\left(iR\omega + \frac{H^2(im+1)}{(m^2+1)} + K^{-1} \right) a_0 - Gr f_0 - Gc g_0 - A R \right) + \frac{\eta^3}{6+6i\omega\gamma} \left(\left(iR\omega + \frac{H^2(im+1)}{(m^2+1)} + K^{-1} \right) a_1 - Gr f_1 - Gc g_1 \right) \quad (43)$$

$$\theta_0(\eta) = f_0 + f_1 + \frac{1}{2} (I\omega Pe + N^2) f_0 \eta^2 + \frac{1}{6} ((I\omega Pe + N^2) f_1 \eta^3 + \dots) \quad (44)$$

$$\phi_0(\eta) = g_0 + g_1 + \frac{1}{2} Sc(\xi + iR\omega) g_0 \eta^2 + \frac{1}{6} Sc(\xi + iR\omega) g_1 \eta^3 + \dots \quad (45)$$

Applying the boundary conditions(35) and (36) into equations (43), (44) and (45) to obtain the constants in the solutions.

The general solutions for velocity, Temperature and mass concentration are obtained from equation (31). The absolute velocity of the flow are given as $R_f = \sqrt{u^2 + v^2}$ where u is the real part of $F(\eta, t)$ and v is the imaginary part of $F(\eta, t)$. The skin friction in term amplitude and phase angle can be derived as $\left(\frac{\partial F(\eta, t)}{\partial \eta}\right)_{y=-0.5} = |F| \cos(\omega t + \beta)$. $|F| = \sqrt{\tau_r^2 + \tau_i^2}$ is the amplitude of the skin friction and $\beta = \tan^{-1} \frac{\tau_i}{\tau_r}$ is the phase angle of the skin friction where τ_r is the real part of $\left(\frac{\partial F(\eta, t)}{\partial \eta}\right)_{y=-0.5}$ and τ_i is the imaginary part of $\left(\frac{\partial F(\eta, t)}{\partial \eta}\right)_{y=-0.5}$. In the temperature profile, the rate of heat transfer (Nu) can be determined as $\left(\frac{\partial T(\eta, t)}{\partial \eta}\right)_{y=-0.5} = |T| \cos(\omega t + \psi)$. The amplitude and the phase angle of Nut-sent number are $|T| = \sqrt{T_r^2 + T_i^2}$ and $\psi = \tan^{-1} \frac{T_i}{T_r}$. T_r and T_i are real part and imaginary part of $\left(\frac{\partial T(\eta, t)}{\partial \eta}\right)_{y=-0.5}$ respectively. Also, the amplitude and phase angle of Sherwood number are derived as $\left(\frac{\partial \phi(\eta, t)}{\partial \eta}\right)_{y=-0.5} = |\phi| \cos(\omega t + \alpha)$ where $|\phi| = \sqrt{\phi_r^2 + \phi_i^2}$ and $\alpha = \tan^{-1} \frac{\phi_i}{\phi_r}$ are amplitude and phase angle respectively. ϕ_r is the real part of $\left(\frac{\partial \phi(\eta, t)}{\partial \eta}\right)_{y=-0.5}$ and ϕ_i is the imaginary part of $\left(\frac{\partial \phi(\eta, t)}{\partial \eta}\right)_{y=-0.5}$.

1. Discussion of the Results

In this section, the numerical results of resultant velocity profile, Temperature profile, concentration profile, skin friction, Nusselt number and sherwood number are discussed and presented graphically. The value of the parameters $\gamma = 0.2$, $\omega = 2$, $Re = 0.3$, $H = 2$, $m = 1$, $Gr = 2$, $K = 0.2$, $N = 1$, $Pe = 1$, $A = 3$, $\xi = 1$, $Sc = 0.3$, $Gc = 2$, $t = 0$ are kept unchanged through out the study unless otherwise stated. The convergence of DTM solution is shown in table(2). Table (3) shows the comparison of present work with K.D. Singh (2014) when the mass concentration of the fluid is neglected. The listed parameters in the table (4) and (5) represent different curves in figure (9),(10)and (11) respectively.

The effect of schmidt number and Grashof number are shown in Fig 1a and Fig 1b. An increase in Schimdt number is seen to lower the fluid flow while an increase in thermal Grashof number increases it due to the presence of buoyancy force. Fig 2a and Fig2b show the effect of mass

Grashof number and Hall current parameter on fluid flow. It is observed that an increase in mass Grashof number enhances the fluid flow. Similarly, an increase in Hall current parameter increases the fluid flow. Fig 3a and Fig 3b depict the effect of Hartman number and porosity parameter on the fluid flow. It is discovered that an increase in Hartman number lower the fluid flow due to the presence of Lorentz force in the magnetic field. On the other hand, an increase in porosity parameter increases the fluid flow as a result of weakness of the resistance caused by porous matrix. The effect of radiation parameter and Oscillation are studied in Fig 4a and Fig 4b respectively. It is noticed that as radiation parameter increases, there is a decrease in fluid flow. Similarly, an increase in oscillation parameter reduces the velocity of the fluid. The response of fluid flow to Peclet number and second grade parameter are shown in Fig 5a and Fig 5b. An increase in Peclet number lower the fluid velocity. The fluid velocity also decreases as second grade parameter is increased due to fluid thickening. In Fig 6a and Fig 6b the effect of Reynold number and chemical reaction are shown. From the result, an increase in Reynold number increases the fluid flow as a result of inertial force dominates the viscous force. However, the fluid velocity decreases as the chemical reaction is increased. The effect of oscillation parameter and Peclet number on the temperature of the fluid are depicted in Fig 7a and Fig 7b respectively. An increase in oscillation parameter reduces the temperature of the fluid. Similarly, an increase in Peclet number decreases the fluid temperature. Fig 8a and Fig 8b show the effects of chemical reaction and schmidt number on the mass concentration. It is observed that an increase in chemical reaction decreases the mass concentration. Also, the mass concentration decreases as the Schmidt number increases.

Table 2: Convergence of constants a_0 , a_1 , f_0 , f_1 , g_0 and g_1 for: $Gr = 2$, $m = 1$, $\gamma = 0.2$, $Sc = 0.3$, $K = 0.2$, $N = 1$, $H = 2$, $Pe = 1$, $Re = 0.3$, $\xi = 1$, $A = 3$, and $Gm = 2$, $\omega = 2$

n	a0	a1	f0	f1	g0	g1
1	0.15250531	0.10662095	0.42352941	0.95389507	0.48170116	0.98760013
2	0.16584630	0.11511400	0.42448761	0.95500143	0.48162590	0.98757006
3	0.16657637	0.11511400	0.42457627	0.95500143	0.48162590	0.98757006
4	0.16657637	0.11545320	0.42457627	0.95503325	0.48162590	0.98757006
5	0.16659812	0.11545320	0.42457689	0.95503325	0.48162590	0.98757006
6	0.16659812	0.11546112	0.42457690	0.95503336	0.48162590	0.98757006
7	0.16659853	0.11546111	0.42457689	0.95503336	0.48162590	0.98757006
8	0.16659853	0.11546124	0.42457689	0.95503336	0.48162590	0.98757006
9	0.16659853	0.11546124	0.42457689	0.95503336	0.48162590	0.98757006
10	0.16659853	0.11546124	0.42457689	0.95503336	0.48162590	0.98757006
11	0.16659853	0.11546124	0.42457689	0.95503336	0.48162590	0.98757006

Table 3: Comparison of present DTM solution of amplitude of skin friction $|F|$ and phase angle $|\phi|$ with the previous work K.D. Singh [21] for:

$$Gr = 5, \gamma = 0.2, m = 1, H = 2, N = 1, Re = 0.5, K = 0.2, Pe = 1, A = 5$$

ω	Exact solution of $ F $ in [21]	DTM solution of $ F $	Exact solution of $ \phi $ in [21]	DTM solution of $ \phi $
0	1.205737600	1.205737601	-0.1059310346	-0.105931048
5	0.8614145193	0.86141451037	-0.8771282494	-0.8771279190
10	0.5281367286	0.5281371948	-1.255480849	-1.255479787
15	0.3510434325	0.3510433038	-1.410199140	-1.410198436
20	0.2570903941	0.2570901205	-1.469707174	-1.469708055
25	0.2024952322	0.2024948194	-1.491795140	-1.491797257

Table 4: Parameters' values for different curves in Figure (9a) and (9b)

Gr	Gm	γ	m	Sc	II	ξ	N	Re	K	Pe	A	curves
2	2	0.2	1	0.3	2	1	1	0.3	0.2	1	3	I
5	2	0.2	1	0.3	2	1	1	0.3	0.2	1	3	II
2	5	0.2	1	0.3	2	1	1	0.3	0.2	1	3	III
2	2	0.4	1	0.3	2	1	1	0.3	0.2	1	3	IV
2	2	0.2	3	0.3	2	1	1	0.3	0.2	1	3	V
2	2	0.2	1	0.94	2	1	1	0.3	0.2	1	3	VI
2	2	0.2	1	0.3	4	1	1	0.3	0.2	1	3	VII

Table 5: Parameters' values for different curves in Figure (10a), (10b), (11a), and (11b)

Nusselt No.	Sherwood No.			
PeN	Sc	ξ	R	curves
1 1	0.30	10.3		I
3 1	0.94	1	0.3	II
1 3	0.3	3	0.3	III
	0.3	10.5		IV

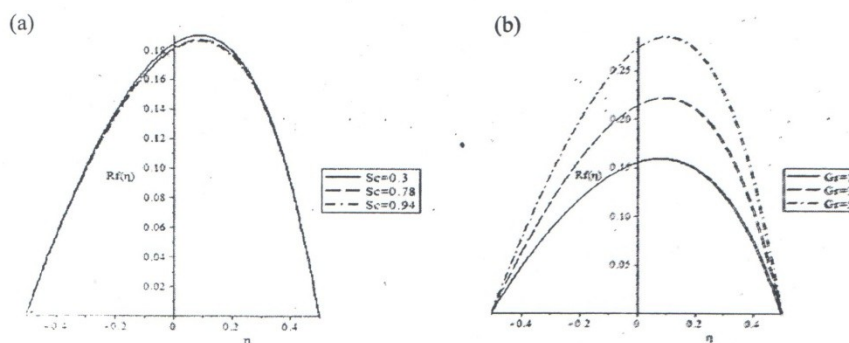


Figure 1: Resultant Velocity with change in (a) Schmidt number (b) Thermal Grashof number

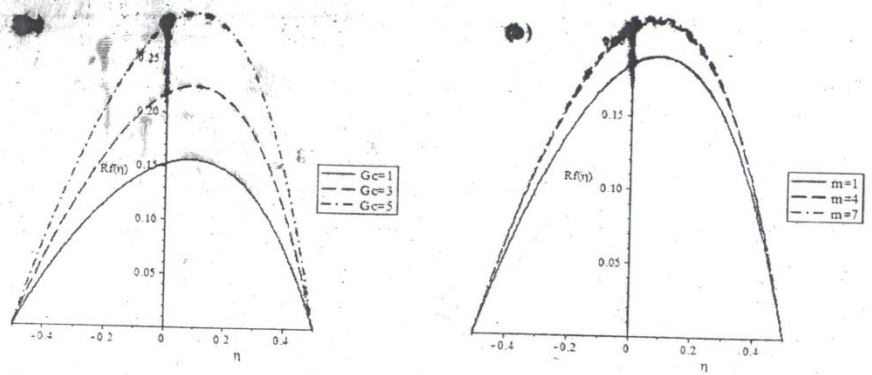


Figure 2: Resultant Velocity with change in (a) mass Grashof number (b) Hall current parameter

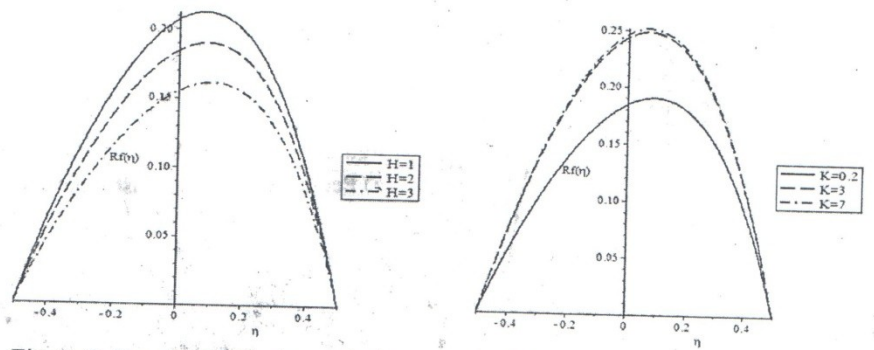
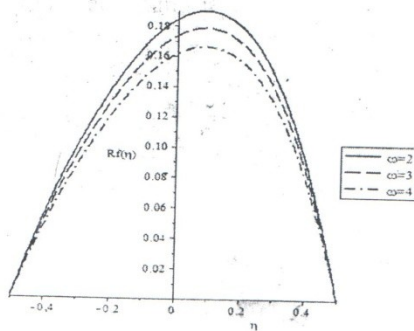


Figure 3: Resultant Velocity with change in (a) Hartman number (b) porosity parameter



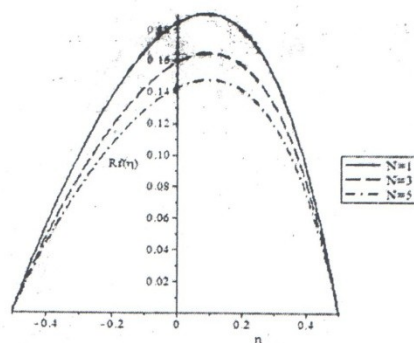


Figure 4: Resultant Velocity with change in (a) radiation parameter
(b) oscillation parameter

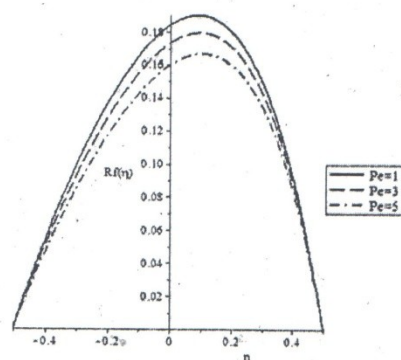
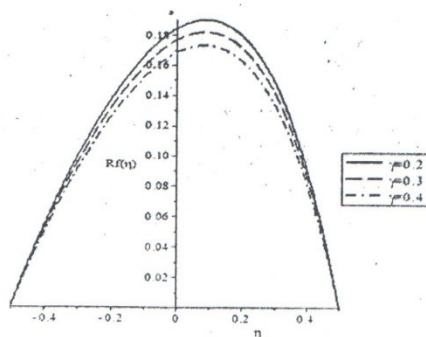


Figure 5: Resultant Velocity with change in
(a) Peclet number (b) elastic parameter



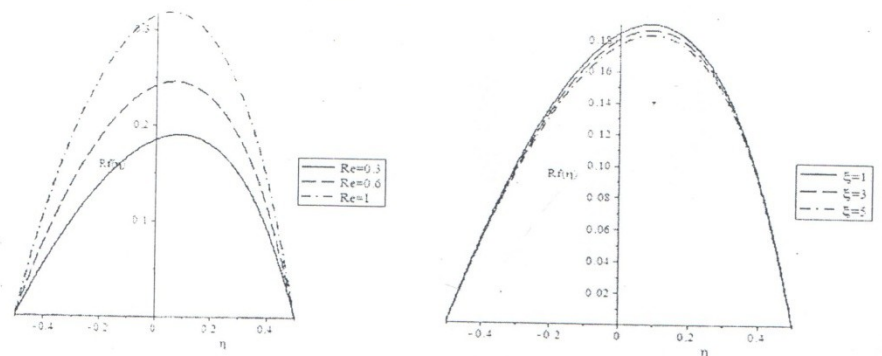


Figure 6: Resultant Velocity with change in (a) Reynold's number
(b) reaction parameter

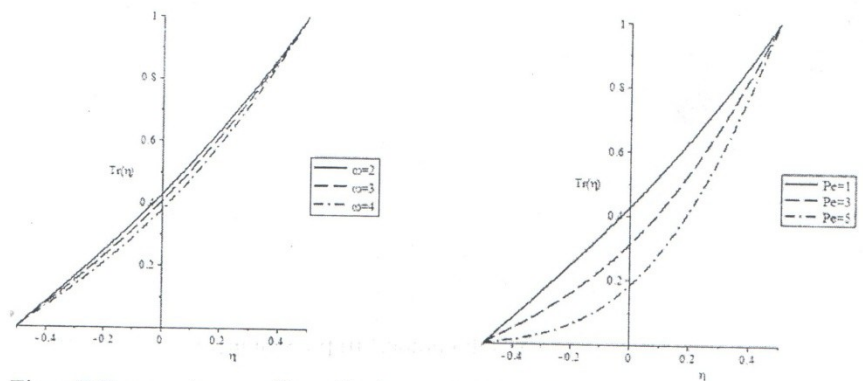


Figure 7: Temperature profile with change in (a) oscillation parameter
(b) Peclet number

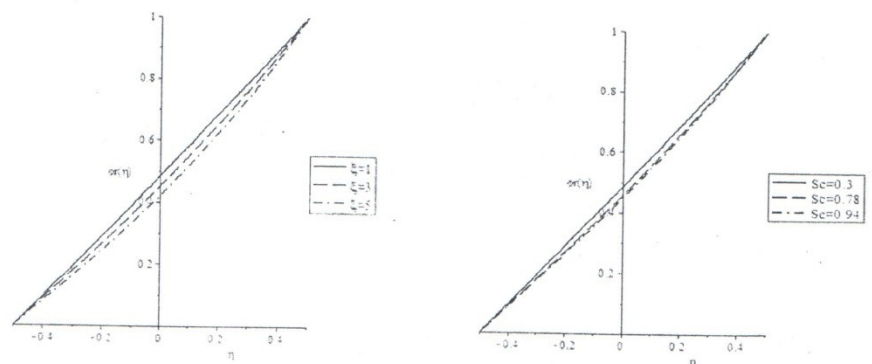


Figure 8: Concentration profile with change in (a) chemical reaction parameter
(b) Schmidt number

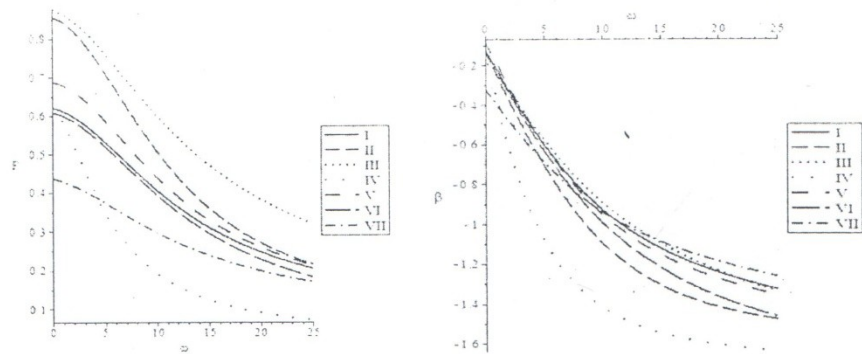


Figure 9: (a) Amplitude of the skin friction (b) Phase angle of the skin friction

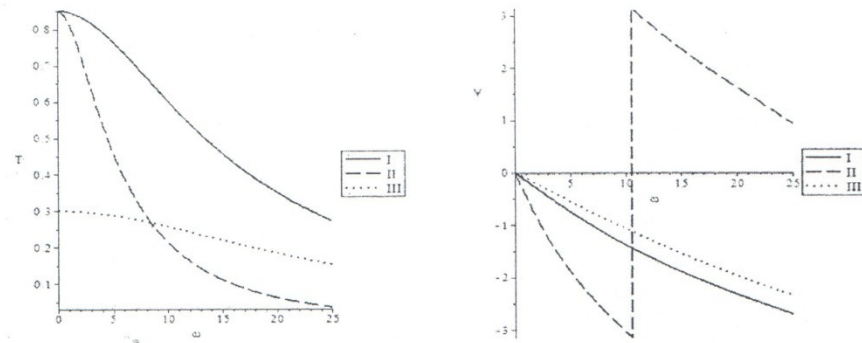


Figure 10: (a) Amplitude of Nuselt number (b) phase angle of Nuselt number

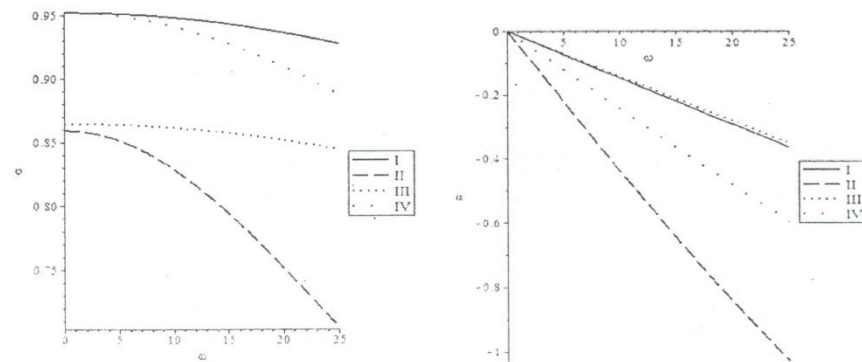


Figure 11: (a) Amplitude of Sherwood number (b) phase angle of Sherwood number

The values of flow parameters Gr , Gm , γ , m , Sc and H on the amplitude of the skin friction T_l at the left plate ($y = -0.5$) and the phase angle of the skin friction against the oscillation

frequency in table 4 are plotted in Fig 9a and Fig 9b. It observed that as the frequency of oscillation is increased, the amplitude of the skin friction decreases. The phase angle of the skin friction is lagged as a result of the fact that the values of the phase angle are negative all through as the frequency of the oscillation is increased. It is evidence from Fig 9a that as there is increase in Gr , Gm and m , the amplitude of the skin friction increases. However, the amplitude decreases as γ , Sc and H increases. Fig 9b depicts that the phase lag increases as Gr , γ , m and Sc increases, while it decreases with an increase in Gm . However, it is interesting to note that as the magnetic term is increased the phase lag increases for some values of ω and then later decreases.

The amplitude of Nusselt number and phase angle are examined in Fig 10a and Fig 10b against the oscillation frequency. As the Peclet number and radiation parameter are increased, the amplitude of Nusselt number decreases. However, Oscillation between phase lag and phase lead is experienced as Peclet number is increased while an increase in radiation parameter decreases the phase lag. Finally, Fig 11a and Fig 11b show the amplitude and phase angle of Sherwood number respectively. In Fig 11a, it is seen that the amplitude of Sherwood number decreases as chemical reaction parameter, Schmidt and Reynold number increased. The phase lag is experienced in Fig 11b as the phase angle is plotted against oscillation parameter. Increase in Sc number and Reynolds number increase the phase lag while an increase of chemical reaction, decreases phase angle.

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