

Steady Blood Flow through Vascular Stenosis under the Influence of Magnetic Field

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Abstract

The influence of magnetic field on a steady blood flow through vascular stenosed artery was investigated by assuming blood to be a Newtonian fluid. The Navier-Stokes equations governing the flow through the axial directions were solved using integral momentum method. The velocity profile graphs obtained shows that as magnetic field increases on the stenosis area the velocity decreases.

Keywords: Magnetic field, Newtonian fluid and stenosis.

NOMENCLATURE:

B_0 Magnetic field	σ Electrical conductivity
L_0 Length of the stenosis	μ Viscous coefficient of blood
M Magnetic parameter	τ_w Wall shear stress
P Pressure	τ_{w0} Wall shear stress in unobstructed tube
$\frac{\partial p}{\partial x}$ Pressure gradient	ρ Density,
Q flow flux,	Re Reynold's number,
R_0 Radius of normal tube,	$R_{(x)}$ Radius of stenosed tube,
$\frac{dR}{dx}$ Slope of the stenosed wall,	(u, w, v) Velocity components in (r, θ, x) ,
U Centerline velocity,	$\bar{U}_0$ Average velocity,
δ Maximum height of the stenosis,	(r, θ, x) Cylindrical polar coordinate,

Introduction

The vitality of the human body depends on a continuous supply of oxygen and nutrients delivered by the blood via the circulatory system. As such, the structural integrity and function of the circulatory system is the keystone of health. Diseases that impede blood flow through the circulatory system can cause serious, potentially

life-threatening effects throughout the body.

Vascular Stenosis (Atherosclerosis) is defined as the localized narrowing or constriction of the blood vessels due to abnormal growth of tissues, which causes resistance to blood flow through the artery. Once constriction is formed, the blood flow is significantly altered and vascular fluid dynamics factors play important role in the development and progression of arterial stenosis as the stenosis continues to enlarge leading to the development of cardiovascular diseases such as angina pectoris, heart attack, cardiac arrest, stroke and so on.

Many authors have reported that the rheologic and fluid dynamics properties of blood and its flow behavior through non-uniform cross-section of the tube could play an important role in the fundamental understanding, diagnosis and treatment of many cardiovascular diseases. Forrester and Young (1970) have studied blood flow in arteries with stenosis by considering blood as a Newtonian fluid. Results of experimental work on models of arterial stenoses were presented by Young and Tsai (1973). They observed that at Reynolds number 1000, the dimensionless pressure drop for severely constricted axisymmetric model is approximately 23 times the corresponding value of moderately constricted model. MacDonald (1979) studied the steady flow through a mild axisymmetric stenosed artery. The pressure drop and wall shear stress distribution for different Reynolds numbers and axial distance were discussed. El-Shahed (2003) showed that in the arterial system of human or animals, it is quite common to find localized narrowing, commonly called stenosis. In that study it was further described that these narrowing are caused by intravascular plaques. These stenosis disturb the normal pattern of blood flow through the artery. Perkkio and Keskinen (1983) studied the effects of concentrated flow of blood through a vessel with stenosis. They found out its important aspect from psychological point of view. It was observed that apparent viscosity increases as the stenosis grows but the same is not true in the absence of stenosis. The viscosity of blood flow has been found to vary with the radius of the artery in blood flow.

Recently, Mistra and Shit (2007), Bisswas and Chak Raborty (2010) and Verma et al. (2011) developed mathematical models for blood flows through arterial segment by taking a velocity slip condition at the constricted wall. Biswas and Laskas (2011) studied the influence of magnetic field and slip velocity on pulsatile blood flow through a constricted artery.

Furthermore, Hazarika and Sharma (2014), investigated two-layered model of blood flow through composite stenosed artery in porous medium under the effect of applied magnetic field by considering blood to be a Newtonian fluid of variable viscosity in the central region and plasma fluid of constant viscosity in the peripheral region of the stenotic tube.

Thus in this present study, we present the mathematical modeling formulated to study the effect of magnetic field on a stenosed vascular artery.

Mathematical Formulation

A mathematical model of the axisymmetric flow of blood in an uniform circular tube with an axisymmetric mild stenosis under the influence of magnetic field was considered by treating blood as a Newtonian fluid. We use cylindrical polar coordinates $(\bar{r}, \bar{\theta}, \bar{x})$, with $\bar{x}$ as the central axis of the artery and $(\bar{u}, \bar{w}, \bar{v})$ denoting the velocity component of blood along the axial, tangential and radial direction respectively. Due to the axisymmetric flow of blood being considered, the flow variation is independent of $\bar{\theta}$. Figure 1 illustrates the geometry of the stenosis together with a standard cylindrical polar coordinate system used for theoretical analysis.

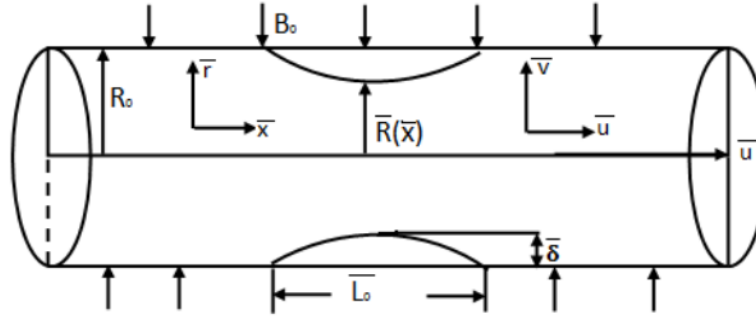


Figure 1: Geometry of Stenosis

The axial coordinate and velocity are $\bar{x}$ and $\bar{u}$ respectively. The radial coordinate and velocity are $\bar{r}$ and $\bar{v}$ respectively. The geometry of the stenosis assumed to be axisymmetric is described by Young (1968) as;

$$\bar{R}(\bar{x}) = R_0 - \frac{\bar{\delta}}{2} \left(1 + \cos \frac{2\pi}{\bar{L}_0} \bar{x} \right) \quad (1)$$

Where $\bar{R}(\bar{x})$ is the radius of the stenosed tube, R_0 is the radius of normal tube, $\bar{\delta}$ is the maximum height of the stenosis, $\bar{x}$ is the axial coordinate and $\bar{L}_0$ is the length of the stenosis. Some dimensional variables are designated by the notation (-) to differentiate them from corresponding dimensionless variables.

Flow Analysis

With this given model, a two-dimensional steady, incompressible blood flow under the influence of externally applied magnetic field through the stenosis vascular is governed by Navier-Stokes equation and the continuity equation given below respectively;

$$\bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{r}} = -\frac{1}{\rho} \left(\frac{\partial \bar{p}}{\partial \bar{x}} \right) + \frac{\mu}{\rho} \left(\frac{\partial^2 \bar{u}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} \right) - \frac{\sigma B_0^2 \bar{u}}{\rho} \quad (2)$$

in the axial direction and

$$\bar{u} \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{r}} = -\frac{1}{\rho} \left(\frac{\partial \bar{p}}{\partial \bar{r}} \right) + \frac{\mu}{\rho} \left(\frac{\partial^2 \bar{v}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\partial^2 \bar{v}}{\partial \bar{x}^2} - \frac{\bar{v}}{\bar{r}^2} \right) \quad (3)$$

in the radial direction.

The continuity equation can be written as

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\bar{v}}{\bar{r}} = 0 \quad (4)$$

where $\bar{u}$ represents the axial velocity, $\bar{v}$ the radial velocity, ρ the density, $\bar{p}$ the pressure, σ the electrical conductivity, B_0 the applied magnetic field, μ the viscosity coefficient of blood, $\bar{x}$ and $\bar{r}$ the axial and radial coordinates respectively.

The above equations can be rewritten in dimensionless form using the following dimensionless transformations

$$\begin{aligned} r &= \frac{\bar{r}}{R_0}, & x &= \frac{\bar{x}}{R_0}, & u &= \frac{\bar{u}}{\bar{U}_0}, & v &= \frac{\bar{v}}{\bar{U}_0}, \\ p &= \frac{\bar{p}}{\rho \bar{U}_0^2}, & R &= \frac{\bar{R}}{R_0}, & U &= \frac{\bar{U}}{\bar{U}_0}, & \delta &= \frac{\bar{\delta}}{R_0}, & L_0 &= \frac{\bar{L}_0}{R_0} \end{aligned} \quad (5)$$

where $\bar{U}$ is the centerline velocity and $\bar{U}_0$ is the average velocity in the unobstructed tube. Using these transformations in equation (1) we derived the geometry of the stenosis in dimensionless form as;

$$R(x) = 1 - \frac{\delta}{2} \left(1 + \cos \frac{2\pi}{L_0} x \right) \quad (6)$$

Similarly, the Navier-Stokes equation are derived as

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = -\frac{\partial p}{\partial x} + \frac{2}{Re} \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial x^2} \right] - \frac{2M^2}{Re} u \quad (7)$$

in the axial direction; and

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial r} = -\frac{\partial p}{\partial r} + \frac{2}{Re} \left[\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial x^2} - \frac{v}{r^2} \right] \quad (8)$$

in the radial direction, where

$$Re = \frac{2R_0 \bar{U}_0 \rho}{\mu} \text{ is Reynolds number and } M = B_0 R_0 \sqrt{\frac{\sigma}{\mu}} \text{ is Magnetic parameter.}$$

The continuity equation then becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r} = 0 \quad (9)$$

It is assumed that the term due to viscous component of the normal stress in the axial direction ($\partial^2 u / \partial x^2$) are negligible. This assumption has been used in the analysis of non-uniform flow. Therefore equation (7) becomes

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = -\frac{\partial p}{\partial x} + \frac{2}{R_e} \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right] - \frac{2M^2}{R_e} u \quad (10)$$

Method of Solution

We employed the momentum integral technique to resolve equation (10) over the cross-section of the tube with boundary condition, $v = 0$, (that is, no slip in the radial velocity at the stenosed wall) and equation (10) was multiply by ∂r to obtain the integral-momentum equation

$$u \frac{\partial u}{\partial x} \partial r + v \frac{\partial u}{\partial r} \partial r = -\frac{\partial p}{\partial x} \partial r + \frac{2}{R_e} \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right] \partial r - \frac{2M^2}{R_e} u \partial r \quad (11)$$

From the continuity equation (9),

$$\frac{\partial u}{\partial x} = -\left(\frac{\partial v}{\partial r} + \frac{v}{r} \right) \quad (12)$$

Substitute equation (12) into equation (11) and multiply through by r we to obtain

$$-ru \frac{\partial v}{\partial r} \partial r - uv \partial r + rv \frac{\partial u}{\partial r} \partial r = -r \frac{\partial p}{\partial x} \partial r + \frac{2r}{R_e} \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right] \partial r - \frac{2M^2}{R_e} ru \partial r \quad (13)$$

From equation (9),

$$\frac{\partial v}{\partial r} = -\left(\frac{\partial u}{\partial x} + \frac{v}{r} \right) \quad (14)$$

Substituting equation (14) into equation (13) we obtain

$$ru \left(\frac{\partial u}{\partial x} + \frac{v}{r} \right) \partial r - uv \partial r + rv \frac{\partial u}{\partial r} \partial r = -r \frac{\partial p}{\partial x} \partial r + \frac{2r}{R_e} \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right] \partial r - \frac{2M^2}{R_e} ru \partial r \quad (15)$$

Since $v = 0$,

$$ru \frac{\partial u}{\partial x} \partial r = -r \frac{\partial p}{\partial x} \partial r + \frac{2r}{R_e} \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right] \partial r - \frac{2M^2}{R_e} ru \partial r \quad (16)$$

Integrating equation (16) over the cross section of the tube at $r = R$

$$\frac{\partial}{\partial x} \int_0^R r u^2 dr = -\frac{\partial p}{\partial x} \int_0^R r dr + \frac{2}{R_e} \int_0^R r \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right] dr - \frac{2M^2}{R_e} u \int_0^R r dr \quad (17)$$

$$\frac{\partial}{\partial x} \int_0^R r u^2 dr = -\frac{R^2}{2} \frac{\partial p}{\partial x} + \frac{2}{R_e} \left[\left(r \frac{\partial u}{\partial r} \right) \Big|_0^R - \int_0^R \frac{\partial u}{\partial r} dr \right] + \int_0^R \frac{\partial u}{\partial r} dr - \frac{M^2 R^2}{R_e} u \quad (18)$$

$$\frac{\partial}{\partial x} \int_0^R r u^2 dr = -\frac{R^2}{2} \frac{\partial p}{\partial x} + \frac{2R}{R_e} \left(\frac{\partial u}{\partial r} \right)_R - \frac{M^2 R^2}{R_e} u \quad (19)$$

Applying the velocity-profile constraints for axisymmetric tube flow as stated by Verma (2009)

$$u = U \quad \text{at} \quad r = 0 \quad (20)$$

$$u = 0 \quad \text{at} \quad r = R \quad (21)$$

$$\frac{\partial u}{\partial r} = 0 \quad \text{at} \quad r = 0 \quad (22)$$

$$\frac{\partial^3 u}{\partial r^3} = 0 \quad \text{at} \quad r = 0 \quad (23)$$

$$\frac{\partial p}{\partial x} = \frac{2}{R_e} \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right] \quad \text{at} \quad r = R \quad (24)$$

Equation (20) is defining the center line velocity, equation (21) is stating the no-slip condition at the wall. While equation (22) defines the regularity condition and it is deduce by considering the forces on a cylindrical element as if the pressure and the inertial forces are to be infinity as the radius of the element tends to zero, the viscous force that is proportional to $\partial u / \partial r$ must tends to zero.

Equation (23) was obtained by eliminating the pressure between equation (2) and equation (3) and considering the resulting equation as r tends to zero.

Equation (24) represents the validity of equation (5) at $r = R$.

In obtaining the velocity profile, the fourth degree polynomial used by Verma (2009) was used. That is,

$$\frac{u}{U} = A + B \left(\frac{r}{R} \right) + C \left(\frac{r}{R} \right)^2 + D \left(\frac{r}{R} \right)^3 + E \left(\frac{r}{R} \right)^4 \quad (25)$$

The velocity constraints (20) – (24) together with (25) give the values A, B, C, D and E as $A = 1$, $B = 0$, $C = -\frac{1}{3}(4 - \lambda)$, $D = 0$, $E = \frac{1}{3}(1 - \lambda)$, respectively.

Substituting these values into equation (25) we obtain,

$$\frac{u}{U} = 1 - \frac{1}{3}(4 - \lambda) \left(\frac{r}{R}\right)^2 + \frac{1}{3}(1 - \lambda) \left(\frac{r}{R}\right)^4 \quad (26)$$

where

$$\lambda = -\frac{R_e R^2}{8U} \frac{\partial p}{\partial x} \quad (27)$$

It is observed that if $\lambda = 1$ it is a parabolic profile corresponding to the Poiseuille profile

$$\frac{u}{U} = 1 - \left(\frac{r}{R}\right)^2 \quad (28)$$

The flow flux Q is given by

$$Q = \pi R^2 U_a = 2\pi \int_0^R r u dr \quad (29)$$

where $U_a = \bar{\bar{U}}/\bar{U}_0$ is non-dimensional average velocity at any cross section.

Substituting equation (26) into the equation (29) we obtain

$$Q = \frac{1}{18} \pi U R^2 (8 + \lambda) \quad (30)$$

Substituting the value of λ from (27) in (30) we obtain the expression for centerline velocity as

$$U = \frac{9}{4\pi R^2} \left[Q + \frac{\pi R_e}{144} R^4 \frac{\partial p}{\partial x} \right] \quad (31)$$

The parameter λ is determined from equation (19) using (25) and (27) hence from (19)

$$\left(\frac{\partial u}{\partial r}\right)_R = \frac{\partial}{\partial r} \left(U \left[1 - \frac{1}{3}(4 - \lambda) \left(\frac{r}{R}\right)^2 + \frac{1}{3}(1 - \lambda) \left(\frac{r}{R}\right)^4 \right] \right)_R \quad (32)$$

$$\left(\frac{\partial u}{\partial r}\right)_R = -\frac{2U}{3R} (2 + \lambda) \quad (33)$$

now putting (33) into (19) we have

$$\frac{\partial}{\partial x} \int_0^R r u^2 dr = -\frac{R^2}{2} \frac{\partial p}{\partial x} - \frac{4U}{3R_e} (2 + \lambda) - \frac{M^2 R^2}{R_e} u \quad (34)$$

also from (26)

$$-\frac{R^2}{2} \frac{\partial p}{\partial x} = \frac{4U\lambda}{R_e} \quad (35)$$

substituting (35) in (34) to obtain

$$\frac{\partial}{\partial x} \int_0^R r u^2 dr = \frac{4U\lambda}{R_e} - \frac{4U}{3R_e} (2 + \lambda) - \frac{M^2 R^2}{R_e} u \quad (36)$$

simplifying (36) to get

$$\lambda = 1 + \frac{3}{8U} \left[M^2 R^2 u + R_e \frac{\partial}{\partial x} \int_0^R r u^2 dr \right] \quad (37)$$

In the velocity profile equation (26) neglecting terms with degree higher than two in r , and $\lambda = 1$, we have the Poiseuille profile

$$u = 2U_a \left[1 - \left(\frac{r}{R} \right)^2 \right] \quad (38)$$

where

$$U_a = -\frac{R_e R^2}{16} \frac{\partial p}{\partial x} \quad (39)$$

is the non-dimensional average velocity at any given cross-section and $\partial p / \partial x < 0$.

Using equation (27), (29) and (31) in (19) we obtain the expression for pressure gradient. From (19)

$$\left(\frac{\partial u}{\partial r} \right)_R = \frac{\partial}{\partial r} \left[2U_a \left[1 - \left(\frac{r}{R} \right)^2 \right] \right]_R \quad (40)$$

$$= -\frac{4U_a}{R} \quad (41)$$

and

$$\frac{\partial}{\partial x} \int_0^R r u^2 dr = \frac{\partial}{\partial x} \int_0^R r \left[2U_a \left[1 - \left(\frac{r}{R} \right)^2 \right] \right]^2 dr \quad (42)$$

on simplifying we have

$$\frac{\partial}{\partial x} \int_0^R r u^2 dr = \frac{2U_a^2}{3} R \frac{dR}{dx} \quad (43)$$

substituting the expressions of $\left(\frac{\partial u}{\partial r}\right)_R$ and $\frac{\partial}{\partial x} \int_0^R r u^2 dr$ into (19) we have

$$\frac{2U_a^2}{3} R \frac{dR}{dx} = -\frac{R^2}{2} \frac{\partial p}{\partial x} - \frac{2R}{R_e} \left(\frac{4U_a}{R}\right) - \frac{M^2 R^2}{R_e} \left[2U_a \left[1 - \left(\frac{r}{R}\right)^2\right]\right] \quad (44)$$

simplifying (44) further to obtain

$$\frac{4U_a^2}{3R} \frac{dR}{dx} = \frac{8U}{R_e R^2} - \frac{16U_a}{R_e R^2} - \frac{4U_a M^2}{R_e} \left[1 - \left(\frac{r}{R}\right)^2\right] \quad (45)$$

substituting for U in (45) above to get

$$\frac{4U_a^2}{3R} \frac{dR}{dx} = \frac{8}{R_e R^2} \left[\frac{9}{4\pi} \frac{1}{R^2} \left(Q + \frac{\pi R_e}{144} R^4 \frac{\partial p}{\partial x}\right)\right] - \frac{16U_a}{R_e R^2} - \frac{4U_a M^2}{R_e} \left[1 - \left(\frac{r}{R}\right)^2\right] \quad (46)$$

simplifying (46) and using $U_a = \frac{Q}{\pi R^2}$ and $U_a^2 = \frac{Q^2}{\pi^2 R^4}$ we obtain

$$\frac{4Q^2}{3\pi^2 R^5} \frac{dR}{dx} = \frac{18Q}{\pi R_e R^4} + \frac{1}{8} \frac{\partial p}{\partial x} - \frac{16Q}{\pi R_e R^4} - \frac{4QM^2}{\pi R_e R^2} \left[1 - \left(\frac{r}{R}\right)^2\right] \quad (47)$$

simplifying (47) further to obtain the required pressure gradient. From (19)

$$\frac{\partial p}{\partial x} = \frac{32Q^2}{3\pi^2 R^5} \frac{dR}{dx} - \frac{16Q}{\pi R_e R^4} + \frac{32QM^2}{\pi R_e R^2} \left[1 - \left(\frac{r}{R}\right)^2\right] \quad (48)$$

Further using equation (27) and (31) in (26) we obtain the expression for velocity profile as follows:

from (26) we have

$$u = U \left[3 - 4 \left(\frac{r}{R}\right)^2 + \left(\frac{r}{R}\right)^4\right] + \lambda U \left[\left(\frac{r}{R}\right)^2 - \left(\frac{r}{R}\right)^4\right] \quad (49)$$

substituting (27) and (31), for U and λ respectively we have (50)

$$u = \frac{9}{4\pi} \frac{1}{R^2} \left[Q + \frac{\pi R_e}{144} R^4 \frac{\partial p}{\partial x}\right] \left[3 - 4 \left(\frac{r}{R}\right)^2 + \left(\frac{r}{R}\right)^4\right] - \frac{R_e R^2}{8} \frac{\partial p}{\partial x} \left[\left(\frac{r}{R}\right)^2 - \left(\frac{r}{R}\right)^4\right] \quad (50)$$

simplifying (50) and substituting (48) for $\frac{\partial p}{\partial x}$ we have

$$u = \frac{R_e R^2}{192} \left\{ \frac{32Q^2}{3\pi^2 R^5} \frac{dR}{dx} - \frac{16Q}{\pi R_e R^4} + \frac{32QM^2}{\pi R_e R^2} \left[1 - \left(\frac{r}{R}\right)^2\right] \right\}$$

$$\left[3 - 12 \left(\frac{r}{R}\right)^2 + 9 \left(\frac{r}{R}\right)^4\right] + \frac{3}{4\pi R^2} \left[3 - 4 \left(\frac{r}{R}\right)^2 + \left(\frac{r}{R}\right)^4\right] \quad (51)$$

The flow flux in normal unobstructed blood vessel, $\bar{Q} = \pi R_0^2 \bar{U}_0$. This gives non-dimensional flow flux $Q_0 = \bar{Q}_0 / R_0^2 \bar{U}_0 = \pi$, which is same for unobstructed blood vessel. Therefore replacing Q by Q_0 i.e. Q by π in equation (48) and (51) we obtain the expression for pressure gradient and velocity profile, respectively, as

$$\frac{\partial p}{\partial x} = \frac{32}{3} \frac{1}{R^5} \frac{dR}{dx} - \frac{16}{R_e R^4} + \frac{32M^2}{R_e R^2} \left[1 - \left(\frac{r}{R}\right)^2\right] \quad (52)$$

and

$$u = \frac{2}{R^2} \left[1 - \left(\frac{r}{R}\right)^2\right] + \frac{1}{18} \frac{R_e}{R^3} \frac{dR}{dx} \left[3 - 12 \left(\frac{r}{R}\right)^2 + 9 \left(\frac{r}{R}\right)^4\right] + \frac{M^2}{6} \left[3 - 15 \left(\frac{r}{R}\right)^2 + 21 \left(\frac{r}{R}\right)^4 - 9 \left(\frac{r}{R}\right)^6\right] \quad (53)$$

The wall shearing stress is expressed as

$$\bar{\tau}_w = \mu \left(\frac{\partial \bar{u}}{\partial \bar{r}}\right)_{\bar{R}} \left[1 + \left(\frac{d\bar{R}}{d\bar{x}}\right)^2\right] \quad (54)$$

where $d\bar{R}/d\bar{x}$ is the slope of the wall. The dimensionless wall shearing stress is defined as

$$\tau_w = \frac{\bar{\tau}_w}{\tau_{w0}}$$

where τ_{w0} is the corresponding shearing stress due to Poiseuille flow in the unobstructed tube. Thus, in terms of dimensionless variables,

$$\tau_w = -\frac{1}{4} \left(\frac{\partial u}{\partial r}\right)_R \left[1 + \left(\frac{dR}{dx}\right)^2\right] \quad (55)$$

Now, from (53) we get

$$\begin{aligned} \left(\frac{\partial u}{\partial r}\right)_R &= -\frac{2}{R^2} \left(\frac{2R}{R^2}\right) - \frac{1}{18} \frac{R_e}{R^3} \frac{dR}{dx} \left(\frac{24R}{R^2} - \frac{36R^3}{R^4}\right) - \frac{M^2}{6} \left(\frac{30R}{R^2} - \frac{84R^3}{R^4} + \frac{54R^5}{R^6}\right) \\ &= -\frac{4}{R^3} - \frac{1}{18} \frac{R_e}{R^3} \frac{dR}{dx} \left(-\frac{12}{R}\right) - \frac{M^2}{6} (0) \end{aligned}$$

$$= -\frac{4}{R^3} + \frac{12 R_e}{18 R^4} \frac{dR}{dx}$$

$$\left(\frac{\partial u}{\partial r}\right)_R = -\frac{4}{R^3} \left[1 - \frac{1 R_e}{6 R} \frac{dR}{dx}\right] \quad (56)$$

Thus, (55) becomes

$$\tau_w = -\frac{1}{4} \left\{ -\frac{4}{R^3} \left[1 - \frac{1 R_e}{6 R} \frac{dR}{dx}\right] \right\} \left[1 + \left(\frac{dR}{dx}\right)^2\right]$$

$$\tau_w = \frac{1}{R^3} \left[1 - \frac{1 R_e}{6 R} \frac{dR}{dx}\right] \left[1 + \left(\frac{dR}{dx}\right)^2\right] \quad (57)$$

At points along the wall where $\tau_w = 0$, flow separation and reattachment will occur. From equation (57), the condition of zero wall shear stress is given as

$$\frac{dR}{dx} = \frac{6R}{R_e} \quad (58)$$

when $\frac{dR}{dx} > \frac{6R}{R_e}$, τ_w is negative, indicating back-flow and the separated region is clearly described.

Discussion of Result

The behaviour of the flow of blood through a compressed vessel with mild stenosis under the influence of magnetic field have been investigated by modeling of blood as a Newtonian fluid. This Study is concerned with the approximate solution for velocity profile, wall shearing stress, pressure gradient and separation phenomena for specific geometry of the stenosis. It involves integral method for the solution of the problem.

Analytic expressions have been computed numerically for different Reynolds number. It has been suggested by Forrester and Young (1970) in the theoretical and experimental studies, the stenosis parameter R_0 , $\bar{L}_0$ and δ are taken to be related as $\bar{L}_0 = 8R_0 = 24\delta$. The computation has been carried out for the location of stenosis defined by $-4 \leq x \leq 4$ for $\delta = 1/3$ and $L_0 = 8$ for different Reynolds number between 10-100.

Figures 1 to 15 are the non-dimensional axial velocity profiles of blood flow in the mild stenosed vessel segment ($\delta = 1/3$) with the influence of magnetic field under

the consideration for Reynolds number 10, 50 and 100 respectively. It is observed that in converging region near the throat of the stenosis, velocity increases for each value of x as magnetic field increases. It is also observed that in converging region near the throat of the stenosis, velocity decreases but increases away from the throat as Reynolds number increases for r/R up to 0.5 and then in reverse order for $r/R > 0.5$, while in diverging region, all above observations are reversed as Reynolds number increases.

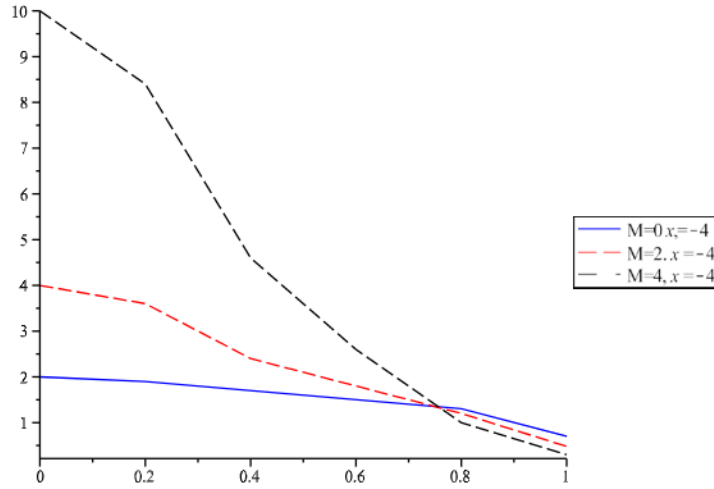


Figure 1: Velocity Profile for $Re=10$ and $\delta=1/3$

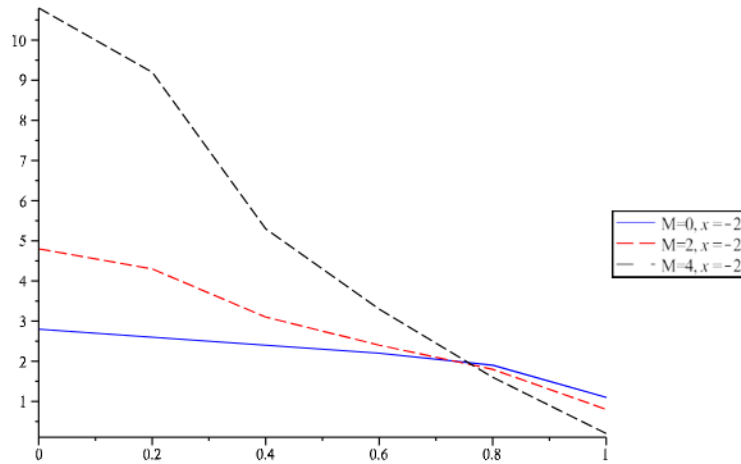


Figure 2: Velocity Profile for $Re=10$ and $\delta=1/3$

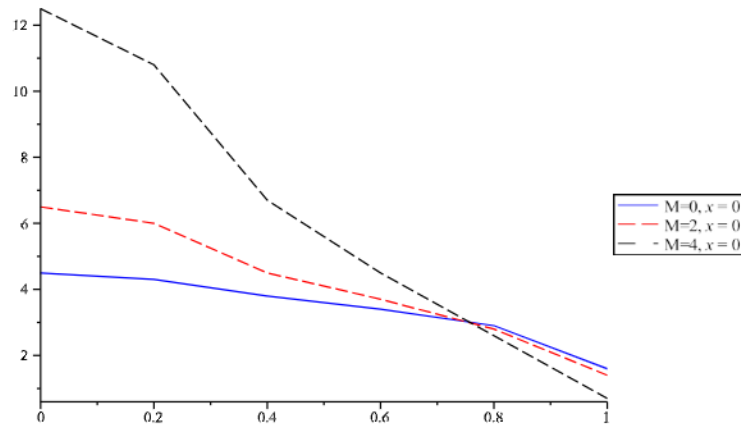


Figure 3: Velocity Profile for $Re=10$ and $\delta=1/3$

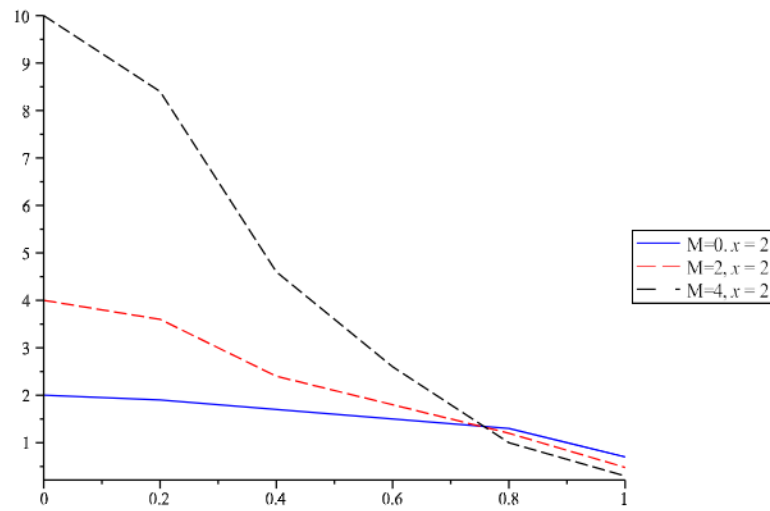


Figure 4: Velocity Profile for $Re=10$ and $\delta=1/3$

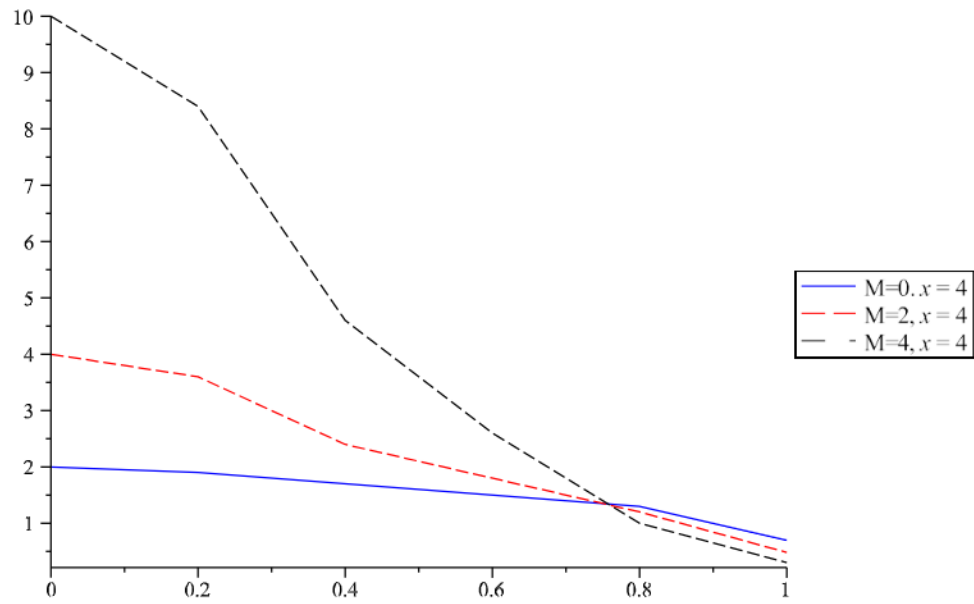


Figure 5: Velocity Profile for $Re=10$ and $\delta=1/3$

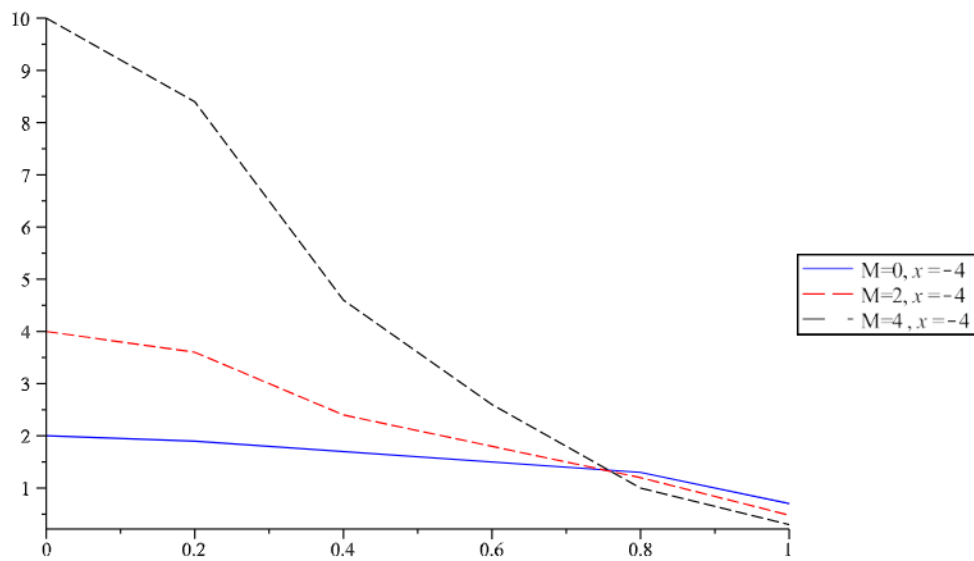


Figure 6: Velocity Profile for $Re=50$ and $\delta=1/3$

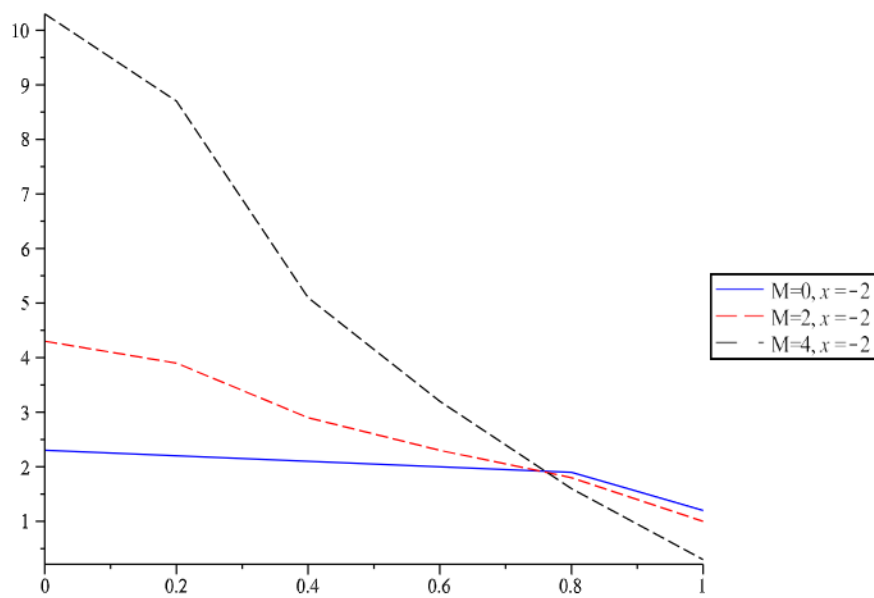


Figure 7: Velocity Profile for $Re=50$ and $\delta=1/3$

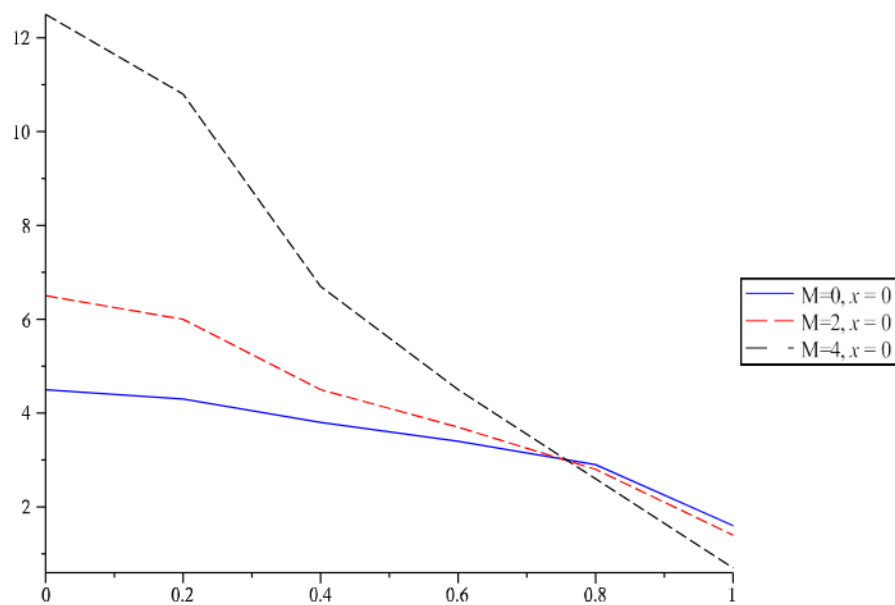


Figure 8: Velocity Profile for $Re=50$ and $\delta=1/3$

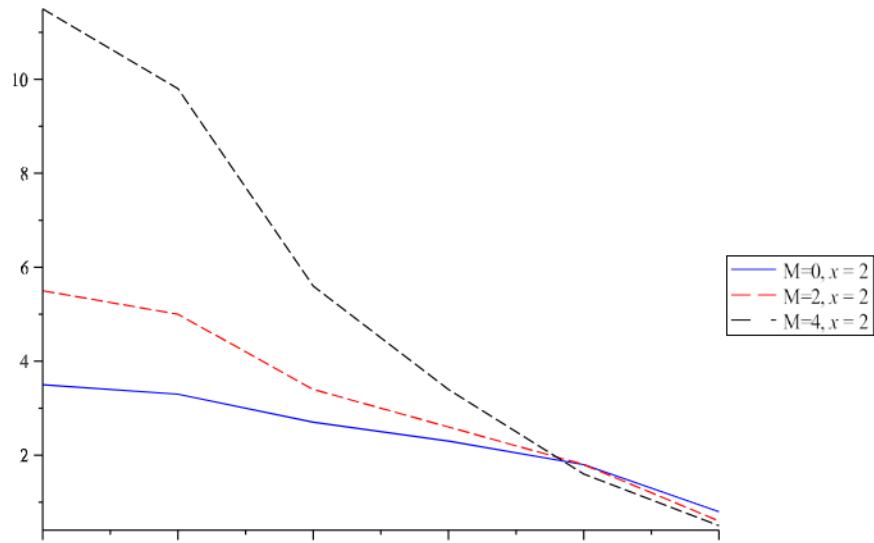


Figure 9: Velocity Profile for $Re=50$ and $\delta=1/3$

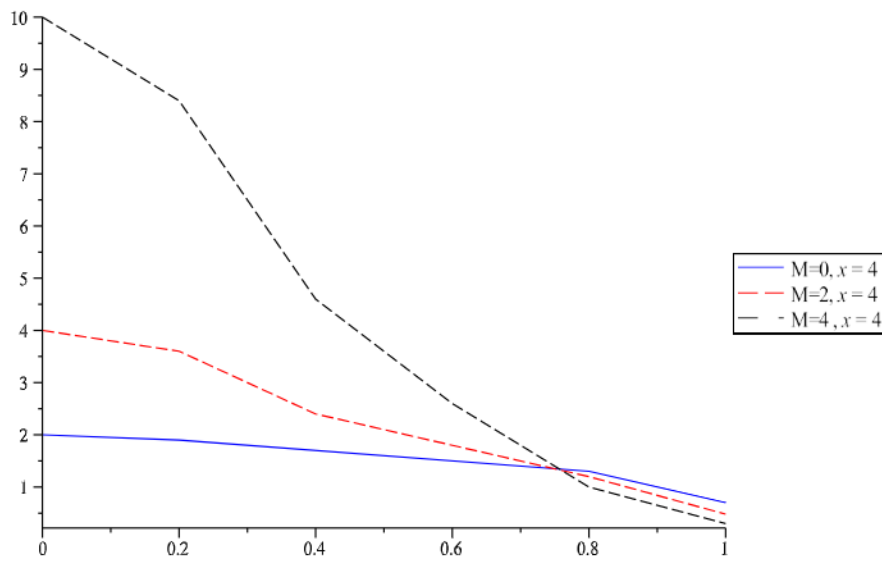


Figure 10: Velocity Profile for $Re=50$ and $\delta=1/3$

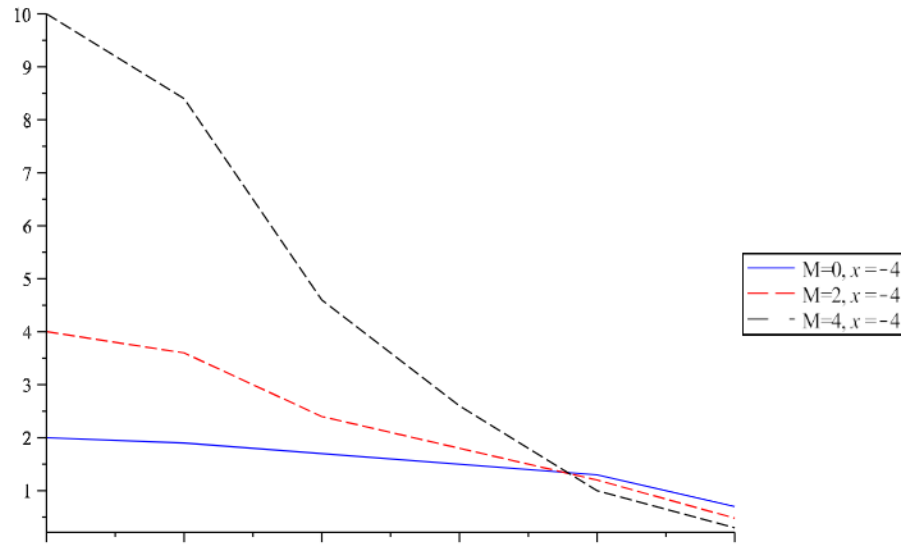


Figure 11: Velocity Profile for $Re=100$ and $\delta=1/3$

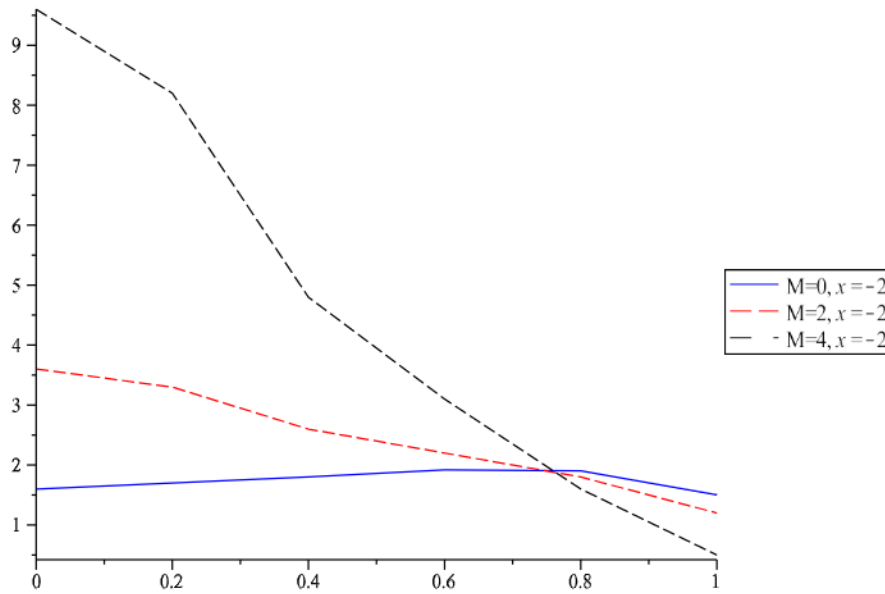


Figure 12: Velocity Profile for $Re=100$ and $\delta=1/3$

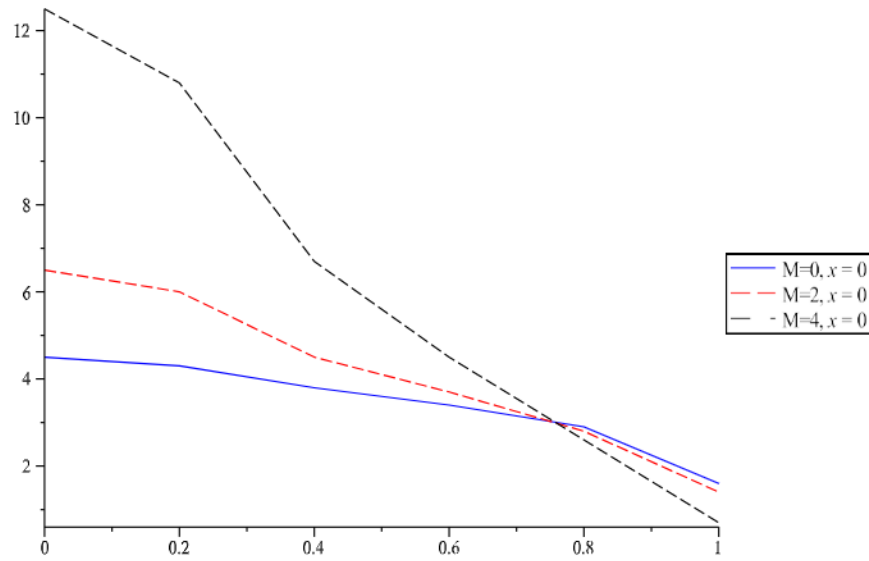


Figure 13: Velocity Profile for $Re=100$ and $\delta=1/3$

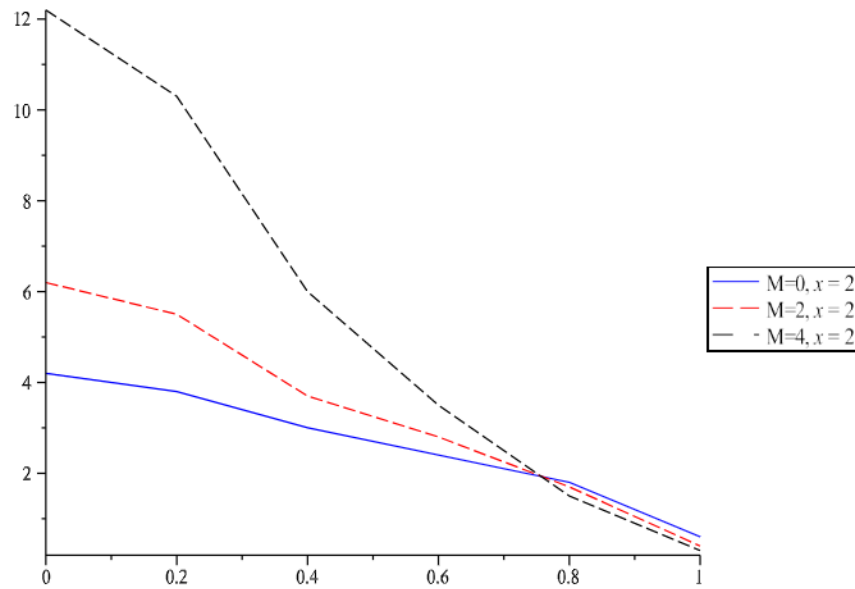


Figure 14: Velocity Profile for $Re=100$ and $\delta=1/3$

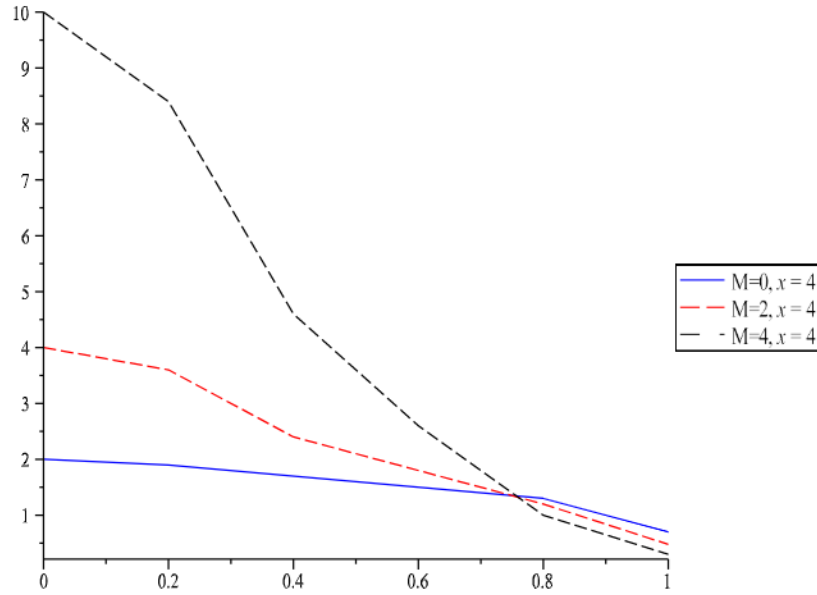


Figure 15: Velocity Profile for $Re=100$ and $\delta=1/3$

Conclusion

In this study, the continuity equation and Navier-Stokes equation have been converted from Cartesian form to cylindrical form. Considering the continuity and Navier-Stokes equation which were derived from the conservation of mass and Cauchy's equation respectively. These equations were then solved using the integral-momentum technique and the obtained results were used to plot the graphs showing the behaviour of the blood at the stenosed part. The effect of mild stenosis in an artery under the influence of magnetic field by considering the blood as a Newtonian fluid. It has been concluded that the velocity of the blood decreases as the size of the stenosis increases for a given Newtonian model of the blood.

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