

THE EFFECT OF VISCOUS DAMPING ON THE DYNAMIC BEHAVIOUR OF RECTANGULAR PLATES RESTING ON ELASTIC FOUNDATION UNDER MOVING LOADS

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ABSTRACT

The effect of viscous damping on the dynamic behaviour of a plate under a moving line load is studied. The problem is formulated within the framework of the classical small-deflection theory of thin plates. The solution to the problem are obtained by transforming the partial differential equation for the problem to coupled ordinary differential equations which were analyzed and evaluated for simply supported boundary conditions. It is shown that the deflection profile of the plate depends on the magnitude of the damping coefficient.

Introduction

The multi-parameter problem of determining the dynamic response of elastic structures, especially bridges, carrying moving loads is a very old and complicated problem. Several authors have reported (and are still reporting) many useful and reliable findings of their work on the dynamic response of railway bridges and later of highway bridges under the influence of moving loads. A comprehensive review of past work on the subject can be found in [1]. Three of the early work in this area are those of Stokes [2], Zimmerman [3] and Krylov [4]. Recently, the dynamic analysis of an isotropic plate continually supported by an elastic foundation traversed by an arbitrary number of concentrated masses was carried out in [5]. It is found that the critical speed of the load which brings about a resonance increase as the values of the foundation moduli increase. An earlier work was carried out by Stanisic et. al [6] where it was concluded that the natural frequency of rectangular plates traversed by moving concentrated forces is greater than that of plates subjected to moving concentrated masses. In [7], the problem of determining the dynamic behaviour of non-Mindlin rectangular plates subjected to uniform partially distributed moving loads was studied. More recently, the dynamic analysis of rectangular plates resting on Pasternak foundation and subjected to uniform partially distributed moving masses was carried out in [8]. It was found that an increase in the area of the distribution of the moving mass causes a reduction in the maximum dynamic deflection. In all these papers, the effect of damping is not taken into consideration. Nevertheless, it is known that in order to have a deep insight into the control and mechanical response of vibrating structures, such as vibrating plates under consideration, it is importantly necessary to carry out a damping analysis of the whole system.

Hence, the problem discussed in what follows is that of the vibration of rectangular, damped plates resting on an elastic foundation under moving non-concentrated loads. The solution is obtained on the basis of a plate theory in which no account is taken of the effects of shear deformation and rotatory inertia. Numerical results are given for various values of damping coefficient.

Key Words: Viscous Damping, Rectangular Plates, Elastic Foundation And Moving Line Load

Formulation Of The Problem

The transverse deflection $Q(x, y)$ of the median plane of a plate according to the classical small deflection theory [9] is given by

$$D\nabla^4 Q(x, y) = q(x, y) \quad (1)$$

where $q(x, y)$ is the load intensity and flexural rigidity D is given by

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

for which E is Young's modulus, h is the plate's thickness and $\nu(<1)$ is the Poisson's ratio.

For a dynamic problem at hand and the inclusion of viscous damping forces of intensity $-C\partial Q/\partial t$, equation (1) becomes

$$D\nabla^4 Q(x, y, t) + \frac{m\partial^2 Q(x, y, t)}{\partial t^2} + C\frac{\partial Q(x, y, t)}{\partial t} = q(x, y, t) \quad (2)$$

where $C = 2dm$, d is the damping coefficient and m is the mass per unit area of the plate.

We shall be concerned with a partially distributed line load which moves with constant speed v parallel to the x -axis. When a transversely loaded plate rests on a yielding foundation, the reaction of the foundation opposes the plate's deflection. For elastic foundation, the resistance is proportional to the deflection thus we have $KW(x, y)$ where K is the modulus of the foundation expressed in force per unit of plate area per unit of deflection. Thus, the equation of the transverse deflection of a damped plate resting on an elastic foundation becomes.

$$D\nabla^4 Q(x, y, t) + \frac{m\partial^2 Q(x, y, t)}{\partial t^2} + C\frac{\partial Q(x, y, t)}{\partial t} + KQ(x, y, t) = q(x, y, t) \quad (3)$$

In particular reference to the type of load in this problem, we are concerned with a uniform partially distributed line load which moves with constant velocity v parallel to the x -axis. Thus, the formulation becomes

$$q(x, y, t) = \frac{1}{\mu} \left[M_l g - M_l \frac{d^2 Q}{dt^2} \right] \left[H\left(x - \xi + \frac{\mu}{2}\right) - H\left(x - \xi - \frac{\mu}{2}\right) \right] \delta(y - y_1) \quad (4)$$

where μ is the length of the line load, M_l is the mass of the load, $\delta(\cdot)$ is the Dirac-delta function, $H(\cdot)$ is the Heaviside Unit function, $\xi = vt$, g is the acceleration due to gravity and

$$\frac{d^2 Q(x, y, t)}{dt^2} = \frac{\partial^2 Q(x, y, t)}{dt^2} + 2v \frac{\partial^2 Q(x, y, t)}{\partial x \partial t} + v^2 \frac{\partial^2 Q(x, y, t)}{\partial x^2} \quad (5)$$

Method Of Solution

Equation (4) may be expressed as

$$\frac{1}{\mu} \left[M_l g - M_l \left(\frac{\partial^2 Q(x, y, t)}{dt^2} + \frac{2v\partial^2 Q(x, y, t)}{\partial x \partial t} + \frac{v^2 \partial^2 Q(x, y, t)}{\partial x^2} \right) \right] \left[H\left(x - \xi + \frac{\mu}{2}\right) - H\left(x - \xi - \frac{\mu}{2}\right) \right] \delta(y - y_1) = \sum_{i=1} \sum_{j=1} P_{ij}(t) X_i(x) Y_j(y) \quad (6)$$

where $P_{ij}(t)$, $i = 1, 2, \dots, j = 1, 2, 3, \dots$ are functions of time to be determined. $X_i(x)$ and $Y_j(y)$ are the deflection curves for i th and j th modes of vibration respectively.

Multiplying both sides of equation (6) by $X_r(x) Y_s(y)$ and taking the integrals along the length l_x and width l_y of the plate, we have

$$\begin{aligned} & \frac{1}{\mu} \left[M_L g \int_0^{l_x} X_r(x) H_L dx \int_0^{l_y} Y_s(y) D_L dy - M_i \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} X_r(x) X_i(x) H_i dx \right. \\ & \left. \int_0^{l_y} Y_s(y) Y_j(y) D_L dy P_{ij}^{\&\&}(t) + 2\nu \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} P_{ij}^{\&\&}(t) \int_0^{l_x} X_i'(x) X_r H_L dx \right. \\ & \left. \int_0^{l_y} Y_s(y) Y_j(y) D_L dy + \nu^2 \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} F_{ij}(t) \int_0^{l_x} X_i''(x) X_r H_L dx \right. \\ & \left. \int_0^{l_y} Y_j(y) Y_s(y) D_L dy = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} P_{ij}(t) \int_0^{l_x} X_i(x) X_r dx \int_0^{l_y} Y_i(y) Y_s(y) dy \right] \quad (7) \end{aligned}$$

where

$$H_L = H\left(x - \xi + \frac{\mu}{2}\right) - H\left(x - \xi - \frac{\mu}{2}\right), D_L = \delta(y - y_i) \text{ and } Q(x, y, t) = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} F_{ij}(t) X_i(x) Y_j(y) \quad (8)$$

Using orthogonality properties $\int_0^{l_x} X_i(x) X_r dx = \begin{cases} 0, & i \neq r \\ a, & i = r \end{cases}$ and

Dirac delta property $\int_0^{l_y} Y_j(y) \delta(y - y_1) dy = Y_j(y_1)$, $0 \leq y_1 \leq l_y$

Equation (7) becomes

$$\begin{aligned} P_{ij}(t) = \frac{1}{\mu a_l} \left[M_L g Y_s(y_1) I_n(X_r) - M_i \left(\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} (I_n(X_r X_i) Y_s(y_1) Y_j(y_1) P_{ij}^{\&\&}(t) \right. \right. \\ \left. \left. + 2\nu \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} P_{ij}^{\&\&}(t) I_n(X_i' X_r) Y_s(y_1) Y_j(y) + \nu^2 \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} F_{ij}(y) Y_j(y_1) Y_s(y_1) I_n(X_i'' X_r) \right) \right] \quad (9) \end{aligned}$$

where $I_n(X_r) = \int_{\xi - \frac{\mu}{2}}^{\xi + \frac{\mu}{2}} X_r dx$, $I_n(X_r X_i) = \int_{\xi - \frac{\mu}{2}}^{\xi + \frac{\mu}{2}} X_r X_i dx$

$$I_n(X_i' X_r) = \int_{\xi - \frac{\mu}{2}}^{\xi + \frac{\mu}{2}} X_i' X_r dx, \quad I_n(X_i'' X_r) = \int_{\xi - \frac{\mu}{2}}^{\xi + \frac{\mu}{2}} X_i'' X_r dx$$

In free vibration of plate, the deflection may be expressed as

$$Q(x, y, t) = Q(x, y) \lambda^{\pm \omega t} \quad (10)$$

where ω is the circular frequency expressed in radians per unit time.

Using equation (10) in a free vibration equation, we have

$$D[X_i^{iv} Y + 2X_i'' Y_j'' + X_i Y_j^{iv}] + K X_i Y_j = \delta_{ij} X_i Y_j \quad (11)$$

where $\delta_{ij} = \omega_{ij}^2 m$ are the eigenvalues.

Simplification of equation (3) by application of the definition of $q(x, y, t)$ in equation (6) and substituting equations (8) and (11) into it gives

$$mI_{ij}^{\&\&}(t) + CI_{ij}^{\&\&}(t) + \delta_{ij}F_{ij}(t) = P_{ij}(t) \quad (12)$$

Thus,

$$mI_{ij}^{\&\&}(t) + CI_{ij}^{\&\&}(t) + \delta_{ij}F_{ij}(t) = -\frac{M_i}{\mu a_1} \left[-gY_s(y/l)I_n(X_r) + \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} (Y_s(y_1)Y_j(y_1)I_{ij}^{\&\&}(t) \right. \\ \left. I_n(X_r X_i) + 2vI_{ij}^{\&\&}(t)Y_s(y_1)I_n(X_i'X_r) + v^2Y_j(y_1)Y_s(y_1)F_{ij}(t)(X_i''X_r) \right] \quad (13)$$

Equation (13) represents the general equation that may be solved for various pertinent boundary conditions of the plate.

Simply Supported Plate As An Illustrative Example

For simply supported boundary conditions of a plate we have the deflection.

$$Q(x, y) = X(x)Y(y) = 2/\sqrt{L_x L_y} \sin \frac{i\pi x}{L_x} \sin \frac{j\pi y}{L_y} \quad (14)$$

with the normalized constant one.

The substitution of equation (14) into a free vibration equation (11) yields

$$\delta_{ij} = D\pi^4 \left\{ \frac{i^2}{l_x^2} + \frac{j^2}{l_y^2} \right\}^2 + k \quad (15)$$

Substituting equation (14) into equation (13), we obtain

$$mI_{ij}^{\&\&}(t) + CI_{ij}^{\&\&}(t) + \delta_{ij}F_{ij}(t) = \frac{4mi}{\mu} \left[\frac{g\sqrt{l_x}}{\sqrt{l_y}i\pi} \sin \left(\frac{s\pi y_1}{l_y} \right) \sin \left(\frac{r\pi \xi}{l_x} \right) \sin \left(\frac{r\pi \mu}{2l_x} \right) \right. \\ \left. - \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \left[\frac{1}{l_x l_y} \sin \left(\frac{j\pi y_1}{l_y} \right) \sin \left(\frac{s\pi y_1}{l_y} \right) I_{ij}^{\&\&}(t) \bar{A}_{ir} + \frac{2_i \pi v}{l_x^2 l_y} I_{ij}^{\&\&}(t) \sin \left(\frac{j\pi y_1}{l_y} \right) \sin \left(\frac{s\pi y_1}{l_y} \right) \bar{C}_{ij} \right. \right. \\ \left. \left. - \frac{(\pi i)^2 v^2}{l_x^3 l_y} F_{ij}(t) \sin \left(\frac{j\pi y_1}{l_y} \right) \sin \left(\frac{s\pi y_1}{l_y} \right) \bar{A}_{ir} \right] \right] \quad (16)$$

where

$$\bar{C}_{ir} = \frac{l_x}{p_1 \pi} \sin \left(\frac{p_1 \pi \xi}{l_x} \right) \sin \left(\frac{p_1 \pi \mu}{2l_x} \right) - \frac{l_x}{d_1 \pi} \sin \left(\frac{d_1 \pi \xi}{l_x} \right) \sin \left(\frac{d_1 \pi \mu}{l_x} \right), \quad i \neq r \\ = \frac{l_x}{2i\pi} \sin \left(\frac{2i\pi \xi}{l_x} \right) \sin \left(\frac{i\pi \mu}{l_x} \right) \quad i = r$$

and

$$\bar{A}_{ir} = \frac{l_x}{\pi d_1} \cos \left(\frac{d_1 \pi \xi}{l_x} \right) \sin \left(\frac{d_1 \pi \mu}{2l_x} \right) - \frac{l_x}{\pi p_1} \cos \left(\frac{p_1 \pi \xi}{l_x} \right) \sin \left(\frac{p_1 \pi \mu}{2l_x} \right), \quad i \neq r \\ = \frac{\mu}{2} - \frac{l_x}{2\pi i} \cos \left(\frac{2i\pi \xi}{l_x} \right) \sin \left(\frac{i\pi \mu}{l_x} \right) \quad i = r$$

$$d_1 = i - r \text{ and } p_1 = i + r$$

Numerical Analysis And Discussion

As for the numerical example considered the differential equation (16) is solved using the central difference formulae of finite difference method. The system of equations resulted from finite difference method has been solved and analyzed for the following data for the purpose of comparison. In particular a rectangular plate of length $l_x = 10$ m and with $l_y = 5$ m was considered. The following values were also used for the plate $E = 2.109 \times 10^7 \text{ Nm}^{-2}$, $y_1 = 2.5$ m, $v = 1.5$ m/s, 2.5 m/s, 7.5 m/s, 10 m/s, $m = 100$ kg and $\nu = 0.2$.

The influence of Winkler foundation with constant of proportionality contributes to the response of the plate by reducing the maximum deflection of the plate as the value k increases as shown in Figure 1 for fixed value c , ν and μ . Neglecting the damping term, by setting $c = 0$, the result agrees to some extent with [8]

Figure 2 shows the mid-deflection of plate for various values of damping coefficient C at different time. The result shows that the plate deflection increases with increase in damping coefficient before the load covers half its distance on the plate. But, the deflection increases with increase in damping coefficient after half its distance on the plate.

Figure 3 shows the deflection of the mid-plate at different time for variation of velocities. It shows that higher velocities produce higher maximum deflections.

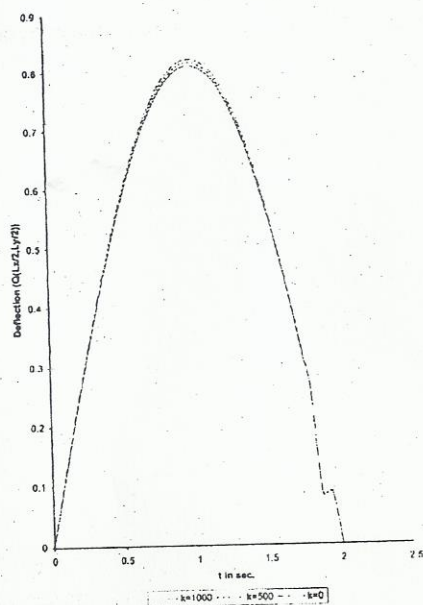


Figure 1 : The Deflection In mm At Different Time For Various Values of k .

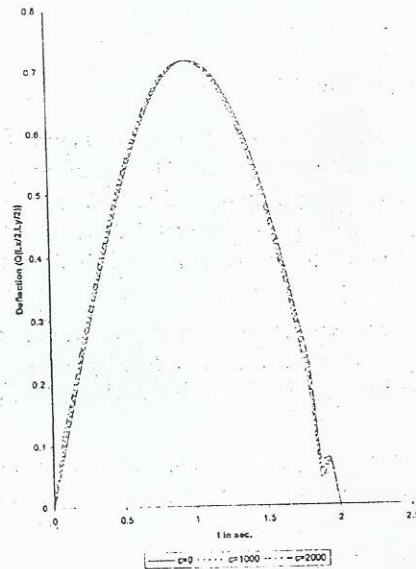


Figure 2 : The Deflection In mm At Different Time For Various Values of c .

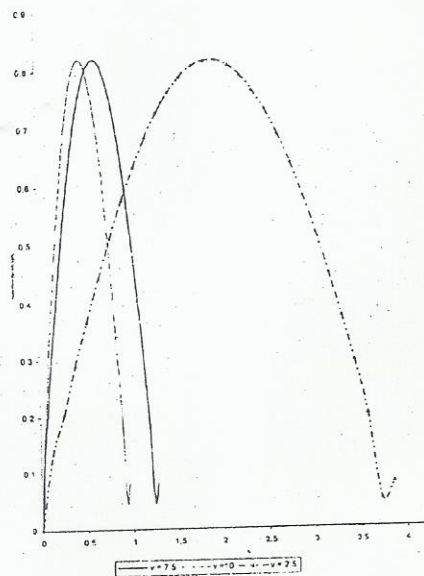


Figure 3 : Time History Of The Mid-Plate's Deflection For Some Velocities.

Conclusion

The effect of viscous damping on the dynamic behaviour of a rectangular plate resting on a foundation under the influence of a moving line load has been carried out. The partial differential equation governing the response of the plate under the moving load was simplified into another set of coupled ordinary differential equations.

In particular, simply supported boundary conditions are used to illustrate the method of solution. The result shows that the presence of foundation reduces the deflection of the plate.

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