

Modelling and Numerical Investigation of Pressure Drop of Viscous Fluid in Elbow Pipe using Colebrook-White Equation

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Abstract

Elbows constitute resistance to flow, increase energy losses, and increase pumping power. This current study presents a mathematical model for analyzing pressure drop in elbow pipes for viscous fluids with flow conditions in turbulent flow regimes. The selected pipes were two 90° 100 mm nominal diameter schedule 40 commercially available steel elbow pipes of different geometries. The mathematical model developed was validated with experimental results, and numerical simulation for the model was conducted using MATLAB with varying parameters such as velocity and friction factor. Results showed that for low viscous flow with velocities ranging from 0.33 to 6 m/s, the pressure drop values obtained were between 0.0128 and 6.1669 kPa

and 0.0087 and 4.2195 kPa for short-radius elbows and long-radius elbows, respectively. For higher viscous fluids with velocities ranging from 46.26 to 100 m/s, pressure drop values obtained were between 431.6507 and 3919.1 kPa and 295.34 and 2681.5 kPa for short-radius and long-radius elbows, respectively. The results of the study can function as a guide in the choice of 90° elbow pipe used in manufacturing and process industries for the transport of the viscous fluids treated.

Keywords

Colebrooke-White equation, elbow pipe, friction factor, pressure drop, turbulent flow, viscous fluids.

1. Introduction

In many engineering applications, pressure drop is an important factor to consider in designing, analyzing, and optimizing pipes and fluid distribution systems (Khamsehchi et al., 2023). Fluids are transported through pipes, but to change the direction at intervals, pipe systems are designed with elbows. Elbows are therefore vital accessories needed in any piping systems as a result of the need for redirection of flow (Helmizar et al., 2026). Elbow pipes are important components of piping systems in chemical, pharmaceutical and petroleum industries and so on. The large eddies are created as a result of the change in direction of fluids which causes significant pressure loss when the flow leaves the elbow (Langwane & Subaschandar, 2021; Wang et al., 2022; Wang et al., 2022). The Darcy-Weisbach formula can be used to quantify pressure drop due to frictional effects in elbow pipes. However, the Darcy friction factor must be known in order to employ the Darcy-Weisbach

formula. The Colebrook-White equation provides the best approximation to the Darcy friction factor for turbulent flow (Arumugam et al., 2022) The Colebrook-White equation is as given in equation (1) (Vasconcellos et al., 2020):

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left[\frac{(\varepsilon/D)}{3.7} + \frac{2.51}{Re\sqrt{f}} \right] \quad (1)$$

Where ε represents the absolute roughness, and D represent the pipe diameter, ε/D is the relative roughness of the pipe, f is the friction factor, and Re is the Reynolds number which signifies the flow regime developed in the pipe; it is valid for the following range of values (Vasconcellos et al., 2020): $4 \times 10^3 < Re < 10^8$ and $0 \leq \varepsilon/D \leq 0.05$. By visual inspection, it can be seen that f appears on both sides of equation (1) which implies that the equation exists in an implicit form. To solve this equation, iterative methods like the Newton-Raphson method can be employed, which takes extra effort to solve. However, explicit versions of the Colebrook-White equation exist that can save computational time while having minimal relative error (Qiu & Ostfeld, 2021). A summary of some explicit models of the Colebrook-White equation can be found elsewhere in (Cahyono, 2022). Inviscid fluids are generally known as fluids without viscosity. However, most fluids have at least a small quantity of viscosity (Rajput, 2011). Various viscous fluids are widely used in industrial process plants and manufacturing. When these viscous fluids are transported through piping systems, they come into contact with elbow of pipes, necessitating the evaluation of their flow characteristics at those points (Garzón et al., 2018).

Helmizar et al., (2026) conducted a Computational Fluid Dynamics study on 90° elbow under different curvatures with an intention of knowing it affects pressure distribution and head loss. The R/D values investigated were five in number ranging from 1.0 to a maximum value of 7.66. The single-phase flow was considered under turbulent flow regimes of $Re=17,631.71$ and $Re=23,888.13$. It was revealed from their result that head loss coefficient represented by (k) varied inversely as the R/D value up to a value of 3.67 after which it increases as the value increases as a result of extended wall friction. Their result also showed that larger ratio of R/D elbow enact smoother flow redirection and higher uniform pressure fields. However, the smaller ratio of R/D have a stronger better energy dissipation and a more robust profile. Saber & Maree, (2022) investigated numerically a two-phase flow of 90° elbow with different inner diameter sizes of 25mm, 50mm, 75mm and 100mm and R/D values of 1,2,4,6 and 10. ANSYS FLUENT was employed for the simulation. Results showed that there were large losses in energy and pressure at higher velocity. Results also showed that Increasing pipe diameter on one side and decreasing R/D on the other resulted in higher pressure drop. Huang et al., (2024) in their study, proposed a novel numerical model to investigate the particle erosion behaviours in 90° elbow pipes. The result of the numerical study was compared with experimental results and it was found to be in good agreement for predicting the erosion rate. Subsequently, influence of various parameters such as particle injection velocity and particle diameters on erosion rate were investigated using the model. Result revealed that increasing the particle diameter resulted into decrease in maximum erosion rate. A Computational Fluid Dynamics study on water-based Titanium Dioxide nanofluids by Nashee, (2024) . ANSYS Fluent software was employed for the

simulation and the laminar flow regime varied from Reynolds number ranging from 100 to 900 and nanoparticle volume fraction of between 0.00 to 0.04. Three types of elbow joint were considered with the same cross-sectional area of 0.636 m^2 . The results revealed that the highest enhancement in heat transfer was generated by triangular cross-section followed by the square shaped and lastly, the circular shaped geometry. Rajkumar et al., (2023) studied both experimentally and numerically the effect of pipe size on erosion measurement for different flow regimes. The results generated from both methods were analysed and compared. Shcherbyna et al., (2025) investigated numerically the energy losses during the transportation of liquid through pipelines. ANSYS CFX software was employed in evaluating energy losses with particular focus on Darcy-Weibach equation. The result from simulation was compared with analytical method result. Crawford et al., (2009) investigated the flow structures and losses in turbulent flow in 90° elbows with ratio of curvature radius to pipe radius (R/r) values of 1.3, 5, and 20. A new equation for predicting pressure drop owing to flow separation was proposed which included elbow pipes with an R/r ratio smaller than 2, while a new composite equation is proposed to estimate pressure losses within a Reynolds number of between 4×10^3 to 3×10^5 . The study concluded that Reynold Stress Model gave the best performance of the models considered. The current work is aimed at investigating the pressure drop characteristics of selected viscous fluids in 90° elbow pipes of specified elbow geometries in the turbulent region with the objective of developing a mathematical model and simulating the fluid flow based on the model parameters.

2.0 Materials and Methods

2.1 The Physical Model

For this study, two commercial steel 90° elbow pipes of circular cross-section having a radius-to-diameter (r/D) ratio of 1 and 1.5 were selected, as they represent short-radius and long-radius elbow geometries. The elbows are presented in Figure 1 (a) and (b). The properties of the elbow pipes are as listed in Table 1. The 90° elbow pipes of circular cross-section were chosen, because of their wide use in the engineering field as compared to elbow pipes of other angles of curvature (Zhang et al., 2013).



Figure 1: 2D Physical models of (a) 90° elbow pipe short radius and (b) 90° elbow pipe long radius.

Table 1: Summary of properties of the elbow pipe considered in the study (McDonald & Magande, 2013)

Pipe Size	Pipe Inside Diameter, d	Material	Average roughness, ϵ
DN100 (4 inch)	102.3 mm	Schedule 40 commercial steel	0.046 mm

Incompressible single-phase fluid flow was employed in this study. Ten (10) viscous fluids were selected for analysis and further grouped into low-viscosity and high-viscosity fluids with, each group having five (5) categories of fluids. The temperature of the flowing fluid was specified at $T = 20^{\circ}\text{C}$ and the density (ρ) and dynamic viscosity (μ) of each viscous fluid were defined in Tables 2(a) and (b).

Table 2(a): Density and dynamic viscosity of group (a) of Low-viscous fluids at 20°C .

Viscous fluid	Density ρ , kg/m^3	Dynamic viscosity μ , kg/m-s
Acetone	785	0.000316
Ammonia	608	0.000220
Crude oil	856	0.007200
Ethanol	789	0.001100
Water	998	0.001000

Table 2(b): Density and dynamic viscosity of group (b) of High-viscous fluids at 20°C (Abulencia & Theodore, 2011; Akgül et al., 2013)

Viscous fluid	Density ρ , kg/m^3	Dynamic viscosity μ , kg/m-s
D-Sorbitol	1487	0.200
Glycerin	1260	1.490
SAE 30 oil	917	0.290
SAE 10 oil	917	0.104
Urea-formaldehyde resin	1227	0.185

2.2 Mathematical Model

The Colebrook-White equation given in equation (1) exists in an implicit form as f exists on both sides of the equation. For this study, an accurate and computationally efficient friction model which is an explicit form of the Colebrook-white equation developed by Offor & Alabi, (2016) is used as shown in equation (2):

$$f = \left(-2 \log_{10} \left(\frac{\varepsilon/D}{a} + \frac{b}{Re} \left(\ln \left(\left(\frac{\varepsilon/D}{c} \right)^d + \left(\frac{a}{Re+g} \right) \right) \right) \right) \right)^h \quad (2)$$

Where,

$a = 3.71$, $b = -1.975$, $c = 3.93$, $d = 1.092$, $e = 7.627$, $g = 395.9$ and $h = -2$.

These coefficients with their parameter bounds were derived using the surface-fitting function in the MATLAB curve-fitting toolbox and were obtained at a 95% confidence level.

Equation (2) is valid for the following ranges of Offor & Alabi, (2016) $4 \times 10^3 \leq Re \leq 10^8$ and $10^{-6} \leq (\varepsilon/D) \leq 0.05$.

Re is the Reynolds number given by equation (3):

$$Re = \frac{VD\rho}{\mu} \quad (3)$$

Where ρ is the density, μ is the dynamic viscosity of the fluid to be analyzed, (ϵ/D) is the relative roughness of the pipe, V is the velocity of the fluid flowing in the pipe, D is the diameter of the pipe being considered.

The Pressure drop for the turbulent flow in the elbow pipe is modelled using the Darcy-Weisbach equation given as equation (4) (Fiorillo et al., 2024):

$$\Delta p = f \frac{L}{D} \rho \frac{V^2}{2} \quad (4)$$

Where f is the friction factor obtained from the explicit friction model in (4), L is the length of the pipe, D is the inside diameter of the pipe, ρ is the density of the viscous fluid, and V is the velocity of flow.

Using the equivalent length method, the elbow pipe is converted to its equivalent straight pipe length using equation (5), given as (Wang et al., 2022):

$$L = \left(\frac{L}{D} \right)_{eq} \times D_i \quad (5)$$

Where $\left(\frac{L}{D} \right)_{eq}$ is the ratio of the equivalent length to pipe diameter for a given fitting (elbow pipe in this case). Table 3 shows the equivalent length of the elbow pipes considered in this study.

Substituting equation (4) into equation (5), gives equation (6):

$$\Delta p = f \frac{\left(\frac{L}{D} \right)_{eq} \times D_i}{D} \rho \frac{V^2}{2} \quad (6)$$

Since the pipe inside diameter is held constant for elbow pipes considered D becomes D_i and equation (6) becomes equation (7):

$$\Delta p = f \left(\frac{L}{D} \right)_{eq} \rho \frac{V^2}{2}$$

(7)

Table 3: Equivalent length values for the elbow pipe geometries (AioFlo, 2014) .

Elbow pipe	$\left(\frac{L}{D} \right)_{eq}$
90° elbow, r/D=1	19
90° elbow, r/D=1.5	13

2.2.1 Confirmation of pipe elbow data for the range of values for the friction factor model

For the explicit friction factor model to be used in this study equation (2) gives a range of values for which its results are accurate. The relative roughness (ϵ/D) of the pipe was found to be 4.49658×10^{-4} which was within the acceptable range for (ϵ/D) values for the friction factor model.

2.2.2 Estimation of velocity values for the range of *Re* values for the friction factor model

In order to evaluate the *Re* values that fit the acceptable range for the friction factor model, the two groups of viscous fluids (low-viscosity and high-viscosity) were treated separately to determine the minimum velocity that will be needed to attain turbulent flow for each group. The estimation of this minimum velocity for turbulent flow is justified as the Colebrook-White equation used in the determination of pressure drop for turbulent flow is given by (Das et al., 2017).

Equation (3) is modified to give the estimation of the velocity:

$$V_m = 4 \times 10^3 \times \frac{\mu}{\rho D}$$

(8)

Where 4×10^3 is the lower limit for Re in equation (2).

The minimum velocity V_m to attain turbulence in the low-viscosity fluids and high-viscosity fluids was determined by applying equation (8) to the two groups of viscous fluids. The maximum values of V_m for the groups of viscous fluids are as given in Table 4, while the values of parameters used for both the low-viscosity fluids and high-viscosity fluids are given in Tables 5 a and b.

Table 4: Maximum values of V_m for the groups of viscous fluids.

	Low-viscosity fluids	High-viscosity fluids
Minimum Velocity V_m , m/s	0.33	46.26

Table 5 (a): Parameter values used for low-viscosity fluids.

Parameter	Symbol	Value
Equivalent length	$\left(\frac{L}{D}\right)_{eq}$	19 and 13
Relative roughness	$\left(\frac{\epsilon}{D}\right)$	4.49658×10^{-4}
Reynolds number	Re	4000 to 10^7
Velocity, m/s	V	0.33 – 6
Friction factor	f	Dependent on Re and $\left(\frac{\epsilon}{D}\right)$ value

Table 5 (b): Parameter values used for high-viscosity fluids.

Parameter	Symbol	Value
Equivalent length	$\left(\frac{L}{D}\right)_{eq}$	19 and 13
Relative roughness	$\left(\frac{\varepsilon}{D}\right)$	4.49658×10^{-4}
Reynolds number	Re	4000 to 10^5
Velocity, m/s	V	46.26 – 100
Friction factor	f	Dependent on Re and $\left(\frac{\varepsilon}{D}\right)$ value

2.3 Model Validation

The mathematical model in the present study was validated with experimental data reported by Crawford et al., (2007) in their work which involved the variation of pressure loss against different elbow geometries at a $Re = 1.00 \times 10^5$, using air held at an ambient temperature of 295-300K. The elbow geometry in current study was modified to conform to the R/r values of the 90° elbow geometries used in their experimental work. It is seen that the curves generated for pressure drop in Figure 2 have similar profiles and an error of 6% from the experimental work of (Crawford et al., 2007).

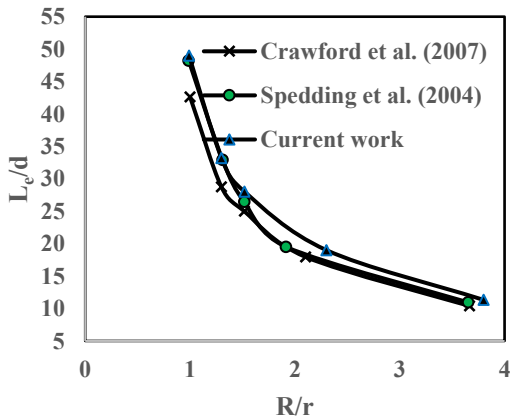


Figure 2: Comparison between Crawford et al., (2007), Spedding et al., (2004) and the current work.

2.4 Flowchart for the Numerical Simulation

The flowchart in Figure 3 gives a visual representation of numerical simulation process carried out.

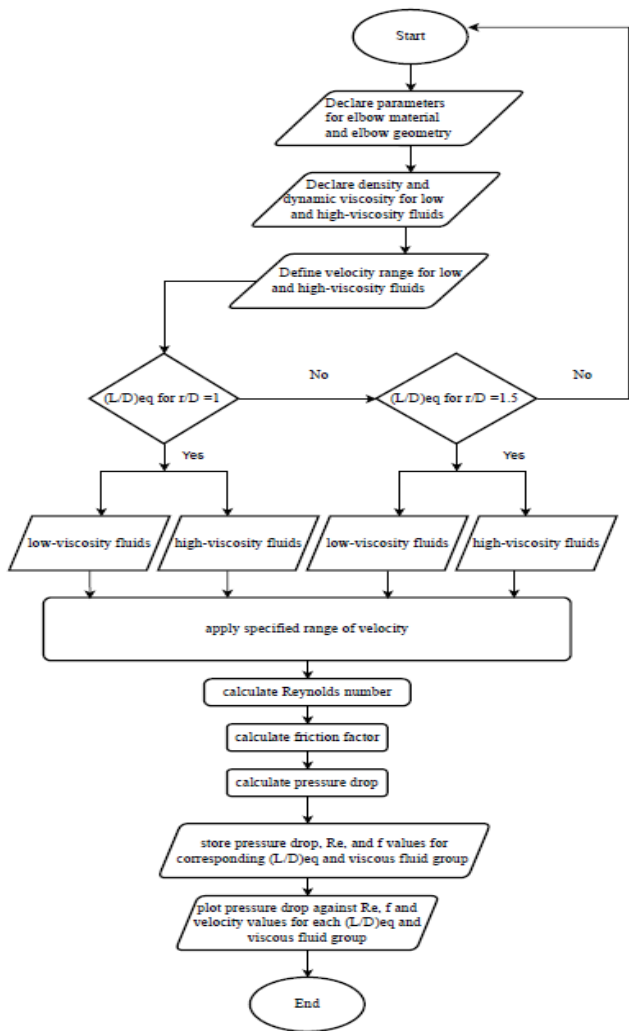


Figure 3: Flowchart for MATLAB simulation process.

3. Results And Discussion

The line graphs presented in Fig. 4 to Fig. 7 were generated from the data derived from the numerical simulation using MATLAB. Values of Δp were plotted against V and Re for the low-viscosity and high-viscosity fluids for the different elbow geometries ($r/D=1$ and $r/D=1.5$).

3.1 Pressure Drop and Velocity

Figures 4 (a) and (b) presents the variation of pressure drop with velocity for low-viscosity fluids in 90° elbows for short radius and long radius elbows. An increase in flow velocity resulted to an increase in the pressure drop across both elbow geometries. The pressure drop developed in the short radius elbow is seen to be higher than that obtained from the long radius elbow.

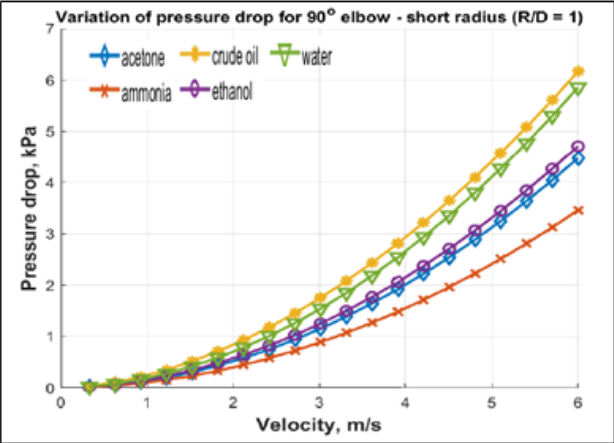
The same trend is also observed in Figure 5 (a) and (b) for high viscosity fluids. Comparing the difference between the low and high viscosity fluids, there was an approximate 40% difference between their pressure drop values. Also, it was observed that the pressure drop range for each low-viscosity fluid is closely associated with their viscosity values.

Crude oil had the highest dynamic viscosity in the group ($\mu = 0.0072$ kg/m-s) and showed a relatively higher pressure drop while ammonia being the smallest ($\mu = 0.00022$ kg/m-s) had the lowest pressure drop range. In the case of high viscosity fluids, the highest pressure drop range across both elbow geometries was recorded for glycerin which

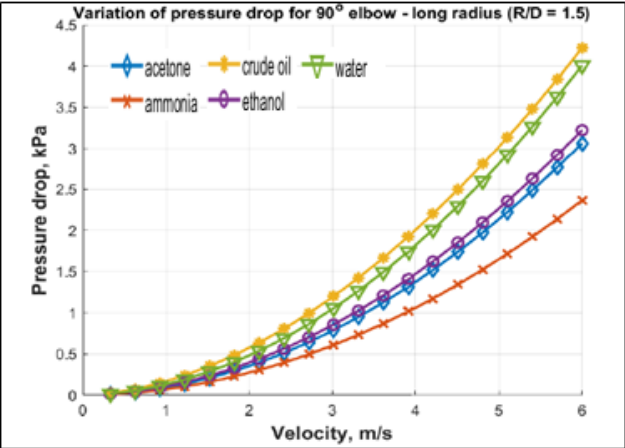
had the highest dynamic viscosity ($\mu = 1.49 \text{ kg/m-s}$), while the lowest was recorded for SAE 10 oil ($\mu = 0.1040 \text{ kg/m-s}$) having the lowest dynamic viscosity in the group. The profiles of pressure drop variation with increasing velocity shown in the presented graphs were found to be consistent with the experimental work of (Kaushal et al., 2005).

2 Pressure Drop and Reynolds number

Figures 6 (a) and (b) presents the variation of pressure drop with Reynolds number for low viscosity fluids in 90° elbow while Figures 7 (a) and (b) presents that of high viscosity fluids in 90° elbow. For both groups of viscous fluids, an increase in Re values increases the value of the pressure drop at different elbow geometries. For low-viscosity fluids, an increase in the Re value of the flow results in an increase in pressure drop. At extremely low Re values, all the low-viscosity fluids experience little drop across the elbow geometries. For high-viscosity fluids, lower Re values gave rise to significant pressure drop values with SAE10 oil recording the lowest pressure drop at lower Re values (SAE 10 oil had the lowest dynamic viscosity of 0.1040 kg/m-s).

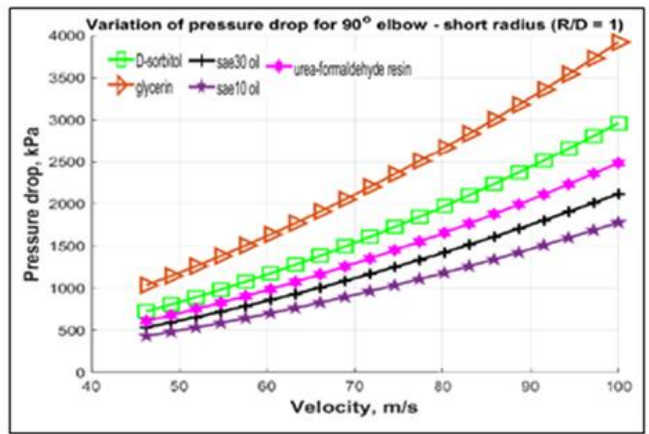


(a)

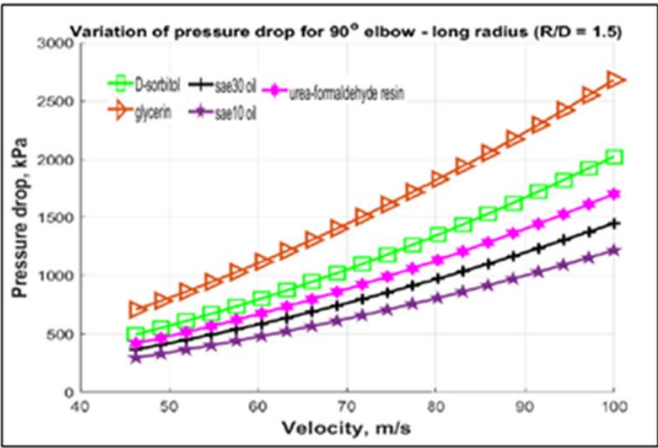


(b)

Figure 4: Variation of pressure drop with velocity for low-viscosity fluids in 90° elbows (a) short radius and (b) long radius.



(a)



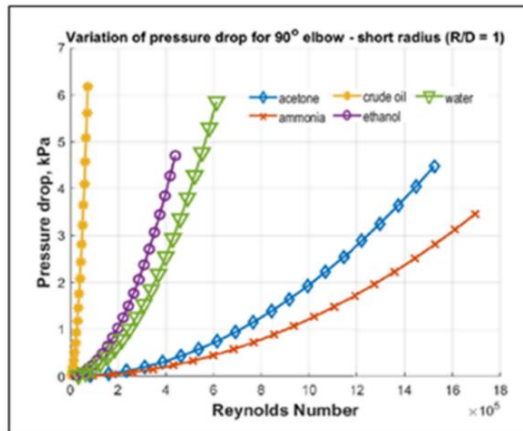
(b)

Figure 5: Variation of pressure drop with velocity for high-viscosity fluids in 90° elbows (a) short radius and (b) long radius.

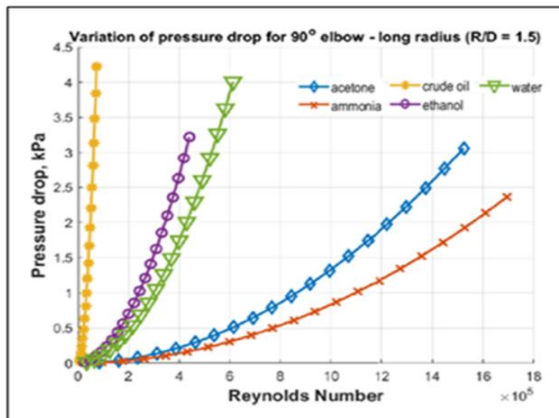
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For high-viscosity fluids, lower Re values gave rise to significant pressure drop values with SAE10 oil recording the lowest pressure drop at lower Re values (SAE 10 oil had the lowest dynamic viscosity of 0.1040 kg/m-s). The profiles of pressure drop variation with increasing Re shown on the presented graphs were found to be consistent with the works of (Minakov et al., 2016; Qi et al., 2022).

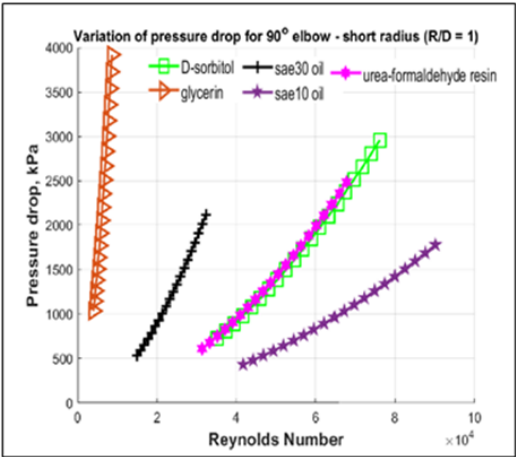


(a)

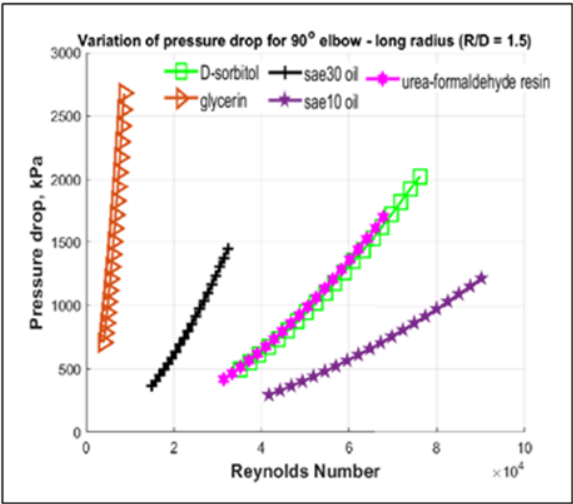


(b)

Figure 6: Variations of pressure drop with Reynolds for low-viscosity fluids in 90° elbows (a) short radius and (b) long radius.



(a)



(b)

Figure 7: Variations of pressure drop with Reynolds number for high-viscosity fluids in 90° elbows (a) short radius and (b) long radius

4. Conclusion

In this study, the mathematical modelling and numerical simulation of pressure drop in elbow pipe of single-phase flow in 90° commercial steel elbow pipe of different geometries using the Colebrook-White equation was carried out for different viscous fluids using MATLAB. The mathematical model developed in the study was validated with experimental data. Elbow geometry was found to play an important role in the determination of pressure drop. Elbow pipes of short radius developed a larger pressure drop across their cross-section when compared with elbow pipes of a long radius. Although density plays an important role in the determination of Re, friction factor and pressure drop values, the viscosity of a fluid is also found to have a pronounced effect on pressure drop. This was found to be true for the different viscous fluids analyzed in this study. The equivalent length method of resolving elbow pipes for pressure drop calculation used in this work is a robust method in the estimation of pressure drops in elbow pipes. This study guides the selection of geometries of 90° elbow pipe for the transport of viscous fluids analyzed in this study. Further studies are recommended to cover a wider range of viscous fluids for both single-phase and multi-phase flows and elbow pipes of different angles of curvature and geometries used in manufacturing and process industries such as the chemical, petroleum, and pharmaceutical industries.

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