

EFFECT OF VARIABLE FLUID PROPERTIES ON RADIATIVE MAGNETOHYDRODYNAMIC FLOW OF A FLUID IN A VERTICAL CHANNEL

BY

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ABSTRACT

The object of this paper is to analyse the effect of temperature dependent fluid's properties on a radiative magnetohydrodynamic flow of a fluid in an open - ended vertical channel and permeated by a uniform transverse magnetic field. The solutions for the temperature difference, velocity and induced magnetic field are obtained in an optically thin limit case using analytical successive approximation method.

It is found that an increase in viscous dissipative heat parameter, when the thermal conductivity is a function of temperature causes corresponding increase in heat transfer, temperature difference but a decrease in flow rate, velocity and induced magnetic field. Also, an increase in viscous dissipative heat parameter, when the viscosity is temperature dependent, results in an increase in flow rate, heat transfer, velocity, induced magnetic field and temperature difference. The result in [2], for constant fluid parameters is obtained as a special case.

1. INTRODUCTION

The heat transfer in an electrically conducting fluid permeated by electromagnetic field has been the focus of many studies. This may be due to its importance in some Engineering designs

Notable among them are the studies of, the magnetic field on forced convective heat transfer in a parallel plate by Siegel [9], heat transfer in magnetohydrodynamic (MHD) flow between parallel plates by Alpher [1], effect of transverse magnetic field on heat transfer to an electrically conducting and thermally radiating fluid flowing in a parallel plate channel by Viskanta [11], the influence of electric and magnetic fields on heat transfer to electrically conducting fluid by Romig [8], variable viscosity flow in channel with high heat generation by Pearson [7], and transient slab with temperature dependent thermal conducting by Sucec and Hedge [10].

The significance of radiation in space application and higher operating temperatures drew the attention of some researchers such as Gupta and Gupta [5] who considered the radiation effect on hydromagnetic convection in a vertical channel, Bestman [4] investigated the laminar convection to hydromagnetic flow of a radiating gas in a rotating medium. Heillievel and Mosa [6] analysed the radiative transfer in horizontal MHD channel flow with buoyancy effects and an axial temperature gradient while Gbadeyan and Andi [4] investigated the combined effects of radiation and viscous dissipation on hydromagnetic fluid flow in a vertical channel.

The above studies on a radiative hydromagnetic flow failed to consider the temperature dependence of viscosity and thermal conductivity without neglecting viscous dissipation function. The effect of these temperature dependence parameters is significant especially in higher operating temperatures.

Therefore, this paper investigates the effect of variable fluid properties on radiative magnetohydrodynamics flow of an electrically conducting fluid in an open vertical channel in the presence of a uniform transverse magnetic field. The analysis is restricted to optically thin limit case.

2. FORMULATION OF THE PROBLEM

The problem concerns the flow of electrically conducting fluid inside an infinite vertical channel formed by two parallel plates of distance $2L$ (L being half width of the channel) apart. The vertical direction is taken as Z -axis and the magnetic field acts normal to the plates which are at $y = \pm L$.

In this paper the following assumptions are adopted.

- (i) The channel walls are electrically non-conducting. Hence, the magnetic field is zero on the walls of the plates.
- (ii) The flow is fully developed laminar flow. As a result, the velocity (V) and induced magnetic field (B) have only vertical components
- (iii) All the physical variables are functions of y except temperature (T) and pressure P , y being the horizontal coordinate normal to the plates.

The temperature inside the fluid is defined as

$$T = \theta + NZ \quad (2.1)$$

where θ is the temperature difference and

N is the vertical temperature gradient.

In order to simplify the subsequent analysis, the effects of variable thermal conductivity and variable viscosity will, henceforth, be separately analysed.

2.1 THE VARIABLE THERMAL CONDUCTIVITY

In this section, the thermal conductivity (K) will be treated as a parameter depending solely on temperature while the viscosity (μ) is held constant. Thus

$$K = K_0 (\alpha\theta + 1)$$

when K_0 and α are constants.

The momentum, energy and magnetic induction equations for this model reduces respectively to the following equations.

$$\frac{\partial P}{\partial Y} = \frac{B}{\mu_0} \frac{dB}{dy} = 0 \quad (2.3)$$

$$\frac{B_0}{\mu_0} \frac{dB}{dy} + \rho g \beta (\theta + NZ) - \frac{\partial P}{\partial z} - \frac{\partial P}{\partial z} + \mu_0 \frac{d^2 V}{dy^2} = 0 \quad (2.4)$$

$$C_p \rho V N = \frac{\partial}{\partial y} \{K_0(\alpha \theta + 1) \frac{\partial T}{\partial y}\} + \mu_0 \left(\frac{\partial v}{\partial y} \right)^2 - \frac{\partial q_R}{\partial y} \quad (2.5)$$

$$\frac{d^2 B}{dy^2} + B_0 \sigma \mu_0 \frac{dv}{dy} = 0 \quad (2.6)$$

where B_0 is applied magnetic field

ρ is the density

q_R is the radiative heat flux

C_p is the specific heat at constant pressure

β is the coefficient of volume expansion

μ_0 is the magnetic permeability

and

σ is the electrical conductivity

Equation (2.5) reduces in view of equation (2.1), to

$$VN = \xi(\alpha \theta + 1) \frac{d^2 \theta}{dy^2} + \xi \alpha \left(\frac{d\theta}{dy} \right)^2 + \frac{v}{C_p} \left(\frac{dv}{dy} \right)^2 - \frac{1}{C_p \rho} \frac{dq}{dy} \quad (2.7)$$

where

$$\xi = \frac{K_0}{\rho C_p} \text{ is the thermal diffusivity and } v = \frac{\mu_0}{\rho} \text{ is the kinematic viscosity.}$$

In optically thin limits as applicable in this paper, the fluid does not absorb its own emitted radiation but does absorb radiation emitted by boundaries. Cogley, Vincenti and Gillies [11] showed that in the optically thin limit for a non-grey gas near equilibrium

$$\frac{\partial q_R}{\partial y} = 4(T - T_w) \int_0^\infty K_{\lambda w} \left(\frac{de_{b\lambda}}{dT} \right)_w d\lambda$$

where K_λ is the absorption coefficient,

$e_{b\lambda}$ is the plank function and the subscript w refers to the values at the wall.

Hence equation (2.7) becomes

$$VN = \xi(\alpha\theta + 1) \frac{d^2\theta}{dy^2} + \xi\alpha \left(\frac{d\theta}{dy} \right)^2 + \frac{v}{C_p} \left(\frac{dv}{dy} \right)^2 - C\theta$$

where

$$C = \frac{4}{C_p \rho} \int_0^\infty K_{\lambda w} \left(\frac{de_{b\lambda}}{dT} \right)_w d\lambda$$

Integrating equation (2.3) with respect to y and differentiating the result with respect to z gives

$$\frac{\partial p}{\partial z} = \rho \frac{\partial f(z)}{\partial z} \quad (2.9)$$

By substituting equation (2.9) into equation (2.4) we have

$$v \frac{d^2 v}{dy^2} + \frac{B_0}{\mu_0 \rho} \frac{dB}{dy} + g\beta\theta = \frac{\partial f(z)}{\partial z} - g\beta NZ \quad (2.10)$$

Each side of equation (2.10) can be equated to a constant. Hence

$$v \frac{d^2 v}{dy^2} + \frac{B_0}{\mu_0 \rho} \frac{dB}{dy} + g\beta\theta = C_1 \quad (2.11)$$

We now introduce the following dimensionless quantities

$$\eta = \frac{y}{L}, \quad U = \frac{v}{V_0}, \quad t = \frac{-\theta}{NL}, \quad b = \frac{B}{B_0} \quad (2.12)$$

and

$$\begin{aligned}
 \text{Hertmann number } (M) &= B_0 L \left(\frac{\sigma}{\nu e} \right)^{\frac{1}{2}} \\
 \text{Grashof number } (G_n) &= \frac{g B N L^3}{\nu \nu_0} \\
 \text{Magnetic Prandtl number } (P_m) &= \nu \mu_0 \sigma \\
 \text{Prandtl number } (P_n) &= \frac{\nu}{\xi} \\
 \text{Reynolds number } (R_n) &= \frac{\rho L V_0}{\mu_0}
 \end{aligned}
 \tag{2.13}$$

The following non-dimensional governing differential equations for the variable thermal conductivity are then obtained by substituting equations (2.12) and (2.13) into equations (2.6), (2.8) and (2.11)

$$\frac{d^2 u}{d\eta^2} + \frac{M^2}{P_m R_n} \frac{db}{d\eta} - G_n \cdot t = C_2 \tag{2.14}$$

$$\frac{\alpha A t}{R_n R_n} \frac{d^2 t}{d\eta^2} - \frac{1}{P_n R_n} \frac{d^2 t}{d\eta^2} + \frac{\alpha A}{P_n R_n} \left(\frac{dt}{d\eta} \right)^2 + E_n \left(\frac{du}{d\eta} \right)^2 \tag{2.15}$$

$$\frac{1}{R_n P_n} \frac{d^2 b}{d\eta^2} + \frac{du}{d\eta} = 0 \tag{2.16}$$

The relevant boundary conditions are

$$t = b = u = 0 \text{ at } \eta = \pm 1$$

Note that in equations (2.14) and (2.15), we have

$$C_2 = \frac{C_1 L^2}{V_0 \nu}, \quad F = \frac{CL}{\nu_0}, \quad E_n = \frac{V_0^2}{C_p N L R_n} \text{ and } A = L\mu$$

2.2 VARIABLE VISCOSITY PARAMETER

In this section, the thermal conductivity is held constant while the viscosity is treated as a parameter depending on temperature. Thus

$$\mu = \mu_0(\alpha\theta + 1)$$

where μ_0 and α are constants. For the variable viscosity, we make use of equations (2.3) and (2.6) but replace equations (2.4) and (2.5) by the following equations.

$$\frac{B_0}{\mu_0} \frac{dB}{dy} + \rho g \beta (\theta + NZ) - \frac{\partial p}{\partial z} + \frac{\partial}{\partial y} \left\{ \mu_0 (\alpha\theta + 1) \frac{dv}{dy} \right\} = 0 \quad (2.17)$$

$$C_p \rho V N = K_0 \frac{\partial^2 T}{\partial y^2} + \mu_0 (\alpha\theta + 1) \left(\frac{dv}{dy} \right)^2 - \frac{\partial q_R}{\partial y} \quad (2.18)$$

respectively.

Simplifying equations (2.3), (2.6), (2.17) and (2.18) as it was done for the case of variable thermal conductivity and using the non-dimensional equations (2.12) and (2.13), the following governing dimensionless equations for the variable viscosity are obtained.

$$\frac{d^2 b}{d\eta^2} + R_n P_m \frac{dy}{d\eta} = 0 \quad (2.19)$$

$$U = \frac{-1}{P_n R_n} \frac{d^2 t}{d\eta^2} - \alpha A E_n t \left(\frac{dy}{d\eta} \right)^2 + E_n \left(\frac{dy}{d\eta} \right)^2 + F.t \quad (2.20)$$

$$- \alpha A t \frac{d^2 U}{dy^2} + \frac{d^2 y}{dy^2} - \alpha A \frac{dt}{dy} \frac{dy}{d\eta} + \frac{M^2}{P_m R_n} \frac{db}{d\eta} - G_n t = C_2 \quad (2.20)$$

Equations (2.19) - (2.21) are to be solved subject to the boundary condition $t = b = u = 0$ at $\eta = -1$ and $+1$

3. APPROXIMATE SOLUTION

(A) The Variable Thermal Conductivity Equations.

Integrating equation (2.16) once with respect to η gives

$$U = C_3 - \frac{1}{P_m R_n} \frac{db}{d\eta} \quad (3.1)$$

By substituting equation (3.1) into equation (2.14) and (2.15)

$$\frac{db}{d\eta} = \frac{1}{M^2} \frac{d^3 b}{d\eta^3} + \frac{P_m R_n}{M^2} (G_n t + C_2) \quad (3.2)$$

$$C_3 - \frac{1}{P_m R_n} \frac{db}{d\eta} - \frac{E_n}{(P_m R_n)^2} \left(\frac{d^2 b}{d\eta^2} \right)^2 = \frac{\alpha A t}{P_m R_n} \frac{d^2 t}{d\eta^2} - \frac{1}{P_m R_n} \frac{d^2 t}{d\eta^2} + \frac{\alpha A}{P_m R_n} \left(\frac{dt}{d\eta} \right)^2 + F t \quad (3.3)$$

The equation (3.2) is then substituted into equation (3.3) and we obtained equation (3.4). The differential equations to be solved are

$$\frac{t''}{P_m R_n} - \frac{\alpha A t''}{P_m R_n} - \frac{\alpha A (t')^2}{P_m R_n} - \left(F + \frac{G_n}{M^2} \right) t - C_4 = \frac{-b''}{M^2 P_m R_n} + E_n \left(\frac{b''}{P_m R_n} \right)^2 \quad (3.4)$$

$$\frac{-b}{P_m R_n} + \frac{M^2 b}{P_m R_n} - G_n t = C_2 \quad (3.5)$$

$$U = C_3 - \frac{b}{P_m R_n} \quad (3.6)$$

$$\text{where } C_4 = \frac{C_2}{M^2} - C_3$$

subject to the boundary conditions $t = u = b = 0$ at $\eta = -1$ and $+1$.

The difficulties involved in solving the above non-linear coupled differential equations make us to resort to using the successive approximation method to solve these differential equations. This method involves keeping certain terms, neglecting some and approximating yet others [2].

In particular, we first neglect the right hand side and the non-linear terms in the equation (3.4) and solve the reduced equation to obtain approximated expression (t_1) for the temperature. This expression for t_1 is substituted into equation (3.5) and the approximated expression (b_1) for the magnetic field is obtained. Then, the approximated velocity (U_1) is also obtained from equation (3.6).

Next, we substitute the solutions into the neglected terms of equation (3.4) and solve for a better approximated expression t_2 . Then, better approximated expressions for b_2 and U_2 can be obtained from equations (3.5) and (3.6) respectively. It shows that t_{n+1} is a better approximation to t_n on proceeding to get $t_2, t_3, \dots, t_{n+1}$ having $|t_{n+1} - t_n| < \xi$ for any $\xi > 0$. The same applies to b_i and U_i . ($i=1,2,3,\dots,n+1$).

Using the method already alluded to, we obtain the following solutions.

$$t = 2[B_1 \cosh 2r\eta + B_2 \cosh 2M\eta + B_3 \eta \sinh rM\eta - (HM^2 + k) \sinh r\eta + B_7 \cosh M\eta + B_9 \cosh (M+r)\eta + B_{11} \cosh (M-r)\eta] + B_{13} + HM^2 \quad (3.7)$$

$$b = 2\left[\left(\frac{A_8}{r} - \frac{A_7}{r^2}\right) \sinh r\eta - \left(\frac{A_{11}}{M^2} + \frac{Q}{Mq}\right) \sinh M\eta + A_7 \frac{\eta}{r} \cosh r\eta + A_{11} \frac{\eta}{M} \cosh M\eta + \frac{A_{13}}{2} r \sinh 2r\eta + \frac{A_5}{2} M \sinh 2M\eta + \frac{A_{13}}{M+r} \sinh (M+r)\eta + \frac{A_{15}}{M-r} \sinh (M-r)\eta\right] + A_{17} \eta \quad (3.8)$$

$$U = \frac{-2}{P_m R_n q} \left\{ \frac{Q}{q} \sinh M\eta + A_3 \sinh 2r\eta + A_5 \sinh 2M\eta + A_{17} \eta (\cosh r\eta) + A_8 \sinh r\eta + A_{11} \eta \sinh M\eta + A_{13} \cosh (M+r)\eta + A_{15} \sinh (M-r)\eta - \frac{C_4}{2} + \frac{B_{13} G_n d}{2 \cdot 2M} \right\} \quad (3.9)$$

$$\text{where } B_1 = \frac{(rM)^2(2\alpha A(HMgp)^2 + P_n R_n E_n d^2)}{3gPrM^2}$$

$$B_3 = \frac{E_n(Mf)^2 P_n R_n}{g^2 f^2 (4M^2 - r^2)}$$

$$B_5 = \frac{-r(\alpha AH^2 M^4 g + G_n H) P_n}{2gp}$$

$$B_7 = \frac{fP_n R_n}{g^2 q}$$

$$B_9 = \frac{2E_n f r d P_n R_n}{g^2 q P(M+2r)}$$

$$B_{11} = \frac{2E_n f r d P_n R_n}{g^2 q P(M-2r)}$$

$$B_{13} = \frac{2E_n M^2((Mfp)^2 + (rqd^2))}{gqp^2(FM^2 + G_n)} - M^2 4$$

and

$$A_3 = \frac{B_1 S}{M^2 - 4r^2}, \quad A_5 = \frac{-B_2 S}{3M^2}, \quad A_7 = \frac{B_5 S}{g}$$

$$A_8 = \frac{-2B_5 r S}{g^2}, \quad A_{11} = \frac{-B_7 S}{2M}, \quad A_{13} = \frac{-B_9 S}{r(2M+r)}$$

$$A_{15} = \frac{B_{11} S}{r(2M-r)}, \quad A_{17} = \frac{P_n R_n}{M^2} (B_{13} G_n + \alpha + C_2)$$

$$\text{also } r = \left[\left(F + \frac{G_n}{M^2} \right) P_n R_n \right]^{\frac{1}{2}}, \quad H = \frac{-C}{M^2 F + G_n}, \quad S = G_n P_m R_n$$

$$d = G_n H M^2, \quad P = 2 \cosh r, \quad g = M^2 - r^2, \quad q = 2 \cosh M, \quad f = C_4' + \frac{C_4^2}{2M^2}$$

$$K = 2[B_1 \cosh 2r + B_3 \cosh 2M + B_5 \sinh r + B_7 \cosh M$$

$$+ B_9 \cosh(M+r) + B_{11} \cosh(M-r) + \frac{B_{13}}{2}]$$

$$Q = 2[P_m R_n \left(\frac{B_{13} C_n}{2M^2} + \frac{d}{2M^2} + \frac{C_4}{2} \right) + A_3 \cosh 2r$$

$$+ A_5 \cosh 2M + A_7 \sinh r + A_{11} \sinh M + A_{13} \cosh(M+r)$$

$$A_{15} \cosh(M-r)]$$

The flow rate is given by $W = \int_{-1}^1 U d\eta$

$$W = \frac{4G_n}{S} \left[\frac{A_3}{2r} \sinh 2r + \frac{A_5}{2M} \sinh 2M + \left(\frac{A_8}{7} - \frac{A_7}{r^2} \right) \cosh r \right.$$

$$+ \frac{A_{13}}{M+r} \cosh(M+r) + \frac{A_{15}}{M-r} \sinh(M-r) + \frac{A_7}{r} \sinh r$$

$$\left. + \frac{A_{11}}{M} \cosh M - \left(\frac{Q}{Mq} + \frac{A_{11}}{M^2} \right) \sinh M \right] - 2 \left(C_4 + \frac{B_{13} G_n}{M^2} + \frac{d}{M^2} \right)$$
(3.10)

The heat-transfer coefficient at the wall due to thermal conduction is given as

$$H = \left\{ \frac{df}{d\eta} \right\}_{\eta=0}$$

$$H = 2rB_1 \sinh 2r - 2MB_3 \sinh 2M - \left(B_5 - r \left(\frac{HM^2 + 1C}{P} \right) \right) \sinh r$$

$$+ rB_7 \cosh r - MB_9 \sinh M - (M-r)B_{11} \sinh(M-r)$$

$$+ (M-r)B_{13} \sinh(M-r)$$
(3.11)

(B) The Variable Viscosity Solutions.

By substituting equation (3.1) into equations (2.20) and 2.21), we have

$$\frac{(\alpha A t - 1)b}{P_m R_n} + \frac{\alpha A t \cdot b}{P_m R_n} + \frac{M^2}{P_m R_n} b - G_n t = C_2 \quad (3.12)$$

$$C_3 \left[\frac{b'}{P_m R_n} - \frac{t''}{P_m R_n} - \frac{\alpha A E_n t (b'')^2}{(P_m R_n)^2} + E_n \left(\frac{b''}{P_m R_n} \right)^2 + F t \right] \quad (3.13)$$

Having substituted b' from equation (3.12) into equation (3.13), the coupled non-linear differential equations to be solved are

$$\begin{aligned} \frac{t}{P_m R_n} - \left(F + \frac{G_n}{M^2} \right) t - C_4 &= \frac{\alpha A}{R_n P_m M^2} (tb) + \frac{b}{M^2 R_n P_m} \\ &- \frac{E_n}{(R_n P_m)^2} (\alpha A t - 1)(b'')^2 \end{aligned} \quad (3.14)$$

$$\frac{-b}{P_m R_n} + \frac{M^2 h}{P_m R_n} - C_2 = \frac{-\alpha (tb)}{P_n P_m} + G_n t$$

$$U = C_3 - \frac{b}{P_m R_n} \quad (3.15)$$

subject to the boundary conditions $b = u = t = 0$ at $\eta = -1$ and $\eta = +1$

The solutions obtained by successive approximation are

$$\begin{aligned} t &= \left[\frac{(K_2 + M^2 h)}{P} \cosh r \eta + a_3 \cosh M \eta + a_5 \cosh 2M \eta + a_7 \cosh (M+r) \eta \right. \\ &+ a_9 \cosh (M-r) \eta + a_{11} \cosh (2M+r) \eta + a_{13} \cosh (2M-r) \eta \\ &\left. + a_1 \eta \sinh r \eta \right] + a_{16} \end{aligned} \quad (3.16)$$

$$\begin{aligned}
b = & 2 \left[\frac{-Q_2}{M_4} \sinh M \eta + \frac{D_2}{r} \sinh r \eta + D - \frac{4}{M} (\eta \cosh M \eta - \frac{\sinh M \eta}{h}) \right] \\
& + \frac{D_6}{2M} \sinh 2M \eta + \frac{D_8}{3M} \sinh 3M \eta + \frac{D_{10}}{r+M} (\eta \cosh(M+r) \eta \\
& - \frac{1}{r+M} \sinh(r+M) \eta) + \frac{D_{23}}{r+M} \sinh(r+M) \eta + \frac{D_{12}}{r-M} (\eta \cosh(r-M) \eta \\
& - \frac{1}{r-M} \sinh(r-M) \eta) + \frac{D_{25}}{r-M} \sinh(r-M) \eta + \frac{D_{14}}{2M+r} \sinh(2M+r) \eta \\
& + \frac{D_{16}}{2M-r} \sinh(2M-r) \eta + \frac{D_{18}}{3M+r} \sinh(3M+r) \eta \\
& + \frac{D_{20}}{3M-r} \sinh(3M-r) \eta + \frac{P_M R_n}{2} \left(\frac{G_n a_{16} + C_2}{M^2} \right) \eta]
\end{aligned} \tag{3.17}$$

$$\begin{aligned}
U = & 2 \left[\frac{C_4}{2} - \frac{Q_2}{q} \cosh M \eta + D_2 \cosh r \eta + D_4 \eta \sinh M \eta \right. \\
& + D_6 \cosh 2M \eta + D_8 \cosh M \eta + D_{10} \eta \sinh(r+M) \eta \\
& + D_{23} \cosh(r+M) \eta + D_{12} \eta \sinh(r-M) \eta + D_{25} \cosh(r-M) \eta \\
& + D_{14} \cosh(2M+r) \eta + D_{16} \cosh(2M-r) \eta + D_{18} \cosh(3M+r) \eta \\
& \left. + D_{20} \cosh(3M-r) \eta + \frac{G_n a_{16}}{M^2} \right]
\end{aligned} \tag{3.18}$$

$$\begin{aligned}
\text{where } a_1 &= \frac{E_n d_1^2 M^2 R}{rP} & a_3 &= \frac{-d_1(RM^2+1)P_n R_n}{g} \\
a_5 &= \frac{E_n M^2 d_1^2 (1-RM^2) E_n P_n}{(2M+r)(2M-r)} & a_7 &= \frac{Rd_1(M+r)P_n R_n}{P(M+2r)} \\
a_9 &= \frac{Rd_1(M-r)R_n P_n}{P(M-2r)} & a_{11} &= \frac{E_n d_1^2 M^3 R R_n P_n}{4(M+r)p} \\
a_{13} &= \frac{E_n R M^2 d_1^2 R_n P_n}{4(M-r)P} & a_{15} &= \frac{2E_n M^4 d_1^2 (1-RM^2)}{(M^2 F + G_n)} - HM^2 \\
a_{16} &= a_{15} + M^2 H & a_{17} &= \frac{K+M^2 H}{P}
\end{aligned}$$

and

$$\begin{aligned}
D_2 &= \frac{L_1(2M-r)a_9 + G_n a_{13})P_n R_n}{(r-M)(3M-r)} \\
D_4 &= -\{L_1(a_{16} - a_{15}) + G_n a_3\} \frac{P_n R_n}{2} \\
D_6 &= -(2L_1 a_3 + G_n a_5) \frac{P_n R_n}{3M} \\
D_8 &= \frac{-3L_1 a_5 P_n R_n}{8M} \\
D_{10} &= -L_1(r+M) \frac{a_1 P_n R_n}{2M+r)r} \\
D_{23} &= \{L_1\{(a_1 - (r+M)(a_{17} + a_{11}))\} + G_n a_7\} \frac{P_n R_n}{M^2 - (r)}
\end{aligned}$$

$$\begin{aligned}
D_{12} &= -L_1(r-M)\frac{a_1 P_m R_n}{(2M-r)} \\
D_{25} &= \frac{\{L_1(r-M)(a_{13}-a_{17}) - a_{17}\} + \{G_n a_{16} + C_2\} P_m R_n}{M^2(r+M)} \\
D_{14} &= \frac{-\{L_1(2M+r)a_7 + G_n a_{11}\} P_m R_n}{(3M+r)(r+M)} \\
D_{16} &= \frac{\{L_1(2M-r)a_9 + G_n a_{13}\} P_m R_n}{(r-M)(3M-r)} \\
D_{18} &= \frac{-L_1(3M+r)a_{11} P_m R_n}{+(2M+r)(4M+r)}
\end{aligned}$$

$$D_{20} = \frac{L_1(3M-r)a_{13} P_m R_n}{r-2M(4M-r)} \quad D_{22} = \frac{(G_n a_{16} + C_2) P_m R_n}{M^2}$$

$$L_1 = d_1 M \alpha A, \quad d_1 = \frac{C_4}{q}, \quad R = \alpha A H.$$

$$\begin{aligned}
\text{Also } Q_2 &= 2[D_2 \cosh r + D_4 \sinh M + D_6 \cosh 2M + D_8 \sinh 3M \\
&\quad + D_{10} \sinh(M+r) + D_{23} \cosh(M+r) + D_{12} \sinh(M-r) + D_{25} \cosh(r-M) \\
&\quad + D_{14} \cosh(2M+r) + D_{16} \cosh(2M-r) + D_{18} \cosh(3M+r) \\
&\quad + D_{20} \cosh(3M-r)] + \left(\frac{G_n a_{16}}{M^2} + C_4 \right) P_m R_n
\end{aligned}$$

The heat transfer coefficient at the wall due to thermal conduction and the flow rate are evaluated by

$$\left[\frac{dt}{d\eta} \right]_{\eta=1} \quad \text{and} \quad \int_{-1}^1 U d\eta \quad \text{respectively.}$$

4. RESULTS AND CONCLUSIONS

It is evidence in the variable viscosity and variable thermal conductivity that temperature difference (θ) increases with an increase in viscous dissipative heat (E_v) as shown in figures (4.1b and

4.1a) and the more the viscous dissipative heat (E_n), the more the velocity (u) for variable viscosity (figure 4.2b) with fixed $M^2 = 10$, $G_n = 3$, $F = 1$ and $C_4 = 0.1$. This increase in velocity for more value of E_n causes the increase in the magnetic field as shown in figure (4.3b). As for the variable thermal conductivity, an increase in viscous dissipation heat (E_n) reduces the velocity as shown in figure (4.2a). This in turn, reduces the magnetic field b with fixed values of M^2 , F , G_n and C_4 figure (4.3a). In the same manner, the flow rate (w) increases with an increase in E_n for variable viscosity and decreases with an increase in E_n for variable thermal conductivity as evidence in Tables (4.1a) and (4.1b).

The effect of radiation is to reduce the influence of natural convection which causes a reduction in the temperature difference between the fluid and the channel walls. This is seen in both variable viscosity and thermal conductivity where t decreases with an increase in F for fixed G_n , M^2 , E_n and C_4 . Figures (4.4a & 4.4b). Due to this effect, the velocity for variable viscosity and variable thermal conductivity is high in accordance to the intensity of fluid radiation. An increase in radiation (F) shows an increase in the velocity with fixed G_n , M^2 , E_n and C_4 and this increase in the velocity (u) and flow rate (w) seems to pull the magnetic line of force thereby causing an increase in the induced magnetic field. Figures (4.6a), (4.6b) and Tables (4.3a), (4.3a), (4.3b).

It is found that an increase in viscous dissipative heat E_n shows an increase in heat-transfer (H) at the wall. Table (4.2a and 4.2b). But, an increase in F indicates a decrease in heat-transfer (H) as shown in Tables (4.4a) and (4.4b).

VARIATION OF FLOW RATE (w) WITH RADIATION (F) AND VISCOUS DISSIPATIVE HEAT (E_n)

TABLE 4.1A:

VARIABLE THERMAL CONDUCTIVITY				
$F = 1$	$E_n = 3$	$E_n = 6$	$E_n = 9$	$E_n = 12$
$F = 1$	0.1354958	0.134592	0.1335825	0.1326259
$F = 2$	0.1361278	0.1345654	0.1354403	0.1327407
$F = 3$	0.1372639	0.1358832	0.1345025	0.1331218
$F = 4$	0.1381782	0.137193	0.1362044	0.1352175

TABLE 4.1B: VARIABLE VISCOSITY

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F	$E_n = 3$	$E_n = 6$	$E_n = 9$	$E_n = 12$
1	0.1571733	0.1670427	0.1769121	0.1867815
2	0.1579355	0.1681747	0.1784141	0.1886534
3	0.1620841	0.1725440	0.1830034	0.1934637
4	0.1636495	0.1740338	0.1844178	0.1948019

**VARIATION OF HEAT TRANSFER (H) RATE AT THE WALL
WITH RADIATION (F) AND VISCOUS DISSIPATIVE HEAT
(E_n)**

TABLE 4.2A.

VARIABLE THERMAL CONDUCTIVITY				
F	$E_n = 3$	$E_n = 6$	$E_n = 9$	$E_n = 12$
1	0.02976314	0.08400632	0.1382495	0.1924927
2	0.28237193	0.08034316	0.1381913	0.1955517
3	0.02140802	0.0797234	0.1380389	0.1963544
4	0.01326925	0.07064538	0.1280215	0.1853976

TABLE 4.2B: VARIABLE VISCOSITY

F	$E_n = 3$	$E_n = 6$	$E_n = 9$	$E_n = 12$
1	0.0621070	0.1003632	0.1386195	0.1768757
2	0.0579838	0.0942901	0.1305965	0.1669029
3	0.0550175	0.0898862	0.1247548	0.1596234
4	0.0573365	0.0866470	0.1202073	0.1537442

VARIATION OF FLOW RATE (W) WITH MAGNETIC FIELD
AND VISCOUS DISSIPATIVE HEAT (E_n)

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TABLE 4.3A.

VARIABLE THERMAL CONDUCTIVITY				
M^2	$E_n = 3$	$E_n = 6$	$E_n = 9$	$E_n = 12$
10	0.13549680	0.1345392	0.1335828	0.1326259
20	0.07734872	0.0772730	0.0771973	0.07712162
30	0.05439213	0.0543780	0.0543640	0.05434996

TABLE 4.3B: VARIABLE VISCOSITY

M^2	$E_n = 3$	$E_n = 6$	$E_n = 9$	$E_n = 12$
10	0.1571733	0.1670427	0.1769121	0.1867815
20	0.0806087	0.0820338	0.0834588	0.0848839
30	0.0554268	0.0558728	0.0563188	0.0567648

VARIATION OF HEAT TRANSFER (H) WITH MAGNETIC
FIELD AND VISCOUS DISSIPATIVE HEAT (E_n)

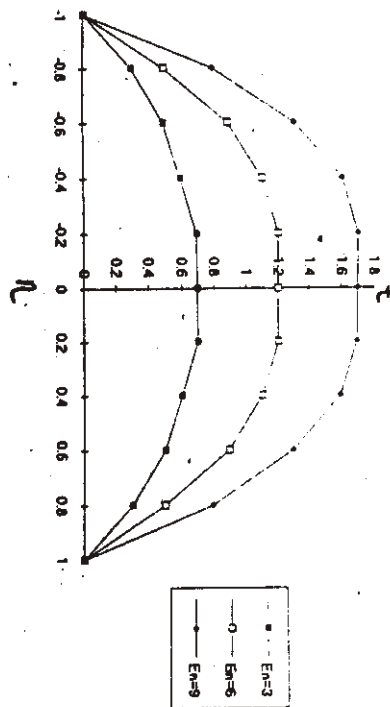
TABLE 4.4A.

VARIABLE THERMAL CONDUCTIVITY				
M^2	$E_n = 3$	$E_n = 6$	$E_n = 9$	$E_n = 12$
10	0.02976314	0.08400632	0.1382495	0.1924927
20	0.00862472	0.02634745	0.0440701	0.0617929
30	0.00421126	0.01348340	0.0229558	0.0320276

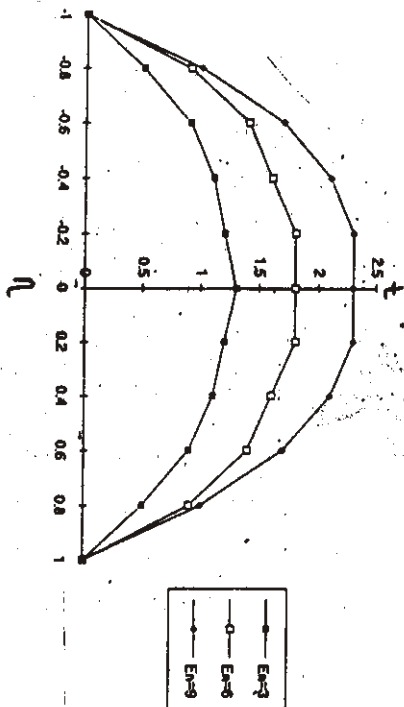
TABLE 4.4B: VARIABLE VISCOSITY

M^2	$E_n = 3$	$E_n = 6$	$E_n = 9$	$E_n = 12$
10	0.06210703	0.10036332	0.1386195	0.1768757
20	0.02361570	0.03814167	0.05266763	0.06719359
30	0.01317515	0.02125241	0.02932968	0.03740695

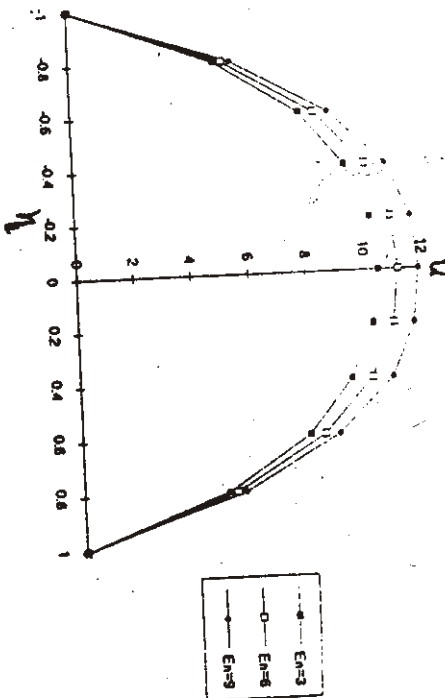
EFFECT OF VISCOS DISSIPATIVE HEAT ON TEMPERATURE
VARIABLE THERMAL CONDUCTIVITY
Figure 4.1a



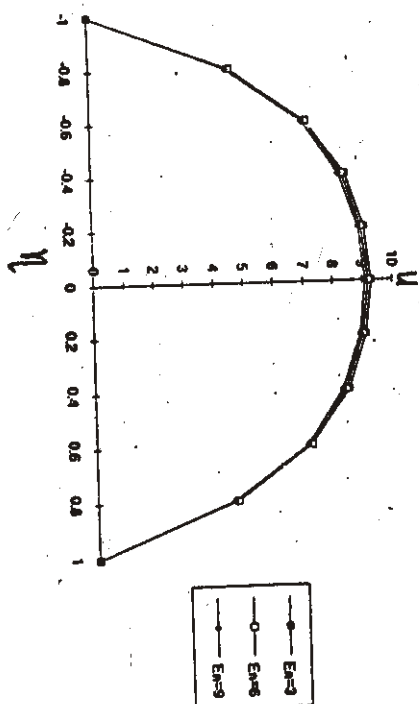
EFFECT OF VISCOS DISSIPATIVE HEAT ON TEMPERATURE
VARIABLE VISCOSITY
Figure 4.1b



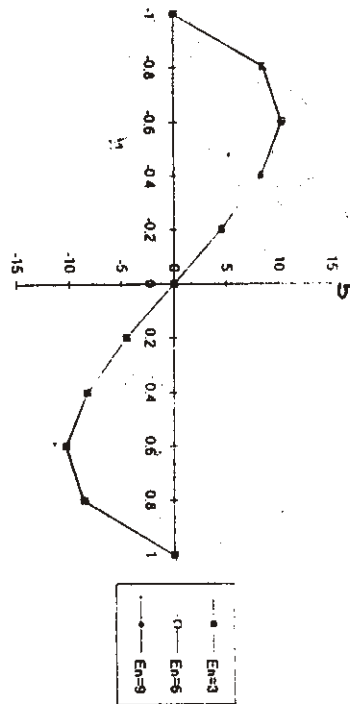
EFFECT OF VISCOS DISSIPATIVE HEAT ON VELOCITY
VARIABLE VISCOSITY
Figure 4.2a



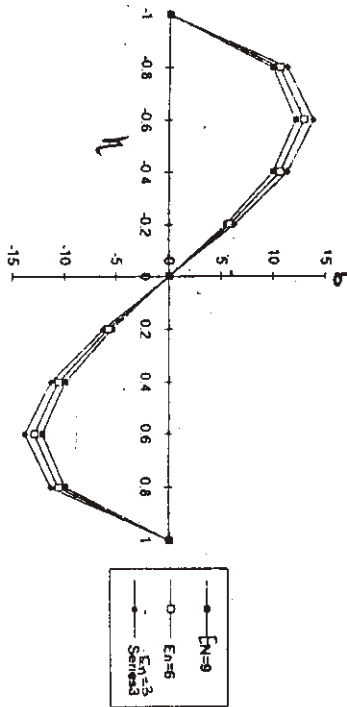
EFFECT OF VISCOS DISSIPATIVE HEAT ON VELOCITY
VARIABLE THERMAL CONDUCTIVITY
Figure 4.2b



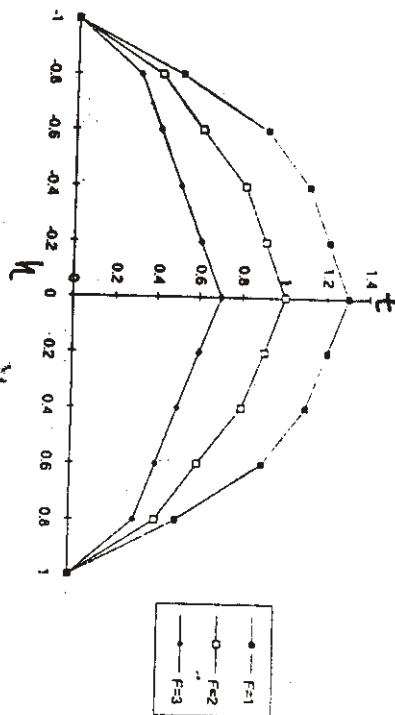
EFFECT OF VISCIOUS DISSIPATIVE HEAT ON MAGNETIC FIELD
VARIABLE THERMAL CONDUCTIVITY
Figure 4.3a



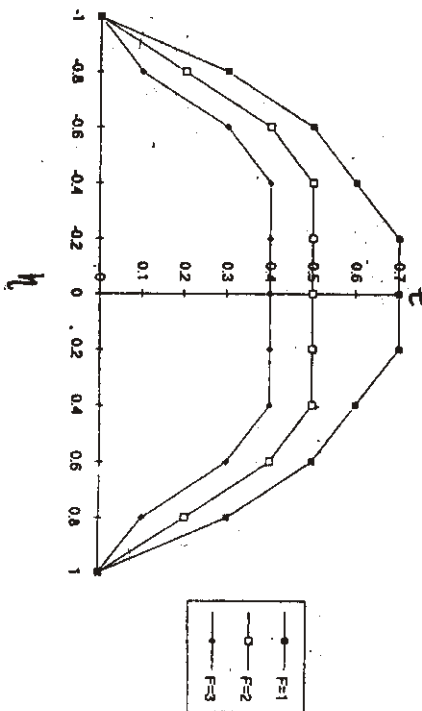
EFFECT OF VISCIOUS DISSIPATIVE HEAT ON MAGNETIC FIELD
VARIABLE VISCOSITY
Figure 4.3b



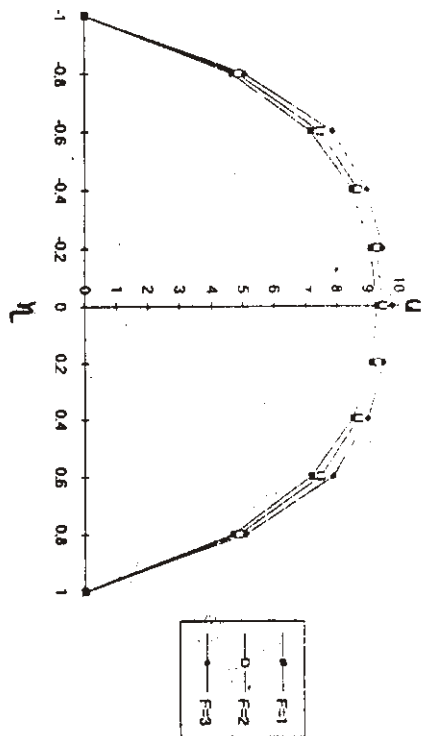
EFFECT OF RADIATION ON TEMPERATURE
VARIABLE VISCOSITY
Figure 4.4a



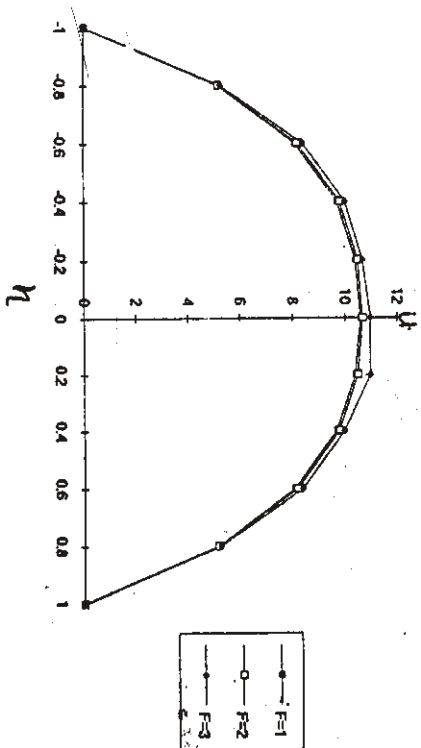
EFFECT OF RADIATION ON TEMPERATURE
VARIABLE THERMAL CONDUCTIVITY
Figure 4.4b



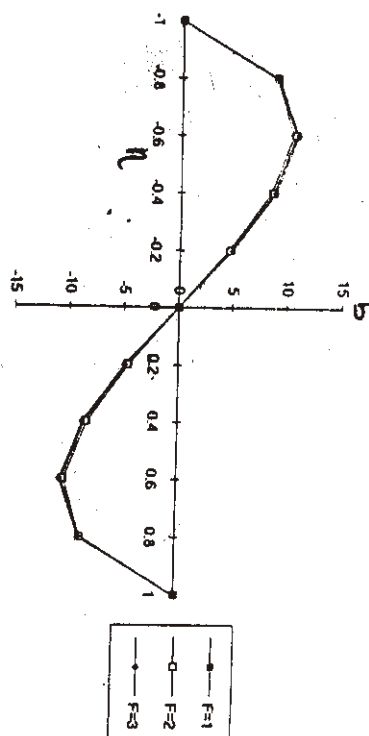
EFFECT OF RADIATION ON VELOCITY
VARIABLE THERMAL CONDUCTIVITY
Figure 4.5a



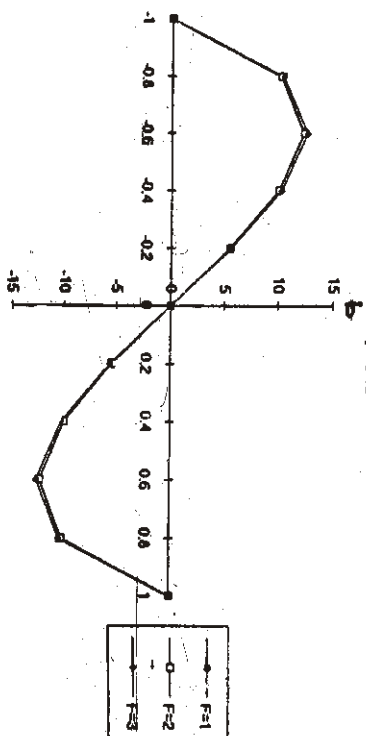
EFFECT OF RADIATION ON VELOCITY
VARIABLE VISCOSITY
Figure 4.5b



EFFECT OF RADIATION ON MAGNETIC FIELD
VARIABLE THERMAL CONDUCTIVITY
Figure 4.6a



EFFECT OF RADIATION ON MAGNETIC FIELD
VARIABLE VISCOSITY
Figure 4.6b



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