

LETTER

Fair Partitioning of the Downlink Resources of an OFDMA-Based Multi-User Multi-Tier Cellular Network Using Fractional Frequency Reuse*

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SUMMARY Orthogonal Frequency Division Multiple Access (OFDMA) is a leading air interface candidate for future generation cellular networks. However, if deployed in a multi-user multi-tier cellular system, it is important to fairly share radio resources such as transmission power and sub-carriers among co-tier and cross-tier users. This paper focuses on a mathematical formulation of cell inner-zone/outer-zone radio resource partitioning variables and considers the case of an FFR-based macrocell underlaid with femtocell. By applying an exhaustive search procedure on the developed formulation, we determine the optimal radio resource partitioning parameter values from the perspectives of macrocell user fairness and femtocell throughput maximization.

key words: resource partitioning, OFDMA, FFR, macrocell, femtocell

1. Introduction

The demand for higher data rates from mobile devices is currently on the rise and is pushing mobile operators to seek new ways of increasing ARPU (Average Revenue Per User) and reducing CAPEX (capital expenditure) and OPEX (operational expenditure) without compromising users' network experience. In this vein, technologies such as OFDMA-based (Orthogonal Frequency Division Multiple Access) RAT (Radio Access Technology) and femtocell (a low power indoor alternative of conventional outdoor macrocell which backhauls user data over residential DSL (Digital Subscriber Line), optical fibers etc. to the operator's core network) are increasingly gaining attention in the telecommunications community. Attractive features of OFDMA such as high spectral efficiency, multi-path tolerance, absence of intra-cell interference, and support for time-frequency domain scheduling for micro-/macro-diversity gain exploitation have made it a viable candidate for beyond 3G (Third Generation) systems which includes LTE (Long Term Evolution) and WiMAX (Worldwide In-

teroperability for Microwave Access) technologies [1], [2].

The femtocell, on the other hand, promises a substantial gain in network-wide throughput since it enables macrocell radio resources to be reused multiple times within the macrocell coverage area. In addition, its indoor placement reduces the separation distance between a UE (user equipment) and its associated base station, leading to better indoor signal quality compared to the conventional outdoor-to-indoor transmission of macrocell networks. In spite of the anticipated throughput gain enabled by these technologies, ensuring that all users receive a fair share of the network resources is another important issue; greedy users must be prevented from consuming excessive network resources and thus starving the other users. It is therefore imperative to ensure a proper network resource partitioning which can jointly reduce inter-cell interference in multi-tier networks (e.g. femtocell underlaid macrocells) and also ensure fairness among the served macrocell users. Fractional Frequency Reuse (FFR) [1]–[3] simplifies spectrum sharing between femtocell and macrocell tiers and is therefore an attractive technique for radio resource partitioning.

FFR separates the cell area into two zones namely, the inner zone (IZ) and the outer zone (OZ), and assigns different reuse factors to these zones. FFR is generally classified into hard-FFR and soft-FFR. Hard-FFR uses separate spectrum for the IZ and the OZ, while in soft-FFR, the IZ can reuse the frequency resources used by its adjacent OZ. To the best of the authors' knowledge, previous studies on FFR have not critically modeled the mutual relationship between the cell area partitioning parameter and the network resource partitioning parameter. This paper, which is an extension of [3], provides a mathematical formulation for these parameters, and by employing an exhaustive search procedure, their optimal values from the perspectives of macrocell user fairness and femtocell throughput maximization are determined. We have focused on improving the macrocell users' fairness and not the femtocell users fairness due to the following reasons.

1. A macrocell's coverage area is usually large, such that, the separation distance between an IZ user and the base station compared to the separation distance between an OZ user and the base station can be large, hence in-

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creasing the chances of unfairness between these two classes of users.

- Macrocells usually serve many users using limited radio resources; hence, the chances of having an unfairly served user is higher compared to femtocells.

In this paper, we have chosen to maximize femtocell throughput because femtocells are deployed in the coverage area of the macrocell and are not given free access to the network resources but are instead allowed to use only the unused macrocell resources in the zones where they (femtocells) are deployed.

Hard-FFR is assumed in this paper in order to reduce the possible interference that the femtocells located in the IZ of the macrocell receive from the macrocell.

The remainder of this paper is organized as follows: In Sect. 2, a formulation describing cellular network resource partitioning for a single cell macrocell system is presented. Section 3 addresses resource partitioning in a multi-tier FFR-based network, while the paper ends with concluding remarks in Sect. 4.

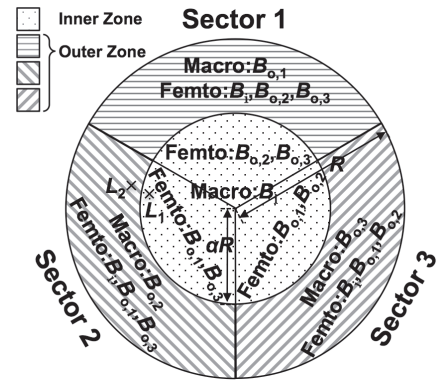
2. Macrocell Network Resource Partitioning

In this section, we establish mathematical relationship between resource partitioning parameters and determine the throughput-fair capacity region of the macrocell.

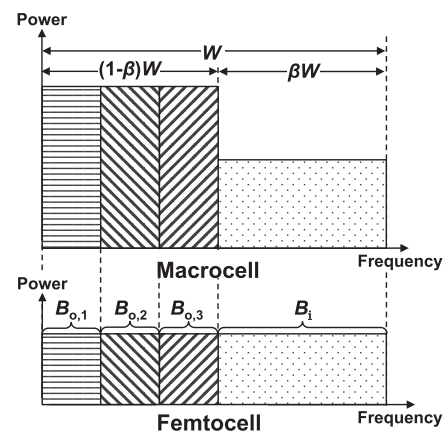
2.1 Mathematical Formulation for Throughput-Fair Resource Partitioning

In cellular networks, if network resources are not adequately shared between the OZ and IZ of a cell, there will be non-uniformity in the per user data rate of UEs located at the OZ and the IZ of the cell. This may stem from the fact that UEs at the IZ of the cell experience low path loss and are also more robust against inter-cell interference compared to those at the OZ who suffer from high interference as well as high path loss. For this reason, it is necessary to determine the optimal resource partitioning ratio that minimizes the margin between the per user capacities of UEs located at the IZ and the OZ of a macrocell. In order to address this radio resource partitioning problem, we consider the downlink of an OFDMA-based macrocell system illustrated in Fig. 1(a), and assume that network resources are partitioned based on FFR-scheme as illustrated in Fig. 1(b).

At the macrocell, the whole bandwidth is divided into two parts and one of the two parts is further divided into three parts, such that, the entire bandwidth is made up of sub-bands $B_{o,1}$, $B_{o,2}$, $B_{o,3}$, and B_i as illustrated in Fig. 1(b). The coverage area of the macrocell is divided into the IZ and the OZ, and sub-band B_i is assigned to the IZ. The OZ of the macrocell is further divided into three parts and the remaining sub-bands ($B_{o,1}$, $B_{o,2}$, and $B_{o,3}$) are shared among the parts, such that, Sectors 1, 2, and 3 of the OZ are assigned sub-bands $B_{o,1}$, $B_{o,2}$, and $B_{o,3}$, respectively, as illustrated in Fig. 1(a). Although, the OZ of the macrocell can



(a) Cell structure



(b) Network resource partitioning architecture

Fig. 1 Cell structure and the network resource partitioning architecture.

be divided into any number of sectoral regions, we have, however, divided it into three sectoral regions just to follow the typical convention adopted in many existing works on FFR (e.g., [4]). Moreover, the division helps to increase the overall network frequency efficiency since more unused sub-bands of the macrocell becomes available to the several femtocells located in the sectorized regions of the macrocell as will be explained shortly.

The patterns in Fig. 1(b) represent the sub-bands that constitute the bandwidth. The amount of power used by the macrocell and the femtocells on each sub-band is also shown in Fig. 1(b). It can be observed in Fig. 1(a) that the sub-bands used by the femtocells that are located in macrocell sub-areas are orthogonal to those sub-bands used by the macrocell in those sub-areas. For example, the femtocells located in the OZ of Sector 2 are prevented from using sub-band $B_{o,2}$ since sub-band $B_{o,2}$ is used by the macrocell users in that region. These femtocells are, however, allowed to use sub-bands B_i , $B_{o,1}$, and $B_{o,3}$ which are not used by the macrocell users in that region as illustrated in Figs. 1(a). Note that the access of these femtocells to sub-bands $B_{o,1}$ and $B_{o,3}$ is possible due to the sectorization of the macrocell OZ and this improves the overall network frequency effi-

ciency as mentioned previously.

The coverage area of the macrocell has a radius denoted as R , and there are N_m macrocell users that are uniformly distributed in the coverage area of the macrocell. P_T denotes the overall macrocell downlink transmission power per Hz, while P_c and P_e are the transmission power per Hz at the IZ and the OZ of the macrocell, respectively. Assuming the system bandwidth is denoted as W , then

$$P_T W = P_c \beta W + P_e (1 - \beta) W, \quad (1)$$

where β is the fraction of the system bandwidth allotted for IZ transmission. Since received power decays with distance according to a path loss exponent n [4], then, inter-zone received power fairness can be realized based on Eq. (2):

$$\frac{P_c}{(\alpha R)^n} = \frac{P_e}{R^n}, \quad (2)$$

where the left and right terms of Eq. (2) are the received powers per Hz of the macrocell users located at the edge region of the IZ and the OZ of the macrocell, respectively, while α is the fraction of R representing the IZ radius. Equation (2) ensures that the received power per Hz of a macrocell user located at the edge of the IZ of the macrocell is equal to the received power per Hz of a macrocell user located at the edge of the OZ of the macrocell. From Eqs. (1) and (2),

$$P_T = P_c \beta + P_e (1 - \beta) = P_e (\alpha^n \beta + 1 - \beta), \quad (3)$$

$$P_e = \frac{P_T}{\alpha^n \beta + 1 - \beta}, \quad (4)$$

and

$$P_c = \alpha^n P_e = \frac{\alpha^n P_T}{\alpha^n \beta + 1 - \beta}. \quad (5)$$

Let $\gamma_{c,u}$ and $\gamma_{e,u}$ denote the SNR (signal to noise ratio) of IZ and OZ macrocell UE, u , respectively, then

$$\gamma_{c,u} = \frac{\alpha^n P_T G_{c,u}}{(\alpha^n \beta + 1 - \beta) \sigma}, \quad (6)$$

while $\gamma_{e,u}$ is

$$\gamma_{e,u} = \frac{P_T G_{e,u}}{(\alpha^n \beta + 1 - \beta) \sigma}, \quad (7)$$

where $G_{c,u}$ and $G_{e,u}$ denote the signal gain (antenna gain and path loss) of the IZ and OZ UE, respectively, while σ denotes the additive white Gaussian noise power per Hz.

Let μ denote macro UE density which can be written as

$$\mu = \frac{N_m}{\pi R^2}, \quad (8)$$

the system bandwidth can be expressed as

$$W = \int_0^{2\pi} \int_0^R W_u \mu r dr d\theta, \quad (9)$$

where W_u is the per UE bandwidth. Based on Shannon's capacity equation, W_u can be expressed as

$$W_u = \frac{C_u}{\log_2(1 + \gamma_u)}, \quad (10)$$

where C_u is the capacity achieved per UE, while γ_u is the corresponding signal to noise ratio of the UE. Substituting for μ and W_u in Eq. (9),

$$\begin{aligned} W &= \int_0^{2\pi} \int_0^R \frac{C_u}{\log_2(1 + \gamma_u)} \frac{N_m}{\pi R^2} r dr d\theta \\ &= \frac{2N_m C_u}{R^2} \int_0^R \frac{1}{\log_2(1 + \gamma_u)} r dr. \end{aligned} \quad (11)$$

Re-arranging Eq. (11), we have

$$C_u = \frac{WR^2}{2N_m \int_0^R \frac{1}{\log_2(1 + \gamma_u)} r dr}. \quad (12)$$

Since β denotes the fraction of the system bandwidth allotted for IZ transmission, the capacity per UE at the IZ, $C_{c,u}$, can be expressed as

$$C_{c,u} = \frac{\beta WR^2}{2N_m \int_0^{\alpha R} \frac{1}{\log_2(1 + \gamma_{u,c})} r dr}, \quad (13)$$

while $C_{e,u}$, which denotes the capacity per UE at the OZ, is

$$C_{e,u} = \frac{(1 - \beta) WR^2}{2N_m \int_{\alpha R}^R \frac{1}{\log_2(1 + \gamma_{u,e})} r dr}. \quad (14)$$

The total macrocell capacity, C_T , is then

$$C_T = N_m \alpha^2 C_{c,u} + N_m (1 - \alpha^2) C_{e,u}. \quad (15)$$

Consider Eqs. (13) and (14), if α is decreased and β is simultaneously increased, $C_{c,u}$ increases while $C_{e,u}$ decreases. Conversely, if α is increased and β is simultaneously decreased, $C_{e,u}$ increases while $C_{c,u}$ decreases. Therefore, α and β can be tuned to achieve any desired level of fairness at the macrocell. In the next section, we discuss how to determine α and β and establish a relationship between them from the perspective of macrocell UE fairness.

2.2 Relationship between α and β at Throughput-Fair Region

It can be observed in Eqs. (13) and (14) that $C_{c,u}$ and $C_{e,u}$ are functions of α and β , hence, they can be represented as $C_{c,u}(\alpha, \beta)$ and $C_{e,u}(\alpha, \beta)$, respectively. Let $f(\alpha, \beta)$ denote the magnitude of the difference between them as shown in Eq. (16):

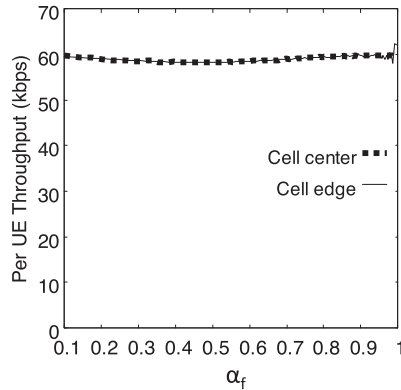
$$f(\alpha, \beta) = |C_{c,u}(\alpha, \beta) - C_{e,u}(\alpha, \beta)|. \quad (16)$$

In order to achieve fairness between the IZ macrocell UEs and the OZ macrocell UEs, we need to find the values of α and β which satisfies

$$f(\alpha, \beta) = 0. \quad (17)$$

Table 1 Configuration parameters.

Parameter	Setting
Sub-carrier spacing	8 kHz
Number of sub-carriers	600
Carrier frequency	2.14 GHz
Transmit power (macrocell, femtocell)	43 dBm, 20 dBm
Antenna gain (macrocell, femtocell, UE)	8 dBi, 3 dBi, 3 dBi
Path loss exponent	3.5
Thermal noise (density, figure)	-134 dBm/Hz, 7 dB
Macro-eNB cell radius	1,000 m
Number of Macro UEs	1,000

**Fig. 2** Per user throughput of macrocell.

It is, however, hard to solve Eq. (17) with respect to α and β explicitly. Therefore, we solve Eq. (18), which is a modified form of Eq. (17), numerically:

$$f(\alpha, \beta) \leq \epsilon, \quad (18)$$

where ϵ is an infinitesimal small number. More precisely, we monotonically increase β in the interval $(0, 1)$, and a corresponding α is selected as α_f from the α space for each β subject to the condition in Eq. (18).

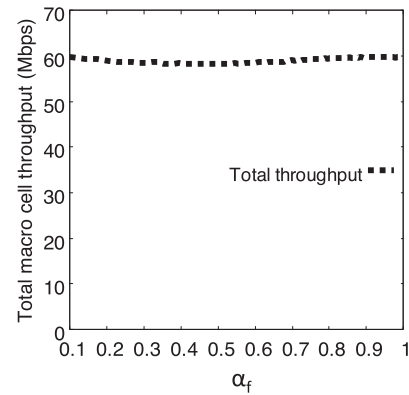
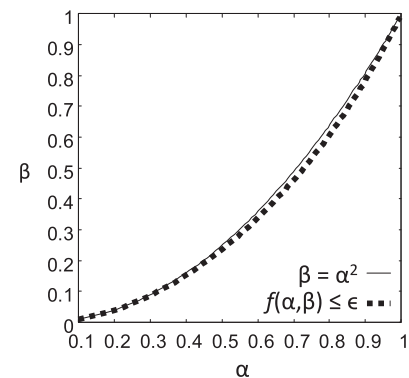
The numerical problem (i.e., Eq. (18)) is solved using *Simpson's* numerical method. Based on *Simpson's* numerical procedure, the values of $C_{c,u}$, $C_{e,u}$, and C_T are computed according to Eqs. (13), (14), and (15), respectively, using the configuration parameters in Table 1.

Figure 2 shows the throughput graphs of the macrocell IZ and OZ UEs (i.e., $C_{c,u}$ and $C_{e,u}$, respectively) plotted against α_f .

As shown in Fig. 2, it can be observed that $C_{c,u}$ and $C_{e,u}$ are approximately equal to each other and are constant regardless of α and β . This demonstrates a high level of fairness among the macrocell UEs. Figure 3 illustrates the graph of total macrocell throughput plotted against α_f , while Fig. 4 shows the graph of β plotted against α . An interesting deduction that can be made is that, at $\alpha = \alpha_f$ (i.e., throughput-fair region),

$$\beta = f(\alpha) \approx \alpha^2, \quad (19)$$

as shown in Fig. 4. This relationship between α and β holds for the path loss exponent, $n = 3.5$, used in this paper. If n is varied, the shape of the graph in Fig. 4 changes and so

**Fig. 3** Total throughput of macrocell.**Fig. 4** Comparison between $f(\alpha, \beta) \leq \epsilon$ graph and the graph of the quadratic equation $\beta = \alpha^2$.

also the representative function. Such function can be determined through a curve fitting procedure (e.g. least square method) and can be used in place of Eq. (19) in the next section. The next section considers a femtocell underlay scenario and discusses how to maximize femtocell throughput while still maintaining macrocell fairness by using Eq. (19).

3. Resource Partitioning in an FFR-Enabled Femtocell Underlaid Macrocell Network

In this section, we assume that a grid network of femtocell is integrated on the macrocell as underlay, and the structure of FFR resource partitioning adopted is as illustrated in Fig. 1(b). We also assume that only one femtocell UE is attached to each femtocell. It can be observed in Figs. 1(a) and (b) that the bandwidth resources used by the inner zone femtocells overlap that used by outer zone macrocell UEs which could result in interference. However, the interference affects only a small subset of the total number of macrocell UEs and femtocell UEs (i.e., UEs at the border region between the IZ and the OZ and also UEs at inter-sector border regions, e.g., Sector 1/Sector 2 border region). For example, mutual interference exists between a macrocell UE using sub-band B_i at location L_1 and a femtocell at location L_2 that is also using sub-band B_i . Actually, the femtocell at this border region (i.e., the femtocell at location L_2) should

sense the resources that are occupied by macrocell UEs in its surrounding so that reuse of such resources by this femtocell can be avoided, but we ignore this in our analysis due to the low number of affected UEs.

From Figs. 1(a) and (b), the size of bandwidth used by each femtocell UE located at the IZ of Sector 1 of the macrocell is

$$\frac{2}{3}(1-\beta)W. \quad (20)$$

In the case of the femtocell UEs located at the OZ of Sector 1 of the macrocell, the bandwidth used by each femtocell UE is

$$\frac{2}{3}(1-\beta)W + \beta W = \frac{(2+\beta)W}{3}. \quad (21)$$

It is noteworthy that we have considered only Sector 1 in Eqs. (20) and (21). This is because symmetry is assumed among the three sectors; hence, the computational results obtained for one of the sectors are applicable to the other sectors. The power transmitted by a serving femtocell (or interfering femtocell), s (or j), to femtocell UE, i , is $P_{s,i}$ (or $P_{j,i}$), while $G_{f,s,i}$ (or $G_{f,j,i}$) denotes the signal gain (antenna gain and the pathloss) between femtocell UE i and its serving femtocell (or interfering femtocell), while N_j denotes the number of neighbor interfering femtocells. Due to the way the bandwidth resources are partitioned and due to the equal distances between adjacent nodes of the grid network, the SINR (signal to interference plus noise ratio) seen at each femtocell is similar. The SINR, γ_i , of UE i can be expressed as

$$\gamma_i = \frac{P_{s,i}G_{f,s,i}}{\sum_{j=1, j \neq s}^{N_j} P_{j,i}G_{f,j,i} + \sigma}. \quad (22)$$

The total number of femtocells at the IZ is

$$N_f \frac{\pi(\alpha R)^2}{\pi R^2} = N_f \alpha^2, \quad (23)$$

where N_f is the total number of femtocells deployed in the network, while the total number of femtocells at the OZ is

$$N_f - N_f \alpha^2 = (1 - \alpha^2)N_f. \quad (24)$$

Combining Eqs. (20), (21), (22), (23), and (24), the total femtocell capacity, C_f , is computed as

$$\begin{aligned} C_f &= \frac{2(1-\beta)W\alpha^2 N_f}{3} \log_2(1 + \gamma_i) \\ &\quad + \frac{(2+\beta)(1-\alpha^2)WN_f}{3} \log_2(1 + \gamma_i) \\ &= \frac{2+\beta-3\alpha^2\beta}{3} N_f W \log_2(1 + \gamma_i). \end{aligned} \quad (25)$$

In order to ensure fairness at the macrocell layer, we replace β with α^2 (i.e., Eq. (19)) in Eq. (25):

$$C_f = \frac{2 + \alpha^2 - 3\alpha^4}{3} N_f W \log_2(1 + \gamma_i). \quad (26)$$

The maximum total throughput of the femtocells, $C_{f,\max}$, is:

$$C_{f,\max} = C_f \Big|_{\frac{\partial C_f}{\partial \alpha} = 0 \text{ and } \frac{\partial^2 C_f}{\partial \alpha^2} < 0}. \quad (27)$$

Solving Eq. (27), $\alpha = \sqrt{\frac{1}{6}}$ and $\beta = \alpha^2 = \frac{1}{6}$. These values of α and β ensure UE fairness at the macrocell layer and at the same time maximizes the total femtocell throughput.

4. Concluding Remarks

Future cellular networks will witness a mass deployment of small-cell base stations (femtocells) as underlay which makes careful consideration of radio resource partitioning among the layers of the network imperative. Since the radio condition is largely dependent on each user's separation distance from the base station, radio resource partitioning can be done with regards to users' distances from their respective base stations.

In this paper, we mathematically established the relationship between base station-to-user distance and radio resource partitioning variables. By applying an exhaustive search procedure on the developed formulation, we determined the optimal radio resource partitioning parameter values from the perspectives of macrocell user fairness and femtocell throughput maximization. It should, however, be noted that the results were obtained for a certain network model.

Acknowledgment

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