

CONSTRUCTION OF OPTIMAL MINIMUM REPLICATE DESIGNS.

By

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B.Sc. (Ilorin), M.Sc(Ilorin)

**A Thesis Submitted to the Department of Statistics, Faculty of
Physical Sciences, University of Ilorin in Partial Fulfillment of
the Requirements for the Award of the Degree of Doctor of
Philosophy in Statistics.**

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CERTIFICATION

This is to certify that this thesis is an original work carried out by **Adijat Bukola AIYELABEGAN** with matriculation number 91/028150 in the department of statistics, Faculty of physical sciences, University of Ilorin. The thesis has been read and approved as meeting the requirements for the award of Doctor of philosophy in statistics of the University of Ilorin, Ilorin, Nigeria.

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DEDICATION

This dissertation is dedicated to my late father and my late husband

ACKNOWLEDGEMENT

Firstly, I would like to express my profound gratitude to my supervisor Prof. R.A Ipinyomi, for his patience, understanding, generous guidance and the support throughout the course of this research and writing of this thesis.

My sincere thanks also goes to Dr Isiaka Oloyede and Dr G.M Oyeyemi for their comments and encouragements, for providing softwares to widen my research work.

I would like thank and appreciate the Tertiary Education Trust Fund TETFUND for sponsoring my Ph.D programme. I am most Grateful.

Finally, I would like to thank my children for supporting me morally and spiritually throughout writing of this thesis.

TABLE OF CONTENTS

Title page	i
Certification	ii
Dedication	iii
Acknowledgement	iv
Abstract	v
Table of Contents	vi
List of Table	viii
List of Figure	xi

CHAPTER ONE: INTRODUCTION

1.1	Definition	5
1.2	Statement of the Problem	7
1.3	Research aim and Objectives	8
1.4	Justification of Study	8

CHAPTER TWO:

Literature Review	10
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CHAPTER THREE: RESEARCH METHODOLOGY

3.1	Introduction	17
3.2	Construction of Optimal Connected Designs	17
3.3	Selection of good designs using optimality criterion	19
3.4	SS-Optimality Criterion	19
3.5	Connections between A-, D- and E-optimality Criteria	21
3.6	Connections between A-, D-, E and SS-optimality Criteria	21
3.7	Advantage of using of SS-Optimality Criteria	22
3.8	Symmetric, Anti-symmetric Designs and their Complementary Designs	22

3.9	Definition	23
3.10	Analysis of Connected optimal designs	25
3.11	Stimulation	25
CHAPTER FOUR: RESULTS		
4.1	Introduction	27
4.2	Mathematical expression on symmetric and anti-symmetric designs	28
4.3	Symmetric, Non-Symmetric and Complementary Designs	45
4.4.	Analysis of Connected Optimal Designs	57
CHAPTER FIVE: CONCLUSION AND RECOMMENDATION		
5.1	Introduction	72
5.2	Conclusion	74
	Reference	76
	Appendix A	79
	Appendix B	80

LIST OF TABLES

Table 4.2.1.	All Designs Generated	32
Table 4.2.2	Partially Balanced Incomplete Block Designs with 2 Associate Classes PBIB (2)	35
Table 4.2.3	Partially Balanced Incomplete Block Design with 3 or More Associate Classes	36
Table 4.2.4.	Selected Good Designs with Minimum Replications	38
Table 4.2.5	Partially Balanced Incomplete Block Designs with one associated PBIBD/1	39
Table 4.3.1	Class of Complementary Designs For Symmetric PBID	45
Table 4.3.2	Class of Anti-symmetric PBIDs	47
Table 4.3.3.	Class of Complementary Designs For Anti-Symmetric PBIDS	50
Table 4.4.1	Treatment and Replication Constraint (Incomplete Block Designs)	52
Table 4.4.2	Block and Block Size Constraint (Incomplete Block Complementary Designs)	54
Table 4.4.3	Unit Constraint	56
Table 4.5.1.	Performance of REML and ML Criterion in Incomplete Block Analysis (9,6,3,2) PBIB(2)	57
Table 4.5.2	Performance of REML and ML Criterion in Incomplete Block Analysis (12,8,3,2) PBIB(5)	58
Table 4.5.3	Performance of REML and ML Criterion in Incomplete Block Analysis (18,12,3,2) PBIB(8)	58
Table 4.5.4	Performance of REML and ML Criterion in Incomplete Block Analysis (21,14,3,2) PBIB(3)	59
Table 4.5.5	Performance of REML and ML Criterion in Incomplete Block Analysis (24,16,3,2) PBIB(13)	59
Table 4.5.6	Performance of REML and ML Criterion in Incomplete Block Analysis (27,18,3,2)PBIB(8)	60

Table 4.5.7	Performance of REML and ML Criterion in Incomplete Block Analysis (16,8,4,2) PBIB(2)	61
Table 4.5.8	Performance of REML and ML Criterion in Incomplete Block Analysis (20,10,4,2)PBIB(4)	61
Table 4.5.9	Performance of REML and ML Criterion in Incomplete Block Analysis (24,12,4,2)PBIB(6)	62
Table 4.5.10	Performance of REML and ML Criterion in Incomplete Block Analysis (28,14,4,2)PBIB(7)	62
Table 4.5.11	Performance of REML and ML Criterion in Incomplete Block Analysis (27,27,3,3) PBIB(26)	63
Table 4.5.12	Performance of REML and ML Criterion in Incomplete Block Analysis (15,20,3,4)PBIB(3)	63
Table 4.5.13	Balance Incomplete Block Analysis (4,4,3,3) PBIB(1)	64
Table 4.5.14	Balance Incomplete Block Analysis $\mathfrak{DS}(8,28,2,7)$ PBIB(1)	64
Table 4.5.15	MSE Criterion Under RML and ML Approaches for Optimal Design (9,9, 3, 3) PBIB(2)	67
Table 4.5.16	MSE Criterion Under RML and ML Approaches for Less Optimal Design (15, 10 ,3 ,2) PBIB(3)	67
Table 4.5.17	MSE Criterion Under RML and ML Approaches for Non-Optimal Design (30 ,20 ,3,2) PBIB(15)	68

LIST OF FIGURES

Figure 4.2.1	Type I, & E- Optimality Criteria for all Designs Generated	40
Figure 4.2.2	Type I, & E- Optimality Criteria in Balanced Incomplete Block Design	41
Figure 4.2.3	Type I, & E- Optimality in Selected Good Designs	42
Figure 4.2.4	Type I, & E- Optimality Criteria in PBIB(2) Designs	43
Figure 4.2.5	Type I, & E- Optimality Criteria PBIB(3) Designs	44

ABSTRACT

A binary block design of size $\mathfrak{D}(t, b, k, r)$ has t treatments, set out in b blocks, each block having k units, and each treatment occurring once or not at all in each block and $r = bk/t$ the number of replicates. The focus in every experimental situation is to search for a design that provides maximum information but utilizes minimum experimental material. A design which can guarantee high precision in estimating parameters with small probabilities of committing Type I and Type II errors is considered useful to the experimenter. Therefore, the aim of this research work was to construct a new class of optimal designs with minimum replication. The specific objectives were to; (i) develop an algorithm for the optimal binary block designs, (ii) assess the binary designs constructed using a new optimality criterion, Minimum Variance $MV = \{\sum_{i=1}^{t-1} (e_i - \bar{e})^2\}$ where e_i is the i^{th} eigenvalue of the information matrix $\mathfrak{D}(t, b, k, r)$ and $\bar{e}$ the mean of the eigenvalues, (iii) classify the obtained designs as partially balanced incomplete block design PBIBD with m associate classes, with $m = 1, 2, \dots$. The best designs being cases with $m = 1$ or 2 , namely PBIBD/1 and PBIBD/2, respectively.

Classes of optimal designs were constructed using the developed algorithm for the following parameter combinations, namely, $2 \leq r \leq t$, $6 \leq t \leq 15$, and $2 \leq k \leq t/2$. The information matrix was obtained for each design $\mathfrak{D}(t, b, k, r)$ and the one with minimum MV was selected. Designs with equal eigenvalues were classified as the PBIBD/1, those with only two distinct eigenvalues as PBIBD/2.

The following were the major findings from this study:

- (i) The algorithm developed had been able to search for the optimal design amongst a class of designs of same size $\mathfrak{D}(t, b, k, r)$.
- (ii) The MV criterion used in design assessment outperformed the well known A-, D-, or the E-Optimality criteria and it was found to be less cumbersome in computation,
- (iii) A new class of optimal designs were constructed and results obtained for parameters combinations $2 \leq r \leq t$, $6 \leq t \leq 15$, and $2 \leq k \leq t/2$.

The study concluded that the algorithm for constructing small size replicates optimal experiments was effective. The study also confirmed that increasing the number of replications or block sizes would lead to improved efficiency but higher cost for the experiment. Thus, the research has opened up efficient designs for pilot survey and for easier computation with less cost. For an experimenter to obtain optimal design for the replication and unit constraint problem, construction of designs and their complementary designs should be considered. Optimal minimum replicate design is recommended for the construction of a better design for a particular anti-symmetric design with an improved frequency distribution of concurrence matrix.

Word Count 440

CHAPTER ONE

INTRODUCTION

Experimentation is an essential part of learning process in order to optimize the variable of interest. To get reliable and objective conclusion from an experiment, statistical procedures are necessary and in fact, statistical experiments are economical. Statistical error is un-avoidable in statistical experiments, but can be controlled. Excessive error in experimental procedures may destroy experimental validity which could affect the accuracy of estimates (Gauch and Zobel 1997). Variations within experimental field are monitored and controlled when optimal design and appropriate model are used with all sources of variation accounted for. A suitable design strategy is capable to improve the accuracy of comparison. Therefore, Incomplete Block Design is appropriate when all the treatments cannot appeared in a block which is common in science, agronomy, medicine, psychology, sociology and other fields, Divecha and Ghosh (1994), Sharma (1996) and Sharma and Fanta (2010). These incomplete block designs are wide and complex therefore; from the planning phase, a researcher should find a design that provides the maximum information among others. To determine the appropriate design to be selected, it is necessary to keep the experimental objective in mind which in turn provides solution to the problem of inference. Failure to find optimal research design leads to inadequacy in inference from such kind of experiment, no matter how sophisticated the statistical tool used. The analysis of incomplete block design is a bit difficult in that treatments and blocks are no longer orthogonal (Bayrak and Buluf 2006). In incomplete block designs only treatment contrasts are estimable Das (2002), because block effects are eliminated and the estimates are the function of treatment effects and error only. Hence, substantial error in an experiment leads to erroneous conclusion. Maximizing the parameters of interest and minimizing the error term at the same time is the major concern. The target of this research is to present a new class of optimal designs with minimum replications as promoted by Sharma and Fanta (2010).

In this research work, optimal incomplete block designs were investigated from a class of obtainable designs which are valuable in general theory of incomplete block designs and in practice. The partially Balanced Incomplete Block Design with m associates denoted by $PBIB/m$, were introduced by Bose and Nair (1939). The class of design avoids the large class of replicates which may be required by a completely balanced design. ***PBIB/2*** and ***PBIB/3*** or more associate classes were constructed using **R-statistical package**. ***PBIB/1*** designs are rare, exist for certain set of design parameters Das (2002) and often need large number of replicates, a great demand on researcher. Ordinarily, an experimenter looks for a design that maximizes information with less experimental units. That is why ***PBIB/2*** and ***PBIB/3*** or more associate

classes were major concern in this research work. Due to the great demand as a result of large units, the most often used design is **PBIB/2** design, it is the simplest extension of **BIB** design, in view of this **BIB** designs can be written as **PBIB/1**. **BIB** designs have attractive property, having all the treatment contrasts estimated with the same accuracy. Core **PBIBD** have different sets of treatment differences with a set estimated with the same accuracy different from other sets. A design that maximizes information in an experiment is called an optimal design. An optimal design is known to produce unbiased estimates, minimum variance, and require minimum number of experimental units. In this research work, four (4) optimality criteria which are **A-**, **D-**, **E-**, and **SS-** were used.

Partially balanced incomplete block designs (PBIBD) were introduced to deal with the challenges of field heterogeneity to a large extent than what Randomized Complete Block Designs (RCBD) could handle when the number of treatment required for the experiment is large. In breeding science, genealogical history of varieties could help in the choice of varieties selection to be used in the experiment, so PBIBD or its complementary design may be a better choice..

Error control is one of the basic principles of experimental design which when failed could cause inaccuracy in estimates Gauch and Zobel (1997). Variations within experimental field are controlled when the right model is used and all sources of variation are accounted for. In a situation where two sources of variation are included which are treatment and blocking with a constraint that all the treatments cannot appear in each block which is common in agronomy, engineering science, medicine, psychology, sociology and other fields, is called **incomplete block design** Divecha and Ghosh (1994), Sharma (1996) and Sharma and Fanta (2010). Incomplete block designs are wide and complex therefore; there is need for selection of appropriate design from the catalogue of obtainable designs. The objective behind an experiment will aid and determine the appropriate design to be selected which will definitely help out in providing solution to the problem of inference. A significant test is a tool that can be used to establish whether or not a difference between treatments is real.

Precision and Canonical efficiency factors

Precision depends mostly on the number of replications and the choice of design in any experiment. Therefore, the issue of replication is important in this work

Canonical efficiency factors of an information matrix from a particular design could be used for comparison among different designs. A design with very good structures in its information matrix will produce high and less variable canonical efficiency factors. This in turn, would rank such a design better than others with low and less variable, low and high variable, high and high variable canonical efficiency factors.

Optimal designs could be compared using **Type-1 optimality**, **E-** and **SS-** optimality criteria to select optimal designs in order to obtain the possible maximum information. Eigenvalues, also recognized as canonical efficiency factors are a function of information matrix Das (2002) and would be used to determine choice of designs.

Selection of good optimal designs using optimality criteria

Statistical criteria are important and required in the selection of experimental designs for estimating statistical models with high precision. These criteria are also called optimality criteria. A non optimal or less optimal design would utilize a large number of experimental resources in the estimation of parameters with the same precision as an optimal design. Careful selection of a design is necessary because, a design selected would guide researcher on how well to collect the right data and analyzed in a manner that a valid and subjective conclusion would be made.

FOUR OPTIMALITY CRITERIA

Rational assessments of optimal designs are only possible after identifying the criteria for evaluation. Constant block size and equireplicate design **d** which is widely used in several fields of research would be considered Bronislaw Ceranka and Malgorzata Graczyk (2007).

E- optimality criterion is a useful criterion, but does not give full detail of the occurrences in the canonical efficiency factors just like **A-**, **D-**, or **SS-** optimality criterion. Individual handling canonical efficiency factors in different ways providing attractive information. These four criteria are combined, in determining optimal designs. **Type-1** optimality criteria produced robust averages in canonical efficiency factors.

E- optimality criterion produces the least or the minimum value from canonical efficiency factors stressing the variation in it without giving this measure. **SS-** optimality criterion produce this measure of variation.

HOMOSCEDASTICITY ASSUMPTION

Homoscedasticity assumption play a serious role in general linear model Bronislaw Ceranka and Malgorzata Graczyk (2008) and Sharma and Fanta (2010). Violation of this assumption, has serious negative effect on the proposed model, the interpretation as well as inconsistency in the optimality criteria. This is the foundation upon which **A-**, **D-** and **SS-** optimality criteria are based.

Complementary designs are common under Balanced Incomplete Block Designs (BIBD). A complementary design of a design denoted as $d_c(t, b, k, r)$ is defined as the position replacement between t treatments and b blocks such that $N = tr = bk$ is of

interest. Complementary designs under symmetric PBIB designs, where $t = b$ and $r = k$, generate the same concurrence matrix. When $t > b$ and $r < k$, complementary design of a design produces different concurrence matrix having different dimension and different content from the original design. The complementary design for a design where $t > b$ and $r < k$ is $b > t$ and $k < r$, with the constraint that $tr = bk$. The complementary design for the design where $t > b$ and $r < k$ and the design where $b > t$ and $k < r$, are both called **anti-symmetric designs** with constraint $tr = bk$. For **symmetric design** or **anti-symmetric design**, it is possible to obtain a design that would utilize the same number of units but different amount of information. The derived design from any of the two type of designs mentioned above is called complementary design.

Under symmetric design, the number of treatments to be compared must be equal to the number of blocks to be considered in the experiment with inevitable constraint that each treatment must be replicated in accordance to the block size. Similarly, under anti-symmetric design, the number of treatments must not be the same as the number of blocks to be considered in the experiment with inevitable constraint that each treatment must not be replicated the same number of time as the block size.

The objective here is to reduce the number of replications and maximize the estimates of the parameters of interest, so the interest lies on $t > b$ and $r < k$, anti-symmetric designs will be of help in the construction of optimal designs through its complementary designs.

A design which could account for assignable causes of variation effectively must be selected from a catalogue of obtainable designs, such designs are often called **optimal error controlled designs**. Optimal designs has properties of unbiasedness, minimum variance, minimum number of experimental units utilization with little cost determined by some statistical criteria. These properties make an experiment cheap to run and providing reliable results. In this thesis, **Type 1-**, **E-** and **SS** optimality criteria were used to select optimal designs and their performances were also assessed. If an experimenter/ researcher fail to search for an optimal design, larger sample size would be required to test whether or not a difference between treatments is real. Minimum variance is the idea of precision. Accuracy is wider than precision due to the effect of biasedness but precision affects accuracy. Mean Square Error (**MSE**) measures accuracy. Accuracy is affected by type I error, type II error and effect size. The smaller the **MSE** the better the design.

In this research work a new class of optimal designs will be presented. Let $d(t, b, k, r)$ represent a block design consisting of t treatments, b blocks of k units replicated r

times. Our objective here is to find a class of design that are optimal for small replication r , where $r = \frac{bk}{t}$ and is an integer; $r = 2, 3, \dots, t - 1$.

1.1 Definitions

Block Design

A block design is a list of t treatments and their corresponding b blocks of sizes $k_1, k_2, \dots, k_b$ respectively such that the treatment i occurs in r_i blocks and the i^{th} and j^{th} treatments occur together in λ_{ij} blocks.

Binary Design

A design of size (t, b, k, r) is called a binary design provided every treatment occurs once or not at all in each block of the experiment, i.e. the incidence matrix of a design $N = n_{ij}$, where $n_{ij} = 0$ or 1

Incidence matrix:

The matrix $N = n_{ij} : (i = 1, 2, \dots, t, j = 1, 2, \dots, b)$ where n_{ij} is the number of times the treatment i occurs in block j is called the incidence matrix.

Information matrix

Let $d \in \mathcal{D}(t, b, k, r)$ be a equireplicate (where $bk/t=r$) design with t -treatments, b -blocks, each of size $k \geq 2$, using Reduced Normal Equation (**RNE**), information matrix is obtained. Let C_d be the information matrix given by

$$C_d = R_d - N_d K_d^{-1} N_d^1$$

Where R_d and K_d are diagonal matrices respectively, the treatment replications $r_{d1}, \dots, r_{dt}$ and the block sizes $k_{d1}, \dots, k_{db}$, and where N_d is the incidence matrix with $(ij)^{th}$ element n_{ij} . We denote the eigenvalues of C_d by $e_{d1}, e_{d2}, \dots, e_{dt-1}, 0$

Connected Design

A design $d \in \mathcal{D}(t, b, k, r)$ is said to be connected provided all possible treatment differences are estimable or if the rank of the information matrix of the design d is $t - 1$.

Partially Balanced Incomplete Block Design with one associate class PBIBD/1

- (i) Let $e_1, e_2, \dots, e_{t-1}$ be the canonical efficiency factors of the information matrix for $\mathfrak{d}(t, b, k, r)$, then $\mathfrak{d}(t, b, k, r)$ is said to be BIBD iff

$$e_1 = e_2 = \dots = e_{t-1} = e$$

Where $0 < e < 1$ is a constant and all treatment differences are estimated with the same variance.

Partially Balanced with 2 Associate Classes (PBIB/2)

Some Pairs of treatments are estimated with the same variance, whereas some other pairs are estimated with the same variance but different from the variance of earlier pairs of treatments. In **PBIB(2)**, every design has two unique eigenvalues (or canonical efficiency factors).

$$0 < e_A \leq 1, \quad 0 < e_B < 1; \quad e_A > e_B$$

Partially Balanced with 3 or more Associate Classes

PBIB(3 or more) is a design that has three or more unique canonical efficiency factors $e_A, e_B, \dots, e_j$;

$0 < e_A \leq 1, \quad 0 < e_B < e_A, \quad 0 < e_j < e_B; \quad j = C, \dots, L$ where L is number of unique canonical efficiency factor.

$$e_A > e_B > \dots > e_L$$

Complementary Design

A design $\mathfrak{d}^*$ having parameters $t_1, b_1, r_1, k_1, \lambda_1$ is said to be a complementary design of a design $\mathfrak{d}$ with parameters t, b, k, r, λ , if $\mathfrak{d}^*$ is obtained by taking in the j^{th} block of $\mathfrak{d}^*$ all those treatments which do not occur in the j^{th} block of $\mathfrak{d}$, $j = 1, 2, \dots, b$.

$$t_1 = b, \quad b_1 = t, \quad r_1 = k, \quad k_1 = r$$

Variance Balanced Design:

A design of size (t, b, k, r, λ) is said to be variance balanced iff the elementary treatment contrasts are estimated with the same accuracy.

Symmetrical Balanced Incomplete Block Designs

A class of balanced incomplete block designs (BIBD) in which the number of experimental treatments is equal to the number of blocks considered in the experiment is called symmetrical BIB designs.

Complete Design

A design is called complete design if every treatment occurs in each block, otherwise it is called incomplete.

Equireplicate Design:

A design is said to be equireplicate if all replications are equal to a constant r .

A-Optimality Criterion

A design $d^* \in \mathcal{D}(t, b, k, r)$ is **A**- optimal if it maximizes

$$\frac{t-1}{\sum_{i=1}^{t-1} e_i} \text{ over all possible designs in } \mathcal{D}(t, b, k, r).$$

D-Optimality Criterion

A design $d^* \in \mathcal{D}(t, b, k, r)$ is **D**- optimal if it maximizes

$$\left\{ \prod_{i=1}^{t-1} e_i \right\}^{\frac{1}{t-1}} \text{ over all possible designs in } \mathcal{D}(t, b, k, r).$$

E-Optimality Criterion

A design $d^* \in \mathcal{D}(t, b, k, r)$ is **E**- optimal design if it maximizes $\min(e_i)$ over all possible designs in $\mathcal{D}(t, b, k, r)$.

MV-optimality Criterion

A design $d^* \in \mathcal{D}(t, b, k)$ is said to be MV-optimal if for any other $d \in \mathcal{D}(t, b, k)$

$$\max_{i \neq j} \text{Var}_{d^*}(\hat{\tau}_i - \hat{\tau}_j) \leq \max_{i \neq j} \text{Var}_d(\hat{\tau}_i - \hat{\tau}_j).$$

1.2 STATEMENT OF THE PROBLEM

The focus in every experimental situation is to search for a design that provides maximum information but utilizes minimum experimental material. A design which can guarantee high precision in estimating parameters with small probabilities of committing Type I and Type II errors is considered useful to the experimenters.

1.3 RESEARCH AIM AND OBJECTIVES

The aim of this research work is to present a new class of optimal designs with minimum replications with objectives, to construct Optimal incomplete block designs with $\min\{\sum(e_i - \bar{e})^2\}$

- (i) Partially Balanced Incomplete Block Design with one associate class PBIBD/1.
- (ii) Partially Balanced Incomplete Block Designs with two associate classes (**PBIBD/2**).
- (iii) Partially Balanced Incomplete Block Designs with three or more associate classes (**PBIBD/3**).
- (iv) Compare optimal, less optimal and non-optimal designs performances.
- (v) Construct symmetric designs and their corresponding complementary designs and evaluate their performances using Type I and E-optimality criteria.
- (vi) Construct non-symmetric designs and their corresponding complementary designs and evaluate their performances using Type I and E-optimality criteria.
- (v) To present competing anti-symmetric designs having varying block and block sizes; and
- (vi) To present competing complementary designs for anti-symmetric designs presented above.

1.4 JUSTIFICATION OF STUDY

A small size design is usually used as pilot survey and pre-test experiments. They are optimal in the sense that nearly equal information is obtained on each treatment comparison.

$$var(\hat{\tau}_i - \hat{\tau}_j) \cong c \text{ or } c_1 \text{ or } c_2$$

$$\text{where } |c_1 - c_2| < \varepsilon$$

Where c_1 and c_2 are the basic contrasts and ε is a small positive number

Small sample size design arrived at a quick decision with limited amount of resources. Small sample size experiment is recommended for the following cases where experiments are conducted such as

- (i) A stage at a time.
- (ii) Each stage is independent from the other
- (iii) Each stage is efficient to proving information each basic contrast.
- (iv) Losing one stage should not adversely affect the overall result.
- (v) Small sample size experiment increases accuracy with high precision that is a result with high efficiency can be obtained in an experimental design with less cost and small units utilization.
- (vi) If a larger number and/or size of the experimental units is considered in an experimental design it is a bit difficult to ensure that they are all equivalent for the purpose of treatment comparisons, the less certain the equivalence of the treatments is, then the less accurate the overall result of the experiment will be.

CHAPTER TWO

LITERATURE REVIEW

A lot of literature has shown that without good design, the information for the conduct of forward looking experiment would be complicated and misleading. Earlier experimental strategy when two sources of variation are present denoted as Treatment and Block is called Randomized Complete Block Design but increase in the number of treatments to be compared leads to increase in variation within the block as well leading to imprecise information. This led to the introduction of Binary Incomplete Block Design experiments. Under the Incomplete Block Design exists Partially Balanced Incomplete Block Design. In recent years, several researchers and experimenters have investigated the use of optimally connected designs. Most especially, Partially Balanced Incomplete Block Experiments have been extensively studied by experts in both academic and empirical realms. The investigations revolve around two aspects namely, analysis of experimental data and construction of partially balanced incomplete block experiments.

Prominent among such researchers is **Lewis and Dean (1971)**: In a book titled “Design and Analysis of Experiment” dealt with symmetric PBIB designs and their corresponding complementary designs where the number of treatments is equal to the the number of blocks which automatically force the number of replications to be equal to the block sizes. It was shown that a symmetric design and its complementary design have the same concurrence matrix.

Bimal (1982): Constructed (M,S) -optimal designs for block size three (3) when the number of treatments is of the form $6t + 3$. Showing a form of uniqueness in the construction of experiment.

Taylor and John (1983): worked on the incidence matrix of binary incomplete block designs with a given concurrence matrix where an algorithm was developed to find a design with a given concurrence matrix. It was discovered that the precision of comparison between treatments in incomplete block designs with equal block sizes depends only on the concurrence matrix.

Kageyama,S(1985).Presented some classes of incomplete block designs such as varianced-balanced, efficiency-balanced and so on and determined the minimum number of experimental units required for an incomplete block designs to be connected given t and b where t is the number of treatments and b is the number of blocks. Another situation was also considered in which the number of replications $r_i \geq 2$ and the block sizes $k_j \geq 2$ for all $i = 1,2, \dots, t, j = 1,2, \dots, b$ under this

constraint , the minimum number of experimental units is determined along with some construction methods.

Sinha Kishore(1987): Worked on Generalized partially balanced incomplete block designs to construct equireplicate balanced block designs and pair-wise balanced designs and balanced incomplete block designs (i.e. BIB designs). The theorems, corollaries and the illustrative examples showed the existence of Generalized partially balanced incomplete block designs with 2 or 3 associate classes with the parameters $t, b_i, r_i, k_i, \lambda_{1i}, \lambda_{2i}$, $i = 1, 2$, implies equireplicate balanced block design with parameters t_1 $b = b_1 + b_2$, $r = r_1 + r_2$, k_1, k_2 , $w = p$

A pair-wise design was obtained if $\lambda_{11} + \lambda_{12} = \lambda_{21} + \lambda_{22} = \lambda_{31} + \lambda_{32}$ when $k_1 = k_2$ a BIB design is obtained.

Mike Jacroux (1987): Considered a general method for finding MV-optimal designs and some sufficient conditions under which a design obtained using the proposed method will be MV-optimal in some experimental situations where t treatments are of interest and are to be compared with some standard treatments in block of size k units. A design is considered to be MV-optimal in such an experimental situation if it minimizes the maximal variance with which the elementary treatment contrasts involving the standard treatments are estimated.

Mike Jacroux et al (1988): Constructed optimal incomplete block designs in classes $\mathfrak{D}(t, b, k)$ where $t < k$ E- and MV-optimality criteria were used in selecting an optimal design in $\mathfrak{D}(t, b, k)$. Several sufficient conditions for designs to be E-and MV-optimal were derived in classes of $\mathfrak{D}(t, b, k)$ where $k > t$ and the information matrix C-matrices were characterized for certain designs which satisfy the sufficient conditions.

Donev (1989): looks into point-exchange algorithms for the construction of experimental design and explains the main drawback of point-exchange algorithms. That is, they require a list of possible design points, called a candidate set, as an input. For design problems involving large numbers of experimental factors, this list becomes too large to handle.

Kraft (1990), Mukerjee and Sinha (1990), Bapat and Dey(1991), Birke Dodge(1991) and Mandal et al (1991) worked on the optimality of connected block designs which is in line with **Mukherjee et al (1986)** when the number of experimental units is small, typically, n_0 or $n_0 + 1$. Very small number of experimental units was the focus, it was reported that the usual argument for arriving at the optimality results no longer work.

Bapat and Dey (1991): use and study **D**- and **E**- criteria in the class of connected block designs, when the number of observations is minimal. It was reported that all designs in the class of competing designs are comparable with respect to the **D**- and **E**- optimality criteria values, and optimal designs are also identified.

Mukherjee et al (1992): examine the study of a minimally connected block design under mixed effects additive model without interaction where the block effects are random with variance σ_b^2 and the treatment effects are fixed. It was reported that designs obtained are E-optimal uniformly by comparing the ratio of the variance components.

Jerzy K. Baksalary and Jaun Hauke(1992): Considered additional properties to determine minimum number of experimental units required for an incomplete block designs to be connected. Since it is known that the minimum number of experimental units for a block design with t treatments and b blocks to be connected is $t + b - 1$. This minimum number may not be attainable particularly when it is required that the design in question should satisfy certain additional conditions on treatment replications and /or pairwise balanced. The problem of determining the minimum number of experimental units in connected block designs satisfying such additional properties is considered. Some of the results derived are extensions of those given by Kageyama(1985), while some other results are new.

Sunanda Bagchis and Ching-Shui Cheng (1993): Established E-optimality of many designs with block size two over the whole class of competing designs. The result identified the **E**- optimality of a class of partially balanced incomplete block designs with a rectangular association scheme. A simple method of construction of highly efficient designs of block size two was also considered.

Dey et al (1995): take the research further and gave special techniques that are needed for identifying optimal designs when the number of experimental units is minimal or nearly minimal. The purpose of this is to present additional optimal results in different classes of connected block designs with n_0 or $n_1 = (n_0 + 1)$ experimental units.

Hedayat et al (1995): Developed a sequential search algorithm for constructing two classes of designs contingently balanced incomplete block designs (i.e, C-BIBD) and virtually balanced incompletely block design for a situation when a balanced incomplete block design of size (t, b, k, λ) does not exist or its existence is not known. The result showed that C-BIB designs can also be used to construct BIB designs by taking multiple permuted copies of it. The performance of C-BIB and V-BIB were

evaluated it is discovered that some V-BIB designs were shown to be highly efficient under **A**-, **D**- and **E**- optimality criteria.

Balasubramanian and Dey (1996): Identify **D**- optimal designs in the classes of connected block designs with fixed block size, when the number of experimental units is one or two more than the minimal number required for the design to be connected. It was reported that an application of these results was made to identify **D**- optimal designs in a class of minimally connected row-column designs. Graph-theoretic methods were employed to arrive at the optimality results.

Olaf Crafft and Schaefer (1997): investigate the features of **A**- optimal block designs in the class of connected block designs with constant block size under the restriction that the number of observations is $v + b$ where v is the number of treatments and b is the number of blocks. Results were obtained through Graph-theoretic methods. Moore-penrose inverse of the C -matrix was obtained in terms of a distance matrix of a graph associated with the design.

John and Sudesh (2000): Are concerned with the class of nearly balanced incomplete block designs with concurrence range l or $\text{NBBD}(l)$ where sufficient conditions for type-1 optimality of $\text{NBBD}(2)$'s were derived. **A**- and **D**- optimality in the families of $\text{NBBD}(2)$ were established in settings where optimal designs were not previously known. It was observed in some cases $\text{NBBD}(3)$'s were found to be uniquely **A**- and **D**- optimal.

Luis, B.Morales (2000): Gives a theorem on 2-concurrence designs where a lower bound of imbalance measure of an equireplicate incomplete block design is attained with certain parameters. The result obtained was used to formulate the construction of regular graph designs as a minimization problem with an objective function. A similar result for general 2-concurrence designs was also obtained which allows one to formulate the construction of these designs as an optimization problem.

Luis Morales (2002): Applies combinatorial approach to constructing resolvable and nonresolvable partially balanced incomplete block designs with 2 associate classes using Tabu Search algorithm. It is discovered that tabu search could also find many $\text{PBIBD}(2)$'s that were previously obtained by other existing methods. The result showed that 2-concurrence designs can be used to formulate the construction of $\text{PBIBD}(2)$'s as a discrete optimization problem where optimal solutions are required rather than close approximations. Random neighborhood search and tabu search were tested on the problems at hand, it was discovered that for easy instances the difference

between neighborhood search and tabu search is not significant. For difficult and hard instances, tabu search gives the best result.

Bo Jin (2003): Developed an algorithm to search for optimal designs with a minimum number of blocks. A binary balanced incomplete block design d in the class of designs $\mathfrak{D}(t, 2, k)$ was constructed with a constraint $\frac{t}{2} < k < t$ using two blocks of the same number of experimental units, Type I optimal designs were identified. The same design is also E-optimal in the class of designs $\mathfrak{D}(t, 2, k)$ when $\frac{t}{2} < k < \frac{5t}{6}$. When there are three blocks of the same number of experimental units it is discovered that a competing design d^* is E- and type 1-optimal in the class $(t, 3, k)$. When $k = \frac{2t}{3}$, $k \geq 3$ an M,S-optimal incomplete block design was also identified.

Ge, Gennian et al (2008): Are concerned with resolvable balanced incomplete block design on points with blocks of size 4, the existence of resolvable balanced incomplete block designs RIBIBDs containing a resolvable balanced incomplete block design as a sub design was investigated. An incomplete RBIBD was obtained by removing the sub designs. Necessary and sufficient conditions for the existence of an incomplete RBIBD were derived. Some results on pair-wise balanced were obtained. The results also showed that existence of pair-wise balanced incomplete implies the existence of an incomplete resolvable balanced incomplete block design.

Bailey, R.A (2009): Studies the existence of a relationship among variance, concurrence, pseudo variance, distance and the canonical efficiency factors of incomplete block designs. Theorems, corollaries and illustrative examples showed that as variance decreases, concurrence matrix gets better or variance increases with distance for all partially balanced incomplete block designs with two associate classes. An example also showed that in partially balanced incomplete block designs with three or more associate classes, variance may not decrease as concurrence increases. In a distance –regular graphs with the same parameters, then the pseudo- variance is a monotonic increasing function of distance. The result established a simple statement linking variance to concurrence or distance for all incomplete block designs in that class.

Alison B. Smith et al (2012) Used a model-based design technique to produce designs for both composite and individual replicates sample. An approach was proposed in which some treatments are tested using individual field replicate samples and others as composite samples. A-optimality criterion was used to minimize the average pairwise prediction error variance of treatment effects, since the interest of the researcher is to estimate the sources of variation rather than the estimate of treatment

effects. The model- based design technique resulted into confounding of multi-phase treatment trials.

Oliveiral, T.A et al (2011): Review construction methods of Balanced Incomplete Block Designs with block repetition (BIBDR). Balanced Incomplete Block Designs with block repetition reduces the cost of experimentation to a greater extent and allow to get reasonable results even when some experimental units are lost. Sufficient conditions for Balanced Incomplete Block Designs were presented, such as (i) $N = tr = bk$, (ii) $\lambda(k - 1) = r(t - 1)$ (iii) $b \geq t$ (Fisher inequality).

Trade-off method was developed by Hedayat and Li(1979), which involves replacement of some blocks with other blocks, so that the general properties of a BIBD is maintained. The basic parameters of a trade-off method are the integers numbers m, k, V, t which represent the volume-sum of all positive entries of a block in the trade, the block dimension, the number of treatments and the strength of the trade. A BIBD with parameters (V, k, t) which contain the block collection of T_1 of a trade $T(t, k, V)$. By replacing in this design the blocks of T_2 for the blocks of T_1 we obtained a new BIBD structure that besides satisfying the same parameters may have support size different from the original design. Another method is juxtaposition method which is based on the definition of complementary designs and it is mostly useful in situations when blocks have large number of plots. The complementary design of a BIBD (t, b, r, k, λ) is a BIBD with parameters $(t^* = t, b^* = b, r^* = r, k^* = t - k, \lambda^* = b - 2r + \lambda)$. Some connections between BIBDR and sampling theory were also presented. Application of BIBDR in education sciences was considered, based on a sample resulting from an inquiry to students.

Davinder, K.G, and the Syed, A.F (2014): Constructed some new partially balanced incomplete block designs using $(s - 1)^2$ replicates, where each of the s rows and s columns of square array of order $s \times s$ are taken as base for constructing PBIB designs. Illustration of some of the new PBIB designs along with two associate classes was presented. It is observed that some newly constructed designs are more efficient, compared to the existing designs based on their respective parameters.

Cakiroglu, S.A (2015): Used exact computer search to construct Regular Graph Designs RGDs by ordering all possible matrices T according to the performance of the corresponding Laplacian matrix $L(x)$ on the A- and D-optimality criteria. The approach was able to find the best Regular Graph Designs in large systems compared to other existing methods by John and Michell (1976), John et al (1976) and Clatworthy (1973).

Mandal, B.N (2015): Used a proposed linear algorithm to construct balanced incomplete block designs. The performance of the algorithm was tested with the aid of illustrative examples, it is discovered that the algorithm is able to generate balanced incomplete block designs very fast in most cases than other algorithms proposed in the literature.

Jyoti Sharma et al (2016): Used semi-regular group divisible design (SRGD) and Regular Group Divisible Design RGD to construct balanced incomplete block design BIBD and partially balanced incomplete block design PBIBD based to their parameters. Theorems, proofs, illustrative examples and tables of designs generated in each case were presented.

Faghihi and Tat (2016): Developed an algorithm to search for ϕ_p -optimal or nearly ϕ_p -optimal incomplete block design. Since the interest of statisticians is to find an optimal incomplete block designs with limited experimental materials. The criterion for optimality in this work is ϕ_p . This algorithm is a variant of exchange and interchange of Jones and Eccleston(1974). The algorithm utilizes the canonical efficiency factors of an information matrix of designs. In most of the cases optimal or nearly optimal designs were found, which shows the good performance of the algorithm.

Franklin, M.F and Payne (1993): Review some of the theory and tools for design construction with a view to providing the statistician and experimenter with a tool kit for building efficient designs. Techniques in the design process are used to determine aliases and patterns of confounding, algorithms for constructing fractional factorial and incomplete block designs and method of randomization. Algorithms in the analysis are used in the computation of canonical efficiency factors, estimating variance components and for assessing general balance.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Introduction

A suitable model for a block design having a factor known as treatment without covariate and interaction, with additivity assumption is generally defined as

$$y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij} \quad (3.11)$$

Where μ is the grand mean of the experiment, α_i is the i^{th} treatment effect with constraint $\sum \alpha_i = 0$, β_j is the j^{th} block effect with constraint $\sum \beta_j = 0$, ε_{ij} is the error term with expectation zero and y_{ij} is the characteristic to be measured in the experiment.

The model above is defined as incomplete block design if and only if $t > k$ an indication that not all treatments are present in any block of the experiment (Sharma and Fanta, 2010).

3.2 Construction Of Optimal Connected Designs

The model (3.1) can be expressed in matrix notation as

$$Y = 1\mu + H'\alpha + \theta'\beta + \varepsilon \quad (3.2.1)$$

Where

Y is the vector of observations, 1 is the vector of 1s, μ is the mean parameter, H is the design matrix for treatments, θ is the design matrix for block, α is the vector of treatment parameters, β is the vector of block parameters and ε is the vector of error terms. HH' is a diagonal matrix of replications, $\theta\theta'$ is a diagonal matrix of block sizes such that $H1$ is a vector of replications and $\theta 1$ is a vector of block sizes.

$N = H\theta' = (n_{ij})$ which reveals the number of times treatment i appear in block j of the experiment and $NN' = (\lambda_{ij})$ reveals the of times treatment i and treatment j appear together in the same block of the experiment.

To simplify model (3.2) above, we have

$$Y = X\theta + \varepsilon \quad (3.2.2)$$

Where X is a matrix containing unity vector of 1s, transpose of design matrix for treatments and transpose of design matrix for block. θ is the vector of design parameters which are μ , α , and β .

$XX'\boldsymbol{\theta}$ produces three linearly dependent equations which are expressed below

$$n\hat{\mu} + r'\hat{\alpha} + k'\hat{\beta} = \mathbb{G} \quad (3.2.3)$$

$$r\hat{\mu} + rI\hat{\alpha} + N\hat{\beta} = \mathbb{T} \quad (3.2.4)$$

$$k\hat{\mu} + N'\hat{\alpha} + kI\hat{\beta} = \mathbb{B} \quad (3.2.5)$$

Since the equations (3.2.3), (3.2.4) and (3.2.5) above are linearly dependent, unique solution cannot be obtained.

Therefore, equation (3.2.4) - $N(kI)^{-1}$ * equation (3.2.5) results in

$$rI\hat{\alpha} - N(kI)^{-1}N'\hat{\alpha} = \mathbb{T} - N(kI)^{-1}\mathbb{B}$$

By factorization

$$(rI - N(kI)^{-1}N')\hat{\alpha} = \mathbb{T} - N(kI)^{-1}\mathbb{B}$$

Equivalently written as

$$A\hat{\alpha} = Q \quad (3.4)$$

Such that

$$A1 = 0$$

Meaning that, every element in A is a contrast.

For the information matrix denoted as C

$$C = \frac{1}{r} * A \quad (3.5)$$

When eigen function is placed on the information matrix

$$EV(C) = e_1, e_2, \dots, e_{t-1}, 0$$

Such that $e_1 \geq e_2 \geq \dots \geq e_{t-1} > 0$

In incomplete block design, treatment effect is of key interest due to the fact that the source of variation denoted as block would be removed from the experiment (Pierre Druilhet, 2004) therefore, we can specify the problem of inference as

$$\mathbf{H}: \boldsymbol{\eta} = \mathbf{L}\boldsymbol{\alpha}$$

Where $\boldsymbol{\eta}$ contains p treatment contrasts, $\mathbf{L}$ is a $p \times t$ matrix with $\mathbf{L}1=0$ with an assumption that all the treatment contrasts are estimable and $\boldsymbol{\alpha}$ is a vector of effects. C is sometimes called information matrix and determined mainly by the incidence matrix denoted as N of a specific design with $rank(C) \leq t - 1$. When $rank(C) =$

$t - 1$ then we have a **connected design** Das (2002). And $Q = T - NK^{-1}B$ is called the adjusted treatment total due to the fact that not all treatments occur in a block and also sometimes not all treatments occur the same number of times in blocks. For this study, classes of optimal designs with the following constraints were constructed

(i) $2 \leq r \leq t$, (ii) $6 \leq t \leq 15$, (iii) $2 \leq k \leq t/2$, and

(iv) $\lambda = r(k - 1)/(t - 1)$

Information matrix is derived from the statistical model depending on the structure of the design in a model therefore, a design with very good structures are selected to obtain the possible maximum information based on some statistical criteria. Eigenvalues, also known as canonical efficiency factors are a function of information matrix (Das, 2002) which would be used to determine the quality of designs while the number of eigenvalues would be used to determine the associate class of designs.

3.3 Selection of good designs using optimality criterion

Knowing the associate class of a design is not enough, it is necessary to find out the quality of such a design in terms of the level of information it provides. The selection of experimental designs for estimating statistical models require statistical criterion. Examples are **A**-, **D**- and **E**. Using these optimality criteria for selecting good designs leads to having optimal designs. Optimal designs assured unbiasedness and minimum variance with less cost. A non optimal design needs a greater number of experimental units in the estimation of parameters with the same precision as an optimal design leading to increase in cost. We proposed **SS** optimality criterion.

3.4 SS – Optimality Criterion

Let $\hat{\eta}$ be the Best Linear Unbiased Estimator (BLUE) of η using a constant block size and equireplicate design $d(t, b, k, r) \in \mathcal{D}$ with an assumption that there is orthogonality between treatment effects such that $\text{var}(\hat{\eta}) = v_d$

Where

$$\begin{aligned}
 V_d &= \begin{bmatrix} v_{11} & 0 & \dots & 0 \\ 0 & v_{22} & \dots & 0 \\ \cdot & \cdot & & \cdot \\ 0 & 0 & & v_{tt} \end{bmatrix} \\
 &= \begin{bmatrix} ss_{11}/df_{11} & 0 & \dots & 0 \\ 0 & ss_{22}/df_{22} & \dots & 0 \\ \cdot & \cdot & \dots & \cdot \\ 0 & 0 & \dots & ss_{tt}/df_{tt} \end{bmatrix} \\
 SS_d &= \begin{bmatrix} ss_{11} & 0 & \dots & 0 \\ 0 & ss_{22} & \dots & 0 \\ \cdot & \cdot & & \cdot \\ 0 & 0 & & ss_{tt} \end{bmatrix}
 \end{aligned}$$

Using the concept of the sum of squares

A-Optimality: A design $d^* \in \mathcal{D}$ is said to be A-optimal in $\mathcal{D}$ iff

$$tr(V_{d^*}) \leq tr(V_d) \text{ for any } d \in \mathcal{D}$$

equivalently

$$tr(C_{d^*}) \geq tr(C_d) \text{ for any } d \in \mathcal{D}$$

D-Optimality: A design $d^* \in \mathcal{D}$ is said to be D-optimal in $\mathcal{D}$ iff

$$det(V_{d^*}) \leq det(V_d) \text{ for any } d \in \mathcal{D}$$

equivalently

$$det(C_{d^*}) \geq det(C_d) \text{ for any } d \in \mathcal{D}$$

E-Optimality: A design $d^* \in \mathcal{D}$ is said to be E-optimal in $\mathcal{D}$ iff

$$\min(\text{eigenvalue}(V_{d^*})) \leq \min(\text{eigenvalue}(V_d)) \text{ for any } d \in \mathcal{D}$$

equivalently

$$\min(\text{eigenvalue}(C_{d^*})) \geq \min(\text{eigenvalue}(C_d)) \text{ for any } d \in \mathcal{D}$$

where $\text{eigenvalue}(C_d) = e_1, e_2, \dots, e_{t-1}$

3.5 Relationships between Type I and E- Optimality Criteria

A design $d^* \in \mathcal{D}$ is said to be A-, D- and E-optimal in $\mathcal{D}$ iff

$$\text{tr}(V_{d^*}) \leq \text{tr}(V_d) \text{ for any } d \in \mathcal{D}$$

$$\det(V_{d^*}) \leq \det(V_d) \text{ for any } d \in \mathcal{D}$$

$$\min(\text{eigenvalue}(V_{d^*})) \leq \min(\text{eigenvalue}(V_d)) \text{ for any } d \in \mathcal{D}$$

Or

$$\text{tr}(C_{d^*}) \geq \text{tr}(C_d) \text{ for any } d \in \mathcal{D}$$

$$\det(C_{d^*}) \geq \det(C_d) \text{ for any } d \in \mathcal{D}$$

$$\min(\text{eigenvalue}(C_{d^*})) \geq \min(\text{eigenvalue}(C_d)) \text{ for any } d \in \mathcal{D}$$

The optimality criterion minimizes variance (V_d) component. As the variance (V_d) component is minimized, the information C_d component is maximized.

Sum of Squares (SS) Optimality

A design $d^* \in \mathcal{D}$ is said to be SS-optimal in $\mathcal{D}$ iff

$$\text{tr}(SS_{d^*}) \leq \text{tr}(SS_d) \text{ for any } d \in \mathcal{D}$$

$$\det(SS_{d^*}) \leq \det(SS_d) \text{ for any } d \in \mathcal{D}$$

$$\min(\text{eigenvalue}(SS_{d^*})) \leq \min(\text{eigenvalue}(SS_d)) \text{ for any } d \in \mathcal{D}$$

3.6 Relationship between Type I, E- and SS- Optimality Criteria

A design $d^* \in \mathcal{D}$ is said to be A-, D-, E- and SS-optimal in $\mathcal{D}$ iff

$$\text{tr}(C_{d^*}) \geq \text{tr}(C_d) \text{ for any } d \in \mathcal{D}$$

$$\det(C_{d^*}) \geq \det(C_d) \text{ for any } d \in \mathcal{D}$$

$$\min(\text{eigenvalue}(C_{d^*})) \geq \min(\text{eigenvalue}(C_d)) \text{ for any } d \in \mathcal{D}$$

$$\text{tr}(SS_{d^*}) \leq \text{tr}(SS_d) \text{ for any } d \in \mathcal{D}$$

An increase in sum of squares is directly proportional to increase in variance with respect to decrease in information.

Note: to have an optimal designs, we seek for two scenarios

1. When $\lambda_{ij} = \lambda$ for any $i \neq j$

2. When $\lambda_{ij} = \lambda_1$ or λ_2 such that $|\lambda_1 - \lambda_2| = 0$ or 1 (only interested in minimal variation)

3.7 Advantage of SS- Optimality Criterion over A,D and E-optimality criteria

A-, D-, and E- optimal Criteria are on assumption that the variations in eigenvalues are close. The **Sum of Squares** optimality gives the estimate of the variation.

3.8 Symmetric, Anti-symmetric Designs and their Complementary Designs

A symmetric of size (t, K, λ) design with parameters is an incident structure satisfying the following six requirements:

- (1). There are t points
- ((2) There b blocks
- (3) Any block is incident with k points
- (4) Any point is incident with k blocks
- (5) Any two blocks are incident with λ points
- (6) Any two points are incident with λ blocks.

In a symmetric design of size (t, K, λ)

1. $(t - 1)\lambda = k(k - 1)$
2. $k^2 - t\lambda = k - \lambda$
3. $(t - k)\lambda = (k - 1)(k - \lambda)$.

The value of $(k - \lambda)$, which occurs in two of the equations above is an extremely important parameter. We set $n = k - \lambda$ and call n the order of the symmetric (t, K, λ) design.

Theorem: (Schutzenberger). Suppose that there exist a symmetric (t, K, λ) design. If t is even, then n must be a square.

The incidence matrix N of a symmetric design is the $t \times t$ matrix whose rows are indexed by points, with the entry in row R and column C being 1 if C and R are incident and 0 otherwise.

The incidence requirements can be expressed in terms of N .

$$NJ = JN = KJ \text{ and } NN' = N'N = (K - \lambda)I = nI + \lambda J$$

I is the identity matrix and J is the matrix with every entry 1.

If N is the incidence matrix of a symmetric (t, K, λ) design then $|det N| =$

$$Kn^{1/2(t-1)}$$

Complementary Designs: By replacing incidence by non-incidence, we obtain the complement of D , denoted by D' . The incidence structure D' is a symmetric (t', k', λ') design where $(t', k', \lambda') = (t, t - k, t - 2k + \lambda)$

3.9 Definitions

We use " $\wedge$ " to show that statement S_1 and S_2 are true if both statements S_1 and S_2 are true indicated as $S_1 \wedge S_2$

Definition 3.9.1: A PBIB design is said to be symmetric if

$$\theta R \emptyset = \emptyset R \theta$$

Where θ indicates function for a PBIB design and $\emptyset$ a function for its complementary design

Proof:

$$\theta R \emptyset = \emptyset R \theta$$

$$\theta R \emptyset = \theta \subseteq \{v, x, y, z\} \wedge \emptyset \subseteq \{v, x, y, z\}$$

$$= \theta \subseteq \{v, x, y, z\} \wedge \{v, x, y, z\} \subseteq \emptyset$$

$$= \theta \subseteq \{v, x, y, z\} \subseteq \emptyset$$

$$= \emptyset \subseteq \{v, x, y, z\} \subseteq \theta$$

$$\therefore \theta R \emptyset = \emptyset R \theta \text{ Proved}$$

$$\text{Also } \emptyset R \theta = \theta R \emptyset.$$

Definition 3.9.2: A PBIB design is said to be nonsymmetrical if

$$\theta R \emptyset \neq \emptyset R \theta$$

$$\theta = \{v, x, y, z\} \text{ and } \emptyset = \{x, v, z, y\}.$$

$$\text{Where } \theta = \{t, b, k, r\} \text{ and } \emptyset = \{t', b', k', r'\}$$

$$\text{Such that } t \neq t', b \neq b', k \neq k' \text{ and } r \neq r'$$

Proof:

$$\theta R \emptyset = \theta \subseteq \{t, b, k, r\} \wedge \emptyset \subseteq \{t', b', k', r'\}$$

$$\theta \subseteq \{t, b, k, r\} \wedge \{t', b', k', r'\} \subseteq \emptyset$$

$$\emptyset \not\subseteq \{t, b, k, r\} \wedge \{t', b', k', r'\} \not\subseteq \theta$$

$$\therefore \theta R \emptyset \neq \emptyset R \theta \text{ proved.}$$

$$\text{Also}$$

$$\emptyset R \theta \neq \theta R \emptyset.$$

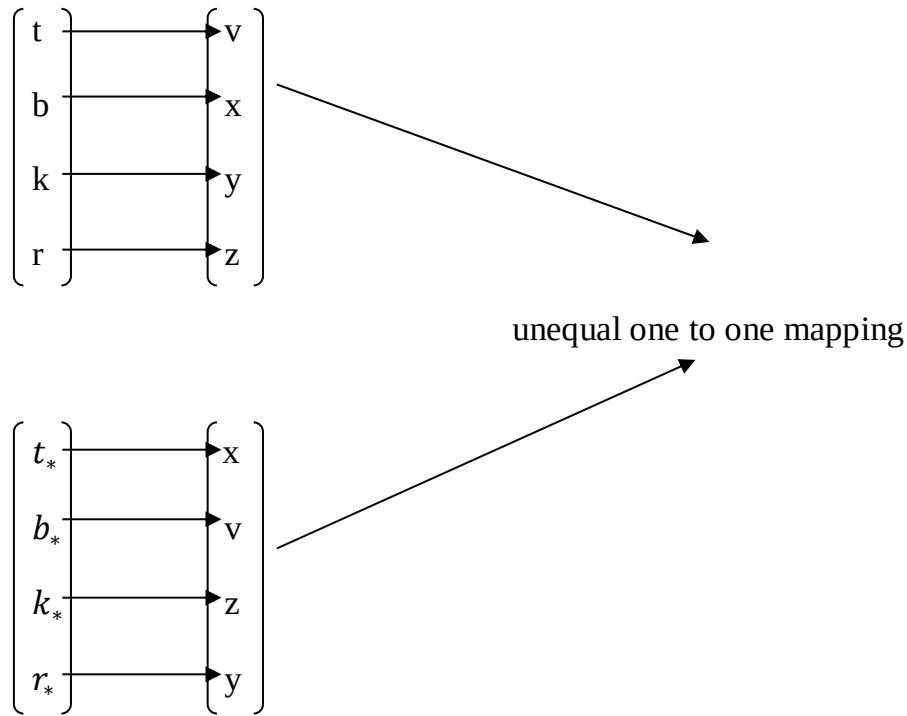
Definition 3.9.3. Let θ and ψ be any two non-empty sets of a design $d(t, b, k, r)$ and its complementary design $d(t_*, b_*, k_*, r_*)$ respectively with a binary operation $*$ satisfying the following conditions where ρ is a constant, e as an identity element, p be any parameter of a design $d(t, b, k, r)$ or its complementary design $d(t_*, b_*, k_*, r_*)$ and p^{-1} as the inverse of p such that

1. $(t * r) * \rho = t * (r * \rho)$ Associative law
 $(t_* * r_*) * \rho = t_* * (r_* * \rho)$ Associative law
2. $p * e = e * p$ Identity law
3. $p * p^{-1} = p^{-1} * p = e$ Inverse law

Then, ψ is a permutation group of θ .

Definition 3.9.4. Given that ψ is a permutation group of θ where the information matrix for θ is $C = rI - k^{-1}NN'$ and for $\psi\theta$ is $C_* = r_*I - k_*^{-1}N_*N_*'$ then, $NN' \neq N_*N_*'$. If $t \neq t_*$ and $b \neq b_*$ this results into having different number of efficient factors and, if $r \neq r_*$ and $k \neq k_*$, this results into having different optimality criteria values otherwise $NN' = N_*N_*'$.

Definition 3.9.5 Let φ be a non-empty set such that $\varphi = \{v, x, y, z\}$ where f is a function. If $f: \theta \rightarrow \varphi$ and $f: \psi \rightarrow \varphi$ where $t \neq t_*$, $b \neq b_*$, $r \neq r_*$ and $k \neq k_*$ then unequal one to one mapping is the result, represented graphically as



Definition 3.9.6 Let θ and ψ be any two non-empty sets of a design $d(t, b, k, r)$ and its complementary design $d(t_*, b_*, k_*, r_*)$ respectively

- i. The relation denoted as R is symmetric in $d(t, b, k, r)$ if and only if

$$\theta R \psi = \psi R \theta$$
- ii. The relation denoted as R is anti-symmetric in $d(t, b, k, r)$ if and only if

$$\theta R \psi \neq \psi R \theta$$

3.10 Analysis of Connected Optimal Designs

The study would be concluded by comparing the performance of those designs constructed using Maximum likelihood estimator (MLE) and Restricted Maximum likelihood estimator (RMLE). Experimental data would be simulated to compare the two estimators mentioned above.

3.11 Simulation

Incomplete block design data were simulated using mean model defined as

$$y_{ij} = \mu_{ij} + e_{ij} ; \quad i = 1, 2, \dots, t, \quad j = 1, 2, \dots, b \quad (1)$$

Where

y_{ij} is the observation from i th treatment and j th block, μ_{ij} is the sum of treatment effect, block effect and the overall mean and e_{ij} is the error from i th treatment and j th block; $e_{ij} \sim N(0, \sigma^2)$. The variability in μ_{ij} depend heavily on model parameters and e_{ij} is added to it in order to have response variable as a random variable. In this experiment, the addition of the e_{ij} to the μ_{ij} varies with changes in variance such that $0 \leq \text{var} \leq 2$ the effects of these error terms would also be observed on both optimal and less optimal designs. Here, treatments and blocks are no longer orthogonal (Bayrak and Buluf, 2006), only treatment contrasts are estimable because block effects are eliminated (Das, 2002).

P-value approach was used in the decision making regardless of whether or not to reject H_0 . This approach gives information about how deep the test statistic enters the rejection region, a plus to a researcher (Hun Myoung Park 2010). Actually, P-value show case the smallest level of significance possible to reject H_0 , this will help an experimenter to take a decision at any recognized level of significance.

Heteroscedasticity between treatments tends to inflate the experimental error (Bronislaw Ceranka and Malgorzata Graczyk, 2008 and Sharma and Fanta, 2010), making the test statistic less sensitive to detect true effects. When interaction occurs between treatment and block and this interaction are not accounted for in the model, it

invalidates the outcome of the experiment. All these and others were considered in the simulation.

The **crossdes** and **agricolae** package in R would be useful in the construction of designs and analysis of data respectively.

Construction of Optimal Anti-symmetric PBIB Designs With Varying Block and Block Size and Their Complementary Designs

It is possible to vary block and block sizes when the number of treatment and the number of replication remain the same considering constraint $tr = bk$.

Methodology

Algebraic Algorithm

Find a number with at least two factors for treatment and replication such that the following constraint holds $tr = bk$

If $t = xy$ and $r = sz$

Such that $x \neq y$ and $s \neq z$

Then $tr = bk$

$$\left. \begin{array}{l} xy \times sz = xs \times yz \\ xy \times sz = xz \times ys \\ xy \times sz = xys \times z \\ xy \times sz = xyz \times s \end{array} \right\} \begin{array}{|c|} \hline \text{Competing designs} \\ \hline \end{array}$$

Stop when repetition starts.

The complementary designs for the set of designs above is position replacement between treatment t and block b and r and k such that $tr = bk$.

CHAPTER FOUR

RESULTS

4.1 INTRODUCTION

Classes of one, two and, three or more associate classes of partially balanced incomplete block designs with the following constraints were constructed

(i) $2 \leq r \leq t$, (ii) $6 \leq t \leq 15$, (iii) $2 \leq k \leq t/2$,

(iv) $\lambda = r(k - 1)/(t - 1)$

A-, D-, E- and SS- optimality criteria were used in the selection of optimal designs.

In this chapter, the conditions for the existence of associate classes are first considered. The result of the study presented gave explanation to those conditions. This would help experimenters to find optimal designs in the catalogue of possible designs. The concept of symmetric and anti-symmetric designs and, their complementary designs can pave ways for finding optimal designs when an experimenter has reached a compromise on the number of units to use for the study.

Corollary 4.1.1

When $r = t - 1$ BIB design is attained provided t is even with A-, D- and E-values equal with minimum sum of squares zero.

Corollary 4.1.2

When $r = t - 2$ and $k = 2$, PBIB(2) is attained provided t is even.

Corollary 4.1.3

A design $d \in \mathcal{D}(t, b, k, r)$ is PBIB(2) if it has a structure defined as

$$d(t, b, k = 2, r = t/2)$$

Corollary 4.1.4

Increase in replications or block sizes or both in a design $d(t, b, k, r)$ improves the frequency distribution of the concurrence matrix.

Proof: Let C be the information matrix given as $C = rI - \frac{1}{K} NN'$

As $r \rightarrow \infty$, $C \rightarrow \infty$ proved

Corollary 4.1.5

A symmetric design $d(t = b, k = r)$ produces at least PBIBD(k) or PBIBD

$$(k - 1)$$

Proof: Let the concurrence pair of treatment be

$$\lambda = \frac{r(k - 1)}{t - 1}$$

$$\lambda(t - 1) = r(k - 1)$$

Since $t = b$ and $k = r$ then

$$\lambda(t - 1) = k(k - 1) \text{ proved.}$$

Corollary 4.1.6

A design $d(t, b, k, r)$ which produces PBIB(k), PBIBD($k + 1$) PBIBD($k + j$) gives A- and D-highest optimality values for PBIBD(k) and lowest to PBIBD($k + j$)

Proof: Let θ and Θ be two designs having the same parameter sets. Let θ_λ and Θ_λ be their concurrent pairs of treatments set respectively.

$$\text{If } n(\theta_\lambda) < n(\Theta_\lambda)$$

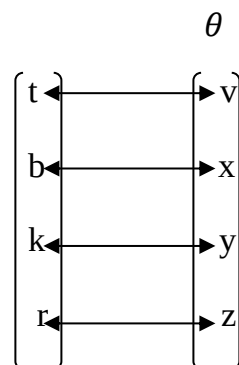
And

$$\theta_\lambda \geq \Theta_\lambda$$

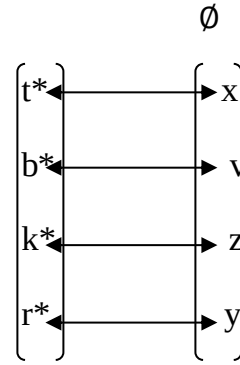
Where the unique elements in θ_λ are fewer than that of Θ_λ .

4.2 Mathematical expression on symmetric and anti-symmetrical designs

Under PBIBD, a design with its complementary designs is constructed. Let a PBIB design parameters be



and its complementary design be



If $\theta R\psi = \psi R\theta$ then we have symmetric designs such that $t = b$ and $k = r$ otherwise it is called nonsymmetrical design i.e.

$$\theta R\psi \neq \psi R\theta.$$

Suppose C_θ is the information matrix for a PBIB design and C_ψ for its complementary design then

$$C_\theta = rI - \frac{1}{k}NN' \text{ and } C_\psi = r'I - \frac{1}{k'}NN'$$

If design d_θ is symmetric then $r = r'$ and $k = k'$ thus

$$C_\theta = C_\psi$$

Where $v = x$ and $t = b$ also $y = z$ and $k = r$. Thus C_θ and C_ψ have the same dimension taking EV as eigenvalue function on the information matrices having the $\text{rank}(C_\theta) = t - 1$ and $\text{rank}(C_\psi) = t - 1$

$$\text{EV}(C_\theta) = \lambda_1, \lambda_2, \dots, \lambda_{t-1}, 0 \text{ where } \lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_{t-1}$$

$$\text{EV}(C_\psi) = \lambda'_1, \lambda'_2, \dots, \lambda'_{t-1}, 0 \text{ where } \lambda'_1 \geq \lambda'_2 \geq \dots \geq \lambda'_{t-1}$$

$$\text{Since } C_\theta = C_\psi$$

$$\lambda_1 = \lambda'_1$$

$$\lambda_2 = \lambda'_2$$

$$\vdots$$

$$\lambda_{t-1} = \lambda'_{t-1}$$

Let A-optimal= $Y_A(.)$, D-optimal= $Y_D(.)$ and E-optimal= $Y_E(.)$ such that A_θ -optimal= $Y_A(C_\theta)$ and A_ψ -optimal= $Y_A(C_\psi)$

$$\text{Thus } Y_A(C_\theta) = Y_A(C_\psi)$$

$$Y_D(C_\theta) = Y_D(C_\psi)$$

$$Y_E(C_\theta) = Y_E(C_\psi)$$

If design d_θ is non-symmetric design then $C_\theta \neq C_\psi$ where $v \neq x$ and $t \neq b$ also $y \neq z$ and $k \neq r$. Thus they do not have the same dimension, taking EV as eigenvalue function on the information matrices

$$EV(C_\theta) = \lambda_1, \lambda_2, \dots, \lambda_{t-1}, 0 \text{ where } \lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_{t-1}$$

$$EV(C_\psi) = \lambda'_1, \lambda'_2, \dots, \lambda'_{t'-1}, 0 \text{ where } \lambda'_1 \geq \lambda'_2 \geq \dots \geq \lambda'_{t'-1}$$

Where $t \neq t'$ and $b \neq b'$ also $r \neq r'$ and $k \neq k'$

$\dim(C_\theta) \neq \dim C_\psi$ where $\dim$ = dimension.

and

$$\text{rank}(C_\theta) \neq \text{rank}(C_\psi)$$

$$\lambda_i \neq \lambda'_i \text{ For at least one } i$$

Therefore

$$Y_A(C_\theta) \neq Y_A(C_\psi)$$

$$Y_D(C_\theta) \neq Y_D(C_\psi)$$

Note :

$$\text{as } r \rightarrow \infty, k \rightarrow 0$$

$$\text{also, as } r \rightarrow 0, k \rightarrow \infty$$

From a relationship in their information matrix

$$k = \frac{NN'}{rI - C_\theta}$$

On account of limit

$$k = \lim_{r \rightarrow \infty} \left(\frac{NN'}{rI - C_\theta} \right) \rightarrow 0$$

Since $r = k'$ and $k = r'$

There exist a relationship between C_θ and C_ψ

Therefore

$$\gamma_E(C_\theta) = \gamma_E(C_\psi)$$

TABLE 4.2.1. ALL DESIGNS GENERATED

This table consist of all designs generated having $r < k$

Design	A	D	E	$\sum(e_i - \bar{e})^2$
$\mathfrak{D}(6,9,2,3)$	0.555	0.574	0.500	0.2
$\mathfrak{D}(6,12,2,4)$	0.576	0.588	0.500	0.075
$\mathfrak{D}(6,15,2,5)$	0.600	0.600	0.600	$6.162e^{-32}$ BIBD
$\mathfrak{D}(6,18,2,6)$	0.588	0.594	0.500	0.033 Good design
$\mathfrak{D}(6,4,3,2)$	0.769	0.784	0.667	0.133
$\mathfrak{D}(6,6,3,3,)$	0.784	0.792	0.667	0.059
$\mathfrak{D}(6,8,3,4)$	0.787	0.793	0.667	0.05 Good design
$\mathfrak{D}(6,10,2,3)$	0.800	0.800	0.800	$1.235e^{-32}$ BIBD
$\mathfrak{D}(8,12,2,3)$	0.487	0.526	0.333	0.380
$\mathfrak{D}(8,18,2,4)$	0.538	0.552	0.500	0.214
$\mathfrak{D}(8,20,2,5)$	0.545	0.557	0.438	0.114
$\mathfrak{D}(8,24,2,6)$	0.560	0.565	0.500	0.047 G design
$\mathfrak{D}(8,28,2,7)$	0.571	0.571	0.571	$9.861e^{-32}$ BIBD
$\mathfrak{D}(8,4,4,2)$	0.807	0.834	0.500	0.232
$\mathfrak{D}(8,6,4,3)$	0.838	0.847	0.666	0.107
$\mathfrak{D}(8,8,4,4)$	0.849	0.853	0.750	0.044
$\mathfrak{D}(8,10,4,5)$	0.854	0.855	0.800	0.017 Good des
$\mathfrak{D}(8,12,4,6)$	0.853	0.855	0.833	0.023
$\mathfrak{D}(8,14,4,7)$	0.857	0.857	0.857	$2.465e^{-32}$ BIBD
$\mathfrak{D}(9,6,3,2)$	0.667	0.707	0.500	0.500
$\mathfrak{D}(9,9,3,3)$	0.727	0.737	0.667	0.167
$\mathfrak{D}(9,12,3,4)$	0.750	0.750	0.750	$4.930e^{-32}$ BIBD
$\mathfrak{D}(10,15,2,3)$	0.454	0.500	0.333	0.556
$\mathfrak{D}(10,20,2,4)$	0.500	0.525	0.345	0.347
$\mathfrak{D}(10,25,2,5)$	0.539	0.540	0.500	0.222
$\mathfrak{D}(10,30,2,6)$	0.530	0.542	0.417	0.139
$\mathfrak{D}(10,35,2,7)$	0.541	0.548	0.428	0.079
$\mathfrak{D}(10,40,2,8)$	0.548	0.552	0.500	0.034 Good design
$\mathfrak{D}(10,45,2,9)$	0.556	0.556	0.556	$3.204e^{-32}$ BIBD
$\mathfrak{D}(10,4,5,2)$	0.850	0.871	0.600	0.245
$\mathfrak{D}(10,6,5,2)$	0.878	0.883	0.800	0.089
$\mathfrak{D}(10,8,5,3)$	0.883	0.886	0.800	0.049
$\mathfrak{D}(10,10,5,5)$	0.886	0.887	0.815	0.025
$\mathfrak{D}(10,12,5,6)^*$	0.886	0.887	0.833	0.022
$\mathfrak{D}(10,14,5,7)$	0.888	0.888	0.857	0.007 Good design
$\mathfrak{D}(10,16,5,8)$	0.887	0.888	0.844	0.014
$\mathfrak{D}(10,18,5,9)$	0.888	0.888	0.844	2.761
$\mathfrak{D}(12,18,2,3)$	0.424	0.482	0.239	0.727
$\mathfrak{D}(12,24,2,4)$	0.479	0.509	0.375	0.477

$\mathfrak{D}(12,30,2,5)$	0.508	0.521	0.400	0.327
$\mathfrak{D}(12,36,2,6)$	0.523	0.532	0.500	0.227
$\mathfrak{D}(12,42,2,7)$	0.523	0.534	0.410	0.156
$\mathfrak{D}(12,48,2,8)$	0.532	0.538	0.500	0.102
$\mathfrak{D}(12,54,2,9)$	0.537	0.541	0.500	0.060
$\mathfrak{D}(12,60,2,10)$	0.541	0.543	0.500	0.027
$\mathfrak{D}(12,66,2,11)$	0.545	0.543	0.500	1.599
$\mathfrak{D}(12,8,3,2)$	0.599	0.665	0.33	0.848
$\mathfrak{D}(12,12,3,3)$	0.680	0.702	0.553	0.404
$\mathfrak{D}(12,16,3,4)$	0.709	0.718	0.668	0.181
$\mathfrak{D}(12,20,3,5)$	0.721	0.724	0.667	0.048
$\mathfrak{D}(12,24,3,6)$	0.723	0.725	0.667	0.034
$\mathfrak{D}(12,28,3,7)$	0.720	0.724	0.631	0.059
$\mathfrak{D}(12,36,3,8)$	0.721	0.724	0.667	0.056
$\mathfrak{D}(12,36,3,9)$	0.722	0.725	0.657	0.041
$\mathfrak{D}(12,44,3,11)$	0.727	0.727	0.727	5.460
$\mathfrak{D}(12,6,4,2)$	0.750	0.785	0.500	0.511
$\mathfrak{D}(12,9,4,3)$	0.805	0.811	0.750	0.136
$\mathfrak{D}(12,12,4,4)$	0.813	0.815	0.750	0.042
$\mathfrak{D}(12,15,4,5)$	0.810	0.814	0.750	0.076
$\mathfrak{D}(12,18,4,6)$	0.812	0.815	0.750	0.053
$\mathfrak{D}(12,21,4,7)$	0.816	0.817	0.786	0.013
$\mathfrak{D}(12,24,4,8)$	0.816	0.817	0.758	0.019
$\mathfrak{D}(12,27,4,9)$	0.815	0.816	0.777	0.025
$\mathfrak{D}(12,30,4,10)$	0.816	0.817	0.800	0.016
$\mathfrak{D}(12,33,4,11)$	0.818	0.817	0.800	1.232
$\mathfrak{D}(12,4,6,2)$	0.880	0.895	0.667	0.242
$\mathfrak{D}(12,6,6,3)$	0.894	0.902	0.722	0.131
$\mathfrak{D}(12,8,6,4)$	0.901	0.905	0.833	0.075
$\mathfrak{D}(12,10,6,5)$	0.905	0.907	0.833	0.042
$\mathfrak{D}(12,12,6,6)$	0.907	0.908	0.888	0.020
$\mathfrak{D}(12,14,6,8)$	0.908	0.908	0.857	0.013
$\mathfrak{D}(12,18,6,9)$	0.907	0.908	0.875	0.013
$\mathfrak{D}(12,20,6,10)$	0.908	0.907	0.900	0.009
$\mathfrak{D}(12,22,6,11)$	0.909	0.909	0.909	$3.698e^{-31}$
$\mathfrak{D}(14,21,2,3)$	0.408	0.469	0.264	0.897
$\mathfrak{D}(14,28,2,4)$	0.460	0.497	0.305	0.605
$\mathfrak{D}(14,35,2,5)$	0.487	0.511	0.319	0.431
$\mathfrak{D}(14,42,2,6)$	0.506	0.521	0.417	0.314
$\mathfrak{D}(14,49,2,7)$	0.520	0.527	0.500	0.231
$\mathfrak{D}(14,56,2,8)$	0.519	0.528	0.397	0.168
$\mathfrak{D}(14,63,2,9)$	0.524	0.531	0.444	0.197
$\mathfrak{D}(14,70,2,10)$	0.528	0.533	0.450	0.0807
$\mathfrak{D}(14,77,2,11)$	0.532	0.535	0.470	0.048

$\mathfrak{D}(14,84,2,12)$	0.535	0.537	0.500	0.224
$\mathfrak{D}(14,91,2,13)$	0.538	0.538	0.538	$1.577e^{-30}$
$\mathfrak{D}(14,4,7,2)$	0.893	0.909	0.571	0.270
$\mathfrak{D}(14,6,7,3)$	0.911	0.917	0.761	0.134
$\mathfrak{D}(14,8,7,4)$	0.917	0.920	0.857	0.066
$\mathfrak{D}(14,10,7,5)$	0.919	0.921	0.857	0.041
$\mathfrak{D}(14,12,7,6)$	0.921	0.922	0.857	0.025
$\mathfrak{D}(14,14,7,7)$	0.922	0.923	0.889	0.013
$\mathfrak{D}(14,16,7,8)$	0.922	0.923	0.877	0.012
$\mathfrak{D}(14,18,7,9)$	0.922	0.923	0.889	0.007
$\mathfrak{D}(14,20,7,10)$	0.922	0.923	0.892	0.009
$\mathfrak{D}(14,22,7,11)$	0.923	0.923	0.896	0.005
$\mathfrak{D}(14,24,7,12)$	0.923	0.923	0.896	0.006
$\mathfrak{D}(14,26,7,13)$	0.923	0.923	0.901	0.002
$\mathfrak{D}(14,28,7,14)$	0.923	0.923	0.902	0.005
$\mathfrak{D}(15,10,3,2)$	0.565	0.641	0.333	1.195
$\mathfrak{D}(15,15,3,3)$	0.660	0.685	0.556	0.635
$\mathfrak{D}(15,20,3,4)$	0.682	0.698	0.583	0.357
$\mathfrak{D}(15,25,3,5)$	0.700	0.706	0.667	0.190
$\mathfrak{D}(15,30,3,6)$	0.707	0.710	0.667	0.079
$\mathfrak{D}(15,35,3,7)$	0.714	0.714	0.714	$1.010e^{-30}$
$\mathfrak{D}(15,6,5,2)$	0.808	0.833	0.600	0.514
$\mathfrak{D}(15,9,5,3)$	0.833	0.845	0.631	0.274
$\mathfrak{D}(15,15,5,5)$	0.850	0.854	0.800	0.082
$\mathfrak{D}(15,18,5,6)$	0.853	0.855	0.780	0.047
$\mathfrak{D}(15,21,5,7)$	0.855	0.856	0.794	0.019
$\mathfrak{D}(15,24,5,8)$	0.855	0.856	0.805	0.027
$\mathfrak{D}(15,27,5,9)$	0.855	0.856	0.808	0.025
$\mathfrak{D}(15,30,5,10)$	0.856	0.857	0.822	0.010
$\mathfrak{D}(15,33,5,11)$	0.856	0.856	0.819	0.009
$\mathfrak{D}(15,36,5,12)$	0.856	0.857	0.821	0.014
$\mathfrak{D}(15,39,5,13)$	0.863	0.857	0.824	0.010
$\mathfrak{D}(15,42,5,14)$	0.857	0.857	0.828	0.002
$\mathfrak{D}(15,45,5,15)$	0.857	0.857	0.830	0.008

**TABLE 4.2.2 PARTIALLY BALANCED INCOMPLETE BLOCK DESIGNS WITH
2 ASSOCIATE CLASSES PBIB (2)**

This table consists of partially balanced incomplete block designs with two associate class when $r < k$

Design	A	D	E	$\Sigma(e_i - \bar{e})^2$
$\mathfrak{D}(6,9,2,3)$	0.556	0.574	0.500	0.200
$\mathfrak{D}(6,12,2,4)$	0.577	0.588	0.500	0.075
$\mathfrak{D}(6,4,3,2)$	0.769	0.784	0.667	0.133
$\mathfrak{D}(6,6,3,3)$	0.784	0.792	0.667	0.059
$\mathfrak{D}(8,16,2,4)$	0.538	0.552	0.500	0.214
$\mathfrak{D}(8,24,2,6)$	0.560	0.565	0.500	0.048
$\mathfrak{D}(8,10,4,5)$	0.854	0.856	0.800	0.017
$\mathfrak{D}(8,12,4,6)$	0.854	0.855	0.833	0.024
$\mathfrak{D}(9,6,3,2)$	0.667	0.707	0.500	0.500
$\mathfrak{D}(9,9,3,3)$	0.727	0.730	0.667	0.167
$\mathfrak{D}(10,15,2,3)$	0.455	0.501	0.333	0.556
$\mathfrak{D}(10,25,2,5)$	0.529	0.540	0.500	0.222
$\mathfrak{D}(10,40,2,8)$	0.549	0.552	0.500	0.035
$\mathfrak{D}(10,6,5,3)$	0.878	0.883	0.800	0.088
$\mathfrak{D}(10,14,5,7)$	0.888	0.888	0.857	0.007
$\mathfrak{D}(12,36,2,6)$	0.523	0.523	0.500	0.227
$\mathfrak{D}(12,48,2,8)$	0.532	0.538	0.500	0.102
$\mathfrak{D}(12,54,2,9)$	0.536	0.541	0.500	0.060
$\mathfrak{D}(12,60,2,10)$	0.541	0.543	0.500	0.027
$\mathfrak{D}(12,16,3,4)$	0.710	0.718	0.667	0.181
$\mathfrak{D}(12,20,3,5)$	0.721	0.724	0.667	0.048
$\mathfrak{D}(12,24,3,6)$	0.723	0.725	0.667	0.034
$\mathfrak{D}(12,40,3,10)$	0.724	0.726	0.700	0.022
$\mathfrak{D}(12,9,4,3)$	0.804	0.811	0.750	0.136
$\mathfrak{D}(12,12,4,4)$	0.813	0.815	0.750	0.043
$\mathfrak{D}(12,21,4,7)$	0.817	0.817	0.786	0.014
$\mathfrak{D}(12,4,6,2)$	0.880	0.895	0.667	0.242
$\mathfrak{D}(12,8,6,4)$	0.902	0.906	0.833	0.075
$\mathfrak{D}(14,49,2,7)$	0.520	0.527	0.500	0.231
$\mathfrak{D}(14,84,2,12)$	0.535	0.537	0.500	0.022
$\mathfrak{D}(14,8,7,4)$	0.918	0.920	0.857	0.066
$\mathfrak{D}(14,10,7,5)$	0.919	0.921	0.857	0.041
$\mathfrak{D}(15,25,3,5)$	0.700	0.706	0.667	0.190
$\mathfrak{D}(15,6,5,2)$	0.808	0.833	0.600	0.514

The table above shows that PBIB designs with two associate classes, some designs utilize large number of units while others low.

**TABLE 4.2.3 PARTIALLY BALANCED INCOMPLETE BLOCK DESIGN WITH
3 OR MORE ASSOCIATE CLASSES**

This table consists of partially balanced incomplete block designs with three or more associate class when $r < k$

Design	A	D	E	$\Sigma(e_i - \bar{e})^2$
$\mathfrak{D}(6,8,3,4)$	0.787	0.794	0.667	0.050
$\mathfrak{D}(8,12,2,3)$	0.488	0.526	0.330	0.381
$\mathfrak{D}(8,20,2,5)$	0.545	0.557	0.438	0.114
$\mathfrak{D}(8,4,4,2)$	0.808	0.834	0.500	0.232
$\mathfrak{D}(8,6,4,3)$	0.838	0.848	0.667	0.107
$\mathfrak{D}(8,8,4,4)$	0.849	0.853	0.750	0.446
$\mathfrak{D}(10,20,2,4)$	0.50	0.525	0.345	0.347
$\mathfrak{D}(10,30,2,6)$	0.530	0.542	0.417	0.139
$\mathfrak{D}(10,35,2,7)$	0.541	0.548	0.429	0.079
$\mathfrak{D}(10,4,5,2)$	0.850	0.871	0.600	0.249
$\mathfrak{D}(10,8,5,4)$	0.883	0.886	0.800	0.489
$\mathfrak{D}(10,10,5,5)$	0.886	0.887	0.815	0.025
$\mathfrak{D}(10,12,5,6)$	0.886	0.888	0.833	0.022
$\mathfrak{D}(10,16,5,8)$	0.887	0.888	0.844	0.013
$\mathfrak{D}(12,18,2,3)$	0.424	0.482	0.240	0.727
$\mathfrak{D}(12,24,2,4)$	0.479	0.509	0.375	0.477
$\mathfrak{D}(12,30,2,5)$	0.502	0.522	0.400	0.327
$\mathfrak{D}(12,42,2,7)$	0.523	0.534	0.411	0.156
$\mathfrak{D}(12,8,3,2)$	0.599	0.665	0.333	0.848
$\mathfrak{D}(12,12,3,3)$	0.68	0.701	0.554	0.404
$\mathfrak{D}(12,28,3,7)$	0.720	0.724	0.639	0.059
$\mathfrak{D}(12,36,3,8)$	0.721	0.724	0.667	0.057
$\mathfrak{D}(12,36,3,9)$	0.723	0.725	0.658	0.0412
$\mathfrak{D}(12,6,4,2)$	0.750	0.786	0.500	0.511
$\mathfrak{D}(12,15,4,5)$	0.810	0.814	0.750	0.076
$\mathfrak{D}(12,18,4,6)$	0.813	0.815	0.750	0.050
$\mathfrak{D}(12,24,4,8)$	0.816	0.817	0.759	0.019
$\mathfrak{D}(12,27,4,9)$	0.816	0.817	0.778	0.025
$\mathfrak{D}(12,30,4,10)$	0.817	0.817	0.800	0.016
$\mathfrak{D}(12,6,6,3)$	0.894	0.902	0.722	0.131
$\mathfrak{D}(12,10,6,5)$	0.905	0.907	0.833	0.424
$\mathfrak{D}(12,12,6,6)$	0.907	0.908	0.89	0.020
$\mathfrak{D}(12,14,6,8)$	0.908	0.908	0.857	0.013
$\mathfrak{D}(12,18,6,9)$	0.908	0.908	0.875	0.013
$\mathfrak{D}(12,20,6,10)$	0.908	0.909	0.900	0.009
$\mathfrak{D}(14,21,2,3)$	0.409	0.469	0.264	0.897
$\mathfrak{D}(14,28,2,4)$	0.460	0.497	0.305	0.606

$\mathfrak{D}(14,35,2,5)$	0.488	0.511	0.319	0.431
$\mathfrak{D}(14,42,2,6)$	0.506	0.521	0.417	0.314
$\mathfrak{D}(14,56,2,8)$	0.519	0.521	0.398	0.168
$\mathfrak{D}(14,63,2,9)$	0.524	0.531	0.444	0.197
$\mathfrak{D}(14,70,2,10)$	0.528	0.533	0.450	0.081
$\mathfrak{D}(14,77,2,11)$	0.532	0.535	0.470	0.049
$\mathfrak{D}(14,4,7,2)$	0.894	0.909	0.570	0.270
$\mathfrak{D}(14,6,7,3)$	0.911	0.917	0.762	0.134
$\mathfrak{D}(14,12,7,6)$	0.921	0.921	0.857	0.025
$\mathfrak{D}(14,14,7,7)$	0.922	0.923	0.889	0.013
$\mathfrak{D}(14,16,7,8)$	0.922	0.923	0.877	0.012
$\mathfrak{D}(14,18,7,9)$	0.922	0.923	0.889	0.007
$\mathfrak{D}(14,20,7,10)$	0.922	0.923	0.892	0.009
$\mathfrak{D}(14,22,7,11)$	0.923	0.923	0.897	0.005
$\mathfrak{D}(14,24,7,12)$	0.923	0.923	0.896	0.006
$\mathfrak{D}(14,26,7,13)$	0.923	0.923	0.901	0.002
$\mathfrak{D}(14,28,7,14)$	0.923	0.923	0.902	0.005
$\mathfrak{D}(15,10,3,2)$	0.545	0.641	0.333	1.190
$\mathfrak{D}(15,15,3,3)$	0.660	0.685	0.5556	0.6349206
$\mathfrak{D}(15,20,3,4)$	0.682	0.698	0.583	0.357
$\mathfrak{D}(15,30,3,6)$	0.707	0.710	0.667	0.079
$\mathfrak{D}(15,9,5,3)$	0.833	0.845	0.631	0.274
$\mathfrak{D}(15,15,5,5)$	0.851	0.854	0.800	0.082
$\mathfrak{D}(15,18,5,6)$	0.853	0.855	0.780	0.048
$\mathfrak{D}(15,21,5,7)$	0.855	0.856	0.790	0.020
$\mathfrak{D}(15,24,5,8)$	0.855	0.856	0.805	0.027
$\mathfrak{D}(15,27,5,9)$	0.855	0.856	0.808	0.025
$\mathfrak{D}(15,30,5,10)$	0.856	0.857	0.822	0.010
$\mathfrak{D}(15,33,5,11)$	0.856	0.857	0.819	0.010
$\mathfrak{D}(15,36,5,12)$	0.855	0.857	0.821	0.014
$\mathfrak{D}(15,39,5,13)$	0.856	0.857	0.824	0.010
$\mathfrak{D}(15,42,5,14)$	0.857	0.857	0.828	0.001
$\mathfrak{D}(15,45,5,15)$	0.856	0.857	0.830	0.008

The table above shows that PBIB designs with three or more associate classes some designs utilize large number of units while others low.

TABLE 4.2.4. SELECTED GOOD DESIGNS WITH SMALL REPLICATIONS

This table consists of optimal partially balanced incomplete block regardless of associate class designs except BIBD when $r < k$

Design	A	D	E	$\Sigma(e_i - \bar{e})^2$
$\mathfrak{D}(6,12,2,4)$	0.577	0.588	0.500	0.075
$\mathfrak{D}(6,6,3,3)$	0.784	0.792	0.667	0.059
$\mathfrak{D}(8,24,2,6)$	0.560	0.566	0.500	0.047
$\mathfrak{D}(8,10,4,5)$	0.854	0.857	0.800	0.017
$\mathfrak{D}(9,9,3,3)$	0.727	0.738	0.667	0.167
$\mathfrak{D}(10,40,2,8)$	0.549	0.552	0.500	0.035
$\mathfrak{D}(10,14,5,7)$	0.888	0.888	0.857	0.007PBIB(3)
$\mathfrak{D}(12,60,2,10)$	0.5409836	0.5432021	0.500	0.272
$\mathfrak{D}(12,40,3,10)$	0.7247059	0.7259622	0.700	0.022
$\mathfrak{D}(12,30,4,10)$	0.816	0.817	0.8	0.016PBIB(3)
$\mathfrak{D}(12,20,6,10)$	0.908	0.909	0.900	0.0090909 PBIB(3)
$\mathfrak{D}(14,8,4,2,12)$	0.535	0.536	0.500	0.022
$\mathfrak{D}(14,28,7,14)$	0.923	0.923	0.901	0.001
$\mathfrak{D}(15,30,3,6)$	0.707	0.710	0.667	0.079
$\mathfrak{D}(15,30,5,11)$	0.856	0.857	0.819	0.010
$\mathfrak{D}(15,42,5,14)$	0.857	0.857	0.829	0.002

The table above shows that good designs are influenced by replications or/ and block sizes.

**TABLE 4.2.5 PARTIALLY BALANCED INCOMPLETE BLOCK DESIGNS
WITH ONE ASSOCIATE PBIBD/1.**

This table consists of optimal balanced incomplete block (BIBD) when $r < k$

Design	A	D	E	$\Sigma(e_i - \bar{e})^2$
$\mathfrak{D}(6,15,2,5)$	0.600	0.600	0.600	$6.1692e^{-32}$
$\mathfrak{D}(6,10,3,5)$	0.800	0.800	0.800	$1.234e^{-32}$
$\mathfrak{D}(8,28,2,7)$	0.571	0.571	0.571	$9.860e^{-32}$
$\mathfrak{D}(8,14,4,7)$	0.857	0.857	0.857	$2.465e^{-32}$
$\mathfrak{D}(9,12,3,4)$	0.750	0.750	0.750	$4.930e^{-32}$
$\mathfrak{D}(10,45,2,9)$	0.560	0.560	0.560	$3.204e^{-32}$
$\mathfrak{D}(10,18,5,9)$	0.890	0.890	0.890	$2.761e^{-32}$
$\mathfrak{D}(12,60,2,11)$	0.550	0.550	0.550	$1.590e^{-32}$
$\mathfrak{D}(12,44,2,11)$	0.727	0.727	0.727	$5.460e^{-32}$
$\mathfrak{D}(12,33,4,11)$	0.818	0.818	0.818	$1.232e^{-32}$
$\mathfrak{D}(12,22,6,11)$	0.909	0.909	0.909	$3.698e^{-32}$
$\mathfrak{D}(14,91,2,13)$	0.538	0.538	0.538	$1.578e^{-32}$
$\mathfrak{D}(15,35,3,7)$	0.714	0.714	0.714	$1.017e^{-30}$

The table above shows that BIB designs are not easy to come by and even when they are found they utilize large number of units.

We used a graphical representations to visualize what is happening in accordance to the tables given above.

Type I, and E- Optimality Criteria for all Designs Generated

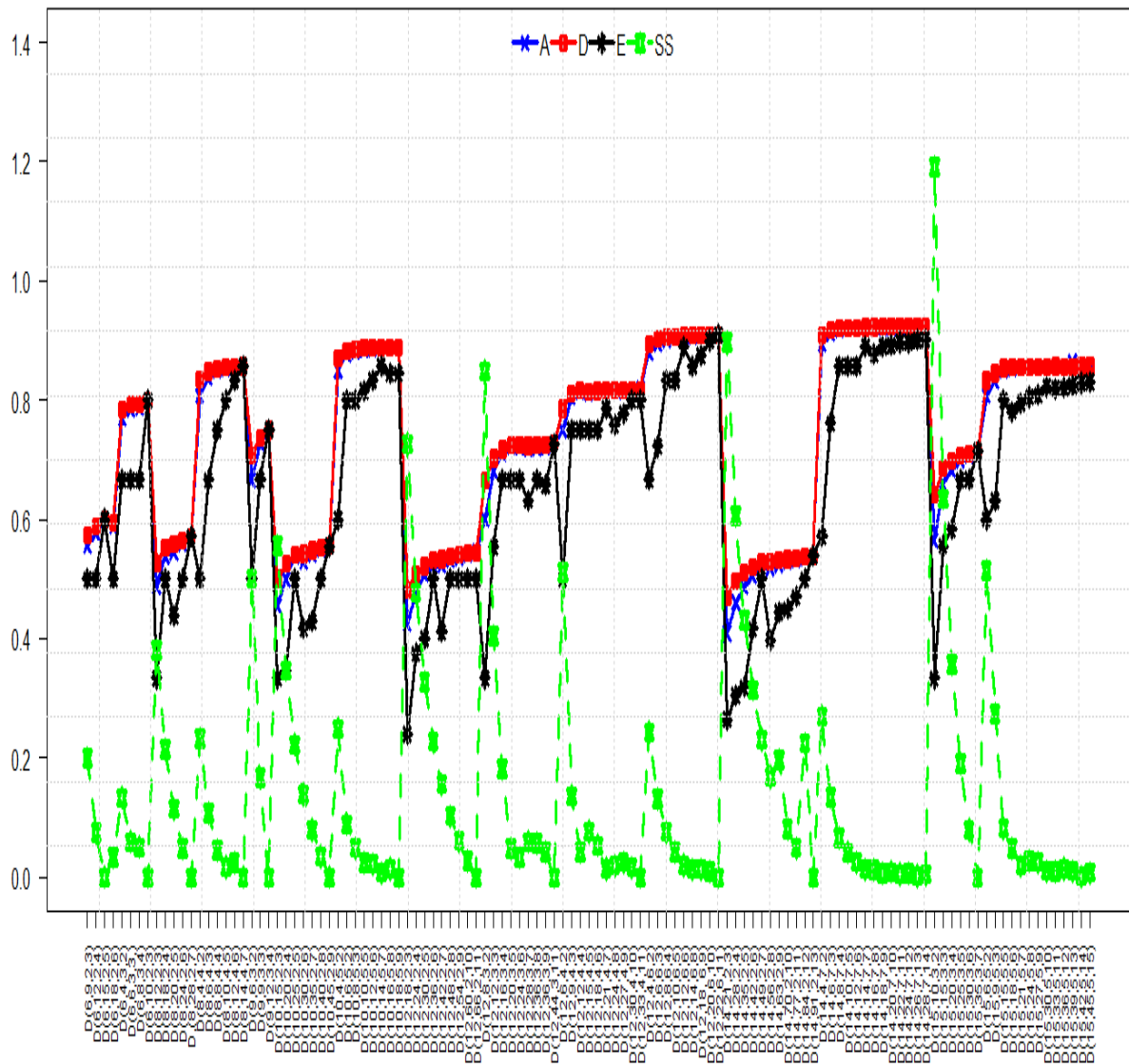


Fig.4.2.1

Type I, and E- Optimality Criteria in Balanced Incomplete Block Designs

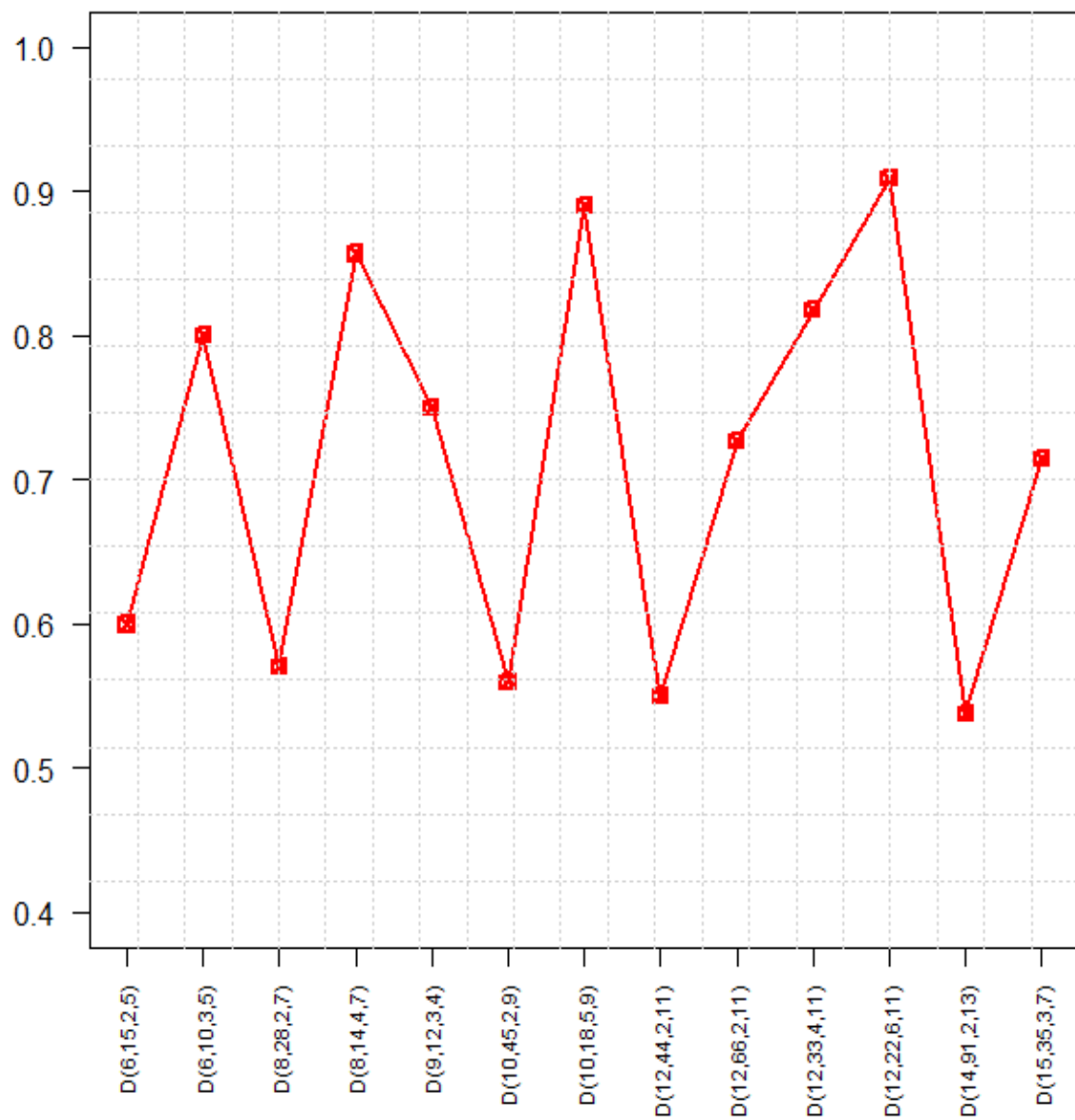


Fig. 4.2.2

Type I, and E- Optimality Criteria in Selected Good Designs

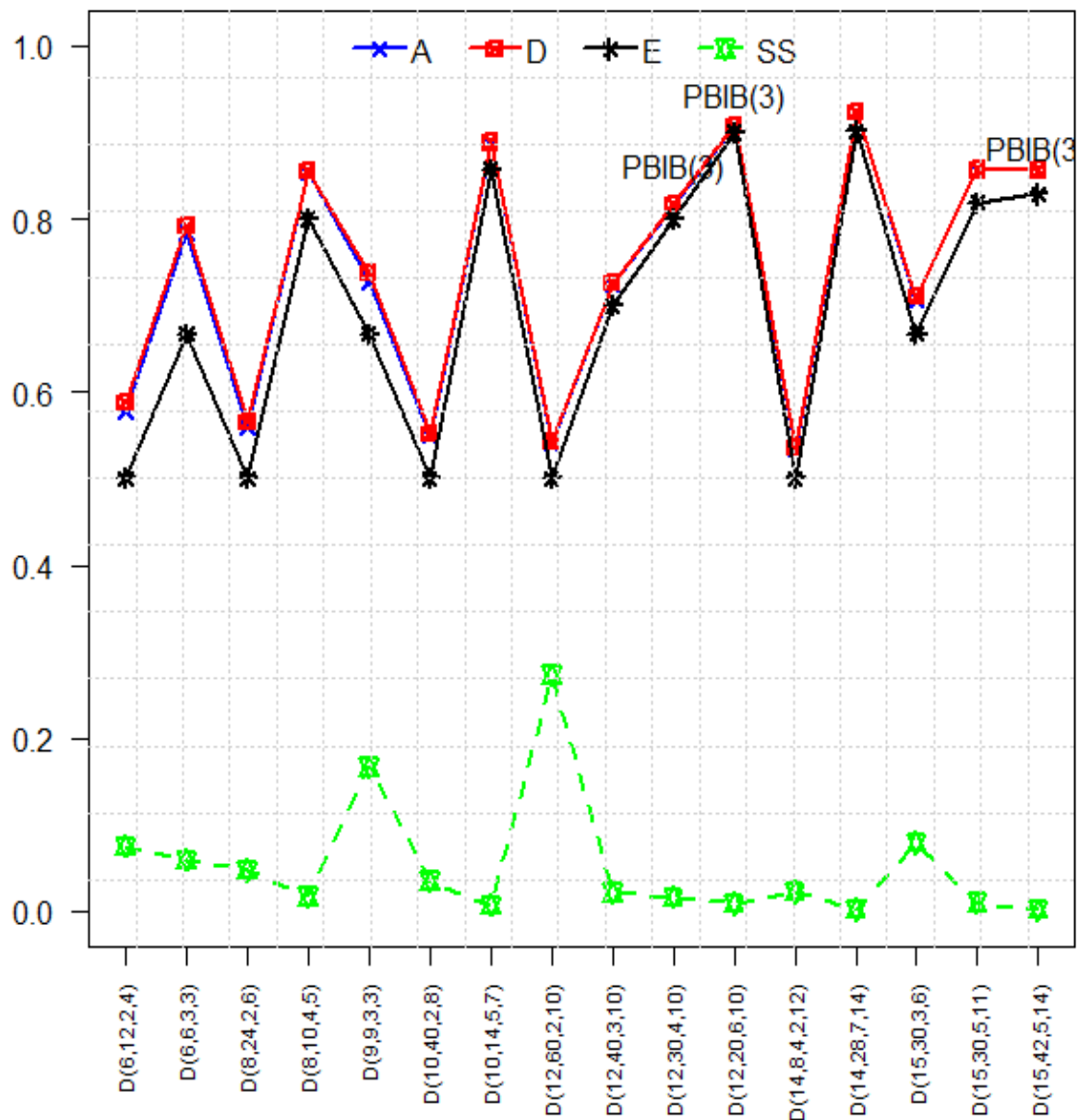


Fig.4.2.3

Type I, and E- Optimality Criteria in PBIB (2) Designs

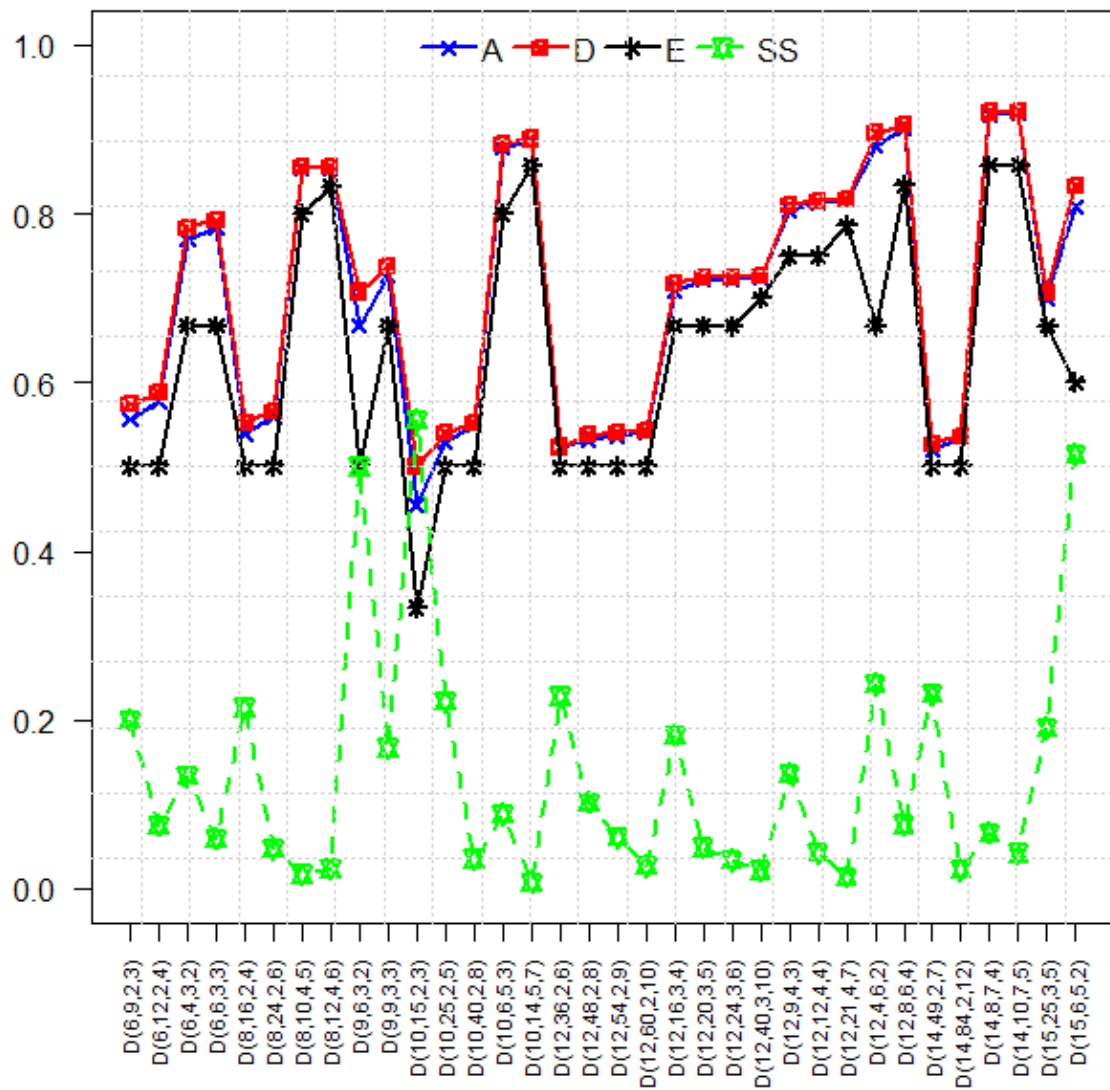


Fig.4.2.4

Type I, and E- Optimality Criteria in PBIB (3) Designs

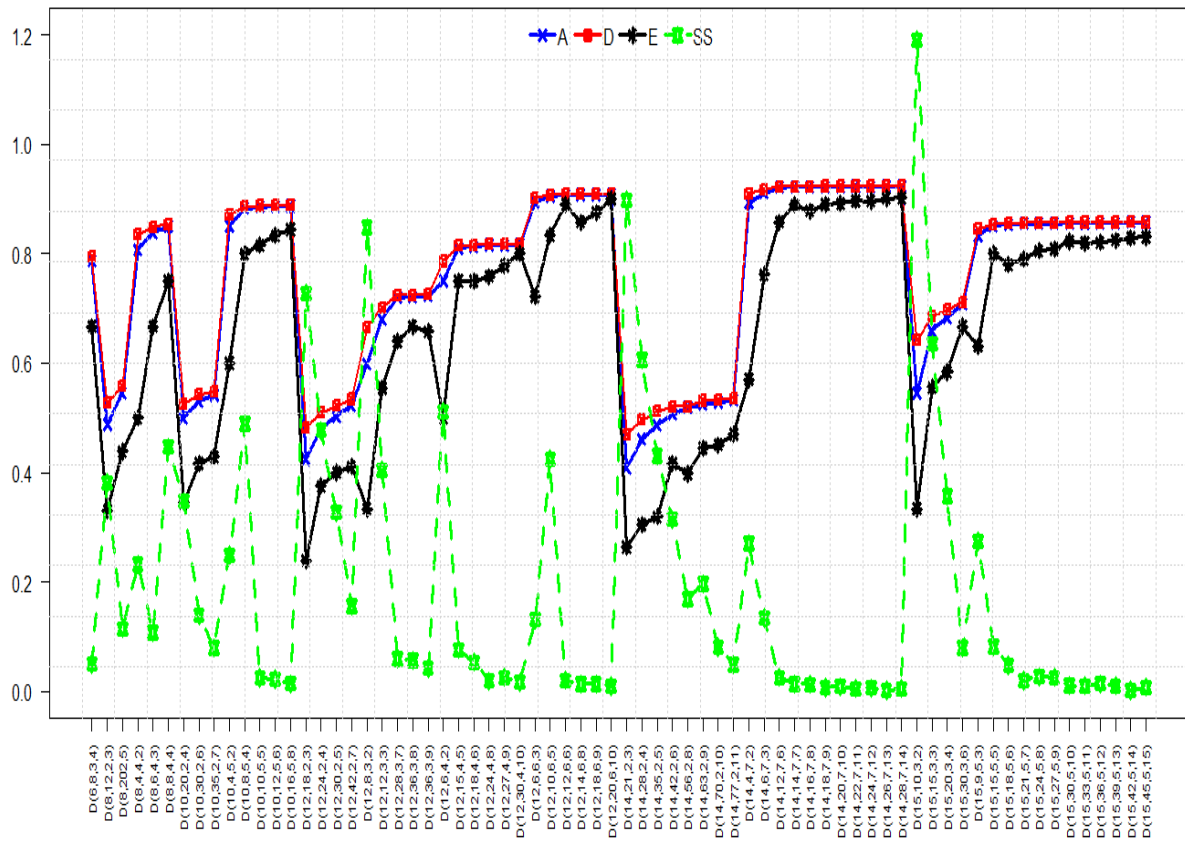


Fig.4.2.5

4.3 SYMMETRIC, NON-SYMMETRIC AND COMPLEMENTARY DESIGNS

Table 4.3.1 Complementary designs for Symmetric PBID

SN	Design	Efficiency factors	Optimality Criteria	Freq. Distr.
1	(6,6,3,3)	0.8888889 0.8888889 0.8888889 0.6666667 0.6666667	Geometric mean (D optimal) 0.7922678 Harmonic mean (A optimal) 0.7843137 Maximizes Min (E optimal) 0.6666667	1 2 3 24 6 6
2	(8,8,4,4)	[1] 0.9633883 0.9633883 0.8750000 0.8750000 0.7866117 0.7866117 0.7500000 [1] 1.0000 0.9375 0.9375 0.8125 0.8125 0.7500 0.7500	Geometric mean (D optimal) 0.8534372 Harmonic mean (A optimal) 0.8497616 Maximizes Min (E optimal) 0.75 Geometric mean (D optimal) 0.8521779 Harmonic mean (A optimal) 0.8472998 Maximizes Min (E optimal) 0.75	1 2 4 16 40 8 1 2 3 4 18 36 2 8
3	(9,9,3,3)	1.0000000 1.0000000 0.6666667 0.6666667 0.6666667 0.6666667 0.6666667 0.6666667	Geometric mean (D optimal) 0.7377879 Harmonic mean (A optimal) 0.7272727 Maximizes Min (E optimal) 0.6666667	0 1 3 18 54 9
4	(10,10,5,5)	0.9600000 0.9447214 0.9447214 0.9047214 0.9047214 0.8552786 0.8552786 0.8152786 0.8152786	Geometric mean (D optimal) 0.887323 Harmonic mean (A optimal) 0.8857495 Maximizes Min (E optimal) 0.8152786	2 3 5 70 20 10
5	(12,12,3,3)	[1] 1.0000000 0.9892458 0.9225306 0.9225306 0.7562293 0.6666667 0.6666667 0.5555556 0.5219138 0.5219138 0.4767472 [1] 1.0000000 1.0000000 0.8888889 0.8888889 0.8888889 0.5555556 0.5555556 0.5555556 0.5555556 0.5555556 0.5555556	Geometric mean (D optimal) 0.7020355 Harmonic mean (A optimal) 0.6777251 Maximizes Min (E optimal) 0.4767472 Geometric mean (D optimal) 0.7027644 Harmonic mean (A optimal) 0.6800618 Maximizes Min (E optimal) 0.5555556	0 1 3 60 72 12 0 1 3 60 72 12

6	(12,12,4,4)	[1] 1.000 0.875 0.875 0.875 0.875 0.750 0.750 0.750 0.750 0.750 0.750 [1] 0.875 0.875 0.875 0.875 0.875 0.875 0.750 0.750 0.750 0.750 0.750	Geometric mean (D optimal) 0.814261 Harmonic mean (A optimal) 0.8105263 Maximizes Min (E optimal) 0.75 Geometric mean (D optimal) 0.8157887 Harmonic mean (A optimal) 0.8133803 Maximizes Min (E optimal) 0.75	0 1 2 4 4 112 16 12 1 2 4 120 12 12
7	(12,12,6,6)	[1] 0.9982065 0.9722222 0.9528549 0.9166667 0.9166667 0.9038167 0.8888889 0.8757546 0.8611111 0.8611111 0.8527007 [1] 0.9722222 0.9722222 0.9528549 0.9528549 0.8888889 0.8888889 0.8888889 0.8888889 0.8888889 0.8527007 0.8527007 [1] 1.0000000 1.0000000 0.8888889 0.8888889 0.8888889 0.8888889 0.8888889 0.8888889 0.8888889 0.8888889 0.8888889	Geometric mean (D optimal) 0.9079461 Harmonic mean (A optimal) 0.9068216 Maximizes Min (E optimal) 0.8527007 Geometric mean (D optimal) 0.9080895 Harmonic mean (A optimal) 0.9070977 Maximizes Min (E optimal) 0.8527007 Geometric mean (D optimal) 0.9081298 Harmonic mean (A optimal) 0.9072165 Maximizes Min (E optimal) 0.8888889	2 3 4 6 38 92 2 12 2 3 6 36 96 12 2 3 6 36 96 12

The same number of efficiency factors, A-, D- and E-optimality criteria values and frequency distribution in the concurrence matrix for symmetric and complementary designs are observed.

Table 4.3.2 Anti-symmetric PBIDs

SN	Design	Efficiency factors	Optimality Criteria	Freq. Distr.
1	(6,4,3,2)	1.0000000 1.0000000 0.6666667 0.6666667 0.6666667	Geometric mean (D optimal) 0.7840527 Harmonic mean (A optimal) 0.7692308 Maximizes Min (E optimal) 0.6666667	0 1 2 6 24 6
2	(8,4,4,2)	(8,4,4) 1.00 1.00 1.00 1.00 0.75 0.75 0.50	Geometric mean (D optimal) 0.8342552 Harmonic mean (A optimal) 0.8076923 Maximizes Min (E optimal) 0.5	0 1 2 12 40 12
3	(8,6,4,3)	1.0000000 1.0000000 0.9166667 0.9166667 0.7500000 0.7500000 0.6666667	Geometric mean (D optimal) 0.8479116 Harmonic mean (A optimal) 0.8384755 Maximizes Min (E optimal) 0.6666667	1 2 3 42 12 10
4	(9,6,3,2)	1.0 1.0 1.0 1.0 0.5 0.5 0.5 0.5	Geometric mean (D optimal) 0.7071068 Harmonic mean (A optimal) 0.6666667 Maximizes Min (E optimal) 0.5	0 1 2 36 36 9
5	(10,4,5,2)	1.0 1.0 1.0 1.0 1.0 1.0 0.8 0.6 0.6	Geometric mean (D optimal) 0.8708282 Harmonic mean (A optimal) 0.8503937 Maximizes Min (E optimal) 0.6	0 1 2 18 64 18
6	(10,6,5,3)	1.0 1.0 1.0 1.0 0.8 0.8 0.8 0.8 0.8	Geometric mean (D optimal) 0.8834075 Harmonic mean (A optimal) 0.8780488 Maximizes Min (E optimal) 0.8	1 2 3 60 30 10
7	(10,8,5,4)	1.0 1.0 0.9 0.9 0.9 0.9 0.8 0.8 0.8	Geometric mean (D optimal) 0.8858496 Harmonic mean (A optimal) 0.8828338 Maximizes Min (E optimal) 0.8	0 1 2 4 2 16 72 10
8	(12,8,3,2)	1.0000000 1.0000000 1.0000000 1.0000000 0.9023689 0.9023689 0.6666667 0.4309644 0.4309644 0.3333333 0.3333333	Geometric mean (D optimal) 0.6647605 Harmonic mean (A optimal) 0.5992218 Maximizes Min (E optimal) 0.3333333	0 1 2 84 48 12
9	(12,6,4,2)	1.00 1.00 1.00 1.00 1.00 1.00 0.75 0.75 0.50 0.50 0.50	Geometric mean (D optimal) 0.7855698 Harmonic mean (A optimal) 0.75 Maximizes Min (E optimal) 0.5	0 1 2 60 72 12
10	(12,9,4,3)	1.00 1.00 1.00 0.75 0.75 0.75 0.75 0.75 0.75 0.75 0.75	Geometric mean (D optimal) 0.8112141 Harmonic mean (A optimal) 0.804878 Maximizes Min (E optimal) 0.75	0 1 3 24 108 12
11	(12,4,6,2)	1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 0.6666667 0.6666667 0.6666667	Geometric mean (D optimal) 0.8953135 Harmonic mean (A optimal) 0.88 Maximizes Min (E optimal) 0.6666667	0 1 2 24 96 24

12	(12,6,6,3)	1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 0.8888889 0.8333333 0.8333333 0.7222222 0.7222222	Geometric mean (D optimal) 0.9021048 Harmonic mean (A optimal) 0.8947286 Maximizes Min (E optimal) 0.7222222	0 1 2 3 4 78 48 14 0 1 2 3 6 72 54 12
13	(12,8,6,4)	1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 0.8333333 0.8333333 0.8333333 0.8333333 0.8333333 0.8333333	Geometric mean (D optimal) 0.9053369 Harmonic mean (A optimal) 0.9016393 Maximizes Min (E optimal) 0.8333333	1 2 3 4 36 84 12 12 0 1 2 3 4 4 24 96 8 12
14	(12,10,6,5)	[1] 1.0000000 1.0000000 0.9666667 0.9333333 0.9333333 0.9333333 0.8666667 0.8666667 0.8333333 0.8333333 0.8333333 [1] 1.0000000 1.0000000 1.0000000 0.9333333 0.9000000 0.9000000 0.8666667 0.8666667 0.8666667 0.8333333 0.8333333	Geometric mean (D optimal) 0.9069748 Harmonic mean (A optimal) 0.9048686 Maximizes Min (E optimal) 0.8333333 Geometric mean (D optimal) 0.9070067 Harmonic mean (A optimal) 0.9049633 Maximizes Min (E optimal) 0.8333333	1 2 3 5 6 84 42 12 1 2 3 5 6 84 42 12
15	(14,4,7,2)	1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 0.7142857 0.7142857 0.5714286	Geometric mean (D optimal) 0.9095436 Harmonic mean (A optimal) 0.8934708 Maximizes Min (E optimal) 0.5714286	0 1 2 34 128 34
16	(14,6,7,3)	1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 0.8571429 0.8571429 0.7619048 0.7619048 0.7619048	Geometric mean (D optimal) 0.9171636 Harmonic mean (A optimal) 0.9109489 Maximizes Min (E optimal) 0.7619048	0 1 2 3 2 114 60 20 0 1 2 3 6 102 72 16 1 2 3 120 54 22 0 1 2 3 8 96 78 14

17	(14,8,7,4)	1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 0.8571429 0.8571429 0.8571429 0.8571429 0.8571429 0.8571429 0.8571429	Geometric mean (D optimal) 0.9203473 Harmonic mean (A optimal) 0.9176471 Maximizes Min (E optimal) 0.8571429	v9 0 1 2 3 4 2 36 132 12 14 0 1 2 3 4 6 24 144 8 14 1 2 3 4 42 126 14 14
18	(14,10,7,5)	1.0000000 1.0000000 1.0000000 1.0000000 0.9142857 0.9142857 0.9142857 0.9142857 0.9142857 0.8571429 0.8571429 0.8571429 0.8571429	Geometric mean (D optimal) 0.9213667 Harmonic mean (A optimal) 0.9196758 Maximizes Min (E optimal) 0.8571429	1 2 3 5 6 114 62 14
19	(15,6,5,2)	1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 0.6 0.6 0.6 0.6 0.6	Geometric mean (D optimal) 0.8332365 Harmonic mean (A optimal) 0.8076923 Maximizes Min (E optimal) 0.6	0 1 2 90 120 15
20	(15,9,5,3)	1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 0.8586257 0.8586257 0.8000000 0.8000000 0.7101802 0.7101802 0.6311941 0.6311941	Geometric mean (D optimal) 0.8451191 Harmonic mean (A optimal) 0.8326351 Maximizes Min (E optimal) 0.6311941	0 1 2 3 48 144 18 15

The different number of efficiency factors, A-, D- and E-optimality criteria values and frequency distribution in the concurrence matrix for symmetric and complementary designs are observed.

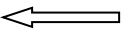
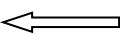
Table 4.3.3. Complementary Designs For Anti-symmetric PBIDs

SN	Design	Efficiency factors	Optimality Criteria	Freq. Distr.
1	(4,6,2,3)	0.6666667 0.6666667 0.6666667	Geometric mean (D optimal) 0.6666667 Harmonic mean (A optimal) 0.6666667 Maximizes Min (E optimal) 0.6666667	1 3 12 4
2	(4,8,2,4)	0.75 0.75 0.50	Geometric mean (D optimal) 0.6551853 Harmonic mean (A optimal) 0.6428571 Maximizes Min (E optimal) 0.5	1 2 4 8 4 4
3	(6,8,3,4)	0.9166667 0.9166667 0.7500000 0.7500000 0.6666667	Geometric mean (D optimal) 0.793763 Harmonic mean (A optimal) 0.7875895 Maximizes Min (E optimal) 0.6666667	1 2 4 12 18 6
4	(6,9,2,3)	1.0 0.5 0.5 0.5 0.5	Geometric mean (D optimal) 0.5743492 Harmonic mean (A optimal) 0.5555556 Maximizes Min (E optimal) 0.5	0 1 3 12 18 6
5	(4,10,2,5)	0.8 0.6 0.6	Geometric mean (D optimal) 0.6603854 Harmonic mean (A optimal) 0.6545455 Maximizes Min (E optimal) 0.6	1 2 5 4 8 4
6	(6,10,3,5)	0.8 0.8 0.8 0.8 0.8	Geometric mean (D optimal) 0.8 Harmonic mean (A optimal) 0.8 Maximizes Min (E optimal) 0.8	2 5 30 6
7	(8,10,4,5)	0.9 0.9 0.9 0.9 0.8 0.8 0.8	Geometric mean (D optimal) 0.855697 Harmonic mean (A optimal) 0.8542373 Maximizes Min (E optimal) 0.8	2 3 5 48 8 8
8	(8,12,2,3)	0.9023689 0.9023689 0.6666667 0.4309644 0.4309644 0.3333333 0.3333333	Geometric mean (D optimal) 0.5264184 Harmonic mean (A optimal) 0.4875622 Maximizes Min (E optimal) 0.3333333	0 1 3 32 24 8
9	(6,12,2,4)	0.75 0.75 0.50 0.50 0.50	Geometric mean (D optimal) 0.5880395 Harmonic mean (A optimal) 0.5769231 Maximizes Min (E optimal) 0.5	0 1 4 6 24 6
10	(9,12,3,4)	0.75 0.75 0.75 0.75 0.75 0.75 0.75 0.75	Geometric mean (D optimal) 0.75 Harmonic mean (A optimal) 0.75 Maximizes Min (E optimal) 0.75	1 4 72 9
11	(4,12,2,4)	0.6666667 0.6666667 0.6666667	Geometric mean (D optimal) 0.6666667 Harmonic mean (A optimal) 0.6666667 Maximizes Min (E optimal) 0.6666667	2 6 12 4
12	(6,12,3,6)	0.8888889 0.8333333 0.8333333 0.7222222 0.7222222	Geometric mean (D optimal) 0.7971964 Harmonic mean (A optimal) 0.7943782 Maximizes Min (E optimal) 0.7222222	2 3 6 18 12 6
13	(8,12,4,6)	1.0000000 0.8333333 0.8333333 0.8333333 0.8333333 0.8333333 0.8333333	Geometric mean (D optimal) 0.8553234 Harmonic mean (A optimal) 0.8536585 Maximizes Min (E optimal) 0.8333333	2 3 6 24 32 8
14	(10,12,5,6)	[1] 1.0000000 0.9333333 0.9000000 0.9000000 0.8666667 0.8666667 0.8666667 0.8333333 0.8333333 [1] 0.9666667 0.9333333 0.9333333 0.9333333 0.8666667 0.8666667 0.8333333 0.8333333	Geometric mean (D optimal) 0.8875455 Harmonic mean (A optimal) 0.8862464 Maximizes Min (E optimal) 0.8333333 Geometric mean (D optimal) 0.8875073	2 3 6 30 60 10 2 3 6 30 60 10

		0.8333333	Harmonic mean (A optimal) 0.8861355 Maximizes Min (E optimal) 0.8333333	
15	(4,14,2,7)	0.7142857 0.7142857 0.5714286	Geometric mean (D optimal) 0.6630841 Harmonic mean (A optimal) 0.6593407 Maximizes Min (E optimal) 0.5714286	2 3 7 8 4 4
16	(6,14,3,7)	0.8571429 0.8571429 0.7619048 0.7619048 0.7619048	Geometric mean (D optimal) 0.7986596 Harmonic mean (A optimal) 0.7973422 Maximizes Min (E optimal) 0.7619048	2 3 7 6 24 6
17	(8,14,4,7)	0.8571429 0.8571429 0.8571429 0.8571429 0.8571429 0.8571429 0.8571429	Geometric mean (D optimal) 0.8571429 Harmonic mean (A optimal) 0.8571429 Maximizes Min (E optimal) 0.8571429	3 7 56 8
18	(10,14,5,7)	0.9142857 0.9142857 0.9142857 0.9142857 0.9142857 0.8571429 0.8571429 0.8571429 0.8571429	Geometric mean (D optimal) 0.8884331 Harmonic mean (A optimal) 0.8879753 Maximizes Min (E optimal) 0.8571429	3 4 7 80 10 10
19	(6,15,2,5)	0.6 0.6 0.6 0.6 0.6	Geometric mean (D optimal) 0.6 Harmonic mean (A optimal) 0.6 Maximizes Min (E optimal) 0.6	1 5 30 6
20	(9,15,3,5)	0.8586257 0.8586257 0.8000000 0.8000000 0.7101802 0.7101802 0.6311941 0.6311941	Geometric mean (D optimal) 0.7449143 Harmonic mean (A optimal) 0.7397758 Maximizes Min (E optimal) 0.6311941	1 2 5 54 18 9

The same number of efficiency factors, A-, D- and E-optimality criteria values and frequency distribution in the concurrence matrix for symmetric and complementary designs are observed.

Table: 4.4.1 Treatment and Replication constraint (Incomplete Block Designs)

Parameters (t,b,k,r) And eigenvalues	Design	Optimality Criteria and Associate Class
8,12,4,6 1.0000000 0.8333333 0.8333333 0.8333333 0.8333333 0.8333333 0.8333333	{2 3 5 8}, {2 5 6 7} {1 4 5 7}, {1 3 6 7} {1 2 4 8}, {2 4 5 6} {4 6 7 8}, {1 3 4 5} {3 5 7 8}, {1 2 3 6} {1 2 7 8}, {3 4 6 8}	Geometric mean (D) 0.8553234 Harmonic mean (A) 0.8536585 Maximizes Min (E) 0.8333333 PBIB(2) 
8,16,3,6 0.9166667 0.8590847 0.8590847 0.7916667 0.7916667 0.7659153 0.7659153	{2 3 8}, {1 4 5}, {1 6 8} {2 4 7}, {5 7 8}, {3 4 6} {2 3 5}, {1 4 8}, {2 5 6} {3 6 7}, {4 5 7}, {3 4 8} {1 2 6}, {1 2 7}, {1 3 7} {5 6 8}	Geometric mean (D) 0.8197391 Harmonic mean (A) 0.8180878 Maximizes Min (E) 0.7659153 PBIB(4)
10,12,5,6 1.0000000 0.9333333 0.9000000 0.9000000 0.8666667 0.8666667 0.8666667 0.8333333 0.8333333	{1 6 8 9 10}, {1 3 4 7 8} {1 4 5 7 9}, {1 3 5 6 10} {3 4 5 6 8}, {1 2 4 9 10} {2 3 4 6 9}, {2 3 7 8 10} {3 5 7 9 10}, {2 4 5 8 10} {1 2 5 6 7}, {2 6 7 8 9}	Geometric mean (D) 0.8875455 Harmonic mean (A) 0.8862464 Maximizes Min (E) 0.8333333 PBIB(5) 
10,15,4,6 0.8333333 0.8333333 0.8333333 0.8333333 0.8333333 0.8333333 0.8333333 0.8333333 0.8333333	{1 4 7 8}, {5 7 8 9} {2 5 6 8}, {3 6 7 10} {1 3 4 5}, {3 4 6 8} {1 5 6 10}, {2 3 5 7} {2 4 6 9}, {1 2 8 10} {1 6 7 9}, {2 4 7 10} {1 2 3 9}, {3 8 9 10} {4 5 9 10}	Geometric mean (D) 0.8333333 Harmonic mean (A) 0.8333333 Maximizes Min (E) 0.8333333 BIB
10,20,3,6 0.8333333 0.8333333 0.8333333 0.8333333 0.6666667 0.6666667 0.6666667 0.6666667 0.6666667	{8 9 10}, {2 3 4}, {1 5 10} {5 6 9}, {3 6 10}, {4 7 10} {3 6 8}, {2 6 9}, {3 5 7} {1 4 8}, {2 8 10}, {3 4 9} {1 5 6}, {4 6 7}, {1 2 9} {1 3 8}, {1 2 7}, {5 7 8} {2 4 5}, {7 9 10}	Geometric mean (D) 0.736173 Harmonic mean (A) 0.7317073 Maximizes Min (E) 0.6666667 PBIB(2)
12,18,4,6 0.9672633 0.9202554 0.8525338 0.8333333 0.8095003 0.8052029 0.7916667 0.7916667 0.7500000 0.7500000 0.7285776	{3 6 7 11}, {1 4 7 9} {7 8 11 12}, {1 5 7 8} {1 5 6 9}, {3 4 8 10} {3 7 9 10}, {3 4 5 12} {5 6 8 10}, {2 5 9 11} {1 2 8 12}, {1 4 10 11} {2 3 8 9}, {2 4 5 11} {9 10 11 12}, {2 4 6 7} {2 6 10 12}, {1 3 6 12}	Geometric mean (D) 0.8153507 Harmonic mean (A) 0.812633 Maximizes Min (E) 0.7285776 PBIB(9)

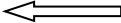
12,24,3,6 0.7777778 0.7777778 0.7777778 0.7777778 0.7777778 0.7777778 0.6666667 0.6666667 0.6666667 0.6666667 0.6666667	{1 2 11}, {5 6 10}, {7 9 12} {1 10 12}, {1 3 9}, {3 7 11} {2 3 8}, {4 8 12}, {5 8 11} {2 4 7}, {4 6 9}, {3 10 12} {7 8 10}, {1 4 6}, {6 11 12} {5 9 11}, {2 6 7}, {3 4 5} {4 10 11}, {1 8 9}, {1 5 7} {3 6 8}, {2 5 12}, {2 9 10}	Geometric mean (D) 0.7251455 Harmonic mean (A) 0.7230047 Maximizes Min (E) 0.6666667 PBIB(2)
12,9,8,6 1.0000 1.0000 1.0000 0.9375 0.9375 0.9375 0.9375 0.9375 0.9375 0.9375 0.9375	{1 2 3 4 5 6 10 11} {1 4 5 8 9 10 11 12} { 1 2 5 6 7 8 11 12} {2 4 6 7 9 10 11 12} {1 3 4 5 6 7 9 12} {2 3 4 5 7 8 10 12} {1 2 3 4 7 8 9 11} {3 5 6 7 8 9 10 11} {1 2 3 6 8 9 10 12}	Geometric mean (D) 0.9541474 Harmonic mean (A) 0.9537572 Maximizes Min (E) 0.9375 PBIB(2) 

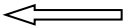
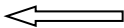
Table: 4.4.2 Block and Block size Constraint (Incomplete Block Complementary Designs)

Parameters (t,b,k,r) And eigenvalues	Design	Optimality Criteria and Associate Class
12,8,6,4 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 0.8333333 0.8333333 0.8333333 0.8333333 0.8333333 0.8333333	[1,] 1 7 8 9 10 12 [2,] 2 4 7 8 9 11 [3,] 2 3 6 8 10 12 [4,] 1 3 4 5 7 10 [5,] 3 4 5 6 8 9 [6,] 1 5 6 9 11 12 [7,] 2 5 6 7 10 11 [8,] 1 2 3 4 11 12	Geometric mean (D) 0.9053369 Harmonic mean (A) 0.9016393 Maximizes Min (E) 0.8333333 PBIB(2)
16,8,6,3 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 0.8888889 0.8121130 0.8121130 0.7222222 0.7222222 0.6878870 0.6878870	[1,] 1 4 10 13 14 15 [2,] 3 8 9 10 11 16 [3,] 2 4 5 6 15 16 [4,] 1 5 8 9 12 15 [5,] 4 6 7 10 11 12 [6,] 2 3 6 9 13 14 [7,] 1 3 5 7 11 14 [8,] 2 7 8 12 13 16	Geometric mean (D) 0.8790841 Harmonic mean (A) 0.8688401 Maximizes Min (E) 0.687887 PBIB(5)
12,10,6,5 1.0000000 1.0000000 1.0000000 0.9333333 0.9000000 0.9000000 0.8666667 0.8666667 0.8666667 0.8333333 0.8333333	[1,] 1 3 6 9 10 11 [2,] 1 2 6 7 8 11 [3,] 2 6 7 9 10 12 [4,] 1 4 8 9 11 12 [5,] 2 3 5 9 11 12 [6,] 1 4 5 6 10 12 [7,] 3 4 7 8 9 10 [8,] 3 5 6 7 8 12 [9,] 1 2 3 4 5 7 [10,] 2 4 5 8 10 11	Geometric mean (D) 0.9070067 Harmonic mean (A) 0.9049633 Maximizes Min (E) 0.8333333 PBIB(5)
15,10,6,4 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 0.8333333 0.8333333 0.8333333 0.8333333 0.8333333 0.8333333 0.8333333 0.8333333 0.8333333	[1,] 1 6 8 9 12 14 [2,] 1 2 3 5 9 13 [3,] 4 5 6 7 9 15 [4,] 1 4 7 10 13 14 [5,] 2 3 6 10 14 15 [6,] 1 3 4 11 12 15 [7,] 2 6 7 11 12 13 [8,] 3 5 7 8 10 12 [9,] 2 4 8 9 10 11 [10,] 5 8 11 13 14 15	Geometric mean (D) 0.8894013 Harmonic mean (A) 0.8860759 Maximizes Min (E) 0.8333333 PBIB(2)
20,10,6,3 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000	[1,] 1 10 11 15 16 19 [2,] 1 3 7 12 14 20 [3,] 1 2 5 11 13 18 [4,] 2 7 9 16 19 20 [5,] 6 8 13 14 17 19	Geometric mean (D) 0.8649489 Harmonic mean (A) 0.8520179 Maximizes Min (E) 0.6666667 PBIB(3)

1.0000000 1.0000000 0.8333333 0.8333333 0.8333333 0.8333333 0.6666667 0.6666667 0.6666667 0.6666667 0.6666667	[6,] 3 4 5 9 13 15 [7,] 3 4 6 12 16 18 [8,] 8 9 15 17 18 20 [9,] 4 6 7 8 10 11	
18,12,6,4 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 0.9471688 0.9471688 0.8333333 0.8333333 0.8333333 0.8028312 0.8028312 0.7500000 0.7500000 0.7500000 0.7500000	[1,] 1 3 6 7 9 18 [2,] 3 7 8 11 12 17 [3,] 2 4 9 11 16 17 [4,] 5 6 12 14 17 18 [5,] 2 7 10 12 13 14 [6,] 1 2 5 8 16 18 [7,] 1 2 3 10 15 17 [8,] 6 9 12 13 15 16 [9,] 4 5 6 10 11 15 [10,] 3 4 5 7 13 16 [11,] 1 8 11 13 14 15 [12,] 4 8 9 10 14 18	Geometric mean (D) 0.8762652 Harmonic mean (A) 0.8701851 Maximizes Min (E) 0.75 PBIB(5)
24,12,6,3 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 0.7777778 0.7777778 0.7777778 0.7777778 0.7777778 0.7777778 0.6666667 0.6666667 0.6666667 0.6666667 0.6666667	[1,] 8 9 11 15 22 23 [2,] 2 9 10 11 13 24 [3,] 1 6 7 13 16 23 [4,] 5 8 16 18 21 24 [5,] 5 6 10 19 20 22 [6,] 1 3 12 14 22 24 [7,] 2 12 15 16 17 20 [8,] 1 4 5 9 17 18 [9,] 3 7 11 14 18 20 [10,] 3 13 17 19 21 23 [11,] 4 7 10 12 15 21 [12,] 2 4 6 8 14 19	Geometric mean (D) 0.8575251 Harmonic mean (A) 0.8451444 Maximizes Min (E) 0.6666667 PBIB(3)
9,12,6,8 0.9375 0.9375 0.9375 0.9375 0.9375 0.9375 0.9375 0.9375	[1,] 1 3 4 5 6 7 [2,] 2 3 4 5 8 9 [3,] 1 2 3 4 6 9 [4,] 2 4 5 6 7 9 [5,] 1 2 3 5 6 8 [6,] 1 3 4 7 8 9 [7,] 1 4 5 6 8 9 [8,] 1 2 6 7 8 9 [9,] 3 5 6 7 8 9 [10,] 1 2 3 5 7 9 [11,] 1 2 4 5 7 8 [12,] 2 3 4 6 7 8	Geometric mean (D) 0.9375 Harmonic mean (A) 0.9375 Maximizes Min (E) 0.9375 BIB

A design that optimizes both block size and replication is the optimal design.

Table: 4.4.3 Unit constraint

PBIB designs	Complementary Designs
8,12,4,6 Geometric mean (D) 0.8553234 Harmonic mean (A) 0.8536585 Maximizes Min (E) 0.8333333 PBIB(2) 	12,8,6,4  Geometric mean (D) 0.9053369 Harmonic mean (A) 0.9016393 Maximizes Min (E) 0.8333333 PBIB(2) 
8,16,3,6 Geometric mean (D) 0.8197391 Harmonic mean (A) 0.8180878 Maximizes Min (E) 0.7659153 PBIB(4)	16,8,6,3 Geometric mean (D) 0.8790841 Harmonic mean (A) 0.8688401 Maximizes Min (E) 0.687887 PBIB(5)
10,12,5,6 Geometric mean (D) 0.8875455 Harmonic mean (A) 0.8862464 Maximizes Min (E) 0.8333333 PBIB(5) 	12,10,6,5  Geometric mean (D) 0.9070067 Harmonic mean (A) 0.9049633 Maximizes Min (E) 0.8333333  PBIB(5)
10,15,4,6 Geometric mean (D) 0.8333333 Harmonic mean (A) 0.8333333 Maximizes Min (E) 0.8333333 BIB	15,10,6,4 Geometric mean (D) 0.8894013 Harmonic mean (A) 0.8860759 Maximizes Min (E) 0.8333333 PBIB(2)
10,20,3,6 Geometric mean (D) 0.736173 Harmonic mean (A) 0.7317073 Maximizes Min (E) 0.6666667 PBIB(2)	20,10,6,3 Geometric mean (D) 0.8649489 Harmonic mean (A) 0.8520179 Maximizes Min (E) 0.6666667 PBIB(3)
12,18,4,6 Geometric mean (D) 0.8153507 Harmonic mean (A) 0.812633 Maximizes Min (E) 0.7285776 PBIB(9)	18,12,6,4 Geometric mean (D) 0.8762652 Harmonic mean (A) 0.8701851 Maximizes Min (E) 0.75 PBIB(5)

12,24,3,6 Geometric mean (D) 0.7251455 Harmonic mean (A) 0.7230047 Maximizes Min (E) 0.6666667 PBIB(2)		24,12,6,3 Geometric mean (D) 0.8575251 Harmonic mean (A) 0.8451444 Maximizes Min (E) 0.6666667 PBIB(3)
12,9,8,6 Geometric mean (D) 0.9541474 Harmonic mean (A) 0.9537572 Maximizes Min (E) 0.9375 PBIB(2) ←	★	9,12,6,8 Geometric mean (D) 0.9375 Harmonic mean (A) 0.9375 Maximizes Min (E) 0.9375 BIB ←

4.4. ANALYSIS OF CONNECTED OPTIMAL DESIGNS

To conclude this study, we observed two important methods which could be considered for the analysis of partially balanced incomplete block designs which are the Maximum Likelihood method and Restricted Maximum Likelihood method using the Mean Square Error as the yardstick for judgement.

Table 4.5.1: Performance of REML and ML Criterion in Incomplete Block Analysis (9,6,3,2) PBIB(2)

A=0.6667	D=0.7071068	E=0.5	SS=0.5
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Random variation RML	Random variation ML	Bahavior of randomness
Df SSq MSq F Pr(>F) trt 8 631.1 78.9 4779 1.095e-09 Re 4 0.01 0.002 Efficiency 0.6666667 Mean = 34.76729 CV = 0.1168622 AIC 62.39154 BIC 64.75824 -2 Res Log Likelihood -19.19577	Df SSq MSq F Pr(>F) trt 8 315.6 39.4 71683 4.865e-10 Re 4 0.002 0.001 Efficiency 0.6666667 Mean = 34.76729 CV = 0.06747032 AIC 25.9395514 BIC 36.6240125 -2 Res Log Likelihood -0.9697757	Min -0.0627 Max 0.09984 Var 0.00188 SS 0.03394 MD 0.03144

The design above is optimal in the sense that the optimality criteria used testified to it which are A= **0.6667**, D= **0.7071068**, E= **0.5** and SS=**0.5**. A- and D- optimality criteria values are greater than 0.5 with low SS optimality criterion value. It is observed that ML performed better than REML due to the fact that ML has the minimum MSE.

Table 4.5.2: Performance of REML and ML Criterion in Incomplete Block Analysis (12,8,3,2)PBIB(5)

A=0.5992218	D=0.6647605	E=0.3333333	SS=0.8484848
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$\mathcal{D}(12,8,3,2)PBIB(5)$	(6,10,11),(2, 7, 8),(3, 5,11),(2, 9,12), (1, 5,12),(1, 6, 8),(3, 4, 7),(4, 9,10)
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Random variation RML					Random variation ML					Behavior of randomness	
Df	SSq	MSq	F	Pr(>F)	Df	SSq	MSq	F	Pr(>F)	Min	-0.07201
trt 11	1605.1	145.9	61752	4.26e-12	trt 11	802.6	72.9	98804	1.32e-12	Max	0.086816
Re 5	0.01	0.002			Re 5	0.00	0.001			Var	0.002017
Efficiency	0.6470588				Efficiency	0.6470588				SS	0.048238
Mean	=33.86624				Mean	= 33.86624				MD	0.03804
CV	= 0.143537				CV	= 0.08023939					
AIC	85.73222				AIC	36.649452					
BIC	93.00582				BIC	54.320259					
-2 Res Log Likelihood	-27.86611				-2 Res Log Likelihood	-3.324726					

The design above is less optimal in the sense that the optimality criteria used testified to it which are A=**0.5992218**, D=**0.6647605**, E=**0.3333333** and SS=**0.8484848**. A- and D- optimality criteria values are greater than 0.5 with medium SS optimality criterion value while E- optimality criterion value is less than 0.5. It is observed that ML performed better than REML due to the fact that ML has the minimum MSE.

Table 4.5.3 Performance of REML and ML Criterion in Incomplete Block Analysis (18,12,3,2)PBIB(8)

A= 0.5323194	D= 0.6233753	E= 0.2397412	SS= 1.529412
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$\mathcal{D}(18,12,3,2)PBIB(8)$	(2, 4,17),(1, 8,12),(9,15,18),(3, 5,10), (2,11,13),(7,14,17),(4, 6,15),(5,11,16), (1,14,16),(7,10,18),(9,12,13),(3, 6, 8)
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Random variation RML	Random variation ML	Bahavior of randomness
Df SSq MSq F Pr(>F) trt 17 2275.5 133.85 49.45 1.192e-05 Re 7 18.95 2.707 Efficiency 0.6296296 Mean = 38.29848 CV = 4.295782 AIC 179.66409 BIC 198.36190 -2 Res Log Likelihood -68.83205	Df SSq MSq F Pr(>F) trt 17 1132.6 66.6 84.7 1.87e-06 Re 7 5.5 0.786 Efficiency 0.6296296 Mean = 38.29848 CV = 2.315508 AIC 219.58374 BIC 252.83764 -2 Res Log Likelihood -88.79187	Min -0.10546 Max 0.091914 Var 0.002936 SS 0.104581 MD 0.045721

Table 4.5.4: Performance of REML and ML Criterion in Incomplete Block Analysis (21,14,3,2)PBIB(3)

A=0.5147059	D= 0.6118411	E= 0.2642977	SS= 1.866667
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$\mathcal{D}(21,14,3,2)PBIB(3)$	(3,12,20),(7, 8,14),(2, 9,16),(13,17,18), (4,10,11),(5, 6,21),(3, 9,15),(1,15,19), (4, 7,19),(1 5,18),(8,20,21),(6,10,16), (11,12,17),(2,13,14)
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Random variation RML	Random variation ML	Bahavior of randomness
Df SSq MSq F Pr(>F) trt 20 2534.96 126.8 49664 < 2.2e-16 Re 8 0.02 0.003 Efficiency 0.625 Mean = 40.69617 CV = 0.124136 AIC 152.56171 BIC 177.63025 -2 Res Log Likelihood -52.28085	Df SSq MSq F Pr(>F) trt 20 1267.5 63.4 86911 < 2.2e-16 Re 8 0.01 0.001 Efficiency 0.625 Mean = 40.69617 CV = 0.06635373 AIC 59.048731 BIC 100.752802 -2 Res Log Likelihood -5.524366	Min -0.10393 Max 0.101144 Var 0.002197 SS 0.091455 MD 0.036081

Table 4.5.5: Performance of REML and ML Criterion in Incomplete Block Analysis (24,16,3,2)PBIB(13)

A=0.4938019	D=0.601624	E=0.1898657	SS=2.202899
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$\mathcal{D}(24,16,3,2)PBIB(13)$	(1, 6,24),(3, 9,18),(5, 10,21),(2, 7,12), (14,18,19),(1,10,17),(8,11,20),(4,13,16), (17,19,22),(7,13,15),(16,20,22),(4,21,23), (,9,15,24),(2, 6,11),(12,14,23),(3, 5, 8)
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Random variation RML	Random variation ML	Behavior of randomness
Df SSq MSq F	Df SSq MSq F	Min -0.10139
Pr(>F)	Pr(>F)	Max 0.097915
trt 23 3063.4 133.19 41960 < 2.2e-16	trt 23 1531.7 66.6 74596 < 2.2e-16	Var 0.002087
Re 9 0.03 0.003	Re 9 0.01 0.001	SS 0.098943
Efficiency 0.6216216	Efficiency 0.6216216	MD 0.034941
Mean = 42.91244	Mean = 42.91244	
CV = 0.1312919	CV = 0.06962824	
AIC 177.92794	AIC 75.26030	
BIC 209.73539	BIC 125.78273	
-2 Res Log Likelihood -61.96397	-2 Res Log Likelihood -10.63015	

Table 4.5.6: Performance of REML and ML Criterion in Incomplete Block Analysis (27,18,3,2)PBIB(8)

A=0.4818781	D=0.5945557	E=0.188967	SS=2.538462
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$\mathcal{D}(27,18,3,2)PBIB(8)$	(4, 9, 24),(5, 6, 7),(9, 22,23),(12,17,26), (11,22,25),(1, 6,23),(8, 13,27),(4,10,18), (2, 5, 16),(12,18,19),(2, 19,25),(14,17,21), (15,16,20),(1, 3,26),(7, 10,27),(14,15,24), (3,13,20),(8, 11,21)
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Random variation RML	Random variation ML	Behavior of randomness
Df SSq MSq F Pr(>F)	Df SSq MSq F Pr(>F)	Min -0.12957
trt 26 3834.0 147.46 67.62 3.383e-08	trt 26 1913.6 73.6 121.18 1.90e-09	Max 0.084308
Re 10 21.8 2.181	Re 10 6.07 0.607	Var 0.002236
Efficiency 0.6190476	Efficiency 0.6190476	SS 0.128564
Mean = 42.17153	Mean = 42.17153	MD 0.037843
CV = 3.501672	CV = 1.847998	
AIC 267.0303	AIC 320.0301	
BIC 305.9054	BIC 379.6996	
-2 Res Log Likelihood -103.5151	-2 Res Log Likelihood -130.0150	

The design above is non optimal in the sense that the optimality criteria used testified to it which are A=0.4818781, D=0.5945557, E=0.188967 and SS=2.538462. D-optimality criterion value is greater than 0.5 with high SS optimality criterion value while A- and E- optimality criteria values are less than 0.5. It is observed that ML performed better than REML due to the fact that ML has the minimum MSE

Table 4.5.7: Performance of REML and ML Criterion in Incomplete Block Analysis (16,8,4,2)Pbib(2)

A= 0.7142857	D= 0.7578583	E= 0.5	SS= 0.9
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$\mathfrak{D}(16,8,4,2)\text{PBIB}(2)$	(1, 5, 9,13),(2, 7, 8,15),(3,10,11,14),(4, 6,12,16), (2, 6,13,14),(5, 7,11,12),(1, 8,10,16),(3, 4, 9, 15)
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Random variation RML	Random variation ML	Behavior of randomness	
Df SSq MSq F Pr(>F)	Df SSq MSq F Pr(>F)	Min	-0.09338
trt 15 2246.13 149.7 90784 < 2.2e-16	trt 15 1123.1 74.9 121045 <2.2e-16	Max	0.087048
Re 9 0.01 0.002	Re 9 0.01 0.001	Var	0.002254
Efficiency 0.7142857	Efficiency 0.7142857	SS	0.069986
Mean = 33.3106	Mean = 33.3106	MD	0.038156
CV = 0.1219228	CV = 0.07466216		
AIC 82.81241	AIC 7.731961		
BIC 97.49159	BIC 35.580944		
-2 Res Log Likelihood -22.40620	-2 Res Log Likelihood 15.134019		

Table 4.5.8: Performance of REML and ML Criterion in Incomplete Block Analysis (20,10,4,2)PBIB(4)

A= 0.6785714	D= 0.7372383	E= 0.3454915	SS= 1.282895
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$\mathfrak{D}(20,10,4,2)\text{PBIB}(4)$	(1, 4, 6,16),(3, 4, 8,14),(6, 7,17,19),(1, 2,10,18), (5,12,15,16),(3, 7,15,18),(2, 9,13,17),(5, 8, 9,11), (12,13,14,20),(10,11,19,20)
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Random variation RML	Random variation ML	Behavior of randomness	
Df SSq MSq F Pr(>F)	Df Sum Sq Mean Sq F Pr(>F)	Min	-0.07877
trt 19 2831 149 71427 < 2.2e-16	trt 19 1415.5 74.5 97400 < 2.2e-16	Max	0.152973
Re 11 0.02 0.002	Re 11 0.01 0.001	Var	0.002223
Efficiency 0.7037037	Efficiency 0.7037037	SS	0.090812
Mean = 34.86016	Mean =34.86016	MD	0.034373
CV = 0.1310182	CV = 0.07933548		
AIC 106.45995	AIC 13.78794		
BIC 129.36180	BIC 52.63217		
-2 Res Log Likelihood -30.22998	-2 Res Log Likelihood 16.10603		

Table 4.5.9: Performance of REML and ML Criterion in Incomplete Block Analysis (24,12,4,2)PBIB(6)

A=0.6580381	D=0.7242399	E=0.375	SS=1.663043
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$\mathfrak{D}(24,12,4,2)PBIB(6)$	(2,10,13,17),(3,13,23,24),(7,10,18,22),(2,12,14,15), (3,9,11,20),(1,4,16,18),(5,6,12,19),(1,5,11,17), (8,16,19,23),(7,8,14,20),(4,9,15,21),(6,21,22,24)
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Random variation RML					Random variation ML				Behavior of randomness	
Df	SSq	MSq	F	Pr(>F)	Df	SSq	MSq	F	Min	-0.08058
trt 23	3738.6	162.6	66420	< 2.2e-16	trt 23	1869.3	81.28	91966	Max	0.126619
Re 13	0.0	0.002			Re 13	0.01	0.001		Var	0.001923
Efficiency	0.6216216				Efficiency	0.6216216			SS	0.090473
Mean	= 40.2932				Mean	= 40.2932			MD	0.030415
CV	= 0.1227756				CV	= 0.07377898				
AIC		131.71108			AIC		21.80031			
BIC		163.51853			BIC		72.32274			
-2 Res Log Likelihood		-38.85554			-2 Res Log Likelihood		16.09984			

Table 4.5.10:Performance of REML and ML Criterion in Incomplete Block Analysis (28,14,4,2)PBIB(7)

A=0.6391613	D=0.7141151	E=0.3048059	SS=2.041667
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$\mathfrak{D}(28,14,4,2)PBIB(7)$	(9,17,21,22),(7,12,19,23),(6,11,14,15),(4,5,10,23), (8,13,16,19),(3,4,24,26),(1,17,20,28),(1,5,8,14), (2,20,24,27),(16,18,22,25),(7,9,11,27),(12,25,26,28), (2,6,10,18),(3,13,15,21)
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Random variation RML					Random variation ML					Behavior of randomness	
Df	SSq	MSq	F	Pr(>F)	Df	Sum Sq	Mean Sq	F	Pr(>F)	Min	-0.08401
trt 27	3845	142.41	81388	< 2.2e-16	trt 27	1922.5	71.2	113943	< 2.2e-16	Max	0.129432
Re 15	0	0.002			Re 15	0.01	0.001			Var	0.001941
Efficiency	0.6923077				Efficiency	0.6923077				SS	0.107219
Mean	= 42.18138				Mean	=42.18138				MD	0.036257
CV	= 0.09916716				CV	= 0.05926374					
AIC		148.53830			AIC		9.186555				
BIC		189.83664			BIC		71.972458				
-2 Res Log Likelihood		-43.26915			-2 Res Log Likelihood		26.406722				

Table 4.5.11:Performance of REML and ML Criterion in Incomplete Block Analysis (27,27,3,3) PBIB(26)

A= 0.5999261	D= 0.6459757	E= 0.3519388	SS= 1.538462
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$\mathfrak{D}(27,27,3,3)$ PBIB(26)	(2,9,18),(1,16,20),(1,3,18),(6,10,12),(16,24,26), (3,5,21),(7,22,27),(5,23,27),(3,6,13),(19,24,25), (7,18,19),(1,10,27),(7,11,14),(4,15,21),(6,22,26), (13,17,25),(5,14,24),(2,8,26),(4,9,16),(8,11,21), (9,12,14),(4,17,22),(11,13,20),(12,15,19), (8,10,25),(2,17,23),(15,20,23)
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Random variation RML					Random variation ML					Behavior of randomness	
Df	SSq	MSq	F	Pr(>F)	Df	SSq	MSq	F	Pr(>F)	Min	-0.09297
trt 26	5396.4	207.6	103.38	< 2.2e-16	trt 26	3592.4	138.2	132.7	< 2.2e-16	Max	0.13043
Re 28	56.2	2.008			Re 28	29.2	1.041			Var	0.002136
Efficiency	0.6842105				Efficiency	0.6842105				SS	0.172074
Mean	= 42.74456				Mean	=42.74456				MD	0.036961
CV	= 3.31487				CV	= 2.387077					
AIC		423.9176			AIC		471.5312				
BIC		483.5871			BIC		543.3647				
-2 Res Log Likelihood		-181.9588			-2 Res Log Likelihood		-205.7656				

Table 4.5.12:Performance of REML and ML Criterion in Incomplete Block Analysis (15,20,3,4)PBIB(3)

A=0.68191	D=0.6974384	E=0.5282626	SS=0.3571429
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$\mathfrak{D}(15,20,3,4)$ PBIB(3)	(1,2,12),(3,12,13),(8,12,14),(1,10,11),(2,4,13), (1,14,15),(2,7,10),(7,13,15),(3,8,11),(1,6,9), (6,8,15),(5,7,11),(7,9,12),(2,5,8),(4,10,14), (5,6,13),(5,9,14),(3,6,10),(4,11,15),(3,4,9)
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Random variation RML					Random variation ML					Behavior of randomness	
Df	SSq	MSq	F	Pr(>F)	Df	SSq	MSq	F	Pr(>F)	Min	-0.07892
trt 14	4124.3	294.6	143912	<2.2e-16	trt 14	3093	220.9	166052	<2.2e-16	Max	0.10876
Re 26	0.1	0.002			Re 26	0.03	0.001			Var	0.001736
Efficiency	0.7241379				Efficiency	0.7241379				SS	0.106226
Mean	= 39.6746				Mean	= 39.6746				MD	0.033529
CV	= 0.1140381				CV	= 0.09194044					
AIC		149.80657			AIC		78.59711				
BIC		182.32650			BIC		116.29532				
-2 Res Log Likelihood		-56.90329			-2 Res Log Likelihood		-21.29856				

Table 4.5.13: Balance Incomplete Block Analysis (4,4,3,3) PBIB(1)

A= 0.8888889	D= 0.8888889	E= 0.8888889	SS=0
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$\mathfrak{D}(4,4,3,3)$	(1 2 4),(2 3 4),(1 3 4),(1 2 3)
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Generalized Least Square Method						Bahavior of randomness	
	Df	SSq	MSq	F	Pr(>F)	Min	-0.0792
trt	3	260.47	86.824	37956	2.6e-11	Max	0.084758
Res	5	0.01	0.002			Var	0.001688
						SS	0.018572
						MD	0.029051
Efficiency 0.8888889							
Mean = 20.24927							
CV = 0.2361947							

Table 4.5.14: Balance Incomplete Block Analysis $\mathfrak{DS}(8,28,2,7)$ PBIB(1)

A= 0.5714286	D= 0.5714286	E= 0.5714286	SS=0
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Least Square Method						Bahavior of randomness	
	Df	SSq	MSq	F	Pr(>F)	Min	-0.09963
trt	7	1714.2	244.89	219585	< 2.2e-16 ***	Max	0.0878
Re	21	0.0	0.00			Var	0.001473
						SS	0.081706
						MD	0.02926
Efficiency 0.5714286							
Mean = 40.85361							
CV = 0.08174294							

In this research work, three major designs and their performances were checked. The first design is considered to be an optimal design due to A-, D- and E- optimality criteria values greater than 0.5 with low SS value. The second design is considered to be a less optimal design due to one of the A-, D- and E- optimality criteria value less than 0.5 having others medium SS value. The third design is considered to be a non optimal design due to two or all of the A-, D- and E- optimality criteria values less than 0.5 with high SS value.

The graphical representation of analyses of the simulated data under Partially Balanced Incomplete Block Designs were also observed comparing ML and REML methods using P-values, AIC and BIC. The analyses were shown below.

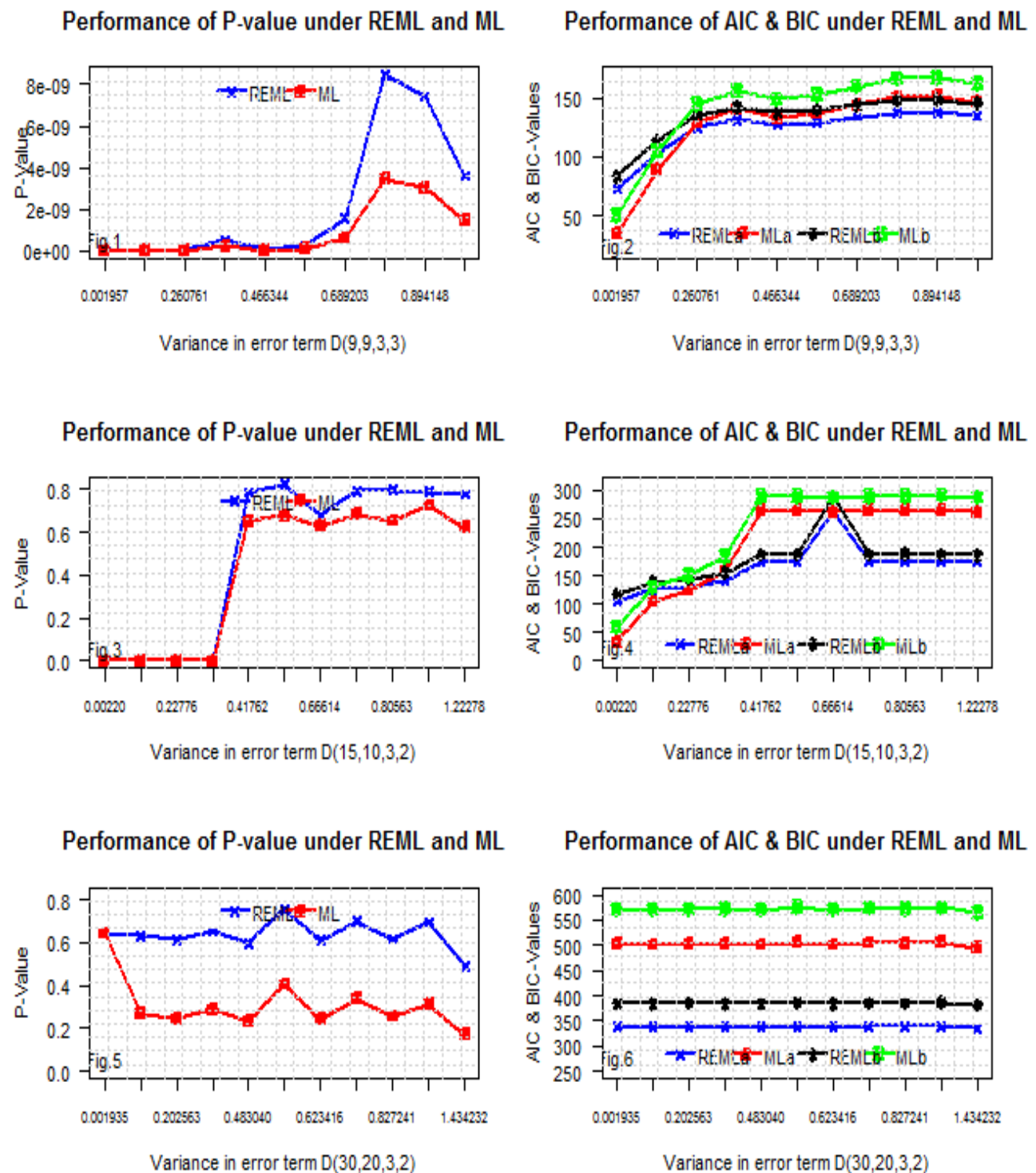


Fig. 4.5.1

Table 4.5.15 MSE Criterion Under RML and ML Approaches for Optimal Design (9,9,3,3) PBIB(2)

A=0.727272	D=0.7377879	E=0.66667	SS=0.166667
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Variation	RML	ML
0.001957	0.002	0.001
0.134975	0.035	0.019
0.260761	0.31	0.174
0.31054	0.604	0.34
0.466344	0.401	0.223
0.527299	0.485	0.27
0.689203	0.737	0.409
0.760583	1.073	0.596
0.894148	1.105	0.614
1.218913	0.84	0.467

ML performs better than **RML** under optimal design due to the fact that it has smaller **MSE** regardless of the amount of variation in the experiment.

Table 4.5.16: MSE Criterion Under RML and ML Approaches for Less Optimal Design (15,10,3,2) PBIB(3)

A=0.545	D=0.64117	E=0.333	SS=1.190476
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Variation	RML	ML
0.0022	0.002	0
0.1225	0.05	0.015
0.22776	0.141	0.042
0.38947	0.749	0.225
0.41762	148.01	46.69
0.51925	147.494	46.321
0.66614	46.32	44.384
0.70578	150.467	48.11
0.80563	149.034	47.086
0.94493	170.04	54.961
1.22278	137.094	43.334

ML performs better than **RML** under less optimal design due to the fact that it has smaller **MSE** regardless of the amount of variation in the experiment.

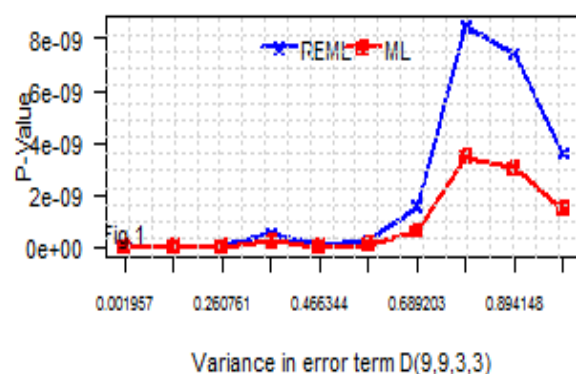
Table 4.5.17: MSE Criterion Under RML and ML Approaches for Non-Optimal Design (30,20,3,2) PBIB(15)

A=0.4706192	D=0.5885551	E=0.1774279	SS=2.873563
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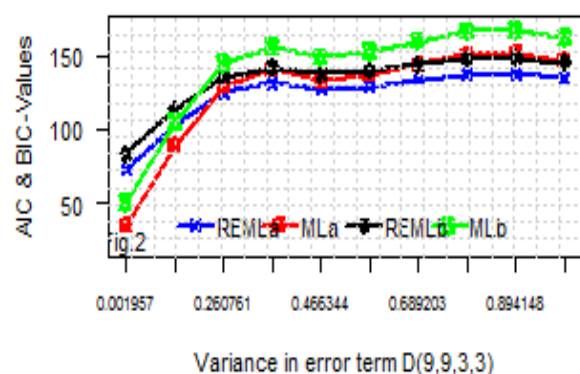
Variation	RML	ML
0.001935	175.11	52.585
0.230646	173.6	52.2
0.301957	173.2	51.9
0.48304	176.8	53.3
0.520981	172.38	51.6
0.623416	186.5	56.08
0.795896	176.1	52.9
0.827241	172.7	51.6
0.961578	156.84	46.79
1.434232	180.74	54.9
6.381211	165.64	49.635

ML performs better than **RML** under non-optimal design due to the fact that it has smaller **MSE** regardless of the amount of variation in the experiment.

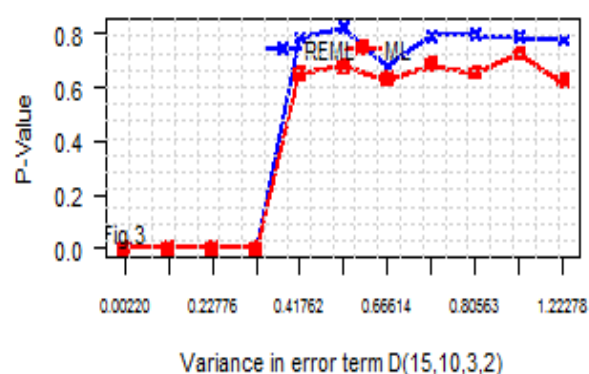
Performance of P-value under REML and ML



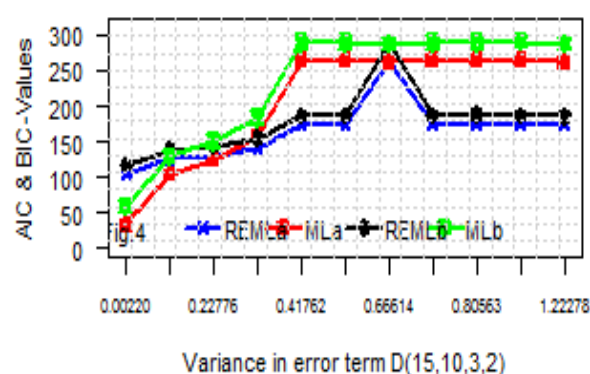
Performance of AIC & BIC under REML and ML



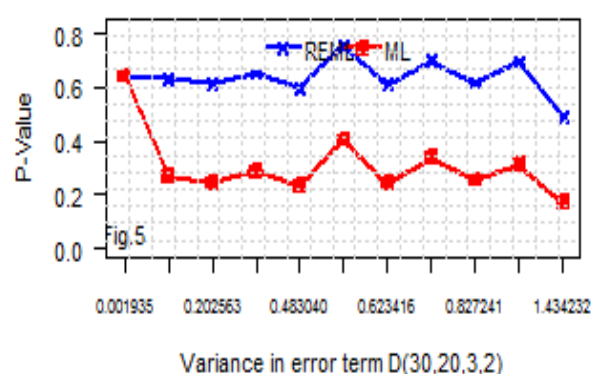
Performance of P-value under REML and ML



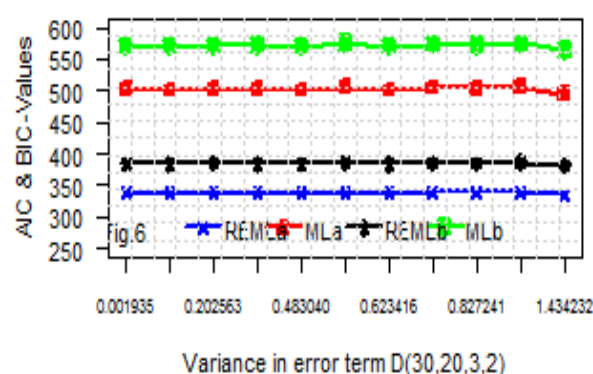
Performance of AIC & BIC under REML and ML



Performance of P-value under REML and ML



Performance of AIC & BIC under REML and ML



In the analysis, it is discovered that **ML** criterion detect true effect faster than **REML**, indicated by the P-value on Table 1, Table 2 and Table3 even as the variance in error term increases. This can be graphically seen in Fig. 1, Fig.3 and Fig. 5. The error term variance has different effect on optimal, less optimal, and non optimal designs. In the presence of true effect for the optimal design, even as the variance in the error term increases, it is more likely to detect it, observed in Fig. 1. In the presence of true effect for a less optimal design, if increase in the error term variance gets to a point, true effect would no longer be detected,

In this study, it is noticed that optimal design helps to control both type I and type II errors by minimizing **MSE** appropriately, determining an appropriate cost effective sample sizes with high precision, are the benefits of the experimenter. Through these, quality conclusions would be made on well planned experiments. Even at increase of error in an experiment, optimal design has the tendency to detect true effect.

This study also testify to the effectiveness of using A-, D-, E- and SS- optimality criteria in selecting optimal designs.

In this work, we are able to solidify that there is high tendency to find significant difference when it is actually there for an optimal design than for non- optimal design according to A-, D- E- and SS- optimality criteria. If a researcher fails to find an optimal design, larger sample size would be required to detect true effect by increasing replications. Less optimal design may lack capability to minimize type I and type II errors thus causing confusion in conclusion, is another problem a researcher may face.

Huge error in experimental procedures affects the outcome of the experiment negatively (Gauch and Zobel 1997), most especially the less optimal and non optimal designs. Experimenter should know that as the inflow of error increases, it is more likely to commit type II error.

REML and **ML** criteria were considered in the analyses above, **ML** often get significant difference faster than **REML**. Often **REML** are considered better than **ML** using **AIC** and **BIC**. Due to inconsistency showed in Fig.4, it would be better to use **ML** when fixed effects only are for consideration since **REML** was developed for Mixed Models Karin Meyer (1987).

Note:

1. As optimality criteria values Type I and E- increase with respective decrease in SS optimality criterion value, better precision would be recorded.
2. Low E- optimality criterion value even when A- and D- optimality criteria values are high will produce less precision with respective high SS optimality criterion value.
3. With Type I, E- and SS optimality criteria values, type I and type II errors can be controlled which also safe difficulties in finding prior information to determine the sample size.
4. It is important to recognize that there is difference between practical significance and statistical significance. ML and REML can be compared to justify the better based on the practical knowledge of the experiment.
5. Experimenter should know that as the inflow of error increases, it is more likely to commit type II error.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1 INTRODUCTION

In this study we presented Balanced Incomplete Block Designs (BIBD), Partially Balanced Incomplete Block Designs with two associates (PBIB/2), Partially Balanced Incomplete Block Designs with 3 or more associates (PBIB/3 or more) with the following constraints

(i) $2 \leq r \leq t$, (ii) $6 \leq t \leq 15$, (iii) $2 \leq k \leq t/2$,

(iv) $\lambda = r(k - 1)/(t - 1)$

A-, D- and E-criteria values were computed with corresponding sum of squares to identify designs that are optimal.

5.2 FINDINGS

From the new class of designs constructed with varying values of replications. It was discovered that as the number of replication increases **Type I**, and **E**- criteria values increases, with corresponding decrease in sum of squares respectively which is, an indication of optimal designs.

Amongst the class of design $\mathcal{D}(t, b, k, r)$ with fixed block sizes and the minimum number of replications, we look at the **Type I** and **E**- efficiency of the designs. When $r = t - 1$ BIB design is attained except for $t=15$, with **Type I** and **E**- criteria values equal and minimum sum of squares zero when $r = t - 2$ optimal designs with higher values of **Type I** criteria values were obtained provided t , k and r are even.

Fig. 4.2.1 showed the relationship among the three optimality criteria and the sum of squares for all the designs generated, we observed that as **Type I** and **E**- optimality criteria values increase the sum of squares decreases.

Fig. 4.2.2 revealed Balanced Incomplete Block Designs. They are found to be optimal due to the fact that they produced **Type I** and **E**- optimality criteria values higher than 0.5 and their corresponding Sum of Squares are zero.

Fig. 4.2.3 revealed that as the block size or/and replication increase there is an increase in the **Type I** and **E**- optimality criteria values. Among the selected optimal designs, only three of the designs are Partially Balanced Incomplete Block Designs with 3 or more associate classes others are Partially Balanced Incomplete Block Designs with 2 associate classes. It would be recalled that design having maximum

Type I and **E**- optimality criteria values indicates optimal designs provided the $\sum(e_i - \bar{e})^2$ is minimum. Designs $\mathcal{D}(12,30,4,10)$, $\mathcal{D}(12,20,6,10)$, $\mathcal{D}(15,42,5,14)$ have the highest **Type I** and **E**- values with their sum of squares close to zero which indicates optimal designs but these designs are partially Balanced Incomplete Block Designs with three or more associates and they utilize a large number of units.

Fig. 4.2.4 revealed that **Type I** and **E**- optimality criteria values are closer, and the designs with high optimality criteria values also have their corresponding sum of squares closer to zero which also indicates optimal designs.

Fig. 4.2.5 shows partially Balanced Incomplete Block Designs with three or more associate classes which reveal that as the number of replication increases we achieved optimal designs.

In this research work, the analysis of Partially Balanced Incomplete Block Designs were observed where moderate P-values suggest little signal against the null hypothesis whereas small P-values suggest that the null hypothesis is false. R package was used for the simulation and the analysis.

In the analysis, it is discovered that **ML** criterion detect true effect faster than **REML**, indicated by the P-value on Table 4.4.1, Table 4.4.2 and Table 4.4.3 even as the variance in error term increases. This can be graphically seen in Fig. 4.4.1 (Fig. 1, Fig.3 and Fig. 5). The error term variance has different effect on optimal, less optimal, and non optimal designs. In the presence of true effect for optimal designs, even as the variance in the error term increases, it is more likely to detect it, observed in Fig. 4.4.1 (Fig. 1.) In the presence of true effect for a less optimal design, if increase in the error term variance gets to a point, true effect would no longer be detected, noticed in Fig. 4.4.1 (Fig.3). It is unlikely to detect true effect under non optimal designs, Fig. 4.4.1 (Fig. 5) shows this.

Akaike Information Criterion (**AIC**) and Bayesian Information Criterion (**BIC**) are useful for model selection. The performances of these two criteria were also observed for the three designs mentioned above. **REML** criterion produces lower **AIC** and **BIC** values for the three designs observed in Fig. 4.4.1 (Fig. 2, Fig. 4 and Fig. 6), but only consistent on average with optimal design even as the error term in variance increases. As error term variance increases the **AIC** and **BIC** increase on a reasonable average. **ML** criterion produces higher **AIC** and **BIC** values for the three designs, but only consistent on average with optimal design and partially for less optimal design even as the error term variance increases. As error term variance increases the **AIC** and **BIC** increase on a reasonable average seen in Fig. 4.4.1 (Fig. 2) for optimal designs. When

the variance in error term increases to a point for less optimal design, the **AIC** and **BIC** reach their peak noticed in Fig. 4.4.1 (Fig. 4). Both **REML** and **ML** criteria produce constant values for **AIC** and **BIC** even as the variance in error term increases but each having unique value.

We are able to construct symmetric and anti-symmetric designs and their corresponding complementary designs. A design $d(t, b, k, r)$ having $t > b$ and $r < k$ with the constraint that $tr = bk$ has the same E-optimal value with its complementary design having $b > t$ and $k < r$. In this study, it is observed that **Type I** optimality criteria values for a design having $t > b$ and $r < k$ is higher than its complementary design having $b > t$ and $k < r$ with an impression that unnecessary increase in blocking does not improve the precision of an experiment. The concept of E-optimality can help in the construction of a better design for a particular anti-symmetric design. Any design $d(t = b, k = r)$ has the same **Type I** and **E**- optimal criteria values with its complementary design and it is known to be symmetric design. We also observed that increase in replications or block sizes or both in a design $d(t, b, k, r)$ improve the frequency distribution of the concurrence matrix.

Algebraic algorithm developed had been able to search for competing anti-symmetric designs with varying block and block sizes and their competing complementary designs for anti-symmetric designs.

The algorithm also presented optimal designs that optimizes both block size and the replication

5.3 CONCLUSION

In conclusion the results revealed that as the number of replication or/and block sizes increase optimal designs were identified. For some design parameter sets where **Type I** optimality values are closer to each other, it can be said that those design parameter sets are **Type I** optimal, which is an indication that most of the designs generated satisfied **Type I** optimality criteria. Among the selected good designs Partially Balanced Incomplete Block Designs with 3 or more associate classes are the best based on some well-defined optimality criteria but they utilize large number of units.

In this study, we presented Complementary designs for Symmetric and Anti-symmetric Partially Balanced Incomplete Block Designs with **Type I** and **E**-optimality criteria values.

Table 4.3.1 shows that the same number of efficiency factors (that is, eigenvalues), **Type I** and **E**- optimality criteria values and frequency distribution in the concurrence matrix for symmetric and their complementary designs are observed.

Table 4.3.2 shows that different number of efficiency factors, **Type I** and **E**- optimality criteria values and frequency distribution in the concurrence matrix for anti-symmetric and their complementary designs are observed.

Note:

1. As optimality criteria values **Type I** and **E**- increase with respective decrease in **SS**- optimality criteria values, better precision would be recorded.
2. Low **E**- optimality criterion value even when **Type I** optimality criteria values are high will produce less precision with respective high **SS** optimality criterion value.
3. With **Type I**, **E**- and **SS** optimality criteria values, type I and type II errors can be controlled which also safe difficulties in finding prior information to determine the sample size.
4. It is important to recognize that there is difference between practical significance and statistical significance. **ML** and **REML** can be compared to justify the better based on the practical knowledge of the experiment.
5. Experimenter should know that as the inflow of error increases, it is more likely to commit type II error.

A design that optimizes block size is the optimal design and when BIB design is found under competing incomplete block designs, it is unlikely to be the optimal except it optimizes both replication and block sizes.

A design that optimizes both block size and replication is the optimal design.

To have a design that optimize both block size and replication among competing PBIB designs under constant number of units in order to have optimal design, $t > b$ and $r < k$ such that $r \geq r^* \forall r^* \in R$ and $k \geq k^* \forall k^* \in K$.

5.4 Contribution to Knowledge

1. Minimizing replications can result in optimal design when block size is optimized;
2. Introduction of new optimality criterion which is **SS criterion**; and

3. BIB design might not be the optimal design when the block size and replications are not optimized.

5.6 Recommendation

Competing PBIB designs could be extended to augmented designs.

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APPENDIX A:

The R program used in the construction of new classes of optimal designs with the following constraint $2 \leq r \leq t$, $6 \leq t \leq 15$, and $2 \leq k \leq t/2$.

This a programme for searching new class of optimal Incomplete Block Designs of size (t, b, k, r) given t, k is an integer between 2 and $t/2$

```
library(crossdes)
```

```
Opimal<- function(x) {
```

```
  gm<-exp(mean(log(x)))
```

```
  hm<-1/mean(1/x)
```

```
  mn<-min(x)
```

```
  ss<-sum((x-mean(x))^2)
```

```

cat("Sum of Squares(Dispersion)",ss,"\n")

cat("Geometric mean (D optimal)",gm,"\n")

cat("Harmonic mean (A optimal)",hm,"\n")

cat("Maximizes Min (E optimal)",mn,"\n")


d1=find.BIB(t,b,k) #Generating IBD or PBIBD

mat=isGYD(d1,tables=TRUE) # Incidence and concurrence matrices

conc=mat$`Concurrence w.r.t. rows`;conc;class(conc)

I=matrix(,byrow=T,nrow=t);r;k

A=r*I-(1/k)*conc #Information matrix

C=A*(1/r);canonicalEF=eigen(C)

canonicalEF$Eigenvalue=canonicalEF$values;EigenValue=Eigenvalue[-t]

Optimal(EigenValue)

```

APPENDIX B:

A R programme for checking the performance of ML and REML criteria in incomplete block analysis.

Data were simulated using mean model defined as

$$y_{ij} = \mu_{ij} + e_{ij}; i = 1, 2, \dots, t, j = 1, 2, \dots, b$$

The addition of the error term e_{ij} to the sum of treatment effect, block effect and the grand mean varies with changes in error term variance such that $0 \leq var \leq 2$. effect of these error term variance were used to classify the optimal designs into optimal, less optimal and non-optimal designs

```

library(agricolae)

#####Experiment D(9,9,3,3)#####

#####

trt<-1:9;t <- length(trt)

# size block k and Blocks s

k<-3;s<-t/k ;r <- 3

outdesign<- design.alpha(trt,k,r,serie=2)

book<-outdesign$book

plots<-book[,1] ;book

bb=c(rep(7,3),rep(22,3),rep(11,3),rep(5,3),rep(35,3),rep(54,3),rep(19,3),rep(0,3),rep(1
7,3))

yt=c(3,23,20,6,10,11,16,4,19,6,20,4,19,23,11,16,3,10,16,23,6,19,10,20,11,4,3)

yb=yt+bb

r=rnorm(27,0,0.05);var(r)

r1=c(0.019734007,-0.078055964,0.005105432,-0.085700523,-0.001076978,-
0.029060119,
0.034833202,-0.004150924,-0.046728399,0.017982175,-0.050106003,-0.044840080,-
0.010222431,
-0.041503852,-0.079065775,0.013279383,0.072987729,-0.002298586,-0.028706615,-
0.049942896,
-0.134245747,-0.033331955,0.003044409,0.007719521,-0.087817800,-0.023520252,-
0.015494703)

min(r1);max(r1);var(r1);sum(r1^2);sum(abs(r1-mean(r1)))/27

book$y1=yb+r1

fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

```

```

fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="ML",test="lsd",alpha=0.05);fit

r=rnorm(27,0,0.3);var(r)

r2=c(-0.453990078,0.208641065,0.280299223,-0.338859811,-0.225797564,-
0.240548531,
-0.761440771,-0.025468520,-0.152054771,-
0.094372503,0.798598765,0.309279159,0.167958373,
0.776253387,-0.001571695,-0.009944148,-0.426842509,-0.019278869,-
0.244544650,-0.226832783,
-0.553033002,0.103499008,-0.054570776,0.454151464,0.191427991,0.352279792,-
0.049262065)

min(r2);max(r2);var(r2);sum(r2^2);sum(abs(r2-mean(r2)))/27

book$y2=yb+r2

fit=PBIB.test(book$block,book$trt,book$replication,book$y2,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

fit=PBIB.test(book$block,book$trt,book$replication,book$y2,3,
method="ML",test="lsd",alpha=0.05);fit

r=rnorm(27,0,0.4);var(r)

r3=c(0.67134928,-0.61432620,-0.47649255,-0.37266319,-0.33207766,-0.34188189,-
0.18408969,
0.07314434,-0.51102735,0.23304864,-0.56341759,-0.40708671,-
0.49710022,0.98032652,
0.09159056,-
0.72966805,0.85037220,0.38258237,0.53830806,0.41669865,0.02568358,-
0.37670424,
-0.32167204,0.61500282,0.10191350,-0.65625849,-0.73716312)

```

```

min(r3);max(r3);var(r3);sum(r3^2);sum(abs(r3-mean(r3)))/27

book$y3=yb+r3

fit=PBIB.test(book$block,book$trt,book$replication,book$y3,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

fit=PBIB.test(book$block,book$trt,book$replication,book$y3,3,
method="ML",test="lsd",alpha=0.05);fit

```

```

r=rnorm(27,0,0.5);var(r)

r4=c(0.60914919,-0.84153061,-0.75211097,0.56107087,-
0.01622987,0.40153356,0.09074914,
-0.02258729,0.29132092,-0.28489198,0.69006928,-0.08680016,-
0.24724648,0.44200846,
-0.65747600,0.69480279,-0.60024187,0.40091816,-0.61658441,0.84832563,-
0.11070842,
0.15008746,-0.01534440,-0.82208072,0.32897773,0.46348604,-1.21314305)

min(r4);max(r4);var(r4);sum(r4^2);sum(abs(r4-mean(r4)))/27

book$y4=yb+r4

fit=PBIB.test(book$block,book$trt,book$replication,book$y4,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

fit=PBIB.test(book$block,book$trt,book$replication,book$y4,3,
method="ML",test="lsd",alpha=0.05);fit

```

```

r=rnorm(27,0,0.6);var(r)

r5=c(0.18597367,-0.30762058,-0.38980141,-0.87055124,-1.20948136,-
0.23316474,0.11375219,

```

```

-0.22185615,0.81010436,0.46682237,0.33180890,-0.32287737,-
0.08589421,0.52861742,0.29956080,

0.42372452,0.52688621,0.95789643,1.49885495,1.01183603,-1.23484848,-
0.74514822,0.29742627,

-0.63733691,0.21059407,-0.80161005,0.41615838)

min(r5);max(r5);var(r5);sum(r5^2);sum(abs(r5-mean(r5)))/27

book$y5=yb+r5

fit=PBIB.test(book$block,book$trt,book$replication,book$y5,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

fit=PBIB.test(book$block,book$trt,book$replication,book$y5,3,
method="ML",test="lsd",alpha=0.05);fit


r=rnorm(27,0,0.7);var(r)

r6=c(-0.45492662,0.45566578,-0.43476620,-0.33167715,0.28126170,-1.96470631,-
1.07931909,

-0.13319733,-0.81641895,-0.34270337,-
0.07947526,0.07452153,0.97905401,0.29661871,

-0.20838473,-1.06218295,0.57898116,0.23720011,-0.04289053,-0.05655385,-
0.31864725,

-1.48603747,1.24491518,0.93725818,-0.24212153,-0.22571816,0.38998030)

min(r6);max(r6);var(r6);sum(r6^2);sum(abs(r6-mean(r6)))/27

book$y6=yb+r6

fit=PBIB.test(book$block,book$trt,book$replication,book$y6,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

fit=PBIB.test(book$block,book$trt,book$replication,book$y6,3,
method="ML",test="lsd",alpha=0.05);fit

```

```

r=rnorm(27,0,0.8);var(r)

r7=c(-0.30666967,-0.81504299,0.96250953,-1.32636228,-
0.25447847,2.03218766,0.88224090,
-0.34679658,1.08190375,1.06203098,-0.27148285,-0.21448309,-
0.11734131,0.01685678,
0.39092815,-
0.62611962,0.67932612,0.69002186,0.39371885,1.16445827,0.58386958,-
0.63766454,
0.18304652,-0.91208127,-0.65874101,-1.35152500,-0.45493291)
min(r7);max(r7);var(r7);sum(r7^2);sum(abs(r7-mean(r7)))/27
book$y7=yb+r7
fit=PBIB.test(book$block,book$trt,book$replication,book$y7,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11
fit=PBIB.test(book$block,book$trt,book$replication,book$y7,3,
method="ML",test="lsd",alpha=0.05);fit

```

```

r=rnorm(27,0,0.9);var(r)

r8=c(1.40127653,0.21042654,-1.45107154,-0.13074089,1.63898671,-0.02572420,-
0.45555325,
-0.55482760,-0.18814329,-0.57546769,-0.61968605,-
0.82550307,0.01602786,1.45014276,
-0.11615147,-0.79919053,-
1.57260957,1.15777950,0.45380717,0.83513058,0.10072780,0.17642731,
-0.64798642,0.31585702,1.66031482,-0.18267796,-0.45982094)
min(r8);max(r8);var(r8);sum(r8^2);sum(abs(r8-mean(r8)))/27
book$y8=yb+r8
fit=PBIB.test(book$block,book$trt,book$replication,book$y8,3,

```

```

method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

fit=PBIB.test(book$block,book$trt,book$replication,book$y8,3,
method="ML",test="lsd",alpha=0.05);fit

r=rnorm(27,0,1.0);var(r)

r9=c(-0.187709980,-0.340749022,0.561047190,-0.277636909,-
0.134233680,0.549726482,
-1.432270602,-1.382251062,-0.080419340,-1.398902151,1.559391539,0.431182590,-
0.482302924,
-0.842794123,0.067118287,1.998665218,-1.104056764,-
0.074736080,0.008383285,0.070659196,
-0.345565146,0.356752810,-1.550914503,-1.777539516,-0.640592163,-
0.615258405,1.490055920)

min(r9);max(r9);var(r9);sum(r9^2);sum(abs(r9-mean(r9)))/27

book$y9=yb+r9

fit=PBIB.test(book$block,book$trt,book$replication,book$y9,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

fit=PBIB.test(book$block,book$trt,book$replication,book$y9,3,
method="ML",test="lsd",alpha=0.05);fit

r=rnorm(27,0,1.0);var(r)

r10=c(1.91528578,0.53900547,-0.24597952,0.05789964,-2.13372793,0.75565845,-
0.74717555,
0.98363398,0.37172277,-0.28464279,-0.10126243,-3.17061561,-1.64127110,-
0.02452360,
1.31298533,-0.80616549,0.13527355,-0.87741502,-
1.80475227,0.50344800,0.03191705,

```

```
-0.30498887,-1.15576429,-1.00291708,-0.25474530,-1.02585602,-1.32014296)
```

```
min(r10);max(r10);var(r10);sum(r10^2);sum(abs(r10-mean(r10)))/27
```

```
book$y10=yb+r10
```

```
fit=PBIB.test(book$block,book$trt,book$replication,book$y10,3,
```

```
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11
```

```
fit=PBIB.test(book$block,book$trt,book$replication,book$y10,3,
```

```
method="ML",test="lsd",alpha=0.05);fit
```

```
#####Experiment D(15,10,3,2)#####
```

```
#####
```

```
trt<-1:15;t <- length(trt)
```

```
# size block k and Blocks s
```

```
k<-3;s<-t/k ;r <- 2
```

```
outdesign<- design.alpha(trt,k,r,serie=2)
```

```
book<-outdesign$book
```

```
plots<-book[,1] ;book
```

```
bb=c(rep(7,3),rep(22,3),rep(11,3),rep(5,3),rep(35,3),rep(54,3),rep(19,3),rep(0,3),
```

```
rep(17,3),rep(39,3))
```

```
yt=c(20,19,4,10,0,30,6,3,8,26,23,2,27,11,16,16,3,19,30,23,11,20,2,10,6,27,26,4,8,0)
```

```
yb=yt+bb
```

```
r=rnorm(30,0,0.05);var(r)
```

```
r1=c(0.040280172,0.027200785,0.079238657,-
```

```
0.021627851,0.042470104,0.019933565,-0.033382471,
```

```

0.049976133,0.048362029,-0.011066508,0.048265133,-
0.078425293,0.013223891,0.039930154,

-0.073785656,-
0.074015380,0.023696204,0.005541606,0.049753775,0.009946163,0.064323704,

-0.045917503,-0.018680228,0.040596203,-0.082191336,-0.063146879,0.045109753,-
0.007832629,

0.060129885,-0.002317256)

min(r1);max(r1);var(r1);sum(r1^2);sum(abs(r1-mean(r1)))/30

book$y1=yb+r1

fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="ML",test="lsd",alpha=0.05);fit

r=rnorm(30,0,0.3);var(r)

r2=c(0.04859497,0.49465340,0.27801089,-0.01516754,0.07276114,0.24257222,-
0.23224101,

-0.29141651,0.39645506,0.27018414,-
0.31003484,0.03121966,0.42144824,0.56201473,0.07535079,

-0.23108899,-0.29704481,0.51469947,-0.12529470,0.45876478,0.26725608,-
0.56603090,0.12833014,

-0.08901919,-0.64236377,-0.55021112,0.49680685,0.37410879,-0.32399936,-
0.08134205)

min(r2);max(r2);var(r2);sum(r2^2);sum(abs(r2-mean(r2)))/30

book$y2=yb+r2

fit=PBIB.test(book$block,book$trt,book$replication,book$y2,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

```

```

fit=PBIB.test(book$block,book$trt,book$replication,book$y2,3,
method="ML",test="lsd",alpha=0.05);fit

r=rnorm(30,0,0.4);var(r)

r3=c(0.22322581,0.61282904,-
0.37547324,0.31444868,1.00718966,0.15962962,0.27163499,
0.66525392,0.41662424,-0.19496632,-0.33843111,-0.40245510,0.49484231,-
0.07257704,
0.24127493,-0.61031477,0.09101484,0.10881549,0.44360269,-
0.96352065,0.77966947,-0.45207511,
-0.63046055,-0.34642292,0.14903988,-0.22247861,-0.17748090,-0.48262311,-
0.19770572,0.63919236)

min(r3);max(r3);var(r3);sum(r3^2);sum(abs(r3-mean(r3)))/30

book$y3=yb+r3

fit=PBIB.test(book$block,book$trt,book$replication,book$y3,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

fit=PBIB.test(book$block,book$trt,book$replication,book$y3,3,
method="ML",test="lsd",alpha=0.05);fit

r=rnorm(30,0,0.6);var(r)

r4=c(0.71746838,-1.11740853,0.24616280,-1.02390751,-0.18362894,0.14508595,-
0.15690221,
-0.41256388,0.75699843,-0.74980163,-0.15967783,-0.57219885,0.89235457,-
0.89359716,
0.16097004,-0.31990375,0.65833888,-0.15287711,0.41802071,-
0.82831468,1.13004682,0.02709239,
0.40381238,-0.87393885,0.50723159,0.46242330,-0.44351480,-0.46705117,-
0.80376789,0.29806150)

```

```

min(r4);max(r4);var(r4);sum(r4^2);sum(abs(r4-mean(r4)))/30

book$y4=yb+r4

fit=PBIB.test(book$block,book$trt,book$replication,book$y4,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

fit=PBIB.test(book$block,book$trt,book$replication,book$y4,3,
method="ML",test="lsd",alpha=0.05);fit


r=rnorm(30,0,0.6);var(r)

r5=c(0.62101943,-1.52984195,0.75363033,-0.27716058,0.24516282,-1.21399727,-
0.78179233,
0.07076150,1.26247227,0.18697399,0.28426568,-0.62403152,-
0.08342923,0.25451993,
1.00324809,0.31607195,-0.91572576,0.75253066,-
0.11658201,0.11849640,0.09820287,
-0.20841024,-0.36026528,-0.47773063,-0.74352216,0.01789323,-0.20850110,-
0.32379434,-0.04332297,-1.02766353)

min(r5);max(r5);var(r5);sum(r5^2);sum(abs(r5-mean(r5)))/30

book$y5=yb+r5

fit=PBIB.test(book$block,book$trt,book$replication,book$y5,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

fit=PBIB.test(book$block,book$trt,book$replication,book$y5,3,
method="ML",test="lsd",alpha=0.05);fit


r=rnorm(30,0,0.7);var(r)

r6=c(0.35994799,0.35608384,-0.99805138,-0.02420185,0.39307610,-1.36414952,-
1.27670039,

```

```

-0.02737906,-1.33383817,0.43310305,-0.05381123,0.12435699,-
0.13499748,0.78161326,

0.74570522,-1.13484944,-0.19827290,-0.16160352,0.26666542,0.26075504,-
0.17675974,

0.27534445,1.34899190,0.31209308,-0.93827563,0.97687128,-
0.72765844,0.63860773,

-0.80030392,-0.53616094)

min(r6);max(r6);var(r6);sum(r6^2);sum(abs(r6-mean(r6)))/30

book$y6=yb+r6

fit=PBIB.test(book$block,book$trt,book$replication,book$y6,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

fit=PBIB.test(book$block,book$trt,book$replication,book$y6,3,
method="ML",test="lsd",alpha=0.05);fit

r=rnorm(30,0,0.8);var(r)

r7=c(-0.10515582,-0.55764638,-0.09483567,-1.22353113,0.34561187,-
0.66799190,0.52451565,

-0.88816776,0.60219625,0.28024984,0.70028405,-
0.77226565,0.73730972,1.67716817,

0.48938989,-1.56972799,-1.12751504,-1.00525881,-0.57285182,0.63258375,-
0.20979671,

1.05739499,1.03914274,-0.02117888,1.19953998,-0.35464589,-0.29103523,-
0.88514289,

0.03399261,0.76688566)

min(r7);max(r7);var(r7);sum(r7^2);sum(abs(r7-mean(r7)))/30

book$y7=yb+r7

fit=PBIB.test(book$block,book$trt,book$replication,book$y7,3,

```

```
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11
```

```
fit=PBIB.test(book$block,book$trt,book$replication,book$y7,3,
```

```
method="ML",test="lsd",alpha=0.05);fit
```

```
r=rnorm(30,0,0.9);var(r)
```

```
r8=c(-0.39427458,0.06678556,0.67961630,-0.44101216,1.34351593,-  
0.09995444,0.57600049,
```

```
0.48471587,-0.21751833,0.51287156,-1.39414440,-1.71631889,-  
0.95794324,1.36670593,
```

```
0.80165277,0.81846601,-0.10975906,-0.49050267,1.40347675,-0.57589710,-  
0.33176749,
```

```
0.30774603,-
```

```
0.36137588,0.60769085,0.73642505,0.64252813,0.97117590,1.04084664,-  
1.48300952,
```

```
0.29593519)
```

```
min(r8);max(r8);var(r8);sum(r8^2);sum(abs(r8-mean(r8)))/30
```

```
book$y8=yb+r8
```

```
fit=PBIB.test(book$block,book$trt,book$replication,book$y8,3,
```

```
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11
```

```
fit=PBIB.test(book$block,book$trt,book$replication,book$y8,3,
```

```
method="ML",test="lsd",alpha=0.05);fit
```

```
r=rnorm(30,0,1);var(r)
```

```

r9=c(-0.91021729,0.58088736,0.87439645,-0.90257312,-0.54123645,0.30844400,-
0.90294503,
-1.73819622,-0.05463297,-0.93696501,0.39943194,-
1.60070987,0.14506174,0.51462591,
1.14998107,0.29536657,-0.79098869,1.52228986,-0.08600261,-0.34233719,-
0.15998500,
-1.42435235,1.84252142,-0.05519652,-0.66034293,0.13455531,-
0.68443818,0.42183993,
-1.52483590,-0.41705746)
min(r9);max(r9);var(r9);sum(r9^2);sum(abs(r9-mean(r9)))/30
book$y9=yb+r9
fit=PBIB.test(book$block,book$trt,book$replication,book$y9,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11
fit=PBIB.test(book$block,book$trt,book$replication,book$y9,3,
method="ML",test="lsd",alpha=0.05);fit

r=rnorm(30,0,1);var(r)

r10=c(-0.03835490,-0.27982352,-0.44345190,-
2.75976712,0.74700148,1.36087304,1.78666285,
0.62593947,-1.20960481,-0.81884488,-0.29341510,0.21997784,-0.05806412,-
0.66032170,
0.24092896,0.27599040,-1.04803959,-0.37144269,0.80560536,-1.29399253,-
0.13614321,
1.75547768,-1.16859878,-0.60676692,0.69832014,0.67863545,0.54025668,-
0.84675837,
-0.25081244,-0.79600236)
min(r10);max(r10);var(r10);sum(r10^2);sum(abs(r10-mean(r10)))/30
book$y10=yb+r10

```

```

fit=PBIB.test(book$block,book$trt,book$replication,book$y10,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11
fit=PBIB.test(book$block,book$trt,book$replication,book$y10,3,
method="ML",test="lsd",alpha=0.05);fit

r=rnorm(30,0,1);var(r)

r11=c(0.80628098,0.63577724,0.77084417,-0.65725796,-1.05905615,-
0.52609470,0.27152086,
1.75129833,0.24056606,1.14803458,-1.15703729,-1.59473048,-0.42715016,-
0.36604601,
-0.79419565,0.14938116,-1.70938887,-0.51904957,-0.47821718,-
0.27115457,0.33455735,
-1.31993401,1.91638505,2.38064689,2.44674104,-0.01663056,-
0.74756793,0.17414681,
-1.02336935,-0.55123259)

min(r11);max(r11);var(r11);sum(r11^2);sum(abs(r11-mean(r11)))/30
book$y11=yb+r11

fit=PBIB.test(book$block,book$trt,book$replication,book$y11,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11
fit=PBIB.test(book$block,book$trt,book$replication,book$y11,3,
method="ML",test="lsd",alpha=0.05);fit

#####Experiment D(30,20,3,2)#####
#####

trt<-1:30;t <- length(trt)

# size block k and Blocks s

```

```

k<-3;s<-t/k;r <- 2

outdesign<- design.alpha(trt,k,r,serie=2)

book<-outdesign$book

plots<-book[,1] ;book

bb=c(rep(7,3),rep(22,3),rep(11,3),rep(5,3),rep(35,3),rep(54,3),rep(19,3),rep(0,3),
rep(17,3),rep(39,3),rep(51,3),rep(31,3),rep(44,3),rep(30,3),rep(8,3),rep(57,3),
rep(48,3),rep(2,3),rep(26,3),rep(14,3))

yt=c(9,7,11,20,25,6,19,29,30,12,5,8,4,22,14,3,32,16,15,27,28,34,10,27,23,26,18,0,1,2,
22,30,8,11,28,1,7,0,5,34,26,16,19,6,4,12,9,14,18,2,27,27,29,20,3,15,23,25,10,32)

yb=yt+bb

r=rnorm(60,0,0.5);var(r)

r1=c(0.4589736707,0.3217421890,-0.7912687388,-
0.7223812406,0.3822875106,0.3070470276,
0.2039671848,-0.2708196778,-
0.1588806486,0.8517019848,0.0284134294,0.3149725039,
-0.1765422043,0.3979787924,-0.0020115075,0.2215814502,0.2333919175,-
0.4012039023,
-0.1490948868,0.5736587337,0.4029289147,0.4739737385,-0.3848888746,-
0.3546417361,
0.5298581868,-0.0996760916,0.0486886719,-0.2225415772,-0.1389933912,-
0.1944204238,
0.2911589464,0.5630098100,-0.1988336005,-0.7661863319,-0.2049372320,-
0.4533172235,
-0.4545277911,-0.8380235391,0.7315849582,0.5644989256,-
0.5818902505,0.4419185193,
0.0976119920,-0.2164735846,-0.1274163211,0.3836108546,-
0.3779934793,0.9448555232,

```

```

0.0975049233,0.7607542695,0.0734114025,-0.3272356011,0.0001482248,-
0.1016439657,

0.2868751464,-1.1529340177,0.2220259156,-0.0274152438,-
0.0881537454,0.6852658638) #0.2260991

r2=c(1.19622134,-0.04726055,-0.39141542,0.41325622,-0.79265556,0.07259193,-
0.17578027,

-0.08737926,-0.59492642,-0.16575295,0.46128831,0.06156930,-
0.14272907,0.06037169,0.31566525,

0.30088308,-0.47273967,-0.77912012,-
0.69522927,0.15930863,0.31595300,1.50982870,0.41447213,

-0.32691437,0.06564833,1.70786086,-0.12937185,0.53828943,-
0.46650221,0.17944387,-0.12415438,

-0.88313766,-0.30527311,-0.19402031,0.47017399,-
0.15630840,0.94412155,0.19115724,0.74706171,

0.33712315,-0.25319607,0.53467834,-0.47754565,-0.48179987,-
0.73207923,0.32094416,0.57946271,

-0.29603051,0.27058762,0.50531959,0.20862156,-0.08479212,0.56881062,-
0.21890453,-0.44992646,

-0.14368102,-0.81093604,-0.73004265,0.45425211,-0.38172731)#0.301957

book$y1=yb+r1;book$y2=yb+r2

min(r1);max(r1);var(r1);sum(r1^2);sum(abs(r1-mean(r1)))/60
min(r2);max(r2);var(r2);sum(r2^2);sum(abs(r2-mean(r2)))/60
range(r1);range(r2)

fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="ML",test="lsd",alpha=0.05);fit

```

```

fit=PBIB.test(book$block,book$trt,book$replication,book$y2,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.16

fit=PBIB.test(book$block,book$trt,book$replication,book$y2,3,
method="ML",test="lsd",alpha=0.05);fit #MSE=0.16


r=rnorm(60,0,0.6);var(r)

r3=c(-1.141684297,-0.511103960,-0.686126109,-0.212212453,-
0.971360055,0.304082399,
-0.672782667,-0.910039826,-0.658982565,-0.096043751,-
0.278659946,0.438069209,0.859367227,
0.970449940,0.438376510,-0.891001181,-0.960350950,0.452943234,-
1.085904147,0.511598187,
-0.783332492,1.002589921,-0.112091299,0.340384782,-0.386101555,-
1.748361391,0.311530054,
0.848121570,-0.145841982,-0.603826007,0.088128668,0.325053905,0.368053333,-
0.373814305,
0.821953436,0.278090633,0.422634390,0.376200275,0.754238709,0.791158877,0.6
97539848,
0.419120786,-0.580540576,-0.006998845,-0.942612785,-1.003463456,0.824032533,-
0.177667434,
0.358520044,-0.597744681,-1.162113043,-0.303547107,0.565664721,0.749816931,-
0.024628274,
-1.344962342,0.602021621,0.342945975,0.060840873,0.893989997)

book$y3=yb+r3

min(r3);max(r3);var(r3);sum(r3^2);sum(abs(r3-mean(r3)))/60

fit=PBIB.test(book$block,book$trt,book$replication,book$y3,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.16

```

```

fit=PBIB.test(book$block,book$trt,book$replication,book$y3,3,
method="ML",test="lsd",alpha=0.05);fit

r=rnorm(60,0,0.7);var(r)

r4=c(1.435084407,-0.637711014,-0.671519373,0.283082571,0.174360183,-
0.204265732,0.206000842,
0.581722651,-0.514842801,-0.136185593,0.039437452,0.674001498,-
1.468229974,0.222978412,
1.143627803,0.278321078,-0.385001625,-0.298539049,-0.642127077,-
0.336021008,0.873897745,
-0.766853655,1.281945938,-0.539947564,1.046988669,-0.608173044,-
0.383102708,0.005627349,
-0.244738894,-1.287277729,-0.432651864,-1.143817045,0.410024079,-
0.961687946,0.961035456,
0.360008128,0.752195707,-0.168763633,-
0.530761534,0.320441440,0.159600600,0.446039722,
-0.542956751,0.436952572,-1.044731346,-1.278933487,-0.520865820,-
0.248708863,0.240026312,
-0.378295866,1.366169668,1.560973552,-
0.141253383,0.066526513,0.067064254,1.455865554,
-0.696139763,-0.125984609,-0.081378257,0.175817627)

book$y4=yb+r4

min(r4);max(r4);var(r4);sum(r4^2);sum(abs(r4-mean(r4)))/60

fit=PBIB.test(book$block,book$trt,book$replication,book$y4,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.16

fit=PBIB.test(book$block,book$trt,book$replication,book$y4,3,
method="ML",test="lsd",alpha=0.05);fit

r=rnorm(60,0,0.8);var(r)

```

```

r5=c(-0.540102537,-0.522264859,-1.649411700,-
0.073635851,0.170188495,0.930370529,
0.652788357,0.356366730,0.091827033,-0.338302601,-1.076483498,-1.230238443,-
0.589962982,
-0.691132690,-0.804563847,1.581458627,0.535919916,-0.269685427,-
0.724507179,0.049714108,
0.150877731,-0.592007590,-1.220360776,1.086896281,-1.074848521,-0.223493322,-
0.262686430,
0.381676711,0.500281881,-
0.370080271,0.780729446,0.446824961,0.272925733,0.130306592,
-0.242157190,-0.496016494,0.410544063,0.805137539,0.546412617,-
1.961294456,1.048925812,
0.265975175,-0.097754167,2.235351587,-0.924964748,-0.559974424,-
0.013588211,0.462446556,
0.392444655,-0.796907988,0.561400097,-
0.599887926,0.619299904,0.436301248,1.066042915,
0.905101614,-0.199828460,-0.180405214,0.003062443,1.324598785)
book$y5=yb+r5
min(r5);max(r5);var(r5);sum(r5^2);sum(abs(r5-mean(r5)))/60
fit=PBIB.test(book$block,book$trt,book$replication,book$y5,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.16
fit=PBIB.test(book$block,book$trt,book$replication,book$y5,3,
method="ML",test="lsd",alpha=0.05);fit

r=rnorm(60,0,0.8);var(r)

r6=c(-0.427253921,-1.293966707,0.333398024,-0.024325417,1.160813061,-
0.096356884,
-0.175229641,-0.447336612,-1.275453106,-0.460023584,0.026999812,0.417605378,-
1.994058853,

```

```

-0.344208797,-1.445716747,-0.581133982,-1.200500080,-0.722122182,-
0.779835327,-0.414796728,

0.602598648,-2.009713997,-1.458697111,-0.156453473,-
0.117129553,0.005252359,0.484187347,

-0.993686873,0.871942618,1.484188673,-0.557918503,-0.659439335,-
0.773721109,0.745909814,

0.349101065,-0.466648715,-0.937393008,-1.534708007,0.734913338,-0.481360493,-
0.547336160,

0.945793201,-2.634279270,-1.545831287,-0.386669523,0.608555719,-
0.457802016,0.447463676,

-0.716039222,-0.877649682,-0.767712311,-0.416225749,-0.373144600,-
0.009371953,0.841727893,

1.149392913,1.602506380,-0.213328696,-0.566370774,-0.333330758)

book$y6=yb+r6

min(r6);max(r6);var(r6);sum(r6^2);sum(abs(r6-mean(r6)))/60

fit=PBIB.test(book$block,book$trt,book$replication,book$y6,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.16

fit=PBIB.test(book$block,book$trt,book$replication,book$y6,3,
method="ML",test="lsd",alpha=0.05);fit

r=rnorm(60,0,0.8);var(r)

r7=c(-0.36704654,0.07524488,-0.16754720,-0.86408663,2.03252106,-
1.01198225,0.38441523,

-0.32044830,-2.07609314,0.30543290,-0.51447408,0.88021116,-0.79866163,-
1.19068331,

-0.17152354,-1.05885945,-0.89222762,0.07559505,1.69808414,0.66698130,-
0.90898682,0.42339980,

1.08455577,-1.29043936,0.40556702,0.97593752,-0.76490417,0.03573186,-
0.95848777,1.39289335,

```

```

0.04668988,-0.55926026,-1.31684353,-0.15287326,0.17635881,-
0.04893556,0.30979130,-1.73523195,

0.05157541,1.89453778,-0.01439785,0.15439204,2.37067562,-0.18001480,-
0.05877035,-0.31945692,

-0.51646341,0.31194999,-
0.04892637,0.11784485,1.86306359,0.15153241,0.15316401,-0.56073350,

-0.38263706,-0.57501453,-0.85392586,-0.87027825,-0.92705910,0.02498628)

book$y7=yb+r7

min(r7);max(r7);var(r7);sum(r7^2);sum(abs(r7-mean(r7)))/60

fit=PBIB.test(book$block,book$trt,book$replication,book$y7,3,

method="REML",test="lsd",alpha=0.05);fit #MSE=0.16

fit=PBIB.test(book$block,book$trt,book$replication,book$y7,3,

method="ML",test="lsd",alpha=0.05);fit

r=rnorm(60,0,1);var(r)

r8=c(1.558604705,-0.073884426,-0.925803815,-
0.837829775,1.205969415,0.090883733,

-1.261730678,-0.459374177,0.232622118,-0.555945148,0.991833841,-
2.181571781,0.282139668,

-0.747403710,0.086819267,-1.766282085,-0.245638890,-0.436777985,-
0.043017404,-1.001697772,

-0.391454625,0.978854598,0.339058124,1.427986975,-
0.845480371,0.165457222,0.830513841,

0.264677217,-1.294180217,-0.055559804,-1.745529328,0.599055444,-0.905931764,-
2.084798637,

-0.274009627,-0.546784451,-0.952257566,1.574848924,-0.333327639,0.453491403,-
0.936093180,

0.856852988,0.146321716,0.065312195,-1.576503167,0.937411480,-
0.007153096,0.917409632,

```

```

-1.240654529,-0.710509483,-1.273108064,1.401583984,-0.196768802,-
0.282471361,-1.210066218,

1.329747610,0.146879186,0.503401853,-1.679367725,-0.727029682)

book$y8=yb+r8

min(r8);max(r8);var(r8);sum(r8^2);sum(abs(r8-mean(r8)))/60

fit=PBIB.test(book$block,book$trt,book$replication,book$y8,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.16

fit=PBIB.test(book$block,book$trt,book$replication,book$y8,3,
method="ML",test="lsd",alpha=0.05);fit

r=rnorm(60,0,1.5);var(r)

r9=c(0.01635545,-0.02117010,1.19335475,0.64819835,-2.16451744,-
0.08237787,0.25368682,

-0.27163590,1.59004258,-2.43306087,1.77804649,0.94947486,-
0.85262913,0.49001664,1.43300570,

-1.11901521,0.76645695,-0.27856416,1.76893068,-1.63650821,-
2.75282493,0.32296216,1.17870902,

-0.39314623,-0.91736501,0.29613360,-0.83727115,0.42267840,-0.48640714,-
0.09809795,

-0.53578132,-0.21564781,-0.30175879,0.68808025,0.35001364,-2.24231095,-
0.18749964,

0.35907811,0.64802155,-0.85148175,1.62700328,-
0.39677711,1.87532780,2.28224190,1.01865180,

-1.51371820,1.43070704,0.10320902,0.97127875,0.19166703,-
1.39648026,0.26190537,0.89679900,

-0.03477915,2.14241021,-1.26030451,-
2.16952721,0.39841720,0.60641479,2.10777662)

book$y9=yb+r9

```

```
min(r9);max(r9);var(r9);sum(r9^2);sum(abs(r9-mean(r9)))/60
```

```
fit=PBIB.test(book$block,book$trt,book$replication,book$y9,3,
```

```
method="REML",test="lsd",alpha=0.05);fit #MSE=0.16
```

```
fit=PBIB.test(book$block,book$trt,book$replication,book$y9,3,
```

```
method="ML",test="lsd",alpha=0.05);fit
```

```
r=rnorm(60,0,2.5);var(r)
```

```
r10=c(0.64245121,0.75452098,-0.08781389,-5.02516548,0.03462379,-  
1.39659748,0.73590487,
```

```
-3.19640933,-2.34005975,-3.31287645,0.13015616,3.90350611,-  
3.02756359,2.91521442,2.91969162,
```

```
-1.21213693,-4.75249650,0.53402440,-0.12283471,-2.10928177,2.14796000,-  
0.15741816,-3.77869739,
```

```
-1.21396784,2.21329631,3.54828002,-0.67651904,-1.09921531,-  
0.71350348,2.11886440,2.73861103,
```

```
2.23164151,2.26796953,2.12918152,-2.50111519,-0.36572833,1.37301655,-  
1.87166687,-2.47698512,
```

```
-3.38783769,-0.80463587,-  
0.47660129,3.43498634,4.05069994,5.39327030,2.28766901,-4.51971427,
```

```
0.88008880,4.19183312,4.24182799,2.08465043,3.19456671,-  
0.80359858,1.03482584,-0.64225717,
```

```
3.23266786,-1.53945074,-0.30197057,3.09799468,0.45873596)
```

```
r=rnorm(60,0,0.3);var(r)
```

```
r10=c(0.003641889,0.542003459,-0.825703736,0.251897092,-  
0.076357065,0.171330122,
```

```
-0.145653878,0.327588310,-0.058980207,-0.279803826,0.320065632,-  
0.336141960,0.237578825,
```

```

-0.373926957,-0.042668243,-0.420898488,0.120574201,-0.029029894,-
0.021253840,0.305250705,

0.119826739,-0.286708523,-0.171777402,0.208820344,0.417035810,-0.163463181,-
0.451910245,

0.452315730,-0.001931775,-0.395195169,0.015705703,0.159041846,0.090204374,-
0.069173023,

-0.224614559,0.061295031,-0.530526724,-0.205481736,-0.745864751,0.117758909,-
0.353762061,

0.090043197,-0.099589628,0.500124957,-0.416668356,0.296400167,0.121773299,-
0.561318186,

-0.230763838,0.326333494,-0.212131338,-0.135936110,0.256168085,0.312518122,-
0.559893360,

-0.355306525,-0.533722881,0.170278886,-0.236861419,-0.428606271)

book$y10=yb+r10

min(r10);max(r10);var(r10);sum(r10^2);sum(abs(r10-mean(r10)))/60

fit=PBIB.test(book$block,book$trt,book$replication,book$y10,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.16

fit=PBIB.test(book$block,book$trt,book$replication,book$y10,3,
method="ML",test="lsd",alpha=0.05);fit

r=rnorm(60,0,0.05);var(r)

r11=c(9.790872e-03,-1.845235e-02,1.356565e-02,-4.420238e-02,5.602581e-02,-
8.617406e-02,

-7.800003e-02,3.180368e-03,4.621108e-02,-1.430731e-02,6.645179e-02,3.376807e-
02,

-1.404830e-02,-3.614131e-02,3.237769e-02,-2.961500e-02,-6.173268e-02,-
4.006609e-02,

1.506845e-02,-5.072958e-02,1.555518e-02,-2.102849e-02,8.663366e-02,3.669771e-02,

```

```

3.116730e-03,-5.393598e-03,-9.731189e-03,8.578423e-02,-3.836480e-03,-2.207501e-02,
8.974404e-05,-3.078041e-03,-1.604027e-02,5.060338e-02,1.007530e-02,1.290942e-02,
-2.369581e-02,3.061493e-02,4.849913e-02,4.172777e-02,-1.773314e-02,2.954503e-03,
-3.003159e-02,-6.964408e-02,2.765667e-02,1.262137e-02,-2.662726e-02,4.139482e-02,
-6.102625e-02,1.290055e-02,-3.600072e-02,-1.029811e-02,-2.686718e-02,-2.957937e-02,
-4.759028e-03,5.376177e-02,1.087977e-01,5.722086e-02,-5.203428e-02,-1.102325e-01)

```

```
book$y11=yb+r11
```

```
min(r11);max(r11);var(r11);sum(r11^2);sum(abs(r11-mean(r11)))/60
```

```
fit=PBIB.test(book$block,book$trt,book$replication,book$y11,3,
```

```
method="REML",test="lsd",alpha=0.05);fit #MSE=0.16
```

```
fit=PBIB.test(book$block,book$trt,book$replication,book$y11,3,
```

```
method="ML",test="lsd",alpha=0.05);fit
```

```
#####Experiment D(15,10,3,2) others#####
```

```
#####
```

```
trt<-1:9;t <- length(trt)
```

```
k<-3;s<-t/k;r <- 2
```

```
outdesign<- design.alpha(trt,k,r,serie=2)
```

```
book<-outdesign$book
```

```
plots<-book[,1] ;book
```

```
bb=c(rep(7,3),rep(22,3),rep(11,3),rep(5,3),rep(35,3),rep(54,3))
```

```
yt=c(20,16,6,4,19,23,11,10,3,16,19,11,23,3,20,10,6,4)
```

```
yb=yt+bb
```

```
r=rnorm(18,0,0.05);var(r)
```

```

r1=c(-0.014202553,-0.014078781,-0.011293268,-
0.032960033,0.033928844,0.062660638,
-0.039710134,-0.059815306,-0.047247668,-0.062712236,0.001385751,-
0.061505438,0.008915553,
-0.044143889,0.099835459,-0.010722668,-0.002244605,0.005054734) #0.2260991

book$y1=yb+r1

min(r1);max(r1);var(r1);sum(r1^2);sum(abs(r1-mean(r1)))/18

fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="ML",test="lsd",alpha=0.05);fit

trt<-1:12;t <- length(trt)

k<-3;s<-t/k;r <- 2

outdesign<- design.alpha(trt,k,r,serie=2)

book<-outdesign$book

plots<-book[,1] ;book

bb=c(rep(7,3),rep(22,3),rep(11,3),rep(5,3),rep(35,3),rep(54,3),rep(19,3),rep(0,3))

yt=c(3,27,6,11,4,20,10,23,30,8,16,19,10,8,27,11,6,19,4,16,23,20,30,3)

yb=yt+bb

r=rnorm(24,0,0.05);var(r)

r1=c(0.086815710,0.044027260,0.025465147,-0.003002413,-0.044356726,-
0.023215602,
0.069564826,-0.062109503,-0.035810436,-0.046151450,0.031000839,0.014953546,-
0.061974536,

```

```

-0.055420782,-0.024751982,0.031784128,-0.025781857,-0.005971531,-
0.072005169,0.039760990,

0.015701363,-0.054021241,0.001251462,-0.056012434) #0.2260991

book$y1=yb+r1

min(r1);max(r1);var(r1);sum(r1^2);sum(abs(r1-mean(r1)))/24

fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="ML",test="lsd",alpha=0.05);fit


library(agricolae)

trt<-1:18;t <- length(trt)

k<-3;s<-t/k;r <- 2

outdesign<- design.alpha(trt,k,r,serie=2)

book<-outdesign$book

plots<-book[,1] ;book


bb=c(rep(7,3),rep(22,3),rep(11,3),rep(5,3),rep(35,3),rep(54,3),rep(19,3),rep(0,3),
rep(17,3),rep(39,3),rep(51,3),rep(31,3))

yt=c(27,30,0,2,1,11,3,19,25,6,4,20,22,23,8,10,26,16,10,11,6,20,0,1,4,26,3,30,25,23,22
,16,19,8,27,2)

yb=yt+bb

r=rnorm(36,0,0.05);var(r)

r1=c(-0.105463683,0.027470999,-
0.084931297,0.054204056,0.059911691,0.017542770,0.047895195,

```

```

0.001564556,0.042593930,0.001882506,-0.007650429,-0.012672426,-
0.077661423,0.027648634,

-0.035998123,-0.004823658,-0.057625647,0.045826727,0.070652557,-0.012907802,-
0.008062959,

-0.065648014,-0.060526986,-0.072375546,-0.084182839,0.037611535,0.084811592,-
0.059122441,

0.055932816,0.091914370,-0.038467349,-0.043154572,0.040011436,-0.018925781,-
0.057460964,

-0.054601976) #0.2260991

book$y1=yb+r1

min(r1);max(r1);var(r1);sum(r1^2);sum(abs(r1-mean(r1)))/36

fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="ML",test="lsd",alpha=0.05);fit

trt<-1:12;t <- length(trt)

k<-3;s<-t/k;r <- 2

outdesign<- design.alpha(trt,k,r,serie=2)

book<-outdesign$book

plots<-book[,1] ;book

bb=c(rep(7,3),rep(22,3),rep(11,3),rep(5,3),rep(35,3),rep(54,3),rep(19,3),rep(0,3))

yt=c(3,27,6,11,4,20,10,23,30,8,16,19,10,8,27,11,6,19,4,16,23,20,30,3)

yb=yt+bb

r=rnorm(24,0,0.05);var(r)

```

```

r1=c(0.086815710,0.044027260,0.025465147,-0.003002413,-0.044356726,-
0.023215602,
0.069564826,-0.062109503,-0.035810436,-0.046151450,0.031000839,0.014953546,-
0.061974536,
-0.055420782,-0.024751982,0.031784128,-0.025781857,-0.005971531,-
0.072005169,0.039760990,
0.015701363,-0.054021241,0.001251462,-0.056012434) #0.2260991

book$y1=yb+r1

min(r1);max(r1);var(r1);sum(r1^2);sum(abs(r1-mean(r1)))/24

fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="ML",test="lsd",alpha=0.05);fit

library(agricolae)

trt<-1:18;t <- length(trt)

k<-3;s<-t/k;r <- 2

outdesign<- design.alpha(trt,k,r,serie=2)

book<-outdesign$book

plots<-book[,1] ;book

bb=c(rep(7,3),rep(22,3),rep(11,3),rep(5,3),rep(35,3),rep(54,3),rep(19,3),rep(0,3),
rep(17,3),rep(39,3),rep(51,3),rep(31,3))

yt=c(27,30,0,2,1,11,3,19,25,6,4,20,22,23,8,10,26,16,10,11,6,20,0,1,4,26,3,30,25,23,22
,16,19,8,27,2)

yb=yt+bb

```

```

r=rnorm(36,0,0.05);var(r)

r1=c(-0.105463683,0.027470999,-
0.084931297,0.054204056,0.059911691,0.017542770,0.047895195,
0.001564556,0.042593930,0.001882506,-0.007650429,-0.012672426,-
0.077661423,0.027648634,
-0.035998123,-0.004823658,-0.057625647,0.045826727,0.070652557,-0.012907802,-
0.008062959,
-0.065648014,-0.060526986,-0.072375546,-0.084182839,0.037611535,0.084811592,-
0.059122441,
0.055932816,0.091914370,-0.038467349,-0.043154572,0.040011436,-0.018925781,-
0.057460964,
-0.054601976) #0.2260991

book$y1=yb+r1

min(r1);max(r1);var(r1);sum(r1^2);sum(abs(r1-mean(r1)))/36

fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

C

outdesign<- design.alpha(trt,k,r,serie=2)

book<-outdesign$book

plots<-book[,1] ;book

bb=c(rep(7,4),rep(22,4),rep(11,4),rep(5,4),rep(35,4),rep(54,4),rep(19,4),rep(0,4),
rep(17,4),rep(39,4),rep(51,4),rep(31,4),rep(44,4),rep(30,4))

yt=c(22,12,1,19,10,11,7,27,16,25,20,4,28,26,32,23,34,14,29,30,8,2,15,18,6,3,9,0,25,1
1,9,
23,30,2,27,22,1,34,15,3,20,19,6,32,18,16,26,7,29,12,4,0,10,28,8,14);table(yt)

yb=yt+bb

```

```

r=rnorm(56,0,0.05);var(r)

r1=c(0.071268324,-0.023799955,-0.024657780,0.010866280,-
0.056263815,0.031410384,
-0.029119700,0.033599405,0.010496073,0.079456663,-0.006643428,0.035927904,-
0.019226964,
0.005347389,-0.059476749,-0.036957885,0.039695357,-0.034749800,0.018039168,-
0.013392115,
-0.045359994,-0.025095306,0.033832768,0.071889084,-0.040632904,-
0.051507284,0.030759865,
0.030740958,0.046606586,0.002652726,0.024675582,-
0.036093542,0.011550665,0.006580305,
-0.005505973,-0.040816918,0.063489335,-0.001977513,0.068432587,-
0.007808198,0.025042544,
-0.074553268,-0.034882652,-0.084007868,-0.052483799,0.026441110,0.048469070,-
0.023759206,
-0.053580542,-0.022582842,0.027017983,0.063397207,-
0.010084553,0.046491724,0.129431537,
-0.021436198) #0.2260991

book$y1=yb+r1

min(r1);max(r1);var(r1);sum(r1^2);sum(abs(r1-mean(r1)))/56

fit=PBIB.test(book$block,book$trt,book$replication,book$y1,4,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

fit=PBIB.test(book$block,book$trt,book$replication,book$y1,4,
method="ML",test="lsd",alpha=0.05);fit

trt<-1:27;t <- length(trt)

# size block k and Blocks s

k<-3;s<-t/k ;r <- 3

```

```

outdesign<- design.alpha(trt,k,r,serie=2)

book<-outdesign$book

plots<-book[,1] ;book

bb=c(rep(7,3),rep(22,3),rep(11,3),rep(5,3),rep(35,3),rep(54,3),rep(19,3),rep(0,3),
rep(17,3),rep(39,3),rep(51,3),rep(31,3),rep(44,3),rep(30,3),rep(8,3),rep(57,3),
rep(48,3),rep(2,3),rep(26,3),rep(14,3),rep(32,3),rep(16,3),rep(49,3),rep(25,3),
rep(28,3),rep(29,3),rep(36,3))

yt=c(1,8,34,10,22,4,6,9,2,29,7,30,27,12,16,20,23,18,25,28,19,14,0,26,15,11,3,8,4,
14,7,25,11,19,29,12,26,3,22,6,18,1,20,16,2,10,15,30,28,23,27,34,0,9,12,20,9,0,11,22,
1,14,10,29,4,15,28,7,3,18,27,34,25,2,26,8,6,23,19,16,30);table(yt)

yb=yt+bb

r=rnorm(81,0,0.05);var(r)

r1=c(0.0229451258,0.0662080785,0.0158690430,0.0344725505,-
0.0227644807,0.1030904895,
-0.0743904071,-0.0284784145,0.0497122108,-0.0261339521,-0.0360575645,-
0.0124639907,
0.1304303303,-0.0082275401,-0.0507817768,-0.0009346505,-
0.0077498345,0.0353630605,
0.0115221653,-0.0167302126,0.0439449273,-0.0225748532,0.0333445358,-
0.0413352283,
0.0094730853,0.1171770939,0.0573014434,0.0837060927,0.0783449442,0.0401196
148,
-0.0198839376,-0.0345706947,-0.0004465528,-0.0491976887,0.0251313682,-
0.0232212055,
0.0546464026,-0.0290101505,-0.0050545844,0.0159340912,0.0060433014,-
0.0102888761,

```

```

0.0638225809,-0.0188350556,-0.0929712246,0.0573844024,0.0320252543,-
0.0010269631,

0.0478575281,0.0099336834,0.1090653287,0.0126984805,-
0.0501532659,0.0271213377,

0.0178800978,0.0411272121,0.0220375123,-0.0094936544,0.0249095549,-
0.0581530047,

-0.0098837026,-0.0111823834,-0.0527678278,-0.0518956262,0.0139001091,-
0.0211777782,

0.0401744537,-0.0120762089,-0.0266435110,-0.0868622229,-
0.0257913736,0.0425580229,

0.0279598190,-0.0250167368,-0.0604687418,-0.0086178935,-0.0685336001,-
0.0387807389,

0.0167071837,-0.0506953891,-0.0314891911) #0.2260991

book$y1=yb+r1

min(r1);max(r1);var(r1);sum(r1^2);sum(abs(r1-mean(r1)))/81

fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11

fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="ML",test="lsd",alpha=0.05);fit


trt<-1:15;t <- length(trt)

# size block k and Blocks s

k<-3;s<-t/k ;r <- 4

outdesign<- design.alpha(trt,k,r,serie=2)

book<-outdesign$book

plots<-book[,1] ;book

```

```

bb=c(rep(7,3),rep(22,3),rep(11,3),rep(5,3),rep(35,3),rep(54,3),rep(19,3),rep(0,3),
rep(17,3),rep(39,3),rep(51,3),rep(31,3),rep(44,3),rep(30,3),rep(8,3),rep(57,3),
rep(48,3),rep(2,3),rep(26,3),rep(14,3))

yt=c(6,16,26,20,30,27,8,11,3,0,10,23,4,2,19,19,26,10,0,20,3,23,6,11,16,2,27,8,30,
4,0,4,26,30,19,11,3,10,27,20,2,6,8,23,16,10,30,6,0,11,2,4,27,23,3,16,19,26,20,8);table
(yt)

yb=yt+bb

r=rnorm(60,0,0.05);var(r)

r1=c(-0.051879157,-0.078922216,0.054512012,-0.048474637,-
0.031396771,0.074281240,
0.035037228,-0.049892249,0.046544941,0.056801365,0.048172823,-
0.073588870,0.055984171,
0.047686593,0.012945273,-0.004864691,-
0.012692040,0.044919601,0.009168935,0.008721107,
-0.020387979,0.067621060,-0.002352731,-0.008798356,0.052807117,-
0.027388782,0.019628299,
-0.033322376,0.046112663,0.051944995,0.011907274,-0.005522156,-0.019927389,-
0.007724711,
-0.016678486,0.032310910,-0.008215439,0.092470750,-0.020375609,-
0.033848280,0.008117869,
-0.048728606,-0.023922638,0.031872972,-0.021468118,-0.012494480,-
0.041777852,0.022239309,
-0.032096038,0.010467977,0.029120439,0.004239701,-
0.047587043,0.014945634,0.020346003,
0.029038976,0.004404549,0.108759665,0.012626017,0.094828666) #0.2260991

book$y1=yb+r1

min(r1);max(r1);var(r1);sum(r1^2);sum(abs(r1-mean(r1)))/60

```

```
fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11
fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="ML",test="lsd",alpha=0.05);fit
```

```
trt<-1:14;t <- length(trt)
# size block k and Blocks s
k<-2;s<-t/k ;r <- 3
outdesign<- design.alpha(trt,k,r,serie=2)
book<-outdesign$book
plots<-book[,1] ;book
```

```
bb=c(rep(7,3),rep(22,3),rep(11,3),rep(5,3),rep(35,3),rep(54,3),rep(19,3),rep(0,3),rep(1
7,3))
yt=c(3,23,20,6,10,11,16,4,19,6,20,4,19,23,11,16,3,10,16,23,6,19,10,20,11,4,3)
yb=yt+bb
r=rnorm(27,0,0.05);var(r)
r1=c()
min(r1);max(r1);var(r1);sum(r1^2);sum(abs(r1-mean(r1)))/27
book$y1=yb+r1
fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="REML",test="lsd",alpha=0.05);fit #MSE=0.22 180.75 df=11
fit=PBIB.test(book$block,book$trt,book$replication,book$y1,3,
method="ML",test="lsd",alpha=0.05);fit
```

Appendix C: A R programme for searching symmetric, anti-symmetric PBIB designs and their complementary designs.

```
#####lambda=2,Trt=4,k=3,b=4,r=3#####

# 4 treatments and k=3 size block

trt<-c("1","2","3","4")

k<-3

outdesign<-design.bib(trt,k,serie=2,seed =41,kinds ="Super-Duper") # seed = 41

print(outdesign$parameters)

book<-outdesign$book

bb=c(rep(7,3),rep(22,3),rep(11,3),rep(5,3))

yt=c(3,16,6,16,11,3,16,6,11,6,11,3);table(yt)

yb=yt+bb

r=rnorm(12,0,0.05);var(r)

r1=c(0.084758207,0.029241899,-0.079203774,-0.025895422,-0.043777900,-
0.003317968,
-0.021797196,0.027128970,0.019647274,0.005118108,0.004005356,-0.004718916)

min(r1);max(r1);var(r1);sum(r1^2);sum(abs(r1-mean(r1)))/12

book$y1=yb+r1

fit=BIB.test(book$block,book$trt,book$y1,test="lsd",alpha=0.05,console=TRUE);fit
#MSE=0.22 180.75 df=11

#####lambda=2,Trt=4,k=3,b=4,r=3#####

# 4 treatments and k=3 size block

trt<-c("1","2","3","4","5","6","7","8")

k<-2
```

```

outdesign<-design.bib(trt,k,serie=2,seed =41,kinds ="Super-Duper") # seed = 41

print(outdesign$parameters)

book<-outdesign$book

bb=c(rep(7,2),rep(22,2),rep(11,2),rep(5,2),rep(35,2),rep(54,2),rep(19,2),rep(0,2),
rep(17,2),rep(39,2),rep(51,2),rep(31,2),rep(44,2),rep(30,2),rep(8,2),rep(57,2),
rep(48,2),rep(2,2),rep(26,2),rep(14,2),rep(32,2),rep(16,2),rep(49,2),rep(25,2),
rep(28,2),rep(29,2),rep(36,2),rep(52,2))

yt=c(23,3,4,20,20,11,4,6,3,20,19,23,11,6,19,11,3,6,16,19,20,23,3,19,4,11,16,4,3,16,4,
23,
16,23,6,20,4,3,16,11,6,16,4,19,20,19,6,19,6,23,20,16,3,11,23,11);table(yt)

yb=yt+bb

r=rnorm(56,0,0.05);var(r)

r1=c(1.986032e-02,-8.160819e-03,-1.264887e-02,-2.377188e-04,3.795242e-
02,1.590653e-02,
-7.803889e-02,-2.786818e-02,-5.110209e-02,-1.208346e-02,1.595612e-02,-
1.989486e-02,
4.537389e-03,2.393936e-02,3.205817e-02,4.191532e-02,-7.849883e-02,3.738678e-
02,-4.045095e-02,
-3.672814e-02,-2.435899e-05,3.891244e-02,-5.445510e-02,2.945644e-02,-
1.228367e-02,
-3.964510e-02,1.821737e-02,1.009200e-03,-2.218346e-02,-1.239345e-02,-
2.095448e-02,
6.644216e-03,8.779997e-02,-1.843301e-03,-2.812353e-02,3.112845e-02,-1.755786e-
03,
7.594061e-02,-9.963365e-02,-7.574059e-02,5.092180e-02,-7.771403e-03,-
5.148326e-02,
-9.875568e-04,3.045468e-02,-5.407682e-03,6.618542e-03,-4.049353e-02,-
2.539729e-02,

```

1.725503e-02,6.769097e-03,6.675480e-02,-4.613829e-02,6.698483e-03,-2.986972e-03,

1.354552e-02)

min(r1);max(r1);var(r1);sum(r1^2);sum(abs(r1-mean(r1)))/56

book\$y1=yb+r1

fit=BIB.test(book\$block,book\$trt,book\$y1,test="lsd",alpha=0.05,console=TRUE);fit
#MSE=0.22 180.75 df=11

Appendix D: R Programme for searching optimal designs under treatment and replication constraint, Block and block size constraint.

Algebraic Algorithm

Find a number with at least three factors for treatment and replication such that the following constraint holds $tr = bk$

If $t = xy$ and $r = sz$

Such that $x \neq y$ and $s \neq z$

Then $tr = bk$

$xy \times sz = xs \times yz$

$xy \times sz = xz \times ys$

$xy \times sz = xys \times z$

$xy \times sz = xyz \times s$

Competing designs

Stop when repetition starts.

The complementary designs for the set of designs above is position replacement between treatment t and block b and r and k such that $tr = bk$.