

EFFICIENCY OF AUDU & ADEWARA (2017) TWO-PHASE FACTOR-TYPE ESTIMATORS WITH TWO AUXILIARY VARIABLES IN SAMPLE SURVEY

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ABSTRACT: In this paper, efficiency of Audu & Adewara ([AA17]) two-phase factor-type estimators with two auxiliary variables for estimating finite population mean were examined using simulation. These estimators were obtained by incorporating some known functions of auxiliary variables X and Z in some existing factor-type estimators. Bias and Mean square error (MSE) of these estimators, $\bar{y}_{FTAA}^{\beta_1(d)}$ and $\bar{y}_{FTAA}^{\beta_2(d)}$ were obtained using Taylor's series expansion. Audu & Adewara ([AA17]) revealed that although $\bar{y}_{FTAA}^{\beta_1(d)}$ and $\bar{y}_{FTAA}^{\beta_2(d)}$, have minimum MSE and high PRE than all other related existing factor-type estimators considered using three life dataset but of these two, which is more efficient and most preferred. The simulation results obtained in this study revealed that $\bar{y}_{FTAA}^{\beta_2(d)}$ is more efficient than $\bar{y}_{FTAA}^{\beta_1(d)}$ and hence, most preferred.

KEYWORDS: Factor-type estimator, Two-phase sampling, Mean square error (MSE), Efficiency.

1. INTRODUCTION

It has been established that when the population mean $\bar{X}$ of auxiliary variables X is not known but X is correlated to another auxiliary variable Z whose population mean $\bar{Z}$ is known, then information on Z can be used to improved the efficiency of estimator at hand ([CS12]). In this study, information on secondary auxiliary variable were incorporated into the work of Shukla ([Shu02]) in addition to power transformation with sole aim of improving Shukla ([Shu02]) factor-type estimator.

Shukla ([Shu02]) suggested a factor-type estimator for population mean under two-phase sampling as:

$$\bar{y}_{FTd} = \bar{y}_2 \frac{(A+C)\bar{x}_1 + fB\bar{x}_2}{(A+fB)\bar{x}_1 + C\bar{x}_2} \quad (1)$$

$$Bias(\bar{y}_{FTd})_I = \bar{Y}P\theta_3[\rho_{xy}C_xC_y - \psi_4C_x^2] \quad (2)$$

$$Bias(\bar{y}_{FTd})_{II} = \bar{Y}P[(\theta_1\psi_3 - \theta_2\psi_4)C_x^2 + \theta_2\rho_{xy}C_xC_y] \quad (3)$$

$$MSE(\bar{y}_{FTd})_I = \bar{Y}^2[\theta_2C_y^2 + \theta_3P^2C_x^2 + 2\theta_3P\rho_{xy}C_xC_y] \quad (4)$$

$$MSE(\bar{y}_{FTd})_{II} = \bar{Y}^2[\theta_2C_y^2 + \theta_4P^2C_x^2 + 2\theta_2P\rho_{xy}C_xC_y] \quad (5)$$

Where:

$$\theta_1 = \frac{1}{n_1} - \frac{1}{N}, \theta_2 = \frac{1}{n_2} - \frac{1}{N}, \theta_3 = \frac{1}{n_2} - \frac{1}{n_1}, \theta_4 = \theta_1 + \theta_2$$

$$A = (d-1)(d-2), \quad B = (d-1)(d-4), \quad C = (d-2)(d-3)(d-4),$$

$$\psi_1 = \frac{A+C}{A+fB+C}, \psi_2 = \frac{fB}{A+fB+C}, \psi_3 = \frac{A+fB}{A+fB+C}, \psi_4 = \frac{C}{A+fB+C}, \quad P = \psi_3 - \psi_1 = \psi_2 - \psi_4$$

d is an unknown positive real number to be estimated i.e $d \in \mathbb{R}^+$

In his work, it was observed that factor-type estimator $\bar{y}_{FTd}$ was more efficient than classical ratio estimator $\bar{y}_r^d$, if $-2C_{yx} < P < 0$ under case I and if $-2C_{yx}(1+\delta)^{-1} < P < 0$ under case II where $\delta = \theta_1\theta_2^{-1}$.

2. AUDU & ADEWARA ([AA17]) ESTIMATORS

Consider a preliminary large sample S_1 of size n_1 drawn from population Ω of size N by SRSWOR and secondary sample S_2 of size $n(n < n_1)$ drawn either of the following manners:

Case I: as a subset from the preliminary sample i.e. $S_2 \subset S_1$.

Case II: as an independent sample from population i.e. $S_2 \subset \Omega$.

Motivated by Choudhury and Singh ([CS12]) and Khan et. al ([KSS12]), Audu & Adewara ([AA17]) suggested the following estimators under two-phase sampling as:

$$\bar{y}_{FTAA}^{\beta_1(d)} = \bar{y}_2 \left[\frac{(A+C)\bar{x}_1 + fB\bar{x}_2}{(A+fB)\bar{x}_1 + C\bar{x}_2} \right] \left[\frac{a_z\bar{z} + b_z}{a_z\bar{z}_1 + b_z} \right]^{\beta_1} \quad (6)$$

$$\bar{y}_{FTAA}^{\beta_2(d)} = \bar{y}_2 \left[\frac{(A+C)\bar{x}_1 + fB\bar{x}_2}{(A+fB)\bar{x}_1 + C\bar{x}_2} \right] \left[\frac{a_z\bar{z}_1 + b_z}{a_z\bar{z}_2 + b_z} \right]^{\beta_2} \quad (7)$$

where $a_x > 0$, $b_x > 0$, $a_z > 0$ and $b_z > 0$ are assumed to be known as either real numbers or (linear or non-linear) functions, $0 < \beta_1 < 1$ and $0 < \beta_2 < 1$

3. PROPERTIES OF AUDU & ADEWARA ([AA17]) FACTOR-TYPE ESTIMATORS

(a). Under Case I ([AA17])

$$Bias\left(\bar{y}_{FTAA}^{\beta_1(d)}\right)_I = \bar{Y} \left[\theta_3 PC_x^2 (\rho_{xy} C_x C_y - \psi_4) + \theta_1 C_z^2 \left(\frac{\beta_1(\beta_1+1)}{2} \delta_z^2 - \beta_1 \delta_z \rho_{yz} C_y C_z \right) \right] \quad (8)$$

$$Bias\left(\bar{y}_{FTAA}^{\beta_1(d)}\right)_{II} = \bar{Y} \left[\theta_3 P \psi_3 C_x^2 + P \theta_2 C_x^2 (C_{yx} - \psi_4) + \beta_1 \delta_z \theta_3 C_z^2 (\delta_z (\beta_1+1) / 2 + PC_{xz}) \right] \quad (9)$$

$$MSE\left(\bar{y}_{FTAA}^{\beta_1(d)}\right)_I = \bar{Y}^2 \left[\theta_2 C_y^2 + \theta_3 C_x^2 P (P + 2C_{yx}) + \theta_1 \beta_1 \delta_z C_z^2 (\beta_1 \delta_z - 2C_{yz}) \right] \quad (10)$$

and

$$MSE\left(\bar{y}_{FTAA}^{\beta_1(d)}\right)_{II} = \bar{Y}^2 \left[\theta_2 C_y^2 + C_x^2 P \left\{ \theta_2 (P + 2C_{yx}) + \theta_3 P \right\} + \theta_3 \beta_1 \delta_z C_z^2 (\beta_1 \delta_z + 2PC_{xz}) \right] \quad (11)$$

(b). Under Case II ([AA17])

$$Bias\left(\bar{y}_{FTAA}^{\beta_2(d)}\right)_I = \bar{Y} \left[\theta_2 PC_{yz} C_z^2 - \theta_1 PC_{yx} C_x^2 - \theta_3 \psi_4 PC_x^2 - \theta_3 \beta_2 \delta_z C_{yz} C_z^2 + \theta_2 \frac{\beta_2(\beta_2+1)}{2} \delta_z^2 C_z^2 + \theta_1 \delta_z^2 C_z^2 \left(\frac{\beta_2(\beta_2-1)}{2} - \beta_2^2 \right) \right] \quad (12)$$

$$Bias\left(\bar{y}_{FTAA}^{\beta_2(d)}\right)_{II} = \bar{Y} \left[\frac{\beta_2(\beta_2+1)}{2} \delta_z^2 \theta_2 C_z^2 + \frac{\beta_2(\beta_2-1)}{2} \delta_z^2 \theta_1 C_z^2 - \beta_2 \delta_z P \theta_2 C_{xz} C_z^2 + \psi_3 P \theta_1 C_x^2 - \psi_4 P \theta_2 C_x^2 + P \theta_2 C_x^2 C_{yx} - \beta_2 \delta_z \theta_2 C_{yz} C_z^2 \right] \quad (13)$$

$$MSE\left(\bar{y}_{FTAA}^{\beta_2(d)}\right)_I = \bar{Y}^2 \left[\delta_2 C_y^2 + \delta_3 C_x^2 P (P - 2C_{yx}) + \delta_3 \beta_2 \delta_z C_z^2 (\beta_1 \delta_z - 2C_{yz} - 2PC_{xz}) \right] \quad (14)$$

and

$$MSE\left(\bar{y}_{FTAA}^{\beta_2(d)}\right)_{II} = \bar{Y}^2 \left[\theta_2 C_y^2 + \theta_2 C_x^2 P(P + 2C_{yx}) + \theta_1 \beta_2 \delta_z C_z^2 (\beta_2 \delta_z - 2PC_{xz}) \right. \\ \left. \theta_2 \delta_z \beta_2 C_z^2 (\delta_z \beta_2 - 2C_{yz} - 2PC_{xz}) + \theta_1 P^2 C_x^2 \right] \quad (15)$$

4. EFFICIENCY COMPARISON

$\bar{y}_{FTAA}^{\beta_2(d)}$ is said to be more efficient than $\bar{y}_{FTAA}^{\beta_1(d)}$ whenever:

(a). For case I:

$$MSE\left(\bar{y}_{FTAA}^{\beta_1(d)}\right)_{II} = \bar{Y}^2 \left[\theta_2 C_y^2 + C_x^2 P \left\{ \theta_2 (P + 2C_{yx}) + \theta_3 P \right\} + \theta_3 \beta_1 \delta_z C_z^2 (\beta_1 \delta_z + 2PC_{xz}) \right] < \\ MSE\left(\bar{y}_{FTAA}^{\beta_2(d)}\right)_I = \bar{Y}^2 \left[\theta_2 C_y^2 + \theta_3 C_x^2 P(P + 2C_{yx}) + \theta_1 \beta_1 \delta_z C_z^2 (\beta_1 \delta_z - 2C_{yz}) \right]$$

and

(b). For case II:

$$MSE\left(\bar{y}_{FTAA}^{\beta_2(d)}\right)_{II} = \bar{Y}^2 \left[\theta_2 C_y^2 + \theta_2 C_x^2 P(P + 2C_{yx}) + \theta_1 \beta_2 \delta_z C_z^2 (\beta_2 \delta_z - 2PC_{xz}) \right. \\ \left. \theta_2 \delta_z \beta_2 C_z^2 (\delta_z \beta_2 - 2C_{yz} - 2PC_{xz}) + \theta_1 P^2 C_x^2 \right] < \\ MSE\left(\bar{y}_{FTAA}^{\beta_1(d)}\right)_I = \bar{Y}^2 \left[\delta_2 C_y^2 + \delta_3 C_x^2 P(P - 2C_{yx}) + \delta_3 \beta_2 \delta_z C_z^2 (\beta_1 \delta_z - 2C_{yz} - 2PC_{xz}) \right]$$

5. EMPIRICAL STUDY USING SIMULATED DATA

In order to justify the results obtained in section three, a numerical simulation study is conducted to investigate the efficiency of these Audu & Adewara ([AA17]) factor-type estimators. The correlation coefficients between the study and auxiliary variables are assumed to be $\rho_{xy} = \pm 0.2$, $\rho_{xy} = \pm 0.5$, $\rho_{xy} = \pm 0.8$ and $\rho_{xy} = \pm 0.99$ using multivariate normal with the following parameters:

$N = 200$, $\bar{Y} = 1.51$, $\bar{X} = 2.02$, $\bar{Z} = 3.05$, $S_y = 0.241$, $S_x = 0.387$, $S_z = 0.239$

Table 1: Bias, MSE and PRE of $\bar{y}_{FTAA}^{\beta_1(d)}$, $\bar{y}_{FTAA}^{\beta_2(d)}$ when $\rho_{xy} = 0.2$ and $\rho_{xy} = 0.5$

Estimators	$\rho_{xy} = 0.2$			$\rho_{xy} = 0.5$		
	Bias	MSE	PRE	Bias	MSE	PRE
CASE I						
$\bar{y}_{FTAA}^{\beta_1(d)}$	6.92X10 ⁻⁴	10.75X10 ⁻⁴	107.1	23.2X10 ⁻⁴	5.41X10 ⁻⁴	157.8
$\bar{y}_{FTAA}^{\beta_2(d)}$	4.81X10 ⁻⁴	10.02X10 ⁻⁴	113.6	31.7X10 ⁻⁴	4.87X10 ⁻⁴	175.2
CASE II						
$\bar{y}_{FTAA}^{\beta_1(d)}$	21.2X10 ⁻⁴	8.52X10 ⁻⁴	133.7	13.9X10 ⁻⁴	5.08X10 ⁻⁴	168.1
$\bar{y}_{FTAA}^{\beta_2(d)}$	16.1X10 ⁻⁴	8.41X10 ⁻⁴	135.3	17.2X10 ⁻⁴	4.52X10 ⁻⁴	188.9

Table 2: Bias, MSE and PRE of $\bar{y}_{FTAA}^{\beta_1(d)}$, $\bar{y}_{FTAA}^{\beta_2(d)}$ when $\rho_{xy} = 0.8$ and $\rho_{xy} = 0.99$

Estimators	$\rho_{xy} = 0.8$			$\rho_{xy} = 0.99$		
	Bias	MSE	PRE	Bias	MSE	PRE
CASE I						
$\bar{y}_{FTAA}^{\beta_1(d)}$	19.8X10 ⁻⁴	2.07X10 ⁻⁴	320.7	28.9X10 ⁻⁴	1.31X10 ⁻⁴	507.4
$\bar{y}_{FTAA}^{\beta_2(d)}$	24.7X10 ⁻⁴	1.99X10 ⁻⁴	333.2	18.4X10 ⁻⁴	1.23X10 ⁻⁴	524.1
CASE II						
$\bar{y}_{FTAA}^{\beta_1(d)}$	3.82X10 ⁻⁴	2.84X10 ⁻⁴	233.6	11.9X10 ⁻⁴	1.58X10 ⁻⁴	421.1
$\bar{y}_{FTAA}^{\beta_2(d)}$	8.42X10 ⁻⁴	2.75X10 ⁻⁴	241.1	1.77X10 ⁻⁴	1.02X10 ⁻⁴	653.4

Table 3: Bias, MSE and PRE of $\bar{y}_{FTAA}^{\beta_1(d)}$, $\bar{y}_{FTAA}^{\beta_2(d)}$ when $\rho_{xy} = -0.2$ and $\rho_{xy} = -0.5$

Estimators	$\rho_{xy} = -0.2$			$\rho_{xy} = -0.5$		
	Bias	MSE	PRE	Bias	MSE	PRE
CASE I						
$\bar{y}_{FTAA}^{\beta_1(d)}$	2.13X10 ⁻⁴	9.03X10 ⁻⁴	125.9	23.2X10 ⁻⁴	4.41X10 ⁻⁴	193.9
$\bar{y}_{FTAA}^{\beta_2(d)}$	2.34X10 ⁻⁴	7.79 X10 ⁻⁴	146.2	31.7X10 ⁻⁴	4.23X10 ⁻⁴	202.0
CASE II						
$\bar{y}_{FTAA}^{\beta_1(d)}$	2.24X10 ⁻⁴	7.93 X10 ⁻⁴	143.6	3.16X10 ⁻⁴	3.84X10 ⁻⁴	224.9
$\bar{y}_{FTAA}^{\beta_2(d)}$	2.28X10 ⁻⁴	4.62 X10 ⁻⁴	246.2	5.31X10 ⁻⁴	2.34X10 ⁻⁴	249.3

Table 4: Bias, MSE and PRE of $\bar{y}_{FTAA}^{\beta_1(d)}$, $\bar{y}_{FTAA}^{\beta_2(d)}$ when $\rho_{xy} = -0.8$ and $\rho_{xy} = -0.99$

Estimators	$\rho_{xy} = -0.8$			$\rho_{xy} = -0.99$		
	Bias	MSE	PRE	Bias	MSE	PRE
CASE I						
$\bar{y}_{FTAA}^{\beta_1(d)}$	37.4X10 ⁻⁴	3.51X10 ⁻⁴	279.5	42.9X10 ⁻⁴	1.58X10 ⁻⁴	421.3
$\bar{y}_{FTAA}^{\beta_2(d)}$	45.1X10 ⁻⁴	3.33X10 ⁻⁴	294.8	45.4X10 ⁻⁴	1.52X10 ⁻⁴	438.2
CASE II						
$\bar{y}_{FTAA}^{\beta_1(d)}$	3.82X10 ⁻⁴	4.47X10 ⁻⁴	210.2	6.30X10 ⁻⁴	2.85X10 ⁻⁴	232.8
$\bar{y}_{FTAA}^{\beta_2(d)}$	8.42X10 ⁻⁴	4.35X10 ⁻⁴	225.6	4.76X10 ⁻⁴	2.53X10 ⁻⁴	262.1

6. RESULTS AND DISCUSSION

Tables 1 - 4 show the biases, MSEs and PRE of $\bar{y}_{FTAA}^{\beta_1(d)}$ and $\bar{y}_{FTAA}^{\beta_2(d)}$ under simple random sampling scheme when the study and auxiliary variables are negatively and positively correlated with $\rho_{xy} = \pm 0.2$, $\rho_{xy} = \pm 0.5$, $\rho_{xy} = \pm 0.8$ and $\rho_{xy} = \pm 0.99$ coefficients respectively. These properties (Bias, MSE and PRE) were computed under cases I and II. The simulation results of the analysis revealed that $\bar{y}_{FTAA}^{\beta_2(d)}$ have minimum MSEs and high PRE and hence, more efficient and most preferred to $\bar{y}_{FTAA}^{\beta_1(d)}$.

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