

Heat transfer with radiation and temperature dependent heat source in MHD free convection flow in a porous medium between two vertical wavy walls

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Abstract

This study was conducted to investigate the two dimensional heat transfer of a free convective MHD flow with radiation and temperature dependent heat source of a viscous incompressible fluid in a porous medium between two vertical wavy walls. The flow is assumed to consist of a mean part and a perturbed part. The perturbed quantities are expressed in terms of exponential series for short wave-length. The resultant differential equations are solved by Differential Transform Method (DTM). The numerical computations are presented graphically to show the salient features of the fluid flow and heat transfer characteristics. The skin friction and Nusselt number are also analyzed for variation of governing parameters.

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Keywords: Wavy walls; A porous medium; Two-dimensional radiation; Differential Transform method

1. Introduction

Free convective flows over wavy wall(s) have received some attentions in recent years because of its engineering and industrial applications in designing ventilating–heating buildings, cooling of electronic components, drying several types of Agricultural products (grain and food). Corrugated surfaces, especially wavy surfaces are often used as flow passages in several applications for controlling the rate of heat transfer. Wavy walls are also employed in medical operations to increase the mass transfer (blood oxygenator).

In view of these applications, Fasogbon [1] presented MHD flows in corrugated channel and the effect of MHD on the fluid flow was reported. Tak and Kumar [2] investigated heat transfer with radiation in MHD free convection between a vertical wavy wall and a flat wall. The influences of radiation and MHD on the fluid flow were presented in the study. Analytical studies of heat and mass transfer by convection in a two dimensional irregular channel were considered by Fasogbon [3]. The author reported that the heat source parameter has an accelerating effect on the fluid

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temperature. Recently, Abubakar [4] investigated natural convective flow and heat transfer in a viscous incompressible fluid with slip effects confined within spirally-enhanced channel and presented that the fluid velocity increasing linearly with the slip effects. In all these studies, the investigations were limited to a long vertical wavy wall with a parallel flat wall.

The study of viscous fluid flow in two vertical wavy channel was investigated by Tak and Kumar [5] and Kumar [6] presented heat transfer with radiation and temperature dependent heat source in MHD free convection flow confined between two vertical wavy walls. The authors reported that the thermal radiation has a linear accelerating effect on the fluid flow while temperature dependent heat source slow down the fluid flow for non porous medium.

The fluid flow through porous medium has been studied extensively due to their applications in soil mechanics, ground water hydrology, water purification processes, industrial filtration processes, ceramic engineering, power metallurgy, agricultural engineering, chemical engineering and petroleum engineering. The importance of such flows has also been recognized in the field of aeronautical engineering in view of its applications to reduce drag. Further, convection flows through porous media are widely used in high temperature heat exchangers, turbine blades and jet nozzle.

In light of various applications of flow through a porous medium, Das, Jana and Makinde investigated [7] an oscillatory MHD convective flow in a vertical channel filled with porous medium with hall and thermal radiation effects. MHD oscillatory slip flow and heat transfer in a channel filled with porous media was considered by Adesanya and Makinde [8]. Makanda, Makinde and Sibanda [9] studied the natural convection of viscoelastic fluid from cone embedded in a porous medium with viscous dissipation. Chinyoka and Makinde [10] presented analysis of non-Newtonian flow with reacting species in a channel filled with porous medium. In these studies, the authors investigations were based on porous medium between two flat walls. However, the MHD free convection flow in the presence of a temperature-dependent heat source in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall in slip flow regime with constant heat flux at the flat wall through a porous medium was considered by Teneja and Jain [11]. All the above studies have neglected the influence of porous medium between two vertical wavy walls.

In this study, we investigated two-dimensional heat transfer of a free convective MHD flow with radiation and temperature dependent heat source in an incompressible fluid in a porous medium between two vertical wavy walls. To achieve the aim of this study, Differential Transform Method was used to obtain the solutions of the nonlinear differential equations governing the flow problem.

2. Formulation of the problem

We consider the two-dimensional steady laminar free convective hydromagnetic flow in a porous medium between long a vertical wavy channel. The x -axis is taken vertically upwards and y -axis perpendicular to it. The wavy walls are represented by $Y = \varepsilon^* \cos(\lambda X)$ and $Y = L + \varepsilon^* \cos(\lambda X)$ respectively, where $\varepsilon^* \ll 1$. This takes place under buoyancy in the presence of radiation and temperature dependent heat source. The equations governing the steady two dimensional flow and heat transfer are as follows:

$$\rho \left(U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) = -\frac{\partial P}{\partial X} + \mu \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) + g\beta(T - T_c) - H_0^2 U - \frac{\mu}{K^*} U \quad (1)$$

$$\rho \left(U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) = -\frac{\partial P}{\partial Y} + \mu \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - \frac{\mu}{K^*} V \quad (2)$$

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (3)$$

$$\rho C_p \left(U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} \right) = k \left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right) - \frac{\partial q_x}{\partial X} - \frac{\partial q_y}{\partial Y} + Q(T_c - T). \quad (4)$$

The boundary conditions relevant to the problem are taken as follows:

$$\left. \begin{aligned} U = 0, \quad V = 0, \quad T = T_c, \quad \text{at } Y = \varepsilon^* \cos(\lambda X) \\ U = 0, \quad V = 0, \quad \frac{\partial T}{\partial Y} = 0, \quad \text{at } Y = L + \varepsilon^* \cos(\lambda X) \end{aligned} \right\} \quad (5)$$

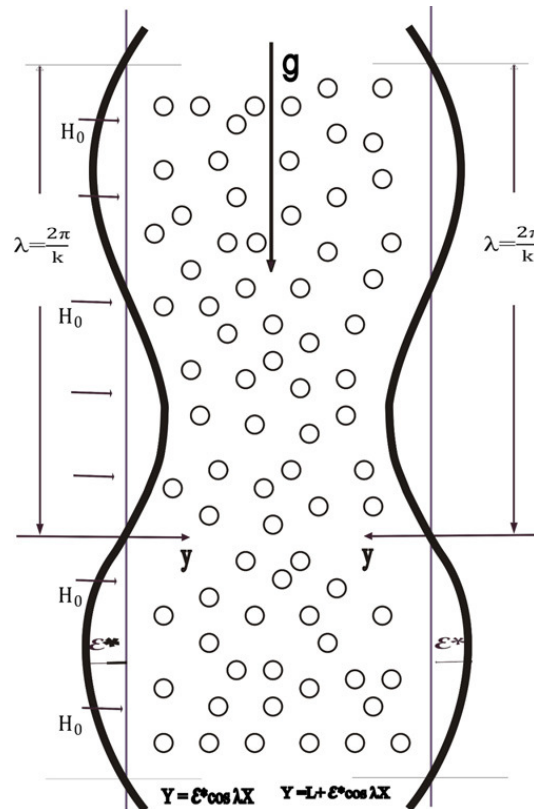


Fig. 1. The flow channel.

where U , V are the velocity components, P is the pressure, ν is the kinematic viscosity, β the coefficient of volume expansion, H_0^2 is the uniform magnetic field, K^* is the porosity parameter, k is the thermal conductivity, ρ is the density of the fluid, c_p is the specific heat at constant pressure, Q is the temperature dependent heat source and T_c is the equilibrium temperature. Also, q_x is radiative heat flux in the x -direction and q_y is radiative heat flux in the y -direction (see Fig. 1).

Following Rosseland approximation, the radiative heat flux in the x and y -directions are modeled as

$$q_x = -\frac{4\sigma\partial T^4}{3R\partial X}, \quad q_y = -\frac{4\sigma\partial T^4}{3R\partial Y} \quad (6)$$

where σ is the Stefan–Boltzmann constant and R is the mean absorption coefficient. If the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature, then the Taylor series T^4 for about T_c , after neglecting higher order terms, is given by

$$T^4 = 4T_c^3 T - 3T_c^4. \quad (7)$$

The following dimensionless parameters are used for Eqs. (1)–(8)

$$\left. \begin{aligned} (x, y) &= \frac{1}{L}(X, Y), & (u, v) &= \frac{L}{\nu}(U, V), & p &= \frac{PL^2}{\rho\nu^2}, & \theta &= \frac{T - T_s}{T_c - T_s}, & Pr &= \frac{\mu c_p}{k}, \\ G &= \frac{g\rho\beta L^4}{kv^2}, & R &= \frac{ka_R}{4T_c}, & \alpha &= \frac{QL^2}{k}, & M &= \frac{H_0 L^2}{\rho\nu}, & K &= \frac{K^*}{\nu L^2}, & \lambda &= \Lambda L, & \varepsilon &= \frac{\varepsilon^*}{L} \end{aligned} \right\} \quad (8)$$

where G is the Grashof number, Pr is the Prandtl, α is the heat source parameter, M is the magnetic parameter, R is radiation parameter, K is the porosity parameter, λ is the dimensionless frequency and ε is the dimensionless amplitude ratio.

Eqs. (1)–(5) in dimensionless form reduce to

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + G\theta - Mu - \frac{1}{K}u \quad (9)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} - \frac{1}{K}v \quad (10)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (11)$$

$$Pr \left(u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} \right) = \omega \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) - \alpha \theta \quad (12)$$

with the following boundary conditions:

$$\left. \begin{aligned} u &= 0, & v &= 0, & \theta &= 1, & \text{at } y &= \varepsilon \cos(\lambda x) \\ u &= 0, & v &= 0, & \frac{\partial \theta}{\partial y} &= 0, & \text{at } y &= 1 + \varepsilon \cos(\lambda x) \end{aligned} \right\} \quad (13)$$

where $\omega = (1 + \frac{4}{3K})$ is the radiation parameter.

We assume that the solution consists of a mean part and a perturbed part so that the velocity and temperature distributions are

$$\left. \begin{aligned} u(x, y) &= u_0(y) + \varepsilon u_1(x, y) \\ v(x, y) &= \varepsilon v_1(x, y) \\ \theta(x, y) &= \theta_0(y) + \varepsilon \theta_1(x, y) \\ p(x, y) &= p_0(x) + \varepsilon p_1(x, y) \end{aligned} \right\} \quad (14)$$

where the perturbed quantities u_1, v_1, θ_1 and p_1 are small compared the mean quantities.

Substituting Eq. (14) into Eqs. (9)–(12) with boundary conditions (13), we obtain the following sets of equations:

Zeroth order equations are:

$$\frac{d^2 u_0}{dy^2} - Mu_0 - \frac{1}{K}u_0 = -G\theta_0 - C \quad (15)$$

$$\omega \frac{d^2 \theta_0}{dy^2} - \alpha \theta_0 = 0 \quad (16)$$

where $C = \frac{dp_0}{dx}$ with corresponding boundary condition

$$\left. \begin{aligned} u_0 &= 0, & \theta_0 &= 1, & y &= 0 \\ u_0 &= 0, & \frac{d\theta_0}{dy} &= 0, & y &= 1 \end{aligned} \right\} \quad (17)$$

and the first order are:

$$u_0 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_0}{\partial y} = -\frac{\partial p_1}{\partial x} + \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} + G\theta_1 - Mu_1 - \frac{1}{K}u_1 \quad (18)$$

$$u_0 \frac{\partial v_1}{\partial x} = -\frac{\partial p_1}{\partial y} + \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} - \frac{1}{K}v_1 \quad (19)$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \quad (20)$$

$$Pr \left(u_0 \frac{\partial \theta_1}{\partial x} + v_1 \frac{\partial \theta_0}{\partial y} \right) = \omega \left(\frac{\partial^2 \theta_1}{\partial x^2} + \frac{\partial^2 \theta_1}{\partial y^2} \right) - \alpha \theta_1 \quad (21)$$

with boundary conditions

$$\left. \begin{aligned} u_1 &= -\frac{du_0}{dy}, & v_1 &= 0, & \theta_1 &= -\frac{d\theta_0}{dy}, & y &= 0 \\ u_1 &= 0, & v_1 &= 0, & \frac{\partial \theta_1}{\partial y} &= 0, & y &= 1 \end{aligned} \right\}. \quad (22)$$

Eqs. (18)–(21) with boundary conditions (22) are simplified by introducing the stream function $\Psi(x, y)$ such that

$$u_1 = -\frac{\partial \Psi}{\partial y}, \quad v_1 = \frac{\partial \Psi}{\partial x}. \quad (23)$$

Note that Eq. (20) is satisfied identically and eliminating p_1 from (18) and (19), reduce to

$$u_0 \left(\frac{\partial^3 \Psi}{\partial x^3} + \frac{\partial^3 \Psi}{\partial x \partial y^2} \right) - \frac{d^2 u_0}{dy^2} \frac{\partial \Psi}{\partial x} - \frac{\partial^4 \Psi}{\partial x^4} - \frac{\partial^4 \Psi}{\partial y^4} - 2 \frac{\partial \Psi^4}{\partial x^2 \partial y^2} + M \frac{\partial^2 \Psi}{\partial y^2} + \frac{1}{K} \frac{\partial^2 \Psi}{\partial y^2} + G \frac{\partial \theta_1}{\partial y} = 0 \quad (24)$$

$$Pr \left(u_0 \frac{\partial \theta_1}{\partial x} + \frac{\partial \Psi}{\partial x} \frac{d\theta_0}{dy} \right) = \omega \left(\frac{\partial^2 \theta_1}{\partial x^2} + \frac{\partial^2 \theta_1}{\partial y^2} \right) - \alpha \theta_1 \quad (25)$$

with boundary conditions

$$\left. \begin{aligned} \frac{\partial \Psi}{\partial y} &= -\frac{du_0}{dy}, & \frac{\partial \Psi}{\partial x} &= 0, & \theta_1 &= -\frac{d\theta_0}{dy}, & y &= 0 \\ \frac{\partial \Psi}{\partial y} &= 0, & \frac{\partial \Psi}{\partial x} &= 0, & \frac{\partial \theta_1}{\partial y} &= 0, & y &= 1 \end{aligned} \right\}. \quad (26)$$

Due to the nature of the wall motion, we assume wave-like solutions of the

$$\Psi(x, y) = \text{Real} \left[\sum_r \left(\psi_r \lambda^r e^{(i\lambda x)} \right) \right] \quad (27)$$

$$\theta_1(x, y) = \text{Real} \left[\sum_r \left(t_r \lambda^r e^{(i\lambda x)} \right) \right] \quad (28)$$

where ($r = 0, 1, \dots$) substituting Eq. (27) into (24) and (28) into (25) with boundary conditions (26), we obtain the following sets of ordinary differential equations:

$$\frac{d^4 \psi_0}{dy^4} - M \frac{d^2 \psi_0}{dy^2} - \frac{1}{K} \frac{d^2 \psi_0}{dy^2} = G \frac{dt_0}{dy} \quad (29)$$

$$\omega \frac{d^2 t_0}{dy^2} - \alpha t_0 = 0 \quad (30)$$

with corresponding boundary conditions

$$\left. \begin{aligned} \psi'_0 &= u'_0, & \psi_0 &= 0, & t_0 &= -\theta'_0, & y &= 0 \\ \psi'_0 &= 0, & \psi_0 &= 0, & t'_0 &= 0, & y &= 1 \end{aligned} \right\} \quad (31)$$

$$\frac{d^4 \psi_1}{dy^4} - M \frac{d^2 \psi_1}{dy^2} - \frac{1}{K} \frac{d^2 \psi_1}{dy^2} + i \left(\psi_0 \frac{d^2 u_0}{dy^2} - u_0 \frac{d^2 \psi_0}{dy^2} \right) = G \frac{dt_1}{dy} \quad (32)$$

$$\omega \frac{d^2 t_1}{dy^2} - \alpha t_1 = i Pr \left(u_0 t_0 + \psi_0 \frac{d\theta_0}{dy} \right) \quad (33)$$

$$\left. \begin{aligned} \psi'_j &= 0, & \psi_j &= 0, & t_j &= 0, & y &= 0 \\ \psi'_j &= 0, & \psi_j &= 0, & t'_j &= 0, & y &= 1 \end{aligned} \right\} \quad (34)$$

for $j \geq 1$.

The zeroth-order Eqs. (15) and (16) are solved with boundary condition (17) to obtain the mean temperature and velocity as follows:

$$\theta_0 = A_2 e^{-\beta y} + A_3 e^{\beta y} \quad (35)$$

$$u_0 = A_8 (e^{\gamma y} - e^{-\gamma y}) + A_{10} e^{-\beta y} + A_{11} e^{\beta y}. \quad (36)$$

3. Differential transform method of solution

Taking the Differential transform of Eqs. (29) and (30) with boundary condition (31), we obtain respectively

$$\psi_0(k+4) = \frac{k!}{(k+4)!} \left(\frac{(k+2)!}{k!} M \psi_0(k+2) + \frac{(k+2)!}{k!} \frac{1}{K} \psi_0(k+2) + G(k+1) T_0(k+1) \right) \quad (37)$$

$$T_0(k+2) = \frac{k!}{(k+2)!} \left(\frac{\alpha T_0(k)}{\omega} \right) \quad (38)$$

and boundary conditions

$$\left. \begin{aligned} \psi_0 &= A_{13}, & \psi_0(1) &= 0, & T_0 &= -A_{12}, & y &= 0 \\ \sum_{k=0}^n k \psi_0(k) &= 0, & \sum_{k=0}^n \psi_0(k) &= 0, & \sum_{k=0}^n k T_0(k) &= 0, & y &= 1 \end{aligned} \right\}. \quad (39)$$

Substituting the exact solutions (35) and (36) into Eqs. (32) and (33) and taking their Differential Transforms give

$$\begin{aligned} \psi_1(k+4) &= \frac{k!}{(k+4)!} \left(\frac{(k+2)!}{k!} M \psi_1(k+2) + \frac{(k+2)!}{k!} \frac{1}{K} \psi_1(k+2) + G(k+1) t_1(k+1) \right. \\ &\quad - i \left(A_{26} \sum_{r=0}^k \frac{\gamma^r}{r!} \psi_0(k-r) - A_{26} \sum_{r=0}^k \frac{(-\gamma)^r}{r!} \psi_0(k-r) \right) + A_{27} \sum_{r=0}^k \frac{(-\beta)^r}{r!} \psi_0(k-r) \\ &\quad \times A_{28} \sum_{r=0}^k \frac{\beta^r}{r!} \psi_0(k-r) + A_8 \sum_{r=0}^k (k-r+1)(k-r+2) \frac{\gamma^r}{r!} \psi_0(k-r+2) \\ &\quad - A_8 \sum_{r=0}^k (k-r+1)(k-r+2) \frac{(-\gamma)^r}{r!} \psi_0(k-r+2) + A_{10} \sum_{r=0}^k (k-r+1) \\ &\quad \times (k-r+2) \frac{(-\beta)^r}{r!} \psi_0(k-r+2) \\ &\quad \left. + A_{11} \sum_{r=0}^k (k-r+1)(k-r+2) \frac{(-\beta)^r}{r!} \psi_0(k-r+2) \right) \end{aligned} \quad (40)$$

$$\begin{aligned} T_1(k+2) &= \frac{k!}{(k+2)!} \left(\frac{\alpha T_1(k)}{\omega} + iPr \left(A_8 \sum_{r=0}^k \frac{\gamma^r}{r!} T_0(k-r) - A_8 \sum_{r=0}^k \frac{(-\gamma)^r}{r!} \psi_0(k-r) \right. \right. \\ &\quad + A_{10} \sum_{r=0}^k \frac{(-\beta)^r}{r!} T_0(k-r) + A_{11} \sum_{r=0}^k \frac{\beta^r}{r!} T_0(k-r) + A_{17} \sum_{r=0}^k \frac{(-\beta)^r}{r!} \psi_0(k-r) \\ &\quad \left. \left. + A_{18} \sum_{r=0}^k \frac{\beta^r}{r!} \psi_0(k-r) \right) \right) \end{aligned} \quad (41)$$

$$\left. \begin{aligned} \psi_1(0) &= 0, & \psi_1(1) &= 0, & T_1 &= 0, & y &= 0 \\ \sum_{k=0}^n k \psi_1(k) &= 0, & \sum_{k=0}^n \psi_1(k) &= 0, & \sum_{k=0}^n k T_1(k) &= 0, & y &= 1 \end{aligned} \right\}. \quad (42)$$

The solutions of (37) and (38) satisfying the boundary conditions (37) with solutions (40) and (41) and boundary conditions (42) are given as

$$t_0 = -A_{12} + \frac{1}{3} \frac{\alpha A_{12} (6\omega + \alpha) y}{\omega (2\omega + \alpha)} - \frac{1}{2} \frac{\alpha A_{12} y^2}{\omega} + \frac{1}{18} \frac{\alpha^2 A_{12} (6\omega + \alpha) y^3}{\omega^2 (2\omega + \alpha)} - \frac{1}{24} \frac{\alpha^2 A_{12} y^4}{\omega^2} \quad (43)$$

$$\psi_0 = A_{13} - \frac{1}{6} A_{14} K y^2 + \frac{1}{3} \frac{A_{15} y^3}{A_{16}} + \left(-\frac{A_{14} MK}{72} - \frac{A_{14} M}{72} + \frac{A_{12} G\alpha (6\omega + \alpha)}{72\omega (2\omega + \alpha)} \right) y^4 \quad (44)$$

$$t_1 = -\frac{1}{3} \frac{A_{25} yi}{A_{24}} + \frac{1}{2} \frac{y^2 i Pr A_{19}}{\omega} + \frac{1}{6} \frac{y^3}{\omega} \left(-\frac{1}{3} \frac{\alpha i A_{25}}{A_{24}} + i Pr A_8 (-A_{17} \beta A_{13} + A_{18} \beta A_{13} + A_{21} + A_{22})) \right) + \frac{1}{12} \frac{y^4}{\omega} \left(\frac{1}{2} \frac{\alpha i Pr A_{19}}{\omega} - \frac{1}{6} \frac{i Pr A_{23}}{A_{24}} \right) \quad (45)$$

$$\psi_1 = \frac{1}{12} K A_{29} y^2 + \frac{1}{6} K A_{29} y^3 + \left(-\frac{A_{29} MK}{144} - \frac{A_{29}}{144} - \frac{1}{24} \left(2 A_{27} + 2 A_{28} - \frac{1}{6} A_{10} A_{14} K - \frac{1}{6} A_{11} A_{14} K \right) + \frac{124}{G} A_{28} \right) y^4. \quad (46)$$

The expressions for the constants are given in the [Appendix](#).

3.1. Fluid pressure

The fluid pressure $P(x, y)$ at any point (x, y) can be obtained from Eqs. (9) and (8)

$$P(x, y) = \int dP = \int \left(\frac{\partial P}{\partial x} dx + \frac{\partial P}{\partial y} dy \right). \quad (47)$$

In view of Eqs. (23), (27) and (28), Eq. (49) becomes

$$P(x, y) - C_1 = x \left(\frac{d^2 u_0}{dy^2} - M u_0 - \frac{1}{K} u_0 + G \theta_0 \right) + \text{Real} \left[u_0 \psi'_0 - \frac{du_0}{dy} \psi_0 + \lambda \left(u_0 \psi'_1 - \frac{du_0}{dy} \psi_1 \right) + i \lambda \left(-\psi_0 + \int \psi_0 dy \right) + i \left(\psi_1''' - M \psi_1' - \frac{1}{K} \psi_1' - G t_1 + \lambda^{-1} \left(\psi_0''' - M \psi_0' - \frac{1}{K} \psi_0' - G t_1 \right) \right) \right] \varepsilon e^{i\lambda x} \quad (48)$$

where C_1 is an arbitrary constant.

Eq. (48) can be written [3] as $\nabla P = P(x, y) - P(x, 1)$, where ∇P is the pressure drop that indicate the differences between the pressure at any point y in the fluid flow field with x fixed. The pressure drop (∇P) is at $\lambda x = 0$ or 2π .

3.2. Skin friction

The shear stress on the wavy boundaries, $y = \varepsilon \cos(\lambda x)$ and $y = 1 + \varepsilon \cos(\lambda x)$ is given as

$$\tau = u'(0) + \varepsilon \left(-\psi_r''(0) \cos(\lambda x) + \lambda \psi_i''(0) \sin(\lambda x) \right) \quad (49)$$

$$\tau = u'(1) + \varepsilon \left(-\psi_r''(1) \cos(\lambda x) + \lambda \psi_i''(1) \sin(\lambda x) \right). \quad (50)$$

3.3. Nusselt number

The rate of heat transfer at the wavy boundaries, $y = \varepsilon \cos(\lambda x)$ and $y = 1 + \varepsilon \cos(\lambda x)$ is given as

$$Nu = -\theta'(0) - \varepsilon \left(-t_r'(0) \cos(\lambda x) + \lambda t_i'(0) \sin(\lambda x) \right) \quad (51)$$

$$Nu = -\theta'(1) - \varepsilon \left(-t_r'(1) \cos(\lambda x) + \lambda t_i'(1) \sin(\lambda x) \right). \quad (52)$$

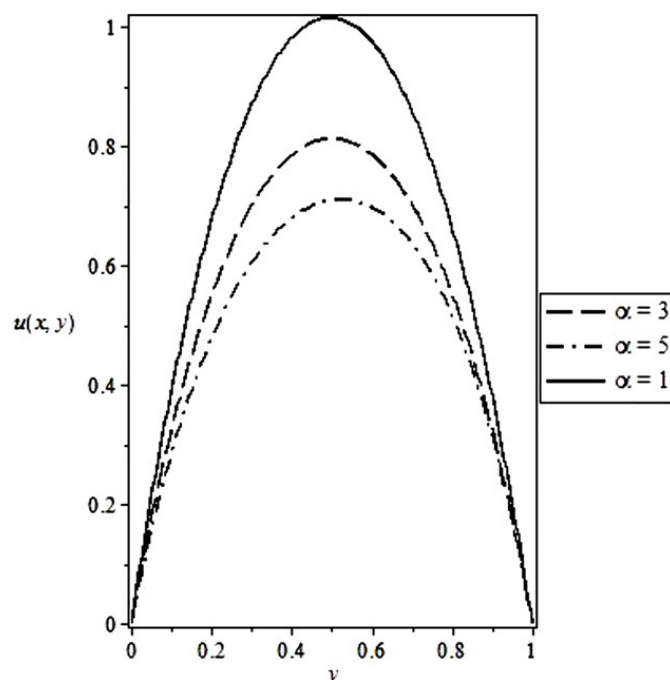


Fig. 2. Velocity profiles for different values of α .

4. Discussion of results

A series of computations has been carried out for the effects of the following parameters: heat source (α), radiation parameter (ω), Hartmann number (M), Grashof number (Gr), porosity parameter (K), frequency (λ) and epsilon (ε) on the velocity, temperature, skin friction as well as Nusselt number. The analysis of the fluid flow for the distribution profiles with the following default values for the parameters, $\alpha = 1.0$, $\omega = 1.0$, $M = 1.0$, $G = 1.0$, $K = 1.0$, $C = 1$, $\lambda = 0.01$, and $x = 1.0$. All the graphs use the default values except otherwise stated.

Figs. 2 and 3 depict the influence of heat source (α) and radiation parameter (ω) respectively on the velocity profile. It is observed that an increase in heat source parameter causes a reduction in the buoyancy effect which reduces the fluid velocity and the velocity profile increases as the radiation parameter (ω) increases, thereby increasing the momentum boundary layer thickness. This is because when the intensity of heat produced through thermal radiation increased, the bond holding the components of the fluid particles together is easily broken and the fluid velocity increased.

The effects of magnetic field parameter (M) are presented in Fig. 4. It is observed that the effects of increasing values of the magnetic field parameter results in decreasing velocity distribution profiles. This is due to the fact that the transverse magnetic field normal to the flow direction creates a drag force due to Lorentz force and hence results in reducing the velocity profiles.

Fig. 5 shows variation of velocity distribution with different values of Grashof number. It can be seen that an increase in Grashof number leads to a rise in velocity profiles. Fig. 6 presents variation of velocity profiles with permeability parameter. It is observed that the presence of permeability parameter reduces the resistance of the porous medium thereby enhance the fluid velocity.

Fig. 7 illustrates the influence of heat source parameter (α) on velocity profiles. It is observed that the velocity of the fluid decreases with increase in the values of the heat source. The variation of temperature profiles for different values of thermal radiation parameter (ω) is shown in Fig. 8. The results show that the temperature profile increases with increase in the thermal radiation parameter and hence there would be an increase of thermal boundary layer thickness.

Fig. 9 illustrates the influence of thermal radiation parameter (ω) on the fluid pressure. It can be that fluid pressure decreases as the radiation parameter increases. Fig. 10 presents the trend of the fluid pressure with variation of heat source parameter (α). It can be seen that an increase in heat source parameter produces a rise in the fluid pressure.

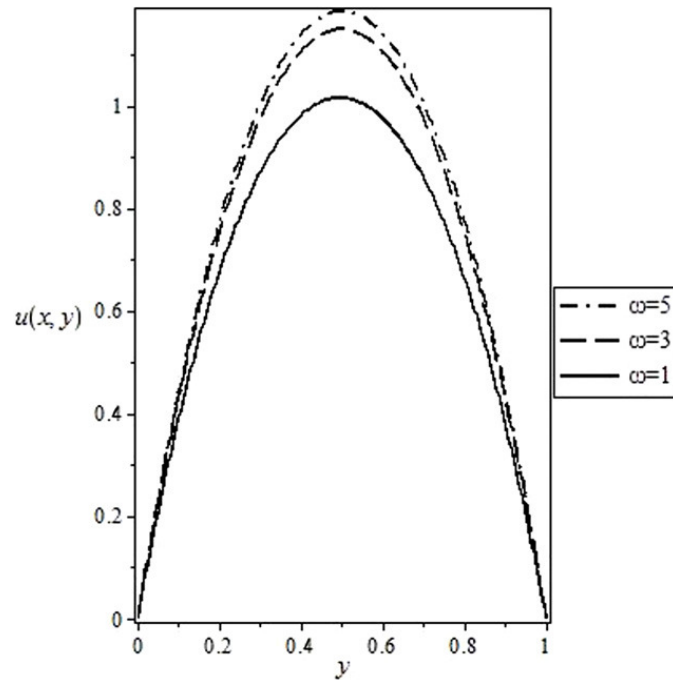
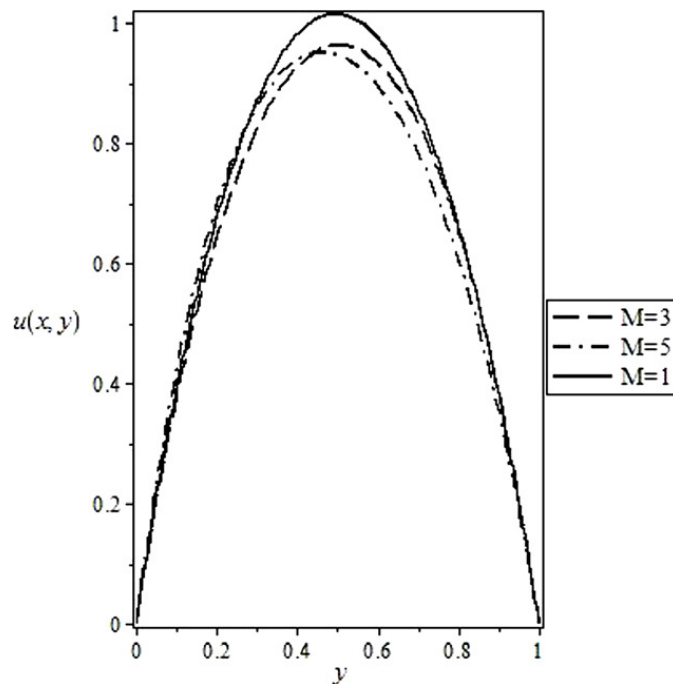
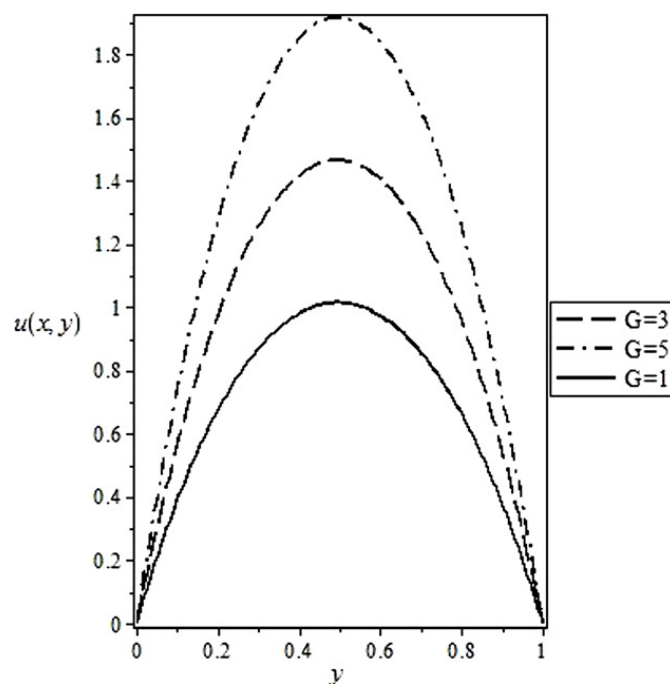
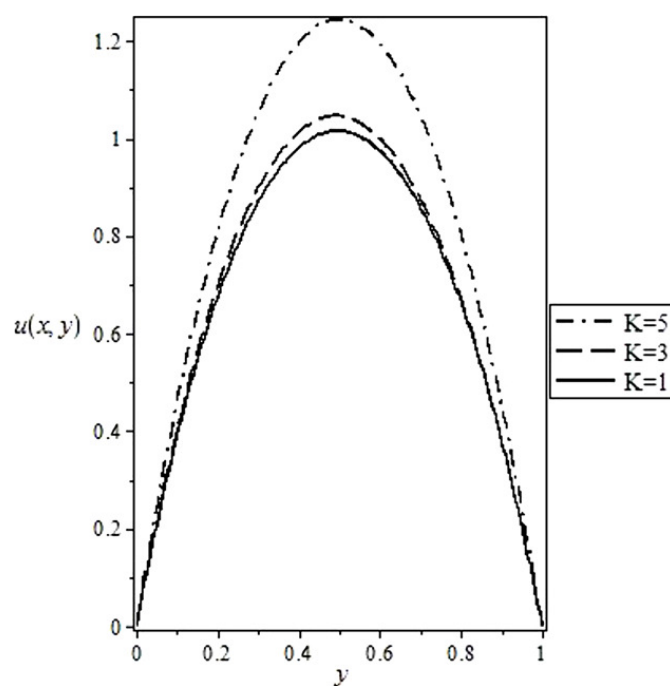
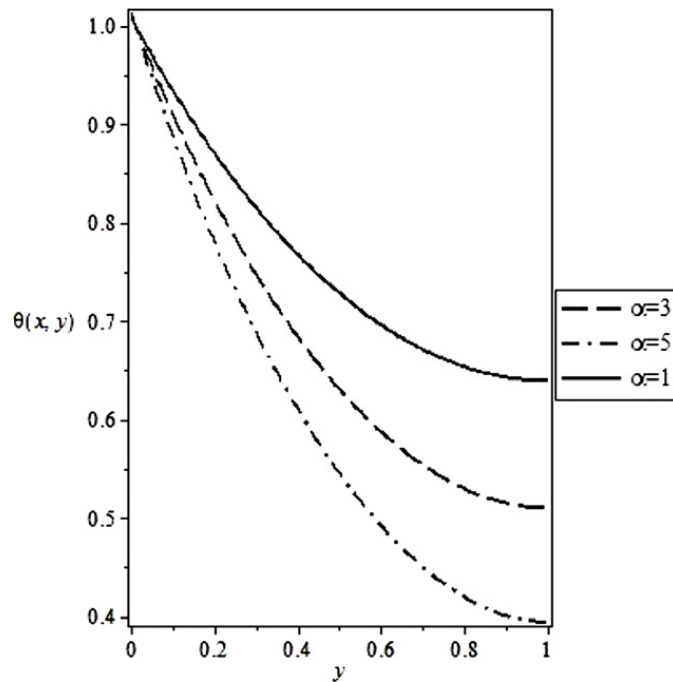
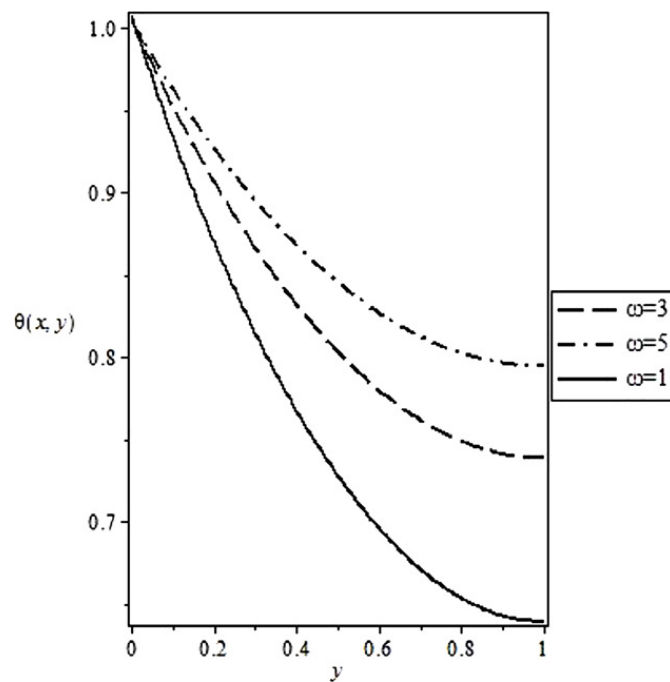
Fig. 3. Velocity profiles for different values of ω .Fig. 4. Velocity profiles for different values of M .

Fig. 11 depicts the fluid pressure with different values of magnetic parameter M . It is observed that an increase in magnetic parameter increases the fluid pressure. The effects of Grashof number on the fluid pressure are presented in Fig. 12. An increase in Grashof number significantly changes the behavior of the fluid pressure. It can be seen that the fluid pressure decreases from $y = 0$ – 0.2 but increases sharply from $y = 0.3$ – 1 .

Fig. 13 represents the variation of permeability parameter on the fluid pressure. It is observed that an increase in the permeability parameter causes a reduction in the fluid pressure distribution profiles.

Fig. 5. Velocity profiles for different values of G .Fig. 6. Velocity profiles for different values of K .

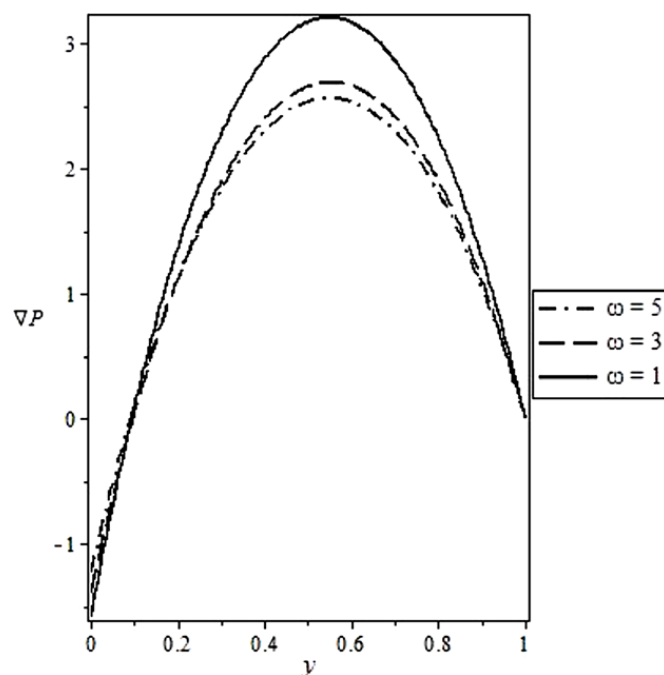
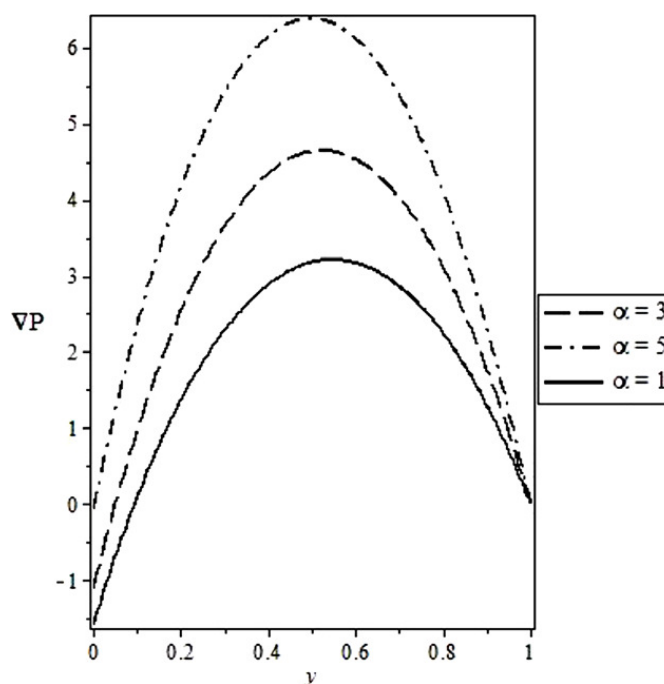
The skin friction coefficient and Nusselt number are expressed in Eqs. (49)–(52) and are shown in Tables 1 and 2 for different values of entering parameters. In order to highlight the contributions of each parameter, one parameter is varied while the rest take default fixed values. It is observed from Table 1 that an increase in any of the parameter ω , G , K and C produces a rise in the skin friction while increase in any of parameter α and M resulted in reduction in the skin friction coefficient at the wall $y = 0$. It can be seen that an increase in any of parameter ω , G , K and C causes a reduction in the skin friction while increase in α or M increases the skin friction at the wall $y = 1$.

Fig. 7. Temperature profiles for different values of α .Fig. 8. Temperature profiles for different values of ω .

As for the Nusselt number shown in Table 2, an increase in α produces corresponding rise in the Nusselt number while an increase in ω causes a reduction in the Nusselt number at the wall $y = 0$. It is observed that increasing α the Nusselt number as α increases while ω causes the Nusselt number to fall at $y = 1$.

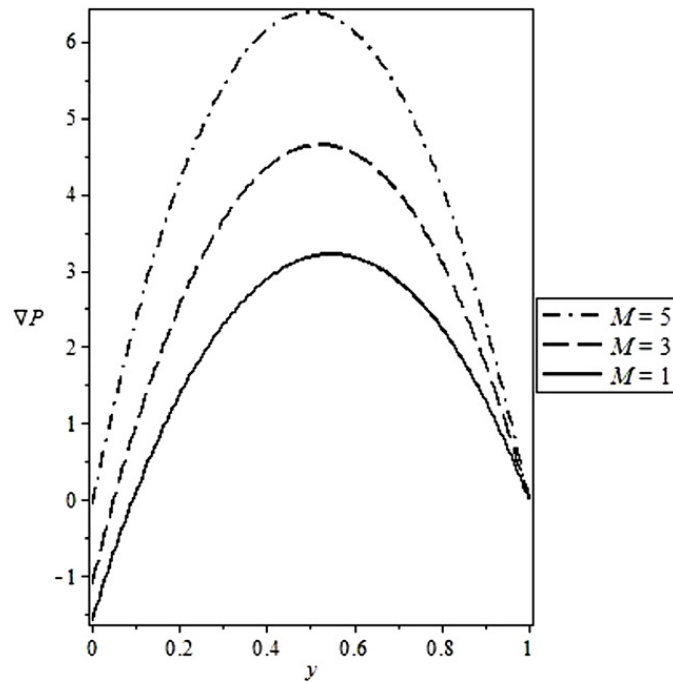
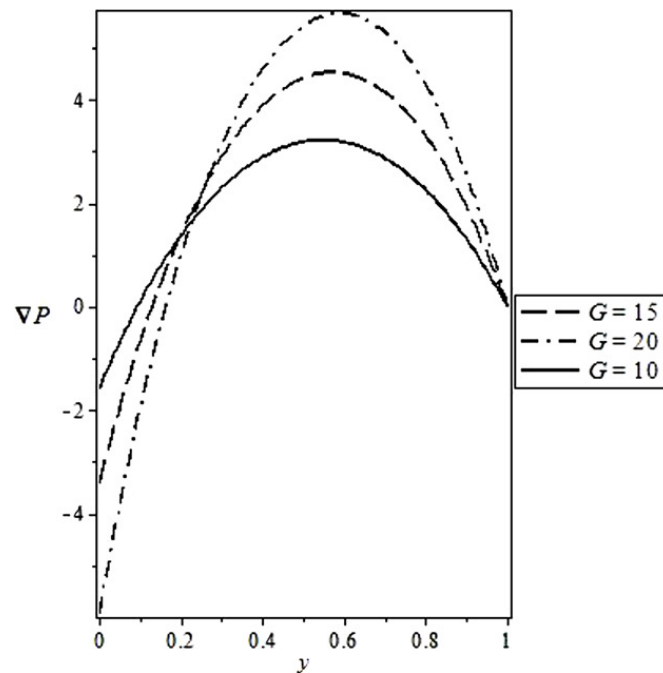
5. Conclusion

The effects of radiation and heat transfer on MHD free convective flow in a porous medium between two vertical wavy walls are presented. The resulting governing equations from the mathematical model of the problem are non-dimensionalized, simplified and solved using Differential Transform Method (DTM). The results are presented

Fig. 9. Fluid pressure for different values of ω .Fig. 10. Fluid pressure for different values of α .

in graphs and tables and discussed for different values of parameters entering into the problem. The following conclusions can be drawn from the results obtained:

- (i) The velocity increases with increase in the value of ω , G , K and C while it decreases with an increasing α and M .
- (ii) The temperature profile increases with corresponding increase in the value of ω whereas the temperature profile falls with increase in the value of α .
- (iii) The fluid pressure profile distribution profiles increase with an increase in the value of α and M whereas it decreases with an increase in ω and K . However, It depicted different behavior as the value of G increases i.e. the fluid pressure decreases from $y = 0$ – 0.2 but increases from $y = 0.3$ – 1 .

Fig. 11. Fluid pressure for different values of M .Fig. 12. Fluid pressure for different values of G .

- (iv) The skin friction increases with increase in the value of ω , G , K and C while an increase in ω or M causes a reduction in the skin friction at $y = 1$. However, the skin friction decreases with increasing ω , G and K whereas α or M causes a reduction in the skin friction at $y = 0$.
- (v) The Nusselt number increases with an increase in the of α while increase in ω causes a fall at $y = 0$. However, an increase in α or ω causes a reduction in the Nusselt number at $y = 1$.

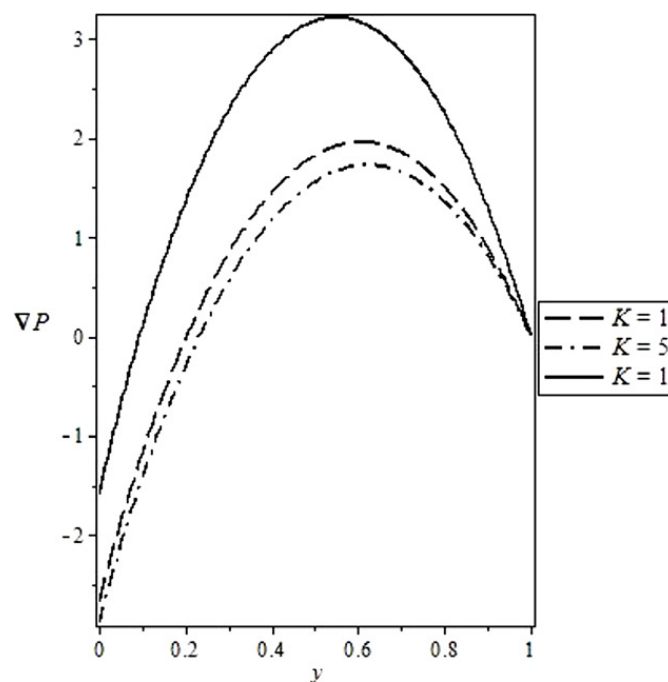
Fig. 13. Fluid pressure for different values of K .

Table 1

The skin friction coefficients for various values of γ , α , ω , M , G and K .

Parameters	$y = 0$	$y = 1$
$\alpha = 1$	4.457812	-4.332163
$\alpha = 3$	3.784857	-3.802260
$\alpha = 5$	3.993055	-3.782640
$\omega = 1$	4.457812	-4.332163
$\omega = 3$	4.867432	-4.851904
$\omega = 5$	4.974579	-5.002667
$M = 1$	4.457812	-4.332163
$M = 3$	4.346947	-4.84436
$M = 5$	5.042031	-4.33216
$G = 10$	4.457812	-4.332163
$G = 15$	6.452219	-6.258578
$G = 20$	8.446625	-8.184994
$K = 1$	4.457812	-4.332163
$K = 3$	4.574880	-4.307417
$K = 5$	4.601471	-4.306132
$C = 1$	4.457812	-4.332163
$C = 3$	5.395806	-5.290827
$C = 5$	6.333818	-6.249496

Table 2

The Nusselt Number for different values of α and ω .

Parameters	$y = 0$	$y = 1$
$\alpha = 1$	0.77172869	-2×10^{-7}
$\alpha = 3$	1.05994957	-4.95×10^{-10}
$\alpha = 5$	1.31555645	$-7. \times 10^{-10}$
$\omega = 1$	0.77172869	-2×10^{-7}
$\omega = 3$	0.55246929	-1.298×10^{-10}
$\omega = 5$	0.43145564	-6.98×10^{-11}

Appendix

$$\begin{aligned}
\beta &= \frac{\sqrt{\alpha}}{\sqrt{\omega}}, & A &= e^{-\beta} + e^{\beta}, & A_1 &= \frac{1}{A}, & A_2 &= A_1 e^{\beta}, & A_3 &= A_1 e^{-\beta}, & A_4 &= \frac{\sqrt{KM+1}}{\sqrt{K}} \\
A_5 &= -2 + e^{-(A_4+\beta)} + e^{-(A_4-\beta)}, & A_6 &= \ln(e)^2 K \beta^2 + KM + 1, & A_7 &= e^{A_4} + e^{-A_4} \\
A_8 &= \frac{A_1 A_5 G K}{A_6 A_7}, & A_9 &= \frac{K G A_1}{A_6}, & A_{10} &= A_9 e^{\beta}, & A_{11} &= A_9 e^{-\beta}, \\
A_{12} &= -A_2 \beta \ln(e) + A_3 \beta \ln(e), & A_{13} &= 2 A_8 \gamma \ln(e) - A_{10} \beta \ln(e) + A_{11} \beta \ln(e), \\
A_{14} &= \frac{(A_{12} G \alpha^2 + 6 A_{12} G \alpha \omega - 216 A_{13} \alpha \omega - 432 A_{13} \omega^2)}{\omega (K M \alpha + 2 K M \omega - 12 K \alpha - 24 K \omega + M \alpha + 2 M \omega)} \\
A_{15} &= A_{12} G K \alpha^2 + 6 A_{12} G K \alpha \omega - 12 A_{13} K M \alpha \omega - 24 A_{13} K M \omega^2 - 72 A_{13} K \alpha \omega \\
&\quad - 144 A_{13} K \omega^2 - 12 A_{13} M \alpha \omega - 24 A_{13} M \omega^2 \\
A_{16} &= \omega (2 \omega + \alpha) (K M - 12 K + M), & A_{17} &= -A_2 \beta \ln(e), & A_{18} &= A_3 \beta \ln(e) \\
A_{19} &= -A_{10} A_{12} - A_{11} A_{12} + A_{13} A_{17} + A_{13} A_{18}, & A_{20} &= \frac{1}{3} \frac{\alpha A_{12} (6 \omega + \alpha)}{\omega (2 \omega + \alpha)} \\
A_{21} &= \frac{1}{3} \frac{A_{10} A_{12} (3 \alpha \beta \omega + 6 \beta \omega^2 + \alpha^2 + 6 \alpha \omega)}{\omega (2 \omega + \alpha)}, \\
A_{22} &= \frac{1}{13} \frac{A_{11} A_{12} (-3 \alpha \beta \omega - 6 \beta \omega^2 + \alpha^2 + 6 \alpha \omega)}{\omega (2 \omega + \alpha)} \\
A_{23} &= 3 A_{10} A_{12} \alpha \beta^2 \omega + 6 A_{10} A_{12} \beta^2 \omega^2 + 3 A_{11} A_{12} \alpha \beta^2 \omega + 6 A_{11} A_{12} \beta^2 \omega^2 - 3 A_{13} A_{17} \alpha \beta^2 \omega \\
&\quad - 6 A_{13} A_{17} \beta^2 \omega^2 - 3 A_{13} A_{18} \alpha \beta^2 \omega - 6 A_{13} A_{18} \beta^2 \omega^2 + 2 A_{10} A_{12} \alpha^2 \beta + 12 A_{10} A_{12} \alpha \beta \omega \\
&\quad - 2 A_{11} A_{12} \alpha^2 \beta - 12 A_{11} A_{12} \alpha \beta \omega - 4 A_{12} A_8 \alpha^2 \gamma - 24 A_{12} A_8 \alpha \gamma \omega + A_{14} A_{17} K \alpha \omega \\
&\quad + 2 A_{14} A_{17} K \omega^2 + A_{14} A_{18} K \alpha \omega + 2 A_{14} A_{18} K \omega^2 + 3 A_{10} A_{12} \alpha^2 + 6 A_{10} A_{12} \alpha \omega \\
&\quad + 3 A_{11} A_{12} \alpha^2 + 6 A_{11} A_{12} \alpha \omega, & A_{24} &= \omega (2 \omega + \alpha) \\
A_{25} &= \alpha Pr A_{19} + 6 Pr A_{19} \omega + 3 Pr A_8 (-\gamma A_{12} + A_{20}) - A_8 (\gamma A_{12} + A_{20}) + (-A_{17} \beta A_{13} \\
&\quad + A_{18} \beta A_{13} + A_{21} + A_{22}) \omega - \frac{1}{3} \frac{Pr \omega A_{23}}{A_{24}} \\
A_{26} &= A_8 \gamma^2 \ln(e)^2, & A_{27} &= A_{10} \beta^2 \ln(e)^2, & A_{28} &= A_{11} \beta^2 \ln(e)^2 \\
A_{29} &= \frac{K (A_{10} A_{14} K i + A_{11} A_{14} K i - 12 A_{27} i + 6 G A_{28} - 12 A_{28} i)}{M K - 12 K + 1}.
\end{aligned}$$

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