



Improved Energy Detection Algorithm for Cognitive Radios in Cooperative Spectrum Sensing

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Abstract

Background: The performance of algorithms used in white space detection in cooperative spectrum sensing is largely dependent on the sensitivity of the cognitive radios used. The sensitivity of the radios is measured in terms of the probability of missed detections and probability of false alarms. **Aim:** This work is aimed at improving on the detection sensitivity of the Conventional Energy Detection (CED) algorithm while preserving its simplicity. **Method:** The detection performance of the CED was improved by avoiding the misdetections caused by instantaneous signal energy drops through the use Modified Energy Detection (MED) and Improved Energy Detection (IED) algorithms in cooperative spectrum sensing. **Results:** The IED scheme proposed reduces the false alarm ratio of the MED scheme while preserving the detection performance improvement attained with respect to the CED method. With a probability of false alarm at 0.1, the MED algorithm increases the CED detection probability from 0.2 to 0.4, while the IED algorithm increases the probability of detection to 0.8, in poor channel conditions where the CED performs unreliably. This fulfils the objective of improving the sensitivity of energy detection in cooperative spectrum sensing

Keywords: Cognitive Radio, Cooperative Spectrum Sensing, Detection Sensitivity. Energy detection.

1 Introduction

Spectrum sensing is one of the cardinal functions of cognitive radios. It is the process whereby unused spectrum is detected and unlicensed (secondary) users are intelligently permitted to utilize the vacant bands which are allocated to licensed (primary) users. Various spectrum sensing algorithms in the context of cognitive radio have been proposed in literature to detect the presence of primary users. Some examples of the existing proposals besides energy detection (Urkowitz, 1967) include: Cyclostationary feature detection (Gardner & Spooner, 1992), matched filter detection (Price & Abramson, 1961), covariance-based detection (Yonghong Zeng & Ying-Chang Liang, 2009), multi-taper spectrum estimation (Thomson, 1982) and filter bank spectrum estimation (Hussein Moradi, Behrouz Farhang, & Carl A. Kutsche, 2014).

The existing solutions provide different trade-offs between required sensing time, complexity and detection capabilities, but their practical applicability depends on how much information is available about the primary user signal. Even though, the performance of Energy Detection is limited by factors such as multipath fading, shadowing and consequently, the hidden terminal problem, it still has the lowest computational and implementation costs which usually makes it the preferred technique. Effort is



therefore made in this research to utilize cognitive radios using the energy detection principle in the cooperative scheme of spectrum sensing.

Some of the factors limiting the performance of energy detection technique have been mitigated with the introduction of the other more detection-sensitive techniques mentioned above as well as when incorporated in cooperative spectrum sensing. But these other solutions have their limitations such as increased complexity and communication overhead. Furthermore, misdetections still occur caused by instantaneous signal energy drop which the conventional cooperative spectrum sensing algorithm has not been able to forestall.

The aim of this work therefore is to improve on the detection sensitivity of the cognitive radios used in cooperative spectrum sensing while preserving the simplicity of the energy detection technique. The objectives pursued to achieve this aim were first to simulate the conventional non-cooperative and cooperative sensing schemes to compare their performance and to validate the advantage cooperative sensing has over the single user sensing technique especially in overcoming the hidden node problem; improving the detection performance of individual cognitive radios by upgrading their sensing schemes with more detection sensitive algorithms; then incorporating the upgraded cognitive radios in cooperative sensing to improve the its sensitivity.

Cognitive radios perform spectrum sensing, analysis and decision making. The scope of the work is however limited to spectrum sensing function of cognitive radios and the work is particularly focused on cooperative spectrum sensing technique of Cognitive Radio Systems.

2 RELATED WORKS

Researchers over the years tried to improve detections sensitivity of energy detection-based spectrum sensing using several techniques. (Fatih, 2010) used the concept of an adaptive threshold for the energy detector to improve the performance of the sensing scheme along with multiple antennas. This produced significant improvements. Several combining techniques for the cognitive radio users in cooperative spectrum sensing were then considered while utilizing different modulation schemes before selecting the use of equal gain combining (EGC) which gave the highest gain. However, the performance of this technique in noisy channels was not fully predictable. Paisana (2012) also evaluated the adaptive threshold energy detector for various spectrum sensing schemes. The results produced some theoretical basis of these techniques. The focus of the work was on the reduction of the complexity of local sensing algorithms while maintaining the efficiency of the system. These techniques were implemented in cooperative spectrum sensing and significant improvements were observed. The selection of the appropriate threshold in low signal-to-noise ratio (SNR) conditions was also a challenge, due to noise uncertainty which makes it difficult to estimate the noise variance.

(Lopez-Benitez & Casadevall F., 2012) proposed and evaluated an improved version of the energy detection algorithm that is able to outperform the classical energy detection scheme while ensuring that the computational complexity does not increase. The experiments were however performed only for single user spectrum sensing. (Eghbali, Hassani, Koohian, & Ahmadian-Attari, 2014) used this improved energy detector for joint detection in a wideband secondary network. The joint detection of sub-bands was formulated as a set of optimization problems which improved opportunistic access for the secondary user but increased the computational demand on each of the cognitive radio units.



(Kanti, Barma, Singh, Roy, & Sen, 2015) proposed an augmented spectrum sensing algorithm where the energy detector's detection is augmented by cyclostationary detection. This however requires some information about the primary users' transmission characteristics which is not always available. More recently, (Vishnu, Dias, Tholeti, & Sheetal, 2018) proposed a two-stage reinforcement learning approach to improve the performance of the cooperative sensing. The method helped minimize the number of sensing operations and reduced the energy required for sensing. This improved the channel sensing and allocation, but demanded computational resources and learning time.

In an effort to improve the detection sensitivity of energy detection, (Salama, Sarker, & Chakrabarty, 2018) utilized some information connected with the channel activity for matched filter detection. The matched filter was then incorporated in the energy detector to reduce signal interference. This produced significant improvements on the performance of the energy detection. The limitation of the technique is that the information used for the matched filtering may not always be available. (Kumar, Thakur, Pandit, & Singh, 2019) further explored optimal threshold selection at low SNR to provide better sensing performance. The results provided better sensing performance compared to previous approaches which used constant false-alarm rate and constant detection rate threshold selection. But provision was not made in the approach for instantaneous SNR drop which sometimes occur.

(Men, Chargé, Wang, & Li, 2019) focused on the management of limited resources while working on wideband signal detection. The Dempster-Shafer decision process was used while aiming at small samples based on students' t distribution. The technique improved the detection of the traditional energy detection method but just with a small sample size. (Tan, Li, Yang, Wu, & Xu, 2019) applied two thresholds, using minimum and maximum noise variances in a two-step cooperative sensing algorithm to improve detection performance. The technique effectively reduced the complexity and time involved in sensing.

The current paper builds on this and former previous studies reviewed by using an adaptive detection threshold which makes provision for low SNR conditions and also instantaneous fluctuations in channel conditions.

3 BACKGROUND

3.1 System Model

The occupancy status of a radio frequency channel may be either busy or idle. Spectrum sensing by the cognitive radio is to detect these states and to take decisions appropriately. After sampling the signal sensed in continuous time the discrete form of the signal is obtained is usually expressed as:

$$y[d] = s[d] + n[d] \quad (1)$$

where

$y[d]$ = received signal in discrete-time

$x[d]$ = primary user signal in discrete-time

$w[d]$ = noise signal in discrete-time

Assumptions:

The above discrete-time signals were sampled at instants $t = dT_s$, with d being a positive integer,

N represents the sample index and

$T_s = 1/f_s$ is the sampling period.

The received signal is then passed through a filter of order K to produce the equations below as used in López-Benítez M. et al (2006):

$$\hat{s}[d] = \sum_k^K f[k].y[d - k] \quad (2)$$

$$\check{n}[d] = \sum_k^K f[k].y[d - k] \quad (3)$$

$$\hat{y}[d] = \sum_k^K f[k].y[d - k] \quad (4)$$

where

$$\hat{y}[d] = \hat{s}[d] + \check{n}[d] \quad (5)$$

The received signal can then be reduced to minimize the processing load using a decimation factor $D \geq 1$ as expressed in equation 6:

$$\hat{y}[z] = \hat{s}[z] + \check{n}[z] \quad (6)$$

where z = number of samples.

The spectrum sensing algorithm then decides on the occupancy of the frequency channel based on $y[n]$. The hypothesis testing problem then arises similar to as:

$$H_0 : \hat{y}[z] = \check{n}[z] \quad z = 0, 1, \dots, Z - 1 \quad (7)$$

$$H_1 : \hat{y}[z] = \hat{s}[z] + \check{n}[z] \quad z = 0, 1, \dots, Z - 1 \quad (8)$$

where

$\hat{y}[z]$ = sample to be analyzed at each instant z ,

$\check{n}[z]$ = noise (not necessarily white Gaussian noise) of variance σ^2 ,

$\hat{s}[z]$ = is the signal the network wants to detect

H_0 = noise-only hypothesis

H_1 = signal plus noise hypothesis

z = the number of samples collected during the signal observation interval

3.2 Methodology

The work was carried out by first simulating the Conventional Energy Detection (CED) as well as the conventional cooperative sensing schemes. The improved cooperative sensing algorithm is then developed to enhance the detection sensitivity of the multiple sensors.

3.2.1 Conventional Energy Detection (CED) Model

An energy detector, also referred to as a *radiometer*, measures the energy on a particular channel on a narrowband portion of the spectrum and compares it to a pre-set threshold to determine the presence of the primary user in the channel (Sachin, 2012). The standardized test statistic T for the energy detector as used previously in literature (Urkowitz, 1967) is:

$$T' = \left(\frac{1}{N_{02}}\right) \int_0^T y^2(t) dt \quad (9)$$

where:

T' = test statistic in during sensing session

y = received signal input

T = sampling instant

N_{02} = two-sided noise power density spectrum

H_1 hypothesis is the outcome if the test statistics exceeds a fixed decision threshold (λ) while H_0 hypothesis occurs when the test statistics is less than the decision threshold. The equations for the decision threshold, probability of false alarm and detection are presented in equations (10 – 12).

$$\lambda = \sqrt{\frac{2}{NQ^{-1}}} (P_{fa}^{CED} + 1) \quad (10)$$

where:

$$P_{fa}^{CED} = Q\left(\frac{\lambda-1}{\sqrt{\frac{2}{N}}}\right) \quad (11)$$

$$P_d^{CED} = Q\left(\frac{\lambda-(1+\gamma)}{\sqrt{\left(\frac{2}{N}\right)(1+\gamma)^2}}\right) \quad (12)$$

The algorithm for the implementation is presented in **Figure 1**.

3.2.2 Modified Energy Detection (MED)

The MED scheme is developed as an improvement over the CED. The energy of the primary signal sometimes drops instantaneously. If sensing is done at this point, using the CED technique, there is high probability that the detection would not be correct. The channel may be predicted as idle, whereas, the primary user is still present. Allocating a secondary user into this channel would very likely cause interference. The MED scheme makes provision for some of these misdetections by computing an additional test for the average signal energy over a certain period of time as used by M. López-Benítez et al., 2012 in equations (13) and (14).

$$T_i^{avg}(T_i) = \frac{1}{P} \sum_{l=1}^P T_{i-P+l}(x_{i-P+l}) \quad (13)$$

$$T_i = [T_{i-P+1}(x_{i-P+1}), (T_{i-P+2}(x_{i-P+2}), \dots, (T_{i-1}(x_{i-1}), T_i(x_i)] \quad (14)$$

where:

- $T_i^{avg}(T_i)$ = average test statistic in the i -th sensing event
- T_i = test statistic vector
- P = number of previous sensing events.

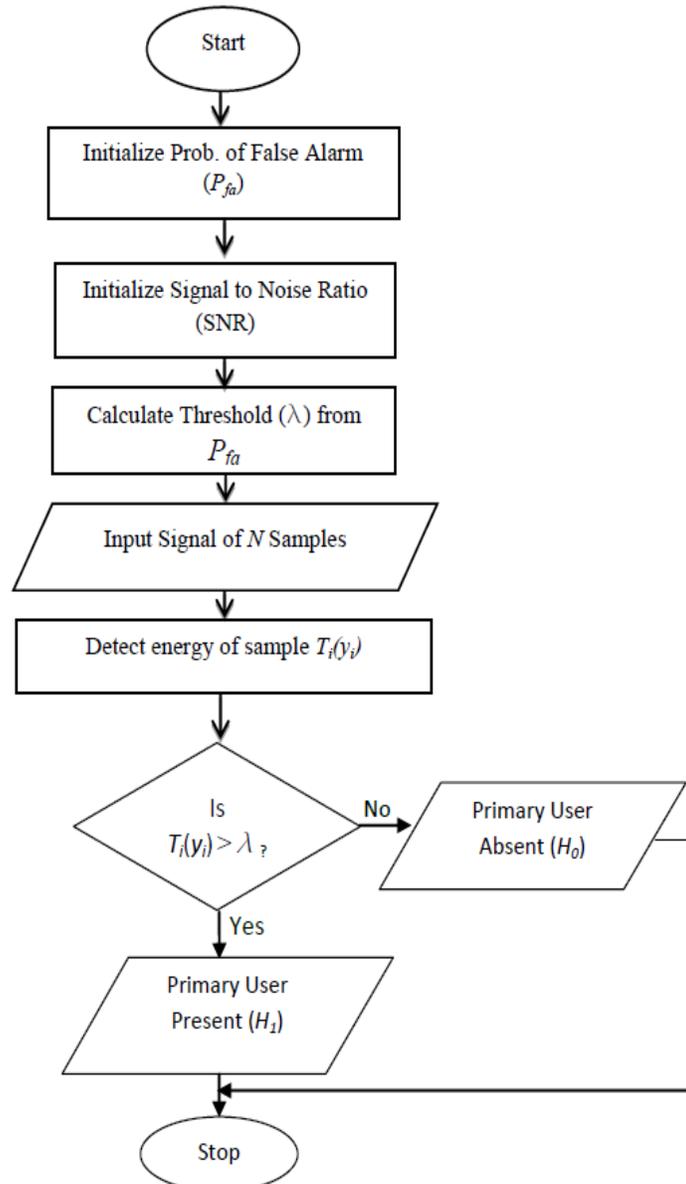


Figure 1: Flowchart for Conventional Energy Detection

The procedure through which the MED will be implemented is presented as a flowchart in **Figure 2(a)**.

3.2.3 Improved Energy Detection (IED)

This scheme is developed to further improve on the performance of the MED scheme especially in making it more effective in spectrum utilization. It involves an extra check to determine the presence or absence of the PU. The first check, similar to the MED algorithm is to examine if $T_i^{avg}(T_i) > \lambda$ to determine if the PU is present in the channel or not during a series of past sensing sessions. The second check which to ensure a more accurate detection is to check the immediate past sensing session $T_{i-1}(x_i - 1)$. This is in a bid to explore the immediate past occupancy status of the channel which the averaging procedure of MED may not be able to identify. The check, $(T_{i-1}(x_i - 1) > \lambda)$ is to confirm the viability of the channel for allocation and is referred to as an Improved Energy Detection (IED) technique. λ is the decision threshold which in the number of samples $N \gg 1$, can be expressed as a Gaussian distribution as proposed in (Lopez-Benitez & Casadevall F., 2012) and used in equation (10):

$T_i^{avg}(T_i)$ as used in equation (13) is normally distributed as an average of independent and identically distributed Gaussian random variables. It can be expressed as (Jain, Kumar, Gangopadhyay, & Debnath, 2015):

$$T_i^{avg}(T_i) \sim N(\mu_{avg}, \sigma_{avg}^2) \quad (15)$$

where:

$$\mu_{avg} = \frac{R}{p}(1 + \gamma) + \frac{p-R}{p} \quad (16)$$

$$\sigma_{avg}^2 = \frac{R}{p^2} \left(\frac{2}{N}(1 + \gamma)^2 \right) + \frac{p-R}{p^2} \quad (17)$$

$$\gamma = \frac{\sigma_s^2}{\sigma_w^2} \quad (18)$$

The probability of detection and false alarm, including the threshold for the IED can therefore be modified as presented in equations (13 – 15)(Lopez-Benitez & Casadevall F., 2012). The algorithm is presented in **Figure 2(b)**.

$$P_d^{IED} = P_d^{CED} + P_d^{CED}(1 - P_d^{CED})Q\left(\frac{\lambda_{IED} - \mu_{avg}}{\sigma_{avg}}\right) \quad (19)$$

$$P_{fa}^{IED} = P_{fa}^{CED} + P_{fa}^{CED}(1 - P_{fa}^{CED})Q\left(\frac{\lambda_{IED} - \mu_{avg}}{\sigma_{avg}}\right) \quad (20)$$

$$\lambda_{IED} = (Q^{-1}(P_{fa,target}^{IED})\sqrt{2N} + N)\sigma_w^2 \quad (21)$$

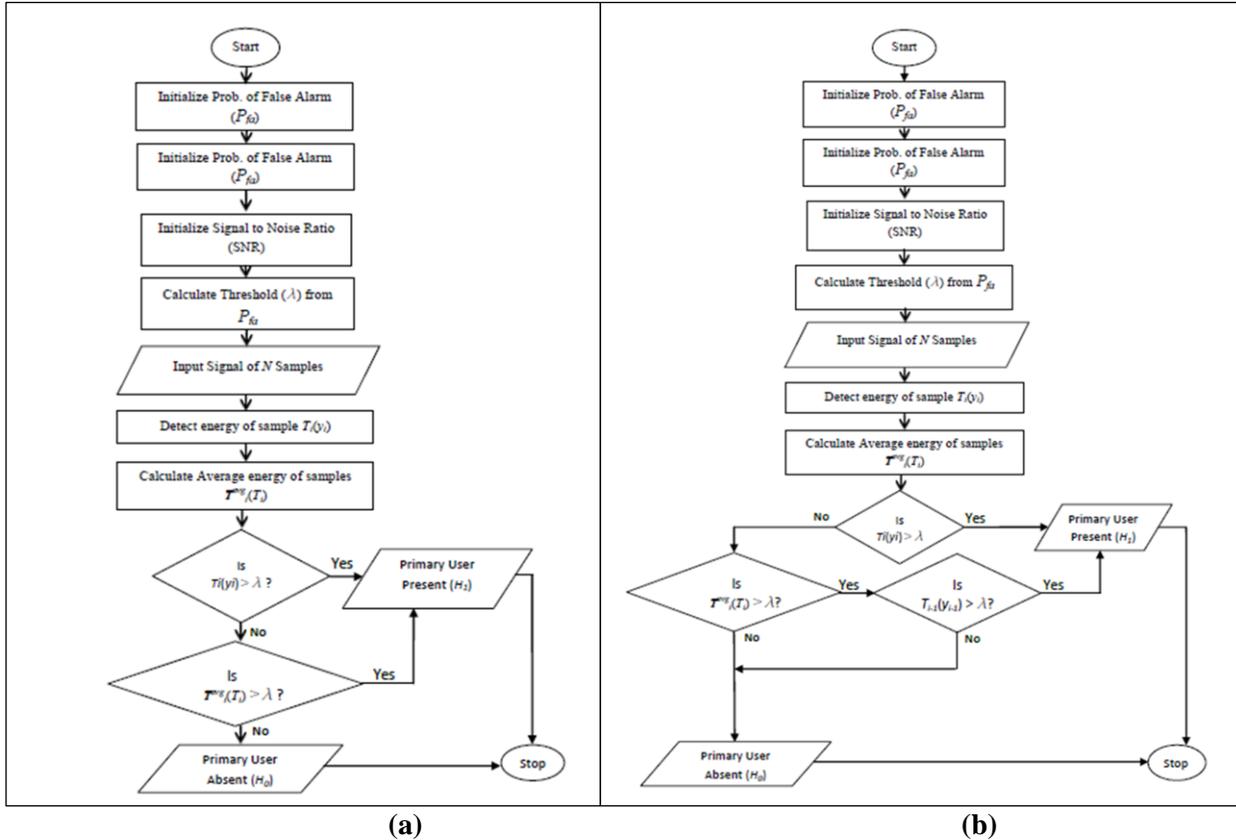


Figure 2: Flowcharts for: Modified Energy Detection (a) and Improved Energy Detection Technique (b)

3.2.4 Improved Cooperative Spectrum Sensing

The sensitivity of the individual cognitive radios is first enhanced using the IED algorithm described previously. Each of these cognitive radios is then organized into a cooperative network such that each CR user orthogonally sends its report to the fusion centre which collates these reports and makes a decision that protects the primary user and utilizes the spectrum efficiently. These reports are sent in binary form and combined at the fusion centre as follows:

$$S = \sum_{i=1}^N S_i \quad (22)$$

where S = Sum of the reports of all the cognitive radios.

The fusion centre then decides using M-out-of-N rule to decide on the occupancy status of the channel. The cooperative decision made is an improved version of the conventional cooperative spectrum sensing due to the enhanced spectrum sensing capability of each of the cognitive radios using the IED technique.

3.3 Algorithm for the Improved Cooperative Spectrum Sensing

These are the steps taken in carrying out the improved cooperative spectrum sensing:

Step 1: Each secondary user makes a local decision using the MED and IED schemes

Step 2: Binary decisions are taken: 1 indicating the presence of the Primary User, 0 indicating the absence of the Primary User

Step 3: The results are then collated

Step 4: Counting and majority rules are used (specifically OR rule) to determine the cooperative decision on the presence of the Primary User.

Step 5: The decision is then broadcast to all neighbours.

4 SIMULATION AND RESULTS

4.1 Simulating the Received Signal

The simulations were carried out using MATLAB version R2017a. Receiver Operations Characteristics (ROC) analysis was employed for the signal detection theory to study the performance of the energy detector in the first set of simulations. ROC analysis is widely used in the signal detection theory due to the fact that it is an ideal technique to analyse the trade-off between the probability of detection (P_d) and the probability of false alarm (P_f). Chi-square distribution was used to analyze the output though it was assumed as Gaussian distribution when the samples are large. A total of a hundred samples were shown for a sensing episode and the energy of the samples varied randomly from 2.07×10^8 to 2.005×10^8 dBm. The simulations were first carried out under a Signal to Noise Ratio (SNR) value of -16 dB and then using other SNR values to vary the noisy and poor signal conditions. A threshold is set based on calculations from the P_f to determine the energy level that would be termed as an H_1 or H_0 hypothesis. The results obtained for the CED, MED and IED for non-cooperative spectrum sensing are presented in Figures 3 – 6.

4.2 Simulations for Cooperative Spectrum Sensing

A total of ten (10) cognitive radios were employed. The cognitive radios in cooperative sensing mode were first encoded with the CED algorithm and compared with the non-cooperative CED technique to create a benchmark for comparison and to aid more accurate inferences. Thereafter, the improved cooperative sensing scheme is then simulated with the MED and IED algorithms. The simulation parameters are given in Table 1. The results obtained are presented in Figure 8.

Table 1. Simulation Parameters

Parameter	Value
SNR variation	-15dB – 15dB
P_f	0.05
No. of agents	10
Operating frequency of PU	1×10^9 Hz
Observation time	1×10^{-4}
Variance of the noise (σ^2_n)	1×10^{-12}
Variance of the received signal (σ^2_s)	$(\sigma^2_n \times 10^{-1})^2$
Operating power	40 mW

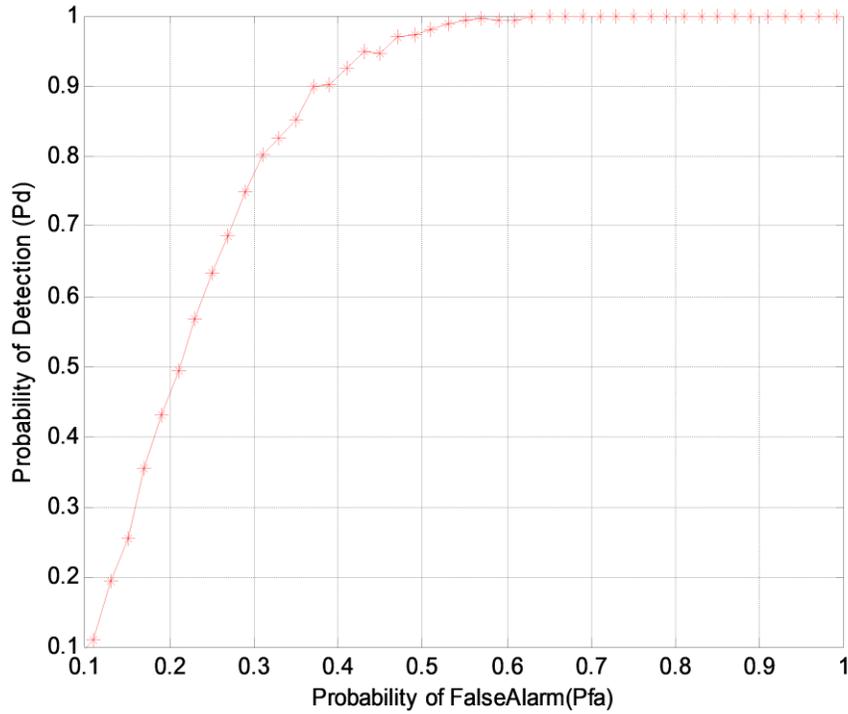


Figure 3: Complementary ROC of CED under AWGN

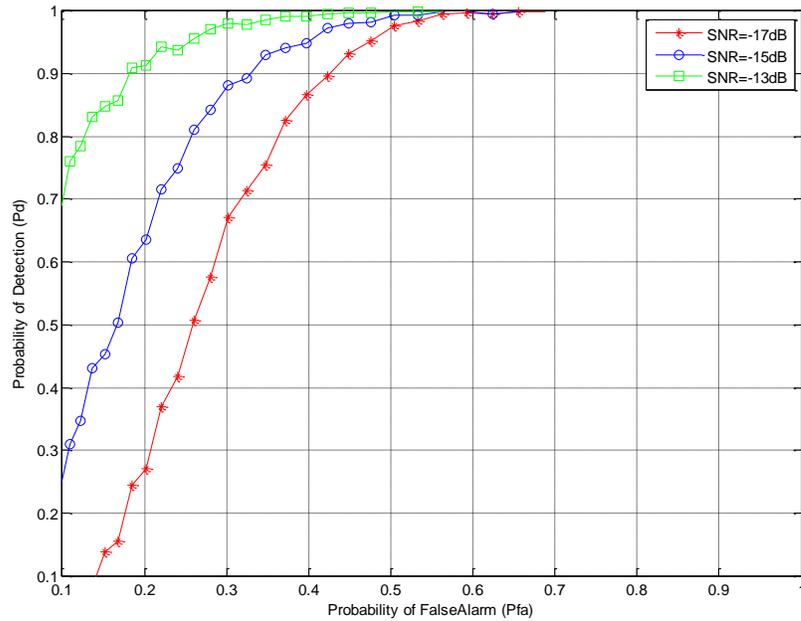


Figure 4: Complementary ROC of CED using various SNR values

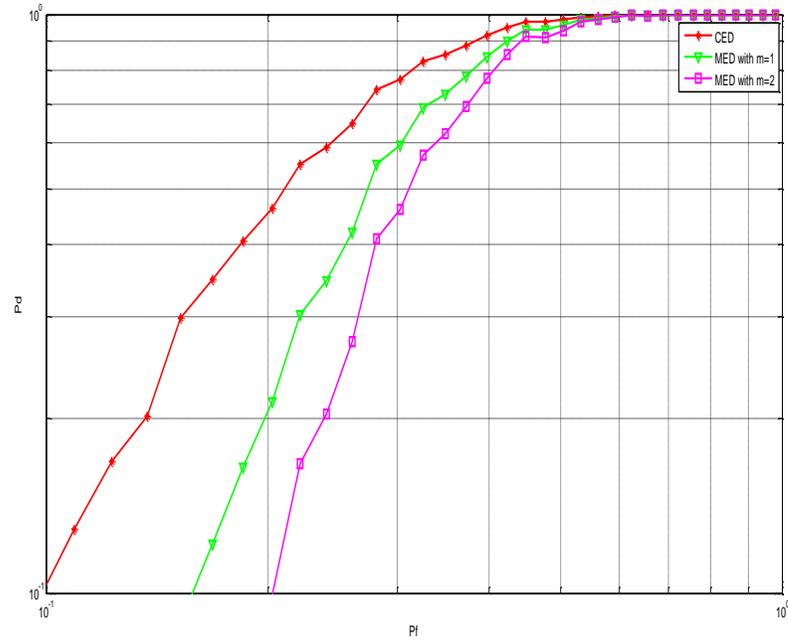


Figure 5: Comparison of CED and MED using a Single Cognitive Radio

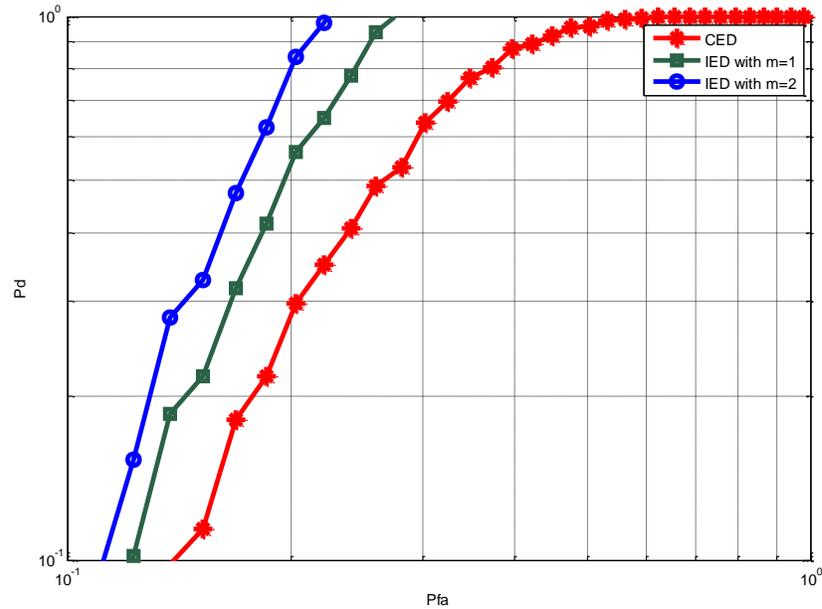


Figure 6: Comparison of the CED and IED Schemes using a single CR

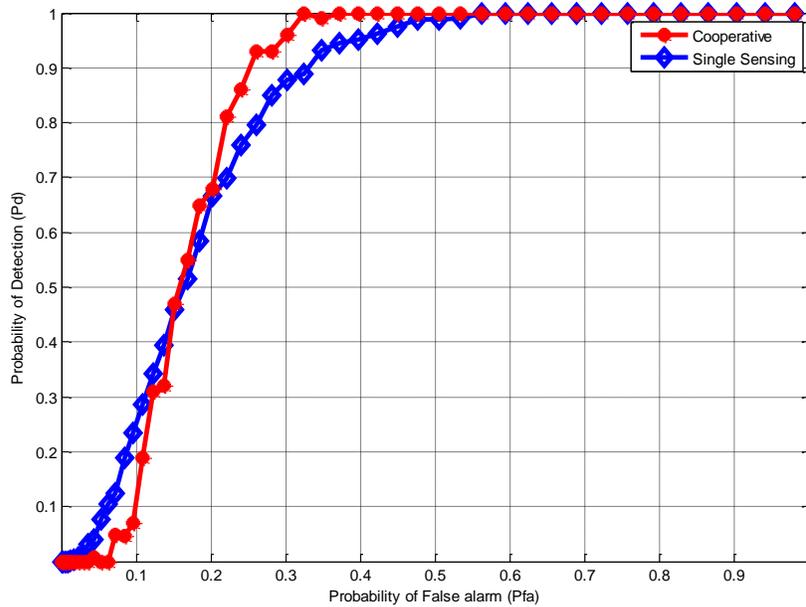


Figure 7: Comparison of Cooperative and Non-Cooperative Spectrum Sensing

CED using a single CR compared with the conventional cooperative spectrum sensing is presented in *Figure 7*.

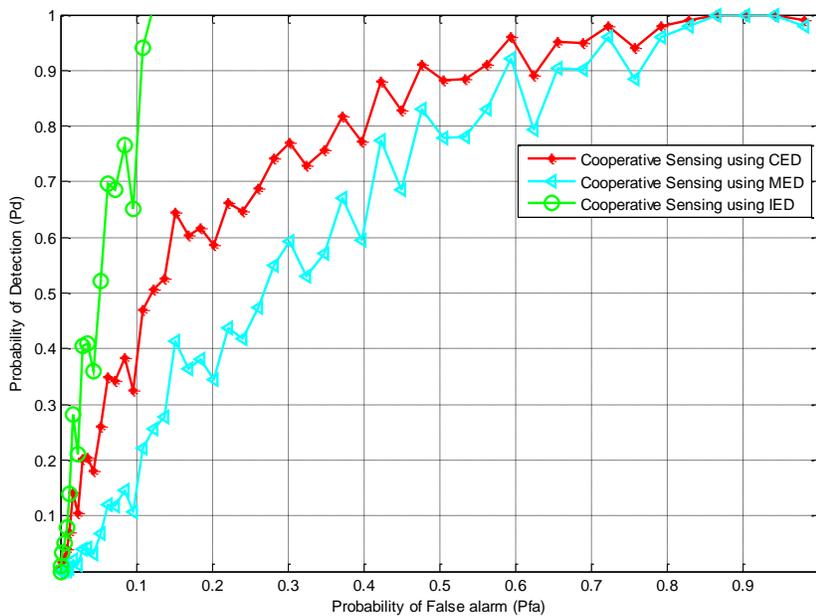


Figure 8: MED and IED schemes in Cooperative sensing

5 DISCUSSION

5.1 Conventional Energy Detection Result

The result presented in Figure 3 shows that the probability of detection (P_d) increases with the probability of false alarm (P_f). The P_d is higher compared to the P_f . Over time, both probabilities become about the same. The effort therefore is to improve on this curve such that per sensing session, there are higher detection probabilities than false alarms and no missed detections. The result obtained above in Figure 4 shows the ROC curve when SNR is -17dB being significantly lower than that of -15dB and -13dB which confirms established postulations that Conventional Energy Detection (CED) technique performs poorly in low signal to noise conditions. This is so because the energy detector (radiometer) finds it difficult to detect the signals when the noise floor is raised. It is therefore not recommended for use when spread spectrum techniques are employed where pseudo-noise codes are used by the users while transmitting. It becomes problematic for the energy detector to differentiate between the primary users' transmission, interference and noise.

However, when the SNR condition increases to about -13dB as shown in Figure 4, the probability of detection becomes much higher than the probability of false alarm per sensing outcome. This therefore confirms that the Conventional Energy Detector performs better in higher SNR conditions.

5.2 Modified Energy Detection Results

It can be observed from the ROC curves for the CED and MED in Figure 5 using a single cognitive radio that there are more probabilities of false alarm than probabilities of detection. The labels $m=1$, $m=2$ indicates the number of episodes of sensing carried out. Therefore, as more episodes of sensing took place, there was an even much lower curve with more probabilities of false alarm than detections. This is expected as some of the detections that were indicated by the CED technique were probably false alarms. This therefore shows that the developed MED technique is more sensitive to errors and safer to use i.e. it keeps the probabilities of missed detection to the barest minimum. But it can also be inferred that this technique does not utilize the spectrum as effectively as desired.

5.3 Improved Energy Detection Results

This result of the IED scheme shown in Figure 6 when compared with the CED indicates that there are more probabilities of detection than probabilities of false alarm per sensing outcome. This technique therefore is more sensitive, safer and also utilizes the spectrum better than the CED and MED.

5.4 Comparison of Non-Cooperative and Cooperative Spectrum Sensing

The result presented in Figure 7 shows the ROC curves of cooperative and non-cooperative spectrum sensing techniques labelled as Cooperative and Single Sensing respectively. The result depicts the performance of the non-cooperative technique which initially seemed better with higher probability of detection at the start. This is expected because in situations where shadowing occurs, the single cognitive radio could have indicated a detection when it was simply noise or vice versa. Therefore, the detections observed at this point are not reliable enough. Cooperative spectrum sensing helps to avoid this. It is also observed from the ROC curves that the cooperative sensing performs better than the non-cooperative technique later and eventually has a higher probability of detection than the non-cooperative technique. This indicates that it would create the avenue to better utilize the spectrum than the non-cooperative spectrum sensing technique. The cooperative spectrum sensing was therefore not just safer and more accurate; it as well ensured a more efficient use of the spectrum.

5.5 Modified and Improved Energy Detection in Cooperative Spectrum Sensing

The result is shown in Figure 8 and it reveals the improvement made on the performance of the cooperative spectrum sensing scheme which is the focus of this research. The cooperative sensing using MED scheme displays the same characteristics as observed when operated in the non-cooperative technique i.e. it was safer and more sensitive. But the IED incorporated in the cooperative sensing gives safer outcome and at the same time, utilizes the spectrum more efficiently. The results reveal that while the probability of false alarm is at 0.1, the CED algorithm can only be reliable for accurate detection, only 20% of the time, in very poor SNR conditions. The MED algorithm however has an increased detection accuracy of about 40% and the IED has a detection probability of 80% in such similar conditions. This therefore satisfies the objective of improving the sensitivity of cooperative spectrum sensing.

6 Conclusion

The MED and IED algorithms provided a more accurate sensing result than the CED. The IED however reduced the probabilities of false alarm and misdetections further than the MED. Incorporating it in cooperative sensing gave very significant improvement on the detection probabilities. This enhanced the overall sensing outcomes of cooperative spectrum sensing in cognitive radio systems.

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