# AN INVESTIGATION INTO THE PERFORMANCE AND ACCURACY OF AUTOMATIC IMAGE REGISTRATION USING EPIPOLAR GEOMETRY 

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#### Abstract

Image registration has being of great importance to a variety of applications in remote sensing, medical diagnosis, computer vision and pattern recognition. Owing to this, several methods have been adopted by different researchers, with a view to enhance the performance and accuracy of image registration process. This paper presents an investigation into the performance of automatic image registration using epipolar geometry. The epipolar geometry was recovered through the estimation of a fundamental matrix from matched image points, rectification transforms were computed using the F-matrix, and the images were registered using epipolar correlation. A Root mean square error (RMSE) of 0.032 and 0.00 was recorded for the correlated points, and the paper concluded that the performance of the image registration process was successful and accurate.


Keywords: Image registration, Epipolar geometry, Fundamental matrix, Image rectification

### 1.0 INTRODUCTION

Image registration is the process of overlaying two or more images of the same scene taken at different times from different viewpoints, with the same or different sensors. It requires the superimposition of one image upon another, so that pixel by pixel comparison can be made (Zitova and Flusser, 2003; Olaleye, 2008).

Image registration is known to be of great importance to a variety of applications in remote sensing, medical diagnosis, computer vision, and pattern recognition. Others are environmental monitoring, change detection, image mosaicking, and weather forecasting (Rami, 2004; Medha et al, 2009). Owing to the numerous importance of image registration, several methods have been adopted by different researchers, with a view to enhance the performance and accuracy of image registration process.

Zitova and Flusser (2003) outlined four basic steps involved in image registration procedure, and they include; feature detection, feature matching, mapping function design, image transformation and resampling. Among these steps, feature matching is considered as the most difficult task, due to the fact that image registration requires an exact pixel-to-pixel matching of the images involved (the sensed and the referenced image). However, various image registration methods have been in existence, and they are mainly centered on the determination of transformations that map points from one image to corresponding points in another image. Some of these methods include the curve matching methods that was used by Blatler et al, (1992), surface matching methods that was used by Chow et al, (2004), correlation based methods which was used by
various researchers such as Min Xu and Varshney (2009), Collignon, (1998), Sanjay et al, (1999). Other methods that have been used for image registration include; the Atlas methods, mutual information based methods, artificial neural networks and the wavelet based methods (Medha et al, 2009).

This paper aims at carrying out an investigation on how automatic image registration will perform when the concept of epipolar geometry is used. The developed algorithm uses the recovered epipolar geometry between two views to determine projective transforms for rectifying the images, and a correlation based method (epipolar correlation) to register the resampled images. The fundamental matrix computed from the registered images served as the transformation matrix of the registration.

### 2.0 METHODOLOGY

### 2.1 ALGORITHM WORKFLOW

The flow chart of the image registration algorithm is shown in figure 1
Fig. 1 Flow chart of the image registration algorithm


### 2.2 FEATURE DETECTION, EXTRACTION AND MATCHING

### 2.2.1 Harris and Stephens corner detector

Corner features were detected and extracted on both images using the Harris and Stephens corner detection and extraction algorithm. It considers the differential of the corner score with respect to the direction, instead of using shifted patches. Corners are generally located in regions with large
intensity variations in every direction and the instrument for detecting it lies in the image derivatives (Yanamadala et al, 2006).

## Mathematical Formulation

Let I be a grayscale 2-dimensional image. Taking an image patch over an area ( $u, v$ ) and shifting it by ( $\mathrm{x}, \mathrm{y}$ ), the weighted sum of squared differences (SSD) between these patches denoted by ' S ' is given by:

$$
\begin{equation*}
S(x, y)=\sum_{u} \sum_{v} w(u, v)(I(u+x, v+y)-I(u, v))^{2} \tag{1.1}
\end{equation*}
$$

Using Tylor series expansion, the term $I(u+x, v+y)$ can be approximated as follows (Wikipedia, 2013);

Let $I_{x}$ and $I_{y}$ be partial derivatives of I, such that;
$I(u+x, v+y)=I(u, v)+I_{x}(u, v) x+I_{y}(u, v) y$
This produces the approximation
$S(x, y)=\sum_{u} \sum_{v} w(u, v)\left(I_{x}(u, v) x+I_{y}(u, v) y\right)^{2}$
Which can be written in matrix form as:
$S(x, y) \approx(x, y) A\binom{x}{y}$
Where A is the structure tensor given as:

$$
A=\sum_{u} \sum_{v} w(u, v)\left[\begin{array}{cc}
I_{x}^{2} & I_{x} I_{y}  \tag{1.5}\\
I_{x} I_{y} & I_{y}^{2}
\end{array}\right]=\left\lceil\begin{array}{lc}
\left\langle I_{x}^{2}\right\rangle & \left\langle I_{x} I_{y}\right\rangle \\
\left\langle I_{x} I_{y}\right\rangle & \left\langle I_{y}^{2}\right\rangle
\end{array}\right]
$$

Equation (1.5) is known as the Harris matrix, and the angle brackets denote averaging (summation over (u, v)).

Based on the fact that a corner is characterized by a large variation of ' S ' (' S ' is defined in equation (1.4)) in all directions of the vector ( $x, y$ ), the Eigen values of A in equation (1.5) can be analyzed and the characterization can be analyzed based on the magnitude of the Eigen values as (Wikipedia, 2013):

1. If $\lambda_{1} \approx 0$ and $\lambda_{2} \approx 0$ then the pixel ( $\mathrm{x}, \mathrm{y}$ ) has no features of interest.
2. If $\lambda_{1} \approx 0$ and $\lambda_{2}$ some large positive value, then an edge is found.
3. If $\lambda_{1}$ and $\lambda_{2}$ have positive values, then a corner is found.

In order to simplify the computation of the Eigen values, the function $M_{c}$ (suggested by Harris and Stephens in 1988) is formulated as (Wikipedia, 2013);
$M_{c}=\lambda_{1} \lambda_{2}-k\left(\lambda_{1}+\lambda_{2}\right)^{2}=\operatorname{det}(A)-\operatorname{trace}^{2}(A)$
Where k is a tunable sensitivity parameter. Thus, Equation (2.6) forms the bases of the Harris and Stephen's corner detector which states that a corner is detected if $\operatorname{det}(A)-\operatorname{trace}^{2}(A)$ is greater than a threshold (Yanamadala et al, 2006; Wikipedia, 2013).

### 2.2.2 Matching Strategy

The Sum of Absolute Difference (SAD) algorithm was used to match the extracted corner points. It measures the similarity between image blocks (templates). It works by taking the absolute pixel difference between each pixel in the original block and corresponding pixel in the block being used for comparison. The differences are summed to create a simple metric of block similarity, the L1 norm of the difference in image or Manhattan distance between image blocks (Wikipedia, 2013). The SAD distance metric is given by;

$$
\begin{equation*}
d_{1}\left(I_{j}, \quad T\right)=\sum_{i=1}^{n}\left|I_{i, j}-T_{i}\right| \tag{1.7}
\end{equation*}
$$

Where; $\mathrm{T}=$ template (block) M by N pixels is correlated with the original image I (original block) for the same size drawn from the search window.

### 2.3 Recovery of epipolar geometry and estimtion of the fundamental matrix

The epipolar geometry was recovered through the computation of fundamental matrix from matched image points using the Random Sample Consensus (RANSAC) algorithm. The RANSAC algorithm sets a distance threshold for the matched points, such that only the points that satisfy the distance threshold are been used to compute the fundamental matrix.

## 2.4 computation of projective transforms and rectification of images

Rectification is an important task in image registration, due to the fact that the images have to be in their ideal form (free from distortion) for a pixel to pixel matching to be possible. The fundamental matrix was used to determine two projective transforms ( H and $\mathrm{H}^{\prime}$ ) for the rectification process.

### 2.4.1 Mathematical formulation of the projective transforms from the Fundamental matrix

Having computed the fundamental matrix from matched image points, the epipoles (e and e') of each image can be obtained through a singular value decomposition (SVD) of the fundamental matrix.

That is, $F=U \operatorname{diag}\left(0, \sigma_{1}, \sigma_{2}\right) V^{T}$

Where; $U=\left[e^{\prime}, u_{1}, u_{2}\right\rfloor$ and $V=\left[e, v_{1}, v_{2}\right\rfloor$
$\sigma_{1}$ and $\sigma_{2}$; are the typically non-zero singular values.
$e=\left(e_{u}, e_{v}, 1\right)$ and $e^{\prime}=\left(e_{u}^{\prime}, e_{v}^{\prime}, 1\right)^{T}$
From the obtained epipoles, the projective transforms can be written in matrix form as;
$H=\left[\begin{array}{ccc}1 & 0 & 0 \\ -e_{v} / e_{u} & 1 & 0 \\ -1 / e_{u} & 0 & 1\end{array}\right]$ and $H^{\prime}=H\left([e]_{x} F+e e^{, T}\right)$
(Hartley, 1999; Mallon and Whelan, 2005)

### 2.5 Registering the Images

Due to the fact that conjugate epipolar lines are collinear and parallel to the image horizontal axes after rectification, the 2D search was reduced to a 1 D search along conjugate epipolar lines. This made it possible for the images to be registered using epipolar correlation, which mapped points in the Sensed image to corresponding epipolar lines in the Reference image.

### 2.6 Accuracy of the Registration

A measure of the accuracy of registration was estimated by making a pixel by pixel comparison of the conjugate image points, and then taking the Root Mean Square Error (RMSE) of the pairwise difference to know how far on average the error is from zero. Thus, a general equation for the RMSE as given in equation (1.8) was used.

The general equation for root mean square error (RMSE) is given by:

$$
\begin{equation*}
R M S E=\sqrt{\frac{\left(\sum_{i=1}^{n}(a i-b i)^{2}\right)}{n}} \tag{1.8}
\end{equation*}
$$

Where n is the total number of estimated values to be evaluated

### 2.7 Program development and Implementation

The algorithm which was programmed in MATLAB was implemented through a developed graphical user interface (GUI), as shown in figure 1.3.


Figure 1.3: Graphical User Interface of the Image registration Algorithm

### 3.0 Results and Discussion

The developed program was experimented with satellite imageries of some parts of University of Lagos, Nigeria. The results obtained are presented as follows;


Figure 1.4: Inputted sensed and Reference image, as displayed by the image axes of the program


Figure 1.5: Extracted and matched corner points on both images after removing outliers using geometric and epipolar constraint.


Figure 1.5: Rectified Sensed and Reference image


Figure 1.6: Registered image
Table 1.1: pixel coordinates of some conjugate points on the Sensed and referenced image after registration.

|  | LEFT IMAGE (Sensed image) |  |  | RIGHT IMAGE (Reference image) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{S} / \mathrm{N}$ | X | Y |  | X | Y |
| 1 | 6 | 229 |  | 6 | 229 |
| 2 | 6 | 242 |  | 6 | 242 |
| 3 | 122 | 201 |  | 122 | 201 |
| 4 | 223 | 241 |  | 223 | 241 |
| 5 | 137 | 204 |  | 136 | 204 |
| 6 | 290 | 229 |  | 290 | 229 |
| 7 | 229 | 46 |  | 229 | 46 |
| 8 | 229 | 240 |  | 229 | 240 |
| 9 | 303 | 236 |  | 303 | 236 |

Computed Root Mean Square Error $($ RMSE $)$ of the pairwise difference of conjugate points $=$ 0.0328 and 0.00.

Computed F-Matrix after registration of the images $=\left[\begin{array}{ccc}0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0\end{array}\right]$

### 3.1 Discussion

From table 1.1, an exact pixel to pixel correspondence of both images has been achieved, and the recorded root mean square error of 0.0328 and 0.00 shows that the extent of mismatch is very minimal. This gives an indication that the images have been accurately registered, and the performance of the image registration process can be described as successful.

### 4.0 Conclusion

The performance of automatic image registration using the method of epipolar geometry is very successful, and accurate, as an exact pixel to pixel comparison of both images is attainable.

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