UNSTEADY FREE CONVECTION OF MAGNETO-HYDRODYNAMIC RADIATIVE FLOW PAST AN INFINITE VERTICAL POROUS PLATE IN A POROUS MEDIUM

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ABSTRACT

The unsteady free convection of magneto-hydrodynamic radiative fluid flow past an infinite vertical porous plate in a porous medium under the limit of an optically thin fluid is investigated. An implicit finite difference scheme of crank-Nicolson type was employed in solving the resulting coupled non-linear partial differential equations to obtain the dimensionless velocity and temperature. Velocity and temperature fields have been studied and their profiles are graphically presented while velocity gradient and Nusselt number are shown in a tabular form. It is observed, among others, that the radiation parameter decreases the velocity and temperature of fluid while the suction/injection parameter leads to an increase in the transient velocity and temperature of fluid. Also, a rise in magnetic field and permeability parameter parameters reduce the velocity significantly and temperature slightly.

KEYWORDS: Unsteady, Thermal radiation, MHD, Viscous dissipation, Porous medium.

INTRODUCTION

The problem of convection flow of a fluid is a natural phenomenon. It is common in science and technology and is being studied by several authors due to its significant applications in industry.

These applications led Illingworth [1], who is one of the earlier researchers, to study transient free convection flow past an infinite vertical isothermal plate with an assumption that Prandtl number is unity. Another similar study was carried out later by Siegel [2]. He investigated the transient free convection from a vertical flat plate and reported that for semi-infinite and doubly vertical plate, initial behaviour of temperature and velocity fields are the same. An investigation on the transient free convection flow past an infinite vertical plate with periodic temperature variation was reported by Soundalgekar *et al.* [3]. They used finite difference method to solve the non-linear coupled partial differential equations governing the model of the problem. Some other similar studies on the subject matter are the computation of natural convection by finite difference methods by Hellums and Churchill [4] and the transient free convection flow of a micro polar fluid past an infinite vertical plate by Soundalgekar *et al.* [5].

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The significance of viscous fluid flow in industrial application led researchers like Nanda and Sharma [6] to carry out their studies on the free convection laminar boundary layers in oscillatory flow while Singh [7] investigated the unsteady hydromagnetic free convection flow past a vertical infinite flat plate. Also, Muthucumaraswamy [8] investigated the natural convection on flow past an impulsively started vertical plate with variable surface heat flux.

However, the viscous dissipation was neglected in all these studies. The importance of viscous dissipation in geophysical fluid flows and also in some industrial operations motivated Soundalgekar *et al.* [9] to study the transient free convection flow of dissipative fluid past an infinite vertical porous plate and obtained the numerical solutions for velocity, temperature, skin friction and Nusselt number. They reported among others that there is a rise in the velocity and temperature of air owing to greater viscous dissipative heat and an increase in the suction parameter led to a decrease in velocity with a fall in the temperature of air.

The above study neglected the radiation, magnetic field and medium permeability's effects on the dissipative fluid. In the light of this, the interaction of radiation with heat transfer of a megnetohydrodynamic, dissipative fluid past an infinite vertical porous plate in a porous medium is presented in this study. The momentum equation includes magnetic and viscous permeability terms while the dissipation and radiation terms are taken into account in the energy equation.

1. FORMULATION OF THE PROBLEM

The problem of the unsteady flow of incompressible fluid past an infinite vertical porous plate, whose physical variables are functions of y' and t', are considered. The x'-axis is taken vertically upward and parallel to the plate while y'-axis is taken perpendicular to it such that $y' \to \infty$ at $t' > 0$. It is assumed that the plate and mass of the fluid are at the same temperature T'_∞ at the initial stage (i e, at $t' \leq 0$). When $t' > 0$, the temperature of the plate is taken as T'_w. The magnetic field, viscous permeability, viscous dissipation and radiation are hereby assumed to play significant roles because of their importance in flow problem. It is also assumed that the fluid properties are constant except for the body force terms in the momentum equation, which are approximated by the Boussinesq relation. With these assumptions, the flow can be governed by the momentum, continuity and energy equations in the form

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} + g\beta[T' - T'_\infty] - \frac{\sigma \beta_o u'}{\rho} - \frac{\upsilon u'}{K_p} \tag{2}$$

$$\rho C_P \left[\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'}\right] = \kappa \frac{\partial^2 T'}{\partial y'^2} + \mu \left[\frac{\partial u'}{\partial y'}\right]^2 - \frac{\partial q_r}{\partial y'} \tag{3}$$

with the following initial and boundary conditions

$$\begin{aligned} &t' \le 0, \quad u' = 0, \quad T' \to T'_\infty \; for\, all\, y' \\ &t' > 0, \quad u' = 0, \quad T' \to T'_w \;\; as\, y' = 0 \\ &\qquad\quad\;\, u' = 0, \quad T' \to T'_\infty \;\; as\, y' \to \infty \end{aligned}$$

where u' and v' are the velocity components in the x' and y' direction respectively, T' is the velocity temperature of the fluid in the boundary layer, t' is the time, β is the volumetric coefficient of thermal expansion, υ is the kinematic viscosity, g is the acceleration due to gravity, κ is the thermal conductivity, K_p is the permeability of the porous medium, μ is the viscosity of the fluid, ρ is the density of the fluid, β_o is the magnetic field strength, q_r is the radiative flux and σ is the electrical conductivity of the fluid.

In an optically thin limit, the fluid does not absorb its own emitted radiation. It implies that there is no self-absorption but rather the fluid absorbs radiation emitted by the boundaries. Cogly, Vincentine and Gillies [10] showed, for non-grey gas near equilibrium in the optically thin limit case, that

$$\frac{\partial q_r}{\partial \hat{y}} = 4(T' - T_\infty)I \tag{4}$$

where $I = \int_0^\infty \kappa_{\lambda_w} \left(\frac{de_{b\lambda}}{d\hat{T}}\right)_w d\lambda$, $\kappa_{\lambda_w}$is the absorption coefficient, $e_{b\lambda}$ is the Planck function and the subscript w indicates the values at the wall.

The following non-dimensional quantities

$$u = \frac{u'}{u_0}, y = \frac{y'}{L}, t = \frac{t'}{t_0}, \delta T = T'_w - T_\infty, \;\; \theta = \frac{T' - T'_\infty}{T'_w - T_\infty}$$

are introduced into equations (1) - (3), where $u_0 = (\upsilon g\beta\delta T)^{\frac{1}{3}}, L = \left[\frac{g\beta\delta T}{\upsilon^2}\right]^{-\frac{1}{3}}, t_0 = \left[\frac{g\beta\delta T}{\upsilon^{-\frac{1}{3}}}\right]^{-\frac{2}{3}}$

and u_0, L and $t_0$are referred to as velocity, length, and time respectively.

The non-dimensional system of governing equations for the fluid motion is obtained as

$$\frac{\partial v}{\partial y} = 0 \tag{5}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \theta - (H + K_1)u \tag{6}$$

$$p_r\left[\frac{\partial \theta}{\partial t} + v\frac{\partial \theta}{\partial y}\right] = \frac{\partial^2 \theta}{\partial y^2} + p_r E\left[\frac{\partial u}{\partial y}\right]^2 - R\theta \tag{7}$$

with the following initial and boundary conditions

$$\begin{aligned} &t \leq 0, \quad u = 0, \quad \theta = 0 \quad \text{for all } y \\ &t > 0, \quad u = 0, \quad \theta = 1 \quad \text{for } y = 0 \\ &\qquad\quad\; u = 0, \quad \theta = 0 \quad \text{as } y \to \infty \end{aligned} \tag{8}$$

where $p_r = \dfrac{\mu C_P}{k}$, $E = \dfrac{u_o^2}{C_P \Delta T}$, $R = \dfrac{4vI}{ku_o}$, $H = \dfrac{\sigma \beta_o^2 t_o}{\rho}$ and $K_1 = \dfrac{vt_o}{k_p}$ are the Prandtl number, Eckert number, radiation parameter, magnetic parameter and viscous permeability parameter respectively.

Integrating equation (5), the velocity of suction/injection (horizontal velocity v) may be expressed as a constant $\lambda = \dfrac{v_o}{u_o}$.

Therefore, equations (5), (6) and (7) are reduced to the following system of equations

$$\frac{\partial u}{\partial t} + \lambda \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y'^2} + \theta - (H + K_1) u \tag{9}$$

$$p_r \left[\frac{\partial \theta}{\partial t} + \lambda \frac{\partial \theta}{\partial y} \right] = \frac{\partial^2 \theta}{\partial y'^2} + p_r E \left[\frac{\partial u}{\partial y} \right]^2 - R\theta \tag{10}$$

with the following boundary conditions

$$\begin{aligned} &t \leq 0, \quad u = 0, \quad \theta = 0 \quad \text{for all } y \\ &t > 0, \quad u = 0, \quad \theta = 1 \quad \text{for } y = 0 \\ &\qquad\quad\; u = 0, \quad \theta = 0 \quad \text{as } y \to \infty \end{aligned} \tag{11}$$

2. METHOD OF SOLUTION AND DISCUSSION

Equations (9) and (10) with boundary conditions (11) are now solved by implicit finite difference schemes of Crank-Nicolson type. The finite difference equations corresponding to these equations are given by

$$\frac{u_{j-1}^{k+1} - u_j^k}{\Delta t} + \lambda \left[\frac{u_{j+1}^{k+1} - u_{j-1}^{k+1} + u_j^k - 1}{4\Delta y} \right] = \left[\frac{u_{j-1}^{k+1} - 2u_j^{k+1} + u_{j+1}^{k+1} + u_{j-1}^k - 2u_j^k + u_{j-1}^k}{2(\Delta y)^2} \right] + \frac{1}{2}\left[\theta_j^{k+1} + \theta_j^k \right] - \frac{(H + \tau)}{2} \left[u_j^{k+1} + u_j^k \right] \tag{12}$$

$$p_r\left[\frac{\theta_j^{k+1}-\theta_j^k}{\Delta t}+\lambda\frac{\left(\theta_{j+1}^{k+1}-\theta_{j-1}^{k+1}+\theta_{j+1}^k-\theta_{j-1}^k\right)}{4\Delta y}\right]$$
$$=\left[\frac{\theta_{j-1}^{k+1}-2\theta_j^{k+1}+\theta_{j+1}^{k+1}+\theta_{j-1}^k-2\theta_j^k+\theta_{j-1}^k}{}\right]+p_rE\left[\frac{u_{j+1}^k-u_{j-1}^k}{2\Delta y}\right]^2-\frac{R}{2}\left[\theta_j^{k+1}+\theta_j^k\right] \quad (13)$$

with the following initial and boundary conditions

$$\left.\begin{array}{llll} t\le 0, & u_{j,o}=0, & \theta_{j,o}=0 & \text{for all } j \text{ except } j=0 \\ t>0, & u_{o,k}=0, & \theta_{o,k}=0 & \text{for } j=0 \\ & u_{m,k}=0, & \theta_{m,k}=0 & \text{for } j\to\infty \end{array}\right\} \quad (14)$$

where m corresponds to ∞.

Equations (12) and (13) can now be written in the following form

$$-A_1u_{j-1}^{k+1}+A_2u_j^{k+1}+A_3u_{j+1}^{k+1}=A_1u_{j-1}^k+A_4u_j^k-A_3u_{j+1}^k+A_5\left[\theta_j^{k+1}+\theta_j^k\right] \quad (15)$$

$$-B_1\theta_{j-1}^{k+1}+B_2\theta_j^{k+1}+B_3\theta_{j+1}^{k+1}=B_1\theta_{j-1}^k+B_4\theta_j^k-B_3\theta_{j+1}^k+B_5\left[u_{j+1}^k-u_{j-1}^k\right] \quad (16)$$

where

$$A_1=r_1\lambda+2r_2,\ A_2=4+4r_2+r_4,\ \ A_3=r_1\lambda-2r_2,\ \ A_4=4-4r_2-r_4,\ \ A_5=r_3$$

$$r_1=\frac{\Delta t}{\Delta y},\ \ r_2=\frac{\Delta t}{(\Delta y)^2},\ \ r_3=2\Delta t,\ \ r_4=r_3(H+K_1)$$

$$B_1=p_rr_1\lambda+2r_2,\ \ B_2=4p_r+4r_2-r_5,\ \ B_3=p_rr_1\lambda-2r_2,\ \ B_4=4p_r-4r_2+r_5, B_5=p_rEr_2$$

$$r_5=r_3R$$

The suffixes j and k correspond to y and t respectively, $\Delta y=y_{j+1}-y_j$ and $\Delta t=t_{k+1}-t_k$.

The implicit finite difference schemes of Crank-Nicolson type resulted to tri-diagonal system of equations that are finally solved by employing Thomas algorithm with the aid of *MATLAB* package to obtain the values of dimensionless velocity (u) and dimensionless temperature (θ) at different time t. The computations are carried out for velocity and temperature for various value of the fluid parameters: radiation (R), Permeability (K_1), suction $(\lambda=V)$, Prandtl number (p_r), Ecker number (E) and magnetic (H) . The results compared well with [9] in the absence of the radiation, permeability and magnetic parameters. Figures 1-9 depict the graphical representation of the solution of equations (9) and (10) with default fixed values $R=2, H=1, K_1=3, E=0.1, p_r=0.71, V=0.3$ and time $t=150$.

Figures 1 and 2 show the velocity and temperature distributions for variation of radiation parameter R. It is noticed that a rise in radiation parameter reduces the magnitude of the dimensionless velocity and temperature of fluid. The suction/injection parameter leads to increase in the transient velocity and temperature of fluid, that is the velocity and temperature of fluid increase with increasing the suction/injection parameter as depicted in Figures 3 and 4. The influence of the injection parameter $V = 0.3$ on transient velocity is noticed clearly in Figures 5a and 5b, where an increase in the Prandtl number leads to an increase in the magnitude of transient velocity and temperature near the porous plate, but a decrease in the velocity and temperature far from the plate (i.e. $y > 1$). However, for suction parameter $V = -0.3$, a rise in the Prandtl number results in a decrease in the velocity and temperature profiles as shown in Figures 6a and 6b.

The influence of dissipation parameter is presented in Figures 7 and 8 where an Eckert number's increment slightly increases velocity and temperature distributions. It is observed that with an increase in the permeability parameter, there is a decrease in the magnitude of velocity and temperature of the fluid as shown in Figures 9 and 10. However, it is seen clearly that variation on the temperature is negligible.

The presence of the magnetic field has significant influence on the velocity profile and little influence on temperature profile of the fluid as shown in Figures 11 and 12. A rise in magnetic parameter reduces the velocity significantly and temperature slightly.

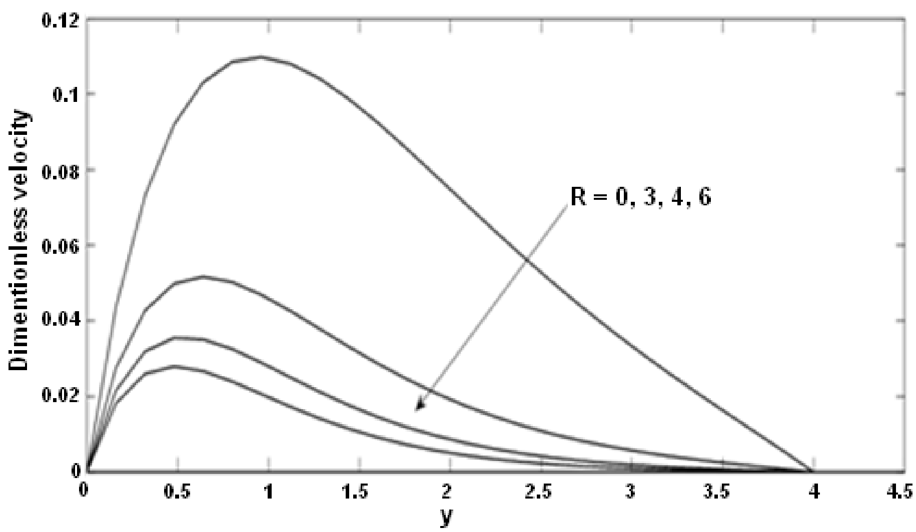

Figure 1. The velocity distribution for variation of radiation parameter *R*

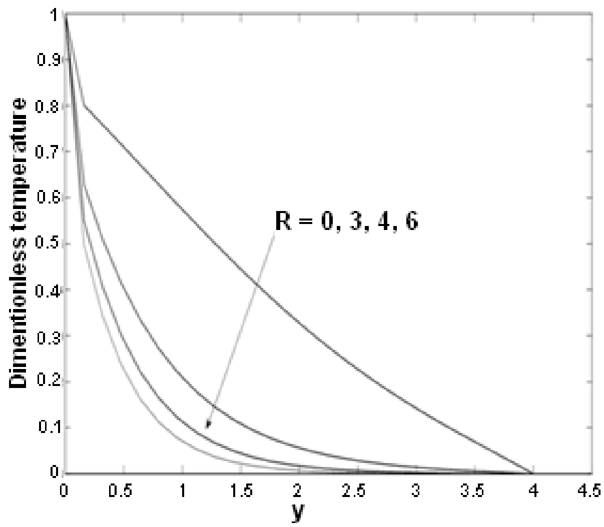

Figure 2. The temperature distribution for variation of radiation parameter *R*

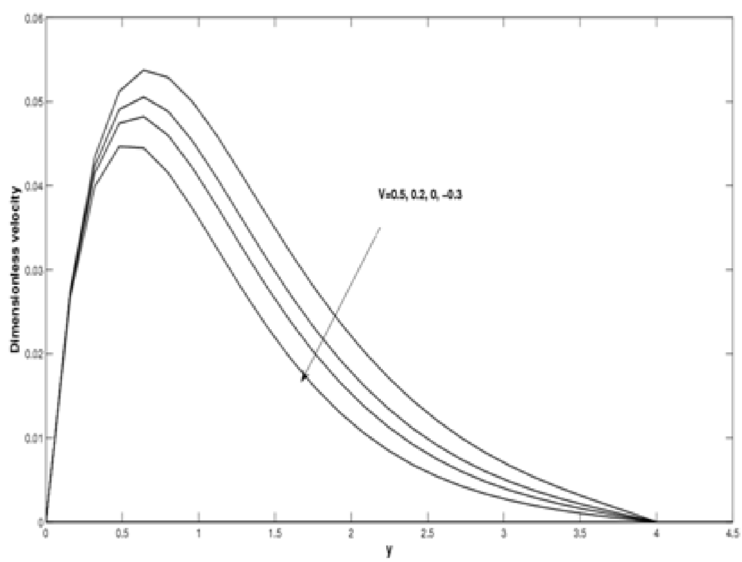

Figure 3. The velocity distribution for variation of suction/injection parameter *V*

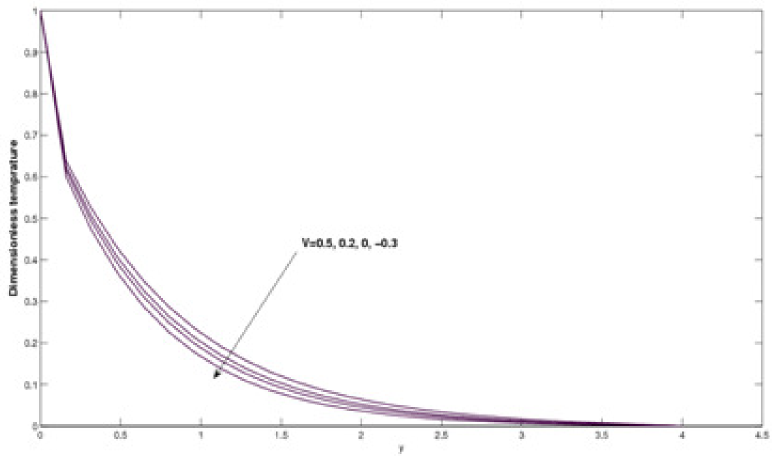

Figure 4. The temperature distribution for variation of suction/injection parameter V

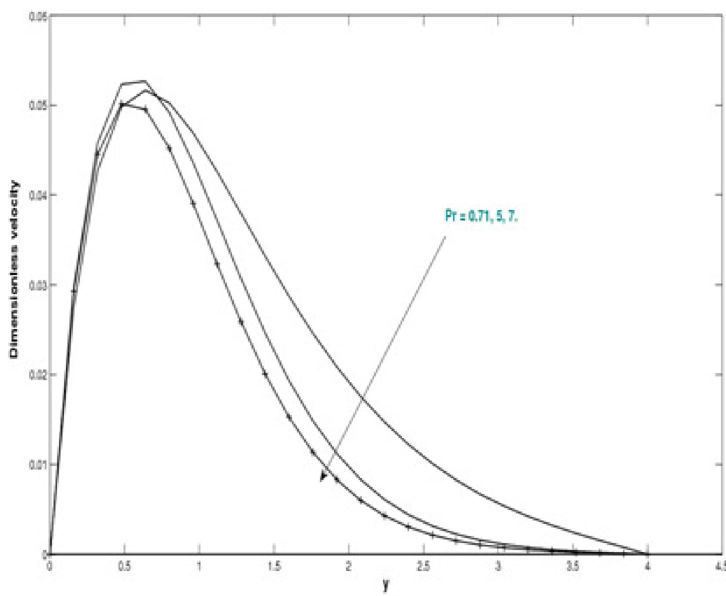

Figure 5a. The velocity distribution for variation of Prandtl number Pr

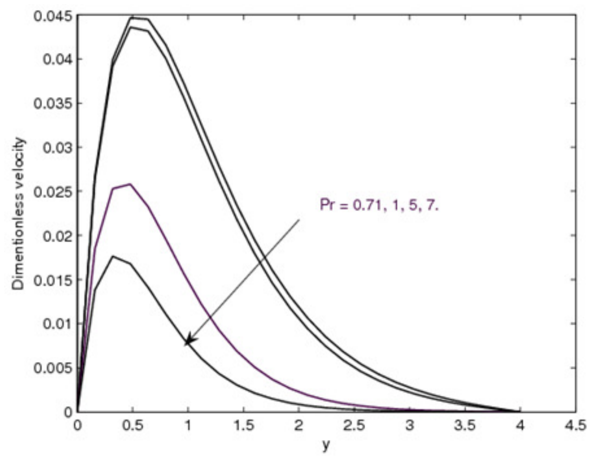

Figure 5b. The velocity distribution for variation of Prandtl number Pr and $V = -0.3$

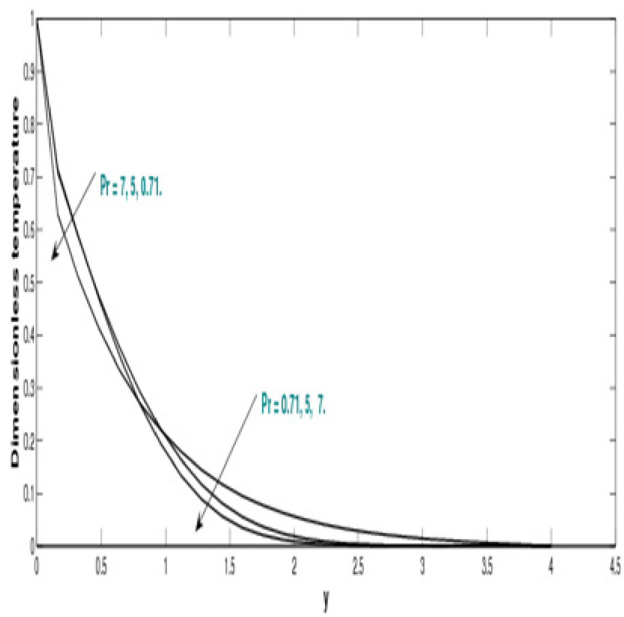

Figure 6a. The temperature distribution for variation of Prandtl number Pr

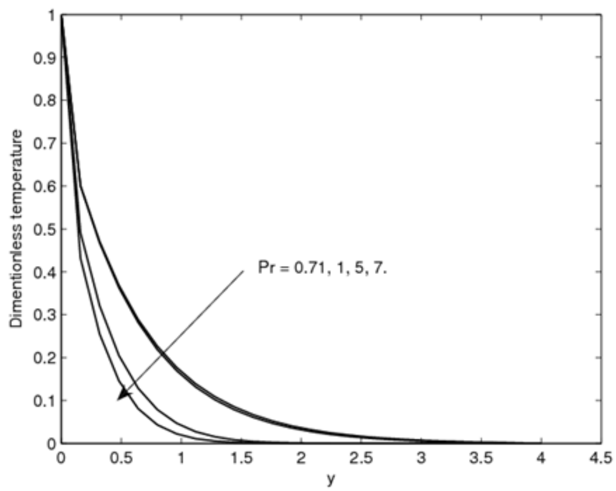

Figure 6b. The temperature distribution for variation of Prandtl number Pr and $V = -0.3$

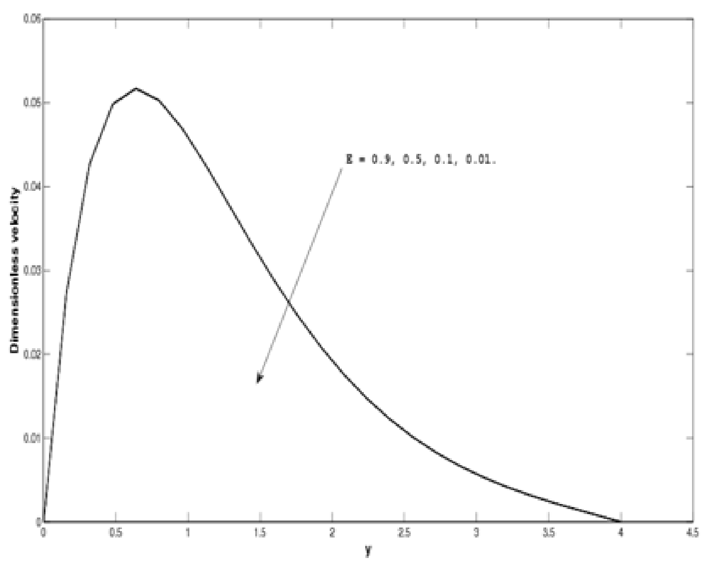

Figure 7. The velocity distribution for variation of Eckert number E

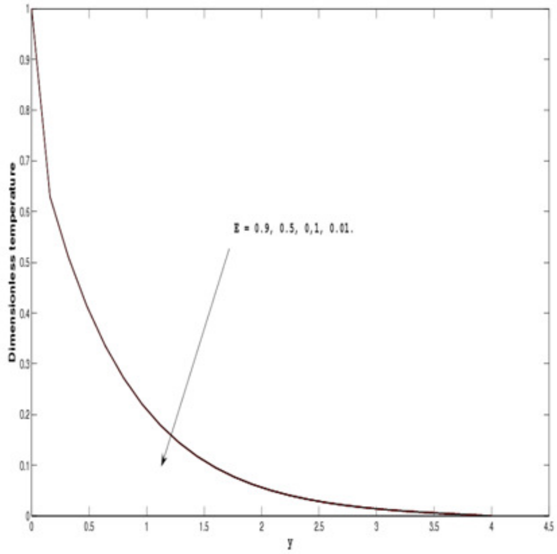

Figure 8. The temperature distribution for variation of Eckert number *E*

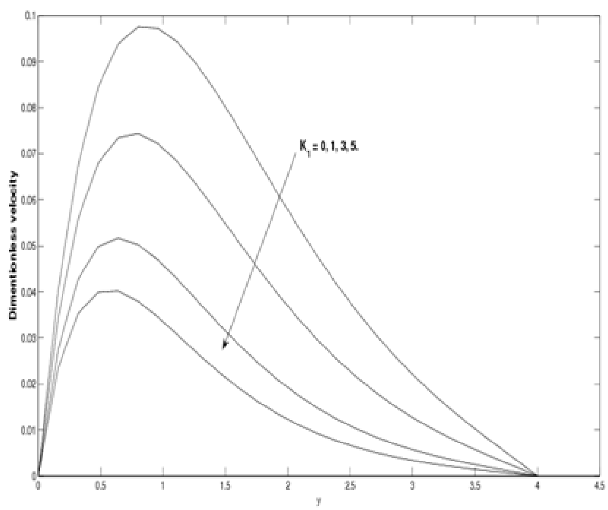

Figure 9. The velocity distribution for variation of the permeability parameter K_1

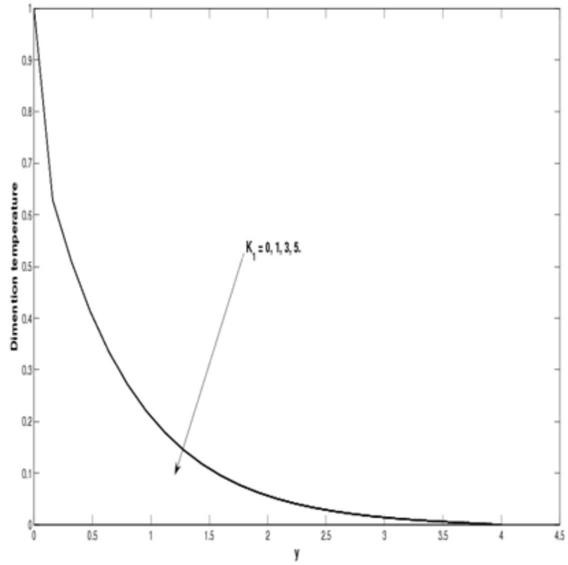

Figure 10. The temperature distribution for variation of permeability parameter K_1

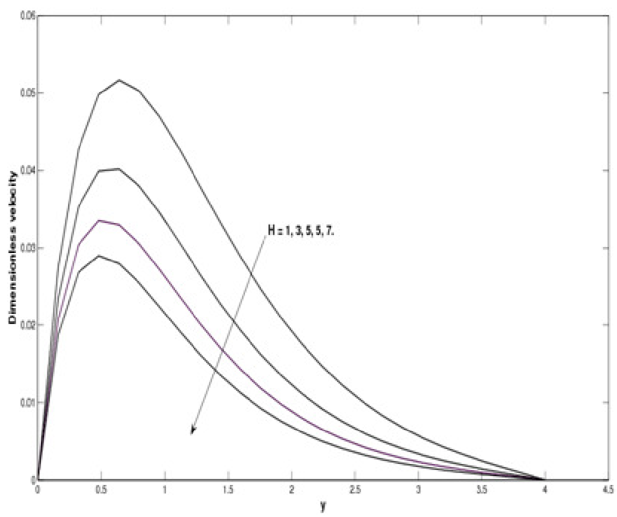

Figure 11. The velocity distribution for variation of the magnetic field parameter H

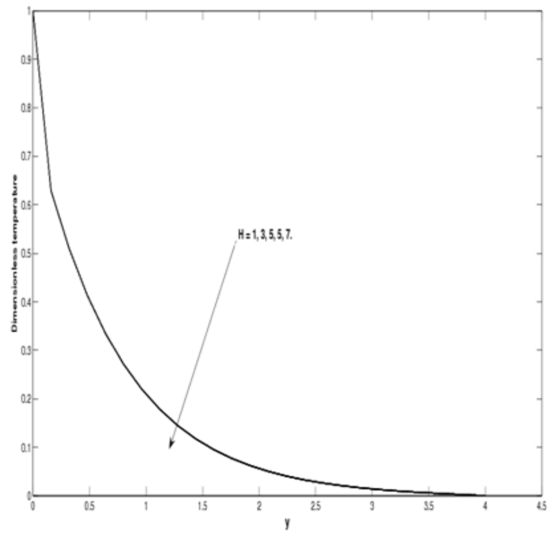

Figure 12. The temperature distribution for variation of the magnetic field parameter *H*

The velocity gradient and Nusselt number with variation of some fluid's parameters are shown in Table 1 with default fixed values $R = 3, H = 1, K_1 = 3, E = 0.1, Pr = 0.71$ and $V = 0.3$.

Fluid parameters	Velocity gradients	Nusselt numbers	Fluid parameters	Velocity gradients	Nusselt numbers
R = 0	0.32609	2.9315	Ec = 0	0.19790	5.3439
R = 3	0.19792	5.3434	Ec = 0.4	0.19795	5.3419
R = 6	0.15806	6.3900	Ec = 0.8	0.19800	5.3400
H = 1	0.19792	5.3434	K_1 = 0	0.26441	5.3427
H = 3	0.17649	5.3436	K_1 = 4	0.18609	5.3435
H = 5	0.16157	5.3437	K_1 = 8	0.15557	5.3437
V = - 0.3	0.20033	5.6396	t = 0.5	0.18867	5.3700
V = - 0.2	0.20019	5.5881	t = 1	0.19712	5.3447
V = - 0.1	0.19994	5.5374	t = 1.5	0.19792	5.3434
V = 0.1	0.19913	5.4387	t = 2	0.19800	5.3434
V = 0..2	0.19858	5.3906	t = 2.5	0.19801	5.3433
V = 0.3	0.19792	5.3434	t = 3	0.19801	5.3433
Pr = 0.71	0.19792	5.3434	t = 4.5	0.19801	5.3433
Pr = 7.0	0.22514	3.8187	t = 6	0.19801	5.3433

Table 1. The Nusselt number and velocity gradient for variation of fluid parameters

A rise in the suction/injection parameter causes the Nusselt number as well as the velocity gradient which is a measure of skin friction to decrease. The viscous dissipation parameter increases the value of velocity gradient and reduces the Nusselt number. It is observed also that the radiation parameter lowers the velocity gradient and increases the Nusselt number. In addition, as the permeability parameter increases, the velocity gradient value reduces and causes a rise in Nusselt number. The presence of magnetic field term reduces the velocity gradient and increases the Nusselt number which is a measure of temperature gradient. For the time $t < 2.5$, it is observed that the Nusselt number decreases and velocity gradient increases with time but for time $t \geq 2.5$, the Nusselt number and velocity gradient remains the same with increase in time.

It was generally observed from the above results that the presence of permeability, suction/injection, magnetic field, viscous dissipation and radiation parameters in the fluid flow model played significant roles by influencing the behaviours of the natural velocity and temperature of the fluid.

CONCLUSIONS

In this paper, we have studied the unsteady free convection of magneto-hydrodynamic, radiative fluid flow past an infinite vertical porous plate in a porous medium under an optically thin limit case. An implicit finite difference scheme of Crank-Nicolson type was employed in solving the resulting coupled partial differential equations. From the study, we make the following conclusions:

(i) The radiation parameter increases the magnitude of the dimensionless velocity and temperature of fluid.

(ii) The suction/injection parameter leads to increase in the transient velocity and temperature of fluid, and the velocity and temperature near the porous plate are affected by injection.

(iii) A rise in magnetic field and permeability parameters reduce the velocity significantly and temperature slightly.

(iv) Eckert number increment slightly increases velocity and temperature distributions.

(v) The Nusselt number reduces with a rise in the suction/injection or viscous dissipation parameter and increases with a rise in the magnetic field, permeability or radiation parameter.

(vi) The velocity gradient increases with a rise in the presence of viscous dissipation parameter and lowers with a rise in magnetic field, permeability, radiation or suction/injection parameter.

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