BOUNDARY VALUE METHODS FOR NUMERICAL SOLUTION OF FOURTH ORDER PROBLEMS IN ORDINARY DIFFERENTIAL EQUATIONS

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Certification

This is to certify that the research reported in this thesis was carried out by SALAMI, Adesina Jimoh with Matriculation Number 97/55EC104 in the De-partment of Mathematics, Faculty of Physical Sciences, University of Ilorin, Ilorin, Nigeria, for the award of Doctor of Philosophy Degree in Mathemat-ics.

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Dedication

This Thesis is dedicated to God Almighty and my father in the Lord, Pastor Emmanuel Oluwasegun Akinloye.

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Abstract

Many real life situations can be modelled mathematically as Ordinary Di er-ential Equations (ODEs) or Partial Di erential Equations (PDEs). Problems arising from these are either of Initial Value Problem (IVP) or Boundary Value Problem (BVP) types in the case of ODEs and Initial Boundary Value Problems (IBVPs) in the case of PDEs. As PDEs are more di cult to solve, techniques have been developed to transform them into ODEs of BVP form. For higher order di erential equations, their solutions are mainly obtained numerically by reducing them to equivalent rst order systems. However, this is not considered e cient if direct method could be found for their so-lution. The main focus of the present study is therefore the development, analysis and application of a class of Boundary Value Methods (BVMs) for direct solution of Fourth Order ODEs of both Initial Value and Boundary Value types. A BVM is a Linear Multistep Method (LMM) coupled with boundary conditions. Also BVMs have the advantage of being self - starting viz-a-viz many other numerical methods. Therefore, the objectives of this study were to: (i) develop polynomial- tted BVMs for solving fourth order ODEs; (ii) develop trigonometrically- tted block BVMs for solving oscilla-tory fourth order ODEs; (iii) analyse the basic properties of the methods developed and these include zero- stability, consistency and convergence of the methods; (iv) implement the methods on speci c fourth order ODEs; and

compare the performance of the proposed methods with those of existing ones.

The problems considered in this work were the IVP :

$$y^{iv} = f(x; y; y^{0}; y^{00}; y^{000}) \quad a \quad x \quad b$$

$$y(a) = y_0; \; y^{0}(a) = y_0^{0}; \; y^{00}(a) = y_0^{00}; \; y^{000}(a) = y_0^{000}$$

and the BVP :

$$y^{iv} = f(x; y; y^{0}; y^{00}; y^{000}) \quad a \quad x \quad b$$

$$y(a) = A_1; \; y^{0}(a) = A_2; \; y^{00}(b) = B_1; \; y^{000}(b) = B_2$$

Trial solution of the type:

$$U(x) = \sum_{=0}^{p+q \; 1} a_r x^{r} \, ' \, y(x)$$

for the polynomial - tted BVMs and

$$U(x) = \sum_{=0}^{p+q \; 3} a_r x^{r} + a_{k+3} \cos wx + a_{k+4} \sin wx \, ' \, y(x)$$

for trigonometric - tted BVMs, where a_r are uniquely determined coe cients,! is the frequency, and p and q are the number of interpolation and collocation points, respectively. Interpolation and collocation were achieved through the sets of equations:

$$u(x_n) = y_n; \; u(x_{n+1}) = y_{n+1} \quad u(x_{n+k}) = y_{n+k}$$

$$u^{iv}(x_n) = f_n; \; u^{iv}(x_{n+1}) = f_{n+1}; \qquad u^{iv}(x_{n+k}) = f_{n+k}:$$

This system of equations was solved by Mathematica 8.0 for a_r; r = 0(1)(k+

The substitution of the resulting a^{0}_{r}s into the trial solution yielded the desired BVMs.

The ndings of the study were the:

derivation of BVMs with polynomial and basis;

derivation of BVMs with trigonometric basis ;

derived methods are consistent, zero stable and convergent;

proposed BVMs that can handle both initial and boundary value prob-lems; and

methods compare favourably well in terms of accuracy with existing methods .

The study concluded that a class of BVMs with polynomial and trigonomet-ric bases derived was developed and successfully implemented on sti and non- sti problems in fourth order ODEs. It is therefore recommended for application in determining the solution of real life problems leading to IVPs and BVPs in Fourth Order ODEs.

Contents

Chapter 1

GENERAL INTRODUCTION

1.1 Background of the Study

The numerical approximations of solutions of Di erential Equations have been and continue to be one of the principal concerns of Numerical Analysis. It is a very active eld of research due to the pressure of the evolving needs of the Mathematical modelization of real world phenomena. At present, new demands such as Hamiltonian problems or even chaotic systems are stimu-lating the search for new and more general methods which are able to handle problems other than classical ones.

Ordinary Di erential Equations occurs in many elds such as Dynamics, Fluid Mechanics, Elasticity, Engineering and Mathematical Physics Prob-lems. It also occur in elds such as Economics, Biology, Psychology, An-thropology, Operations Research etc. Most of these equations do not have analytic solutions, the approximate solution is sought by numerical methods. This can be obtained to high level of accuracy and with a reliable bound on

the error between approximate and analytic solutions. Several numerical methods such as nite di erence method, shooting method, nite element method, Linear multi step methods, etc, have been developed based on the nature of the Di erential Equation whose solution is being sought. Because of the numerous importance of Di erential Equations, many researchers have carried out a lot of work on the solution of such problems . Direct block code by Gamel and Zayed (2004), Boundary Value Methods via Multistep meth-ods by Jator and Lee (2012) and a host of others.

Furthermore, Dickmu , et al used trigonometrically tted extended black-ward di erentiation formulae for sti and oscillatory initial value problems. We are motivated in this direction to extend the idea of boundary value method to fourth order ordinary di erential equations using both polynomi-als and trigonometric functions as bases.

1.2 De nition of Relevant Terms

1.2.1 Di erential Equations

A di erential equation is an equation in terms of derivatives of an unknown function to be determined. A di erential equation can either be an Ordinary Di erential Equation (ODE) which involves only one independent variables or a Partial Di erential Equation (PDE) which has two or more independent variables .

1.2.2 Initial Value Problem

An initial value problem is an ordinary di erential equation whose conditions are speci ed at the intial point only or the beginning of an interval.

1.2.3 Boundary Value Problem

A Boundary Value Problem (BVP) is a di erential equation either ordinary or partial with at least one condition speci ed at the end points.

1.2.4 Boundary Value Method

A boundary value method is a Linear Multistep Method (LMM) coupled with boundary conditions. It is the method derived by distributing the additional conditions needed by LMMs to consistency de ne the approximation of the solution of a di erential equation partly at the beginning and at end of an interval.

1.2.5 Approximate Solution

An Approximate Solution is the expression obtained after the unknown con-stants have been found and substituted back into an assumed solution. It is called approximate solution because it is reasonably close to the exact solution. It is denoted by y_N in this thesis and taken as an exact represen-tation of the exact solution, where N is the degree of approximation used in the calculation. Approximate solutions are usually sought when information needed to get exact solution is not given.

1.2.6 Zero Stability

A boundary value method

$$u(x) = y_0 + {}_{v\ 1}\, y_{n+v\ 1} + y_{v\ n+v} + {}_{v+1}\, y_{n+v+1} + h^4 \sum_{=0}^{k} {}_{x_i}\, f_{i\ n+i} \qquad (1.1)$$

is said to be zero stable if its characteristics polynomial

$$(R) = \det \ {}_{=1} A^{(i)} R^{1\ i\#} \qquad = 0 \qquad (1.2)$$

$$X_i^{1}$$

satis es jR_jj 1; j = 1; 2; ; 4k and for those roots with jR_jj= 1, the multiplicity does not exceed 4 where A is a 4k 4k matrix.

1.2.7 Consistency

A boundary value method (1.1) is said to be consistent if

(i) it has order p > 4

$$\sum_{r=0}^{x^k} {}_r = 0$$

1.2.8 Convergence

This is the property that determines whether the solution generated by a numerical method approaches the exact or theoretical solution as the step length h tends to zero. Generally, a numerical method is said to be convergent if it is zero stable and consistent.

1.2.9 Collocation Method

This is a method of evaluating a given di erential equation after assuming an approximate solution which is then substituted back into the given problem and evaluated at same equally spaced points in order to nullify the unknown constants in the assumed solution.

1.2.10 Taylor's Series

Let f(x) be a function with derivatives of all orders around an interior point a in the interval (x_0; x_1). The Taylor's series of f(x) generated at x = a is given by

$$f(x) = f(a) + \frac{f^{0}(a)}{1!}(x\ \ a) + \frac{f^{00}(a)}{2!}(x\ \ a)^2 + \frac{f^{000}(a)}{3!}(x\ \ a)^3 + \frac{f^{(k)}(a)}{k!}(x\ \ a)^k \quad (1.3)$$

1.3 Aim and Objectives of the Thesis

The aim of this study is to develop an alternative method for solving fourth order ordinary di erential equations without necessarily reducing to a system of lower order.

The objectives of this thesis are to:

construct a polynomial- tted BVM for solving fourth order ODEs;

construct a trignometrically- tted block BVM for solving oscillatory fourth order ODEs.

analyse the basic properties of the methods developed and this includes zero- stability, consistency and convergence of the methods;

implement the methods on some fourth order ODEs; and

compare the performance of the proposed methods with those of exist-ing ones.

1.4 Organization of the Thesis

Basically, the thesis consists of ve chapters. Chapter one contains the gen-eral introduction, some relevant de nitions, aim and objectives of the re-search. Chapter two deals with relevant and related literature review. In chapter three, formulation of the proposed method and derivation for poly-nomial and trigonometric bases will be obtain for various values of step length k. Chapter four deals with the analysis of basic properties of the methods obtained in chapter three. Several problems were also used to test the perfor-mance of the methods discussed. Chapter ve gives the general conclusion, contribution to knowledge and recommendation for further studies.

1.5 Limitation of the Study

This study is limited to the following:

only continuously di erentiable functions in the interval of integration were considered.

derivation of the methods are restricted to polynomials and trigono-metric bases.

only block implicit methods were considered in this research.

Chapter 2

RELATED LITERATURE REVIEW

2.1 Introduction

This chapter deals with review of contributions in the solution of fourth order di erential equation of the form

$$y^{iv} = f(x; y; y^{0}; y^{00}; y^{000}) \quad a \quad x \quad b$$

subject to initial conditions

$$y(a) = y_0 \; ; \; y^{0}(a) = y_0^{0} \; ; \; y^{00}(a) = y_0^{00} \; ; \; y^{000}(a) = y_0^{000} \tag{2.1}$$

or boundary conditions

$$y(a) = A_1; \qquad y^{0}(a) = A_2; \; y^{00}(b) = B_1 \; ; \; y^{000}(b) = B_2$$

2.2 Review of Existing Methods For Fourth Order Boundary Value Problems

According to Hollevoet , et al (2011), fourth order boundary value problems are solved by means of exponentially tted method of di erent orders. These methods depend on a parameter constructed following a six-step ow chart. Srivastava, et al (2012) used non-polynomial quintic spline functions to construct numerical algorithms for fourth order boundary value problems of the form

$$y^{iv}(x) = f(x; y); \quad a \quad x \quad b \tag{2.2}$$
$$y(a) = A_0; \; y^{00}(a) = B_0; \; y(b) = A_1; \; y^{00}(b) = B_1:$$

This approach generalizes non - polynomial spline algorithms and provides a solution at every point of the range of integration.

Siddiqi and Akram (2003) devised a di erence scheme with quintic spline function to solve the problem (2.2) with boundary conditions.

$$y(a) = A_1; \qquad y^{0}(a) = A_2$$
$$y^{00}(b) = B_1; \qquad y^{000}(b)= B_2 \tag{2.3}$$

Twizell and Tiraizi, (1985) solved (2.2) by multi-derivative methods.
Kumar and Sonali (2014) applied an initial value technique to solve fourth order singularly perturbed boundary layer at the left of the underlying inter-val.

In this method, a fourth order singularly perturbed boundary value problem is reduced to three initial value problems and Runge-Kutta fourth order method was used to solve these problems numerically.

Usmani (1983) developed nite di erence methods for a continuous approx-imation to the solution of a two- point boundary value problem involving a fourth order linear di erential equation of the type

$$\left[L + f(x)g(x)\right] y(x) = g(x); \quad L = \frac{d^4}{dx^4} \qquad a < x < b \tag{2.4}$$

with boundary conditions

$$y(a) = A_1; \;\; y(b) = A_2; \;\; y^0(a) = B_1; \;\; y^0(b) = B_2$$

where A_1; A_2; B_1; B_2 are constants.

Kasim , et al (2015), used a Runge-Kutta type method for directly solving special fourth order ordinary di erential equations

$$y^{iv}(x) = f(x; y) \tag{2.5}$$

with initial conditions

$$y(x_0) = y_0; \;\; y^0(x_0) = y_0{}^0; \;\; y^{00}(x_0) = y_0{}^{00}; \;\; y^{000}(x_0) = y_0{}^{000}$$

The other conditions of the method were derived up to order 5 based on three stage fourth and fth order Runge-Kutta type methods.

2.3 Review on Collocation Method

Collocation methods were rst studied in the late 1930s to solve IVPs in ODEs. It was shown that collocation is continuous piecewise polynomial spaces that lead to a class of implicit high order Runge-Kutta method. Collocation methods involve the determination of an approximate solution in a suitable set of functions called trial solution. This method is one of the

Methods for solving ODEs and PDEs. Several authors have employed the technique of collocation in di erent ways to solve di erential equation prob-lems.

Agarwal (1986), solved higher order di erential equations using collocation method.

Collocation methods were used over the past decades in seeking solution to a broad class of problems covering Ordinary and Partial di erential equations, functional equations and intergro-di erential equations. The popularity of such methods is due to their conceptual simplicity, wide applicability and ease of implementation. The collocation method is not new but dates back in 1930s according to Kantorovich and Akilov (1964). The method of Kan-torovich was actually a method of lines collocation procedure for the solution of partial di erential equations in two variables with collocation being applied in one variable for each xed value of the second. Collocation at the roots of orthogonal polynomials is often called orthogonal collocation; particularly in the chemical engineering literature, Fox (1968). Chebyshev orthogonal col-location methods are described by Fox and Parker (1968) for the solution of ordinary and partial di erential equations together with integral equations. Spectral collocation methods are closely related to this form of collocation, Henrici (1962). There is a rapid development as reported in the literature on the use of spline collocation methods for the numerical solution of rst order ODEs. The multistep collocation techniques involve obtaining an approxi-mate solution of a set of functions or of a linear combination of functions known as the trial function The analytic solution of an initial value problem is assumed to be approximated by the basis function. The linear combina-

tion of these basis functions is required to satisfy the approximation at some certain grid points called the collocation points.

2.4 Review of Block Methods

In view of the advantages mentioned in sections 2.2 and 2.3, many researchers concentrated their e orts on advancing the numerical solution of ODEs. One of the outcome is the development of a class of methods called Block method. This method generates approximate solution at di erent grid points in the interval of integration simultaneously. It is less expensive in terms of the num-ber of function evaluations compared to LMMs and Runge- Kutta methods. This method was rst proposed by Milne (1953), who advocated their use only as a means of obtaining starting values for predictor-corrector algo-rithms.

Lambert(1973), developed a two-step fourth order explicit block method. Implicit one- step block method was analyzed by Shampine and Watts (1969) Chu and Hamilton (1987), suggested a generalization of the LMMs to a class of multiblock methods where step values are all obtained together in a single block accomplished by allocating parallel tasks on separate processes.

Ken , et al (2009) also developed explicit-implicit three-point block methods for the direct solution of special second order ODEs.

Fatunla (1991), proposed block method for the solution of special second order ODEs.

Jator (2009), developed a self starting block LMM for the solution of second

order ODEs of the form

$$y^{00} = f(x; y; y^{0}) \tag{2.6}$$

Dickmu , et al (2016) constructed modi ed methods based on the form

$$u(x) = \sum_{r=0}^{k} a_r x^r + a_k \sin wx + a_{k+1} \cos wx \; ' \; y(x)$$

subject to solve initial conditions

$$y(a) = y_0; \qquad y^{0}(a) = y_0^{0}$$

or boundary conditions

$$y(a) = y_0; \;\; y(b) = y_N$$

Salami A.J and Adeniyi R.B (2017) developed a class of block methods for solving fourth order Initial and boundary value problems in ordinary di erential equations directly without reducing to a system of lower order equations. Majid and Suleiman (2007) have adopted block methods where the derivative function was interpolated using Lagrange interpolation.

Higino and Kalogiratou (2015) developed a hybrid block method for the numerical integration of general second-order initial value problems. The method considers two intra-step points which are selected adequately in or-der to optimize the local truncation errors of the main formulae for the solution and the derivative at the nal point of the block.

These di erent methods have their desirable qualities. Thus, the method proposed in this research is one that combines these qualities for the solution of (2.1).

Folaranmi (2017) developed a class of continuous hybrid k step block meth-ods for solving general third order IVPs of the form

$$y^{000} = f(x; y; y^{0}; y^{00}); \qquad y^{0}(a) = \qquad ; \; y^{0}(a) = \qquad ; \; y^{000}(a) = \quad a \qquad x \qquad b$$

Two kinds of orthogonal polynomials namely Chebyshev and constructed or-thogonal polynomials were used as basis functions for the trial solution. Direct methods for solving (2.1) which were more e cient than the method of reduction to system of rst order ordinary di erential equations have been studied by many scholars including; Adeyefa (2014), Anake et al (2012), Ade-sanya et al (2012), Awoyemi et al (2011), Adesanya, Anake and Udoh (2008), Vigo-Aguiar and Ramos (2006), Yusuph and Onumanyi (2005), Awoyemi and Kayode (2005), Simos (2002), Fatunla (1988), Chawla and Sharma (1985), Twizel and Khalid (1984), Jeltsch (1976), Hairer and Wanner (1976), Gear (1971), Henrici (1962) to mension a few. The direct methods are self-starting methods which are formulated in terms of linear multistep methods called block methods. The block method provides the traditional advantage of one-step methods, e.g., Runge-Kutta methods, of being self-starting and permit-ting easy change of step length Lambert (1973). Another important feature of the block approach is that all the discrete schemes are of uniform order and are obtained from a single continuous formula in contrast to the non-self-starting predictor-corrector approach. This method was used by Anake et al (2013) to derive a class of one-step hybrid methods for the numerical solution of second order ordinary di erential equation with power series as the basis function.

2.5 Review on Boundary Value Methods

Lee and Fudziah(2014) constructed hybrid collocation method with three o - step points for direct solution of fourth order ordinary di erential equations. Interpolation and collocation techniques were applied on basis polynomial to generate the main and additional methods.

Amodio and Iavernaro (2006), used interpolation and collocation procedures to develop a symmetric boundary value method for second order initial and boundary value problems.

Amodio and Mazzia (1995), applied multistep methods as a Boundary Valued Method to solve Ordinary Di erential equations.

Annamana Mazzia and Francescal Mazzia (1997) used Extended Trapezoidal Rules (ETRs) which is a class of BVMs with the same number of initial and nal additional conditions. It was used to solve a system of ODEs of the form:

$$y^{00}(t) = f(t; y(t)); \qquad t_0 \qquad t \qquad T$$

Jator (2016), constructed a continuous linear multistep method with trigono-metric coe cients from which a symmetric main method and additional methods were produced. The main and additional methods whose coe cients depend on the frequency and step length were then applied as a trigonomet-ric Symmetric Boundary Valued Method (SBVM) to solve systems of second order initial and boundary value problems of the form $y^{00} = f(x; y)$ without st reducing the di erential equation to an equivalent rst order system.

Brugnano and Trigiante (1998) combined methods of the form

$$y_{n+4} + {}_0y_n + {}_1y_{n+1} + {}_2y_{n+2} + {}_3y_{n+3} = h^2({}_0f_n + {}_1f_{n+1} + {}_2f_{n+2}$$

$$+ {}_3f_{n+3} + {}_4f_{n+4}) \quad (n = 0; 1; \quad N \quad 4) \tag{2.7}$$

together with

$$y_3 = \hat{}_0y_0 + \hat{}_1y_1 + \hat{}_2y_2 + \hat{}_3y_3 + h^2({}_0\hat{f}_0 + {}_1\hat{f}_1 + {}_2\hat{f}_2$$

$$\hat{}_3f_3 + \hat{}_4f_4)$$

$$hy_0^0 = \quad \bar{}_0y_0 + \bar{}_1y_1 + \bar{}_2y_2 + \bar{}_3y_3 = h^2({}_0\bar{f}_0 + {}_1\bar{f}_1 + {}_2\bar{f}_2$$

$$\bar{}_3f_3 + \bar{}_4f_4) \tag{2.8}$$

and

$$y_{N\ 3} = \hat{}_0y_N + \hat{}_1y_{N\ 1} + \hat{}_2y_N \quad {}_2 \quad h^2 \quad \hat{} \quad (\quad {}_0\ {}_N \quad \hat{} \quad {}_1\ {}_N\ {}_1 \quad \hat{} \quad {}_2\ {}_N\ {}_2$$

$$f_{3\ N\ 3} + \hat{f}_{4\ N\ 4}) \tag{2.9}$$

as boundary value methods to solve

$$y^{00} = f(x; y); \; y(x_0) = y_0^0(x_0) = A \; x_0 \quad x \quad x_N = y_0^0 \tag{2.10}$$

were constructed. Most of these methods require the use of known pivotal points generated through the use of a set of so called Pivot formulae which are known as predictors. Implementation of the method in predictor-corrector mode is very costly as subroutines are very complicated to write because of the special technique required to supply staring values and for varying the step size and which leads to longer computational time and more human e ort.

Most of existing methods for solution of fourth order ODEs are de cient in

handling such equations directly without recourse to reduction to systems of lower order equations. This research intends to construct a BVM that can solve fourth order ODEs directly without reducing to systems of lower order equations. The BVM will be self starting and can handle both IVPs and BVPs.

The next chapter details the presentation of the methodology of this research , the speci cation of the derived method and the analysis.

Chapter 3

RESEARCH METHODOLOGY

3.1 Introduction

This chapter deals with the development of continuous LMMs from which several numerical schemes are obtained. Two basis functions namely; polyno-mials and trigonometric are used to obtain a class of Boundary value method in block form.

The basic idea is to approximate the exact solution y(x) of (2.1) in the par-tition

$$_N = [a = x_0 < x_1 < \quad < x_n = b]$$

of the interval [a; b] by using polynomial basis function

$$U(x) = \sum_{=0}^{k+4} a_r x^r \, ' \, y(x) \tag{3.1}$$

and trigonometric basis function

$$U(x) = \sum_{=0}^{k+4} a_r x^r + a_{k+3} \cos !x + a_{k+4} \sin !x \; ' \; y(x) \qquad (3.2)$$

where a_r; a_{k+3}; a_{k+4} are coe cients to be uniquely determined and ! is the frequency. These functions satisfy respectively.

$$U^{iv}(x) = \sum_{=0}^{k+4} r(r \;\; 1)(r \;\; 2)(r \;\; 3)a_r x^{r \;\; 4} = f(x; u; u^0; u^{00}; u^{000}) \qquad (3.3)$$

and

$$U^{iv}(x) = \sum_{=0}^{k+4} r(r \qquad 1)(r \;\; 2)(r \;\; 3)a_r x^{r \;\; 4} + !^4 a_{k+3} \cos !x + !^4 a_{k+4} \sin !x$$

$$= f(x; u; u^0; u^{00}; u^{000}) \qquad (3.4)$$

3.2 Derivation of Methods

In this section, a k- step LMM is constructed using polynomial and trigonom-etry as basis functions so as to develop BVMs. Basically, we consider method of the form (1.1) where

$$v = \begin{cases} \frac{k}{2}; & k(\text{even}) \\ \frac{k+1}{2}; & k(\text{odd}) \end{cases}$$

An approximation to the exact solution y(x) is sought by the interpolating functions (3.1) & (3.2) and collocation functions (3.3) & (3.4). The condi-tion that the interpolation functions (3.1) & (3.2) coincide with the analytic solution at the points x_0; x_1; x_k is imposed. These lead to k + 5 equations.

3.3 Speci cation of BVMs

Considering the continuous scheme (1.1) and taking speci c cases k = 4; k = 6 and k = 8

3.3.1 Case k = 4; v = 2

$$u(x) = {}_0(x)y_n + {}_1(x)y_{n+1} + {}_2(x)y_{n+2} + {}_3(x)y_{n+3} + h^4 \; {}^{h}_{i} \; {}_0(x)f_n + {}_1(x)f_{n+1}$$

$${}_2(x)f_{n+2} + {}_3(x)f_{n+3} + {}_4(x)f_{n+4} \qquad (3.5)$$

From (3.1)

$$y(x) \text{ ' } U(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + a_8x^8 \quad (3.6)$$

Using (3.1) & (3.3), the following system of equations are obtained.

$$\begin{matrix} 0 & 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 \end{matrix} \begin{matrix} 1 \\ a_0 \end{matrix} \begin{matrix} 1 \\ 0 \end{matrix} \begin{matrix} y_n \end{matrix} \begin{matrix} 1 \end{matrix}$$

$$\begin{matrix} 1 & x_{n+2} & x_{n+2}^2 & x_{n+2}^3 & x_{n+2}^4 & x_{n+2}^5 & x_{n+2}^6 & x_{n+2}^7 & x_{n+2}^8 \\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6 & x_{n+1}^7 & x_{n+1}^8 \\ 1 & x_{n+3} & x_{n+3}^2 & x_{n+3}^3 & x_{n+3}^4 & x_{n+3}^5 & x_{n+3}^6 & x_{n+3}^7 & x_{n+3}^8 \\ 0 & 0 & 0 & 0 & 24 & 120x_n & 360x_n^2 & 840x_n^3 & 1680x_n^4 \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+1} & 360x_{n+1}^2 & 840x_{n+1}^3 & 1680x_{n+1}^4 \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+2} & 360x_{n+2}^2 & 840x_{n+2}^3 & 1680x_{n+2}^4 \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+3} & 360x_{n+3}^2 & 840x_{n+3}^3 & 1680x_{n+3}^4 \end{matrix} \begin{matrix} a_0 \\ a_1 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{matrix} = \begin{matrix} y_{n+2} \\ y_{n+1} \\ y_{n+3} \\ f_n \\ f_{n+1} \\ f_{n+2} \\ f_{n+3} \end{matrix}$$

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 24 & 120x_{n+4} & 360x_{n+4}^{2} & 840x_{n+4}^{3} & 1680x_{n+4}^{4}
\end{bmatrix}
\begin{bmatrix} a_8 \end{bmatrix}
=
\begin{bmatrix} f_{n+4} \end{bmatrix}
$$

The solution of this system yields the value of a_k; $k = 0(1)8$; which are substituted back into (3.6) and which after some simpli cation leads to the

continuous scheme of the form

$$y(x) = \frac{1}{20160h^4} f_{n+2} (209h^4 + 162h^3x + 53h^2x^2 \quad 30hx^3 + 3x^4)x(h \quad x)(2h \quad x)(3h \quad x) + \frac{1}{30240h^4} f_{n+3}(7h^4 + 60h^3x + 19h^2x^2 \quad 24hx^3 + 3x^4)x(h \quad x)(2h \quad x)(3h \quad x) \quad \frac{1}{120960h^4} f_{n+4}(h^4 + 42h^3x + 13h^2x^2 \quad 18hx^3 + 3x^4)x(h \quad x)(2h \quad x)(3h \quad x) \quad \frac{1}{30240h^4} f_{n+1}x(h \quad x)(2h \quad x)(3h \quad x)(905h^4 \quad 115h^2x^2 + 36hx^3 \quad 3x^4 \quad 96h^3x) \quad \frac{1}{120960h^4} f_n x(h \quad x)(2h \quad x)(3h \quad x)(193h^4 + 205h^2x^2 \quad 42hx^3 + 3x^4 \quad 390h^3x) + \frac{(h \quad x)(2h \quad x)(3h \quad x)y_n}{6h^3} + \frac{x(2h \quad x)(3h \quad x)y_{n+1}}{2h^3} \quad \frac{x(h \quad x)(3h \quad x)y_{n+2}}{2h^3} + \frac{x(h \quad x)(2h \quad x)f_{n+3}}{6h^3} \tag{3.7}$$

The main formula is obtained by evaluating (3.7) at x_{n+4} to get

$$y_{n+4} = \frac{f_n h^4}{720} + \frac{31f_{n+1}h^4}{180} + \frac{79f_{n+2}h^4}{120} + \frac{31f_{n+3}h^4}{180} \quad \frac{f_{n+4}h^4}{720} \quad y_n + 4y_{n+1} \quad 6y_{n+2} + 4y_{n+3} \tag{3.8}$$

The continuous scheme (3.7) is di erentiated and evaluated at x_n; x_{n+1}; x_{n+2}; x_{n+3} and x_{n+4} to give, respectively, the following expressions;

$$y_n^0 = \frac{193f_nh^3}{20160} \quad \frac{181f_{n+1}h^3}{1008} \quad \frac{209f_{n+2}h^3}{3360} + \frac{f_{n+3}h^3}{720} \quad \frac{f_{n+4}h^3}{20160} \quad \frac{11y_n}{6h} + \frac{3y_{n+1}}{h} \quad \frac{3y_{n+2}}{2h} + \frac{y_{n+3}}{3h}$$

$$y_n^0{}_{+1} = \frac{31f_nh^3}{60480} + \frac{727f_{n+1}h^3}{15120} \quad \frac{397f_{n+2}h^3}{10080} \quad \frac{13f_{n+3}h^3}{3024} + \frac{41f_{n+4}h^3}{60480} \quad \frac{y_n}{3h} \quad \frac{y_{n+1}}{2h} + \frac{y_{n+2}}{h} \quad \frac{y_{n+3}}{6h}$$

$$y_{n+2}^{0} = \frac{11f_n h^3}{12096} - \frac{493f_{n+1}h^3}{15120} + \frac{79f_{n+2}h^3}{1440} - \frac{59f_{n+3}h^3}{15120} + \frac{41f_{n+4}h^3}{60480} + \frac{y_n}{6h} - \frac{y_{n+1}}{h} + \frac{y_{n+2}}{2h} - \frac{y_{n+3}}{3h}$$

$$y_{n+3}^{0} = \frac{23f_nh^3}{20160} + \frac{311f_{n+1}h^3}{5040} + \frac{121f_{n+2}h^3}{672} + \frac{47f_{n+3}h^3}{5040} + \frac{f_{n+4}h^3}{20160} - \frac{y_n}{3h} + \frac{3y_{n+1}}{2h} - \frac{3y_{n+2}}{h} + \frac{11y_{n+3}}{6h}$$

$$y_{n+4}^{0} = \frac{151f_nh^3}{60480} + \frac{679f_{n+1}h^3}{2160} + \frac{12793f_{n+2}h^3}{10080} + \frac{7489f_{n+3}h^3}{15120} + \frac{85f_{n+4}h^3}{12096} - \frac{11y_n}{6h} + \frac{7y_{n+1}}{h} - \frac{19y_{n+2}}{2h} + \frac{13y_{n+3}}{3h}$$

$$y_n^{00} = \frac{4463f_nh^2}{60480} + \frac{10531f_{n+1}h^2}{15120} + \frac{1327f_{n+2}h^2}{10080} + \frac{283f_{n+3}h^2}{15120} + \frac{241f_{n+4}h^2}{60480} + \frac{2y_n}{h^2} - \frac{5y_{n+1}}{h^2} - \frac{4y_{n+2}}{h^2} + \frac{y_{n+3}}{h^2}$$

$$y_{n+1}^{00} = \frac{157f_nh^2}{60480} + \frac{1187f_{n+1}h^2}{15120} + \frac{17f_{n+2}h^2}{10080} + \frac{11f_{n+3}h^2}{15120} + \frac{11f_{n+4}h^2}{60480} + \frac{y_n}{h^2} - \frac{2y_{n+1}}{h^2} + \frac{y_{n+2}}{h^2}$$

$$y_{n+2}^{00} = \frac{11f_nh^2}{60480} - \frac{53f_{n+1}h^2}{15120} + \frac{773f_{n+2}h^2}{10080} - \frac{53f_{n+3}h^2}{15120} + \frac{11f_{n+4}h^2}{60480} - \frac{y_{n+1}}{h^2} + \frac{2y_{n+2}}{h^2} - \frac{y_{n+3}}{h^2}$$

$$y_{n+3}^{00} = \frac{73f_nh^2}{60480} + \frac{2593f_{n+1}h^2}{15120} + \frac{6619f_{n+2}h^2}{10080} + \frac{1417f_{n+3}h^2}{15120} - \frac{241f_{n+4}h^2}{60480} - \frac{y_n}{h^2} + \frac{4y_{n+1}}{h^2} - \frac{5y_{n+2}}{h^2} + \frac{2y_{n+3}}{h^2}$$

$$y_{n+4}^{00} = \frac{409f_nh^2}{60480} + \frac{5491f_{n+1}h^2}{15120} + \frac{14599f_{n+2}h^2}{10080} + \frac{15739f_{n+3}h^2}{15120} + \frac{859f_{n+4}h^2}{12096} - \frac{2y_n}{h^2} + \frac{7y_{n+1}}{h^2} - \frac{8y_{n+2}}{h^2} + \frac{3y_{n+3}}{h^2}$$

$$y_n^{000} = \frac{53f_nh}{160} - \frac{23f_{n+1}h}{20} + \frac{f_{n+2}h}{16} - \frac{f_{n+3}h}{10} + \frac{3f_{n+4}h}{160} - \frac{y_n}{h^3} + \frac{3y_{n+1}}{h^3} - \frac{3y_{n+2}}{h^3} + \frac{y_{n+3}}{h^3}$$

$$y_{n+1}^{000} = \frac{5f_n h}{288} \quad \frac{91f_{n+1}h}{360} \quad \frac{73f_{n+2}h}{240} + \frac{17f_{n+3}h}{360} \quad \frac{11f_{n+4}h}{1140} \quad \frac{y_n}{h^3} + \frac{3y_{n+1}}{h^3} \quad \frac{3y_{n+2}}{h^3} + \frac{y_{n+3}}{h^3}$$

$$y_{n+2}^{000} = \frac{13f_n h}{1440} + \frac{41f_{n+1}h}{180} + \frac{79f_{n+2}h}{240} \quad \frac{f_{n+3}h}{18} + \frac{11f_{n+4}h}{1140} \quad \frac{y_n}{h^3} + \frac{3y_{n+1}}{h^3} \quad \frac{3y_{n+2}}{h^3} + \frac{y_{n+3}}{h^3}$$

$$y_{n+3}^{000} = \frac{f_n h}{160} + \frac{f_{n+1}h}{8} + \frac{77f_{n+2}h}{80} + \frac{17f_{n+3}h}{40} \quad \frac{3f_{n+4}h}{160} \quad \frac{y_n}{h^3} + \frac{3y_{n+1}}{h^3} \quad \frac{3y_{n+2}}{h^3} + \frac{y_{n+3}}{h^3}$$

$$y_{n+4}^{000} = \frac{29f_n h}{1440} + \frac{49f_{n+1}h}{180} + \frac{143f_{n+2}h}{240} + \frac{119f_{n+3}h}{90} + \frac{95f_{n+4}h}{288} \quad \frac{y_n}{h^3} + \frac{3y_{n+1}}{h^3} \quad \frac{3y_{n+2}}{h^3} + \frac{y_{n+3}}{h^3}$$

3.3.2 Case k = 6; v = 3

$$u(x) = {}_0(x)y_n + + {}_2(x)y_{n+2} + {}_3(x)y_{n+3} + {}_4(x)y_{n+4} + h^4 \; h \; {}_0(x)f_n + {}_1(x)f_{n+1} \; {}_i$$

$${}_2(x)f_{n+2} + {}_3(x)f_{n+3} + {}_4(x)f_{n+4} + {}_5(x)f_{n+5} + {}_6(x)f_{n+6} \qquad (3.9)$$

From (3.1)

$$y(x) \; ' \; U(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + a_8x^8 + a_9x^9 + a_{10}x^{10} \qquad (3.10)$$

Using (3.1) & (3.3), a system of 11 equations are obtained.

$$
\begin{bmatrix}
1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 & x_n^9 & x_n^{10} \\
1 & x_{n+2} & x_{n+2}^2 & x_{n+2}^3 & x_{n+2}^4 & x_{n+2}^5 & x_{n+2}^6 & x_{n+2}^7 & x_{n+2}^8 & x_{n+2}^9 & x_{n+2}^{10} \\
1 & x_{n+3} & x_{n+3}^2 & x_{n+3}^3 & x_{n+3}^4 & x_{n+3}^5 & x_{n+3}^6 & x_{n+3}^7 & x_{n+3}^8 & x_{n+3}^9 & x_{n+3}^{10} \\
1 & x_{n+4} & x_{n+4}^2 & x_{n+4}^3 & x_{n+4}^4 & x_{n+4}^5 & x_{n+4}^6 & x_{n+4}^7 & x_{n+4}^8 & x_{n+4}^9 & x_{n+4}^{10} \\
0 & 0 & 0 & 0 & 24 & 120x_n & 360x_n^2 & 840x_n^3 & 1680x_n^4 & 3024x_n^5 & 5040x_n^6 \\
0 & 0 & 0 & 0 & 24 & 120x_{n+1} & 360x_{n+1}^2 & 840x_{n+1}^3 & 1680x_{n+1}^4 & 3024x_{n+1}^5 & 5040x_{n+1}^6 \\
0 & 0 & 0 & 0 & 24 & 120x_{n+2} & 360x_{n+2}^2 & 840x_{n+2}^3 & 1680x_{n+2}^4 & 3024x_{n+2}^5 & 5040x_{n+2}^6 \\
0 & 0 & 0 & 0 & 24 & 120x_{n+3} & 360x_{n+3}^2 & 840x_{n+3}^3 & 1680x_{n+3}^4 & 3024x_{n+3}^5 & 5040x_{n+3}^6 \\
0 & 0 & 0 & 0 & 24 & 120x_{n+4} & 360x_{n+4}^2 & 840x_{n+4}^3 & 1680x_{n+4}^4 & 3024x_{n+4}^5 & 5040x_{n+4}^6 \\
 & & & & & & 360x_{n+5}^2 & 840x_{n+5}^3 & 1680x_{n+5}^4 & 3024x_{n+5}^5 & 5040x_{n+5}^6 \\
0 & 0 & 0 & 0 & 24 & 120x_{n+6} & 360x_{n+6}^2 & 840x_{n+6}^3 & 1680x_{n+6}^4 & 3024x_{n+6}^5 & 5040x_{n+6}^6
\end{bmatrix}
\begin{bmatrix} 1 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \end{bmatrix}
=
\begin{bmatrix} 0 \\ y_{n+2} \\ y_{n+3} \\ y_{n+4} \\ f_n \\ f'_{n+1} \\ f'_{n+2} \\ f'_{n+3} \\ f'_{n+4} \\ f'_{n+5} \\ f'_{n+6} \end{bmatrix}
$$

The solution of this system yields the value of a_k; k = 0(1)10; which are substituted back into(3.10) and which after some simpli cation leads to the

continuous scheme of the form.

$y(x) = \frac{1}{1814400h^6}\,{}^{n+5}\,(2h\ x)(3h\ x)(4h\ x)x\quad 66h^6\quad 730h^5x$

$\frac{1}{725760h^6}\,{}^{n+4}\,(2h\ x)(3h\ x)(4h\ x)x\quad + 234h^4x^2 + 530h^3x^3\quad 300h^2x^4 + 53hx^5\quad 3x^6\ \#$

$117h^6\quad 905h^5x$

$\frac{1}{1814400h^6}\,{}^{n+1}\,(2h\ x)(3h\ x)(4h\ x)x\quad + 309h^4x^2 + 679h^3x^3\quad 363h^2x^4 + 58hx^5\quad 3x^6\ \#$

$23490h^6\quad 10394h^5x$

$\frac{1}{3628800h^6}\,{}^{n+6}\,(2h\ x)(3h\ x)(4h\ x)x\quad 2166h^4x^2 + 2530h^3x^3\quad 660h^2x^4 + 73hx^5\quad 3x^6\ \#$

$15h^6\quad 203h^5x$

$\frac{1}{181440h^6}\,{}^{n+3}\,(2h\ x)(3h\ x)(4h\ x)x\quad + 63h^4x^2 + 145h^3x^3\quad 85h^2x^4 + 16hx^5\quad x^6\ \#$

$1002h^6\quad 162h^5x$

$\frac{1}{3628800h^6}\,{}^{n}\,(2h\ x)(3h\ x)(4h\ x)x\quad + 210h^4x^2 + 330h^3x^3\quad 148h^2x^4 + 21hx^5\quad x^6\ \#$

$1233h^6\quad 3605h^5x$

$\frac{1}{725760h^6}\,{}^{n+2}\,(2h\ x)(3h\ x)(4h\ x)x\quad + 3297h^4x^2\quad 1325h^3x^3 + 265h^2x^4\quad 26hx^5 + x^6\ \#$

$16683h^6 + 4873h^5x$

$405h^4x^2\quad 1571h^3x^3 + 543h^2x^4\quad 68hx^5 + 3x^6\quad \#+$

$$\frac{(2h\ \ x)(3h\ \ x)(4h\ \ x)y_n}{24h^3} + \frac{x(12h^2\ \ 7hx + x^2)y_{n+2}}{4h^3}$$

$$\frac{x(8h^2\ 6hx + x^2)y_{n+3}}{3h^3} + \frac{x(6h^2\ \ 5hx + x^2)y_{n+4}}{8h^3} \qquad (3.11)$$

The BVMs are obtained by evaluating (3.11) at x_{n+6}, x_{n+1} and x_{n+5}

$$y_{n+6} = \frac{f_n h^4}{1680} + \frac{13f_{n+1}h^4}{84} + \frac{2311f_{n+2}h^4}{1680} + \frac{617f_{n+3}h^4}{210} + \frac{2311f_{n+4}h^4}{1680} + \frac{13f_{n+5}h^4}{84} + \frac{f_{n+6}h^4}{1680}\ \ y_n + 9y_{n+2}\ \ 16y_{n+3} + 9y_{n+4} \qquad (3.12)$$

$$y_{n+1} - \frac{f_n h^4}{3780}\ \ \frac{143f_{n+1}h^4}{3360}\ \ \frac{3343f_{n+2}h^4}{20160}\ \ \frac{313f_{n+3}h^4}{7560}\ \ \frac{f_{n+4}h^4}{1120} + \frac{f_{n+5}h^4}{2016}\ \ \frac{f_{n+6}h^4}{12096} + \frac{y_n}{4} + \frac{5y_{n+2}}{2}\ \ y_{n+3} + \frac{y_{n+4}}{4} \qquad (3.13)$$

$$y_{n+5} - \frac{f_n h^4}{15120} + \frac{79f_{n+1}h^4}{2016} + \frac{461f_{n+2}h^4}{1344} + \frac{131f_{n+3}h^4}{189} + \frac{359f_{n+4}h^4}{2016} \frac{13f_{n+5}h^4}{3360} + \frac{5f_{n+6}h^4}{12096}\ \ \frac{y_n}{4} + \frac{5y_{n+2}}{2}\ \ 5y_{n+3} + \frac{15y_{n+4}}{4} \qquad (3.14)$$

The continuous scheme (3.7) is di erentiated and evaluated at x_n; x_{n+1}; x_{n+2}; x_{n+3}; x_{n+4}; x_{n+5} and x_{n+6} to give, respectively, the following expressions;

$$y_n^0 = \frac{137f_n h^3}{16800}\ \ \frac{87f_{n+1}h^3}{280}\ \ \frac{5561f_{n+2}h^3}{10080}\ \ \frac{167f_{n+3}h^3}{1260} + \frac{13f_{n+4}h^3}{3360}\ \ \frac{11f_{n+5}h^3}{12600} + \frac{f_{n+6}h^3}{10080}\ \ \frac{13y_n}{12h} + \frac{5y_{n+2}}{h}\ \ \frac{5y_{n+3}}{3h} + \frac{5y_{n+4}}{4h}$$

$$y_n^{0+1} = \frac{5f_n h^3}{36288} + \frac{3359f_{n+1}h^3}{50400} + \frac{3103f_{n+2}h^3}{24192} + \frac{173f_{n+3}h^3}{18144} + \frac{11f_{n+4}h^3}{2016}\ \ \frac{5f_{n+5}h^3}{3024} + \frac{401f_{n+6}h^3}{1814400}\ \ \frac{11y_n}{24h} + \frac{y_{n+2}}{4h} + \frac{y_{n+3}}{3h}\ \ \frac{y_{n+4}}{8h}$$

$$y^0_{n+2} = \frac{83f_nh^3}{907200} + \frac{977f_{n+1}h^3}{75600} + \frac{421f_{n+2}h^3}{4032} + \frac{1199f_{n+3}h^3}{22680}$$

$$\frac{277f_{n+4}h^3}{6048000} + \frac{3f_{n+5}h^3}{2800} \quad \frac{109f_{n+6}h^3}{90720000} \quad \frac{y_n}{12h} \quad \frac{y_{n+2}}{h} \quad \frac{4y_{n+3}}{+3h} \quad \frac{y_{n+4}}{4h}$$

$$y_{n+3}^0 = \frac{f_nh^3}{6300} \quad \frac{67f_{n+1}h^3}{12600} \quad \frac{827f_{n+2}h^3}{13440} \quad \frac{617f_{n+3}h^3}{10080} + \frac{17f_{n+4}h^3}{4032}$$

$$\frac{19f_{n+5}h^3}{16800} + \frac{3f_{n+6}h^3}{22400} \quad \frac{y_n}{+24h} \quad \frac{3y_{n+2}}{4h} \quad \frac{y_{n+3}}{+3h} \quad \frac{3y_{n+4}}{+8h}$$

$$y_{n+4}^0 = \frac{11f_nh^3}{64800} + \frac{149f_{n+1}h^3}{10260000} + \frac{103f_{n+2}h^3}{8640} + \frac{311f_{n+3}h^3}{1620} + \frac{103f_{n+4}h^3}{10080}$$

$$\frac{f_{n+5}h^3}{37800000} \quad \frac{19f_{n+6}h^3}{45360000} \quad \frac{y_n}{12h} \quad \frac{y_{n+2}}{+h} \quad \frac{8y_{n+3}}{3h} \quad \frac{7y_{n+4}}{+4h}$$

$$y^0_{n+5} = \frac{47f_nh^3}{907200} + \frac{439f_{n+1}h^3}{6048} + \frac{25201f_{n+2}h^3}{40320} + \frac{121301f_{n+3}h^3}{90720} + \frac{15187f_{n+4}h^3}{30240} +$$

$$\frac{3f_{n+5}h^3}{700} + \frac{149f_{n+6}h^3}{36280} \quad \frac{11y_n}{24h} + \frac{17y_{n+2}}{4h} \quad \frac{23y_{n+3}}{3h} + \frac{31y_{n+4}}{8h}$$

$$y^0_{n+6} = \frac{11f_nh^3}{20160} + \frac{4247f_{n+1}h^3}{25200} + \frac{5993f_{n+2}h^3}{4032} + \frac{557f_{n+3}h^3}{168} + \frac{8233f_{n+4}h^3}{4032} + \frac{2411f_{n+5}h^3}{50400} \quad \frac{887f_{n+6}h^3}{100800} \quad \frac{13y_n}{12h} \quad \frac{9y_{n+2}}{h} \quad \frac{44y_{n+3}}{3h} \quad \frac{27y_{n+4}}{4h}$$

$$y_n^{00} = \frac{19763f_nh^2}{30240} + \frac{71683f_{n+1}h^2}{75640} + \frac{7543f_{n+2}h^2}{8640} + \frac{499f_{n+3}h^2}{1512} + \frac{4127f_{n+4}h^2}{60480} + \frac{229f_{n+5}h^2}{10800} \quad \frac{877f_{n+6}h^2}{302400} \quad \frac{3y_n}{4h^2} \quad \frac{7y_{n+2}}{2h^2} + \frac{4y_{n+3}}{h^2} \quad \frac{5y_{n+4}}{4h^2}$$

$$y_{n+1}^{00} = \frac{643f_{n+5}h^2}{302400} + \frac{421f_nh^2}{21600} + \frac{7f_{n+6}h^2}{2h^2} + \frac{479f_{n+1}h^2}{h^2} + \frac{y_n}{h^2} + \frac{2y_{n+2}}{h^2} + \frac{2y_{n+3}}{2h^2} + \frac{2021f_{n+2}h^2}{y_{n+4}} + \frac{37f_{n+3}h^2}{} + \frac{53f_{n+4}h^2}{}$$

$$y_{n+2}^{00} = \frac{2f_nh^2}{4725} + \frac{f_{n+5}h^2}{945} + \frac{31f_{n+1}h^2}{1575} + \frac{f_{n+6}h^2}{9450} + \frac{17f_{n+2}h^2}{4h^2} + \frac{y_n}{2h^2} + \frac{y_{n+2}}{4h^2} + \frac{y_{n+4}}{f_{n+4}h^2} + \frac{2f_{n+3}h^2}{}$$

$$y_{n+3}^{00} = \frac{233f_{n+4}h^2}{43200} + \frac{f_nh^2}{60480} + \frac{97f_{n+5}h^2}{302400} + \frac{97f_{n+1}h^2}{43200} + \frac{233f_{n+2}h^2}{h^2} + \frac{y_{n+2}}{h^2} + \frac{461f_{n+3}h^2}{h^2} + \frac{2y_{n+3}}{h^2} + \frac{y_{n+4}}{h^2}$$

$$y_{n+4}^{00} = \frac{13f_nh^2}{302400} + \frac{991f_{n+1}h^2}{25200} + \frac{4147f_{n+2}h^2}{12096} + \frac{1739f_{n+3}h^2}{2520} + \frac{97f_{n+4}h^2}{} + \frac{547f_{n+5}h^2}{} + \frac{173f_{n+6}h^2}{} + \frac{y_n}{4h^2} + \frac{5y_{n+2}}{2h^2} + \frac{4y_{n+3}}{h^2} + \frac{7y_{n+4}}{4h^2}$$

$$y''_{n+5} = \frac{47f_nh^2}{75600} + \frac{3251f_{n+1}h^2}{43200} + \frac{499f_{n+2}h^2}{720} + \frac{1039f_{n+3}h^2}{672} + \frac{3863f_{n+4}h^2}{3780}$$

$$\frac{8279f_{n+5}h^2}{\begin{array}{r}1\\0\\0\\8\\0\\+0\end{array}} \quad \frac{47f_{n+6}h^2}{\begin{array}{r}1\\8\\9\\0\\0\end{array}} \quad y_n \quad 4y_{n+2} \quad 6y_{n+3} \quad 3y_{n+4}$$

$$2h^2 \quad +h^2 \quad h^2 \quad +2h^2$$

$$y_{n+6}'' = \frac{53f_nh^2}{21600} + \frac{5189f_{n+1}h^2}{\begin{array}{r}3\\7\\8\\0\\+0\end{array}} \quad \frac{5827f_{n+2}h^2}{\begin{array}{r}6\\0\\4\\+8\end{array}} \quad \frac{9577f_{n+3}h^2}{\begin{array}{r}3\\7\\8\\+0\end{array}} \quad \frac{57599f_{n+4}h^2}{\begin{array}{r}3\\0\\2\\4\\+0\end{array}}$$

$$\frac{5747f_{n+5}h^2}{\begin{array}{r}5\\4\\0\\+0\end{array}} \quad \frac{9949f_{n+6}h^2}{\begin{array}{r}1\\5\\1\\2\\0\\+0\end{array}} \quad 3y_n \quad 11y_{n+2} \quad 8y_{n+3} \quad 13y_{n+4}$$

$$4h^2 \quad + 2h^2 \quad h^2 \quad + 4h^2$$

$$y_n^{000} = \frac{36791f_nh}{120960} + \frac{42967f_{n+1}h}{30240} + \frac{13729f_{n+2}h}{120960} + \frac{29f_{n+3}h}{48} + \frac{31999f_{n+4}h}{120960}$$

$$\frac{2519f_{n+5}h}{30240} + \frac{277f_{n+6}hy_n}{241924h^3} + \frac{3y_{n+2}}{2h^3} + \frac{2y_{n+3}}{h^3} + \frac{3y_{n+4}}{4h^3}$$

$$y_{n+1}^{000} = \frac{461f_nh}{40320} + \frac{10411f_{n+1}h}{30240} + \frac{106651f_{n+2}h}{120960} + \frac{241f_{n+3}h}{15120} + \frac{8423f_{n+4}h}{120960}$$

$$+ \frac{91f_{n+5}h}{4320} + \frac{341f_{n+6}h}{120960} + \frac{y_n}{4h^3} + \frac{3y_{n+2}}{2h^3} + \frac{2y_{n+3}}{h^3} + \frac{3y_{n+4}}{4h^3}$$

$$y_{n+2}^{000} = \frac{49f_nh}{17280} + \frac{2153f_{n+1}h}{30240} + \frac{12673f_{n+2}h}{120960} + \frac{3823f_{n+3}h}{15120} + \frac{1235f_{n+4}h}{24192}$$

$$\frac{407f_{n+5}h}{3024} + \frac{67f_{n+6}hy_n}{40320\,4h^3} + \frac{3y_{n+2}}{2h^3} + \frac{2y_{n+3}}{h^3} + \frac{3y_{n+4}}{4h^3}$$

$$y_{n+3}^{000} = \frac{199f_nh}{120960} + \frac{773f_{n+1}h}{30240} + \frac{1399f_{n+2}h}{3456} + \frac{617f_{n+3}h}{1680} + \frac{7367f_{n+4}h}{120960}$$

$$+ \frac{397f_{n+5}h}{30240} + \frac{181f_{n+6}h}{120960} + \frac{y_n}{4h^3} + \frac{3y_{n+2}}{2h^3} + \frac{2y_{n+3}}{h^3} + \frac{3y_{n+4}}{4h^3}$$

$$y_{n+4}^{000} = \frac{61f_nh}{40320} + \frac{1577f_{n+1}h}{30240} + \frac{35423f_{n+2}h}{120960} + \frac{14929f_{n+3}h}{15120} + \frac{7753f_{n+4}h}{17280}$$

$$\frac{983f_{n+5}h}{30240} + \frac{361f_{n+6}hy_n}{120960\,4h^3} + \frac{3y_{n+2}}{2h^3} + \frac{2y_{n+3}}{h^3} + \frac{3y_{n+4}}{4h^3}$$

$$y_{n+5}^{000} = \frac{359f_nh}{120960} + \frac{533f_{n+1}h}{30240} + \frac{50021f_{n+2}h}{120960} + \frac{2173f_{n+3}h}{3024} + \frac{148249f_{n+4}h}{120960}$$

$$+ \frac{11581f_{n+5}h}{30240} + \frac{13f_{n+6}h}{1152} + \frac{y_n}{4h^3} + \frac{3y_{n+2}}{2h^3} + \frac{2y_{n+3}}{h^3} + \frac{3y_{n+4}}{4h^3}$$

$$y_{n+6}^{000} = \frac{1367f_nh}{120960} + \frac{527f_{n+1}h}{4320} + \frac{9599f_{n+2}h}{120960} + \frac{2249f_{n+3}h}{1680} + \frac{55327f_{n+4}h}{120960} + \frac{44137f_{n+5}h}{30240} + \frac{36809f_{n+6}h}{120960} \quad \frac{y_n}{4h^3} + \frac{3y_{n+2}}{2h^3} \quad \frac{2y_{n+3}}{h^3} + \frac{3y_{n+4}}{4h^3} +$$

3.3.3 Case k = 8; v = 4

$$u(x) = {}_0(x)y_n + {}_3(x)y_{n+3} + {}_4(x)y_{n+4} + {}_5(x)y_{n+5} + h^4 \; h \; {}_0(x)f_n + {}_1(x)f_{n+1}$$

$$ {}_2(x)f_{n+2} + {}_3(x)f_{n+3} + {}_4(x)f_{n+4} + {}_5(x)f_{n+5} + {}_6(x)f_{n+6} + {}_7f_{n+7}$$

$$ + {}_8f_{n+8} \qquad (3.15)$$

From (3.1)

$$y(x) \text{ ' } U(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + a_8x^8$$

$$+ a_9x^9 + a_{10}x^{10} + a_{11}x^{11} + a_{12}x^{12} \qquad (3.16)$$

Using (3.1) & (3.3), a system of 13 equations are obtained.

$$A\underline{x} = \underline{b}$$

where

$$
A = \begin{pmatrix}
1 & x & x^2 & x^3 & x^4 & x^5 & x^6 & x^7 & x^8 & x^9 & x^{10} & x^{11} & x^{12} \\
1 & x_{n+3} & (x_{n+3})^2 & (x_{n+3})^3 & (x_{n+3})^4 & (x_{n+3})^5 & (x_{n+3})^6 & (x_{n+3})^7 & (x_{n+3})^8 & (x_{n+3})^9 & (x_{n+3})^{10} & (x_{n+3})^{11} & (x_{n+3})^{12} \\
1 & x_{n+4} & (x_{n+4})^2 & (x_{n+4})^3 & (x_{n+4})^4 & (x_{n+4})^5 & (x_{n+4})^6 & (x_{n+4})^7 & (x_{n+4})^8 & (x_{n+4})^9 & (x_{n+4})^{10} & (x_{n+4})^{11} & (x_{n+4})^{12} \\
1 & x_{n+5} & (x_{n+5})^2 & (x_{n+5})^3 & (x_{n+5})^4 & (x_{n+5})^5 & (x_{n+5})^6 & (x_{n+5})^7 & (x_{n+5})^8 & (x_{n+5})^9 & (x_{n+5})^{10} & (x_{n+5})^{11} & (x_{n+5})^{12} \\
0 & 0 & 0 & 0 & 24 & 120x_n & 360x_n^2 & 840x_n^3 & 1680x_n^4 & 3024x_n^5 & 5040x_n^6 & 7920x_n^7 & 11880x_n^8 \\
0 & 0 & 0 & 0 & 24 & 120(x_{n+1}) & 360(x_{n+1})^2 & 840(x_{n+1})^3 & 1680(x_{n+1})^4 & 3024(x_{n+1})^5 & 5040(x_{n+1})^6 & 7920x^7_{n+1} & 11880x^8_{n+1} \\
0 & 0 & 0 & 0 & 24 & 120(x_{n+2}) & 360(x_{n+2})^2 & 840(x_{n+2})^3 & 1680(x_{n+2})^4 & 3024(x_{n+2})^5 & 5040(x_{n+2})^6 & 7920(x_{n+2})^7 & 11880(x_{n+2})^8 \\
0 & 0 & 0 & 0 & 24 & 120(x_{n+3}) & 360(x_{n+3})^2 & 840(x_{n+3})^3 & 1680(x_{n+3})^4 & 3024(x_{n+3})^5 & 5040(x_{n+3})^6 & 7920(x_{n+3})^7 & 11880(x_{n+3})^8 \\
0 & 0 & 0 & 0 & 24 & 120(x_{n+4}) & 360(x_{n+4})^2 & 840(x_{n+4})^3 & 1680(x_{n+4})^4 & 3024(x_{n+4})^5 & 5040(x_{n+4})^6 & 7920(x_{n+4})^7 & 11880(x_{n+4})^8 \\
0 & 0 & 0 & 0 & 24 & 120(x_{n+5}) & 360(x_{n+5})^2 & 840(x_{n+5})^3 & 1680(x_{n+5})^4 & 3024(x_{n+5})^5 & 5040(x_{n+5})^6 & 7920(x_{n+5})^7 & 11880(x_{n+5})^8 \\
0 & 0 & 0 & 0 & 24 & 120(x_{n+6}) & 360(x_{n+6})^2 & 840(x_{n+6})^3 & 1680(x_{n+6})^4 & 3024(x_{n+6})^5 & 5040(x_{n+6})^6 & 7920(x_{n+6})^7 & 11880(x_{n+6})^8 \\
0 & 0 & 0 & 0 & 24 & 120(x_{n+7}) & 360(x_{n+7})^2 & 840(x_{n+7})^3 & 1680(x_{n+7})^4 & 3024(x_{n+7})^5 & 5040(x_{n+7})^6 & 7920(x_{n+7})^7 & 11880(x_{n+7})^8 \\
0 & 0 & 0 & 0 & 24 & 120(x_{n+8}) & 360(x_{n+8})^2 & 840(x_{n+8})^3 & 1680(x_{n+8})^4 & 3024(x_{n+8})^5 & 5040(x_{n+8})^6 & 7920(x_{n+8})^7 & 11880(x_{n+8})^8
\end{pmatrix}
$$

$$x = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \end{bmatrix} \text{ and } b = \begin{bmatrix} y_n \\ y_{n+1} \\ y_{n+2} \\ y_{n+3} \\ y_{n+4} \\ y_{n+5} \\ f_n \\ f_{n+1} \\ f_{n+2} \\ f_{n+3} \\ f_{n+4} \\ f_{n+5} \\ f_{n+6} \\ f_{n+7} \\ f_{n+8} \end{bmatrix}$$

The solution of this system yields the value of a_k; $k = 0(1)12$; which are substituted back into(3.10) and which after some simpli cation leads to the continuous scheme of the form.

$$
\begin{aligned}
y(x) = {} & \frac{1}{958003200h^8} f_n'' (3h - x)(4h - x)(5h - x)x(129917h^8 - 385464xh^7 \\
& + 435725x^2h^6 - 244704x^3h^5 + 76373x^4h^4 - 13908x^5h^3 + 1472x^6h^2 \\
& - 84x^7h + 2x^8) + \frac{1}{958003200h^8} f_{n+8}'' (3h - x)(4h - x)(5h \\
& - x)x(1603h^8 - 15488xh^7 + 8563x^2h^6 + 9832x^3h^5 - 10901x^4h^4 \\
& + 4012x^5h^3 - 704x^6h^2 + 60x^7h - 2x^8) + \frac{1}{239500800h^8} f_{n+6}'' (3h \\
& - x)(4h - x)(5h - x)x(9553h^8 - 146742xh^7 + 81169x^2h^6 + 93090x^3h^5 \\
& - 100991x^4h^4 + 35634x^5h^3 - 5876x^6h^2 + 462x^7h - 14x^8) \\
& + \frac{1}{95800320h^8} f_{n+4}'' (3h - x)(4h - x)(5h - x)x(323963h^8 + 291196xh^7 \\
& - 112693x^2h^6 - 141116x^3h^5 + 149699x^4h^4 - 47936x^5h^3 + 7088x^6h^2 \\
& - 504x^7h + 14x^8) + \frac{1}{119750400h^8} f_{n+7}'' (3h - x)(4h - x)(5h \\
& - x)x(1597h^8 - 17806xh^7 + 9853x^2h^6 + 11306x^3h^5 - 12419x^4h^4 \\
& + 4490x^5h^3 - 767x^6h^2 + 63x^7h - 2x^8) + \frac{1}{119750400h^8} f_{n+1}'' (3h \\
& - x)(4h - x)(5h - x)x(715549h^8 - 473386xh^7 + 9277x^2h^6 \\
& - 113870x^3h^5 - 53603x^4h^4 + 11582x^5h^3 - 1343x^6h^2 + 81x^7h - 2x^8)
\end{aligned}
\tag{3.17}
$$

$$\frac{1}{119750400h^8} f''_{n+5}(3h\ \ x)(4h\ \ x)(5h\ \ x)x(3571h^8$$
$$178742xh^7 + 98515x^2h^6 + 112978x^3h^5\ \ 120461x^4h^4$$
$$+ 41026x^5h^3\ \ 6449x^6h^2 + 483x^7h\ \ 14x^8)^{\#}$$

$$\frac{1}{119750400h^8} f''_{n+3}(3h\ \ x)(4h\ \ x)(5h$$
$$x)x(1935571h^8 + 20334xh^7 + 223891x^2h^6$$
$$+ 203574x^3h^5\ \ 195437x^4h^4 + 56694x^5h^3\ \ 7793x^6h^2 \qquad (3.18)$$
$$+ 525x^7h\ \ 14x^8)^{\#}\ \ \frac{1}{239500800h^8} f''_{n+2}(3h\ \ x)(4h$$
$$x)(5h\ \ x)x(3834191h^8 + 516962xh^7\ \ 426961x^2h^6$$
$$373942x^3h^5 + 266783x^4h^4\ 67630x^5h^3 + 8564x^6h^2$$
$$546x^7h + 14x^8)^{\#} + \frac{(3h\ x)(4h\ x)(5h\ x)y_n}{60h^3}$$
$$\frac{x(20h^2\ \ 9xh + x^2)y_{n+3}}{6h^3} \frac{x(15h^2\ \ 8xh + x^2)y_{n+4}}{4h^3}$$
$$\frac{x(12h^2\ \ 7xh + x^2)y_{n+5}}{10h^3}$$

The BVMs are obtained by evaluating (3.18) at x_{n+1}; x_{n+2} and x_{n+5}; x_{n+6}; x_{n+7}

$$y_{n+8} = \frac{23f_nh^4}{18144} + \frac{731f_{n+1}h^4}{22680} + \frac{12121f_{n+2}h^4}{9072} + \frac{102889f_{n+3}h^4}{4536} + \frac{143921f_{n+4}h^4}{9072} + \frac{102889f_{n+5}h^4}{4536} + \frac{12121f_{n+6}h^4}{9072} + \frac{731f_{n+7}h^4}{22680} + \frac{23f_{n+8}h^4}{18144}$$

$$y_{n+7} = \frac{13f_nh^4}{5184} + \frac{169f_{n+1}h^4}{5184} + \frac{2753f_{n+2}h^4}{5184} + \frac{4729f_{n+3}h^4}{5184} + \frac{158381f_{n+4}h^4}{5184} + \frac{88531f_{n+5}h^4}{5184} + \frac{4093f_{n+6}h^4}{25920} + \frac{f_{n+7}h^4}{2592} + \frac{7f_{n+8}h^4}{51842} + \frac{2y_n}{5} + 7y_{n+3} - 14y_{n+4} + \frac{42y_{n+5}}{51840} + \frac{64800}{5}$$

$$y_{n+6} = \frac{19f_nh^4}{2419200} + \frac{979f_{n+1}h^4}{2419200} + \frac{16097f_{n+2}h^4}{2419200} + \frac{27589f_{n+3}h^4}{2419200} + \frac{174301f_{n+4}h^4}{2419200} + \frac{53777f_{n+5}h^4}{302400} + \frac{107f_{n+6}h^4}{24192} + \frac{43f_{n+7}h^4}{60480} + \frac{5f_{n+8}h^4}{96768} + \frac{y_n}{10} + 2y_{n+3} - \frac{9y_{n+4}}{2} + \frac{16y_{n+5}}{5} + \frac{60480}{241920}$$

$$y_{n+1} = \frac{61f_nh^4}{3628800} + \frac{1171f_{n+1}h^4}{3628800} + \frac{68317f_{n+2}h^4}{3628800} + \frac{40679f_{n+3}h^4}{3628800} + \frac{42701f_{n+4}h^4}{3628800} + \frac{4463f_{n+5}h^4}{453600} + \frac{613f_{n+6}h^4}{181440} + \frac{67f_{n+7}h^4}{90720} + \frac{11f_{n+8}h^4}{145152} + \frac{2y_n}{5} + 2y_{n+3} - 2y_{n+4} + \frac{3y_{n+5}}{5}$$

$$y_{n+2} = \frac{467 f_n h^4}{7257600} + \frac{559 f_{n+1} h^4}{3628800} + \frac{50089 f_{n+2} h^4}{3628800} + \frac{10009 f_{n+3} h^4}{3628800} + \frac{52781 f_{n+4} h^4}{3628800} + \frac{2279 f_{n+5} h^4}{907200} + \frac{193 f_{n+6} h^4}{362880} + \frac{13 f_{n+7} h^4}{181440} + \frac{f_{n+8} h^4}{207360} + \frac{y_n}{10} + 2y_{n+3} + \frac{3y_{n+4}}{2} + \frac{2y_{n+5}}{5}$$

The continuous scheme (3.18) is di erentiated and evaluated at x_n; x_{n+1}; x_{n+2}; x_{n+3} x_{n+8}

to give, respectively, the following expressions;

$$y_n^0 = \frac{129917 f_n h^3}{15966720} - \frac{715549 f_{n+1} h^3}{1995840} + \frac{3834191 f_{n+2} h^3}{3991680} - \frac{175961 f_{n+3} h^3}{181440} + \frac{323963 f_{n+4} h^3}{1596672} - \frac{3571 f_{n+5} h^3}{1995840} + \frac{9553 f_{n+6} h^3}{3991680} - \frac{1597 f_{n+7} h^3}{229} + \frac{f_{n+8} h^3}{47} - \frac{y_n}{10} + \frac{y_{n+3}}{15} - \frac{y_{n+4}}{6} + \frac{y_{n+5}}{5}$$

$$y_{n+1}^0 = -\frac{46297 f_n h^5}{479001600} + \frac{3648797 f_{n+1} h^5}{59875200} + \frac{2388599 f_{n+2} h^5}{23950080} - \frac{842207 f_{n+3} h^5}{11975040} + \frac{18569 f_{n+4} h^5}{47900160} - \frac{590089 f_{n+5} h^3}{5987520} + \frac{43361 f_{n+6} h^3}{10886400} - \frac{1619 f_{n+7} h^3}{171072} + \frac{1379 f_{n+8} h^3}{1368576} - \frac{13 y_n}{30 h} + \frac{5 y_{n+3}}{6 h} - \frac{y_{n+4}}{2 h} + \frac{y_{n+5}}{10 h}$$

$$y_{n+2}^{0} = \frac{1411 f_n h^3}{14968800} + \frac{86147 f_{n+1} h^3}{2993760} + \frac{7216061 f_{n+2} h^3}{29937600} + \frac{991511 f_{n+3} h^3}{2993760} + \frac{391621 f_{n+4} h^3}{5987520}$$

$$+ \frac{6529 f_{n+5} h^3}{14968800} \quad \frac{2213 f_{n+6} h^3}{5987520} + \frac{1597 f_{n+7} h^3}{14968800} \quad \frac{71 f_{n+8} h^3}{5987520} \quad \frac{11 y_n}{60h} \quad \frac{2y_{n+3}}{3h}$$

$$+ \frac{5y_{n+4}}{4h} \quad \frac{2y_{n+5}}{5h}$$

$$y_{n+3}^{0} = \frac{2887 f_n h^3}{159667200} + \frac{2017 f_{n+1} h^3}{362880} + \frac{352447 f_{n+2} h^3}{7983360} + \frac{2829559 f_{n+3} h^3}{19958400} + \frac{143873 f_{n+4} h^3}{2280960}$$

$$\frac{17213 f_{n+5} h_3}{3991680} + \frac{14599 f_{n+6} h_3}{159667200} \quad \frac{1447 f_{n+7} h_3}{30h} + \frac{5513 f_{n+8} h_3}{6h} \quad \frac{y_n}{28512007983360} \quad \frac{7y_{n+3}}{}$$

$$+ \frac{5y_{n+4}}{2h} \quad \frac{5y_{n+5}}{10h}$$

$$y_{n+4}^{0} = \frac{7571 f_n h^3}{239500800} \quad \frac{90821 f_{n+1} h^3}{29937600} \quad \frac{1231687 f_{n+2} h^3}{59875200} \quad \frac{2414651 f_{n+3} h^3}{29937600} \quad \frac{143921 f_{n+4} h^3}{2177280}$$

$$+ \frac{151093 f_{n+5} h^3}{29937600} \quad \frac{101623 f_{n+6} h^3}{59875200} + \frac{10411 f_{n+7} h^3}{29937600} \quad \frac{8077 f_{n+8} h^3}{239500800} + \frac{y_n}{60h} \quad \frac{2y_{n+3}}{3h}$$

$$+ \frac{y_{n+4}}{4h} + \frac{2y_{n+5}}{5h}$$

$$y_{n+5}^{0} = \frac{2903 f_n h^3}{95800320} + \frac{68669 f_{n+1} h^3}{11975040} + \frac{1022851 f_{n+2} h^3}{23950080} + \frac{1883141 f_{n+3} h^3}{11975040} + \frac{1928743 f_{n+4} h^3}{9580032}$$

$$+ \frac{113111 f_{n+5} h^3}{11975040} + \frac{9307 f_{n+6} h^3}{23950080} \quad \frac{29 f_{n+7} h^3}{155520} + \frac{2137 f_{n+8} h^3}{95800320} \quad \frac{y_n}{30h} + \frac{5y_{n+3}}{6h}$$

$$\frac{5y_{n+4}}{2h} + \frac{17y_{n+5}}{10h}$$

$$y_{n+6}^{0} = \frac{467 f_n h^3}{13305600} + \frac{24481 f_{n+1} h^3}{831600} + \frac{9067 f_{n+2} h^3}{36960} + \frac{8641 f_{n+3} h^3}{10395} + \frac{1847911 f_{n+4} h^3}{1330560}$$

$$+ \frac{46247 f_{n+5} h^3}{92400} + \frac{1627 f_{n+6} h^3}{415800} + \frac{8 f_{n+7} h^3}{10395} \quad \frac{f_{n+8} h^3}{14080} \quad \frac{11 y_n}{60h} + \frac{10y_{n+3}}{3h}$$

$$\frac{27y_{n+4}}{4h} + \frac{18y_{n+5}}{5h}$$

$$y_{n+7}^{0} = \frac{21953f_nh^3}{479001600} + \frac{847597f_{n+1}h^3}{11975040} + \frac{6259559f_{n+2}h^3}{10886400} + \frac{23659021f_{n+3}h^3}{11975040}$$

$$+ \frac{164664193f_{n+4}h^3}{47900160} + \frac{121916051f_{n+5}h^3}{59875200} + \frac{2295565f_{n+6}h^3}{4790016} + \frac{532523f_{n+7}h^3}{59875200}$$

$$\frac{571f_{n+8}h^3}{13685760} + \frac{13y_n}{30h} + \frac{41y_{n+3}}{6h} + \frac{25y_{n+4}}{2h} + \frac{61y_{n+5}}{10h}$$

$$y_{n+8}^{0} = \frac{263f_nh^3}{239500800} + \frac{152129f_{n+1}h^3}{1197504} + \frac{2500891f_{n+2}h^3}{2395008} + \frac{106440791f_{n+3}h^3}{29937600}$$

$$+ \frac{153673759f_{n+4}h^3}{23950080} + \frac{12310981f_{n+5}h^3}{2721600} + \frac{24035687f_{n+6}h^3}{11975040} + \frac{414643f_{n+7}h^3}{855360}$$

$$+ \frac{281791f_{n+8}h^3}{34214400} \quad \frac{47y_n}{60h} + \frac{34y_{n+3}}{3h} \quad \frac{79y_{n+4}}{4h} + \frac{46y_{n+5}}{5h}$$

$$y_n^{00} = \frac{4176277f_nh^2}{68428800} + \frac{62033963f_{n+1}h^2}{59875200} + \frac{21312751f_{n+2}h^2}{17107200} + \frac{89751797f_{n+3}h^2}{59875200}$$

$$\frac{2245499f_{n+4}h^2}{47900160} + \frac{1556051f_{n+5}h^2}{8553600} \quad \frac{9253511f_{n+6}h^2}{11975040} + \frac{1143419f_{n+7}h^2}{59875200}$$

$$\frac{1004621f_{n+8}h^2}{4790016005h^2} + \frac{2y_n}{h^2} \quad \frac{3y_{n+3}}{h^2} + \frac{4y_{n+4}}{5h^2} \quad \frac{7y_{n+5}}{5h^2}$$

$$y_{n+1}^{00} = \frac{1006667f_nh^2}{479001600} + \frac{1881827f_{n+1}h^2}{59875200} + \frac{10668601f_{n+2}h^2}{17107200} + \frac{36165539f_{n+3}h^2}{59875200}$$

$$+ \frac{8572363f_{n+4}h^2}{47900160} \quad \frac{1655431f_{n+5}h^2}{59875200} + \frac{1267351f_{n+6}h^2}{119750400} \quad \frac{147871f_{n+7}h^2}{59875200}$$

$$+ \frac{125893f_{n+8}h^2}{479001600} + \frac{3y_n}{10h^2} \quad \frac{2y_{n+3}}{h^2} + \frac{3y_{n+4}}{2h^2} \quad \frac{4y_{n+5}}{5h^2}$$

$$y_{n+2}^{00} = \frac{125299f_nh^2}{479001600} \quad \frac{2202121f_{n+1}h^2}{59875200} \quad \frac{21378899f_{n+2}h^2}{119750400} + \frac{6113297f_{n+3}h^2}{59875200}$$

$$+ \frac{243689f_{n+4}h^2}{9580032} + \frac{55871f_{n+5}h^2}{8553600} \quad \frac{52109f_{n+6}h^2}{17107200} + \frac{46367f_{n+7}h^2}{59875200}$$

$$\frac{41021f_{n+8}h^2}{479001600} + \frac{y_n}{5h^2} \quad \frac{y_{n+3}}{h^2} + \frac{y_{n+4}}{h^2} \quad \frac{y_{n+5}}{5h^2}$$

$$y_{n+3}^{00} = \frac{40163f_nh^2}{479001600} \quad \frac{904891f_{n+1}h^2}{59875200} \quad \frac{16978481f_{n+2}h^2}{119750400} \quad \frac{3012997f_{n+3}h^2}{8553600}$$

$$\frac{3655853f_{n+4}h^2}{479001600} + \frac{157391f_{n+5}h^2}{5987520} \quad \frac{54233f_{n+6}h^2}{11975040} + \frac{2543f_{n+7}h^2}{59875200}$$

$$\frac{563f_{n+8}h^2}{479001600} + \frac{y_n}{10h^2} \quad \frac{y_{n+4}}{2h^2} + \frac{2y_{n+5}}{5h^2}$$

$$y_{n+4}'' = \frac{1747f_nh^2}{479001600} - \frac{3133f_{n+1}h^2}{59875200} + \frac{50641f_{n+2}h^2}{119750400} - \frac{242899f_{n+3}h^2}{59875200}$$

$$- \frac{3638891f_{n+4}h^2}{479001600} - \frac{242899f_{n+5}h^2}{59875200} + \frac{50641f_{n+6}h^2}{119750400} - \frac{3133f_{n+7}h^2}{59875200}$$

$$+ \frac{1747f_{n+8}h^2}{479001600} + \frac{y_{n+3}}{h^2} - \frac{2y_{n+4}}{h^2} + \frac{y_{n+5}}{h^2}$$

$$y_{n+5}'' = \frac{787f_nh^2}{68428800} + \frac{138209f_{n+1}h^2}{8553600} + \frac{15945487f_{n+2}h^2}{119750400} + \frac{27320087f_{n+3}h^2}{59875200}$$

$$+ \frac{4905613f_{n+4}h^2}{6842880} + \frac{6071717f_{n+5}h^2}{59875200} - \frac{139823f_{n+6}h^2}{17107200} + \frac{60029f_{n+7}h^2}{59875200}$$

$$- \frac{34091f_{n+8}h^2}{479001600} + \frac{y_n}{10h^2} - \frac{2y_{n+3}}{h^2} + \frac{7y_{n+4}}{2h^2} - \frac{8y_{n+5}}{5h^2}$$

$$y_{n+6}'' = \frac{28877f_nh^2}{479001600} + \frac{1976207f_{n+1}h^2}{59875200} + \frac{31634677f_{n+2}h^2}{119750400} + \frac{54716489f_{n+3}h^2}{59875200}$$

$$+ \frac{11029819f_{n+4}h^2}{6842880} + \frac{60438689f_{n+5}h^2}{59875200} + \frac{10620541f_{n+6}h^2}{119750400} - \frac{272281f_{n+7}h^2}{59875200}$$

$$+ \frac{137443f_{n+8}h^2}{479001600} - \frac{y_n}{5h^2} + \frac{3y_{n+3}}{h^2} - \frac{3y_{n+4}}{h^2}$$

$$- \frac{11y_{n+5}}{5h^2}$$

$$y_{n+7}'' = \frac{20587f_nh^2 + 2746889f_{n+1}h^2 + 7038073f_{n+2}h^2 + 79832657f_{n+3}h^2}{6842880059875200171072005987520 0}$$

$$+ \frac{24511559f_{n+4}h^2}{9580032} + \frac{16807661f_{n+5}h^2}{8553600} + \frac{122679367f_{n+6}h^2}{119750400} + \frac{4776587f_{n+7}h^2}{59875200}$$

$$- \frac{988451f_{n+8}h^2}{479001600} - \frac{3y_n}{10h^2} + \frac{4y_{n+3}}{h^2} - \frac{13y_{n+4}}{2h^2} + \frac{14y_{n+5}}{5h^2}$$

$$y_{n+8}'' = \frac{980333f_nh^2}{479001600} + \frac{5003099f_{n+1}h^2}{59875200} + \frac{7820767f_{n+2}h^2}{17107200} + \frac{119543141f_{n+3}h^2}{59875200}$$

$$+ \frac{149735077f_{n+4}h^2}{47900160} + \frac{198402581f_{n+5}h^2}{59875200} + \frac{213188137f_{n+6}h^2}{119750400} + \frac{65893643f_{n+7}h^2}{59875200}$$

$$+ \frac{29258227f_{n+8}h^2}{479001600} - \frac{2y_n}{5h^2} + \frac{3y_{n+3}}{h^2} - \frac{8y_{n+4}}{h^2} + \frac{17y_{n+5}}{5h^2}$$

$$y_n^{000} = \frac{12397f_nh}{43200} \quad \frac{57077f_{n+1}h}{36288} + \frac{177481f_{n+2}h}{1814400} \quad \frac{540979f_{n+3}h}{302400}$$

$$+ \frac{53767f_{n+4}h}{51840} \quad \frac{652483f_{n+5}h}{907200} + \frac{7201f_{n+6}h}{24192} \quad \frac{65783f_{n+7}h}{907200}$$

$$+ \frac{2047f_{n+8}h}{259200} \quad \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} \quad \frac{\partial y_{n+4}}{2h^3} + \frac{\partial y_{n+5}}{5h^3}$$

$$y_n^{000}{}_{+1} = \frac{28669f_nh}{3628800} \quad \frac{620303f_{n+1}h}{1814400} \quad \frac{44267f_{n+2}h}{37800} \quad \frac{89639f_{n+3}h}{362880} \quad \frac{126943f_{n+4}h}{362880}$$

$$+ \frac{89401f_{n+5}h}{604800} \quad \frac{3769f_{n+6}h}{64800} + \frac{3553f_{n+7}h}{259200} \quad \frac{353f_{n+8}h}{241920}$$

$$\frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} \quad \frac{\partial y_{n+4}}{2h^3} + \frac{\partial y_{n+5}}{5h^3}$$

$$y_n^{000}{}_{+2} = \frac{1321f_nh}{907200} + \frac{1607f_{n+1}h}{43200} \quad \frac{502423f_{n+2}h}{1814400} \quad \frac{662233f_{n+3}h}{907200}$$

$$+ \frac{319f_{n+4}h}{24192} \quad \frac{54659f_{n+5}h}{907200} + \frac{41611f_{n+6}h}{1814400} \quad \frac{1597f_{n+7}h}{302400}$$

$$+ \frac{143f_{n+8}h}{259200} \quad \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} \quad \frac{\partial y_{n+4}}{2h^3} + \frac{\partial y_{n+5}}{5h^3}$$

$$y_n^{000}{}_{+3} = \frac{671f_nh}{1209600} + \frac{17681f_{n+1}h}{1814400} + \frac{3959f_{n+2}h}{22680} \quad \frac{407f_{n+3}h}{86400}$$

$$\frac{78527f_{n+4}h}{3628800} + \frac{89611f_{n+5}h}{1814400} \quad \frac{2453f_{n+6}h}{15120} + \frac{1243f_{n+7}h}{3628800} \quad \frac{1231f_{n+8}h}{3628800}$$

$$\frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} \quad \frac{\partial y_{n+4}}{2h^3} + \frac{\partial y_{n+5}}{5h^3}$$

$$y_n^{000}{}_{+4} = \frac{61f_nh}{181440} + \frac{17939f_{n+1}h}{907200} + \frac{69571f_{n+2}h}{604800} + \frac{473191f_{n+3}h}{907200}$$

$$+ \frac{143921f_{n+4}h}{362880} \quad \frac{587f_{n+5}h}{8640} + \frac{33707f_{n+6}h}{1814400} \quad \frac{3319f_{n+7}h}{907200}$$

$$+ \frac{211f_{n+8}h}{604800} \quad \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} \quad \frac{\partial y_{n+4}}{2h^3} + \frac{\partial y_{n+5}}{5h^3}$$

$$y_{n+5}^{000} = \frac{1277f_nh}{3628800} + \frac{307f_{n+1}h}{24192} + \frac{16991f_{n+2}h}{113400} + \frac{733501f_{n+3}h}{1814400}$$

$$+ \frac{122123f_{n+4}h}{120960} + \frac{831659f_{n+5}h}{1814400} \quad \frac{743f_{n+6}h}{18144} + \frac{3853f_{n+7}h}{604800}$$

$$\frac{281f_{n+8}h}{518400} \quad \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} \quad \frac{\delta y_{n+4}}{2h^3} + \frac{\delta y_{n+5}}{5h^3}$$

$$y_{n+6}^{000} = \frac{163f_nh}{302400} + \frac{2773f_{n+1}h}{129600} + \frac{28687f_{n+2}h}{259200} + \frac{31081f_{n+3}h}{60480}$$

$$+ \frac{283057f_{n+4}h}{362880} + \frac{1073789f_{n+5}h}{907200} + \frac{248281f_{n+6}h}{604800} \quad \frac{19127f_{n+7}h}{907200}$$

$$+ \frac{533f_{n+8}h}{362880} \quad \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} \quad \frac{\delta y_{n+4}}{2h^3} + \frac{\delta y_{n+5}}{5h^3}$$

$$y_{n+7}^{000} = \frac{763f_nh}{518400} + \frac{4369f_{n+1}h}{1814400} + \frac{7249f_{n+2}h}{37800} + \frac{554909f_{n+3}h}{1814400} + \frac{11851f_{n+4}h}{10368}$$

$$+ \frac{423769f_{n+5}h}{604800} + \frac{591809f_{n+6}h}{453600} + \frac{649543f_{n+7}h}{1814400} \quad \frac{1363f_{n+8}h}{172800}$$

$$\frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} \quad \frac{\delta y_{n+4}}{2h^3} + \frac{\delta y_{n+5}}{5h^3}$$

$$y_{n+8}^{000} = \frac{7153f_nh}{907200} + \frac{26801f_{n+1}h}{302400} \quad \frac{59531f_{n+2}h}{362880} + \frac{1064039f_{n+3}h}{907200}$$

$$\frac{29509f_{n+4}h}{120960} + \frac{2034493f_{n+5}h}{907200} + \frac{9277f_{n+6}h}{259200}$$

$$+ \frac{13729f_{n+7}h}{8640} + \frac{520697f_{n+8}h}{1814400} \quad \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} \quad \frac{\delta y_{n+4}}{2h^3}$$

$$+ \frac{\delta y_{n+5}}{5h^3}$$

3.4 Speci cation of Trignometrically Fitted Block Method

The continuous scheme (1.1) and speci c cases k = 4; k = 6 & k = 8 is considered again.

3.4.1 Case k = 4; v = 2

$$u(x) = {}_0(x;!)y_n + {}_1(x;!)y_{n+1} + {}_2(x;!)y_{n+2} + {}_3(x;!)y_{n+3} + h^4 \frac{h}{i}\,{}_0(x;!)f_n$$

$$+ {}_1(x;!)f_{n+1}\ {}_2(x;!)f_{n+2} + {}_3(x;!)f_{n+3} + {}_4(x;!)f_{n+4} \qquad (3.19)$$

From (3.2)

$$y(x)\ '\ U(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7\cos !x$$

$$+ a_8\sin !x \qquad (3.20)$$

Using (3.2) & (3.4), a system of 9 equations are obtained.

$$\begin{bmatrix} 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6 & \cos[wx_{n+1}] & \sin[wx_{n+1}] \\ 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & \cos[wx_n] & \sin[wx_n] \\ 1 & x_{n+2} & x_{n+2}^2 & x_{n+2}^3 & x_{n+2}^4 & x_{n+2}^5 & x_{n+2}^6 & \cos[wx_{n+2}] & \sin[wx_{n+2}] \\ 1 & x_{n+3} & x_{n+3}^2 & x_{n+3}^3 & x_{n+3}^4 & x_{n+3}^5 & x_{n+3}^6 & & \\ 0 & 0 & 0 & 0 & 24 & 120x_n & 360x_n^2 & w^4\cos[wx_n] & w^4\sin[wx_n] \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+1} & 360x_{n+1}^2 & w^4\cos[wx_{n+1}] & w^4\sin[wx_{n+1}] \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+2} & 360x_{n+2}^2 & w^4\cos[wx_{n+2}] & w^4\sin[wx_{n+2}] \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+3} & 360x_{n+3}^2 & w^4\cos[wx_{n+3}] & w^4\sin[wx_{n+3}] \\ & & & & & 120x_{n+4} & 360x_{n+4}^2 & w^4 & w^4 \end{bmatrix} \begin{bmatrix} a_1 \\ a_0 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{bmatrix} = \begin{bmatrix} y_{n+1} \\ y_n \\ y_{n+2} \\ y_{n+3} \\ f_n \\ f_{n+1} \\ f_{n+2} \\ f_{n+3} \\ f_{n+4} \end{bmatrix}$$

Solving these equations, substituting for a_0; a_1; ; a_8 in (3.19) and simplifying gives

$$_0(x;!) = (h\ \ x)(2h\ \ x)(3h\ \ x)$$

$$_1(x;!) = \frac{x(6h^2\ \ 5hx + x^2)}{6h^3}$$

$$_2(x;!) = \frac{x(3h^2\ \ 4hx + x^2)}{2h^3}$$

$$_3(x;!) = \frac{x(2h^2\ \ 3hx + x^2)}{6h^3}$$

$$_0(x;!) = \frac{1}{5760\,h^3 w^4}\csc\frac{hw}{2}\,{}_3$$

$$h^3\ \ w^4 x\ \ h^3\ \ 11h^2x\ \ 6hx^2 + x^3\ \ + 24(1\ \ 2\cos[hw]$$

$$(15\ \ (\ 24\ \ 6\ \ + hw\ \ {}_2\ \ {}_2$$

$$+2\cos[2hw])\cos[wx])\csc\ 2\ + 2\ \ 6h\ \ 5hx + x$$

$$240x + h(120\ \ w^4(h\ \ x)x\ 25h^2\ \ 9hx + x^2\ \ + 240(h\ \ x)\cos[hw])\sin\frac{hw}{2}$$

$$+ 360h^3(1 + 2\cos[hw] + 2\cos[2hw])\sec\frac{hw}{2}\sin[wx]$$

$$_1(x;!) = \frac{1}{5760h^3w^4}\csc\frac{hw}{2}\,{}_3\ \ 60h^3(\ 24\ \ w^4x(\ 6h^3 + 11h^2x\ \ 6hx^2 + x^3$$

$$+ 12(1\ \ 2\cos[hw] + 3\cos[2hw])\cos[wx])\csc\frac{hw}{2}\ + 2(2h\ \ x)$$

$$360h^2 + 3h\ \ 960 + h^4w^4\ \ x +\ \ 1080\ \ 13h^4w^4\ \ x^2 + 16h^3w^4x^3\ \ 7h^2w^4x^4 + hw^4x^5$$

$$\sin\frac{hw}{2}\ \ + 3h^2\ \ 4hx + x^2\ \ 360 + hw\,x^4\ 25h^2\ \ 9hx + x^2\ \ \sin\frac{3hw}{2}$$

$$720h^3(2 + 4\cos[hw] + 3\cos[2hw])\sec\frac{hw}{2}\sin[wx]$$

$$_2(x;!) = \frac{1}{2880h^3w^4}((2h\ \ x)x^4\ \ )(3h\ \ x)\ 47h^2 + 6hx\ \ x^2\ \ \cos[hw]$$

$$+ (\ \ 3h + x)(\ \ h + x)\ \ 540x + h\ \ 1800 + w\ \ (h\ \ x\ \ 4\ \ 2h\ \ 6hx + x^2\ \ \cos[2hw]$$

$$+1080h^3\cos[w(2h\ \ x)]\ \ \csc\frac{hw}{2}$$

$$_3(x; !) = \frac{1}{5760h^3w^4} \csc\frac{hw}{2}^{3} \quad 60h^3 \quad 24\ w^4x\ \ 6h^3 + 11h^2x\ \ 6hx^2 \quad + x^3 \quad \csc\frac{hw}{2}$$

$$720h^3(\ 1 + 2\cos[hw] + \cos[2hw])\cos[wx]\csc\frac{hw}{2} + 2(2h\ \ x)$$

$$(75h^3w^3x + 360x^2 \quad 127h^4w^4x^2 + 64h^3w^3x^3 + hw^4x^3 \quad h^2 \quad 1080 + 13w^4x^4)\sin \quad 2 + \frac{hw}{}$$

$$3h^2 \quad 4hx + x^2 \quad 120 + hw^4x\,h^2 \quad 3hx + x^2 \quad \sin\frac{hw}{3}$$

$$720h^3(2 + 4\cos[hw] + \cos[2hw])\sec\frac{hw}{2}\sin[wx]$$

$$_4(x; !) = \frac{1}{5760h^2w^4}\csc\frac{hw}{2}^{2} \quad 2 \quad 2h^2 \quad 3hx + x^2 \quad 360 + w^4(3h\ \ x)x\,h^2 \quad 3hx + x^2$$

$$h^2 \quad 24\ \ w^4x \quad 6h^3 + 11h^2x \quad 6hx^2 + x^3 \quad + 24(\ \ 1 + 2\cos[hw])\cos[wx]$$

$$+ 15\frac{hw}{}^{2}$$

$$\csc\ \ 2 \quad + 48(2\cot[hw] + \csc[hw])\sin[wx]))$$

The main method is

$$y_{n+4} = y_n + 4y_{n+1} \quad 6y_{n+2} + 4y_{n+3} + \frac{f_{n+2} \quad 8h^2 \quad 2880h + h \quad 1800 + 165h^4w^4 \quad \cos[hw] + 1080h^3 \cos[2hw] + 3h^2 \quad 2160h \quad 2h \quad 180 \quad 40h^4w^4 \quad 2880h^3w^4 \quad \csc\frac{hw}{2} \quad 4}{2880h^3w^4}$$

$$+ \frac{1}{5760h^3w^4} f_{n+3} \csc^2\frac{hw}{2} \quad 60h^3 \quad 24 \quad 24h^4w^4 \csc^3\frac{hw}{2} \quad 720h^3(1 + 2\cos[hw] + \cos[2hw])\cos[4hw]$$

$$\csc\frac{hw}{4h} \quad 5760h^2 + 3388h^6w^4 \quad h^2 \quad 1080 + 3328h^4w^4 \sin^2 \quad 2 + 3h^2 120 + 20h^4w^4 \sin^3 \frac{hw}{2} \quad 720h^3(2 + 4\cos[hw] + \cos[2hw])\sec^2\frac{hw}{2} \sin[4hw] + \frac{1}{5760h^3w^4} f_n$$

$$\csc^3\frac{hw}{2} \quad 15h^3 \quad 24 \quad 24h^4w^4 + 24(1 \quad 2\cos[hw] + 2\cos[2hw])\cos[4hw] \csc^2\frac{hw}{2} + 4h^2 \quad 960 + h\,120 + 60h^4w^4 \quad 720h\cos[hw]\sin\frac{hw}{2} + 360h^3(1 + 2\cos[hw] + 2\cos[2hw])\sec^2\frac{hw}{2} \sin[4hw])$$

$$\csc^2\frac{hw}{2} \quad \frac{1}{5760h^3w^4} f_{n+1} \csc^3\frac{hw}{2} \quad 60h^3 \quad 24 \quad 24h^4w^4 + 12(1 \quad 2\cos[hw] + 3\cos[2hw])\cos[4hw] \quad 4h360h^2 + 256h^6w^4 + 16h^2 \quad 1080 \quad 13h^4w^4 + 12h^2 \quad 960 + h^4w^4 \sin^2\frac{hw}{2} + 3h^2 \quad 360 + 20h^4w^4 \sin^2\frac{3hw}{2} \quad 720h^3(2 + 4\cos[hw] + 3\cos[2hw])\sec^2\frac{hw}{2} \sin[4hw]$$

$$f_{n+4} \csc^2\frac{hw}{2} \quad 12h^2 \quad 360 \quad 20 \quad h^4w^4 + 15h^2 \quad 24 \quad 24h^4w^4 \quad 5760h^2w^4 + 24(1 + 2\cos[hw])\cos[4hw])\csc^2\frac{hw}{2} + 48(2\cot[hw] + \csc[hw])\sin[4hw]))$$

$$5760h^2w^4$$

The derivative formulae are;

$$y'_n = \frac{11y_n}{6h} + \frac{3y_{n+1}}{h} - \frac{3y_{n+2}}{2h} + \frac{y_{n+3}}{3h} :$$

$$\frac{+ f_{n+4} \csc[\frac{hw}{2}]^2 (2160h - 12h^5w^4 + 15h^2(6h^3w^4 \csc[\frac{hw}{2}]^2 + 48w(2\mathrm{Cot}[hw] + \csc[hw]))) - 5760h^2w^4}{+ f_{n+2} \csc[\frac{hw}{2}]^3 (90h^6w^4 \csc[\frac{hw}{2}] + 360h^3w(1 + 2\cos[hw] + 2\cos[2hw]) :}$$

$$\frac{\sec[\frac{hw}{2}] + 12h^2(240 - 25h^4w^4 - 240\cos[hw]) \sin[\frac{hw}{2}]}{5760h^3w^4}$$

$$10h(120h + 240h\cos[hw]) \frac{\sin[\frac{hw}{2}]}{5760h^3w^4} + \frac{1}{5760h^3w^4} f_{n+3} \csc[\frac{hw}{2}]^3 (360hw^6 \csc[\frac{hw}{2}]^4$$

$$720h^3w(2 + 4\cos[hw] + \cos[2hw]) \sec[\frac{hw}{2}] - 2(1080h^2 \sin[\frac{hw}{2}] - 360h^2 \sin[\frac{3hw}{2}])$$

$$4h(75h^5w^4 \sin[\frac{hw}{2}] + 480h \sin[\frac{3hw}{2}] + 3h^5w^4 \sin[\frac{3hw}{2}]))$$

$$+ \frac{1}{5760h^3w^4} f_{n+1} \csc[\frac{hw}{2}]^3 (360h^6w^4 \csc[\frac{hw}{2}]$$

$$720h^3w(2 + 4\cos[hw] + 3\cos[2hw]) \sec[\frac{hw}{2}] - 2(360h^2 \sin[\frac{hw}{2}] - 1080h^2 \sin[\frac{3hw}{2}])$$

$$+ 4h(3h(960 + h^4w^4) \sin[\frac{hw}{2}] + 1440h \sin[\frac{3hw}{2}] + 75h^5w^4 \sin[\frac{3hw}{2}]))$$

$$+ \frac{f_{n+2} \csc[\frac{hw}{2}]^4 (2h^2(1800 + 141h^4w^4)}{2880h^3w^4} \frac{1}{2880h^3w^4} \cos[hw] + 1440h^2 \cos[2hw]$$

$$+ 3h^2(540 + 180\cos[2hw] - 4h^4w^4 \cos[2hw]) + 1080h^3w \sin[2hw])$$

$$
\begin{aligned}
y^{0}_{n+1} = {} & \frac{y_n}{3h} \quad \frac{y_{n+1}}{2h} + \frac{y_{n+2}}{h} \quad \frac{y_{n+3}}{6h} \\
& + \frac{f_{n+2} \csc[\frac{hw}{2}]^4 (h^2(720 \;\; 104h^4w^4) \cos[hw] \;\; 2h(540h + h(\;\; 180 \;\; 7h^4w^4)}{2880h^3w^4} \\
& + \frac{\cos[2hw]) + 1080h^3 w \sin[hw])}{2880h^3w^4} \\
& + \frac{1}{5760h^3w^4} f_n \csc[\frac{hw}{2}]^3 (360h^3 w \cos[hw](1 + 2\cos[hw] + 2\cos[2hw]) \sec[\frac{hw}{2}] \\
& \qquad 2160h^2 \sin[\frac{hw}{2}] + 4h^2(240 + 17h^4w^4 \quad 240\cos[hw]) \sin[\frac{hw}{2}] \\
& \qquad 15h^3 \csc[\frac{hw}{2}](\;\; 2h^3w^4 \quad 24w(1 \;\; 2\cos[hw] + 2\cos[2hw]) \sin[hw])) \\
& + \frac{f_{n+4} \csc[\frac{hw}{2}]^2}{5760h^2w^4} \frac{1}{5760h^2w^4} (2h(\;\; 360 \quad 2h^4 w^4) \\
& \qquad + 15h^2(48w\cos[hw](2\mathrm{Cot}[hw] + \csc[hw]) \\
& \qquad + \csc[\frac{hw}{2}]^2 \frac{(\;\; 2h^3w^4 \quad 24w(\;\; 1 + 2\cos[hw]) \sin[hw])))}{5760h^2w^4} \\
& \qquad + \frac{1}{5760h^3w^4} f_{n+3} \csc[\frac{hw}{2}]^3 (120h^6w^4 \csc[\frac{hw}{2}] \\
& \qquad 720h^3 w \cos[hw](2 + 4\cos[hw] + \cos[2hw]) \sec[\frac{hw}{2}] \\
& \qquad 2(360h^2 + 13h^6w^4 \; h^2(1080 + 13h^4w^4)) \sin[\frac{hw}{2}] \\
& \qquad 720h^3 w(\;\; 1 + 2\cos[hw] + \cos[2hw]) \csc[\frac{hw}{2}] \sin[hw] \\
& \qquad 2h((720h \;\; 34h^5w^4) \sin[\frac{hw}{2}] \;\; 2h(\;\; 120 \;\; h^4w^4) \sin[\frac{3hw}{2}])) \\
& \frac{1}{5760h^3w^4} f_{n+1} \csc[\frac{hw}{2}]^3 (\;\; 720h^3 w \cos[hw](2 + 4\cos[hw] + 3\cos[2hw]) \sec[\frac{hw}{2}] \\
& \qquad 2(\;\; 360h^2 + 10h^6w^4 + h^2(1080 \quad 13h^4w^4) \\
& \qquad 3h^2(\;\; 960 + h^4w^4)) \sin[\frac{hw}{2}] 60h^3 \csc[\frac{hw}{2}](\;\; 2h^3w^4 \\
& \qquad 12w(1 \;\; 2\cos[hw] + 3\cos[2hw]) \; 12w(1 \;\; 2\cos[hw] + 3\cos[2hw]) \sin[hw]) \\
& \qquad 2h((25h^5w^4 + 2h(1080 \;\; 13h^4w^4) + 3h(\;\; 960 + h^4w^4)) \sin[\frac{hw}{2}] \; 2h(\\
& \qquad 360 + 17h^4w^4) \sin[\frac{3hw}{2}]))
\end{aligned}
$$

$$y'_{n+2} = \frac{y_n}{6h} - \frac{y_{n+1}}{h} + \frac{y_{n+2}}{2h} + \frac{y_{n+3}}{3h} + \frac{f_{n+2}(2h(1440h + h(1800 - 55h^4w^4)) \cos[hw]h^2 - (540 - (180 - 20h^4w^4) \cos[2hw])) \csc[\frac{hw}{2}]^4}{2880h^3w^4}$$

$$+ \frac{1}{5760h^3w^4} f_{n+3} \csc[\frac{hw}{2}]^3 (-120h^6w^4 \csc[\frac{hw}{2}] - 720h^3w \cos[2hw](2 + 4\cos[hw] + \cos[2hw]) \sec[\frac{hw}{2}] - 2((1440h^2 + 186h^6w^4 - h^2(1080 + 208h^4w^4)) \sin[\frac{hw}{2}] - h^2(-120 - 2h^4w^4) \sin[\frac{3hw}{2}]) + 720h^3w(-1 + 2\cos[hw] + \cos[2hw]) \csc[\frac{hw}{2}] \sin[2hw])$$

$$+ \frac{1}{5760h^3w^4} f_n \csc[\frac{hw}{2}]^3 (360h^3w \cos[2hw](1 + 2\cos[hw] + 2\cos[2hw]) \sec[\frac{hw}{2}] - 2h(480h + h(120 + 22h^4w^4) - 240h\cos[hw]) \sin[\frac{hw}{2}] - 15h^3 \csc[\frac{hw}{2}](2h^3w^4 - 24w(1 - 2\cos[hw] + 2\cos[2hw]) \sin[2hw]))$$

$$\frac{1}{5760h^3w^4} f_{n+1} \csc[\frac{hw}{2}]^3 (-720h^3w \cos[2hw](2 + 4\cos[hw] + 3\cos[2hw]) \sec[\frac{hw}{2}] - 2((-360h^2 + 48h^6w^4 + 4h^2(1080 - 13h^4w^4) + 6h^2(-960 + h^4w^4)) \sin[\frac{hw}{2}] - h^2(-360 + 22h^4w^4) \sin[\frac{3hw}{2}]) - 60h^3 \csc[\frac{hw}{2}](2h^3w^4 - 12w(1 - 2\cos[hw] + 3\cos[2hw]) \sin[2hw]))$$

$$+ \frac{f_{n+4} \csc[\frac{hw}{2}]^2 (-2h(-360 - 2h^4w^4) + 15h^2(48w \cos[2hw](2\mathrm{Cot}[hw] - 15h^2(48w \cos[2hw](2\mathrm{Cot}[hw] \csc[hw]) + \csc[\frac{hw}{2}]^2] (2h^3w^4 - 24w(-1 + 2\cos[hw]) \sin[2hw])))}{h^2}$$

$$5760h^2w^4$$

$$y^{0}_{n+3} = \frac{y_n}{3h} + \frac{3y_{n+1}}{2h} - \frac{3y_{n+2}}{h} + \frac{11y_{n+3}}{6h}$$

$$+ \frac{f_{n+2}\csc[\frac{hw}{2}]^4}{2880h^3w^4}\frac{1}{2880h^3w^4}h \quad 1440h^2\cos[hw] \;\; 3h^2(720 + 112h^4w^4)\cos[hw]$$

$$+ \frac{1}{5760h^3w^4}f_{n+3}\csc[\frac{hw}{2}]^3(360h^6w^4\csc[\frac{hw}{2}] \quad 720h^3w(2 + 4\cos[hw] + \cos[2hw])\cos[3hw]\sec[\frac{hw}{2}]$$

$$+ 2h(1620h \quad h(180 \quad 33h^4w^4)\cos[2hw]) \;\; 1080h^3w\sin[hw])$$

$$2(3240h^2 + 1053h^6w^4 \quad h^2(1080 + 1053h^4w^4))\sin[\frac{hw}{2}] \;\; 2h((2160h$$

$$+ 42h^5w^4)\sin[\frac{hw}{2}] + 2h(120 + 3h^4w^4)\sin[\frac{3hw}{2}])$$

$$+ \frac{1}{5760h^3w^4}f_n\csc[\frac{hw}{2}]^3(360h^3w(1 + 2\cos[hw] + \cos[2hw])\csc[\frac{hw}{2}]\sin[3hw])$$

$$+ \frac{1}{5760h^3w^4}f_n\csc[\frac{hw}{2}]^3(360h^3w(1 + 2\cos[hw] + 2\cos[2hw])\cos[3hw]\sec[\frac{hw}{2}]$$

$$2h(720h + h(120 + 42h^4w^4) \;\; 480h\cos[hw])\sin[\frac{hw}{2}]$$

$$15h^3\csc[\frac{hw}{2}](6h^3w^4 \quad 24w(1 \quad 2\cos[hw] + 2\cos[2hw])\sin[3hw]))$$

$$\frac{1}{5760h^3w^4}f_{n+1}\csc[\frac{hw}{2}]^3(720h^3w(2 + 4\cos[hw] + 3\cos[2hw])\cos[3hw]\sec[\frac{hw}{2}]$$

$$2(360h^2 + 108h^6w^4 + 9h^2(1080 \quad 13h^4w^4) + 9h^2(960 + h^4w^4))\sin[\frac{hw}{2}]$$

$$2h((81h^5w^4 + 6h(1080 \quad 13h^4w^4) + 3h(960 + h^4w^4))\sin[\frac{hw}{2}]$$

$$+2h(\ 360 + 21h^4w^4)\sin[\frac{3hw}{2}])\qquad 60h^3\csc[\frac{hw}{2}](\ 6h^3w^4$$

$$12w(1\ \ 2\cos[hw] + 3\cos[2hw])\sin[3hw]))$$

$$+\frac{f_{n+4}\csc[\frac{hw}{2}]^2(2160h + 12h^5w^4 + 15h^2(48w\cos[3hw](2\mathrm{Cot}[hw] + \csc[hw])}{5760h^2w^4}$$

$$+\frac{\csc[\frac{hw}{2}]^2(\ 6h^3w^4\quad 24w(\ 1 + 2\cos[hw])\sin[3hw])))}{5760h^2w^4}$$

$$y^0_{n+4} = \frac{11y_n}{6h} + \frac{7y_{n+1}}{h}\quad \frac{19y_{n+2}}{2h} + \frac{13y_{n+3}}{3h}$$

$$\frac{1}{2880h^3w^4}f_{n+2}\csc[\frac{hw}{2}]^4(\ 8h^2(720 + 214h^4w^4)\cos[hw]$$

$$6h(2880h + h(\ 1800 + 165h^4w^4))\cos[hw]$$

$$3h^2(540 + 4h^4w^4\cos[2hw]\ (\ 180\ \ 40h^4w^4)\cos[2hw])$$

$$4h(2160h\ \ 2h(\ 180\ \ 40h^4w^4)\cos[2hw])\ \ 1080h^3w\sin[2hw])$$

$$+\frac{1}{5760h^5w^4}f_{n+3}\csc[\frac{hw}{2}]^3(3000h^6w^4\csc[\frac{hw}{2}]$$

$$720h^3w(2 + 4\cos[hw] + \cos[2hw])\cos[4hw]\sec[\frac{hw}{2}]$$

$$4h((2880h + 83h^5w^4)\sin[\frac{hw}{2}] + 75h^5w^4\sin[\frac{3hw}{2}] + 4h(\ 120 + 20h^4w^4)$$

$$\sin[\frac{3hw}{2}])\ 2((5760h^2 + 3388h^6w^4\ h^2(1080 + 3328h^4w^4))\sin[\frac{hw}{2}]$$

$$+ 3h^2(\ 120 + 20h^4w^4)\sin[\frac{3hw}{2}])$$

$$+ 720h^3w(\ 1 + 2\cos[hw] + \cos[2hw])\csc[\frac{hw}{2}]\sin[4hw])$$

$$+\frac{1}{5760h^5w^4}f_n\csc[\frac{hw}{2}]^3(360h^3w(1 + 2\cos[hw] + 2\cos[2hw])\cos[4hw]\sec[\frac{hw}{2}]$$

$$4h^2(240 + 23h^4w^4\quad 240\cos[hw])\sin[\frac{hw}{2}]$$

$$6h(960h + h(120 + 60h^4w^4)\ \ 720h\cos[hw])\sin[\frac{hw}{2}]$$

$$15h^3\csc[\frac{hw}{2}](\ 50h^3w^4\quad 24w(1\ \ 2\cos[hw] + 2\cos[2hw])\sin[4hw]))$$

$$\frac{1}{5760h^3w^4}f_{n+1}\csc[\frac{hw}{2}]^3(\ 720h^3w(2 + 4\cos[hw] + 3\cos[2hw])\cos[4hw]\sec[\frac{hw}{2}]$$

$$4h((256h^5w^4 + 8h(1080\ \ 13h^4w^4) + 3h(\ 960 + h^4w^4))\sin[\frac{hw}{2}]$$

$$3h^5w^4\sin[\frac{3hw}{2}] + 4h(\ 360 + 20h^4w^4)\sin[\frac{3hw}{2}])$$

$$2((360h^2 + 256h^6w^4 + 16h^2(1080\ 13h^4w^4) + 12h^2(960 + h^4w^4))\sin[\tfrac{hw}{2}]$$

$$+ 3h^2(360 + 20h^4w^4)\sin[\tfrac{3hw}{2}])$$

$$60h^3\csc[\tfrac{hw}{2}](50h^3w^4 \quad 12w(1\ \ 2\cos[hw] + 3\cos[2hw])\sin[4hw]))$$

$$\frac{1}{5760h^2w^4} f_{n+4}\csc[\tfrac{hw}{2}]^2(540h^5w^4 \quad 10h(360\ \ 20h^4w^4)$$

$$+ 15h^2(48w\cos[4hw](2\mathrm{Cot}[hw] + \csc[hw])$$

$$+ \frac{\csc[\frac{hw}{2}]^2(50h^3w^4 \quad 24w(1 + 2\cos[hw])\sin[4hw]))}{5760h^2w^4}$$

$$y''_n = \frac{1}{2880h^3w^4} f_{n+2}(4h(720 \quad 170h^4w^4)\cos[hw] \quad 2h(1800 + 141h^4w^4)\cos[hw]$$

$$720h\cos[2hw]\ \ 1080h^3w^2\cos[2hw]\ \ 60h^5w^4\cos[2hw]\ \ 8h(540 + 180\cos[2hw]$$

$$4h^4w^4\cos[2hw]))\csc[\tfrac{hw}{2}]^4 + \frac{f_{n+4}\csc[\frac{hw}{2}]^2(1440 + 116h^4w^4 + 15h^2(22h^2}{5760h^2w^4}$$

$$\frac{w^4 \quad 24w^2(1 + 2\cos[hw]))\csc[\frac{hw}{2}]^2)}{5760h^2w^4} + \frac{1}{5760h^3w^4} f_n\csc[\tfrac{hw}{2}]^3(15h^3(22h^2w^4$$

$$24w^2(1\ 2\cos[hw] + 2\cos[2hw]))\csc[\tfrac{hw}{2}] + 816h^5w^4\sin[\tfrac{hw}{2}]\ 20h(240$$

$$25h^4w^4\ 240\cos[hw])\sin[\tfrac{hw}{2}] + 4(120h + 240h\cos[hw])\sin[\tfrac{hw}{2}])$$

$$\frac{1}{5760h^3w^2} f_{n+3}\csc[\tfrac{hw}{2}]^3(1320h^5w^4\csc[\tfrac{hw}{2}] + 720h^3w^2(1 + 2\cos[hw]$$

$$\cos[2hw])\csc[\tfrac{hw}{2}] + 4h((720\ 254h^4w^4)\sin[\tfrac{hw}{2}]\ 240\sin[\tfrac{3hw}{2}]\ 26h^4w^4$$

$$\sin[\tfrac{3hw}{2}]) \cdot 4(75h^5w^4\sin[\tfrac{hw}{2}] + 480h\sin[\tfrac{3hw}{2}]$$

$$+ 3h^5w^4\sin[\tfrac{3hw}{2}])) + \frac{1}{5760h^3w^4} f_{n+1}\csc[\tfrac{hw}{2}]^3(60h^3(22h^2w^4 \quad 12w^2(1\ \ 2\cos[hw]$$

$$+ 3\cos[2hw]))\csc[\tfrac{hw}{2}] + 4h(2(1080 \quad 13h^4w^4)\sin[\tfrac{hw}{2}] \quad 720\sin[\tfrac{3hw}{2}]$$

$$254h^4w^4\sin[\tfrac{3hw}{2}]) \quad 4(3h(960 + h^4w^4)\sin[\tfrac{hw}{2}] + 1440h\sin[\tfrac{3hw}{2}] + 75h^5w^4\sin[\tfrac{3hw}{2}]))$$

$$y''_{n+1} = \frac{y_n}{h^2} - \frac{2y_{n+1}}{h^2} + \frac{y_{n+2}}{h^2}$$

$$+ \frac{f_{n+2}(1080h + 2160h\cos[hw] - 1080h^3w^2\cos[hw]) - 1}{2880h^3w^4} \frac{5 \quad 4}{2880h^3w^4 \; h88h \; 4W \; 4\cos[hw] + 2h(\;180 \;\; 7h\,w\,)\cos[2hw]}$$

$$4h(540 \;\; 11h^4w^4\cos[2hw] \;\; (\;180 \;\; 7h^4w^4)\cos[2hw]))\csc[\frac{hw}{2}]^4$$

$$+ \frac{1}{5760h^3w^4} f_n \csc[\frac{hw}{2}]^3 (15h^3(2h^2w^4$$

$$24w^2\cos[hw](1 \;\; 2\cos[hw] + 2\cos[2hw]))\csc[\frac{hw}{2}] + 1440h\sin[\frac{hw}{2}]$$

$$80h^5w^4\sin[\frac{hw}{2}] \;\; 12h(240 + 17h^4w^4 \;\; 240\cos[hw])\sin[\frac{hw}{2}]$$

$$360h^3w^2(1 + 2\cos[hw] + 2\cos[2hw])\sec[\frac{hw}{2}]\sin[hw])$$

$$\frac{f_{n+4}\csc[\frac{hw}{2}]^2}{} \frac{1}{} \quad 12h_{4}w \;\; 4 \quad 4(\;360 \;\; 2h_{4} \;\; w_{4}\,)$$

$$15h^2((2h^2w^4 \;\; 24w^2\cos[hw](\;1 + 2\cos[hw]))\csc[\frac{hw}{2}]^2 \quad 48w_2(2\mathrm{Cot}[hw]$$

$$+ \frac{\csc[hw])\sin[hw]))}{5760h^2w^4} \quad + \frac{1}{5760h^3w^4} f_{n+3}\csc[\frac{hw}{2}]^3 (\;120h^5w^4\csc[\frac{hw}{2}]$$

$$720h^3w^2\cos[hw](\;1 + 2\cos[hw] + \cos[2hw])\sec[\frac{hw}{2}]$$

$$720h^3w^2(2 + 4\cos[hw] + \cos[2hw])\sec[\frac{hw}{2}]\sin[hw]$$

$$2h((720 \;\; 6h^4w^4)\sin[\frac{hw}{2}] + 8h^4w^4\sin[\frac{3hw}{2}] + 2(\;120 \;\; h^4w^4)\sin[\frac{3hw}{2}]) \; 4((720h \;\; 34h^5w^4)\sin[\frac{hw}{2}] \; 2h(\;120 \;\; h^4w^4)\sin[\frac{3hw}{2}]))$$

$$+ \frac{1}{5760h^3w^4} f_{n+1}\csc[\frac{hw}{2}]^3 (\;60h^3(2h^2w^4$$

$$12w^2\cos[hw](1 \;\; 2\cos[hw] + 3\cos[2hw]))\csc[\frac{hw}{2}]$$

$$720h^3w^2(2 + 4\cos[hw] + 3\cos[2hw])\sec[\frac{hw}{2}]\sin[hw]$$

$$2h((32h^4w^4 + 2(1080 \;\; 13h^4w^4))\sin[\frac{hw}{2}] \;\; 40h^4w^4\sin[\frac{3hw}{2}]$$

$$2(\;360 + 17h^4w^4)\sin[\frac{3hw}{2}])$$

$$4((25h^5w^4 + 2h(1080 \;\; 13h^4w^4) + 3h(\;960 + h^4w^4))\sin[\frac{hw}{2}]$$

$$2h(\;360 + 17h^4w^4)\sin[\frac{3hw}{2}]))$$

$$
\begin{aligned}
y''_{n+2} = {} & \frac{h^2}{y_{n+1}} - \frac{h^2}{2y_{n+2}} + \frac{h^2}{y_{n+3}} + \frac{2880h^3w^4\,h}{1}\, f_{n+2} \\
& \frac{1}{hw}\Big(2160h - 1080h^3w^2 - 4h(720 - 2h^4w^4)\cos[hw] - 2(1440h - 28h^5w^4\cos[2hw])\csc\left[\frac{hw}{2}\right]^4 \\
& + \frac{1}{2880h^3w^4}\, f_{n+3}\csc\left[\frac{hw}{2}\right]^3\Big(-120h^5w^4\csc\left[\frac{hw}{2}\right] \\
& + 720h^3w^2\cos[2hw](1 + 2\cos[hw] + \cos[2hw])\csc\left[\frac{hw}{2}\right] \\
& - 4((1440h - h^5w^4)\sin\left[\frac{hw}{2}\right] - h^5w^4\sin\left[\frac{3hw}{2}\right]) \\
& + 720h^3w^2(2 + 4\cos[hw] + \cos[2hw])\sec\left[\frac{hw}{2}\right]\sin[2hw]\Big) \\
& + \frac{1}{5760h^3w^4}\, f_n\csc\left[\frac{hw}{2}\right]^3\Big(15h^3 - (2h^2w^4 \\
& - 24w^2\cos[2hw](1 - 2\cos[hw] + 2\cos[2hw]))\csc\left[\frac{hw}{2}\right] \\
& - 4h(240 + 23h^4w^4 - 240\cos[hw])\sin\left[\frac{hw}{2}\right] \\
& + 4(480h + h(120 + 22h^4w^4) - 240h\cos[hw])\sin\left[\frac{hw}{2}\right] \\
& - 360h^3w^2(1 + 2\cos[hw] + 2\cos[2hw])\sec\left[\frac{hw}{2}\right]\sin[2hw]\Big) \\
& + \frac{1}{5760h^3w^4}\, f_{n+1}\csc\left[\frac{hw}{2}\right]^3\Big(-60h^3(2h^2w^4 \\
& - 12w^2\cos[2hw](1 - 2\cos[hw] + 3\cos[2hw]))\csc\left[\frac{hw}{2}\right] \\
& - 4((48h^5w^4 + 4h(1080 - 13h^4w^4) + 3h(-960 + h^4w^4))\sin\left[\frac{hw}{2}\right] - h^5w^4\sin\left[\frac{3hw}{2}\right]) \\
& + 720h^3w^2(2 + 4\cos[hw] + 3\cos[2hw])\sec\left[\frac{hw}{2}\right]\sin[2hw]\Big) \\
& + \frac{1}{5760h^2w^4}\, h\, f_{n+4}\csc\left[\frac{hw}{2}\right]^2\Big(-12h^4w^4 - 4(-360 - 2h^4w^4) \\
& + 15h^2((2h^2w^4 - 24w^2(1 + 2\cos[hw])\cos[2hw])\csc\left[\frac{hw}{2}\right]^2 \\
& - 48w^2(2\mathrm{Cot}[hw] + \csc[hw])\sin[2hw]))\Big)
\end{aligned}
$$

$$y''_{n+3} = \frac{y_n}{h^2} + \frac{4y_{n+1}}{h^2} \quad \frac{5y_{n+2}}{h^2} + \frac{2y_{n+3}}{h^2}$$

$$\frac{1}{2880h^3w^4} f_{n+2}(3240h \quad 720h\cos[hw] \quad 1080h^3w^2\cos[hw] \quad 336h^5w^4\cos[hw]$$

$$8h(720 + 112h^4w^4)\cos[hw] \quad 2h(\quad 180 \quad 33h^4w^4)\cos[2hw]$$

$$4h(540 + 11h^4w^4\cos[2hw] \quad (\quad 180 \quad 33h^4w^4)\cos[2hw]))\csc[\frac{hw}{2}]^4$$

$$+\frac{1}{5760h^3w^4} f_{n+3}\csc[\frac{hw}{2}]^3(1320h^5w^4\csc[\frac{hw}{2}]$$

$$720h^3w^2(\quad 1 + 2\cos[hw] + \cos[2hw])\cos[3hw]\csc[\frac{hw}{2}]$$

$$2h((720 + 34h^4w^4)\sin[\frac{hw}{2}] + 40h^4w^4\sin[\frac{3hw}{2}] + 2(\quad 120 + 3h^4w^4)\sin[\frac{3hw}{2}])$$

$$4((2160h + 42h^5w^4)\sin[\frac{hw}{2}] + 2h(\quad 120 + 3h^4w^4)\sin[\frac{3hw}{2}])$$

$$720h^3w^2(2 + 4\cos[hw] + \cos[2hw])\sec[\frac{hw}{2}]\sin[3hw])$$

$$+\frac{1}{5760h^3w^4} f_n\csc[\frac{hw}{2}]^3(15h^3(\quad 22h^2w^4$$

$$24w^2(1 \quad 2\cos[hw] + 2\cos[2hw])\cos[3hw])\csc[\frac{hw}{2}]$$

$$+ 4h(240 + 17h^4w^4 \quad 240\cos[hw])\sin[\frac{hw}{2}]$$

$$+ 4(720h + h(120 + 42h^4w^4) \quad 480h\cos[hw])\sin[\frac{hw}{2}]$$

$$360h^3w^2(1 + 2\cos[hw] + 2\cos[2hw])\sec[\frac{hw}{2}]\sin[3hw])$$

$$+\frac{1}{5760h^3w^4} f_{n+1}\csc[\frac{hw}{2}]^3(\quad 60h^3(\quad 22h^2w^4$$

$$12w^2(1 \quad 2\cos[hw] + 3\cos[2hw])\cos[3hw])\csc[\frac{hw}{2}]$$

$$2h((72h^4w^4 + 2(1080 \quad 13h^4w^4))\sin[\frac{hw}{2}] \quad 8h^4w^4\sin[\frac{3hw}{2}]$$

$$+ 2(\quad 360 + 21h^4w^4)\sin[\frac{3hw}{2}])$$

$$4((81h^5w^4 + 6h(1080 \quad 13h^4w^4) + 3h(\quad 960 + h^4w^4))\sin[\frac{hw}{2}]$$

$$+ 2h(\quad 360 + 21h^4w^4)\sin[\frac{3hw}{2}])$$

$$+ 720h^3w^2(2 + 4\cos[hw] + 3\cos[2hw])\sec[\frac{hw}{2}]\sin[3hw])$$

$$+\frac{1}{5760h^2w^4} f_{n+4}\, h\csc[\frac{hw}{2}]^2(1440 + 116h^4w^4$$

$$+ 15h^2(\quad 22h^2w^4 \quad 24w^2(\quad 1 + 2\cos[hw])\cos[3hw])\csc[\frac{hw}{2}]^2$$

$$48w^2(2\mathrm{Cot}[hw] + \csc[hw])\sin[3hw]))$$

$$
\begin{aligned}
y''_{n+4} = {} & \frac{2y_n}{h^2} + \frac{7y_{n+1}}{h^2} - \frac{8y_{n+2}}{h^2} + \frac{3y_{n+3}}{h^2} + \frac{1}{2880h^3w^4} f_{n+2} \Big(4320h - 704h^5w^4 \cos[hw] \\
& - 12h(720 + 214h^4w^4) \cos[hw] - 2(2880h + h(-1800 + 165h^4w^4)) \cos[hw] \\
& - 1080h^3w^2 \cos[2hw] - 60h^5w^4 \cos[2hw] - 4h(-180 - 40h^4w^4) \cos[2hw] \\
& - 8h(540 + 4h^4w^4 \cos[2hw] - (-180 - 40h^4w^4) \cos[2hw])) \csc\left[\frac{hw}{2}\right]^4 \\
& + \frac{1}{5760h^3w^4} f_{n+3} \csc\left[\frac{hw}{2}\right]^3 \Big(4200h^5w^4 \csc\left[\frac{hw}{2}\right] \\
& - 720h^3w^2(-1 + 2 \cos[hw] + \cos[2hw]) \cos[4hw] \csc\left[\frac{hw}{2}\right] \\
& - 4h((720 + 66h^4w^4) \sin\left[\frac{hw}{2}\right] + 254h^4w^4 \sin\left[\frac{3hw}{2}\right] + 2(-120 + 20h^4w^4) \sin\left[\frac{3hw}{2}\right]) \\
& - 4((2880h + 83h^5w^4) \sin\left[\frac{hw}{2}\right] + 75h^5w^4 \sin\left[\frac{3hw}{2}\right] + 4h(-120 + 20h^4w^4) \sin\left[\frac{3hw}{2}\right]) \\
& - 720h^3w^2(2 + 4 \cos[hw] + \cos[2hw]) \sec\left[\frac{hw}{2}\right] \sin[4hw]) \\
& + \frac{1}{5760h^3w^4} f_n \csc\left[\frac{hw}{2}\right]^{33} \Big(15h^2 - (-70h^2w^4 \\
& - 24w^2(1 - 2 \cos[hw] + 2 \cos[2hw]) \cos[4hw]) \csc\left[\frac{hw}{2}\right] + 80h^5w^4 \sin\left[\frac{hw}{2}\right] \\
& - 12h(240 + 23h^4w^4 - 240 \cos[hw]) \sin\left[\frac{hw}{2}\right] \\
& - 4(960h + h(-120 + 60h^4w^4) - 720h \cos[hw]) \sin\left[\frac{hw}{2}\right] \\
& - 360h^3w^2(1 + 2 \cos[hw] + 2 \cos[2hw]) \sec\left[\frac{hw}{2}\right] \sin[4hw]) \\
& + \frac{1}{5760h^3w^4} f_{n+1} \csc\left[\frac{hw}{2}\right]^3 \Big(-60h^3(-70h^2w^4 \\
& - 12w^2(1 - 2 \cos[hw] + 3 \cos[2hw]) \cos[4hw]) \csc\left[\frac{hw}{2}\right] \\
& - 4h((320h^4w^4 + 2(1080 - 13h^4w^4)) \sin\left[\frac{hw}{2}\right] + 26h^4w^4 \sin\left[\frac{3hw}{2}\right] \\
& + 2(-360 + 20h^4w^4) \sin\left[\frac{3hw}{2}\right]) \\
& - 4((256h^5w^4 + 8h(1080 - 13h^4w^4) + 3h(-960 + h^4w^4)) \sin\left[\frac{hw}{2}\right] \\
& + 3h^5w^4 \sin\left[\frac{3hw}{2}\right] + 4h(-360 + 20h^4w^4) \sin\left[\frac{3hw}{2}\right]) \\
& + 720h^3w^2(2 + 4 \cos[hw] + 3 \cos[2hw]) \sec\left[\frac{hw}{2}\right] \sin[4hw]) \\
& + \frac{1}{5760h^2w^4} h f_{n+4} \csc\left[\frac{hw}{2}\right]^2 \Big(1716h^4w^4 - 4(-360 - 20h^4w^4) \\
& + 15h^2((-70h^2w^4 - 24w^2(-1 + 2 \cos[hw]) \cos[4hw]) \csc\left[\frac{hw}{2}\right]^2 \\
& - 48w^2(2\mathrm{Cot}[hw] + \csc[hw]) \sin[4hw]))
\end{aligned}
$$

$$
\begin{aligned}
y'''_n = {} & \frac{y_n}{h^3} + \frac{3y_{n+1}}{h^3} - \frac{3y_{n+2}}{h^3} + \frac{y_{n+3}}{h^3} \\
& + \frac{f_{n+4}\csc[\frac{hw}{2}]^2\left(540h^3w^4 + 15h^2(36hw^4 - \csc[\frac{hw}{2}]^2 - 48w^3(2\mathrm{Cot}[hw] + \csc[hw]))\right)}{5760h^3w^4} \\
& + \frac{1}{5760h^3w^4}\, h f_n \csc[\frac{hw}{2}]^3 \Big(540h^4w^4\csc[\frac{hw}{2}] \\
& \qquad - 360h^3w^3(1 + 2\cos[hw] + 2\cos[2hw])\sec[\frac{hw}{2}] - 2760h^4w^4\sin[\frac{hw}{2}] \\
& \qquad + 12(240 - 25h^4w^4 - 240\cos[hw])\sin[\frac{hw}{2}]\Big) \\
& + \frac{1}{5760h^3w^4} f_{n+1}\csc[\frac{hw}{2}]^3\Big(-2160h^4w^4\csc[\frac{hw}{2}] \\
& \qquad - 720h^3w^3(2 + 4\cos[hw] + 3\cos[2hw])\sec[\frac{hw}{2}] \\
& \qquad - 4h(96h^3w^4\sin[\frac{hw}{2}] + 384h^3w^4\sin[\frac{3hw}{2}]) \\
& \qquad - 6(2(1080 - 13h^4w^4)\sin[\frac{hw}{2}] - 720\sin[\frac{3hw}{2}] - 254h^4w^4\sin[\frac{3hw}{2}])\Big) \\
& + \frac{1}{5760h^3w^4} f_{n+3}\csc[\frac{hw}{2}]^3\Big(-2160h^4w^4\csc[\frac{hw}{2}] \\
& \qquad + 720h^3w^3(2 + 4\cos[hw] + \cos[2hw])\sec[\frac{hw}{2}] \\
& \qquad - 6((720 - 254h^4w^4)\sin[\frac{hw}{2}] - 240\sin[\frac{3hw}{2}] - 26h^4w^4\sin[\frac{3hw}{2}])\Big) \\
& - \frac{1}{2880h^3w^2} f_{n+2}\csc[\frac{hw}{2}]^4\Big(1080 + 240h^4w^4\cos[hw] - 6(720 - 170h^4w^4)\cos[hw] \\
& \qquad - 360\cos[2hw] + 376h^4w^4\cos[2hw] + 4(540 + 180\cos[2hw] - 4h^4w^4\cos[2hw]) \\
& \qquad - 1080h^3w^3\sin[2hw]\Big)
\end{aligned}
$$

$$y^{000}_{n+1} = \frac{y_n}{h^3} + \frac{3y_{n+1}}{h^3} - \frac{3y_{n+2}}{h^3} + \frac{y_{n+3}}{h^3} + \frac{1}{2880h^3w^4} f_{n+2} \csc\left[\frac{hw}{2}\right]^4 (1080 + 36h^4w^4 \cos[hw]$$

$$6(720 - 104h^4w^4) \cos[hw] - 118h^4w^4 \cos[2hw] - 2(-180 - 7h^4w^4) \cos[2hw]$$

$$4(540 - 11h^4w^4 \cos[2hw] - (-180 - 7h^4w^4) \cos[2hw]) - 1080h^3w^3 \sin[hw])$$

$$+\frac{1}{5760h^3w^4} f_n \csc\left[\frac{hw}{2}\right]^3 (-360h^3w^3 \cos[hw](1 + 2\cos[hw] + 2\cos[2hw]) \sec\left[\frac{hw}{2}\right]$$

$$504h^4w^4 \sin\left[\frac{hw}{2}\right] + 12(240 + 17h^4w^4 - 240\cos[hw]) \sin\left[\frac{hw}{2}\right]$$

$$15h^3 \csc\left[\frac{hw}{2}\right](12hw^4 + 24w^3(1 - 2\cos[hw] + 2\cos[2hw]) \sin[hw]))$$

$$\frac{1}{5760h^2w^4} \frac{h}{2} f_{n+4} \csc\left[\frac{hw}{2}\right]^2 (60h^3w^4 + 15h^2(-48w^3 \cos[hw](2\mathrm{Cot}[hw] + \csc[hw])$$

$$+ \csc\left[\frac{hw}{2}\right]^2 (12hw^4 + 24w^3(1 + 2\cos[hw]) \sin[hw]))$$

$$+\frac{1}{5760h^3w^4} f_{n+3} \csc\left[\frac{hw}{2}\right]^3 (-720h^4w^4 \csc\left[\frac{hw}{2}\right]$$

$$720h^3w^3 \cos[hw](2 + 4\cos[hw] + \cos[2hw]) \sec\left[\frac{hw}{2}\right]$$

$$720h^3w^3(-1 + 2\cos[hw] + \cos[2hw]) \csc\left[\frac{hw}{2}\right] \sin[hw]$$

$$2h(132h^3w^4 \sin\left[\frac{hw}{2}\right] - 12h^3w^4 \sin\left[\frac{3hw}{2}\right])$$

$$6((720 - 6h^4w^4) \sin\left[\frac{hw}{2}\right] + 8h^4w^4 \sin\left[\frac{3hw}{2}\right] + 2(-120 - h^4w^4) \sin\left[\frac{3hw}{2}\right]))$$

$$\frac{1}{5760h^3w^4} f_{n+1} \csc\left[\frac{hw}{2}\right]^3 (720h^3w^3 \cos[hw](2 + 4\cos[hw] + 3\cos[2hw]) \sec\left[\frac{hw}{2}\right]$$

$$60h^3 \csc\left[\frac{hw}{2}\right](12hw^4 + 12w^3(1 - 2\cos[hw] + 3\cos[2hw]) \sin[hw])$$

$$2h(-12h^3w^4 \sin\left[\frac{hw}{2}\right] + 132h^3w^4 \sin\left[\frac{3hw}{2}\right])$$

$$6((32h^4w^4 + 2(1080 - 13h^4w^4)) \sin\left[\frac{hw}{2}\right] - 40h^4w^4 \sin\left[\frac{3hw}{2}\right]$$

$$2(-360 + 17h^4w^4) \sin\left[\frac{3hw}{2}\right]))$$

$$y^{000}_{n+2} = \frac{y_n}{h^3} + \frac{3y_{n+1}}{h^3} - \frac{3y_{n+2}}{h^3} + \frac{y_{n+3}}{h^3} + \frac{f_{n+2}(1080 - 672h^4w^4\cos[hw] - 6(720 - 2h^4w^4)\cos[hw] - 2(-180 - 20h^4w^4)\cos[2hw] + 4(540 - (-180 - 20h^4w^4)\cos[2hw]))\csc[\frac{hw}{2}]^4}{2880h^3w^4}$$

$$+ \frac{1}{5760h^3w^4} f_{n+3}\csc[\frac{hw}{2}]^3(720h^4w^4\csc[\frac{hw}{2}] + 720h^3w^3\cos[2hw](2 + 4\cos[hw] + \cos[2hw])\sec[\frac{hw}{2}]$$

$$- 6((720 + 50h^4w^4)\sin[\frac{hw}{2}]$$

$$- 6h^4w^4\sin[\frac{3hw}{2}] + 2(-120 - 2h^4w^4)\sin[\frac{3hw}{2}])$$

$$- 720h^3w^3(-1 + 2\cos[hw] + \cos[2hw])\csc[\frac{hw}{2}]\sin[2hw])$$

$$+ \frac{1}{5760h^3w^4} f_n\csc[\frac{hw}{2}]^3(-360h^3w^3\cos[2hw](1 + 2\cos[hw] + 2\cos[2hw])\sec[\frac{hw}{2}]$$

$$+ 24h^4w^4\sin[\frac{hw}{2}]$$

$$+ 12(240 + 23h^4w^4 - 240\cos[hw])\sin[\frac{hw}{2}]$$

$$+ 15h^3\csc[\frac{hw}{2}](-12hw^4 + 24w^3(1 - 2\cos[hw] + 2\cos[2hw])\sin[2hw])) + \frac{1}{5760h^3w^4} f_{n+1}$$

$$\csc[\frac{hw}{2}]^3(720h^3w^3\cos[2hw](2 + 4\cos[hw]$$

$$- 3\cos[2hw])\sec[\frac{hw}{2}]6((16h^4w^4 + 2(1080 - 13h^4w^4))\sin[\frac{hw}{2}]$$

$$- 6h^4w^4\sin[\frac{3hw}{2}] + 2(-360 + 22h^4w^4)\sin[\frac{3hw}{2}])$$

$$- 60h^3\csc[\frac{hw}{2}](-12hw^4 + 12w^3(1 - 2\cos[hw] + 3\cos[2hw])\sin[2hw]))$$

$$\frac{f_{n+4}\csc[\frac{hw}{2}]^2(-60h^3w^4 + 15h^2(-48w^3\cos[2hw](2\mathrm{Cot}[hw] + \csc[hw]) + \csc[\frac{hw}{2}]^2(-12hw^4 + 24w^3(-1 + 2\cos[hw])\sin[2hw])))}{5760h^2w^4}$$

$$y^{000}_{n+3} = \frac{y_n}{h^3} + \frac{3y_{n+1}}{h^3} - \frac{3y_{n+2}}{h^3} + \frac{y_{n+3}}{h^3} + \frac{1}{2880h^3w^4} f_{n+2} \csc\left[\frac{hw}{2}\right]^4$$

$$(1080 - 1308h^4w^4 \cos[hw] - 6(720 + 112h^4w^4) \cos[hw] + 118h^4w^4 \cos[2hw]$$

$$- 2(-180 - 33h^4w^4) \cos[2hw] + 4(540 + 11h^4w^4 \cos[2hw] - (-180 - 33h^4w^4)$$

$$\cos[2hw]) + 1080h^3w^3 \sin[hw])$$

$$\frac{1}{5760h^3w^4} f_{n+3} \csc\left[\frac{hw}{2}\right]^3 (2160h^4w^4 \csc\left[\frac{hw}{2}\right] + 720h^3w^3(2 + 4\cos[hw] + \cos[2hw])$$

$$\cos[3hw] \sec\left[\frac{hw}{2}\right] - 2h(-12h^3w^4 \sin\left[\frac{hw}{2}\right] + 132h^3w^4 \sin\left[\frac{3hw}{2}\right])$$

$$- 6((720 + 34h^4w^4) \sin\left[\frac{hw}{2}\right] + 40h^4w^4$$

$$\sin\left[\frac{3hw}{2}\right] + 2(-120 + 3h^4w^4) \sin\left[\frac{3hw}{2}\right])$$

$$- 720h^3w^3(-1 + 2\cos[hw] + \cos[2hw]) \csc\left[\frac{hw}{2}\right] \sin[3hw]) + \frac{1}{5760h^3w^4} f_n$$

$$\csc\left[\frac{hw}{2}\right]^3 (-360h^3w^3(1 + 2\cos[hw] + 2\cos[2hw]) \cos[3hw] \sec\left[\frac{hw}{2}\right] - 24h^4w^4 \sin\left[\frac{hw}{2}\right]$$

$$+ 12(240 + 17h^4w^4$$

$$240 \cos[hw]) \sin\left[\frac{hw}{2}\right]$$

$$+ 15h^3 \csc\left[\frac{hw}{2}\right](-36hw^4 + 24w^3(1 - 2\cos[hw] + 2\cos[2hw]) \sin[3hw]))$$

$$+ \frac{1}{5760h^3w^4} f_{n+1}$$

$$\csc\left[\frac{hw}{2}\right]^3 (720h^3w^3(2 + 4\cos[hw] + 3\cos[2hw]) \cos[3hw] \sec\left[\frac{hw}{2}\right]$$

$$- 2h(132h^3w^4 \sin\left[\frac{hw}{2}\right] - 12h^3w^4 \sin\left[\frac{3hw}{2}\right])$$

$$- 6((72h^4w^4 + 2(1080 - 13h^4w^4)) \sin\left[\frac{hw}{2}\right] - 8h^4w^4 \sin\left[\frac{3hw}{2}\right]$$

$$+ 2(-360 + 21h^4w^4) \sin\left[\frac{3hw}{2}\right]) - 60h^3 \csc\left[\frac{hw}{2}\right]$$

$$(-36hw^4 + 12w^3(1 - 2\cos[hw] + 3\cos[2hw]) \sin[3hw]))$$

$$+ \frac{f_{n+4} \csc\left[\frac{hw}{2}\right]^2 (540h^3w^4 + 15h^2(-48w^3 \cos[3hw](2\mathrm{Cot}[hw]}{5760h^2w^4}$$

$$+ \frac{\csc[hw]) + \csc\left[\frac{hw}{2}\right]^2 (-36hw^4 + 24w^3(-1 + 2\cos[hw]) \sin[3hw])))}{5760h^2w^4}$$

$$y^{000}_{n+4} = \frac{y_n}{h^3} + \frac{3y_{n+1}}{h^3} \quad \frac{3y_{n+2}}{h^3} + \frac{y_{n+3}}{h^3} + \frac{1}{2880h^3w^4} f_{n+2} \csc\left[\frac{hw}{2}\right]^4$$

$$(1080 \quad 1296h^4w^4 \cos[hw] \quad 6(720 + 214h^4w^4) \cos[hw] \quad 376h^4w^4 \cos[2hw]$$

$$2(\ 180 \quad 40h^4w^4) \cos[2hw] + 4(540 + 4h^4w^4 \cos[2hw] \quad (\ 180 \quad 40h^4w^4)$$

$$\cos[2hw]) + 1080h^3w^3 \sin[2hw])$$

$$\frac{1}{5760h^3w^4} f_{n+3} \csc\left[\frac{hw}{2}\right]^3 (3600h^4w^4 \csc\left[\frac{hw}{2}\right] + 720h^3w^3(2 + 4\cos[hw] + \cos[2hw])$$

$$\cos[4hw] \sec\left[\frac{hw}{2}\right] \quad 4h(96h^3w^4 \sin\left[\frac{hw}{2}\right] + 384h^3w^4 \sin\left[\frac{3hw}{2}\right])$$

$$6((720 + 66h^4w^4) \sin\left[\frac{hw}{2}\right] + 254h^4w^4$$

$$\sin\left[\frac{3hw}{2}\right] + 2(\ 120 + 20h^4w^4) \sin\left[\frac{3hw}{2}\right])$$

$$720h^3w^3(\ 1 + 2\cos[hw] + \cos[2hw]) \csc\left[\frac{hw}{2}\right] \sin[4hw]) + \frac{1}{5760h^3w^4} f_n$$

$$\csc\left[\frac{hw}{2}\right]^3 (\ 360h^3w^3 (1 + 2\cos[hw] + 2\cos[2hw]) \cos[4hw] \sec\left[\frac{hw}{2}\right] + 504h^4w^4 \sin\left[\frac{hw}{2}\right]$$

$$+ 12(240 + 23h^4w^4 \quad 240\cos[hw])$$

$$\sin\left[\frac{hw}{2}\right] + 15h^3 \csc\left[\frac{hw}{2}\right](\ 60hw^4 + 24w^3(1 \quad 2\cos[hw] + 2\cos[2hw]) \sin[4hw]))$$

$$+ \frac{1}{5760h^3w^4} f_{n+1} \csc\left[\frac{hw}{2}\right]^3$$

$$(720h^3w^3(2 + 4\cos[hw] + 3\cos[2hw]) \cos[4hw] \sec\left[\frac{hw}{2}\right]$$

$$4h(384h^3w^4 \sin\left[\frac{hw}{2}\right] + 96h^3w^4 \sin\left[\frac{3hw}{2}\right]) \quad 6((320h^4w^4$$

$$2(1080 \quad 13h^4w^4)) \sin\left[\frac{hw}{2}\right] + 26h^4w^4 \sin\left[\frac{3hw}{2}\right] + 2(\ 360 + 20h^4w^4) \sin\left[\frac{3hw}{2}\right])$$

$$60h^3 \csc\left[\frac{hw}{2}\right](\ 60hw^4 + 12w^3$$

$$(1 \quad 2\cos[hw] + 3\cos[2hw]) \sin[4hw]))$$

$$+ \frac{f_{n+4} \csc\left[\frac{hw}{2}\right]^2 (3300h^3w^4 + 15h^2(\ 48w^3 \cos[4hw](2\mathrm{Cot}[hw]}{5760h^2w^4}$$

$$+ \frac{\csc[hw]) + \csc\left[\frac{hw}{2}\right]^2 (\ 60hw^4 + 24w^3(\ 1 + 2\cos[hw]) \sin[4hw]))}{5760h^2w^4}$$

3.4.2 Case k = 6; v = 3

$$u(x) = \ _0(x;\ !)y_n + \ _2(x;\ !)y_{n+2} + \ _3(x;\ !)y_{n+3} + \ _4(x;\ !)y_{n+4} + h^4 \ \ ^h_0(x;\ !)f_n$$

$$_1(x;\ !)f_{n+1} \ _2(x;\ !)f_{n+2} + \ _3(x;\ !)f_{n+3} + \ _4(x;\ !)f_{n+4} + \ _5(x;\ !)f_{n+5}$$

$$+ \ _6(x;\ !)f_{n+6} \qquad (3.21)$$

From (3.2)

$$y(x) \,'\, U(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + a_8x^7 + a_9 \cos !x + a_{10} \sin !x \qquad (3.22)$$

Using (3.2) & (3.4), a system of 11 equations are obtained.

$$
\begin{bmatrix}
0 & 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 & \cos[wx_n] & \sin[wx_n] & 1 & 0 & a_0 & 1 & 0 & y_n & 1 \\
1 & x_{n+2} & (x_{n+2})^2 & (x_{n+2})^3 & (x_{n+2})^4 & & (x_{n+2})^5 & (x_{n+2})^6 & (x_{n+2})^7 & (x_{n+2})^8 & \cos[wx_{n+2}] & \sin[wx_{n+2}] & & & a_1 & & & y_{n+2} & \\
1 & x_{n+3} & (x_{n+3})^2 & (x_{n+3})^3 & (x_{n+3})^4 & & (x_{n+3})^5 & (x_{n+3})^6 & (x_{n+3})^7 & (x_{n+3})^8 & \cos[wx_{n+3}] & \sin[wx_{n+3}] & & & a_2 & & & y_{n+3} & \\
1 & x_{n+4} & (x_{n+4})^2 & (x_{n+4})^3 & (x_{n+4})^4 & & (x_{n+4})^5 & (x_{n+4})^6 & (x_{n+4})^7 & (x_{n+4})^8 & \cos[wx_{n+4}] & \sin[wx_{n+4}] & & & a_3 & & & y_{n+4} & \\
0 & 0 & 0 & 0 & 24 & 120x_n & 360x_n^2 & 840x_n^3 & 1680x_n^4 & w^4 \cos[wx_n] & w^4 \sin[wx_n] & & & & a_4 & & & f_n & \\
0 & 0 & 0 & 0 & 24 & 120x_{n+1} & 360x_{n+1}^2 & 840x_{n+1}^3 & 1680x_{n+1}^4 & w^4 \cos[wx_{n+1}] & w^4 \sin[wx_{n+1}] & & & & a_5 & & & f_{n+1} & \\
0 & 0 & 0 & 0 & 24 & 120(x_{n+2}) & 360(x_{n+2})^2 & 840(x_{n+2})^3 & 1680(x_{n+2})^4 & w^4 \cos[wx_{n+2}] & w^4 \sin[wx_{n+2}] & & & & a_6 & & & f_{n+2} & \\
0 & 0 & 0 & 0 & 24 & 120(x_{n+3}) & 360(x_{n+3})^2 & 840(x_{n+3})^3 & 1680(x_{n+3})^4 & w^4 \cos[wx_{n+3}] & w^4 \sin[wx_{n+3}] & & & & a_7 & & & f_{n+3} & \\
0 & 0 & 0 & 0 & 24 & 120(x_{n+4}) & 360(x_{n+4})^2 & 840(x_{n+4})^3 & 1680(x_{n+4})^4 & w^4 \cos[wx_{n+4}] & w^4 \sin[wx_{n+4}] & & & & a_8 & & & f_{n+4} & \\
0 & 0 & 0 & 0 & 24 & 120(x_{n+5}) & 360(x_{n+5})^2 & 840(x_{n+5})^3 & 1680(x_{n+5})^4 & w^4 \cos[wx_{n+5}] & w^4 \sin[wx_{n+5}] & & & & a_9 & & & f_{n+5} & \\
0 & 0 & 0 & 0 & 24 & 120(x_{n+6}) & 360(x_{n+6})^2 & 840(x_{n+6})^3 & 1680(x_{n+6})^4 & w^4 \cos[wx_{n+6}] & w^4 \sin[wx_{n+6}] & & & & a_{10} & & & f_{n+6} & \\
\end{bmatrix}
$$

Solving these equations, substituting for a_0; a_1; ; a_{10} in (3.22) and simplifying gives

$$_0(x) = \frac{(2h\ x)(3h\ x)(4h\ x)}{24h^3}$$

$$_2(x) = \frac{(2h\ x)(3h\ x)(4h\ x)}{24h^3}$$

$$_3(x) = x\ 8h\ \frac{6hx + x^2}{8h^3}$$

$$_4(x) = x\ 6h\ \frac{5hx + x^2}{}$$

$$_0(x) = \frac{3}{2}\ x(\ 2h + x)\ 5124h^6w^4\ \ 6337h^5w^4x + 6208h^4w^4x^2\ \ 3196h^3w^4x^3\ \ 1935360h^4w^4 + 3w^4x^6\ \ 6hx\ \ 2100 + 13w^4x^4\ + h^2\ \ 57960 + 754w^4x^4\ \ \cos\frac{hw}{2}\ x\ 12h^2\ \ 7hx + x^2\ \ 1610h^5w^4 + 425h^4w^4x\ \ 1309h^3w^4x^2 + 461h^2w^4x^3\ + x)\ \ 57hx^3 + 3x^4\ \ \frac{hw}{7\ 2}\ 60480h^4\cos\frac{hw}{2}\ \ \frac{7hw}{2}\ wx\ \ \csc\frac{hw}{5}$$

$$_1(x) = \frac{1}{1935360h^4w^4}\csc\left[\frac{hw}{2}\right]^5(7560h^4(\ 24\ w^4x(\ 24h^3 + 26h^2x\ 9hx^2 + x^3)\ 8(\ 2 + 4\cos[hw]\ 4\cos[2hw] + 5\cos[3hw])\cos[wx])\csc\left[\frac{hw}{2}\right](3h\ x)(\ 8(\ 25200h^3 + 8h^2(\ 7560 + 3917h^4w^4)x + 30h(588\ 925h^4w^4)x^2\ 5751h^4w^4x^3 + 867h^3w^4x^4\ \ 481h^2w^4x^5 + 66hw^4x^6\ \ 3w^4x^7)\sin\left[\frac{hw}{2}\right](8h^2\ 6hx + x^2)((5040h\ 43h^4w^4x + 135h^3w^4x^2\ 115h^2w^4x^3 + 33hw^4x^4\ 3w^4x^5)\sin\left[\frac{3hw}{2}\right] + (25200h\ 1771h^4w^4x + 1287h^3w^4x^2\ 403h^2w^4x^3 + 57hw^4x^4\ 3w^4x^5)\sin\left[\frac{5hw}{2}\right]) + 60480h^4(3 + 6\cos[hw] + 6\cos[2hw] + 5\cos[3hw])\sec\left[\frac{hw}{2}\right]\sin[wx])$$

$$_2(x) = \frac{1}{3870720h^4w^4}\csc\left[\frac{hw}{2}\right]^5\Big(37800h^4\big(24\ w^4x\big(24h^3 + 26h^2x\ 9hx^2 + x^3\big)$$

$$+ 8\big(1 + 2\cos[hw]\ 2\cos[2hw] + 4\cos[3hw]\big)\cos[wx]\big)\csc\left[\frac{hw}{2}\right] + 2(3h\ x)$$

$$h^2x(211680 + 169w^4x^4) + h(65520x^2 + 51w^4x^6))\sin\left[\frac{hw}{2}\right]+$$

$$(8h^2\ 6hx + x^2)(5(5040h + 5957h^4w^4x\ 729h^3w^4x^2\ 235h^2w^4x^3$$

$$57hw^4x^4\ 3w^4x^5)\sin\left[\frac{3hw}{2}\right]\ 4(12600h + 593h^4w^4x + 495h^3w^4x^2\ 289h^2w^4x^3$$

$$51hw^4x^4\ 3w^4x^5)\sin\left[\frac{5hw}{2}\right]))\ 302400h^4(3 + 6\cos[hw] + 6\cos[2hw]$$

$$4\cos[3hw])\sec\left[\frac{hw}{2}\right]\sin[wx])$$

$$_3(x) = \frac{1}{1935360h^4w^4}(201600hx(8h^2\ 6hx + x^2)$$

$$5(3h\ x)x(30104h^6w^4\ 16170h^5w^4x\ 315h^4w^4x^2\ 105h^3w^4x^3 + 3w^4x^6$$

$$63hx(720 + w^4x^4) + 35h^2(4320 + 11w^4x^4))\cos[hw] + (2h\ x)(3h\ x)(4h\ x)($$

$$8w^4x(731h^4 + 423h^3x + 133h^2x^2\ 45hx^3 + 3x^4)\cos[2hw]$$

$$3(179h^4w^4x\ 207h^3w^4x^2 + 203h^2w^4x^3 + 3w^4x^5\ 15h(560 + 3w^4x^4))\cos[3hw])$$

$$604800h^4\cos[w(3h\ x)])\csc\left[\frac{hw}{2}\right]^6$$

$$
\begin{aligned}
{}_4(x) = {} & \frac{1}{3870720h^4w^4}\csc\left[\frac{hw}{2}\right]^5 \Big(37800h^4 \Big(24^4 wx\big(24h^3 + 26hx^2 - 9hx^2 + x^3\big) \\
& 8(1 - 2\cos[hw] + 2\cos[2hw] + 2\cos[3hw])\cos[wx]\Big)\csc\left[\frac{hw}{2}\right] \\
& 2(3h - x)\Big(5\big(40320h^3 + 56h^2(2700 + 851h^4w^4)x + 6h(9240 - 6929h^4w^4)x^2 \\
& 8451h^4w^4x^3 + 1137h^3w^4x^4 - 601h^2w^4x^5 + 75hw^4x^6 - 3w^4x^7\big)\sin\left[\frac{hw}{2}\right] \\
& (8h^2 - 6hx + x^2)\big(5(10080h + 1637h^4w^4x - 873h^3w^4x^2 + 53h^2w^4x^3 \\
& 33hw^4x^4 - 3w^4x^5)\sin\left[\frac{3hw}{2}\right] + 4(6300h + 55h^4w^4x - 171h^3w^4x^2 + 145h^2w^4x^3 \\
& 39hw^4x^4 + 3w^4x^5)\sin\left[\frac{5hw}{2}\right]\big)\Big) - 302400h^4 \\
& (3 + 6\cos[hw] + 6\cos[2hw] + 2\cos[3hw])\sec\left[\frac{hw}{2}\right]\sin[wx]\Big)
\end{aligned}
$$

$$
\begin{aligned}
{}_5(x) = {} & \frac{1}{1935360h^4w^4}\csc\left[\frac{hw}{2}\right]^5 \Big(7560h^4 \Big(24^4 wx\big(24h^3 + 26hx^2 - 9hx^2 + x^3\big) \\
& 8(2 - 4\cos[hw] + 4\cos[2hw] + \cos[3hw])\cos[wx]\Big)\csc\left[\frac{hw}{2}\right] \\
& (3h - x)\Big(8\big(10600h^6w^4x - 10470h^5w^4x^2 + 2439h^4w^4x^3 - 3w^4x^7 + 60hx^2(210 + w^4x^4) \\
& 9h^3(560 + 67w^4x^4) - h^2x(30240 + 373w^4x^4)\big)\sin\left[\frac{hw}{2}\right] \\
& +(8h^2 - 6hx+x^2)\big((25200h - 1771h^4w^4x+1287h^3w^4x^2 - 403h^2w^4x^3 +57hw^4x^4 - 3w^4x^5) \\
& \sin\left[\frac{3hw}{2}\right] + (5040h - 43h^4w^4x + 135h^3w^4x^2 - 115h^2w^4x^3 + 33hw^4x^4 - 3w^4x^5) \\
& \sin\left[\frac{5hw}{2}\right]\big)\Big) + 60480h^4(3 + 6\cos[hw] + 6\cos[2hw] + \cos[3hw])\sec\left[\frac{hw}{2}\right]\sin[wx]\Big)
\end{aligned}
$$

$$
\begin{aligned}
{}_6(x) = {} & \frac{1}{3870720h^4w^4}\csc\left[\frac{hw}{2}\right]^5\Big(2520h^4\Big(24 - w^4x\big(24h^3 + 26h^2x - 9hx^2 + x^3\big) \\
& + 24(1 - 2\cos[hw] + 2\cos[2hw])\cos[wx]\Big)\csc\left[\frac{hw}{2}\right] \\
& + +6x(6h^2 - 5hx + x^2)\Big(2068h^5w^4 - 1321h^4w^4x - 35h^3w^4x^2 + 191h^2w^4x^3 + 3w^4x^5 \\
& 45h(112 + w^4x^4)\Big)\sin\left[\frac{hw}{2}\right] - 2(4h + x)(3h + x)(2h + x)\Big(43h^4w^4x \\
& 135h^3w^4x^2 + 115h^2w^4x^3 + 3w^4x^5 - 3h(1680 + 11w^4x^4)\Big)\sin\left[\frac{3hw}{2}\right] \\
& 60480h^4(1 + 2\cos[hw] + 2\cos[2hw])\sec\left[\frac{hw}{2}\right]\sin[wx]\Big)
\end{aligned}
$$

The main formula is,

$$
\begin{aligned}
y_{n+6} = {} & y_n + 9y_{n+2} \quad 16y_{n+3} + 9y_{n+4} \\
& + \frac{1}{1935360h^4w^4} {}'_{n+3}(9676800h^4 \quad 90h^2(39032h^6w^4 \quad 378h^2(\ 720 + 1296h^4w^4) \\
& \qquad 35h^2(\ 4320 + 14256h^4w^4)) \cos[hw] + 604800h^4 \cos[3hw] \\
& \qquad 24h^3(\ 36624h^5w^4 \cos[2hw] + 3(60798h^5w^4 \ 15h(560 + 3888h^4w^4)) \\
& \cos[3hw])) \csc[\frac{hw}{2}]^6 + \frac{1}{1935360h^4w^4} f_n(24h^2(\ 359778h^6w^4 \\
& \qquad 36h^2(2100 + 16848h^4w^4) + h^2(57960 + 977184h^4w^4)) \\
& \cos[\frac{hw}{2}] \, 54h^3(79940h^5w^4 \quad 63h(160 + 1296h^4w^4)) \cos[\frac{3hw}{2}] \ 60480h^4 \cos[\frac{5hw}{2}] \\
& \qquad + 12h^3(798h^5w^4 \\
& \cos[\frac{5hw}{2}] \ 5040h \cos[\frac{7hw}{2}])) \csc[\frac{hw}{2}]^5 \csc[hw] + \frac{1}{3870720h^4w^4} f_{n+6} \csc[\frac{hw}{2}]^5 \\
& (\ 2520h^4(\ 24 \quad 144h^4w^4 + 24(1 \quad 2\cos[hw] + 2\cos[2hw]) \cos[6hw]) \csc[\frac{hw}{2}] \\
& \qquad + 432h^3(57466h^5w^4 \ 45h(112 + 1296h^4w^4)) \\
& \sin[\frac{hw}{2}] \qquad 48h^3(43566h^5w^4 \quad 3h(1680 + 14256h^4w^4)) \sin[\frac{3hw}{2}] \\
& \qquad 60480h^4(1 + 2\cos[hw] + 2\cos[2hw]) \sec[\frac{hw}{2}] \\
& \sin[6hw]) + \frac{1}{1935360h^4w^4} f_{n+5} \csc[\frac{hw}{2}]^5(7560h^4(\ 24 \ 144h^4w^4 \\
& \qquad + 8(2 \qquad 4\cos[hw] + 4\cos[2hw] + \cos[3hw]) \\
& \cos[6hw]) \csc[\frac{hw}{2}] \\
& \qquad 3h(\ 8(\ 626304h^7w^4 + 2160h^3(210 + 1296h^4w^4) + 9h^3(\ 560 + 86832h^4w^4) \\
& \quad 6h^3(30240 + 483408h^4w^4)) \\
& \sin[\frac{hw}{2}] + 8h^2((25200h \qquad 798h^5w^4) \sin[\frac{3hw}{2}] + (5040h \ 798h^5w^4) \sin[\frac{5hw}{2}])) \\
& \qquad + 60480h^4(3 + 6\cos[hw] \\
& \qquad \frac{hw}{}
\end{aligned}
$$

$$+\frac{1}{3870720h^4w^4}f_{n+4}\,h\csc\left[\frac{hw}{2}\right]^5\Big(37800h^4\Big(24-144h^4w^4+8(1-2\cos[hw]$$

$$-2\cos[2hw]+2\cos[3hw])\cos[6hw]\Big)\csc\left[\frac{hw}{2}\right]-6h\Big(5\Big(40320h^3+1284984h^7w^4$$

$$-216h^3(9240-6929h^4w^4)+336h^3(-2700+851h^4w^4)\Big)$$

$$\sin\left[\frac{hw}{2}\right]+8h^2\Big(5\Big(10080h+9282h^5w^4\Big)\sin\left[\frac{3hw}{2}\right]+4\Big(6300h-1722h^5w^4\Big)\sin\left[\frac{5hw}{2}\right]\Big)\Big)$$

$$-302400h^4(3+6\cos[hw]+6\cos[2hw]+2\cos[3hw])\sec\left[\frac{hw}{2}\right]\sin[6hw]\Big)\Big)$$

$$+\frac{1}{3870720h^4w^4}f_{n+2}\,h\csc\left[\frac{hw}{2}\right]^5\Big(37800h^4\Big(24-144h^4w^4+8(-1+2\cos[hw]-2\cos[2hw]$$

$$+4\cos[3hw])\cos[6hw]\Big)\csc\left[\frac{hw}{2}\right]-6h\Big(5\Big(210336h^7w^4-9h^3(8960+133488h^4w^4)$$

$$-6h^3(211680+219024h^4w^4)+h(2358720h^2+2379456h^6w^4)\Big)\sin\left[\frac{hw}{2}\right]+8h^2\Big(5\Big(5040h+9282h^5w^4\Big)\sin\left[\frac{3hw}{2}\right]-$$

$$4(12600h+1722h^5w^4)\sin\left[\frac{5hw}{2}\right]\Big)\Big)-302400h^4(3+6\cos[hw]+6\cos[2hw]+4\cos[3hw])\sec\left[\frac{hw}{2}\right]\sin[6hw]\Big)\Big)$$

$$+\frac{1}{1935360h^4w^4}f_{n+1}\csc\left[\frac{hw}{2}\right]^5\Big(7560h^4\Big(24-144h^4w^4$$

$$-8(-2+4\cos[hw]-4\cos[2hw]+5\cos[3hw])\cos[6hw]\Big)\csc\left[\frac{hw}{2}\right]-3h\Big(8\Big(25200h^3+865080h^7w^4$$

$$-1080h^3(588-925h^4w^4)+48h^3(-7560+3917h^4w^4)\Big)\sin\left[\frac{hw}{2}\right]$$

$$-8h^2\Big((5040h-798h^5w^4)\sin\left[\frac{3hw}{2}\right]+(25200h-798h^5w^4)\sin\left[\frac{5hw}{2}\right]\Big)\Big)+60480h^4(3+6\cos[hw]$$

$$-6\cos[2hw]+5\cos[3hw])\sec\left[\frac{hw}{2}\right]\sin[6hw]\Big)$$

The additional formulae are

$$y_{n+1} = \frac{y_n}{4} + \frac{3y_{n+2}}{2} y_{n+3} + \frac{y_{n+4}}{4} + \frac{1}{1935360h^4w^4} f_{n+3}(604800h^4 + 10h^2(13517h^6w^4 \quad 63h^2(\ 720 + h^4w^4) + 35h^2$$

$$(\ 4320 + 11h^4w^4)) \cos[hw] + 604800h^4 \cos[2hw] + 6h^3(1736h^5w^4 \cos[2hw] + 3(178h^5w^4 \quad 15h(560 + 3h^4w^4))$$

$$\cos[3hw])) \csc[\frac{hw}{2}]^6 + \frac{1}{1935360h^4w^4} \quad f_n(\ h^2(1802h^6w^4 \quad 6h^2(2100 + 13h^4w^4) + h^2(57960 + 754h^4w^4)) \cos[\frac{hw}{2}]$$

$$9h^3(1190h^5w^4 \quad 63h(160 + h^4w^4)) \cos[\frac{3hw}{2}] \quad 60480h^4 \cos[\frac{5hw}{2}] \quad 3h^3(833h^5w^4 \cos[\frac{5hw}{2}] \quad \frac{}{2}$$

$$5040h \cos[\frac{7hw}{2}])) \csc[\frac{hw}{2}]^5 \csc[hw] + \frac{1}{3870720h^4w^4} f_{n+6} \csc[\frac{hw}{2}]^5(\ 2520h^4(\ 24 + 6h^4w^4$$

$$+ 24 \cos[hw](1 \quad 2 \cos[hw] + 2 \cos[2hw])) \csc[\frac{hw}{2}] + 12h^3(906h^5w^4 \quad 45h(112 + h^4w^4)) \sin[\frac{hw}{2}] \quad 60480h^4$$

$$(1 + 2 \cos[hw] + 2 \cos[2hw]) \sec[\frac{hw}{2}] \sin[hw] + 12h^3(26h^5w^4$$

$$3h(1680 + 11h^4w^4)) \sin[\frac{3hw}{2}]) + \frac{1}{1935360h^4w^4} f_{n+1} \csc[\frac{hw}{2}]^5(7560h^4(\ 24 + 6h^4w^4 + 8 \cos[hw]$$

$$(\ 2 + 4 \cos[hw] \quad 4 \cos[2hw] + 5 \cos[3hw])) \csc[\frac{hw}{2}] + 60480h^4(3 + 6 \cos[hw] + 6 \cos[2hw] + 5 \cos[3hw]) \sec[\frac{hw}{2}] \sin[hw]$$

$$2h(\ 8(\ 25200h^3 + 6200h^7w^4 + 30h^3(588\ 925h^4w^4) + 8h^3(\ 7560 + 3917h^4w^4)) \sin[\frac{hw}{2}] + 3h^2((5040h + 7h^5w^4)\ 2$$

$$\sin[\frac{3hw}{2}] + (25200h \quad 833h^5w^4) \sin[\frac{5hw}{2}])) + \frac{1}{3870720h^4w^4} f_{n+4} \csc[\frac{hw}{2}]^5(\ 37800h^4(\ 24 + 6h^4w^4$$

$$8 \cos[hw](1 \quad 2 \cos[hw] + 2 \cos[2hw] + 2 \cos[3hw])) \csc[\frac{hw}{2}] \quad 302400h^4(3 + 6 \cos[hw] + 6 \cos[2hw] + 2 \cos[3hw])$$

$$\sec[\frac{hw}{2}] \sin[hw] + 4h(5(\ 40320h^3 + 9059h^7w^4 + 6h^3 \quad (9240 \quad 6929h^4w^4) + 56h^3(\ 2700 + 851h^4w^4)) \sin[\frac{hw}{2}]$$

$$+ 3h^2(5(\ 10080h + 847h^5w^4) \sin[\frac{3hw}{2}] + 4(\ 6300h \quad 7h^5w^4) \sin[\frac{5hw}{2}])) + \frac{1}{1935360h^4w^4} \Gamma_{n+5}$$

$$\csc[\frac{hw}{2}]^5(7560h^4(\ 24 + 6h^4w^4 + 8 \cos[hw](2 \quad 4 \cos[hw] + 4 \cos[2hw] + \cos[3hw])) \csc[\frac{hw}{2}] + 60480h^4(3 + 6 \cos[hw]$$

$$6 \cos[2hw] + \cos[3hw]) \sec[\frac{hw}{2}] \sin[hw] + 2h(\ 8(2566h^7w^4 + 60h^3(210 + h^4w^4) + 9h^3(\ 560 + 67h^4w^4)\ 2$$

$$h^3 \quad (30240 + 373h^4w^4)) \sin[\frac{hw}{2}] + 3h^2((25200h \quad 833h^5w^4) \sin[\frac{3hw}{2}] + (5040h + 7h^5w^4) \sin[\frac{5hw}{2}])) + \frac{1}{3870720h^4w^4} \Gamma_{n+2}$$

$$\csc[\frac{hw}{2}]^5(\ 37800h^4(\ 24 + 6h^4w^4 + 8 \cos[hw](\ 1 + 2 \cos[hw] \quad 2 \cos[2hw] + 4 \cos[3hw])) \csc[\frac{hw}{2}] \quad 302400h^4(3$$

$$6 \cos[hw] + 6 \cos[2hw] + 4 \cos[3hw]) \sec[\frac{hw}{2}] \sin[hw] + 4h(5(3586h^7w^4 \quad 9h^3(8960 + 103h^4w^4)\ 2$$

$$h(65520h^2 + 51h^6w^4)) \sin[\frac{hw}{2}] + 3h^2(5(\ 5040h + 5047h^5w^4) \sin[\frac{3hw}{2}] \quad 4(12600h + 847h^5w^4)$$

$$\sin[\frac{5hw}{2}]))$$

$$y_{n+5} = \frac{y_n}{4} + \frac{5y_{n+2}}{2} - 5y_{n+3} + \frac{15y_{n+4}}{4} + \frac{1}{1935360h^4w^4} f_{n+3}(3024000h^4 - 50h^2(-24871h^6w^4 - 315$$

$$h^2(-720 + 625h^4w^4) + 35h^2(-4320 + 6875h^4w^4)) \cos[hw] + 604800h^4 \cos[2hw] - 6h^3(-38360h^5w^4 \cos[2hw]$$

$$+ 3(30470h^5w^4 - 15h(560 + 1875h^4w^4)) \cos[3hw])) \csc[\tfrac{hw}{2}]^6 + \frac{1}{1935360h^4w^4} f_n(15h^2(-223986h^6w^4 - 30h^2(2100 + 8125h^4w^4)$$

$$+ h^2(57960 + 471250h^4w^4)) \cos[\tfrac{hw}{2}] - 60480h^4 \cos[\tfrac{3hw}{2}] - 15h^3(38010h^5w^4 - 63h(160 + 625h^4w^4)) \cos[\tfrac{3hw}{2}] + 3h^3(805h^5w^4$$

$$\cos[\tfrac{5hw}{2}] - 5040h \cos[\tfrac{7hw}{2}])) \csc[\tfrac{hw}{2}]^5 \csc[hw] + \frac{1}{3870720h^4w^4} f_{n+6} \csc[\tfrac{hw}{2}]^5(-2520h^4(-24 - 30h^4w^4 + 24$$

$$(1 - 2\cos[hw] + 2\cos[2hw]) \cos[5hw]) \csc[\tfrac{hw}{2}] + 180h^3(27838h^5w^4 - 45h(112 + 625h^4w^4))$$

$$\sin[\tfrac{hw}{2}] - 12h^3(20590h^5w^4 - 3h(1680 + 6875h^4w^4)) \sin[\tfrac{3hw}{2}] - 60480h^4(1 + 2\cos[hw] + 2\cos[2hw]) \sec[\tfrac{hw}{2}]$$

$$\sin[5hw]) + \frac{1}{1935360h^4w^4} f_{n+5} \csc[\tfrac{hw}{2}]^5(7560h^4(-24 - 30h^4w^4 + 8(2 - 4\cos[hw] + 4\cos[2hw] + \cos[3hw])$$

$$\cos[5hw]) \csc[\tfrac{hw}{2}] - 2h(-8(-138250h^7w^4 + 1500h^3(210 + 625h^4w^4) + 9h^3(-560 + 41875h^4w^4) - 5h^3(30240 + 233125h^4w^4))$$

$$\sin[\tfrac{hw}{2}] + 3h^2((25200h - 805h^5w^4) \sin[\tfrac{3hw}{2}] + (5040h + 35h^5w^4) \sin[\tfrac{5hw}{2}])) + 60480h^4(3 + 6\cos[hw]$$

$$+ 6\cos[2hw] + \cos[3hw]) \sec[\tfrac{hw}{2}] \sin[5hw]) + \frac{1}{3870720h^4w^4} f_{n+4} \csc[\tfrac{hw}{2}]^5(-37800h^4(-24 - 30h^4w^4$$

$$8(1 - 2\cos[hw] + 2\cos[2hw] + 2\cos[3hw]) \cos[5hw]) \csc[\tfrac{hw}{2}] - 4h(5(-40320h^3 + 826375h^7w^4 + 150h^3(9240 - 6929h^4w^4) - 2$$

$$280h^3(-2700 + 851h^4w^4)) \sin[\tfrac{hw}{2}] + 3h^2(5(-10080h + 4235h^5w^4) \sin[\tfrac{3hw}{2}] + 4(-6300h - 875h^5w^4) \sin[\tfrac{5hw}{2}]))$$

$$302400h^4(3 + 6\cos[hw] + 6\cos[2hw] + 2\cos[3hw]) \sec[\tfrac{hw}{2}] \sin[5hw]) + \frac{1}{3870720h^4w^4} f_{n+2} \csc[\tfrac{hw}{2}]^5$$

$$(-37800h^4(-24 - 30h^4w^4 + 8(-1 + 2\cos[hw] - 2\cos[2hw] + 4\cos[3hw]) \cos[5hw]) \csc[\tfrac{hw}{2}] - 4h(5(323330h^7w^4 - 9h^3($$

$$8960 + 64375h^4w^4) - 5h^3(211680 + 105625h^4w^4) + h(1638000h^2 + 796875h^6w^4)) \sin[\tfrac{hw}{2}] + 3h^2(5(-5040h + 8435h^5w^4)$$

$$\sin[\tfrac{3hw}{2}] - 4(12600h + 1715h^5w^4) \sin[\tfrac{5hw}{2}])) - 302400h^4(3 + 6\cos[hw] + 6\cos[2hw] + 4\cos[3hw]) \sec[\tfrac{hw}{2}] \sin[5hw])$$

$$+ \frac{1}{1935360h^4w^4} f_{n+1} \csc[\tfrac{hw}{2}]^5(7560h^4(-24 - 30h^4w^4 + 8(-2 + 4\cos[hw] - 4\cos[2hw] + 5\cos[3hw]) \cos[5hw])$$

$$\csc[\tfrac{hw}{2}] - 2h(-8(-25200h^3 + 554500h^7w^4 + 750h^3(588 - 925h^4w^4) + 40h^3(-7560 + 3917h^4w^4)) \sin[\tfrac{hw}{2}] + 3h^2((5040h$$

$$+ 35h^5w^4) \sin[\tfrac{3hw}{2}] + (25200h - 805h^5w^4) \sin[\tfrac{5hw}{2}])) + 60480h^4(3 + 6\cos[hw] + 6\cos[2hw] + 5\cos[3hw])$$

$$\sec[\tfrac{hw}{2}] \sin[5hw])$$

The derivative formulae are:

$$y_n^0 = \frac{13y_n}{12h} + \frac{3y_{n+2}}{h} - \frac{8y_{n+3}}{3h} + \frac{3y_{n+4}}{4h} + \frac{1}{3870720h^4w^4} f_{n+6} \csc\left[\frac{hw}{2}\right]^5$$

$$\left(60480h^7w^4 \csc\left[\frac{hw}{2}\right] 60480h^4w(1 + 2\cos[hw] + 2\cos[2hw]) \sec\left[\frac{hw}{2}\right] + 36h^2\left(5040h + 2068h^5w^4\right) \sin\left[\frac{hw}{2}\right]\right.$$

$$+ 262080h^3 \sin\left[\frac{3hw}{2}\right] + 2064h^7w^4 \sin\left[\frac{3hw}{2}\right]) + \frac{1}{3870720h^4w^4} f_{n+2} \csc\left[\frac{hw}{2}\right]^5 \left(907200h^7w^4\right.$$

$$\csc\left[\frac{hw}{2}\right] 302400h^4w(3 + 6\cos[hw] + 6\cos[2hw] + 4\cos[3hw]) \sec\left[\frac{hw}{2}\right] 2\left(403200h^3 \sin\left[\frac{hw}{2}\right] + 8h^2\left(25200\right.\right.$$

$$\left.\left.h \sin\left[\frac{3hw}{2}\right] 50400h \sin\left[\frac{5hw}{2}\right]\right)\right)$$

$$+ 6h\left(5\left(211680h^2 + 13096h^6w^4\right) \sin\left[\frac{hw}{2}\right] 6h\left(25200h \sin\left[\frac{3hw}{2}\right] 50400h \sin\left[\frac{5hw}{2}\right]\right) + 8h^2\left(29785h^4w^4\right.\right.$$

$$\left.\left.\sin\left[\frac{3hw}{2}\right] 2372h^4w^4 \sin\left[\frac{5hw}{2}\right]\right)\right) + \frac{1}{1935360h^4w^4} f_{n+1} \csc\left[\frac{hw}{2}\right]^5 \left(181440h^7w^4 \csc\left[\frac{hw}{2}\right]\right.$$

$$+ 60480h^4w(3 + 6\cos[hw] + 6\cos[2hw] + 5\cos[3hw]) \sec\left[\frac{hw}{2}\right] 201600h^3 \sin\left[\frac{hw}{2}\right] 8h^2\left(5040h \sin\left[\frac{3hw}{2}\right]\right.$$

$$\left.+ 25200h \sin\left[\frac{5hw}{2}\right]\right) + 3h\left(64h^2\left(7560 + 3917h^4w^4\right) \sin\left[\frac{hw}{2}\right] 6h\left(5040h \sin\left[\frac{3hw}{2}\right] + 25200h \sin\left[\frac{5hw}{2}\right]\right)\right.$$

$$\left.+ 8h^2\left(43h^4w^4 \sin\left[\frac{3hw}{2}\right] 1771h^4w^4 \sin\left[\frac{5hw}{2}\right]\right)\right)\right) + \frac{1}{1935360h^4w^4} f_{n+5} \csc\left[\frac{hw}{2}\right]^5 \left(181440\right.$$

$$\left.h^7w^4 \csc\left[\frac{hw}{2}\right] + 60480h^4w(3 + 6\cos[hw] + 6\cos[2hw] + \cos[3hw]) \sec\left[\frac{hw}{2}\right] 40320h^3 \sin\left[\frac{hw}{2}\right]\right.$$

$$8h^2\left(25200h \sin\left[\frac{3hw}{2}\right] + 5040h \sin\left[\frac{5hw}{2}\right]\right) + 3h\left(8\left(30240h^2 + 10600h^6w^4\right) \sin\left[\frac{hw}{2}\right] 6h\left(25200h \sin\left[\frac{3hw}{2}\right]\right.\right.$$

$$\left.\left.+ 5040h \sin\left[\frac{5hw}{2}\right]\right) + 8h^2\left(1771h^4w^4 \sin\left[\frac{3hw}{2}\right] 43h^4w^4 \sin\left[\frac{5hw}{2}\right]\right)\right)\right) + \frac{1}{3870720h^4w^4} f_{n+4}$$

$$\csc\left[\frac{hw}{2}\right]^5 \left(907200h^7w^4 \csc\left[\frac{hw}{2}\right] 302400h^4w(3 + 6\cos[hw] + 6\cos[2hw] + 2\cos[3hw]) \sec\left[\frac{hw}{2}\right]\right.$$

$$2\left(201600h^3 \sin\left[\frac{hw}{2}\right] + 8h^2\left(50400h \sin\left[\frac{3hw}{2}\right] 25200h \sin\left[\frac{5hw}{2}\right]\right)\right) + 6h\left(280h^2\left(2700 + 851h^4w^4\right)\right.$$

$$\sin\left[\frac{hw}{2}\right] 6h\left(50400h \sin\left[\frac{3hw}{2}\right] 25200h \sin\left[\frac{5hw}{2}\right]\right) + 8h^2\left(8185h^4w^4 \sin\left[\frac{3hw}{2}\right] + 220h^4w^4\right.$$

$$\left.\left.\sin\left[\frac{5hw}{2}\right]\right)\right)\right) + \frac{1}{1935360h^4w^4} f_{n+3} \csc\left[\frac{hw}{2}\right]^6 \left(1612800h^3 + 15h\left(151200h^2 + 30104h^6w^4\right) \cos[hw]\right.$$

$$+ 655200h^3 \cos[3hw] + 24h^3\left(5848h^4w^4 \cos[2hw] + 537h^4w^4 \cos[3hw]\right) + 604800h^4w \sin[3hw]) + \frac{1}{1935360h^4w^4} f_n$$

$$\csc\left[\frac{hw}{2}\right]^5 \csc[hw]\left(2h\left(57960h^2 + 5124h^6w^4\right) \cos\left[\frac{hw}{2}\right] 18h^2\left(10080h + 1610h^5w^4\right) \cos\left[\frac{3hw}{2}\right]\right.$$

$$\left.21252h^7w^4 \cos\left[\frac{5hw}{2}\right] 65520h^3 \cos\left[\frac{7hw}{2}\right] 60480h^4w \sin\left[\frac{7hw}{2}\right]\right)$$

$$y^{0}_{n+1} = \frac{11y_n}{24h} + \frac{y_{n+2}}{4h} + \frac{y_{n+3}}{3h} \quad \frac{y_{n+4}}{8h} + \frac{1}{3870720h^{7}w^{7}}$$

$$y_{n+6}\csc\frac{hw}{2}^{5} \quad 60480h^{4}w\cos[hw](1 + 2\cos[hw] + 2\cos[2hw])\sec\frac{hw}{2}$$

$$11796h^{7}w^{7}\sin^{2}6h^{2} \quad 906h^{5}w^{4} \quad 45h\,112 + h^{4}w^{4} \quad \sin^{2}hw$$

$$2520h^{4}\csc^{2}hw \quad 5h^{3}w^{4} \quad 24w(1 \quad 2\cos[hw] + 2\cos[2hw])\sin[hw]$$

$$+12h^{7}w^{4}\sin^{3}hw \quad 2 \quad 22h^{2} \quad 26h^{5}w^{4} \quad 3h\,1680 + 11h^{4}w^{4} \quad \sin^{2}3hw \quad +$$

$$\frac{\quad 4\ 4\ y_{n+3}\csc\frac{hw}{\ }^{6} \quad 201600h^{3} + 10h^{2} \quad 15809h^{5}w^{4} \quad 63h \quad 720 + h^{4}w^{4}}{1935360h\ w2}$$

$$\cos[hw] + 5h\ 13517h^{6}w^{4} \quad 63h^{2} \quad 720 + h^{4}w^{4} + 35h^{2} \quad 4320 + 11h^{4}w^{4} \quad \cos[hw]+$$

$$+6h^{3}\ 2792h^{4}w^{4}\cos[2hw] + 627h^{4}w^{4}\cos[3hw]\ 11h^{2}\ 1736h^{5}w^{4}\cos[2hw] + 3\ 178h^{5}w^{4}\ 15h\ 560 +$$

$$3h^{4}w^{4}\cos[3hw] + 604800h^{4}w\sin[2hw] +$$

$$\frac{\quad}{1935360h^{4}w^{4}2}y_n\csc\frac{hw}{\ }^{5} \quad \csc[hw] \quad h^{2} \quad 787h^{5}w^{4} \quad 6h\,2100 + 13h^{4}w^{4}$$

$$\cos\frac{hw}{3\ 2} \quad 579h^{7}w^{4}\cos\frac{3hw}{2} + \frac{3}{2}h^{2} \quad 833h^{5}w^{4}\cos\frac{5hw}{2}$$

$$5040h\cos\frac{hw}{2} \quad 60480h^{4}w\sin\frac{5hw}{2} +$$

$$\frac{1}{1935360h^{7}w^{4}} \quad f_{n+1}\csc\frac{hw}{2}^{5} \quad 60480h^{4}w\cos[hw](3 + 6\cos[hw]$$

$$+ 6\cos[2hw] + 5\cos[3hw])\sec\frac{hw}{2} \quad + 8 \quad 25200h^{5} + 6200h^{7}w^{7} + 30h^{5} \quad 588 \quad 925h^{7}w^{7}$$

$$+8h^{3} \quad 7560 + 3917h^{4}w^{4} \quad \sin\frac{hw}{2} +$$

$$+ 7560h^{7}\csc^{2} \quad 5h^{5}w^{7} \quad 8w(\ 2 + 4\cos[hw] \quad 4\cos[2hw] + 5\cos[3hw])\sin[hw]$$

$$3h^{2} \quad 5040h + 7h^{5}w^{4}\sin^{3}hw \quad 2 + 25200h \quad 833h^{5}w^{4}\sin^{5hw}2$$

$$+ 2h \quad 8\ 18691h^{6}w^{4} + 60h^{2} \quad 588 \quad 925h^{4}w^{4} + 8h^{2} \quad 7560 + 3917h^{4}w^{4} \quad \sin^{hw}2$$

$$+ 3h^{2}h^{4}w^{4}\sin^{3hw}2 \quad 193h^{4}w^{4}\sin^{hw}5$$

$$4h \quad 5040h + 7h^{5}w^{4}\sin^{3hw}2 + \quad 25200h \quad 833h^{5}w^{4}$$

$$\sin\frac{5hw}{2} \quad +$$

$$= \frac{1}{3870720h^4w^4} f_{n+4} \csc\frac{hw}{2} \quad 302400h^4w\cos[hw](3 + 6\cos[hw] + 6\cos[2hw + 2\cos[3hw])$$

$$\sec\frac{hw}{2} \quad 37800h^4 \csc\frac{hw}{2} \quad 5h^3w^4$$

$$8w(1 \quad 2\cos[hw] + 2\cos[2hw] + 2\cos[3hw])\sin[hw]) + 4h \quad 5 \quad 27325h^5w^4$$

$$+12h^2 \quad 9240 \quad 6929h^4w^4 + 56h^2 \quad 2700 + 851h^4w^4 \quad \sin\frac{hw}{2} +$$

$$3h^2 \quad 835h^4w^4 \sin^3\frac{hw}{2} + 28h^4w^4 \sin\frac{5hw}{2}$$

$$4h \quad 5 \quad 10080h + 847h^5w^4 \sin^3\frac{hw}{2} + 4 \quad 6300h \quad 7h^5w^4 \sin\frac{5hw}{2}$$

$$2 \quad 5 \quad 40320h^3 + 9059h^7w^4 + 6h^3 \quad 9240 \quad 6929h^4w^4 + 56h^3 \quad 2700 + 851h^4w^4$$

$$\sin\frac{hw}{2} + 3h^2 \quad 5 \quad 10080h + 847h^5w^4 \sin\frac{3hw}{2}$$

$$+4 \quad 6300h \quad 7h^5w^4 \sin\frac{5hw}{2} + \frac{1}{1935360h^4w^4} f_{n+5}$$

$$\csc\frac{hw}{2} \quad 60480h^4w\cos[hw](3 + 6\cos[hw] + 6\cos[2hw]$$

$$+ \cos[3hw])\sec\frac{hw}{2} + 8 \quad 2566h^5w^4 \quad + 60h^3 \quad 210 + h^4w^4 + 9h^3 \quad 560 + 67h^4w^4 \quad h^3$$

$$30240 + 373h^4w^4 \sin\frac{hw}{2} + 7560h^4 \csc\frac{hw}{2}$$

$$5h^3w^4 \quad 8w(2 \quad 4\cos[hw] + 4\cos[2hw] + \cos[3hw])\sin[hw] \quad 3h^2$$

$$+ 2h \quad 8 \quad 1884h^6w^4 + 120h^2 \quad 210 + h^4w^4 \quad h^2 \quad 30240 + 373h^4w^4 \sin^5\frac{3hw}{2} + 2$$

$$3h^2 \quad 193h^4w^4 \sin^3\frac{hw}{2} \quad h^4w^4 \sin\frac{5hw}{2}$$

$$4h \quad 25200h \quad 833h^5w^4 \sin^3\frac{hw}{2} + 5040h + 7h^5w^4$$

$$\sin\frac{5hw}{2} + \frac{1}{3870720h^4w^4} f_{n+2}$$

$$\csc\frac{hw}{2}^{5} \quad 302400h^4 w \cos[hw](3 + 6\cos[hw] + 6\cos[2hw]$$

$$+ 4\cos[3hw])\sec\frac{hw}{2} \quad 37800h^4 \csc\frac{hw}{2}$$

$$5h^3w^4 \quad 8w(\ 1 + 2\cos[hw] \quad 2\cos[2hw] + 4\cos[3hw])\sin[hw] + 4h^5 \quad 3024h^6w^4 \quad h^2$$

$$= 211680 + 169h^4w^4 + h \quad 131040h + 306h^5w^4 \sin \frac{2}{hw} +$$

$$3h^2 \quad 20035\,h^4w^4 \sin^3 \frac{2}{hw} \quad 3620h^4w^4 \sin \frac{2}{5hw}$$

$$4h \quad 5 \quad 5040h + 5047h^5w^4 \sin^3 \frac{2}{hw}^4 \quad 12600h + 847h^5w^4$$

$$\sin_{h^3} \frac{\frac{hw}{5}}{2} \quad 2\ 5\ 3586h^7w^4 \quad 9h^3 \quad 8960 + 103h^4w^4$$

$$211680 + 169h^7w^7 + h_{65520h^2 + 51h^6w^4} \sin \frac{hw}{2}$$

$$+ 3h^2 \quad 5 \quad 5040h + 5047h^5w^4 \sin 3 \frac{hw}{2}$$

$$\frac{5hw}{4} \quad 12600h + 847h^5w^4 \sin \frac{2}{72}$$

$$y^{0}_{n+2} = \frac{y_n}{12h\ h} \quad \frac{y_{n+2}}{} + \frac{4y_{n+3}}{3h} \quad \frac{y_{n+4}}{4h} + \frac{1}{1935360h^4w^4} f3$$

$$\frac{hw}{2}\ {}^{6} \quad \text{Csc} \quad 806400h^3 + 10h^2 \quad 9826h^5w^4 \quad 63h \quad 720 + 16h^4w^4 \quad \text{Cos}[hw] \quad 5h \quad 4144h^6w^4 \quad 126$$

$$2 \quad 4 \quad 4 \quad 2 \quad 4320 + 176h^4w^4 \quad \text{Cos}[hw] \quad 2h^2 \quad 5360h^5w^4\text{Cos}[2hw]$$

$$h \quad 720 + 5 \quad 16h^4w + 35h \quad 4 \quad 4 \quad 4 \quad \text{Cos}[3hw] \quad + 604800h^4w\sin[hw]$$

$$1250h\ w \quad 15h\ 560 + 48h\ w$$

$$+ \frac{}{1935360h^4w^4} \quad f_n\text{Csc}\ \frac{}{2} \quad \text{Csc}[hw] \quad 2h \quad 8094h^6w^4 \quad 12h^2 \quad 2100 + 208h^4w^4$$

$$+h^2 \quad 57960 + 12064h^4w^4 \quad \text{Cos} \quad 2 \quad + 6330h^4w^4\text{Cos} \quad 2 \quad +$$

$$hw \quad \frac{3hw}{}$$

$$6h^2 \quad 1008h^5w^4 \quad 63h\ 160 + 16h^4w^4 \quad \text{Cos} \quad 3\ 2 \quad + h^2 \quad 802h^5w^4\text{Cos} \quad 2$$

$$hw \quad \frac{5hw}{}$$

$$5040h\text{Cos} \quad 2 \quad 60480h^4w\sin \quad 2 \quad + \quad 3870720h^4w^4 \quad {}^{n+6}$$

$$hw \quad \frac{}{} \quad \frac{3hw}{} \quad \frac{1}{}$$

$$\frac{hw}{2}\ {}^{5} \quad \text{Csc} \quad 60480h^4w\text{Cos}[2hw](1 + 2\text{Cos}[hw] + 2\text{Cos}[2hw])$$

$$\text{Sec}\ 2 \quad 12h^2 \quad 910h^4w^4 \quad 45h\ 112 + 16h^4w^4 \quad \sin \quad 2$$

$$hw \quad \frac{hw}{}$$

$$4h^2 \quad 562h^5w^4 \quad 3h\ 1680 + 176h^4w^4 \quad \sin \quad 3\ 2 \quad 2520h^4\text{Csc} \quad 2$$

$$hw \quad \frac{}{} \quad \frac{hw}{}$$

$$\frac{hw}{} \quad 4h^3w^4 \quad 24w(1\ 2\text{Cos}[hw] + 2\text{Cos}[2hw])\sin[2hw] \quad \frac{1}{} + \frac{w}{4\ 4} \quad {}^{n+1}$$

$$\text{Csc} \quad \frac{}{2} \quad 60480h^4w\text{Cos}[2hw](3 + 6\text{Cos}[hw] + 6\text{Cos}[2hw]$$

$$hw$$

$$+5\text{Cos}[3hw])\text{Sec} \quad \frac{}{2} \quad + 8 \quad 25200h^3 + 48328h^4w^4 + 120h^3 \quad 588 \quad 925h^4w^4 \quad + 16h^3 \quad 7560 + 3917h^4w^4$$

$$\sin \quad 2 \quad + 7560h^4\text{Csc} \quad 2 \quad 4h^3w^4 \quad 8w(\ 2 + 4\text{Cos}[hw] \quad 4\text{Cos}[2hw]$$

$$hw \quad \frac{}{} \quad \frac{hw}{}$$

$$+ 5$$

$$hw$$

$$\text{Cos}[3hw])\sin[2hw]) + h \quad 8 \quad 69604h^3w^4 + 120h^2 \quad 588 \quad 925h^4w^4 \quad + 8h^2 \quad 7560 + 3917h^4w^4$$

$$3hw$$

$$\sin \quad \frac{}{2} \quad 2h \quad 5040h \quad 34h^5w^4 \quad \sin \quad \frac{}{2}$$

$$+ 25200h \quad 802h^{5}w^{4} \quad \sin \frac{5^{hw}}{2} \quad \frac{1}{+1935360h^{4}w^{4}} f_{n+5}$$

hw 5

Csc 2 60480h^4 wCos[2hw](3 + 6Cos[hw] + 6Cos[2hw] + Cos[3hw])

Sec 2 + 8 1552h w + 240h 210 + 16h w + 9h 560 + 1072h w

hw

2h^3 30240 + 5968h^4 w^4 sin 2 + 7560 h^4 Csc 2

hw hw

4h^3 w^4 8w(2 4Cos[hw] + 4Cos[2hw] + Cos[3hw]) sin[2hw] + hw

h 8 252h^6 w^4 + 240h^2 210 + 16h^4 w^4 h^2 30240 + 5968h^4 w^4 sin 2 +

2h 25200h 802h w sin 3 2 + 5040h 34h w sin 2

hw 5hw

1 5 hw

3870720h w f_{n+4} Csc 2 302400h^4 wCos[2hw](3 + 6Cos[hw] + 6Cos[2hw] + 2Cos[3hw])Sec hw

10 40320h + 70984h w + 24h 9240 6929h w + 112h 2700 + 851h w sin 2

hw

37800h^4 Csc hw 2 4h^3 w^4 8w(1 2Cos[hw] + 2Cos[2hw] + 2Cos[3hw]) sin[2hw]

+2h 5 102772h w + 24h 9240 6929h w + 56h 2700 + 851h w sin 2

hw

2h 5 10080h + 638h^5 w^4 sin 3 2 + 4 6300h + 58h^5 w^4 sin 2

hw 5hw

1 5 hw

+ 3870720h^4 w^4 f_{n+2} Csc 2 302400h^4 wCos[2hw](3 + 6Cos[hw] + 6Cos[2hw]

hw

+ 4Cos[3hw])Sec 2 10 16976h w^4 9h 8960 + 1648h^4 w^4 2h 211680 + 2704h^4 w^4

+h 262080h + 3264h w sin 2 37800h Csc 2

hw hw

4h^3 w^4 8w(1 + 2Cos[hw] 2Cos[2hw] + 4Cos[3hw]) sin[2hw] + 2h 5 8364h^6 w^4 h^2 211680 + 2704h^4 w^4

hw 2h 5 5040h + 7934h^5 w^4 3

hw

5hw

4 12600h + 1574h^5 w^4 sin 2

$$y^{0}_{n+3} = \frac{y_n}{24h} - \frac{3y_{n+2}}{4h} + \frac{y_{n+3}}{3h} + \frac{3y_{n+4}}{8h} + \frac{1}{1935360h^4w^4} f_{n+3} \Big(201600h^3 \quad 15h\big(21889h^6w^4 \quad 189h^2$$

$$(720 + 81h^4w^4) + 35h^2(4320 + 891h^4w^4)\big) \cos[hw] + h^2\big(18312h^5w^4 \cos[2hw] + 3(4884h^5w^4 \quad 15h(560$$

$$+ 243h^4w^4)\big) \cos[3hw]\big)\Big) \csc[\tfrac{hw}{2}]^6 + \frac{1}{1935360h^4w^4} f_n \csc[\tfrac{hw}{2}]^5 \csc[hw]\Big(3h^2\big(5153h^5w^4 \quad 6h(2100$$

$$+ 1053h^4w^4)\big) \cos[\tfrac{hw}{2}] + 4h\big(42120h^6w^4 \quad 18h^2(2100 + 1053h^4w^4) + h^2(57960 + 61074h^4w^4)\big) \cos[\tfrac{3hw}{2}] + \tfrac{1}{2}h^2$$

$$(4280h^5w^4 \quad 63h(160 + 81h^4w^4)\big) \cos[\tfrac{hw}{2}] \quad \tfrac{1}{2}h^2(723h^5w^4 \cos[\tfrac{5hw}{2}] \quad 5040h \cos[\tfrac{7hw}{2}]\Big)$$

$$60480h^4w \sin[\tfrac{hw}{2}]\Big) + \frac{1}{3870720h^4w^4} f_{n+6} \csc[\tfrac{hw}{2}]^5\Big(60480h^4w(1 + 2\cos[hw] + 2\cos[2hw]) \cos[3hw]$$

$$\sec[\tfrac{hw}{2}] + 18h^2(3676h^5w^4 \quad 45h(112 + 81h^4w^4)\big) \sin[\tfrac{hw}{2}] + 2h^2(2748h^5w^4 \quad 3h(1680 + 891h^4w^4)\big) \sin[\tfrac{3hw}{2}]$$

$$2520h^4 \csc[\tfrac{hw}{2}](3h^3w^4 \quad 24w(1 \quad 2\cos[hw] + 2\cos[2hw]) \sin[3hw]\big)\Big) + \frac{1}{1935360h^4w^4} f_{n+5} \csc[\tfrac{hw}{2}]^5$$

$$(60480h^4w \cos[3hw](3 + 6\cos[hw] + 6\cos[2hw] + \cos[3hw]) \sec[\tfrac{hw}{2}] + 8\big(3138h^7w^4 + 540h^3$$

$$(210 + 81h^4w^4) + 9h^3\big(560 + 5427h^4w^4\big) \quad 3h^3(30240 + 30213h^4w^4)\big) \sin[\tfrac{5hw}{2}] + h^2\big((25200h \quad 723h^5w^4) \sin[\tfrac{3hw}{2}]$$

$$+ (5040h \quad 75h^5w^4) \sin[\tfrac{hw}{2}]\big) + 7560h^4 \csc[\tfrac{hw}{2}](3h^3w^4 \quad 8w(2 \quad 4\cos[hw] + 4\cos[2hw] + \cos[3hw]) \sin[3hw]\big)\Big)$$

$$+ \frac{1}{3870720h^4w^4} f_{n+4} \csc[\tfrac{hw}{2}]^5\Big(302400h^4w \cos[3hw](3 + 6 \quad \cos[hw] + 6\cos[2hw] + 2\cos[3hw]) \sec[\tfrac{hw}{2}]2$$

$$(5\big(40320h^3 + 222345h^7w^4 + 54h^3(9240 \quad 6929h^4w^4) + 168h^3\big(2700 + 851h^4w^4)\big) \sin[\tfrac{3hw}{2}] \quad h^2\big(5\big(10080h + 429h^5w^4)$$

$$\sin[\tfrac{5hw}{2}] + 4\big(6300h + 111h^5w^4) \sin[\tfrac{hw}{2}]\big)\Big) 37800h^4 \csc[\tfrac{hw}{2}](3h^3w^4 \quad 8w(1 \quad 2\cos[hw] + 2\cos[2hw]$$

$$+ 2\cos[3hw]) \sin[3hw]\big)\Big) + \frac{1}{3870720h^4w^4} f_{n+4} \csc[\tfrac{hw}{2}]^5\Big(302400h^4w \cos[3hw](3 + 6\cos[hw] + 6\cos[2hw] + 4\cos[3hw]) \sec[\tfrac{hw}{2}]$$

$$2\big(5(78546h^7w^4 \quad 9h^3(8960 + 8343h^4w^4) \quad 3h^3(211680 + 13689h^4w^4) + h(589680h^2 + 37179h^6w^4)\big) \sin[hw] \quad h^2\big(5\big(5040h \quad 2$$

$$+ 8853h^5w^4) \sin[\tfrac{3hw}{2}] \quad 4(12600h + 1833h^5w^4) \sin[\tfrac{5hw}{2}]\big)\Big) \quad 37800h^4 \csc[\tfrac{hw}{2}](3h^3w^4 \quad 8w(\quad 1 + 2\cos[hw] \quad 2\cos[2hw]$$

$$+ 4\cos[3hw]) \sin[3hw]\big)\Big) + \frac{1}{1935360h^4w^4} f_{n+1} \csc[\tfrac{hw}{2}]^5\Big(60480h^4w \cos[3hw](3 + 6\cos[hw] + 6\cos[2hw]$$

$$5\cos[3hw]) \sec[hw] + 8\big(25200h^3 + 150174h^7w^4 + 270h^3(588 \quad 925h^4w^4) + 24h^3\big(7560 + 3917h^4w^4\big)\big) \quad 2$$

$$\sin[\tfrac{hw}{2}] + h^2\big((5040h \quad 75h^5w^4) \sin[\tfrac{3hw}{2}] + (25200h \quad 723h^5w^4) \sin[\tfrac{5hw}{2}]\big)$$

$$7560h^4 \csc[hw](3h^3w^4 \quad 8w(\quad 2 + 4\cos[hw] \quad 4\cos[2hw] + 5\cos[3hw]) \sin[3hw]\big)\Big) \quad 2$$

$$y_{n+4} = \frac{y_{n+2}}{12h} + \frac{8y_{n+3}}{3h} + \frac{7y_{n+4}}{4h} + \frac{1}{1935360h^4w^4} f_n \csc^5\frac{hw}{2}$$

$$\csc[hw]\left(8h^2\left(21503h^6w^6 + 6h\left(113152h^6w^6 + 24h^2\left(2100 + 3328h^6w^6\right)\cos^2 hw\right)\right)\right.$$

$$+h^2\left(57960 + 193024h^4w^4\right)\cos^2 hw + 6h^2\left(14942h^5w^4 + 63h\left(160 + 256h^4w^4\right)\cos^2 3hw\right.$$

$$+h^2\left(764h^5w^4\cos^5 2hw + 5040h\cos^2 7hw + 60480h^4w\sin^2 hw\right.$$

$$+\frac{1}{1935360h^4w^4} f_{n+3}\csc^6\frac{hw}{2}\left(1612800h^3\left(20h^2\left(28750h^5w^4 + 63h\left(720 + 256h^4w^4\right)\cos[hw]\right.\right.\right.$$

$$\left.\left.\left.25h\left(34048h^6w^4 + 252h^2\left(720 + 256h^4w^4\right) + 35h^2\left(4320 + 2816h^4w^4\right)\cos[hw]\right)\right)\right.\right.$$

$$\left.\left.+3\left(13468h^5w^4 + 15h\left(560 + 768h^4w^4\right)\cos[3hw]\right)\right)\right.$$

$$\left.\left.604800h^4w\sin[hw] + \frac{1}{3870720h^6w^6} f_{n+6}\csc^5\frac{hw}{2}\right.\right.$$

$$\left.\left.60480h^6w(1 + 2\cos[hw] + 2\cos[2hw])\cos[4hw]\sec^2 hw\right.\right.$$

$$\left.\left.\left(112 + 256h^4w^4\right)\sin^2 hw + 4h^2\left(8444h^5w^4 + 3h\left(1680 + 2816h^4w^4\right)\sin^2 3hw\right.\right.\right.$$

$$\left.\left.\left.2520h^4\csc^2 hw + 8h^3w^4\left(24w(1 + 2\cos[hw] + 2\cos[2hw])\sin[4hw] + 1935360h^4w^4 f_{n+5}\csc^5\frac{hw}{2}\right.\right.\right.$$

$$\left.\left.\left.60480h^4w(3 + 6\cos[hw] + 6\cos[2hw] + \cos[3hw])\cos[4hw]\sec^2 hw\right.\right.\right.$$

$$\left.\left.\left.+8\left(18176h^7w^4 + 960h^3\left(210 + 256h^6w^6 + 9h^3\left(560 + 17152h^6w^6 + 4h^3\left(30240 + 95488h^6w^6\right)\sin^2 hw\right.\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.+2h\left(25200h + 764h^5w^4\sin^3 hw\right) + 5040h + 4h^5w^4\right.\right.\right.\right.$$

sin 2 + 7560h^4 csc 2 8h^3w^4 8w(2 4 cos[hw] + 4 cos[2hw] + cos[3hw]) sin[4hw]
5hw hw

1 hw 5 hw
+ 3870720h^4w^4 f_{n+4} csc 2 302400h^4w(3 + 6 cos[hw] + 6 cos[2hw] + 2 cos[3hw]) cos[4hw] sec 2
10 40320h^3 + 474560h^2w^2 + 96h^3 9240 6929h^4w^4 + 224h^3 2700 + 851h^4w^4 sin 2
hw

2h 5 302224h^6w^4 + 48h^2 9240 6929h^4w^4 + 56h^2 2700 + 851h^4w^4 sin 2
hw

+2h 5 10080h + 1348h^5w^4 sin 3 2 + 4 6300h 148h^5w^4 sin 2
hw 5hw

hw 1 hw 5
37800h^4 csc 2 8h^3w^4 8w(1 2 cos[hw] + 2 cos[2hw] + 2 cos[3hw]) sin[4hw] + 3870720h^4w^4 f_{n+2} csc 2
hw
302400h^4w(3 + 6 cos[hw] + 6 cos[2hw] + 4 cos[3hw]) cos[4hw] sec 2 2 10 201472h^2w^4 9h^3 8960 + 26368h^4w^4
4h^3 211680 + 43264h^2w^2 + h 1048320h^2 + 208896h^3w^4 sin 2h 5 267384h^2w^2 h^2
hw

211680 + 43264h^4w^4 + h 524160h + 313344h^5w^4 sin 2 + 2h 5 5040h + 8644h^5w^4
hw

sin 3 2 4 12600h + 1780h^5w^4 sin 2 37800h^4 csc 2 1 hw 5
hw 5hw hw

8h^3w^4 8w(1 + 2 cos[hw] 2 cos[2hw] + 4 cos[3hw]) sin[4hw] + 1935360h^4w^4 f_{n+1} csc 2
hw
60480h^4w(3 + 6 cos[hw] + 6 cos[2hw] + 5 cos[3hw]) cos[4hw] sec 2 + 8 25200h^3 + 318656h^2w^4 + 480h^3
588 925h^2w^2 + 32h^3 7560 + 3917h^2w^2 sin 2 h 8 201808h^2w^2 + 240h^2 588 925h^2w^2
hw

+8h^2 7560 + 3917h^4w^4 sin 2 + 2h 5040h + 4h^5w^4 sin 2 + 25200h 764h^5w^4
hw 3hw

sin 5 2 + 7560h^4 csc 2 8h^3w^4 8w(2 + 4 cos[hw] 4 cos[2hw] + 5 cos[3hw]) sin[4hw]
hw hw

$$y_{n+5}^{0} = \frac{24h}{11y_n} + \frac{4h}{17y_{n+2}} \quad \frac{3h}{23y_{n+3}} + \frac{8h}{31y_{n+4}} \quad \frac{1}{1935360h^4w^4} + f_n \csc \frac{hw}{2}$$

$$\csc[hw] \quad 15h^2 \quad 54293h^5w^4 \quad 6h\ 2100 + 8125h^4w^4 \cos \frac{hw}{2} + 8h \quad 223986h^6w^4 \quad 30h^2 \quad 2100 + 8125h^4w^4$$

$$+h^2 \quad 57960 + 471250h^4w^4 \cos \frac{hw}{2} + 3225h^7w^4 \cos \frac{3hw}{2} \quad 2h^2 \quad 38010h^5w^4 \quad 63h$$

$$160 + 625h^4w^4 \cos \frac{hw}{2} + 3h^7w^4 \cos \frac{5hw}{2} + 2h^2 \quad 805h^5w^4 \cos \frac{5hw}{2}$$

$$\frac{1}{1935360h^4w^4} f_{n+3} \csc \frac{hw}{2}$$

$$4636800h^3 \quad 50h^2 \quad 64055h^5w^4 \quad 63h \quad 720 + 625h^4w^4 \cos[hw] \quad 35h \quad 24871h^6w^4 \quad 315h^2 \quad 720 + 625h^4w^4$$

$$+35h \quad 4320 + 6875h^4w^4 \cos[hw] \quad 6h \quad 38360h\,w \cos[2hw]$$

$$+3 \quad 30470h^5w^4 \quad 15h\ 560 + 1875h^4w^4 \cos[3hw] \quad 604800h^4w \sin[2hw] + \frac{1}{3870720h^4w^4} f_{n+6} \csc \frac{hw}{2}$$

$$60480h^4w(1 + 2\cos[hw] + 2\cos[2hw]) \cos[5hw] \sec \frac{hw}{2} \quad 84780h^7w^4 \sin \frac{3hw}{2} + 186h^2 \quad 27838h^5w^4 \quad 45h$$

$$\sin \frac{hw}{2} \quad 2520h^4 \csc \frac{hw}{2} \quad 61h^3w^4 \quad 24w(1 \quad 2\cos[hw] + 2\cos[2hw]) \sin[5hw]$$

$$+ \frac{1}{1935360h^4w^4} f_{n+5} \csc \frac{hw}{2} \quad 60480h^4w(3 + 6\cos[hw] + 6\cos[2hw] + \cos[3hw]) \cos[5hw] \sec \frac{hw}{2}$$

$$+ 8 \quad 138250h^7w^4 + 1500h^3 \quad 210 + 625h^4w^4 + 9h^3 \quad 560 + 41875h^4w^4 \quad 5h^3 \quad 30240 + 233125h^4w^4$$

sin 2 3h2 25200h 805h5w4 sin 2 + 5040h + 35h5w4 sin 2
hw 3hw 5hw

2h 8 120300h6w4 + 600h2 210 + 625h4w4 h2 30240 + 233125h4w4 sin 2
hw

+ 3h2 h4w4 sin 3 2 193h4w4 sin 2 + 4h 25200 h 805h5w4 sin 2
hw 5hw 3hw

+ 5040h + 35h5w4 sin 5 2 + 7560h4 csc 2 61h3w4 8w(2 4 cos[hw] + 4 cos[2hw]
hw hw

1
+ cos[3hw]) sin[5hw])) + 3870720h7w7 fn+4 csc hw/2 5 302400h4w(3 + 6 cos[hw] + 6 cos[2hw]

+ 2 cos[3hw]) cos[5hw] sec hw/2 4h 5 402325h6w4 + 60h2 9240 6929h4w4 + 56h2 2700 + 851h4w4
20035h7w7 sin 3620h7w7 sin 2

sin 2 + 3h2
hw

+4h 5 10080h + 4235h5w4 sin 3 2 + 4 6300h 875h5w4 sin 2
hw 3hw 5hw

2 5 40320h3 + 826375h7w4 + 150h3 9240 6929h4w4 + 280h3 2700 + 851h4w4 sin 2
hw 5hw

+3h2 5 10080h + 4235h5w4 sin3 2 + 4 6300h 875h5w4 sin 2
hw 5hw

1 hw hw 5 hw
37800h4 csc 2 61h3w4 8w(1 2 cos[hw] + 2 cos[2hw] + 2 cos[3hw]) sin[5hw]

+ 3870720h7w7 fn+2 csc 2 302400h4w(3 + 6 cos[hw] + 6 cos[2hw] + 4 cos[3hw]) cos[5hw] sec 2
4h 5 821664h9w7 h2 211680 + 105625h7w7 + h 655200h + 956250h9w7 sin 2
hw

+ 3h2 835h4w4 sin 3 2 + 28h4w4 sin 2 + 4h 5 5040h + 8435h5w4
hw 5hw

sin 3 2 4 12600h + 1715h5w4 sin 2 2 5 323330h7w4 9h3 8960 + 64375h4w4
hw 5hw

5h3 211680 + 105625h4w4 + h 1638000h2 + 796875h6w4 sin 2 + 3h2 5 5040h + 8435h5w4
hw

sin 2 4 12600h + 1715h5w4 sin 2 37800h4 csc 2 1 hw 5
3hw 5hw hw

61h3w4 8w(1 + 2 cos[hw] 2 cos[2hw] + 4 cos[3hw]) sin[5hw] + 1935360h4w4 fn+1 csc 2
hw

60480h4w(3 + 6 cos[hw] + 6 cos[2hw] + 5 cos[3hw]) cos[5hw] sec 2 + 8 25200h3 + 554500h7w4 + 750h3 588 925h4w4
+40h9 7560 + 3917h7w7 sin 2 3h2 5040h + 35h5w4 sin 2 +25200h 805h9w7
hw 3hw

sin 5 2 2h 8 271075h6w4 + 300h2 588 925h4w4 + 8h2 7560 + 3917h4w4 sin 2
hw hw

+ 3h2 193h4w4 sin 3 2 h4w4 sin 2 + 4h 5040h + 35h5w4 sin 2
hw 5hw 3hw

+25200h 805h5w4 sin 3 2 + 7560h4 csc 2 61h3w4 8w(2 + 4 cos[hw] 4 cos[2hw]
hw hw

+5 cos[3hw]) sin[5hw]))

$$y_{n+6}^{0} = \frac{13y_n}{12h} + \frac{9y_{n+2}}{h} \quad \frac{44y_{n+3}}{3h} + \frac{27y_{n+4}}{4h} + \frac{1}{1935360h^4w^4} f_n \csc^5\frac{hw}{2}$$

$$\csc[hw] \quad 24h^2 \quad 110063h^3w^2 \quad 6h \quad 2100 + 16848h^2w^2 \cos \quad + 10h \quad 359778h^3w^2 \quad 36h^2$$

$$2100 + 16848h^4w^4 + h^2 \quad 57960 + 977184h^4w^4 \quad \cos^2\frac{hw}{2} + 26298h^7w^4 \cos^2\frac{hw}{2}$$

$$54h^2 \quad 79940h^5w^4 \quad 63h\ 160 + 1296h^4w^4 \quad \cos^3\frac{hw}{2} + 516\,h^7w^4 \cos^2\frac{3hw}{2} + 13h^2$$

$$798h^5w^4 \cos^5\frac{hw}{2} \quad 5040h \cos^2\frac{7hw}{2} + 60480h^4w \sin^2\frac{5hw}{2}$$

$$+ \frac{1}{1935360h^4w^4} f_{n+3} \csc^6\frac{hw}{2} \quad 8870400h^3 \quad 90h^2 \quad 114726h^5w^4 \quad 63h \quad 720 + 1296h^4w^4 \quad \cos[hw]$$

$$45h\ 39032h^6w^4\ 378h^2\ 720 + 1296h^4w^4 + 35h^2\ 4320 + 14256h^4w^4 \cos[hw]\ 24h^3\ 5848h^4w^4 \cos[2hw] + 537h^4w^4$$

$$\cos[3hw]\ 26h^2\ 36624h^5w^4 \cos[2hw] + 3\ 60798h^5w^4\ 15h\ 560 + 3888h^4w^4 \cos[3hw]$$

$$604800h^4w \sin[3hw] \quad + \frac{1}{3870720h^4w^4} f_{n+6} \csc^5\frac{hw}{2} \quad 60480h^4w(1 + 2\cos[hw] + 2\cos[2hw]) \cos[6hw] \sec^2\frac{hw}{2}$$

$$238896h^2w^2 \sin \quad + 324h^2 \quad 57466h^3w^2 \quad 45h\ 112 + 1296h^2w^2 \quad \sin$$

$$85008h^7w^4 \sin^3\frac{hw}{2} \quad 52h^2 \quad 43566h^5w^4 \quad 3h\ 1680 + 14256h^4w^4 \quad \sin^2\frac{3hw}{2}$$

$$2520h^4 \csc^5\frac{hw}{2} \quad 180h^3w^4 \quad 24w(1\ 2\cos[hw] + 2\cos[2hw]) \sin[6hw] \quad + \frac{1}{1935360h^4w^4} f_{n+5}$$

$$\csc^2\frac{hw}{2} \quad 60480h^4w(3 + 6\cos[hw] + 6\cos[2hw] + \cos[3hw]) \cos[6hw] \sec^2\frac{hw}{2} + 8 \quad 626304h^7w^4 + 2160h^3$$

$$210 + 1296h^2w^2 \quad + 9h^3 \quad 560 + 86832h^2w^2 \quad 6h^3 \quad 30240 + 483408h^2w^2 \quad \sin \quad 8h^2 \quad 25200h \quad 798h^5w^4$$

$$\sin^3\frac{hw}{2} + \quad 5040h \quad 798h^5w^4 \sin^2\frac{5hw}{2} \quad 3h \quad 8 \quad 377804h^6w^4 + 720h^2 \quad 210 + 1296h^4w^4$$

$$h^2 \quad 30240 + 483408h^4w^4 \sin^2\frac{hw}{2} + 8h^2 \quad 43h^4w^4 \sin^2\frac{3hw}{2} \quad 1771h^4w^4 \sin^2\frac{5hw}{2}$$

$$+ 6h \quad 25200\,h \quad 798h^5w^4 \sin^3\frac{hw}{2} + 5040h \quad 798h^5w^4 \sin^2\frac{5hw}{2} + 7560h^4$$

$$\csc\frac{hw}{2}\quad 180h^3w^4\quad 8w(2\quad 4\cos[hw] + 4\cos[2hw] + \cos[3hw])\sin[6hw] \quad + \quad \frac{1}{3870720h^4w^4}f_{n+4}$$

$$\csc\frac{hw}{2}^{5}\quad 302400h^4w(3 + 6\cos[hw] + 6\cos[2hw] + 2\cos[3hw])\cos[6hw]\sec\frac{hw}{2}\ 6h\quad 5\ 520020h^6w^4$$

$$+72h^2\quad 9240\quad 6929h^4w^4 + 56h^2\quad 2700 + 851h^4w^4\quad \sin\frac{hw}{2} + 8h^2\quad 29785h^4w^4\sin\frac{3hw}{2}$$

$$2372h^4w^4\sin\frac{hw}{2} + 6h\ 5\quad 10080h + 9282h^5w^4\quad \sin\frac{3hw}{2} + 4\quad 6300h\quad 1722h^5w^4$$

$$\sin \frac{5\,hw}{2} \quad 2\,5 \quad 40320h^3 + 1284984h^7w^4 + 216h^3 \quad 9240 \quad 6929h^4w^4 + 336h^3 \quad 2700 + 851h^4w^4$$

$$\sin \frac{hw}{2} \quad + 8h^2 \quad 5 \quad 10080h + 9282h^5w^4 \sin \frac{3hw}{2}$$

$$\sin \frac{hw}{5\,2} \quad 37800h^4 \csc \frac{2}{hw} \quad 180h^3w^4 \quad 8w(1 \quad 2\cos[hw] + 2\cos[2hw] + 2\cos[3hw])\sin[6hw]$$

$$+ \frac{1}{6h\,3870720h^4w^4} f_{n+2} \csc \frac{hw^5_{hw}}{2} \quad 302400h^4w(3 + 6\cos[hw] + 6\cos[2hw] + 4\cos[3hw])\cos[6hw]\sec \frac{}{2}$$

$$hw \quad 5 \quad 2057084h^5w^4 \quad h^2 \quad 211680 + 219024h^4w^4 + h\,786240h + 2379456h^5w^4 \quad \sin \frac{}{2} +$$

$$\frac{8h^2}{hw} \quad 8185h^4w^4 \sin^3 \frac{}{2} + 220h^4w^4 \sin \frac{}{2} + 6h\,5 \quad 5040h + 9282h^5w^4$$

$$\sin \frac{}{3hw\,2} \quad 412600h + 1722h^5w^4 \sin \frac{5\,2}{hw} \quad 2\,5\,210336h^7w^4 \quad 9h^3 \quad 8960 + 133488h^4w^4$$

$$\frac{6h^3}{hw} \quad 211680 + 219024h^4w^4 + h \quad 2358720h^2 + 2379456h^6w^4 \quad \sin \frac{}{2} + 8h^2\,5 \quad 5040h + 9282h^5w^4$$

$$\sin \frac{}{hw\,3\,2} \quad 412600h + 1722h^5w^4 \sin \frac{}{5hw} \quad 37800h^4 \csc \frac{2}{hw} \quad 1 \quad hw \quad 5$$

$$180h^3w^4 \quad 8w(\ 1 + 2\cos[hw] \quad 2\cos[2hw] + 4\cos[3hw])\sin[6hw] + \frac{}{hw} \quad 60480h^3w(3 + 6\cos[hw] + 6\cos[2hw] + 5\cos[3hw])\cos[6hw]\sec \frac{}{2} \quad + 8 \quad 1935360h^4w^4 \quad f_{n+1} \csc \frac{}{2} \quad 25200h^3 + 865080h^2w^4 + 1080h^3 \quad 588 \quad 925h^4w^4$$

$$+48h^3 \quad 7560 + 3917h^4w^4 \quad \sin \frac{}{hw} \quad 2 \quad 8h^2 \quad 5040h \quad 798h^5w^4 \sin \frac{}{3hw} \quad 2$$

$$+ 25200h \quad 798h^5w^4 \quad \sin \frac{hw}{2} \quad 3h \quad 8 \quad 352836h^6w^4 + 360h^2 \quad 588 \quad 925h^4w^4 + 8h^2 \quad 7560 + 3917h^4w^4$$

$$\sin \frac{}{hw} \quad 2 + 8h^2 \quad 1771h^4w^4 \sin \frac{2}{3hw} \quad 43h^4w^4 \sin \frac{2}{5hw} \quad +$$

$$6h\,5040h \quad 798h^5w^4 \sin^3 \frac{}{2} + 25200h \quad 798h^5w^4 \sin \frac{2}{5hw}$$

$$+7560h^4 \csc \frac{}{hw} \quad 2 \quad 180h^3w^4 \quad 8w(\ 2 + 4\cos[hw] \quad 4\cos[2hw] + 5\cos[3hw])\sin[6hw]$$

$$y''_n = \frac{3y_n}{4h^2} - \frac{7y_{n+2}}{2h^2} + \frac{4y_{n+3}}{h^2} - \frac{5y_{n+4}}{4h^2} + \frac{1}{1935360h^4w^4} f_{n+3} (2419200h^2 + 30h(45360h \quad 16170h^5w^4):$$

$$\cos[hw] \quad 10(151200h^2 + 30104h^6w^4) \cos[hw] \quad :453600h^2 \cos[3hw] \quad 604800h^4w^2 \cos[3hw]:$$

$$24h^3(6768h^3w^4 \cos[2hw] \; 1242h^3w^4 \cos[3hw]) \; 52h^2(5848h^4w^4 \cos[2hw] + 537h^4w^4 \cos[3hw])) \csc[\frac{hw}{2}]^6 : 2$$

$$+ \frac{1}{1935360h^4w^4} f_n(4h(12600h \quad 6337h^5w^4)$$

$$\cos[\frac{hw}{2}] + 2(57960h^2 + 5124h^6w^4) \cos[\frac{hw}{2}] \; 15300h^6w^4 \cos[\frac{3hw}{2}] + 21h(10080h + 1610h^5w^4)$$

$$\cos[\frac{3hw}{2}] + :76934h^6w^4 \cos[\frac{5hw}{2}] + 45360h^2 \cos[\frac{7hw}{2}] + 60480h^4w^2 \cos[\frac{7hw}{2}]) \csc[\frac{hw}{2}]^5$$

$$: \csc[hw] + \frac{f_{n+6} \csc[\frac{hw}{2}]^5(2520h^4(52h^2w^4 \quad 24w^2(1 \quad 2 \cos[hw] + 2 \cos[2hw])) \csc[\frac{hw}{2}]:}{3870720h^4w^4} :$$

$$\frac{95112h^6w^4 \sin[\frac{hw}{2}] \; 60h(5040h + 2068h^5w^4) \sin[\frac{hw}{2}] \; 181440h^2 \sin[\frac{3hw}{2}]}{3870720h^4w^4}$$

$$: \frac{17432h^6w^4 \sin[\frac{3hw}{2}])}{3870720h^4w^4} \quad \frac{1}{+3870720h^4w^4} f_{n+2} \csc[\frac{hw}{2}]^5 (37800h^4(52h^2w^4 \quad 8w^2(1 + 2 \cos[hw]:$$

$$:: \; 2 \cos[2hw] + 4 \cos[3hw])) \csc[\frac{hw}{2}] + 6h(5(131040h^2 \quad 33612h^5w^4) \sin[\frac{hw}{2}] + ::$$

$$:2(25200h \sin[\frac{3hw}{2}] \; 50400h \sin[\frac{5hw}{2}]) + 8h^2(7290h^3w^4 \sin[\frac{3hw}{2}]::$$

$$3960h^3w^4 \sin[\frac{5hw}{2}]) \; 12h(29785h^4w^4 \sin[\frac{3hw}{2}] \; 2372h^4w^4 \sin[\frac{5hw}{2}]))$$

$$:4(5(211680h^2 + 13096h^6w^4) \sin[\frac{hw}{2}] \; 6h(25200h \sin[\frac{3hw}{2}] \; 50400h \sin[\frac{5hw}{2}]::$$

$$+ 8h^2 (29785h^4w^4 \sin[\frac{3hw}{2}] \quad 2372h^4w^4 \sin[\frac{5hw}{2}]))$$

$$+ \frac{1}{1935360h^4w^4} f_{n+1} \csc[\frac{hw}{2}]^5(7560h^4(52h^2w^4 \quad 8w^2(2 + 4 \cos[hw] \; 4 \cos[2hw]$$

hw

+5 cos[3hw])) csc $\frac{}{2}$ + 128h² 7560 + 3917h⁴w⁴ sin $\frac{hw}{2}$ + 12h5040h sin $\frac{3hw}{2}$

+25200h sin 2 16h² 43h⁴w⁴ sin 1771h⁴w⁴ sin 2

5hw hw 5hw

+ 3h 480h 588 925h⁴w⁴ sin 2 + 2 5040h sin 2 + 25200h sin 2

hw 3hw 5hw

+ 8h² 270h³w⁴ sin 3 2 + 2574h³w⁴ sin 2 12h43h⁴w⁴ sin 2

hw 5hw 3hw

$\frac{5hw}{2}$ $\frac{1}{1935360h^4w^4}$ f_{n+5} csc $\frac{hw}{2}$ 5 7560h⁴ 52h²w⁴ 8w²(2 4 cos[hw]

1771h⁴w⁴ sin

hw

+4 cos[2hw] + cos[3hw])) csc $\frac{}{2}$ + 16 30240h² + 10600h⁶w⁴ sin $\frac{hw}{2}$ 12h25200h sin $\frac{3hw}{2}$

+5040h sin 2 16h² 1771h⁴w⁴ sin 3 2 43h⁴w⁴ sin + 2 +

5hw hw 5hw

3h 8 25200h 20940h⁵w⁴ sin 2 + 2 25200h sin 2 + 5040h sin 2

hw 3hw 5hw

+ 8h² 2574h³w⁴ sin 3 2 + 270h³w⁴ sin 2 12h1771h⁴w⁴ sin 2

hw 5hw 3hw

$\frac{5hw}{2}$ $\frac{1}{3870720h^4w^4}$ f_{n+4} csc $\frac{hw}{2}$ 5 37800h⁴ 52h²w⁴ 8w²(1 2 cos[hw]

43h⁴w⁴ sin

+2 cos[2hw] + 2 cos[3hw])) csc 2 + 6h 60h 9240 6929h⁴w⁴ sin + 2 50400h sin 2

hw hw 3hw

25200h sin 2 + 8h² 8730h³w⁴ sin 3 2 1368h³w⁴ sin 2

5hw hw 5hw

12h 8185h⁴w⁴ sin 3 2 + 220h⁴w⁴ sin 2 4 280h² 2700 + 851h⁴w⁴

hw 5hw

sin 2 6h 50400h sin 2 25200h sin 2 + 8h² 8185h⁴w⁴ sin 3 2

hw 3hw 5hw hw

+220h⁴w⁴ sin $\frac{5hw}{2}$

$$y_{n+1}'' = \frac{y_n}{2h^2} \quad \frac{2y_{n+2}}{h^2} + \frac{2y_{n+3}}{h^2} \quad \frac{y_{n+4}}{2h^2} + \frac{1}{1935360h^4w^4}\Big(_{n+3} \quad 1209600h^2 + 21900h^6w^4\cos[hw] + 10h \quad 15809h^5w^4 \quad 63h$$

$$720 + h^4w^4 \quad \cos[hw] \quad 10 \qquad 13517h^6w^4 \quad 63h^2 \quad 720 + h^4w^4 + 35h^2 \quad 4320 + 11h^4w^4 \quad \cos[hw]$$

$$604800h^4w^2\cos[2hw] + 6h^3 \qquad 9312h^3w^4\cos[2hw] + 972h^3w^4\cos[3hw] \qquad 22h^2 \qquad 2792h^4w^4\cos[2hw] + 627h^4w^4\cos[3hw]$$

$$12h\,1736h^5w^4\cos[2hw] + 3\,178h^5w^4\,15h\,560 + 3h^4w^4\cos[3hw]\csc\frac{hhw}{2}\Big(_6$$

$$+ \frac{1}{1935360h^4w^4}\Big(_{n} \quad 818h^6w^4\cos\Big(\frac{hw}{2}\Big) + 2 \quad 1802h^6w^4 \quad 6h^2 \quad 2100 + 13h^4w^4 + h^2 \qquad 57960 + 754h^4w^4$$

$$\cos\Big(\frac{hw}{2}\Big) + 8073h^6w^4\cos\Big(\frac{3hw}{2}\Big) + 12h \quad 1190h^5w^4 \quad 63h\,160 + h^4w^4\cos\Big(\frac{3hw}{2}\Big)$$

$$+ 60480h^4w^2\cos\Big(\frac{5hw}{2}\Big) + 4463h^6w^4\cos\Big(\frac{5hw}{2}\Big) \quad 6h\,833h^5w^4\cos\Big(\frac{hw}{5}\Big) \quad 5040h\cos\Big(\frac{7hw}{2}\Big)$$

$$\csc\Big(\frac{hw}{2}\Big)^5 \quad \csc[hw] + \frac{1}{3870720h^4w^4}\Big(_{n+6}\csc\Big(\frac{hw}{2}\Big)^5 \quad 2520h^4 \quad 10h^2w^4 \quad 24w^2\cos[hw](1 \quad 2\cos[hw] + 2\cos[2hw])$$

$$\csc\Big(\frac{hw}{2}\Big) + 18948h^6w^4\sin\Big(\frac{hw}{2}\Big) \quad 24h \quad 906h^5w^4 \quad 45h \quad 112 + h^4w^4 \quad \sin\Big(\frac{hw}{2}\Big)$$

$$+ 60480h^4w^2(1 + 2\cos[hw] + 2\cos[2hw])\sec\Big(\frac{hw}{2}\Big)\sin[hw] + 964h^6w^4\sin\Big(\frac{3hw}{2}\Big) + 24h\,26h^5w^4$$

$$3h \quad 1680 + 11h^4w^4 \quad \sin\Big(\frac{3hw}{2}\Big) + \frac{1}{1935360h^4w^4}\Big(_{n+1}\csc\Big(\frac{hw}{2}\Big)^5 \quad 7560h^4$$

$$10h^2w^4 \quad 8w^2\cos[hw](\ 2 + 4\cos[hw] \quad 4\cos[2hw] + 5\cos[3hw])\ \csc\Big(\frac{hw}{2}\Big) + 16\,18691h^6w^4 + 60h^2 \quad 588 \quad 925h^4w^4$$

$$+ 8h^2 \quad 7560 + 3917h^4w^4 \quad \sin\Big(\frac{hw}{2}\Big) \quad 60480h^4w^2(3 + 6\cos[hw] + 6\cos[2hw] + 5\cos[3hw])\sec\Big(\frac{hw}{2}\Big)\sin[hw]$$

$$6h^2h^4w^4\sin\Big(\frac{h3hw}{2}\Big) \quad \frac{193}{2}h^4w^4\sin\Big(\frac{h5hw}{2}\Big) + \frac{8h}{2} \quad 5040h + 7h^5w^4 \quad \sin\Big(\frac{h3hw}{2}\Big)$$

$$+ 25200h \quad 833h^5w^4 \quad \sin\Big(\frac{hw}{5}\Big) + 2h \quad 837144h^5w^4 + 60h\,588 \quad 925h^4w^4 \quad \sin\Big(\frac{hw}{2}\Big)$$

$$+ 3h^2 84h^3w^4\sin\Big(\frac{3hw}{2}\Big) + 780h^3w^4\sin\Big(\frac{hw}{5}\Big) \quad 8h \quad h^4w^4\sin\Big(\frac{3hw}{2}\Big)$$

$$193h^4w^4\sin\Big(\frac{5hw}{2}\Big) + 2 \quad 5040h + 7h^5w^4\sin\Big(\frac{hw}{3}\Big) + \quad 25200h \quad 833h^5w^4$$

$$\sin\Big(\frac{h5hw}{2}\Big) + \frac{1}{3870720h^4w^4}\Big(_{n+4}\csc\Big(\frac{h5hw}{2}\Big) \quad 37800h^4 \quad 10h^2w^4 \quad 8w^2$$

cos[hw](1 2 cos[hw] + 2 cos[2hw] + 2 cos[3hw]) csc + 302400h^{4}w^{2} (3 + 6 cos[hw] + 6 cos[2hw] + 2 cos[3hw]) sec sin[hw]

+ 4h 5 54454h^{5}w^{4} + 12h 9240 6929h^{4}w^{4} sin + 3h^{2}5460h^{3}w^{4} sin

+ 480h^{3}w^{4} sin 8h 835h^{4}w^{4} sin + 28h^{4}w^{4} sin

+ 5 10080h + 847h^{5}w^{4} sin + 46300h 7h^{5}w^{4} sin

4 5 27325h^{6}w^{4} + 12h^{2} 9240 6929h^{4}w^{4} + 56h^{2}2700 + 851h^{4}w^{4} sin —

+ 28h^{4}w^{4} sin 4h 5 10080h + 847h^{5}w^{4} sin + 4 6300h 7h^{5}w^{4} sin +1935360h^{4}w^{4}

f$_{n+5}$ csc 7560h^{4} 10h^{2}w^{4} 8w^{2} cos[hw](2 4 cos[hw] + 4 cos[2hw] + cos[3hw]) csc

16 1884h^{6}w^{4} + 120h^{2} 210 + h^{4}w^{4} h^{2} 30240 + 373h^{4}w^{4} sin 60480h^{4}w^{2}(3 + 6 cos[hw] 2

6 cos[2hw] + cos[3hw]) sec sin[hw] 6h^{2}193h^{4}w^{4} sin h^{4}w^{4} sin

+ 8h 25200h 833h^{5}w^{4} sin + 5040h + 7h^{5}w^{4} sin + 2h 8 4976h^{5}w^{4} + 120

h 210 + h^{4}w^{4} sin + 3h^{2}780h^{3}w^{4} sin 84h^{3}w^{4} sin

8h 193h^{4}w^{4} sin h^{4}w^{4} sin + 2 25200h 833h^{5}w^{4}

sin + 5040h + 7h^{5}w^{4} sin + f$_{n+2}$ csc

37800h^{4} 10h^{2}w^{4} 8w^{2} cos[hw](1 + 2 cos[hw] 2 cos[2hw] + 4 cos[3hw]) csc + 302400h^{4}w^{2}(3 + 6 cos[hw] 2

+ 6 cos[2hw] + 4 cos[3hw]) sec sin[hw] + 4h 5 4448h^{5}w^{4} + h 131040 + 1530h^{4}w^{4} sin

+ 3h^{2} 11220h^{3}w^{4} sin + 768h^{3}w^{4} sin 8h 20035h^{4}w^{4} sin

3620h^{4}w^{4} sin + 2 55040h + 5047h^{5}w^{4} sin 4 12600h + 847h^{5}w^{4}

sin 4 53024h^{6}w^{4} h^{2} 211680 + 169h^{4}w^{4} + h 131040h + 306h^{5}w^{4}

sin + 3h^{2} 20035h^{4}w^{4} sin 3620h^{4}w^{4} sin

4h 55040h + 5047h^{5}w^{4} sin 12600h + 847h^{5}w^{4} sin —

$$y_{n+2}'' = \frac{4h^2 2h^2}{y_n} \frac{y_{n+2}}{} + \frac{4h^2}{y_{n+4}} + \frac{1935360h^4w^4}{1} f_{n+3} \quad 604800h^4w^2 \cos[hw] + 79500h^6w^4 \cos[hw]$$

$$10h9826h^5w^4 \quad 63h \quad 720 + 16h^4w^4 \quad \cos[hw] \quad 10 \qquad 4144h^6w^4 \quad 126h^2 \quad 720 + 16h^4w^4 \quad +$$

$$35h^2 4320 + 176h^4w^4 \quad \cos[hw] \quad 4h^2 \quad 10856h^4w^4 \cos[2hw] + 1761h^4w^4 \cos[3hw]$$

$$6h5360h^5w^4 \cos[2hw] + 3\ 1250h^5w^4 \quad 15h \quad 560 + 48h^4w^4 \cos[3hw] \csc \frac{hhw\ i_6}{2}$$

$$+\frac{1}{1935360h^4w^4} f_n \quad 4h145h^5w^4 \quad 6h \quad 2100 + 208h^4w^4 \quad \cos \frac{hw}{h\ 2\ i}$$

$$28094h^6w^4 \quad 12h^2 \quad 2100 + 208h^4w^4 \quad + \quad h^2 \quad 57960 + 12064h^4w^4 \cos \frac{hhw\ i}{} + 60480h^4w^2 \quad 2$$

$$\cos \frac{\pi^3 hw}{2\ i} \quad 14880h^6w^4 \cos \frac{\pi 3hw}{2} + 3h \quad 1008h^5w^4 \quad 63h \quad 160 + 16h^4w^4$$

$$\cos \frac{\pi^3 hw}{2\ i} \quad 250h^6w^4 \cos \frac{\pi 5hw}{2} \quad 3h\ 802h^5w^4 \cos \frac{\pi 5hw}{2\ i} \quad 5040h \cos \frac{\pi 7hw}{2} \quad i$$

$$\csc \frac{\pi hw}{2\ i}^5 \quad \csc[hw] + \frac{1}{3870720h^4w^4} f_{n+6} \csc \frac{\pi hw}{2\ i}^5 \quad 2520h^4 \quad 8h^2w^4 \quad 24w^2 \cos[2hw]$$

$$(1 \quad 2\cos[hw] + 2\cos[2hw]) \csc \frac{\pi hw}{2} + 8856h^6w^4 \sin \frac{\pi hw}{2} + 12h\ 910h^5w^4 \quad 45h\ 112 + 16h^4w^4$$

$$\sin \frac{h hw}{2} i \quad 536h^6w^4 \sin \frac{h 3hw}{2} i + 12h \quad 562h^5w^4\ 3h \quad 1680 + 176h^4w^4 \sin \frac{h hw}{2\ i}$$

$$+ 60480h^4w^2(1 + 2\cos[hw] + 2\cos[2hw]) \sec \frac{\pi hw}{2} \sin[2hw] + \frac{1}{1935360h^4w^4} f_{n+1} \csc \frac{\pi hw}{2\ i}^5$$

$$7560h^4 \quad 8h^2w^4 \quad 8w^2 \cos[2hw](\ 2 + 4\cos[hw] \quad 4\cos[2hw] + 5\cos[3hw]) \csc \frac{n hw}{2} + 16 \quad 69604h^6w^4$$

$$120h^2\ 588\ 925h^4w^4 + 8h^2\ 7560 + 3917h^4w^4 \sin \frac{hhw\ i}{} \quad 60480h^4w^2(3 + 6\cos[hw] + 6\cos[2hw]\ 2$$

$$+ 5\cos[3hw]) \sec \frac{\pi hw}{2} \sin[2hw] + 4h \quad 5040h \quad 34h^5w^4 \sin \frac{\pi 3hw}{2\ i} + \quad 25200h \quad 802h^5w^4 \sin \frac{\pi 5hw}{2} \quad i$$

$$+ h \quad 8 \quad 61316h^5w^4 + 60h\ 588 \quad 925h^4w^4 \sin \frac{h hw}{2\ i}\ 4h \quad 67h^4w^4 \sin \frac{\pi 3hw}{2\ i}$$

$$+ 125h^4w^4 \sin \frac{\pi 5hw}{2} + 2 \quad 5040h\ 34h^5w^4 \sin \frac{\pi 3hw}{2\ i} + \quad 25200h\ 802h^5w^4$$

$$\sin \frac{\pi hw}{2\ i} + \frac{1}{1935360h^4w^4} f_{n+5} \csc \frac{\pi hw}{2\ i}^5 \quad 7560h^4 \quad 8h^2w^4 \quad 8w^2$$

$$\cos[2hw](2\ \ 4\cos[hw] + 4\cos[2hw] + \cos[3hw]) \quad \csc \frac{h hw\ i}{2} + 16252h^6w^4 + 240h^2 \quad 210 + 16h^4w^4$$

$$h^2\ 30240 + 5968h^4w^4 \sin \frac{h hw\ i}{2} \quad 60480h^4w^2(3 + 6\cos[hw] + 6\cos[2hw] + \cos[3hw]) \sec \frac{h hw\ i}{2} \sin[2hw]$$

$$+ 4h \quad 25200h \quad 802h^5w^4 \sin \frac{h 3hw}{2\ i} + \quad 5040h \quad 34h^5w^4 \sin \frac{h 5hw}{2\ i}$$

$$+ h \quad 8 \quad 440h^5w^4 + 120h \quad 210 + 16h^4w^4 \sin \frac{\pi hw}{2} \quad i \quad 4h\ 125h^4w^4 \sin \frac{\pi 3hw}{2\ i}$$

$$67h^4w^4 \sin \frac{h}{2\ i} + 2 \quad 25200h \quad 802h^5w^4 \sin \frac{h}{2\ i} + 5040h \quad 34h^5w^4$$

$$\sin\frac{5hw}{2} + \frac{1}{3870720h^4w^4} f_{n+4} \csc\frac{hw}{2}^5 \; 37800h^4 \; 8h^2w^4 \; 8w^2 \cos[2hw](1 \; 2\cos[hw] + 2\cos[2hw]$$

$$+2\cos[3hw])) \csc\frac{hw}{2} + 302400h^4w^2(3 + 6\cos[hw] + 6\cos[2hw] + 2\cos[3hw]) \sec\frac{hw}{2} \sin[2hw]$$

$$+ 2h \; 5 \; 91796h^5w^4 + 12h \; 9240 \; 6929h^4w^4 \; \sin\frac{hw}{2} \; 4h \; 2015h^4w^4 \sin\frac{3hw}{2}$$

$$+412h^4w^4 \sin\frac{hw}{2} + 2^5 \; 10080h + 638h^5w^4 \; \sin\frac{3hw}{2} + 4 \; 6300h + 58h^5w^4 \sin\frac{5hw}{2}$$

$$4 \; 5 \; 102772h^6w^4 + 24h^2 \; 9240 \; 6929h^4w^4 + 56h^2 \; 2700 + 851h^4w^4 \; \sin\frac{hw}{2}$$

$$2h \; 5 \; 10080h + 638h^5w^4 \; \frac{hw}{} \; 6300h + 58h^5w^4 \; \frac{5hw}{}$$

$$+ \frac{1}{3870720h^4w^4} f_{n+2} \csc\frac{hw}{2}^5 \; 37800h^4 \; 8h^2w^4 \; 8w^2 \cos[2hw](\; 1 + 2\cos[hw] \; 2\cos[2hw] + 4\cos[3hw])$$

$$\csc\frac{hw}{2} + 302400h^4w^2(3 + 6\cos[hw] + 6\cos[2hw] + 4\cos[3hw]) \sec\frac{hw}{2} \sin[2hw] + 2h \; 5 \; 21592h^5w^4$$

$$+h \; 131040 + 24480h^4w^4 \; \sin\frac{hw}{2} \; 4h \; 9025h^4w^4 \sin\frac{3hw}{2} \; 1988h^4w^4 \sin\frac{5hw}{2}$$

$$+2 \; 5 \; 5040h + 7934h^5w^4 \sin\frac{3hw}{2} \; 4 \; 12600h + 1574h^5w^4 \; \sin\frac{5hw}{2}$$

$$4 \; 5 \; 8364h^6w^4 \; h^2 \; 211680 + 2704h^4w^4 + h \; 262080h + 9792h^5w^4 \; \sin\frac{hw}{2} \; 2h \; 5 \; 5040h + 7934h^5w^4$$

$$\sin\frac{3hw}{2} \; 4 \; 12600h + 1574h^5w^4 \sin\frac{5hw}{2}$$

$$y_{n+3}'' = \frac{h^2}{y_{n+2}} \quad \frac{h^2}{2y_{n+3}} + \frac{h^2}{y_{n+4}} + \frac{1}{1935360h^4w^4} f_{n+3} \; 1209600h^2 \quad 604800h^4w^2 \quad 30h \; 4647h^5w^4$$

$$63h \quad 720 + 81h^4w^4 \quad \cos[hw] \quad 10 \quad 21889h^6w^4 \quad 189h^2 \quad 6 \quad 720 + 81h^4w^4 + 35h^2 \quad 4320 + 891h^4w^4 \quad \cos[hw]$$

$$+ 2h^2 \quad 14024h^4w^4 \cos[2hw] + 2319h^4w^4 \cos[3hw] \quad \csc\left(\frac{hw}{2}\right) + \frac{1}{1935360h^4w^4} f_n \quad 60480h^4w^2$$

$$\cos\left(\frac{hw}{2}\right) + 4470h^6w^4 \cos\left(\frac{hw}{2}\right) + 8h \quad 5153h^5w^4 \quad 6h \quad 2100 + 1053h^4w^4 \cos\left(\frac{hw}{2}\right) + 2 \quad 42120h^6w^4$$

$$18h^2 \quad 2100 + 1053h^4w^4 + h^2 \quad 57960 + 61074h^4w^4 \cos\left(\frac{hw}{2}\right) \quad 5139h^6w^4 \cos\left(\frac{3hw}{2}\right)$$

$$6h \quad 4280h^5w^4 \quad 63h \; 160 + 81h^4w^4 \cos\left(\frac{3hw}{2}\right) + 11h^6w^4 \cos\left(\frac{5hw}{2}\right) \csc\left(\frac{hw}{2}\right) \csc[hw] + \frac{1}{3870720h^4w^4}$$

$$f_{n+6} \csc\left(\frac{hw}{2}\right)^5 \; 2520h^4 \quad 2h^2w^4 \quad 24w^2(1 \quad 2\cos[hw] + 2\cos[2hw])\cos[3hw] \csc\left(\frac{hw}{2}\right)$$

$$684h^6w^4 \sin\left(\frac{hw}{2}\right) + 48h \quad 3676h^5w^4 \quad 45h \; 112 + 81h^4w^4 \sin\left(\frac{hw}{2}\right) \quad 44h^6w^4 \sin\left(\frac{3hw}{2}\right)$$

$$+ 60480h^4w^2(1 + 2\cos[hw] + 2\cos[2hw]) \sec\left(\frac{\quad}{2}\right) \sin[3hw] + \frac{1}{1935360h^4w^4} f_{n+5} \csc\left(\frac{hw}{2}\right)^5 \; 7560h^4$$

$$2h^2w^4 \quad 8w^2 \cos[3hw](2 \quad 4\cos[hw] + 4\cos[2hw] + \cos[3hw]) \quad \csc\left(\frac{hw}{2}\right) + 16 \; 916h^6w^4$$

$$+ 360h^2 \quad 210 + 81h^4w^4h^2 \quad 30240 + 30213h^4w^4 \quad \sin\left(\frac{\quad}{2}\right) + 2h^2 \quad 11h^4w^4 \sin\left(\frac{3hw}{2}\right)$$

$$+ 11h^4w^4 \sin\left(\frac{5hw}{2}\right) \quad 60480h^4w^2(3 + 6\cos[hw] + 6\cos[2hw] + \cos[3hw]) \sec\left(\frac{hw}{2}\right) \sin[3hw]$$

$$+ \frac{1}{3870720h^4w^4} f_{n+4} \csc\left(\frac{hw}{2}\right)^5 \; 37800h^4 \quad 2h^2w^4 \quad 8w^2 \cos[3hw](1 \quad 2\cos[hw] + 2\cos[2hw] + 2\cos[3hw]) \csc\left(\frac{hw}{2}\right)$$

$$5\,201609h^6w^6 + 36h^2 \; 9240 \; 6929h^4w^4 + 56h^2 \; 2700 + 851h^4w^4 \sin \quad h^2 \quad 895h^4w^4 \sin$$

$$212h^4w^4 \sin\left(\frac{5}{2}\right) + 302400h^4w^2(3 + 6\cos[hw] + 6\cos[2hw] + 2\cos[3hw]) \sec\left(\frac{3hw}{2}\right) \sin[3hw]$$

$$+ \frac{1}{3870720h^4w^4} f_{n+2} \csc\left(\frac{hw}{2}\right)^5 \; 37800h^4 \quad 2h^2w^4 \quad 8w^2 \cos[3hw](\quad 1 + 2\cos[hw] \quad 2\cos[2hw] + 4\cos[3hw])$$

$$\csc\left(\frac{hw}{2}\right) \quad 5\,60848h^6w^6 \quad h^2 \; 211680 + 13689h^4w^4 + h \; 393120 + 74358h^2w^4 \sin$$

$$h^2 \quad 895h^4w^4 \sin\left(\frac{3hw}{2}\right) \quad 212h^4w^4 \sin\left(\frac{5hw}{2}\right) + 302400h^4w^2(3 + 6\cos[hw] + 6\cos[2hw] + 4\cos[3hw])$$

$$\sec\left(\frac{hw}{2}\right) \sin[3hw] + \frac{1}{1935360h^4w^4} f_{n+1} \csc\left(\frac{hw}{2}\right)^5 \; 7560h^4 \quad 2h^2w^4 \quad 8w^2 \cos[3hw](\quad 2 + 4\cos[hw] \quad 4\cos[2hw]$$

$$+5\cos[3hw])) \csc \quad 2 \quad 16\,135027h^6w^6 + 180h^2 \; 588 \; 925h^4w^4 + 8h^2 \; 7560 + 3917h^4w^4 \sin$$

$$+ 2h^2 \quad 11h^4w^4 \sin\left(\frac{3}{2}\right) + 11h^4w^4 \sin\left(\frac{5hw}{2}\right) \quad 60480h^4w^2(3 + 6\cos[hw] + 6\cos[2hw] + 5\cos[3hw])$$

$$\sec\left(\frac{hw}{2}\right) \sin[3hw]$$

$$y_n'' + 4 = \frac{y_n}{4h^2} + \frac{5y_{n+2}}{2h^2} - \frac{4y_{n+3}}{h^2} + \frac{7y_{n+4}}{4h^2} + \frac{1}{1935360h^4w^4}\Big(f_{n+3}\,2419200h^2$$

$$604800h^4w^2\cos[hw]\;263400h^6w^4\cos[hw]\;50h\,28750h^5w^4\;63h\;720 + 256h^4w^4\;\cos[hw]\;10\;34048h^6w^4$$

$$2\qquad 4\;4\qquad 2\qquad 4\;4\qquad 2\qquad 4\qquad 4\;\cos[2hw] + 1761h^4w^4\cos[3hw]$$

$$252h\;720 + 256h\;w + 35h\;4320 + 2816hw\;\cos[hw]\;4h\;10856h\;w\;hw\;6$$

$$6h\;31264h^5w^4\cos[2hw] + 3\;13468h^5w^4\;15h\;560 + 768h^4w^4\;\cos[3hw]\;\csc\frac{hw}{2}$$

$$1935360h^4w^4\qquad 60480h^4w^2\cos\qquad 2 + 29440h^3w^2\cos\qquad 2 + 12h\;21503h^3w^2\qquad 6h\;2100 + 3328h^2w^2$$

$$\cos\frac{1}{hw}\Big(2 + 2\;113152h^6w^4\;24h^2\;2100 + 3328h^4w^4 + h^2\;57960 + 193024h^4w^4\;\cos\frac{hw}{2}$$

$$+ 2484h^6w^4\cos\frac{hw}{2}\;3\;15h\;14942h^5w^4\;63h\;160 + 256h^4w^4\;\cos\frac{3hw}{2}\;5$$

$$+134h^6w^4\cos\frac{5hw}{2} + 3h\;764h^5w^4\cos\frac{5hw}{2}\;5040h\cos\frac{7hw}{2}\;\csc\frac{hw}{2}\;\csc[hw]$$

$$+ \frac{1}{3870720h^4w^4\sin\frac{hw}{2}}\Big(f_{n+6}\csc\frac{hw}{2}\;5\;2520h^4\;28h^2w^4\;24w^2(1\;2\cos[hw] + 2\cos[2hw])\cos[4hw]\;\csc\frac{hw}{2}$$

$$32712h^3w^3\sin\qquad 2 + 84h\qquad 11520h^3w^3\qquad 45h\;112 + 256h^3w^3\qquad \sin\qquad 2 + 1000h^3w^3$$

$$\sin\frac{hw}{2}\;3\;2\;12h\;8444h^5w^4\;3h\;1680 + 2816h^4w^4\;\sin\frac{3hw}{2} + 60480h^4w^2(1 + 2\cos[hw] + 2\cos[2hw])\sec\frac{hw}{2}\;\sin[4hw] +$$

$$\frac{1}{1935360h^4w^4}\Big(f_{n+5}\csc\frac{hw}{2}\;5\;7560h^4\;28h^2w^4\;8w^2(2\;4\cos[hw] + 4\cos[2hw] + \cos[3hw])\cos[4hw]\;\csc\frac{hw}{2}$$

$$+ 16\qquad 23928h^3w^3 + 480h^2\qquad 210 + 256h^3w^3\qquad h^2\qquad 30240 + 95488h^3w^3\qquad \sin\qquad 2\qquad 4h\qquad 25200h\;764h^3w^3$$

$$\sin\frac{hw}{2}\;3\;2 + 5040h + 4h^5w^4\;\sin\frac{5hw}{2}\;h\;8\;23012h^5w^4 + 12h\;210 + 256h^4w^4\;\sin\frac{hw}{2}$$

$$+ 4h\;67h^4w^4\;\sin\frac{hw}{2}\;3\;2 + 125h^4w^4\sin\frac{5hw}{2} + 2\;25200h\;764h^5w^4\;\sin\frac{3hw}{2}$$

$$+ 5040h + 4h^5w^4\;\sin\frac{hw}{2}\;5\;2\;60480h^4w^2(3 + 6\cos[hw] + 6\cos[2hw] + \cos[3hw])\sec\frac{hw}{2}\;\sin[4hw] +$$

$$\frac{4\;4\;f_{n+4}\csc\frac{hw}{2}\;5\;37800h^4\;28h^2w^4\;8w^2(1\;2\cos[hw] + 2\cos[2hw] + 2\cos[3hw])\cos[4hw]\csc\frac{hw}{2}}{3870720h\,w22}$$

$$2h\qquad 5\;98824h^3w^3 + 12h\;9240\;6929h^3w^3\;\sin\qquad 2 + 4h\qquad 9025h^3w^3\sin\qquad 2$$

$$1988h^4w^4 \sin\left(\frac{5hw}{2}\right) + 2\ 5\ 10080h + 1348h^5w^4 \sin\left(\frac{3hw}{2}\right) + 46300h\ 148h^5w^4$$

$$\sin\left(h\frac{5hw}{2}\right)\ 4\ 5\ 302224h^6w^4 + 48h^2\ 9240\ 6929h^4w^4 + 56h^2 2700 + 851h^4w^4 \sin\left(h\frac{hw}{2}\right)$$

$$+ 2h\ 5\ 10080h + 1348h^5w^4 \sin\left(\pi\frac{hw}{3}\right) + 46300h\ 148h^5w^4 \sin\left(\pi\frac{5hw}{2}\right)$$

$$+ 302400h^4w^2(3 + 6\cos[hw] + 6\cos[2hw] + 2\cos[3hw]) \sec\left(\pi\frac{hw}{2}\right) \sin[4hw] + \frac{1}{3870720h^4w^4} f_{n+2} \csc\left(h\frac{hw}{2}\right)^5$$

$$37800h^4\ 28h^2w^4\ 8w^2(1 + 2\cos[hw]\ 2\cos[2hw] + 4\cos[3hw])\cos[4hw]\ \csc\left(\frac{hhw}{2}\right)$$

$$2h\ 5\ 381764h^5w^4 + h\ 131040 + 391680h^4w^4$$

$$\sin\left(\pi\frac{hw}{2}\right) + 4h\ 2015h^4w^4 \sin\left(h\frac{3hw}{2}\right) + 412h^4w^4 \sin\left(\pi\frac{5hw}{2}\right) + 2\ 55040h + 8644h^5w^4$$

$$\sin\left(\pi\frac{3hw}{2}\right)\ 4\ 12600h + 1780h^5w^4 \sin\left(\pi\frac{5hw}{2}\right)\ 4\ 5267384h^6w^4\ h^2\ 211680 + 43264h^4w^4$$

$$+ h\ 524160h + 313344h^5w^4 \sin\left(\pi\frac{hw}{2}\right) + 2h\ 5\ 5040h + 8644h^5w^4 \sin\left(\pi\frac{3hw}{2}\right)$$

$$4\ 12600h + 1780h^5w^4 \sin\left(h\frac{5hw}{2}\right) + 302400h^4w^2(3 + 6\cos[hw] + 6\cos[2hw] + 4\cos[3hw]) \sec\left(h\frac{hw}{2}\right) \sin[4hw] +$$

$$\frac{1}{1935360h^4w^4} f_{n+1} \csc\left(\pi\frac{hw}{2}\right)^5 7560h^4\ 28h^2w^4\ 8w^2(2 + 4\cos[hw]\ 4\cos[2hw] + 5\cos[3hw])\cos[4hw] \csc\left(\pi\frac{hw}{2}\right)$$

$$16\ 201808h^6w^4 + 240h^2\ 588\ 925h^4w^4 +$$

$$8h^2 7560 + 3917h^4w^4 \sin\left(\frac{hhw}{2}\right)\ 4h\ 5040h +$$

$$4h^5w^4\ 2$$

$$\sin\left(h\frac{hw}{3}\right) + 25200h\ 764h^5w^4 \sin\left(h\frac{5hw}{2}\right)\ h\ 8\ 66664h^5w^4 + 60h\ 588\ 925h^4w^4$$

$$\sin\left(\pi\frac{hw}{2}\right) + 4h\ 125h^4w^4 \sin\left(\pi\frac{hw}{3}\right)\ 67h^4w^4 \sin\left(\pi\frac{5hw}{2}\right) + 2\ 5040h + 4h^5w^4$$

$$\sin\left(h\frac{hw}{3}\right) + 25200h\ 764h^5w^4 \sin\left(h\frac{5hw}{2}\right)\ 60480h^4w^2(3 + 6\cos[hw] + 6\cos[2hw]$$

$$+ 5\cos[3hw]) \sec\left(h\frac{hw}{2}\right) \sin[4hw]$$

$$y_n^{00} + 5 = \frac{y_n}{2h^2} + \frac{4y_{n+2}}{h^2} - \frac{6y_{n+3}}{h^2} + \frac{5y_{n+4}}{2h^2} + \frac{1}{1935360h^4w^4} f_{n+3} \; 3628800h^2 \; 523500h^6w^4$$

$$\cos[hw] \; 70h \; 64055h^5w^4 \; 63h \; 720 + 625h^4w^4 \cos[hw] \; 10 \; 24871h^6w^4 \; 315h^2 \; 720 + 625h^4w^4 + 35h^2 \; 4320 +$$

$$6875h^4w^4 \cos[hw] \; 604800h^4w^2 \cos[2hw] \; 6h^3 \; 9312h^3w^4 \cos[2hw] \; 972h^3w^4 \cos[3hw]$$

$$22h^2 \; 2792h^4w^4 \cos[2hw] + 627h^4w^4 \cos[3hw] \; 12h \; 38360h^5w^4 \cos[2hw] + 3 \; 30470h^5w^4 \; 15h \; 560 + 1875h^4w^4$$

$$\frac{hw}{6h} \cos[3hw])) \csc^6 \frac{2}{2100 + 8125h^2w^2 \cos} + \frac{1}{1935360h^4w^4} f_n \; 59790h^6w^4 \cos \frac{hw}{2} + 16h \; 54293h^5w^4$$

$$\frac{hw}{\cos} \; 2 + 60480h^4w^2 \cos \frac{2}{3hw} + 13785h^6w^4 \cos \frac{2}{3hw} \; 24h \; 38010h^5w^4$$

$$63h \; 160 + 625h^4w^4 \cos \frac{3 \; 2}{hw} \; 241h^6w^4 \cos \frac{2}{5hw} + 6h \; 805h^5w^4 \cos \frac{2}{5hw}$$

$$5040h \cos \frac{2}{2520h^2 - 82h^2w^2 - 24w^2(1 - 2\cos[hw] + 2\cos[2hw])\cos[5hw]} \csc \frac{2}{hw} \csc[hw] + \frac{7hw}{3870720h^4w^4} f_{n+6} \csc^5 \frac{2}{hw} \; 1 \; 236412h^6w^4 \sin \frac{2}{hw} \; hw^5$$

$$+ 120h \; 27838h^5w^4 \; 45h \; 112 + 625h^4w^4 \sin \frac{2}{hw} \; 17852h^6w^4 \sin \frac{3 \; 2}{hw} \; 24h \; 20590h^5w^4 \; 3h$$

$$1680 + 6875h^4w^4 \sin^3 \frac{hw}{2} + 60480h^4w^2(1 + 2\cos[hw] + 2\cos[2hw]) \sec \frac{hw}{2} \sin[5hw]$$

$$+ \frac{1}{1935360h^4w^4} f_{n+5} \csc \frac{2}{hw} \; 7560h^4 \; 82h^2w^4 \; 8w^2(2 \; 4\cos[hw] + 4\cos[2hw] + \cos[3hw])\cos[5hw]$$

$$\csc \frac{2}{hw} + 16 \; 120300h^2w^2 + 600h^2 \; 210 + 625h^2w^2 \; h^2 \; 30240 + 233125h^2w^2 \sin \frac{2}{hw}$$

$$6h^2 \; h^4w^4 \sin^5 \frac{2}{hw} \; 193h^4w^4 \sin \frac{2}{5hw} \; 8h \; 25200h \; 805h^5w^4 \sin \frac{2}{3hw}$$

$+ 5040h + 35h^5w^4 \sin \frac{\frac{hw}{2}}{} \quad 2h \quad 8 \quad 43120h^5w^4 + 120h\ 210 + 625h^4w^4$

$\sin \frac{2}{hw} + 3h^2 \qquad 84h^3w^4 \sin \frac{2}{3hw} \qquad 780h^3w^3 \sin \frac{2}{5hw} + 8h \quad h^4w^4 \sin$

$\frac{2}{3hw} \quad 193h^4w^4 \sin \frac{2}{hw} + 2 \qquad 25200h \quad 805h^5w^4 \sin \frac{2}{3hw}$

$+ 5040h + 35h^5w^4 \quad \sin 5 \frac{2}{hw} \quad 60480h^4w^2(3 + 6\cos[hw] + 6\cos[2hw] + \cos[3hw]) \sec \frac{2}{hw}$

$\sin[5hw]) + \frac{1}{3870720h^4w^4} f_{n+4} \csc \frac{\frac{hw}{2}}{} \quad 37800h^4 \quad 82h^2w^4 \quad 8w^2(1 \quad 2\cos[hw] + 2\cos[2hw] + 2\cos[3hw]) \cos[5hw]$

$\csc \frac{2}{hw} 4h \qquad 5\ 104630h^3w^3 + 12h\ 9240\ 6929h^3w^3 \sin \qquad \frac{2}{hw} +$

$3h^2 \quad 11220h^3w^4 \sin 3 \frac{2}{hw} \quad 768h^3w^4 \sin \frac{2}{5hw} + 8h \qquad 20035h^4w^4 \sin \frac{2}{3hw}$

$3620h^4w^4 \sin \frac{5\ 2}{hw} + 2 \quad 5 \quad 10080h + 4235h^5w^4 \sin \frac{2}{3hw} + 4 \quad 6300h \quad 875h^5w^4$

$$\sin\left(h\frac{5hw}{2}\right) \quad 4 \quad 5\ 402325h^6w^4 + 60h^2\ 9240\ 6929h^4w^4 + 56h^2\ 2700 + 851h^4w^4 \sin\left(h\frac{hw}{2}\right)$$

$$+ 3h^2\ 20035h^4w^4 \sin\left(\frac{\pi}{2}\right) \quad 3620h^4w^4 \sin\left(\frac{\pi\, 5hw}{2}\right) + 4h\ 5\ 10080h + 4235h^5w^4 \sin\left(\frac{\pi\, 3hw}{2}\right)$$

$$+ 4\ 6300h \quad 875h^5w^4 \sin\left(\frac{\pi\, 5hw}{2}\right) + 302400h^4w^2(3 + 6\cos[hw] + 6\cos[2hw] + 2\cos[3hw]) \sec\left(\frac{\pi\, hw}{2}\right)$$

$$\sin[5hw] + \frac{1}{3870720h^4w^4} f_{n+2} \csc\left(\frac{\pi\, hw}{2}\right) \quad 5\ 37800h^4\ 82h^2w^4\ 8w^2(\ 1 + 2\cos[hw]\ 2\cos[2hw]$$

$$+ 4\cos[3hw])\cos[5hw]\ \csc\left(\frac{\pi\, hw}{2}\right) \quad 4h\ 5\ 908992h^5w^4 + h\ 131040 + 956250h^4w^4 \sin\left(\frac{\pi\, hw}{2}\right) +$$

$$3h^2\ 5460h^3w^4 \sin\left(\frac{\pi\, 3hw}{2}\right) \quad 480h^3w^4 \sin\left(\frac{\pi\, 5hw}{2}\right) + 8h\ 835h^4w^4 \sin\left(\frac{\pi\, 3hw}{2}\right)$$

$$+ 28h^4w^4 \sin\left(\frac{\pi\, 5hw}{2}\right) + 2\ 5\ 5040h + 8435h^5w^4 \sin\left(\frac{\pi\, 3hw}{2}\right) \quad 4 \quad 12600h + 1715h^5w^4$$

$$\sin\left(h\frac{5hw}{2}\right) \quad 4 \quad 5\ 821664h^6w^4\ h^2 \quad 211680 + 105625h^4w^4 + h\ 655200h + 956250h^5w^4$$

$$\sin\left(\frac{\pi\, hw}{2}\right) + 3h^2\ 835h^4w^4 \sin\left(\frac{\pi\, 3hw}{2}\right) + 28h^4w^4 \sin\left(\frac{\pi\, 5hw}{2}\right) + 4h\ 5\ 5040h + 8435h^5w^4 \sin\left(\frac{\pi\, 3hw}{2}\right)$$

$$4 \quad 12600h + 1715h^5w^4 \sin\left(\frac{\pi\, 5hw}{2}\right) + 302400h^4w^2(3 + 6\cos[hw] + 6\cos[2hw] + 4\cos[3hw]) \sec\left(\frac{\pi\, hw}{2}\right) \sin[5hw] + \frac{1}{1935360h^4w^4}$$

$$f_{n+1} \csc\left(h\frac{hw}{2}\right)^5 \quad 7560h^4\ 82h^2w^4\ 8w^2(\ 2 + 4\cos[hw]\ 4\cos[2hw] + 5\cos[3hw])\cos[5hw] \csc\left(h\frac{hw}{2}\right)$$

$$+ 16\ 271075h^6w^4 + 300h^2\ 588\ 925h^4w^4 + 8h^2\ 7560 + 3917h^4w^4 \sin\left(\frac{\pi}{2}\right) \quad 6h^2\ 193h^4w^4 \sin\left(\frac{\pi\, 3hw}{2}\right)$$

$$h^4w^4 \sin\left(\frac{\pi\, 5hw}{2}\right) \quad 8h \quad 5040h + 35h^5w^4 \sin\left(\frac{\pi\, 3hw}{2}\right) + 25200h\ 805h^5w^4$$

$$\sin\left(\frac{\pi\, 5hw}{2}\right) \quad 2h\ 8\ 73880h^5w^4 + 60h\ 588\ 925h^4w^4 \sin\left(h\frac{hw}{2}\right)$$

$$+ 3h^2 \quad 780h^3w^4 \sin\left(\frac{\pi\, 3hw}{2}\right) + 84h^3w^4 \sin\left(\frac{\pi\, 5hw}{2}\right) + 8h \quad 193h^4w^4 \sin\left(\frac{\pi\, 3hw}{2}\right)$$

$$h^4w^4 \sin\left(\frac{\pi\, 5hw}{2}\right) + 2 \quad 5040h + 35h^5w^4 \sin\left(\frac{\pi\, 5hw}{2}\right) + 25200h\ 805h^5w^4$$

$$\sin\left(\frac{h\, hw}{2}\right) \quad 60480h^4w^2(3 + 6\cos[hw] + 6\cos[2hw] + 5\cos[3hw]) \sec\left(\frac{h\, hw}{2}\right) \sin[5hw]$$

$$y_{n+6}'' = \frac{3y_n}{4h^2} + \frac{11y_{n+2}}{2h^2} \quad \frac{8y_{n+3}}{h^2} + \frac{13y_{n+4}}{4h^2} + \frac{1}{1935360h^4w^4} f_{n+3} \quad 4838400h^2 \quad 575100h^6w^4 \cos[hw]$$

$$90h \quad 114726h^5w^4 \quad 63h \quad 720 + 1296h^4w^4 \quad \cos[hw] \quad 10 \quad 39032h^6w^4 \quad 378h^2 \quad 720 + 1296h^4w^4$$

$$+ 35h^2 \; 4320 + 14256h^4w^4 \quad \cos[hw] \quad 604800h^4w^2 \cos[3hw] \quad 24h^3 \quad 6768h^3w^4 \cos[2hw] + 1242h^3w^4 \cos[3hw]$$

$$52h \quad 5848h^2w^4 \quad 6 \quad \cos[2hw] + 537h^4w^4 \cos[3hw] \quad 18h \quad 36624h^5w^4 \cos[2hw] + 3 \quad 60798h^5w^4 \quad 15h \; 560 + 3888h^4w^4$$

$$\cos[3hw] \quad \csc\left(\frac{hw}{2}\right) + \frac{1}{1935360h^4w^4} f_n \; 66432h^6w^4 \cos\left(\frac{hw}{2}\right) + 20h \quad 110063h^5w^4 \quad 6h \; 2100 + 16848h^4w^4 \quad \cos\left(\frac{hw}{2}\right)$$

$$+ 2 \; 359778h^6w^4 \quad 36h^2 \quad 2100 + 16848h^4w^4 + h^2 \quad 57960 + 977184h^4w^4 \quad \cos\left(\frac{hw}{2}\right) + 67608h^6w^4 \cos\left(\frac{3hw}{2}\right)$$

$$33h \; 79940h^5w^4 \quad 63h \; 160 + 1296h^4w^4 \quad \cos(h3hw) + \frac{60480h^4w^2}{2} \cos(h5hw) + \frac{4358h^6w^4}{2} \cos(h5hw) \quad \frac{}{2}$$

$$+ 9h \quad 798h^5w^4 \cos\left(\frac{hw}{2}\right) \quad 5040h \cos\left(\frac{7hw}{2}\right) \csc\left(\frac{hw}{2}\right) \csc[hw] + \frac{1}{3870720h^4w^4} f_{n+6}$$

$$\csc\left(\frac{hw}{2}\right) \; 2520h^4 \quad 160h^2w^4 \quad 24w^2(1 \quad 2\cos[hw] + 2\cos[2hw])\cos[6hw] \quad \csc\left(\frac{hw}{2}\right) \quad 217512h^6w^4$$

$$\sin\left(\frac{hw}{2}\right) + 156h \; 57466h^5w^4 \quad 45h \; 112 + 1296h^4w^4 \quad \sin(h3hw) \quad \left(\frac{hw}{2}\right) \quad 307736h^6w^4 \sin\left(\frac{3hw}{2}\right)$$

$$36h \; 43566h^5w^4 \quad 3h \; 1680 + 14256h^4w^4 \quad \sin(h3hw) + \frac{60480h^4w^2}{2}(1 + 2\cos[hw] + 2\cos[2hw]) \sec \quad \frac{}{2}$$

$$\sin[6hw] \quad + \frac{1}{1935360h^4w^4} f_{n+5} \csc\left(\frac{hw}{2}\right) \quad 7560h^4 \quad 160h^2w^4 \quad 8w^2(2 \quad 4\cos[hw] + 4\cos[2hw] + \cos[3hw])\cos[6hw]$$

$$\csc\left(\frac{hw}{2}\right) + 16 \; 377804h^6w^4 + 720h^2 \quad 210 + 1296h^4w^4 \quad h^2 \; 30240 + 483408h^4w^4 \quad \sin\left(\frac{hw}{2}\right)$$

$$16h^2 \; 43h^4w^4 \quad \sin(h3hw) \quad \frac{1771h^4w^4}{2} \sin(h5hw) \quad \frac{12h}{2} \quad 25200h \quad 798h^5w^4 \quad \sin(h3hw) \quad \frac{}{2}$$

$$+ \quad 5040h \quad 798h^5w^4 \quad \sin\left(\frac{5hw}{2}\right) \quad 3h \quad 8 \quad 86496h^5w^4 + 120h \quad 210 + 1296h^4w^4 \quad \sin\left(\frac{hw}{2}\right)$$

$$+ 8h^2 \quad 270h^3w^4 \sin\left(\frac{3hw}{2}\right) \quad 2574h^3w^4 \sin\left(\frac{5hw}{2}\right) + 12h \quad 43h^4w^4 \sin\left(\frac{3hw}{2}\right)$$

$$1771h^4w^4 \sin\left(\frac{5hw}{2}\right) + 2 \quad 25200h \quad 798h^5w^4 \quad \sin\left(\frac{hw}{2}\right) + \quad 5040h \quad 798h^5w^4$$

sin 5 2 60480h^{4}w^2(3 + 6 cos[hw] + 6 cos[2hw] + cos[3hw]) sec 2 sin[6hw] + 3870720h^{4}w^4 f_{n+4}
hw hw 1

csc 2 37800h^4 160h^{2}w^4 8w^2(1 2 cos[hw] + 2 cos[2hw] + 2 cos[3hw]) cos[6hw]csc 2
hw 5hw

6h 5 135324h^{5}w^4 + 12h 9240 6929h^{4}w^4 sin 2 + 8h^2 7290h^{3}w^4 sin 2
hw 3hw

+3960h^{3}w^4 sin 5 2 + 12h 29785h^{4}w^4 sin 2 2372h^{4}w^4 sin 2 + 2 5 10080h + 9282h^{5}w^4
hw 3hw 5hw

sin 3 2 + 4 6300h 1722h^{5}w^4 sin 2 4 5 520020h^{6}w^4 + 72h^2 9240 6929h^{4}w^4
hw 5hw

+56h^2 2700 + 851h^{4}w^4 sin 2 + 8h^2 29785h^{4}w^4 sin 2 2372h^{4}w^4 sin 2
hw 3hw 5hw

+ 6h 5 10080h + 9282h^{5}w^4 sin 5 2 + 4 6300h 1722h^{5}w^4 sin 5 2 + 302400h^{4}w^2(3 + 6 cos[hw]+
hw 5hw

hw

6 cos[2hw] + 2 cos[3hw]) sec 2 sin[6hw] + 1 hw f_{n+2} csc 2
37800h^4 160h^{2}w^4 8w^2(1 + 2 cos[hw] 2 cos[2hw] + 4 cos[3hw]) cos[6hw] csc 3870720h^{4}w^4
hw

2 6h 5 1881168h^{5}w^4
+h 131040 + 1982880h^{4}w^4 sin 2 + 8h^2 8730h^{3}w^4 sin 2 + 1368h^{3}w^4 sin 2
hw 3hw 5hw

+ 12h 8185h^{4}w^4 sin 2 + 220h^{4}w^4 sin 2 + 2 5 5040h + 9282h^{5}w^4 sin 3 2
3hw 5hw hw

$$4\ 12600h + 1722h^5w^4 \sin \frac{hw}{2} \quad 4\ 5\ \ 2057084h^6w^4\ \ h^2\ \ 211680 + 219024h^4w^4$$

$$+h\ 786240h + 2379456h^{\cdot}w^{\cdot} \sin \qquad 8185h^4w^4 \sin \frac{2}{3hw} + 220h^{\cdot}w^{\cdot} \sin \frac{2}{5hw} + 8h^2$$

$$+ 6h\ \ 5\ \ 5040h + 9282h^5w^4 \sin \frac{3\ 2}{hw}\ 4\ 12600h + 1722h^5w^4 \sin \frac{2}{5hw}$$

$$+302400h^4w^2(3 + 6\cos[hw] + 6\cos[2hw] + 4\cos[3hw]) \sec \frac{hw}{2} \sin[6hw] + \frac{1}{1935360h^{\cdot}w^{\cdot}} f_{n+1} \csc \frac{hw}{2}\ {}^{5}$$

$$7560h^{\cdot} \quad 160h^{\cdot}w^{\cdot} \quad 8w^{\cdot}(\ 2 + 4\cos[hw]\ \ 4\cos[2hw] + 5\cos[3hw])\cos[6hw]\csc \qquad 352836h^{\cdot}w^{\cdot}$$

$$+ 360h^2\ \ 588\ \ 925h^4w^4 \quad +8h^2\ \ 7560 + 3917h^4w^4 \sin \frac{2}{hw}\ \ 16h^2\ \ 1771h^4w^4 \sin \frac{2}{3hw}\ \ 43h^4w^4 \sin \frac{2}{5hw}$$

$$12h\ \ 5040h\ \ 798h^5w^4 \sin \frac{3\ 2}{hw} + 25200h\ \ 798h^5w^4 \sin \frac{2}{5hw}\ 3h\ \ 8\ 89964h^5w^4$$

$$+60h\ 588\ \ 925h^4w^4 \sin \frac{2}{hw} + 8h^2 2574h^3w^4 \sin \frac{2}{3hw}\ 270h^3w^4 \sin \frac{2}{5hw}$$

$$+ 12h1771h^4w^4 \sin \frac{3}{2hw}\ 43h^4w^4 \sin \frac{2}{5hw} + 2\ \ 5040h\ \ 798h^5w^4 \sin \frac{2}{3hw}$$

$$+ 25200h\ \ 798h^5w^4 \sin \frac{5hw}{2}\ \ 60480h^4w^2(3 + 6\cos[hw] + 6\cos[2hw]$$

$$+5\cos[3hw]) \sec \frac{hw}{2} \sin[6hw]$$

$$y^{000}_n = \frac{y_n}{4h^3} + \frac{3y_{n+2}}{2h^3} - \frac{2y_{n+3}}{h^3} + \frac{3y_{n+4}}{4h^3} + \frac{1}{3870720h^4w^4} f_{n+6} \csc\left[\frac{hw}{2}\right]^5$$

$$\Big(136080h^5w^4 \csc\left[\frac{hw}{2}\right] + 60480h^4w^3(1 + 2\cos[hw] + 2\cos[2hw]) \sec\left[\frac{hw}{2}\right] + 230220h^5w^4 \sin\left[\frac{hw}{2}\right] + 36\Big(5040h + 2068h^5w^4\Big)$$

$$\sin\left[\frac{hw}{2}\right] + 60480h \sin\left[\frac{3hw}{2}\right] + 79884h^5w^4 \sin\left[\frac{3hw}{2}\right]\Big) + \frac{1}{3870720h^4w^4} f_{n+2} \csc\left[\frac{hw}{2}\right]^5$$

$$\Big(2041200h^5w^4 \csc\left[\frac{hw}{2}\right] + 302400h^4w^3(3 + 6\cos[hw] + 6\cos[2hw] + 4\cos[3hw]) \sec\left[\frac{hw}{2}\right] + 6h\Big(218970h^4w^4 \sin\left[\frac{hw}{2}\right]$$

$$+ 8h^2\Big(7050h^2w^4 \sin\left[\frac{3hw}{2}\right] + 6936h^2w^4 \sin\left[\frac{5hw}{2}\right]\Big) - 18h\Big(7290h^3w^4 \sin\left[\frac{3hw}{2}\right] - 3960h^3w^4 \sin\left[\frac{5hw}{2}\right]\Big)$$

$$+ 6\Big(29785h^4w^4 \sin\left[\frac{3hw}{2}\right] - 2372h^4w^4 \sin\left[\frac{5hw}{2}\right]\Big)\Big)\Big) - 6\Big(5\Big(131040h - 33612h^5w^4\Big) \sin\left[\frac{hw}{2}\right] + 2\Big(25200h \sin\left[\frac{3hw}{2}\right]$$

$$- 50400h \sin\left[\frac{5hw}{2}\right]\Big) + 8h^2\Big(7290h^3w^4 \sin\left[\frac{3hw}{2}\right] - 3960h^3w^4 \sin\left[\frac{5hw}{2}\right]\Big) - 12h\Big(29785h^4w^4 \sin\left[\frac{3hw}{2}\right]$$

$$- 2372h^4w^4 \sin\left[\frac{5hw}{2}\right]\Big)\Big)\Big) + \frac{1}{1935360h^4w^4} f_{n+1} \csc\left[\frac{hw}{2}\right]^5 \Big(408240h^5w^4 \csc\left[\frac{hw}{2}\right] - 60480h^4w^3$$

$$(3 + 6\cos[hw] + 6\cos[2hw] + 5\cos[3hw]) \sec\left[\frac{hw}{2}\right] + 1440h\Big(588925h^4w^4\Big) \sin\left[\frac{hw}{2}\right] - 6\Big(5040h \sin\left[\frac{3hw}{2}\right]$$

$$+ 25200h \sin\left[\frac{5hw}{2}\right]\Big) - 24h^2\Big(270h^3w^4 \sin\left[\frac{3hw}{2}\right] + 2574h^3w^4 \sin\left[\frac{5hw}{2}\right]\Big) + 36h\Big(43h^4w^4 \sin\left[\frac{3hw}{2}\right]$$

$$- 1771h^4w^4 \sin\left[\frac{5hw}{2}\right]\Big) + 3h\Big(276048h^4w^4 \sin\left[\frac{hw}{2}\right] + 8h^2\Big(690h^2w^4 \sin\left[\frac{3hw}{2}\right] - 2418h^2w^4 \sin\left[\frac{5hw}{2}\right]\Big)$$

$$- 18h\Big(270h^3w^4 \sin\left[\frac{3hw}{2}\right] + 2574h^3w^4 \sin\left[\frac{5hw}{2}\right]\Big) + 6\Big(43h^4w^4 \sin\left[\frac{3hw}{2}\right] - 1771h^4w^4 \sin\left[\frac{5hw}{2}\right]\Big)\Big)\Big)$$

$$+ \frac{1}{1935360h^4w^4} f_{n+5} \csc\left[\frac{hw}{2}\right]^5 \Big(408240h^5w^4 \csc\left[\frac{hw}{2}\right] - 60480h^4w^3(3 + 6\cos[hw] + 6\cos[2hw] + \cos[3hw])$$

$$\sec\left[\frac{hw}{2}\right] + 24\Big(25200h - 20940h^5w^4\Big) \sin\left[\frac{hw}{2}\right] - 6\Big(25200h \sin\left[\frac{3hw}{2}\right] + 5040h \sin\left[\frac{5hw}{2}\right]\Big) - 24h^2\Big(2574h^3w^4$$

$$\sin\left[\frac{3hw}{2}\right] + 270h^3w^4 \sin\left[\frac{5hw}{2}\right]\Big) + 36h\Big(1771h^4w^4 \sin\left[\frac{3hw}{2}\right] - 43h^4w^4 \sin\left[\frac{5hw}{2}\right]\Big) + 3h\Big(117072h^4w^4$$

$$\sin\left[\frac{hw}{2}\right] + 8h^2\Big(2418h^2w^4 \sin\left[\frac{3hw}{2}\right] - 690h^2w^4 \sin\left[\frac{5hw}{2}\right]\Big) - 18h\Big(2574h^3w^4 \sin\left[\frac{3hw}{2}\right] + 270h^3w^4$$

$$\sin\left[\frac{5hw}{2}\right]\Big) + 6\Big(1771h^4w^4 \sin\left[\frac{3hw}{2}\right] - 43h^4w^4 \sin\left[\frac{5hw}{2}\right]\Big)\Big)\Big) + \frac{1}{3870720h^4w^4} f_{n+4} \csc\left[\frac{hw}{2}\right]^5$$

$$\Big(2041200h^5w^4 \csc\left[\frac{hw}{2}\right] + 302400h^4w^3(3 + 6\cos[hw] + 6\cos[2hw] + 2\cos[3hw]) \sec\left[\frac{hw}{2}\right] + 6h\Big(253530h^4w^4 \sin\left[\frac{hw}{2}\right]$$

$$+ 8h^2\Big(1590h^2w^4 \sin\left[\frac{3hw}{2}\right] + 3480h^2w^4 \sin\left[\frac{5hw}{2}\right]\Big) - 18h\Big(8730h^3w^4 \sin\left[\frac{3hw}{2}\right] - 1368h^3w^4 \sin\left[\frac{5hw}{2}\right]\Big)$$

$$+ 6\Big(8185h^4w^4 \sin\left[\frac{3hw}{2}\right] + 220h^4w^4 \sin\left[\frac{5hw}{2}\right]\Big)\Big) - 6\Big(60h\Big(9240 - 6929h^4w^4\Big) \sin\left[\frac{hw}{2}\right] + 2\Big(50400h \sin\left[\frac{3hw}{2}\right]$$

$$- 25200h \sin\left[\frac{5hw}{2}\right]\Big) + 8h^2\Big(8730h^3w^4 \sin\left[\frac{3hw}{2}\right] - 1368h^3w^4 \sin\left[\frac{5hw}{2}\right]\Big) - 12h\Big(8185h^4w^4 \sin\left[\frac{3hw}{2}\right]$$

$$+ 220h^4w^4 \sin\left[\frac{5hw}{2}\right]\Big)\Big)\Big) + \frac{1}{1935360h^4w^4} f_{n+3} \csc\left[\frac{hw}{2}\right]^6 \Big(1209600h - 28350h^5w^4 \cos[hw] - 30\Big(45360h$$

$$- 16170h^5w^4\Big) \cos[hw] + 151200h \cos[3hw] + 24h^3\Big(6384h^2w^4 \cos[2hw] + 3654h^2w^4 \cos[3hw]\Big)$$

$$- 78h^2\Big(6768h^3w^4 \cos[2hw] - 1242h^3w^4 \cos[3hw]\Big) + 54h\Big(5848h^4w^4 \cos[2hw] + 537h^4w^4 \cos[3hw]\Big) - 604800h^4w^3 \sin[3hw]\Big)$$

$$+ \frac{1}{1935360h^4w^4} f_n \csc\left[\frac{hw}{2}\right]^5 \csc[hw]\Big(74496h^5w^4 \cos\left[\frac{hw}{2}\right] + 6\Big(12600h - 6337h^5w^4\Big) \cos\left[\frac{3hw}{2}\right] + 168147h^5w^4 \cos\left[\frac{5hw}{2}\right]$$

$$- 9\Big(10080h + 1610h^5w^4\Big) \cos\left[\frac{7hw}{2}\right] - \frac{177219h^5w^4}{2} \cos\left[\frac{5hw}{2}\right] - \frac{15120h}{2} \cos\left[\frac{7hw}{2}\right] + 60480h^4w^3 \sin\left[\frac{7hw}{2}\right]\Big)$$

$$y_n^{000} + 1 = \frac{y_n}{4h^3} + \frac{3y_{n+2}}{2h^3} \quad \frac{2y_{n+3}}{h^3} + \frac{3y_{n+4}}{4h^3} + \frac{1}{3870720h^7w^7} f_{n+6} \csc^5\frac{hw}{2} + 63000h^5w^4 \sin^2\frac{hw}{2} + 36\ 906h^5w^4 \quad 45h\ 112 + h^4w^4$$

$$60480h^7w^7 \cos[hw](1 + 2\cos[hw] + 2\cos[2hw]) \sec^2\frac{hw}{2}$$

$$\sin\frac{hw}{2}\ 2520h^4 \csc\frac{hw}{2} \quad 30hw^4 + 24w^3(1 \quad 2\cos[hw] + 2\cos[2hw]) \sin[hw] \quad 4536h^5w^4 \sin^6\frac{3hw}{2}$$

$$12 \quad 26h^5w^4 \quad 3h\ 1680 + 11h^4w^4 \quad \sin^3\frac{}{2} + \frac{}{1935360h^4w^4} f_{n+3} \csc\frac{}{2}$$

$$1209600h + 84750h^5w^4 \cos[hw] \quad 30 \quad 15809h^5w^4 \quad 63h \quad 720 + h^4w^4 \quad \cos[hw] + 6h^3 \quad 816h^2w^4 \cos[2hw] + 954h^2w^4 \cos[3hw]$$

$$33h^2 \quad 9312h^3w^4 \cos[2hw] + 972h^3w^4 \cos[3hw] + 36h \quad 2792h^4w^4 \cos[2hw] + 627h^4w^4 \cos[3hw] \quad 6\ 1736h^5w^4 \cos[2hw]$$

$$+3\ 178h^5w^4 \quad 15h\ 560 + 3h^4w^4 \quad \cos[3hw] \quad 604800h^4w^3 \sin[2hw] + \frac{1}{1935360h^7w^7} f_n \csc^5\frac{hw}{2}$$

$$\csc[hw] \quad 5400h^3w^2 \cos \quad 2 \quad + 6 \quad 787h^3w^2 \quad 6h\ 2100 + 13h^2w^2 \quad \cos \quad 48132h^3w^2 \cos^2\frac{hw}{2}\ \sin^2\frac{3hw}{2}$$

$$9\ 1190h^5w^4 \quad 63h\ 160 + h^4w^4 \quad \cos^3 hw \quad 2 \quad 17535h^5w^4 \cos \quad 2\ 15120h \cos^2 5hw \quad 2\ 7hw$$

$$+ 6\cos[2hw] + 5\cos[3hw]) \sec^5 hw \quad 2 \quad + 24 \quad \frac{1}{37144h^5w^4 + 60h\ 588 \quad 925h^4w^4} \sin^5\frac{hw}{4}$$

$$+ 7560h^4 \csc^2 hw \quad 30hw^4 + 8w^3(\ 2 + 4\cos[hw] \quad 4\cos[2hw] + 5\cos[3hw]) \sin[hw] \quad 9h^2 \sin^2\frac{hw}{2} \quad 84h^3w^4 \sin^2\frac{3hw}{2}$$

$$+780h^3w^4 \sin^3 hw \quad 2 \quad + 24hh^4w^4 \sin^2\frac{3hw}{2} \quad 193h^4w^4 \sin^2\frac{5hw}{2} \quad 6 \quad 5040h + 7h^5w^4$$

$$\sin^3 hw \quad 2 \quad + 25200h \quad 833h^5w^4 \sin^2\frac{5hw}{2} \quad + 2h \quad 269952h^4w^4 \sin^2\frac{hw}{2}$$

$$+ 3h^2 \quad 78h^2w^4 \sin^3 hw \quad 2 \quad 1230h^2w^4 \sin^2\frac{5hw}{2} \quad 12h84h^3w^4 \sin^2\frac{3hw}{2}$$

$$+780h^3w^4 \sin^5 hw \quad 2 \quad + 6\ h^4w^4 \sin^2\frac{3hw}{2} \quad 193h^4w^4 \sin^2\frac{5hw}{2}$$

$$+ \frac{1}{1935360h^4w^4 f_{n+5}} \csc^5\frac{hw}{2} \quad 60480h^4w^3 \cos[hw](3 + 6\cos[hw] + 6\cos[2hw] + \cos[3hw]) \sec^2\frac{hw}{2}$$

+ 24 4976h^{5}w^{4} + 120h 210 + h^{4}w^{4} sin 2 + 7560h^{4} csc 2 30hw^{4} + 8w^{3}(2 4 cos[hw]
hw hw

+4 cos[2hw] + cos[3hw]) sin[hw]) 9h^{2} 780h^{3}w^{4} sin 2 84h^{3}w^{4} sin 5 2
3hw hw

+24h193h^{4}w^{4} sin 2 h^{4}w^{4} sin 2 6 25200h 833h^{5}w^{4} sin 2
hw 5hw 3hw

+ 5040h + 7h^{5}w^{4} sin 5 2 + 2h106368h^{4}w^{4} sin 2 + 3h^{2} 1230h^{2}w^{4} sin 2
hw hw 3hw

78h^{2}w^{4} sin 2 12h 780h^{3}w^{4} sin 2 84h^{3}w^{4} sin 2 + 6 193h^{4}w^{4}
hw 3hw 5hw

3hw 5hw 1 hw 5
sin 2 h^{4}w^{4} sin 2 + 3870720h^{4}w^{4} f$_{n+4}$ csc 2 302400h^{4}w^{3}
hw

cos[hw](3 + 6 cos[hw] + 6 cos[2hw] + 2 cos[3hw]) sec 2 37800h^{4} csc 2 30hw^{4} + 8w^{3}(1 2 cos[hw] + 2 cos[2hw]
+2 cos[3hw]) sin[hw]) + 4h 251520h^{4}w^{4} sin 2 + 3h^{2} 4650h^{2}w^{4} sin 2 + 456h^{2}w^{4} sin 2
hw 3hw 5hw

12h 5460h^{3}w^{4} sin 3 2 + 480h^{3}w^{4} sin 2 + 6 835h^{4}w^{4} sin 2
hw 5hw 3hw

+28h^{4}w^{4} sin 2 6 5 54454h^{5}w^{4} + 12h 9240 6929h^{4}w^{4} sin 2
hw hw

+ 3h^{2}5460h^{3}w^{4} sin 3 2 + 480h^{3}w^{4} sin 2 8h 835h^{4}w^{4} sin 2 + 28h^{4}w^{4}
hw 5hw 3hw

sin 5 2 + 2 5 10080h + 847h^{5}w^{4} sin 2 + 4 6300h 7h^{5}w^{4} sin 2
hw 3hw 5hw

1 hw 5 hw
+ 3870720h^{4}w^{4} f$_{n+2}$ csc 2 302400h^{4}w^{3} cos[hw](3 + 6 cos[hw] + 6 cos[2hw] + 4 cos[3hw]) sec 2
37800h^{4} csc 2 30hw^{4} + 8w^{3}(1 + 2 cos[hw] 2 cos[2hw] + 4 cos[3hw]) sin[hw] + 4h 84480h^{4}w^{4} sin 2
hw hw

+ 3h^{2}1110h^{2}w^{4} sin 3 2 + 2760h^{2}w^{4} sin 2 12h11220h^{3}w^{4} sin 2
hw 5hw 3hw

+768h^{3}w^{4} sin 5 2 + 6 20035h^{4}w^{4} sin 2 3620h^{4}w^{4} sin 2
hw 3hw 5hw

6 5 4448h^{5}w^{4} + h 131040 + 1530h^{4}w^{4} sin 2 + 3h^{2} 11220h^{3}w^{4} sin 2
hw 3hw

+768h^{3}w^{4} sin 5 2 8h 20035h^{4}w^{4} sin 2 3620h^{4}w^{4} sin 2
hw 3hw 5hw

+2 5 5040h + 5047h^{5}w^{4} sin 2 4 12600h + 847h^{5}w^{4} sin 2
hw 5hw

$$y_{n+2} = \frac{3y_{n+2}}{4h^3} + \frac{2y_{n+3}}{2h^3} + \frac{3y_{n+4}}{h^3} + \frac{1}{4h^3}\left(\frac{1}{1935360h^4w^4} f_{n+3} \csc\frac{hw}{2}\right)\left(1209600h^6 - 63150h^5w^4\cos[hw] + 1209600h^3 - 63150h^5w^4\cos[hw] + 30h^4 - 9826h^5w^4 + 63h^4 - 720 + 16h^4w^4\cos[hw] + 6h^2 - 6096h^5w^4\cos[2hw] + 1026h^5w^4\cos[3hw] + 18h^5 - 10856h^5w^4\cos[2hw] + 1761h^5w^4\cos[3hw] + 6h^6 - 5360h^5w^4\cos[2hw] + 3h^6 - 1250h^5w^4\cos[3hw] + 15h^6 - 560 + 48h^4w^4 - 604800h^4w^3\sin[hw] + \frac{1}{1935360h^4w^4} f_n \csc\frac{hw}{2}\csc[hw] - 4704h^5w^4\cos\frac{hw}{2} - 145h^2w^2\cos\frac{hw}{2} - 6h^2(2100 + 208h^2w^2\cos\frac{hw}{2} - 5607h^2w^2\cos\frac{3hw}{2}) - 1008h^2w^2\cos\frac{3hw}{2} - 63h^2(160 + 16h^2w^2\sin\frac{3hw}{2}) + 3549h^5w^4\cos\frac{5hw}{2} - 15120h^3\cos\frac{7hw}{2} + 60480h^4w^3\sin\frac{3hw}{2} + \frac{1}{3870720h^4w^4} f_{n+6} \csc\frac{hw}{2}(32796h^2w^2\sin\frac{hw}{2} + 36910h^5w^4 - 45h^2(112 + 16h^2w^2\sin\frac{hw}{2}) + 2340h^5w^4\sin\frac{3hw}{2}) - 562h^5w^4 - 3h^2(1680 + 176h^4w^4\sin\frac{hw}{2}) - 2520h^4\csc\frac{hw}{2} - 6hw^4 + 24w^3(1 - 2\cos[hw] + 2\cos[2hw])\sin[2hw])) + \frac{1}{1935360h^4w^4} f_{n+5} \csc\frac{hw}{2}\sec\frac{hw}{2}(440h^2w^2 + 120h^2(210 + 16h^2w^2\sin\frac{hw}{2}) + 7560h^2\csc\frac{hw}{2}) - 60480h^4w^3\cos[2hw](3 + 6\cos[hw] + 6\cos[2hw] + \cos[3hw]) - 6hw^4 + 8w^3(2 - 4\cos[hw] + 4\cos[2hw] + \cos[3hw])\sin[2hw] + 12h^5(125h^4w^4\sin\frac{hw}{2} - 67h^4w^4\sin\frac{5hw}{2}) - 6(25200h^6 - 802h^5w^4\sin\frac{3hw}{2}) + 5040h^5(34h^5w^4\sin\frac{5hw}{2} + h^5(12624h^4w^4\sin\frac{3hw}{2} - 6h^3w^4\sin\frac{5hw}{2} + 6(125h^4w^4\sin\frac{5hw}{2} - 67h^4w^4\sin\frac{5hw}{2}) + \frac{1}{1935360h^4w^4} f_{n+1} \csc\frac{hw}{2}(60480h^4w^3\cos[2hw](3 + 6\cos[hw]$$

+ 6 cos[2hw] + 5 cos[3hw]) sec 2 + 2461316h^5 w^4 + 60h 588 925h^4 w^4 sin 2 + 7560h^4 csc 2
hw hw hw

6hw^4 + 8w^3(2 + 4 cos[hw] 4 cos[2hw] + 5 cos[3hw]) sin[2hw] + 12h 67h^4 w^4 sin 2 + 125h^4 w^4 sin 5 2
3hw hw

6 5040h 34 h^5 w^4 sin 3 2 + 25200 h 802h^5 w^4 sin 2 + h 111696h^4 w^4
hw 5hw

sin 2 6h6h^3 w^4 sin 2 6h^3 w^4 sin 2 + 6 67h^4 w^4 sin 2
hw 3hw 5hw 3hw

+125h^4 w^4 sin 5hw 1 hw 5
2 + 3870720h^4 w^4 f_{n+4} csc 2 302400h^4 w^3 cos[2hw](3 + 6 cos[hw]

hw hw
+ 6 cos[2hw] + 2 cos[3hw]) sec 2 37800h^4 csc 2 6hw^4 + 8w^3(1 2 cos[hw] + 2 cos[2hw] + 2 cos[3hw]) sin[2hw]
+ 2h 114810h^4 w^4 sin 2 6h 30h^3 w^4 sin 2 + 24h^3 w^4 sin 2 + 6 2015h^4 w^4
hw 3hw 5hw

sin 3 2 + 412h^4 w^4 sin 2 6^5 91796h^5 w^4 + 12h 9240 6929h^4 w^4
hw 5hw

sin 2 4h 2015h^4 w^4 sin 2 + 412h^4 w^4 sin 2 + 2 5 10080h + 638h^5 w^4
hw 3hw 5hw

3hw hw 5
sin 2 + 4 6300h + 58h^5 w^4 sin 2 + 1 3870720h^4 w^4 f_{n+2} csc 2 (302400h^4 w^3 cos[2hw]
hw

hw
302400h^4 w^3 cos[2hw](3 + 6 cos[hw] + 6 cos[2hw] + 4 cos[3hw]) sec 2 37800h^4 csc 2 6hw^4 2 + 2040h^3 w^4
+ 8w^3(1 + 2 cos[hw] 2 cos[2hw] +4 cos[3hw]) sin[2hw]) + 2h 11910h^5 w^4 sin 6h 10110h^5 w^4 sin
hw 3hw

sin 5 2 + 6 9025h^4 w^4 sin 2 1988h^4 w^4 sin 2 6^5 21592h^5 w^4 + h 131040 + 24480h^4 w^4
hw 3hw 5hw

sin 2 4h 9025h^4 w^4 sin 2 1988h^4 w^4 sin 2 + 2 5 5040h + 7934h^5 w^4
hw 3hw 5hw

sin 3 2 4^12600h + 1574h^5 w^4 sin 2
hw 5hw

$$y^{000}_{n+3} = \frac{y_n}{4h^3} + \frac{3y_{n+2}}{2h^3} - \frac{2y_{n+3}}{h^3} + \frac{3y_{n+4}}{4h^3} + \frac{1}{1935360h^4w^4} f_{n+3} (1209600h - 554850h^5w^4 \cos[hw]$$

$$30(4647h^5w^4 - 63h(720 + 81h^4w^4)) \cos[hw] - 6(18312h^5w^4 \cos[2hw] + 3(4884h^5w^4 - 15h(560 + 243h^4w^4))$$

$$\cos[3hw]) \csc[\frac{hw}{2}]^6 + \frac{1}{1935360h^4w^4} f_n \csc[\frac{hw}{2}]^5 \csc[hw](26016h^5w^4 \cos[\frac{hw}{2}] + 6(5153h^5w^4$$

$$6h(2100 + 1053h^4w^4)) \cos[\frac{hw}{2}] + 16974h^5w^4 \cos[\frac{3hw}{2}] - 9(4280h^5w^4 - 63h(160 + 81h^4w^4)) \cos[\frac{3hw}{2}]$$

$$+ 1953h^5w^4 \cos[\frac{5hw}{2}] - 15120h \cos[\frac{7hw}{2}] + 60480h^4w^3 \sin[\frac{hw}{2}]) + \frac{1}{3870720h^4w^4} f_{n+6} \csc[\frac{hw}{2}]^5$$

$$(60480h^4w^3(1 + 2\cos[hw] + 2\cos[2hw]) \cos[3hw] \sec[\frac{hw}{2}] + 4176h^5w^4 \sin[\frac{hw}{2}] + 36(3676h^5w^4 - 45h(112 + 81h^4w^4))$$

$$\sin[\frac{hw}{2}] - 864h^5w^4 \sin[\frac{3hw}{2}] - 12(2748h^5w^4 - 3h(1680 + 891h^4w^4)) \sin[\frac{3hw}{2}] - 2520h^4 \csc[\frac{3hw}{2}]$$

$$(18hw^4 + 24w^3(1 - 2\cos[hw] + 2\cos[2hw]) \sin[3hw])) + \frac{1}{1935360h^4w^4} f_{n+5} \csc[\frac{hw}{2}]^5 (60480h^4w^3 \cos[3hw]$$

$$(3 + 6\cos[hw] + 6\cos[2hw] + \cos[3hw]) \sec[\frac{hw}{2}] + 24(7872h^5w^4 + 120h(210 + 81h^4w^4)) \sin[\frac{hw}{2}] + 3h^2(144h^3w^4 \sin[\frac{3hw}{2}]$$

$$+ 144h^3w^4 \sin[\frac{5hw}{2}]) - 6((25200h - 723h^5w^4) \sin[\frac{3hw}{2}] + (5040h - 75h^5w^4) \sin[\frac{5hw}{2}]) + 7560h^4 \csc[\frac{hw}{2}]$$

$$(18hw^4 + 8w^3(2 - 4\cos[hw] + 4\cos[2hw] + \cos[3hw]) \sin[3hw])) + \frac{1}{3870720h^4w^4} f_{n+4} \csc[\frac{hw}{2}]^5 (302400h^4w^3 \cos[3hw]$$

$$(3 + 6\cos[hw] + 6\cos[2hw] + 2\cos[3hw]) \sec[\frac{hw}{2}] - 6(5(102006h^5w^4 + 12h(9240 - 6929h^4w^4)) \sin[\frac{hw}{2}] - h^2(5760h^3w^4$$

$$\sin[\frac{3hw}{2}] - 1296h^3w^4 \sin[\frac{5hw}{2}]) + 2(5(10080h + 429h^5w^4) \sin[\frac{3hw}{2}] + 4(6300h + 111h^5w^4) \sin[\frac{5hw}{2}]))$$

$$37800h^4 \csc[\frac{hw}{2}](18hw^4 + 8w^3(1 - 2\cos[hw] + 2\cos[2hw] + 2\cos[3hw]) \sin[3hw])) + \frac{1}{3870720h^4w^4} f_{n+2} \csc[\frac{hw}{2}]^5$$

$$(302400h^4w^3 \cos[3hw](3 + 6\cos[hw] + 6\cos[2hw] + 4\cos[3hw]) \sec[\frac{hw}{2}] - 6(5(124224h^5w^4 + h(131040 + 123930h^4w^4))$$

$$\sin[\frac{hw}{2}] - h^2(5760h^3w^4 \sin[\frac{3hw}{2}] + 1296h^3w^4 \sin[\frac{5hw}{2}]) + 2(5(5040h + 8853h^5w^4) \sin[\frac{3hw}{2}]$$

$$4(12600h + 1833h^5w^4) \sin[\frac{5hw}{2}])) - 37800h^4 \csc[\frac{hw}{2}](18hw^4 + 8w^3(1 + 2\cos[hw] - 2\cos[2hw] + 4\cos[3hw]) \sin[3hw]))$$

$$+ \frac{1}{1935360h^4w^4} f_{n+1} \csc[\frac{hw}{2}]^5 (60480h^4w^3 \cos[3hw](3 + 6\cos[hw] + 6\cos[2hw] + 5\cos[3hw]) \sec[\frac{hw}{2}]$$

$$+ 24(67176h^5w^4 + 60h(588 - 925h^4w^4)) \sin[\frac{hw}{2}] + 3h^2(144h^3w^4 \sin[\frac{3hw}{2}] - 144h^3w^4 \sin[\frac{5hw}{2}]) - 6((5040h - 75h^5w^4)$$

$$\sin[\frac{3hw}{2}] + (25200h - 723h^5w^4) \sin[\frac{5hw}{2}]) + 7560h^4 \csc[\frac{hw}{2}](18hw^4 + 8w^3(2 + 4\cos[hw] - 4\cos[2hw] + 5\cos[3hw]) \sin[3hw]))$$

3y 2y 3y

y_n ooo n+2 n+3 n+4 1 hw 5 hw 3hw

y_{n+4} = $4h^3$ + $2h^3$ h^3 + $4h^3$ + $1935360h^4w^4$ f_n csc 2 csc[hw] $77184h^5w^4$ cos 2 $5895h^5w^4$ cos 2

+6 $21503h^5w^4$ 6h 2100 + $3328h^4w^4$ cos 2 9 $14942h^5w^4$ 63h 160 + $256h^4w^4$ cos 3 2

hw hw

cos 2 2 + 2 6

hw 7hw hw 1 hw

3 4 4 4 4 4 $31264h^5w^4$ cos[2hw] + 3 $13468h^5w^4$

1209600h $965550h^5w^4$ cos[hw] 30 $28750h^5w^4$ 63h 720 + $256h^4w^4$ cos[hw] $6h^2$ $6096h^5w^4$ cos[2hw]

1026h w cos[3hw] 18h 10856h w cos[2hw] + 1761h w 1cos[3hw] 6 hw 5

15h 560 + $768h^4w^4$ cos[3hw] + $604800h^4w^3$ sin[hw] $3870720h^4w^4$ f_{n+6} csc 2 $60480h^4w^3$)

hw hw hw

(1 + 2 cos[hw] + 2 cos[2hw] cos[4hw] sec 2 $114660h^5w^4$ sin 2 + 36 $11520h^5w^4$ 45h 112 + $256h^4w^4$ sin 2

+ $4572h^5w^4$ sin 2 12 $8444h^5w^4$ 3h 1680 + $2816h^4w^4$ sin 3 2 $2520h^4$ csc 2

3hw hw hw

1 hw 5

$42hw^4$ + $24w^3$(1 2 cos[hw] + 2 cos[2hw]) sin[4hw] + $1935360h^4w^4$ f_{n+5} csc 2 $60480h^4w^3$

hw hw

(3 + 6 cos[hw] + 6 cos[2hw] + cos[3hw]) cos[4hw] sec 2 + 24 $23012h^5w^4$ + 120h 210 + $256h^4w^4$ sin 2

12h $67h^4w^4$ sin 3 2 + $125h^4w^4$ sin 2 6 25200h $764h^5w^4$ sin 2

hw 5hw 3hw

+ 5040h + $4h^5w^4$ sin 5 2 h $111696h^4w^4$ sin 2 + 6h $6h^3w^4$ sin 2

hw hw 3hw

+$6h^3w^4$ sin 5 2 + $667h^4w^4$ sin 2 + $125h^4w^4$ sin 2

hw 3hw 5hw

5 hw 1

$7560h^4$ csc 2 $42hw^4$ + $8w^3$(2 4 cos[hw] + 4 cos[2hw] + cos[3hw]) sin[4hw] + $3870720h^4w^4$ f_{n+4}

hw hw

csc 2 $302400h^4w^3$(3 + 6 cos[hw] + 6 cos[2hw] + 2 cos[3hw]) cos[4hw] sec 2 2h $11910h^4w^4$

sin 2 + 6h $10110h^5w^4$ sin 2 $2040h^3w^4$ sin 2 + 6 $9025h^4w^4$

hw 3hw 5hw

sin 2 1988h⁴w⁴ sin 5 2 6 5 98824h⁵w⁴ + 12h 9240 6929h⁴w⁴
3hw hw

sin 2 + 4h 9025h⁴w⁴ sin 2 1988h⁴w⁴ sin 2 + 2 5 10080h + 1348h⁵w⁴
hw 3hw 5hw

sin 3 2 + 4 6300h 148h⁵w⁴ sin 2 37800h⁴ csc 2 1 hw 5
hw 5hw hw

42hw⁴ + 8w³(1 2 cos[hw] + 2 cos[2hw] + 2 cos[3hw]) sin[4hw] + 3870720h⁴w⁴ f_{n+2} csc 2 302400h⁴w³
hw

(3 + 6 cos[hw] + 6 cos[2hw] + 4 cos[3hw]) cos[4hw] sec 2 2h 114810h⁴w⁴ sin 2 + 6h 30h³w⁴
hw 2 hw

sin 3 2 24h³w⁴ sin 2 + 62015h⁴w⁴ sin 2 + 412h⁴w⁴ sin 2
hw 5hw 3hw 5hw

6 5 381764h⁵w⁴ + h 131040 + 391680h⁴w⁴ sin 2 + 4h2015h⁴w⁴ sin 2
hw 3hw

+412h⁴w⁴ sin 5 2 + 2 5 5040h + 8644h⁵w⁴ sin 2 4 12600h + 1780h⁵w⁴
hw 3hw

sin 2 37800h⁴ csc 2 42hw⁴ + 8w³(1 + 2 cos[hw] 2 cos[2hw] + 4 cos[3hw]) sin[4hw]
5hw hw

+ 1 hw 5 hw
+ 24 1935360h⁴w⁴ f_{n+1} csc 2 60480h⁴w³(3 + 6 cos[hw] + 6 cos[2hw] + 5 cos[3hw]) cos[4hw] sec 2
hw 66664h⁷w⁷ + 60h 588 925h⁷w⁷ sin 2 12h 125h⁴w⁴ sin 2 67h⁴w⁴

sin 5 2 6 5040h + 4h⁵w⁴ sin 2 + 25200h 764h⁵w⁴ sin 2
hw 3hw 3hw 5hw

h12624h⁴w⁴ sin 2 + 6h 6h³w⁴ sin 2 + 6h³w⁴ sin 2 + 6 125h⁴w⁴
hw 3hw 5hw

sin 3 2 67h⁴w⁴ sin 2 + 7560h⁴ csc 2 42hw⁴ + 8w³(2 + 4 cos[hw] 4 cos[2hw]
hw 5hw hw

+5 cos[3hw]) sin[4hw]))

$$y_{n+5} = \frac{3y_{n+2}}{4h^3} + \frac{2y_{n+3}}{2h^3} + \frac{3y_{n+4}}{4h^3} + \frac{1}{1935360h^4w^4} f_n \csc\frac{hw}{2}$$

$$\csc[hw]\ 85224h^5w^5\cos\frac{hw}{2} + 6\ 54293h^5w^5\ 6h\ 2100 + 8125h^5w^5\cos\frac{hw}{2} + 34272h^5w^4$$

$$\cos\frac{3hw}{2}\ 9\ 38010h^5w^4\ 63h\ 160 + 625h^4w^4\cos\frac{hw}{2} + 1281h^5w^4\cos\frac{5hw}{2}\ 15120h\cos\frac{7hw}{2}$$

$$\frac{1}{720\ 9312h^3w^4\cos[2hw]\ 972h^3w^4\cos[3hw]\ 36h^4w^4\ 2792h^4w^4\cos[2hw] + 627h^4w^4\cos[3hw]\ 6\ 38360h^5w^4\cos[2hw]}$$

$$+3\ 30470h^5w^4\ 15h\ 560 + 1875h^4w^4\cos[3hw] + 604800h^4w^3\sin[2hw] + \frac{1}{3870720h^4w^4} f_{n+6}\csc\frac{hw}{2}$$

$$60480h^4w^3(1 + 2\cos[hw] + 2\cos[2hw])\cos[5hw]\sec\frac{hw}{2}\ 315000h^5w^4\sin\frac{2}{3hw}\ 112 + 625h^4w^4\sin\frac{2}{3hw}\ 80136h^5w^5\sin\frac{2}{3hw}\ 1220590h^5w^5\ 3h\ 1680 + 6875h^5w^5 + 36\ 27838h^5w^4\ 45h$$

$$\sin\frac{3hw}{2}\ 2520h^4\csc\frac{5hw}{2}\ 66hw^4 + 24w^3(1\ 2\cos[hw] + 2\cos[2hw])\sin[5hw]\ \frac{1}{1935360h^4w^4}$$

$$f_{n+5}\csc\frac{hw}{2}\ 60480h^4w^3(3 + 6\cos[hw] + 6\cos[2hw] + \cos[3hw])\cos[5hw]\sec\frac{hw}{2} + 24\ 43120h^5w^4 + 120h$$

$$24h\ h^4w^4\sin\frac{3hw}{2}\ 193h^4w^4\sin\frac{2}{5hw}\ 625200h\ 805h^5w^4\sin\frac{2}{3hw}$$

$$+\ 5040h + 35h^5w^4\sin^5\frac{2hw}{2}\ 2h269952h^4w^4\sin\frac{2}{3hw} + 3h^2 78h^2w^4\sin\frac{2}{3hw}$$

$$1230h^2w^4\sin^5\frac{2hw}{2} + 12h\ 84h^3w^4\sin\frac{2}{3hw}\ 780h^3w^4\sin\frac{2}{5hw}$$

$$+6h^4w^4\sin^5\frac{2hw}{2}\ 193h^4w^4\sin\frac{2}{5hw} + 7560h^4\csc\frac{2}{hw}\ 66hw^4 + 8w^3(2\ 4\cos[hw]$$

$$+4\cos[2hw] + \cos[3hw])\sin[5hw])) + \frac{1}{3870720h^4w^4} f_{n+4}\csc\frac{hw}{2}\ 302400h^4w^3(3 + 6\cos[hw] + 6\cos[2hw]$$

$$+ 2\cos[3hw])\cos[5hw]\sec\frac{hw}{2}\ 4h^2 + 3h^2\ 1110h^2w^4\sin\frac{2}{3hw}$$

$$+2760h^2w^4\sin^5\frac{2hw}{2} + 12h\ 11220h^3w^4\sin\frac{2}{3hw}\ 768h^3w^4\sin\frac{2}{5hw}$$

+6 20035h⁴w⁴ sin³ 2 3620h⁴w⁴ sin 2 6 5 104630h⁵w⁴ + 12h 9240 6929h⁴w⁴
hw 5hw

sin 2 + 3h² 11220h³w⁴ sin 2 768h³w⁴ sin 5 2 + 8h 20035h⁴w⁴
hw 3hw hw

sin 2 3620h⁴w⁴ sin 2 + 2 5 10080h + 4235h⁵w⁴ sin 5 2
3hw 5hw hw

+4 6300h 875h⁵w⁴ sin 5 2 37800h⁴ csc 2 66hw⁴ + 8w³(1 2 cos[hw] + 2 cos[2hw]
hw hw

+2 cos[3hw]) sin[5hw])) + 1 / 3870720h⁴w⁴ f_{n+2} csc hw/2 / 4650h²w⁴ sin 5 302400h⁴w³(3 + 6 cos[hw] + 6 cos[2hw] + 4 cos[3hw]) cos[5hw]
sec 2 4h / 251520h⁴w⁴ sin 2 + 3h² 2 + 456h²w⁴ sin
hw hw 3hw 5hw

+ 12h 5460h³w⁴ sin 2 480h³w⁴ sin 5 2 + 6 835h⁴w⁴ sin 2
3hw hw 3hw

+28h⁴w⁴ sin 5 2 6 5 908992h⁵w⁴ + h 131040 + 956250h⁴w⁴ sin 2
hw hw

+ 3h² 5460h³w⁴ sin 5 2 480h³w⁴ sin 2 + 8h 835h⁴w⁴ sin 2
hw 5hw 3hw

+28h⁴w⁴ sin 5 2 + 2 5 5040h + 8435h⁵w⁴ sin 2 4 12600h + 1715h⁵w⁴
hw 3hw

sin 2 37800h⁴ csc 2 66hw⁴ + 8w³(1 + 2 cos[hw] 2 cos[2hw] + 4 cos[3hw]) sin[5hw]
5hw hw

+ 1 / 1935360h⁴w⁴ f_{n+1} csc hw/2 5 / hw 60480h⁴w³(3 + 6 cos[hw] + 6 cos[2hw] + 5 cos[3hw]) cos[5hw] sec 2
+ 24 / 73880h⁵w⁴ + 60h 588 925h⁴w⁴ sin 2 / 9h² 780h⁵w⁴ sin 2 + 84h⁵w⁴ sin 2
hw 3hw 5hw

24h193h⁴w⁴ sin³ 2 h⁴w⁴ sin 2 6 5040h + 35h⁵w⁴ sin 2
hw 5hw 3hw

+ 25200h 805h⁵w⁴ sin 2 2h106368h⁴w⁴ sin 2 + 3h² 1230h²w⁴ sin 2
5hw hw 3hw

78h²w⁴ sin 2 + 12h780h³w⁴ sin³ 2 + 84h³w⁴ sin 2 + 6 193h⁴w⁴
5hw hw 5hw

sin 3 2 h⁴w⁴ sin 2 + 7560h⁴ csc 2 66hw⁴ + 8w³
hw 5hw hw

(2 + 4 cos[hw] 4 cos[2hw] + 5 cos[3hw]) sin[5hw]))

$$y_{n+6} = \frac{3y_{n+2}}{4h^3} + \frac{2y_{n+3}}{h^3} + \frac{3y_{n+4}}{4h^3} + \frac{1}{1935360h^4w^4} f_n \csc^5\frac{hw}{2}$$

$$y_n^{000} = 2h^3 \left(51360h^3w^3 \cos[hw] + 110063h^3w^3 \cos\frac{hw}{2}\right) + 6h\left(2100 + 16848h^2w^2 \cos\frac{hw}{2}\right)$$

$$+ 83853h^5w^4 \cos^3\frac{hw}{2} + 22365h^5w^4 \cos^5\frac{hw}{2} + 79940h^5w^4 \cos^2\frac{hw}{2} + 63h\left(160 + 1296h^4w^4 \cos\frac{3hw}{2}\right) + 15120h \cos\frac{7hw}{2} + 60480h^4w^3 \sin\frac{5hw}{2} + \frac{1935360h^4w^4}{1} f_{n+3}$$

$$\csc^6\frac{hw}{2}\left(1209600h + 546750h^5w^4 \cos[hw] + 30\left(114726h^5w^4 + 63h\left(720 + 1296h^4w^4 \cos[hw]\right) + 24h^3\left(6384h^4w^4 \cos[2hw] + 3654h^4w^4 \cos[3hw]\right)\right) + 78h^2\left(6768h^4w^4 \cos[2hw] + 1242h^4w^4 \cos[3hw]\right) + 54h\left(5848h^4w^4 \cos[2hw] + 537h\,w \cos[3hw]\right)\right) + 6\left(36624h\,w \cos[2hw] + 3\left(60798h\,w + 15h\left(560 + 3888h\,w \cos[3hw]\right) + 604800h\,w \sin[3hw]\right)\right)$$

$$+ \frac{1}{3870720h^3w^3} f_{n+6} \csc^2\frac{hw}{2} + 36\left(60480h^4w^3(1 + 2\cos[hw] + 2\cos[2hw])\cos[6hw]\sec^2\frac{hw}{2} + 57466h^3w^3 \sin\frac{hw}{2}\right) + 45h\left(112 + 1296h^2w^2 \sin\frac{3hw}{2}\right)\left(2520h^4 \csc^2\frac{hw}{2}\right)$$

$$\sin^3\frac{hw}{2}\left(12\left(43566h^5w^4 + 3h\left(1680 + 14256h^4w^4 \sin\frac{3hw}{2}\right)\right) + 90hw^4 + 24w^3(1 + 2\cos[hw] + 2\cos[2hw])\sin[6hw] + \frac{1}{1935360h^4w^4} f_{n+5} \csc^5\frac{hw}{2}\left(60480h^4w^3(3 + 6\cos[hw] + 6\cos[2hw] + \cos[3hw])\cos[6hw]\sec^2\frac{hw}{2} + 24\left(86496h^5w^4 + 120h\left(210 + 1296h^4w^4 \sin^2\frac{3hw}{2}\right)\right)\sin^2\frac{5hw}{2}\right)\right)$$

$$+ 24h^2\left(270h^3w^4 \sin^3\frac{hw}{2} + 2574h^3w^3 \sin^2\frac{5hw}{2}\right) + 1771h^4w^4 \sin^5\frac{hw}{2}\left(6\left(25200h + 798h^5w^4 \sin^2\frac{3hw}{2}\right) + 5040h + 798h^5w^4 \sin^2\frac{5hw}{2}\right)$$

$$+ 18h\left(270h^3w^4 \sin^3\frac{hw}{2} + 2574h^3w^4 \sin^2\frac{5hw}{2}\right) + 6\left(43h^4w^4 \sin^2\frac{3hw}{2} + 1771h^4w^4 \sin^2\frac{5hw}{2}\right) + 7560h^4 \csc^2\frac{hw}{2}\left(90hw^4 + 8w^3(2 + 4\cos[hw] + 4\cos[2hw] + \cos[3hw])\sin[6hw]\right)$$

$$+ \frac{1}{3870720h^3w^3} f_{n+4} \csc^5\frac{hw}{2}\left(302400h^4w^3(3 + 6\cos[hw] + 6\cos[2hw] + 2\cos[3hw])\cos[6hw]\sec^2\frac{hw}{2} + 6h\left(7290h^3w^4 \sin^3\frac{3hw}{2} + 3960h^3w^4 \sin^2\frac{5hw}{2} + 6\left(29785h^4w^4 \sin^2\frac{3hw}{2}\right)\right)\right)$$

$$2372h^4w^4 \sin \frac{5\ 2}{hw} \quad 6 \quad 5 \quad 135324h^5w^4 + 12h\ 9240 \quad 6929h^4w^4 \sin \frac{2}{hw}$$

$$+ 8h^2 \quad 7290h^3w^4 \sin \frac{2}{3hw} + 3960h^3w^4 \sin \frac{2}{5hw} + 12h \quad 29785h^4w^4 \sin \frac{2}{hw}$$

$$2372h^4w^4 \sin \frac{5\ 2}{hw} + 2 \quad 5 \quad 10080h + 9282h^5w^4 \sin \frac{2}{3hw} + 4 \quad 6300h \quad 1722h^5w^4$$

$$\sin \frac{2}{5hw} \quad 37800h^4 \csc \frac{2}{hw} \quad 90hw^4 + 8w^3(1 \quad 2\cos[hw] + 2\cos[2hw] + 2\cos[3hw])\sin[6hw]$$

$$+ \frac{1}{\substack{6h \\ hw}} \frac{\substack{3870720h^4w^4 \\ 253530h^2w^2 \sin}}{\substack{2 \\ 1590h^2w^2 \sin}} f_{n+2} \csc \frac{hw}{2} \frac{\substack{5 \\ hw}}{\substack{2 \\ + 8h^2}} \quad 302400h^4w^3(3 + 6\cos[hw] + 6\cos[2hw] + 4\cos[3hw])\cos[6hw] \sec \frac{2}{\substack{2 \\ + 3480h^2w^4 \sin}} \frac{2}{\substack{3hw \\ 5hw \\ + 18h\ 8730h^3w^4}}$$

$$\sin \frac{3\ 2}{hw} + 1368h^3w^4 \sin \frac{2}{5hw} + 6 \quad 8185h^4w^4 \sin \frac{2}{3hw} + 220h^4w^4 \sin \frac{2}{5hw}$$

$$6 \quad 5\ 1881168h^5w^4 + h\ 131040 + 1982880h^4w^4 \sin \frac{2}{hw} + 8h^2 \quad 8730h^3w^4 \sin \frac{2}{hw}$$

$$+1368h^3w^4 \sin \frac{2}{5hw} + 12h \quad 8185h^4w^4 \sin \frac{2}{3hw} + 220h^4w^4 \sin \frac{5\ 2}{hw} + 2\ (5\ (\quad 5040h + 9282h^5w^4 \sin \frac{2}{3hw}$$

$$+9282h^5w^4 \sin \frac{2}{3hw} \quad 4 \quad 12600h + 1722h^5w^4 \sin \frac{2}{5hw} \quad 37800h^4 \csc \frac{2}{hw} \quad 90hw^4 + 8w^3(\quad 1 + 2\cos[hw] \quad 2\cos[2hw]$$

$$+ 4\cos[3hw])\sin[6hw] + \frac{1\ hw}{\substack{+ 24\ 89964h^3w^2 \\ +60h\ 588\ 925h^4w^4 \sin}} f_{n+1} \csc \frac{5}{2} \frac{\substack{60480h^4w^3 \\ 24h^2 \\ 2574h^3w^4 \sin}}{\substack{2 \\ 270h^3w^4 \sin}} (3 + 6\cos[hw] + 6\cos[2hw] + 5\cos[3hw])\cos[6hw] \sec \frac{hw}{2} \frac{2}{\substack{3hw \\ 5hw}}$$

$$36h \quad 1771h^4w^4 \sin \frac{3\ 2}{hw} \quad 43h^4w^4 \sin \frac{2}{5hw} \quad 6 \quad 5040h \quad 798h^5w^4 \sin \frac{2}{3hw}$$

$$+ 25200h \quad 798h^5w^4 \sin \frac{2}{5hw} \quad 3h \quad 117072h^4w^4 \sin \frac{2}{hw} + 8h^2 \quad 2418h^2w^4 \sin \frac{3\ 2}{hw}$$

$$690h^2w^4 \sin \frac{2}{5hw} + 18h\ 2574h^3w^4 \sin \frac{2}{hw} \quad 270h^3w^4 \sin \frac{2}{5hw} + 6\ 1771h^4w^4$$

$$\sin \frac{3\ 2}{hw} \quad 43h^4w^4 \sin \frac{2}{5hw} + 7560h^4 \csc \frac{2}{hw} \quad 90hw^4 + 8w^3(\quad 2 + 4\cos[hw]$$

$$4\cos[2hw] + 5\cos[3hw])\sin[6hw]))$$

3.4.3 Case k = 8; v = 4

$$u(x) = \ {}_0(x; !)y_n + \ {}_3(x; !)y_{n+3} + \ {}_4(x; !)y_{n+4} + \ {}_5(x; !)y_{n+5}$$

$$h^4 \ {}_0(x; !)f_n + \ {}_1(x; !)f_{n+1} + \ {}_2(x; !)f_{n+2} + \ {}_3(x; !)f_{n+3} + \ {}_4(x; !)f_{n+4}$$

$$+ \ {}_5(x; !)f_{n+5} + \ {}_6(x; !)f_{n+6} + \ {}_7f_{n+7} + \ {}_8(x; !)f_{n+8} \qquad (3.23)$$

$$y(x) \,'\, U(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7$$
$$+ a_8x^8 + a_9x^9 + a_{10}x^{10} + a_{11}\cos !x + a_{12}\sin !x \qquad (3.24)$$

following the established procedure, a system of 13 equations are obtained.

Solving these equations, substituting for a_0; a_1; ; a_{12} in (3.24) and simplifying gives

$$_0(x; !) = \frac{(3h\ \ x)(4h\ \ x)(5h\ \ x)}{60h^3}$$

$$_3(x; !) = \frac{x(20h^2\ \ 9hx + x^2)}{6h^3}$$

$$(x; !)_4 = \frac{x(15h^2\ \ 8hx + x^2)}{4h^3}$$

$$_5(x; !) = \frac{x(12h^2\ \ 7hx + x^2)}{10h^3}$$

$$_0(x) = \frac{1}{464486400h^6w^4}\,\text{"}\,\mathrm{Csc}\left[\frac{\ }{2}\right]^7\mathrm{Csc}[hw](5(3h\ \ x)x(21960h^8w^4$$

$$94338h^7w^4x\ \ 143361h^6w^4x^2 + 93333h^5w^4x^3\ \ 723h^2w^4x^6 + 42hw^4x^7$$
$$w^4x^8 + h^4(616896\ \ 32897w^4x^4) + 9h^3x(\ \ 12096 + 725w^4x^4))\mathrm{Cos}\left[\frac{hw}{2}\right]$$
$$9x(20h^2\ \ 9hx + x^2)(\ \ 24096h^7w^4 + 20713h^6w^4x\ \ 21483h^5w^4x^2$$
$$12817h^4w^4x^3 + 511h^2w^4x^5\ \ 36hw^4x^6 + w^4x^7$$
$$h^3(67200\ \ 3591w^4x^4))\mathrm{Cos}\left[\frac{3hw}{2}\right] + (3h\ \ x)(4h\ \ x)(5h$$
$$x)(\ 5w^4x(\ 9660h^6\ 7299h^5x{+}7836h^4x^2\ 2643h^3x^3\ {+}424h^2x^4\ 33hx^5\ {+}x^6)\mathrm{Cos}\left[\frac{5hw}{2}\right]$$
$$+w^4x(25122h^6\ 24507h^5x{+}11802h^4x^2\ 3075h^3x^3\ {+}442h^2x^4\ 33hx^5\ {+}x^6)\mathrm{Cos}\left[\frac{7hw}{2}\right]$$
$$60480h^3\mathrm{Cos}\left[\frac{9hw}{2}\right]) + 3628800h^6\mathrm{Cos}\left[\frac{9hw}{2}\right]\mathrm{Cos}[wx]$$
$$+ 3628800h^6\mathrm{Sin}\left[\frac{9hw}{2}\right]\mathrm{Sin}[wx])$$

$$
{}_1(x) = \frac{1}{928972800h^6w^4}\,''\ \mathrm{Csc}\ \frac{hw}{2}\ {}^{8}\ \mathrm{Sec}\ \frac{hw}{2}\ \ 2x\ \ 13834440h^9w^4
$$

$$
15333618h^8w^4x + 6213296h^7w^4x^2 + 9885h^2w^4x^7\ \ 560hw^4x^8 + 13w^4x^9
$$

$$
30240h^3x^2\ \ 68 + 3w^4x^4\ \ 4032h^5\ \ 8220 + 259w^4x^4
$$

$$
+ 56h^4x\ \ 311040 + 7993w^4x^4\ \ \mathrm{Cos}\ \frac{hw}{2}
$$

$$
2x(\ 4h + x)\ \ 1045260h^8w^4 + 3737127h^7w^4x\ \ 1559040h^6w^4x^2
$$

$$
389760h^5w^4x^3 + 13342h^2w^4x^6\ \ 857hw^4x^7 + 22w^4x^8
$$

$$
+ 14h^3x\ \ 164160\ \ 7393w^4x^4\ + 336h^4\ \ 33120 + 1159w^4x^4\ \ \mathrm{Cos}\ \frac{\frac{hw}{3}}{2}
$$

$$
+ (3h\ \ x)(4h\ \ x)(5h
$$

$$
x)\ \ 10w^4x\ \ 86220h^6 + 24797h^5x + 3852h^4x^2\ \ 3379h^3x^3 + 692h^2x^4\ \ 61hx^5 + 2x^6\ \ \mathrm{Cos}\ \frac{5hw}{2}
$$

$$
498h^6w^4x\ \ 1589h^5w^4x^2 + 1722h^4w^4x^3 + 202h^2w^4x^5\ \ 23hw^4x^6 + w^4x^7\ \ 5h^3\ \ 12096 + 169w^4x^4
$$

$$
25122h^6w^4x\ \ 24507h^5w^4x^2 + 11802h^4w^4x^3 + 442h^2w^4x^5\ \ 33hw^4x^6 + w^4x^7\ \ 15h^3\ \ 28224 + 205w
$$

$$
+ 3628800h^{\circ}\ \ \mathrm{Cos}\ \ {}_7 2\ \ + 7\mathrm{Cos}\ \ {}_9 2\ \ \mathrm{Cos}[wx]
$$

$$
+ 3628800h^6\ \mathrm{Sin}\ \ {}_7 2\ \ + 7\mathrm{Sin}\ \ {}_9 2\ \ \mathrm{Sin}[wx]\ \ \#
$$

$$
{}_2(x) = \frac{1}{232243200h^6w^4}\,''\,\mathrm{Csc}\left[\frac{hw}{2}\right]^7 \mathrm{Csc}[hw]\;\; 7x\;\; 243720h^9w^4
$$

$$
575574h^8w^4x\;\; 1941h^7w^4x^2 + 2730h^2w^4x^7\;\; 165hw^4x^8 + 4w^4x^9
$$

$$
90h^3x^2\;\; 10752\;\; 251w^4x^4\;\; 22176h^5\;\; 660 + 7w^4x^4
$$

$$
+ 28h^4x\;\; 285120 + 3323w^4x^4\;\; \mathrm{Cos}\left[\frac{hw}{2}\right] + (4h
$$

$$
x)\;\; 7x\;\; 3495780h^8w^4\;\; 1547421h^7w^4x\;\; 31232h^6w^4x^2\;\; 7808h^5w^4x^3 + 3166h^2w^4x^6\;\; 221hw^4x^7 +
$$

$$
+5w^4(3h\;\; x)(5h\;\; x)x\;\; 20988h^6\;\; 8835h^5x\;\; 684h^4x^2 + 1581h^3x^3\;\; 452h^2x^4 + 51hx^5\;\; 2x^6\;\; \mathrm{Cos}\frac{5}{}
$$

$$
+ 15h^2\;\; 8hx+x^2\;\; 7\;\; 61833h^6w^4x\;\; 18513h^5w^4x^2\;\; 1887h^4w^4x^3\;\; 397h^2w^4x^5 + 33hw^4x^6\;\; w^4x^7 +
$$

$$
+ 12700800h^6\;\; \mathrm{Cos}\left[\frac{7hw}{2}\right] + 3\mathrm{Cos}\left[\frac{9hw}{2}\right]\;\; \mathrm{Cos}[wx]
$$

$$
+ 12700800h^6\;\; \mathrm{Sin}\left[\frac{7hw}{2}\right] + 3\mathrm{Sin}\left[\frac{9hw}{2}\right]\;\; \mathrm{Sin}[wx]\;\; \#
$$

$$
{}_3(x) = \frac{1}{464486400h^6w^4}\,''\,\mathrm{Csc}\left[\frac{hw}{2}\right]^7 \mathrm{Csc}[hw]\;\; 70x\;\; 2351880h^9w^4
$$

$$
1456866h^8w^4x + 256648h^7w^4x^2 + 633h^2w^4x^7\;\; 40hw^4x^8 + w^4x^9
$$

$$
288h^3x^2\;\; 1260\;\; 17w^4x^4\;\; 28224h^5\;\; 180 + w^4x^4
$$

$$
+ 280h^3x\;\; 10368 + 65w^4x^2\;\; \mathrm{Cos}\left[\frac{hw}{2}\right]\;\; + (4h
$$

$$
x)\;\; 14x\;\; 1744020h^8w^4\;\; 787779h^7w^4x\;\; 33280h^6w^4x^2\;\; 8320h^5w^4x^3 + 2846h^2w^4x^6\;\; 211hw^4x^7 +
$$

$$
+10w^4(3h\;\; x)(5h\;\; x)x\;\; 366036h^6 + 1303h^5x\;\; 10020h^4x^2\;\; 2009h^3x^3 + 788h^2x^4\;\; 71hx^5 + 2x^6\;\; \mathrm{Co}
$$

$$
+ 15h^2\;\; 8hx+x^2\;\; 7\;\; 51354h^6w^4x\;\; 78047h^5w^4x^2 + 306h^4w^4x^3\;\; 2254h^2w^4x^5 + 211hw^4x^6\;\; 7w^4x
$$

$$
+ 25401600h^6\;\; 3\mathrm{Cos}\left[\frac{7hw}{2}\right] + 5\mathrm{Cos}\left[\frac{9hw}{2}\right]\;\; \mathrm{Cos}[wx]
$$

$$
+ 25401600h^6\;\; 3\mathrm{Sin}\left[\frac{7hw}{2}\right] + 5\mathrm{Sin}\left[\frac{9hw}{2}\right]\;\; \mathrm{Sin}[wx]\;\; \#
$$

$$_{4}(x) = \frac{46448640h^{6}w^{4}}{1} \quad '' \; 3175200h^{3}x \; 15h^{2} \; 8hx + x^{2}$$

$$+(4 \quad h \quad x \quad x \quad 7 \quad 587970 \quad {}^{8}_{h}w^{4} \quad 217224 \quad {}^{7}_{h} \quad {}^{4}_{wx+9856h\,w} \quad {}^{6} \quad {}^{4}x^{2} \quad +2464 \quad {}^{5}_{h}w^{4}x^{3} \quad hw \quad {}^{8} \quad {}^{2}_{489h\,w} \quad {}^{4} \quad {}^{6}_{x} \quad {}^{4}_{+36hw}x^{7}$$

$$+ 12700800h \operatorname{Cos}[w(4h \quad x)] \operatorname{Csc} \quad 2 \quad \#$$

$$_{5}(x) =$$

$$\frac{1}{464486400h^{6}w^{4}} \operatorname{Csc} \frac{hw}{2} \quad {}^{7} \operatorname{Csc}[hw] \quad 70x \quad 2351880h^{9}w^{4} \quad 1456866h^{8}w^{4}x + 256648h^{7}w^{4}x^{2} + 63$$

$$+(4h \quad x) \quad 14x \; 3495780h^{8}w^{4} \quad 1547421h^{7}w^{4}x \quad 31232h^{6}w^{4}x^{2} \quad 7808h^{5}w^{4}x^{3} + 3166h^{2}w^{4}x^{6} \quad 221h$$

$$+ 25401600h^{6} \quad 5\operatorname{Cos} \quad 7 \frac{hw}{2} \quad + 3\operatorname{Cos} \quad 9 \frac{hw}{2} \quad \operatorname{Cos}[wx]$$

$$+ 25401600h^{6} 5\operatorname{Sin} \quad 7 \frac{hw}{2} \quad + 3\operatorname{Sin} \quad 9 \frac{hw}{2} \quad \operatorname{Sin}[wx] \quad \#$$

$_6(x)$

$$= \frac{1}{116121600h^{o}w^{4}} \text{"Csc}\left(\frac{hw}{2}\right)^{7} \text{Csc}[hw]\left(\frac{1}{2}\right) 7x\ 6763320h^{9}w^{4}\ 4214754h^{8}w^{4}x+765893h^{7}w^{4}x^{2}+23$$

$$+ 6350400h^{o}\ 3\text{Cos}\left(\frac{7}{2}\frac{hw}{}\right) + \text{Cos}\left(\frac{9}{2}\frac{hw}{}\right) \text{Cos}[wx]$$

$$+ 6350400h^{6}\ 3\text{Sin}\left(\frac{7}{2}\frac{hw}{}\right) + \text{Sin}\left(\frac{9}{2}\frac{hw}{}\right) \text{Sin}[wx]\ \text{"}$$

$_7(x) =$

$$\frac{1}{464486400h^{6}w^{4}} \text{"Csc}\left(\frac{hw}{2}\right)^{7} \text{Csc}[hw]\ 2x\ 15285960h^{9}w^{4}\ 9586482h^{8}w^{4}x+1802208h^{7}w^{4}x^{2}+7$$

$$+2(4h\ x)x\ 3484260h^{8}w^{4}\ 1596807h^{7}w^{4}x\ 111104h^{6}w^{4}x^{2}\ 27776h^{5}w^{4}x^{3}+9182h^{2}w^{4}x^{6}\ 727hw$$

$$+(3h\ x)(4h\ x)(5h\ x)\ 10w^{4}x\ 20988h^{6}+8835h^{5}x+684h^{4}x^{2}\ 1581h^{3}x^{3}+452h^{2}x^{4}\ 51hx^{5}$$

$$+ 3628800h^{o}\ 7\text{Cos}\left(\frac{7}{2}\frac{hw}{}\right) + \text{Cos}\left(\frac{9}{2}\frac{hw}{}\right) \text{Cos}[wx]$$

$$+ 3628800h^{6}\ 7\text{Sin}\left(\frac{7}{2}\frac{hw}{}\right) + \text{Sin}\left(\frac{9}{2}\frac{hw}{}\right) \text{Sin}[wx]\ \text{"}$$

$_8(x)$

$$= \frac{1}{464486400h^{6}w^{4}} \text{"Csc}\left(\frac{hw}{2}\right)^{7} \text{Csc}[hw]\ 5(5h\ x)x\ 170424h^{8}w^{4}\ 73434h^{7}w^{4}x+6175h^{6}w^{4}x^{2}+123$$

$$9x\ 12h^{2}\ 7hx+x^{2}\ 32240h^{7}w^{4}+4193h^{6}w^{4}x+2891h^{5}w^{4}x^{2}+1337h^{4}w^{4}x^{3}+287h^{2}w^{4}x^{5}\ 28hw$$

$$+(3h\ x)(4h\ x)(5h\ x)\ 5w^{4}x\ 4332h^{6}\ +2659h^{5}x+60h^{4}x^{2}\ 557h^{3}x^{3}+184h^{2}x^{4}\ 23hx^{5}+x^{6}$$

$$+ 3628800h^{6}\text{Cos}\left(\frac{7hw}{2}\right) \text{Cos}[wx] + 3628800h^{6}\text{Sin}\left(\frac{7hw}{2}\right) \text{Sin}[wx]\ \text{"}$$

The main method is

$$y_{n+8} = y_n + 16y_{n+3} \quad 30y_{n+4} + 16y_{n+5}$$

$$+ \frac{1}{46448640h^6w^4} \; {}^{\prime}{}_{n+4} \quad 381024000h^6 + 12700800h^6\mathrm{Cos}[4hw]$$

$$4h \; 56h \; 68725630h^8w^4 + 3360h^4 \; 1152 + 28672h^4w^4 \; 280h^4 \; 7344 + 94208h^4w^4 \; \mathrm{Cos}[hw]$$

$$11664240h^9w^4\mathrm{Cos}[2hw] + 6780240h^9w^4\mathrm{Cos}[3hw]$$

$$_{hw\,8}$$

$$+30h^2 \quad 5792360h^7w^4 \quad 20h^3 \quad 5292 + 290816h^4w^4 \quad \mathrm{Cos}[4hw] \quad \mathrm{Csc}$$

$$\frac{1}{+464486400h^6w^4 f_{n+8}\mathrm{Csc}\frac{hw}{2}} \quad {}_7 \quad \mathrm{Csc}[hw] \quad 120h^2 \quad 42119000h^8w^4 \; {}_{hw}$$

$$+ h^4 \quad 60480 \; 13438976h^4w^4 \quad + 24h^4 \quad 20160 + 2330624h^4w^4 \quad \mathrm{Cos}\frac{hw}{2}$$

$$1440h^3 \quad 5032408h^7w^4 + 35h^3 \quad 1152 \; 143360h^4w^4 \quad \mathrm{Cos}\frac{3}{2}$$

$$+ 3470928h^7w^4 \quad 5h^3 \quad {}_{12096 + 692224h^4w^4} \quad 60h^3 \quad 90400h^7w^4\mathrm{Cos} \quad {}_{\mathrm{Cos}} \quad 7\,2\,\frac{5hw}{2}$$

$$hw$$

$$+ 3628800h^6\mathrm{Cos}\frac{hw}{2}\mathrm{Cos}[8hw] + 3628800h^6\mathrm{Sin}\frac{7hw}{2} \quad \mathrm{Sin}[8hw]$$

$$\frac{1}{+464486400h^6w^4 f_n\mathrm{Csc}\frac{hw}{2}} \quad {}_7 \quad \mathrm{Csc}[hw](\; 200h^2(\; 78838888h^8w^4$$

$$h^4 \; 616896 \; 134746112h^4w^4 \; +72h^4(\; 12096 + 2969600h^4w^4))\mathrm{Cos}\frac{hw}{2}$$

$$864h^3 \quad 14733416h^4w^4 \quad + h^3 \quad 67200 \; {}_{14708736h^4w^4} \quad \mathrm{Cos}\frac{3hw}{2}$$

$$60h^3 \quad 90400h^7w^4\mathrm{Cos}\frac{5}{2} + 9808h^7w^4\mathrm{Cos}\frac{7}{2}$$

$$_{60480h^3\mathrm{Cos}} \quad {}_{9\,2} \quad {}_{+\,3628800h^6\mathrm{Cos}} \quad {}_{92} \quad \mathrm{Cos}[8hw]$$

$$hw \qquad hw$$

$$3628800h^6\mathrm{Sin}\frac{9hw}{2}\mathrm{Sin}[8hw]$$

$$+\frac{1}{116121600h^{6}w^{4}}f_{n+6}\mathrm{Csc}\left[\frac{hw}{2}\right]^{7}\mathrm{Csc}[hw]\left(\frac{1}{2}\left(56h\left(2928715112h^{9}w^{4}+5760h^{5}\left(8064\left(806912h^{4}w^{4}\right.\right.\right.\right.$$

$$4h\left(56h\left(393278076h^{9}w^{4}+16h^{4}\left(423360\left(36589568h^{4}w^{4}+16h^{4}\left(196560+12111872h^{4}w^{4}\right)\right.\right.\right.\right.\right.\right.\mathrm{Cos}$$

$$+6350400h^{6}\left(3\mathrm{Cos}\left[\frac{7hw}{2}\right]+\mathrm{Cos}\left[\frac{9hw}{2}\right]\right)\mathrm{Cos}[8hw]$$

$$+6350400h^{6}\left(3\mathrm{Sin}\left[\frac{7hw}{2}\right]+\mathrm{Sin}\left[\frac{9hw}{2}\right]\right)\mathrm{Sin}[8hw]$$

$$\frac{1}{464486400h^{6}w^{4}}f_{n+7}\mathrm{Csc}\left[\frac{hw}{2}\right]^{7}\mathrm{Csc}[hw]\left(16h\left(8436251960h^{9}w^{4}\right.\right.$$

$$645120h^{5}\left(132+20480h^{4}w^{4}\right)4032h^{5}\left(2460+274432h^{4}w^{4}\right)$$

$$+448h^{5}\left(155520+13078528h^{4}w^{4}\right))\cos\left[\frac{hw}{2}\right]$$

$$64h^{2}\left(1220853292h^{8}w^{4}+112h^{4}\left(112320\left(15822848h^{4}w^{4}\right)\right.\right.$$

$$112h^{4}\left(47520+4943872h^{4}w^{4}\right))\cos\left[\frac{3hw}{2}\right]$$

$$60h^{3}\left(919360h^{7}w^{4}\cos\left[\frac{5hw}{2}\right]\right.$$

$$\left(12605008h^{7}w^{4}\left(15h^{3}\left(28224+839680h^{4}w^{4}\right)\right)\cos\left[\frac{7hw}{2}\right]\right.$$

$$\left(3470928h^{7}w^{4}\left(5h^{3}\left(12096+692224h^{4}w^{4}\right)\right)\cos\left[\frac{9hw}{2}\right]\right)$$

$$3628800h^{6}\left(7\cos\left[\frac{7hw}{2}\right]+\cos\left[\frac{9hw}{2}\right]\right)\cos[8hw]$$

$$3628800h^{6}\left(7\mathrm{Sin}\left[\frac{7hw}{2}\right]+\mathrm{Sin}\left[\frac{9hw}{2}\right]\right)\mathrm{Sin}[8hw]\Big)$$

$$+\frac{1}{232243200h^6w^4} f_{n+2}\mathrm{Csc}[\frac{hw}{2}]^7\mathrm{Csc}[hw](56h(3498091880h^9w^4 + 5760h^5(10752\ \ 1028096h^4w^4)$$

$$22176h^5(\ \ 660 + 28672h^4w^4) + 224h^5(\ \ 285120 + 13611008h^4w^4))\cos[\frac{hw}{2}]$$

$$4h(56h(452260476h^8w^4 + 16h^4(544320\ 43921408h^4w^4) + 16h^4(\ 317520 + 15757312h^4w^4))$$

$$\mathrm{Cos}[\frac{3hw}{2}] + 6895200h^9w^4\mathrm{Cos}[\frac{5hw}{2}] + 15h^2(7(\ 8111608h^7w^4 + 105h^3(\ 288 + 77824h^4w^4))$$

$$\mathrm{Cos}[\frac{7hw}{2}]\ \ (\ \ 28754584h^7w^4 + 10h^3(63504 + 2879488h^4w^4))\cos[\frac{9hw}{2}]))$$

$$+ 12700800h^6(\cos[\frac{7hw}{2}] + 3\cos[\frac{9hw}{2}])\cos[8hw] + 12700800h^6(\mathrm{Sin}[\frac{7hw}{2}] + 3\mathrm{Sin}[\frac{9hw}{2}])\mathrm{Sin}[8hw])$$

$$\frac{1}{928972800h^5w^6{}^4}\quad h\ h_4\ i_4\ \mathrm{Csc}\ \frac{hw}{2}{}^8\ \mathrm{Sec}\ \frac{hw}{2}\quad 16h\ 13368753464h\ w\quad 4\ 4$$

1935360h   68 + 12288h w   4032h   8220 + 1060864h w

$+ 448h^5\ 311040 + 32739328h^4w^4\quad \mathrm{Cos}\,h\,\frac{hw}{2}'\ 64h^2\quad 1798880812h^8w^4$

$112h^4\ \ 164160\ \ 30281728h^4w^4\ \ + 336h^4\quad 33120$

$4747264h^4w^4\ \ \mathrm{Cos}\ h\frac{3hw}{2}i\ 2\ 60h^3\ 919360h^7w^4\mathrm{Cos}\ h\frac{5hw}{2}i$

$15\quad 3470928h^7w^4\ 5h^3\ 12096 + 692224h^4w^4\ \mathrm{Cos}\ h\frac{7hw}{2}i + 12605008h^7w^4\ 2$

$h^5\ \ 28224 + 839680h^4w^4\ \ \mathrm{Cos}\ h\frac{9hw}{2}i + 3628800h^6\ \ \mathrm{Cos}\ h\frac{7hw}{2}i$

$+7\mathrm{Cos}\ h\frac{9}{2}i\ \ \mathrm{Cos}[8hw]+3628800h^6\ \ \mathrm{Sin}\ h\frac{7}{2}i\ +7\mathrm{Sin}\ h\frac{9}{2}i\ \ \mathrm{Sin}[8hw]\ i$

The additional formulae are:

$$y_{n+7} = \frac{2y_n}{5} + 7y_{n+3} \quad 14y_{n+4} + \frac{42y_{n+5}}{5}$$

$$+\frac{1}{46448640h^{o}w^{4}} f_{n+4} \quad 177811200h^{6} + 12700800h^{6}\mathrm{Cos}[3hw]$$

$$3h\ 49h\ 33252116h^{8}w^{4} + 2940h^{4}\ 1152 + 16807h^{4}w^{4}\ 280h^{4}\ 7344 + 55223h^{4}w^{4}\ \mathrm{Cos}[hw]$$

$$7430752h^{9}w^{4}\mathrm{Cos}[2hw] + 3232152h^{9}w^{4}\mathrm{Cos}[3hw]$$

$$hw_{8}$$

$$+16h^{2} \quad 3386334h^{7}w^{4} \quad 20h^{3} \quad 5292 + 170471h^{4}w^{4} \quad \mathrm{Cos}[4hw] \quad \mathrm{Csc}$$

$$\frac{1}{464486400h^{6}w^{4}} \quad \frac{hw}{2}_{7} \quad f_{n+8}\mathrm{Csc}\ 2 \quad \mathrm{Csc}[hw] \quad 70h^{2} \quad 20558956h^{8}w^{4}\ \frac{hw}{2}$$

$$+ h^{4} \quad 60480 \quad 7877681h^{4}w^{4} + 21h^{4} \quad 20160 + 1366169h^{4}w^{4} \quad \mathrm{Cos}\ \frac{hw}{2}$$

$$756h^{3} \quad 2950341h^{7}w^{4} + 35h^{3} \quad 1152 \quad 84035h^{4}w^{4} \quad \mathrm{Cos}\ \frac{3}{2}$$

$$+ 2028901h^{7}w^{4} \quad 5h^{3} \quad 24h^{3} \quad 33530h^{7}w^{4}\mathrm{Cos}\ \frac{5hw}{2}$$

$$12096 + 405769h^{4}w^{4} \quad \mathrm{Cos}\ 72 \quad \frac{hw}{}$$

$$+ 3628800h^{6}\mathrm{Cos}\ \frac{hw}{2} \quad \mathrm{Cos}[7hw] + 3628800h^{6}\mathrm{Sin}\ \frac{hw}{2} \quad \mathrm{Sin}[7hw]$$

$$\frac{1}{464486400h^{6}w^{4}} \quad \frac{hw}{2} \quad \mathrm{Csc}[hw]140h^{2} \quad h^{4} \quad 30565366h^{o}w^{4} \quad 78985697h^{4}w^{4}$$

$$+ 63h^{4} \quad 12096 + 1740725h^{4}w^{4} \quad \mathrm{Cos}\ \frac{hw}{2}$$

$$378h^{3} \quad 8641015h^{7}w^{4} + h^{3} \quad 67200 \quad 8621991h^{4}w^{4} \quad \mathrm{Cos}\ \frac{3}{2}$$

$$24h^{3} \quad 80360h^{7}w^{4}\mathrm{Cos}\ \frac{5hw}{2} \quad + 9842h^{7}w^{4}\mathrm{Cos}\ \frac{7hw}{2}$$

$$60480h^{3}\mathrm{Cos} \quad 92 \quad \frac{hw}{} \quad + 3628800h^{6}\mathrm{Cos} \quad 92 \quad \frac{hw}{} \quad \mathrm{Cos}[7hw]$$

$$3628800h^{6}\mathrm{Sin}\ \frac{9hw}{} \mathrm{Sin}[7hw]\ 2$$

$$\frac{1}{116121600h^6w^4}\, f_{n+6}\mathrm{Csc}\left[\frac{hw}{2}\right]^7 \mathrm{Csc}[hw]\left(\frac{1}{2}\right.$$

$$49h\,1234455982h^9w^4 + 4410h^5 \quad 8064\ 472997h^4w^4 \quad +2016h^5 \quad 3420$$

$$3h\ \ 49h\,187395374h^8w^4 + 14h^4\ 423360\ 21448133h^4w^4 + 16h^4\ 196560 + 7099757h^4w^4\ \mathrm{Cos}\left[\frac{3hw}{2}\right] + 3171560$$

$$+ 6350400h^6\ 3\mathrm{Cos}\left[\frac{7hw}{2}\right] + \mathrm{Cos}\left[\frac{9hw}{2}\right]\ \mathrm{Cos}[7hw]$$

$$+ 6350400h^6\ 3\mathrm{Sin}\left[\frac{7hw}{2}\right] + \mathrm{Sin}\left[\frac{9hw}{2}\right]\ \mathrm{Sin}[7hw]$$

$$\frac{1}{464486400h^5w^4}\, f_{n+7}\mathrm{Csc}\left[\frac{hw}{2}\right]^7 \mathrm{Csc}[hw]^5\ 14h\,3562899904h^9w^4$$

$$493920h^4\ 132 + 12005h^4w^4\ 4032h^4\ 2460 + 160867h^4w^4$$

$$+ 392h^2\ 155520 + 7666393h^8w^4\ \mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$42h\ 112320\ 9275063h^4w^4$$

$$+ 112h^4\ 585698326h^4w^4 + 98\ 47520 + 2898007h^4w^4\ \mathrm{Cos}\left[\frac{3hw}{2}\right]\ 24h^3\ 316190h^7w^4\ \mathrm{Cos}\left[\frac{5hw}{2}\right]$$

$$+ 7392917h^7w^4\ 15h^3\ 28224 + 492205h^4w^4\ \mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$+ 2028901h^7w^4\ 5h^3\ 12096 + 405769h^4w^4\ \mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$+ 3628800h^6\ 7\mathrm{Cos}\left[\frac{7hw}{2}\right] + \mathrm{Cos}\left[\frac{9hw}{2}\right]\ \mathrm{Cos}[7hw]$$

$$+ 3628800h^6\ 7\mathrm{Sin}\left[\frac{7hw}{2}\right] + \mathrm{Sin}\left[\frac{9hw}{2}\right]\ \mathrm{Sin}[7hw]$$

$$+ \frac{1}{232243200h^6w^4}\, f_{n+2}\mathrm{Csc}\left[\frac{hw}{2}\right]^7 \mathrm{Csc}[hw](49h(1462184842h^9w^4$$

$$4410h^5(10752\ 602651h^4w^4)\ 22176h^5(\ 660 + 16807h^4w^4)$$

$$196h^5(\ 285120 + 7978523h^4w^4))\mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$3h(49h(213517858h^8w^4 + 14h^4(544320\ 25745923h^4w^4)$$

$$16h^4(\ 317520 + 9236647h^4w^4))\ \mathrm{cos}\left[\frac{3hw}{2}\right] + 1264760h^9w^4\ \mathrm{Cos}\left[\frac{5hw}{2}\right]$$

$$8h^2(7(\ 4735052h^7w^4 + 105h^3(\ 288 + 45619h^4w^4))\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$(\ 16838878h^7w^4 + 10h^3(63504 + 1687903h^4w^4))\mathrm{Cos}\left[\frac{9hw}{2}\right]))$$

$$12700800h^6(\mathrm{Cos}\left[\frac{7hw}{2}\right] + 3\mathrm{Cos}\left[\frac{9hw}{2}\right])\mathrm{Cos}[7hw]$$

$$12700800h^6(\mathrm{Sin}\left[\frac{7hw}{2}\right] + 3\mathrm{Sin}\left[\frac{9hw}{2}\right])\mathrm{Sin}[7hw])$$

$$\frac{1}{464486400h^6w^4}\, f_{n+5}\mathrm{Csc}\left[\frac{hw}{2}\right]^7 \mathrm{Csc}[hw](490h(335793856h^9w^4$$

$$+14112h^5(1092\ 40817h^4w^4)\ 4032h^5(\ 876 + 16807h^4w^4) + 392h^5(\ 41472 + 780325h^4w^4))\mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$3h(98h(213517858h^8w^4 + 14h^4(907200\ 25745923h^4w^4)$$

$$+16h^4(453600 + 9236647h^4w^4))\mathrm{Cos}[\frac{3hw}{2}]$$

$$55984880h^9w^4\mathrm{Cos}[\frac{5hw}{2}]$$

$$8h^2(7(9528939h^7w^4 + 5h^3(60480 + 1870379h^4w^4))\mathrm{Cos}[\frac{7hw}{2}]$$

$$5(8370439h^7w^4 \quad h^3(254016 + 8329069h^4w^4))\mathrm{Cos}[\frac{9hw}{2}]))$$

$$25401600h^6(5\mathrm{Cos}[\frac{7hw}{2}]$$

$$3\mathrm{Cos}[\frac{9hw}{2}])\mathrm{Cos}[7hw]$$

$$25401600h^6(5\mathrm{Sin}[\frac{7hw}{2}]$$

$$3\mathrm{Sin}[\frac{9hw}{2}])\mathrm{Sin}[7hw]$$

$$\frac{1}{464486400h^6w^4}f_{n+3}\mathrm{Csc}[\frac{hw}{2}]\,\mathrm{Csc}[hw](490h(335793856h^7w^9$$

$$14112h^5(1260 \quad 40817h^4w^4) \quad 28224h^5(180 + 2401h^4w^4)$$

$$1960h^5(10368 + 156065h^4w^4))\mathrm{Cos}[\frac{hw}{2}]$$

$$3h(98h(187395374h^8w^4 + 14h^4(1028160 \quad 21448133h^4w^4)$$

$$16h^4(574560 + 7099757h^4w^4))\mathrm{Cos}[\frac{3hw}{2}] + 72284240h^9w^4\mathrm{Cos}[\frac{5hw}{2}]$$

$$8h^2(7(22183707h^7w^4 + 5h^3(36288 + 4391429h^4w^4))\mathrm{Cos}[\frac{7hw}{2}]$$

$$5(13046999h^7w^4 \quad 7h^3(60480 + 1855973h^4w^4))\mathrm{Cos}[\frac{9hw}{2}]))$$

$$25401600h^6(3\mathrm{Cos}[\frac{7hw}{2}] + 5\mathrm{Cos}[\frac{9hw}{2}])\mathrm{Cos}[7hw]$$

$$25401600h^6(3\mathrm{Sin}[\frac{7hw}{2}] + 5\mathrm{Sin}[\frac{9hw}{2}])\mathrm{Sin}[7hw])$$

$$y_{n+6} = \frac{y_n}{10} + 2y_{n+3} \quad \frac{9y_{n+4}}{2} + \frac{18y_{n+5}}{5}$$

$$+\frac{1}{46448640h^{6}w^{4}} f_{n+4} \; (57153600h^{6} + 12700800h^{6}\mathrm{Cos}[2hw]$$

$$2h(42h(\; 14245038h^{8}w^{4} + 2520h^{4}(1152 + 9072h^{4}w^{4}) \; 280h^{4}(7344 + 29808h^{4}w^{4}))\mathrm{Cos}[hw]$$

$$2843316h^{9}w^{4}\mathrm{Cos}[2hw] + 1153116h^{9}w^{4}\mathrm{Cos}[3hw]$$

$$6h^{2}(1818486h^{7}w^{4} \quad 20h^{3}(5292 + 92016h^{4}w^{4}))\mathrm{Cos}[4hw]))\mathrm{Csc}[\frac{hw}{2}]^{8}$$

$$+\frac{1}{464486400h^{6}w^{4}} f_{n+8}\mathrm{Csc}[\frac{hw}{2}]^{7}\mathrm{Csc}[hw](\; 30h^{2}(\; 8879040h^{8}w^{4}$$

$$h^{4}(60480 \;\; 4252176h^{4}w^{4}) + 18h^{4}(\; 20160 + 737424h^{4}w^{4}))\mathrm{Cos}[\frac{hw}{2}]$$

$$324h^{3}(1591066h^{7}w^{4} + 35h^{3}(1152 \;\; 45360h^{4}w^{4}))\mathrm{Cos}[\frac{3hw}{2}$$

$$] \; 6h^{3}(7740h^{7}w^{4}\mathrm{Cos}[\frac{5hw}{2}]$$

$$(1095336h^{7}w^{4} \quad 5h^{3}(12096 + 219024h^{4}w^{4}))\mathrm{Cos}[\frac{7hw}{2}])$$

$$3628800h^{6}\mathrm{Cos}[\frac{7hw}{2}]\mathrm{Cos}[6hw] + 3628800h^{6}\mathrm{Sin}[\frac{7hw}{2}]\mathrm{Sin}[6hw])$$

$$+\frac{1}{464486400h^{6}w^{4}} f_{n}\mathrm{Csc}[\frac{hw}{2}]^{7}\;\mathrm{Csc}[hw](\; 90h^{2}(\; 8067672hw^{8}{}^{4}$$

$$h^{4}(616896 \;\; 42634512h^{4}w^{4}) + 54h^{4}(\; 12096 + 939600h^{4}w^{4}))\mathrm{Cos}[\frac{hw}{2}]$$

$$108h^{3}(4669122h^{7}w^{4} + h^{3}(67200 \;\; 4653936h^{4}w^{4}))\mathrm{Cos}[\frac{3hw}{2}]$$

$$6h^{3}(80820h^{7}w^{4}\mathrm{Cos}[\frac{5hw}{2}] + 9792h^{7}w^{4}\mathrm{Cos}[\frac{7hw}{2}] \;\; 60480h^{3}\mathrm{Cos}[\frac{9hw}{2}])$$

$$3628800h^{6}\mathrm{Cos}[\frac{9hw}{2}]\mathrm{Cos}[6hw] + 3628800h^{6}\mathrm{Sin}[\frac{9hw}{2}]\mathrm{Sin}[6hw])$$

$$+\frac{1}{116121600h^{6}w^{4}}f_{n+6}\mathrm{Csc}\left[\frac{hw}{2}\right]^{7}\mathrm{Csc}[hw]\left(\frac{1}{2}(42h(452465568h^{9}w^{4}\right.$$

$$3240h^{5}(8064\ \ 255312h^{4}w^{4}) + 2016h^{5}(3420\ \ 63504h^{4}w^{4})$$

$$1176h^{5}(\ \ 25920 + 426384h^{4}w^{4}))\mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$2h(42h(77816322h^{8}w^{4} + 12h^{4}(423360\ \ 11577168h^{4}w^{4})$$

$$16h^{4}(\ 196560 + 3832272h^{4}w^{4}))\mathrm{Cos}\left[\frac{3hw}{2}\right] + 1153980h^{9}w^{4}\mathrm{Cos}\left[\frac{5hw}{2}\right] + 3h^{2}$$

$$(7(\ 157806h^{7}w^{4} + 5h^{3}(\ 18144 + 32400h^{4}w^{4}))\mathrm{Cos}\left[\frac{7hw}{2}\right] + (3798306h^{7}w^{4}$$

$$\left.10h^{3}(21168 + 379728h^{4}w^{4}))\mathrm{Cos}\left[\frac{9hw}{2}\right]\right)))$$

$$6350400h^{6}\left(3\mathrm{Cos}\left[\frac{7hw}{2}\right] + \mathrm{Cos}\left[\frac{9hw}{2}\right]\right)\mathrm{Cos}[6hw]$$

$$6350400h^{6}\left(3\mathrm{Sin}\left[\frac{7hw}{2}\right] + \mathrm{Sin}\left[\frac{9hw}{2}\right]\right)\mathrm{Sin}[6hw])$$

$$\frac{1}{464486400h^{6}w^{4}} f_{n+7}\mathrm{Csc}\left[\frac{hw}{2}\right]^{7}\mathrm{Csc}[hw](12h(1308392604h^{9}w^{4}$$

$$362880h^{5}(\ 132 + 6480h^{4}w^{4})\ \ 4032h^{5}(\ 2460 + 86832h^{4}w^{4})$$

$$336h^{5}(\ 155520 + 4138128h^{4}w^{4}))\mathrm{Cos}\left[\frac{hw}{2}\right]\ \ 24h^{2}(245737530h^{8}w^{4}$$

$$84h^{4}(112320\ \ 5006448h^{4}w^{4}) + 112h^{4}(\ 47520 + 1564272h^{4}w^{4}))\mathrm{Cos}\left[\frac{3hw}{2}\right]$$

$$6h^{3}(139320h^{7}w^{4}\mathrm{Cos}\left[\frac{5hw}{2}\right] + (3994992h^{7}w^{4}\ 15h^{3}(28224 + 265680h^{4}w^{4}))\mathrm{Cos}\left[\frac{7hw}{2}\right] + (1095336h^{7}w^{4}\ 5h^{3}(12096 + 219024h^{4}w^{4}))\mathrm{Cos}\left[\frac{9hw}{2}\right])$$

$$+ 3628800h^{6}(7\mathrm{Cos}\left[\frac{7hw}{2}\right] + \mathrm{Cos}\left[\frac{9hw}{2}\right])\mathrm{Cos}[6hw]$$

$$+ 3628800h^{6}(7\mathrm{Sin}\left[\frac{7hw}{2}\right] + \mathrm{Sin}\left[\frac{9hw}{2}\right])\mathrm{Sin}[6hw])$$

$$+ \frac{1}{232243200h^{6}w^{4}} f_{n+2}\mathrm{Csc}\left[\frac{hw}{2}\right]^{7}\mathrm{Csc}[hw](42h(530539272h^{9}w^{4}$$

$$3240h^{5}(10752\ \ 325296h^{4}w^{4})\ \ 22176h^{5}(\ 660 + 9072h^{4}w^{4})$$

$$168h^{5}(\ 285120 + 4306608h^{4}w^{4}))\mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$2h(42h(87325110h^{8}w^{4} + 12h^{4}(544320\ \ 13897008h^{4}w^{4})$$

$$16h^{4}(\ 317520 + 4985712h^{4}w^{4}))\mathrm{Cos}\left[\frac{3hw}{2}\right] + 208980h^{9}w^{4}\mathrm{Cos}\left[\frac{5hw}{2}\right]$$

$$3h^{2}(7(\ 2530422h^{7}w^{4} + 105h^{3}(\ 288 + 24624h^{4}w^{4}))\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$(\ 9070578h^{7}w^{4} + 10h^{3}(63504 + 911088h^{4}w^{4}))\mathrm{Cos}\left[\frac{9hw}{2}\right]))$$

$$12700800h^{6}(\mathrm{Cos}\left[\frac{7hw}{2}\right] + 3\mathrm{Cos}\left[\frac{9hw}{2}\right])\mathrm{Cos}[6hw]$$

$$12700800h^{6}(\mathrm{Sin}\left[\frac{7hw}{2}\right] + 3\mathrm{Sin}\left[\frac{9hw}{2}\right])\mathrm{Sin}[6hw])$$

$$\frac{1}{464486400h^{6}w^{4}}\ \ f_{n+5}\mathrm{Csc}\ \ \frac{hw}{2}\ \ {}^{7}\ \ \mathrm{Csc}[hw]\ \ \ 420h\ 122942556h^{9}w^{4} + 10368h^{5}\ 1092\ \ 22032h^{4}w^{4}\ \ \ 4032h^{5}\ \ \ 876+$$

$$2h\ \ \ 84h\ 87325110h^{8}w^{4} + 12h^{4}\ \ 907200\ \ 13897008h^{4}w^{4}$$

$$+16h^{4}\ \ 453600+4985712h^{4}w^{4}\ \ \ \mathrm{Cos}\ \ \frac{7hw}{2}\ \ \ \frac{hw}{2}\ \ +12297960h^{9}w^{4}\mathrm{Cos}\ \ \frac{hw}{2}\ \ +3h^{2}\ \ \ 7\ \ 5124024h^{7}w^{4} + 5h^{3}\ \ 60480+$$

$$+ 25401600h^{6}\ \ \ 5\mathrm{Cos}\ \ \frac{7hw}{2}\ \ + 3\mathrm{Cos}\ \ \frac{9hw}{2}\ \ \mathrm{Cos}[6hw]$$

$$+ 25401600h^{6}\ \ \ 5\mathrm{Sin}\ \ \frac{7hw}{2}\ \ + 3\mathrm{Sin}\ \ \frac{9hw}{2}\ \ \mathrm{Sin}[6hw]$$

$$\frac{1}{464486400h^{6}w^{4}}\quad h\quad \frac{hw}{2}^{7}\quad \mathrm{Csc}^{4}\quad h\,w^{4}\quad \mathrm{Csc}[hw]^{5}\quad h^{5}\quad 420h\ 122942556h^{5}w^{4}\quad h\,w^{4}$$

$$+\ 10368_{5}\quad 1260\quad 22032\quad 4\quad 4\ 28224\ hw\ 180 + 1296$$

$$+\ 1680h\quad 10368 + 84240h\ w\quad \mathrm{Cos}\ \frac{}{2}$$

$$2h\quad 84h\ 77816322h^{8}w^{4} + 12h^{4}1028160\ 11577168h^{4}w^{4} + 16h^{4}\quad 574560 + 3832272h^{4}w^{4}\quad \mathrm{Cos}$$

$$25497720h^{9}w^{4}\mathrm{Cos}\ \frac{5hw}{2}$$

$$+3h^{2}\quad 7\quad 12077712h^{7}w^{4} + 5h^{5}\quad hw\quad 36288 + 2370384h^{4}w^{4}\quad hw\quad \mathrm{Cos}\ \frac{7hw}{2}\quad +5\quad 7067736h^{7}w^{4}\quad 7h^{5}\quad 6048$$

$$+\ 25401600h^{6}\quad 3\mathrm{Cos}\ \frac{7}{2}\ + 5\mathrm{Cos}\ \frac{9}{2}\quad \mathrm{Cos}[6hw]\quad i$$

$$+\ 25401600h^{6}\quad 3\mathrm{Sin}\ \frac{7}{2}\ + 5\mathrm{Sin}\ \frac{9}{2}\quad \mathrm{Sin}[6hw]\quad hw\quad hw$$

$$\frac{1}{928972800h^{6}w^{4}}\,(-1)^{n+1}\,\mathrm{Csc}\left[\frac{hw}{2}\right]^{8}\mathrm{Sec}\left[\frac{hw}{2}\right]\Big(12h\,(2103103836h^{9}w^{4}-1088640h^{5}-68+3888h^{4}w^{4})$$

$$-24h^{2}\,(-300594654h^{8}w^{4}+84h^{4}-164160)\,(9581328h^{4}w^{4}+336h^{4}-33120+1502064h^{4}w^{4})\,\mathrm{Cos}\left[\frac{3hw}{2}\right]$$

$$-6h^{3}\,(-769320h^{7}w^{4}\mathrm{Cos}\left[\frac{5hw}{2}\right]+1095336h^{7}w^{4}-5h^{3}-12096+219024h^{4}w^{4})\,\mathrm{Cos}\left[\frac{7hw}{2}\right]+3994992$$

$$+3628800h^{6}\,\mathrm{Cos}\left[\frac{7hw}{2}\right]+7\,\mathrm{Cos}\left[\frac{9hw}{2}\right]\mathrm{Cos}[6hw]$$

$$+3628800h^{6}\,\mathrm{Sin}\left[\frac{7hw}{2}\right]+7\,\mathrm{Sin}\left[\frac{9hw}{2}\right]\mathrm{Sin}[6hw]\Big)$$

$$y_{n+1} = \Big(\frac{5}{2y_n} + 2y_{n+3} \quad 2y_{n+4} + \frac{5}{3y_{n+5}}$$

$$+ \frac{1}{46448640h^{o}w^{4}} f_{n+4} \quad 25401600h^{6} + 12700800h^{6}\mathrm{Cos}[3hw]$$

$$+3h\ 7h\ 382612h^{8}w^{4} + 420h^{4}\ 1152 + 7h^{4}w^{4} \quad 280h^{4}\ 7344 + 23h^{4}w^{4} \quad \mathrm{Cos}[hw] + 1209824h^{9}w^{4}\mathrm{C}$$

$$\frac{1}{464486400h^{6}w^{4}} f_{n+8}\mathrm{Csc} \quad \frac{hw}{2}\ \mathrm{Csc}[hw]\ 20h^{2} \quad 104090h^{o}w^{4} + h^{4}\ 60480\ 3281h^{4}w^{4} + 3h^{4} \quad 2$$

$$54h^{3} \quad 23559h'w^{4} + 35h^{3}\ 1152\ 35h^{4}w^{4}\ \mathrm{Cos}\ \frac{hw}{2}$$

$$3628800h^{6}\mathrm{Cos}[hw]\mathrm{Cos}\ \frac{7hw}{2}$$

$$+24h^{4} \quad 10040h'w^{4}\mathrm{Cos} \quad \frac{hw}{2} + \quad 811h'w^{4}\ 5h^{4} \quad 12096 + 169h^{4}w^{4} \quad \mathrm{Cos} \quad \frac{hw}{2}$$

$$3628800h^{6}\mathrm{Sin}[hw]\mathrm{Sin}\ \frac{7hw}{2}$$

$$+ \frac{1}{464486400h^{o}w^{4}} f_{n}\mathrm{Csc}\ \frac{hw}{2}\ \mathrm{Csc}[hw]\ 10h^{2}\ 65588h^{8}w^{4} + h^{4}\ 616896\ 32897h^{4}w^{4} + 9h^{4}\ 12$$

$$108h^{3} \quad 11573h'w^{4} + h^{3} \quad 67200\ 3591h^{4}w^{4}\ \mathrm{Cos}\ \frac{3hw}{2}$$

$$3628800h^{6}\mathrm{Cos}[hw]\mathrm{Cos}\ \frac{9hw}{2}$$

$$+24h^{4} \quad 56870h'w^{4}\mathrm{Cos} \quad \frac{hw}{2} \quad +9752h'w^{4}\mathrm{Cos} \quad \frac{hw}{2} \quad 60480h^{4}\mathrm{Cos} \quad \frac{hw}{2} \Big)$$

$$+ 3628800h^{6}\mathrm{Sin}[hw]\mathrm{Sin}\ \frac{9hw}{2}$$

$$+ \frac{1}{116121600h^{o}w^{4}} f_{n+6}\mathrm{Csc}\ \frac{hw}{2}\ \mathrm{Csc}[hw]\ 6350400h^{6}\mathrm{Cos}[hw]\ 3\mathrm{Cos}\ \frac{7hw}{2} + \mathrm{Cos}\ \frac{9hw}{2}$$

$$+ \frac{1}{2}\ 7h\ 3316678h^{9}w^{4} + 90h^{5}\ 8064197h^{4}w^{4} + 2016h^{5}\ 3420\ 49h^{4}w^{4} + 196h^{5} \quad 25920 + 329h^{4}w$$

$$+ 6350400h^{o}\mathrm{Sin}[hw] \quad 3\mathrm{Sin}\ \frac{7hw}{2} + \mathrm{Sin}\ \frac{9hw}{2}$$

$$\frac{1}{464486400h^{6}w^{4}}\ f_{n+7}\ \text{"}\ \text{Csc}\ \frac{hw}{2}\ \text{Csc}[hw]\ 2h\ 7508224h^{9}w^{4}\ 10080h^{5}\ 132+5h^{4}w^{4}\ 4032h^{5}$$

$$+6h^{2}\ 1757050h^{0}w^{4}+14h^{4}\ 112320\ 3863h^{4}w^{4}\ +112h^{4}\ 47520+1207h^{4}w^{4}\ \text{Cos}\ \frac{hw}{2}^{3}$$

$$+3628800h^{6}\text{Cos}[hw]\ 7\text{Cos}\ \frac{7}{2}\ +\ \text{Cos}\ \frac{9}{2}\ 28224+205h^{4}w^{4}\ \text{Cos}\ \frac{hw}{2}^{7}\ +\ 811h^{7}w^{4}\ 5$$

$$+24h^{0}\ 126470h^{7}w^{4}\text{Cos}\ \frac{hw}{2}^{5}\ +\ 12827h^{7}w^{4}\ 15h^{0}\ \#$$

$$+3628800h^{6}\text{Sin}[hw]\ 7\text{Sin}\ \frac{hw}{2}^{7}\ +\ \text{Sin}\ \frac{hw}{2}^{9}$$

$$+\frac{1}{232243200h^{6}w^{4}}f_{n+2}\mathrm{Csc}\frac{hw}{2}\ \mathrm{Csc}[hw]\ \ 7h332482h^{9}w^{4}+90h^{5}\ 10752\ \ 251h^{4}w^{4}$$

$$22176h^{5}\qquad 660+7h^{4}w^{4}\qquad +28h^{5}\qquad 285120+3323h^{4}w^{4}\qquad \mathrm{Cos}\frac{hw}{2}$$

$$+12700800h^{6}\mathrm{Cos}[hw]\ \ \mathrm{Cos}\frac{7hw}{2}+3\mathrm{Cos}\frac{9hw}{2}$$

$$+3h\ \ 7h\ \ \ 1912270h^{8}w^{4}+2h^{4}544320\ \ 10723h^{4}w^{4}+16h^{4}\ \ 317520+3847h^{4}w^{4}\ \ \mathrm{Cos}\frac{3hw}{2}$$

$$505880h^{9}w^{4}\mathrm{Cos}\frac{5hw}{2}$$

$$+8h^{2}\qquad 7\qquad 41068h^{7}w^{4}+105h^{3}\ \ 288+19h^{4}w^{4}\qquad \mathrm{Cos}\frac{72}{hw}\qquad 21422h^{7}w^{4}+10h^{3}63504+703h^{4}w^{4}\ \ \mathrm{Cos}\frac{9hw}{2}$$

$$+12700800h^{6}\mathrm{Sin}[hw]\ \mathrm{Sin}\frac{2}{7hw}+3\mathrm{Sin}\frac{92}{hw}$$

$$\frac{1}{464486400h^{6}w^{4}}f_{n+5}\mathrm{Csc}\qquad \frac{hw}{2}^{7}\ \ \mathrm{Csc}[hw]\ \ 70h\ 1152256h^{3}w^{4}+288h^{5}\qquad \frac{hw}{2}\qquad 1092\ \ 17h^{4}w^{4}$$

$$4032h^{5}\ \ 876+7h^{4}w^{4}+56h^{5}\ \ 41472+325h^{4}w^{4}\ \ \mathrm{Cos}\frac{hw}{2}$$

$$+25401600h^{6}\mathrm{Cos}[hw]\qquad 5\mathrm{Cos}\frac{72}{hw}\ +3\mathrm{Cos}\frac{9hw}{2}$$

$$+3h\ \ 14h\ \ \ 1912270h^{8}w^{4}+2h^{4}907200\ \ 10723h^{4}w^{4}+16h^{4}\ \ 453600+3847h^{4}w^{4}\ \ \mathrm{Cos}\frac{3hw}{2}$$

$$12182960h^{9}w^{4}\mathrm{Cos}\frac{5hw}{2}$$

$$+8h^{2}\qquad 7\qquad 26331h^{7}w^{4}+5h^{3}\ \ 60480+779h^{4}w^{4}\qquad \mathrm{Cos}\frac{72}{hw}\qquad +5\ \ 3241h^{7}w^{4}\ \ h^{3}\qquad 254016+3469h^{4}w^{4}\ \ \mathrm{Cos}\frac{9hw}{2}$$

$$+25401600h^{6}\mathrm{Sin}[hw]\ \ 5\mathrm{Sin}\frac{2}{7hw}+3\mathrm{Sin}\frac{92}{hw}$$

$\frac{1}{464486400h^{8}w^{4}}$ (f_{n+3}Csc $\frac{hw}{2}$ ^{7} Csc[hw] 70h $1152256h^{9}w^{4}$ $+288h^{5}$ 1260 $17h^{4}w^{4}$ $28224h^{5}$

+ $25401600h^{8}$Cos[hw] 3Cos $\frac{7}{}$2 + 5Cos $\frac{9}{}$2

hw hw

hw hw $\frac{{}_{3}hw}{2}$

+3h 14h $917282h^{8}w^{4}$ $+2h^{4}$ 1028160 $8933h^{4}w^{4}$ $+16h^{4}$ $574560+2957h^{4}w^{4}$ Cos $\frac{hw}{2}$ +284

$\frac{7}{2}$ $\frac{9}{2}$

+ $25401600h^{6}$Sin[hw] 3Sin $\frac{7}{2}$ + 5Sin $\frac{9}{2}$

1 hw ^{8} hw

$\frac{}{928972800h^{8}w^{4}}$ f_{n+1}Csc $\frac{hw}{2}$ Sec $\frac{hw}{2}$ 2h $4723456h^{9}w^{4}$ $30240h^{5}$ $68+3h^{4}w^{4}$ $4032h^{5}$

hw hw hw

$+6h^{2}$ $755574h^{8}w^{4}$ $+14h^{4}$ 164160 $7393h^{4}w^{4}$ $+336h^{4}$ $33120+1159h^{4}w^{4}$ Cos $\frac{{}^{3}hw}{2}$

+ $3628800h^{6}$Cos[hw] Cos $\frac{7}{2}$ + 7Cos $\frac{9}{2}$

$+24h^{5}$ $603170h^{7}w^{4}$Cos 5 2 + $811h^{7}w^{4}$ $5h^{5}$ $12096+169h^{4}w^{4}$ Cos 7 2 + $12827h^{7}w^{4}$ 15

hw hw

+ $3628800h^{6}$Sin[hw] Sin $\frac{7\,2}{hw}$ + 7Sin $\frac{9\,2}{hw}$)

$$y_{n+2} = \Big(10\, y_n + 2 y_{n+3} \quad 3 y_{n+4}^{2} + 2 y_{n+5}^{5}$$

$$+ \frac{1}{46448640 h^{6} w^{4}} f_{n+4} \quad 19051200 h^{6} + 12700800 h^{6} \mathrm{Cos}[2hw]$$

$$+2 \quad 14 \quad 185714 \quad +840 \quad {}^{7}$$

$$h \quad h \quad h^{8} w^{4} \quad h^{4} \quad 1152 + 112 h^{4} w^{4} \quad 280 h^{4} \quad 7344 + 368 h^{4} w^{4} \quad \mathrm{Cos}[hw] + 510468 h^{9} w$$

$$\frac{1}{464486400 h^{6} w^{4}} \; {}^{+}{}_{n+8} \mathrm{Csc} \quad 2 \; \mathrm{Csc}[hw] \quad 30 h^{2} \quad 40024 h^{8} w^{4} + h^{4} \quad 60480 \quad 52496 h^{4} w^{4} + 6 h^{4} \quad 2$$

$$\frac{hw}{2}$$

$$36 h^{3} 5926 h^{7} w^{4} + 35 h^{3} \quad 1152 \quad 560 h^{4} w^{4} \; \mathrm{Cos}\frac{hw}{2}$$

$$3628800 h^{6} \mathrm{Cos}[2hw] \mathrm{Cos}\frac{7hw}{2}$$

$$+6h^{5} \quad 9580 h^{7} w^{4} \mathrm{Cos} \quad {}^{5\,2} \frac{hw}{} \quad + \quad 13536 h^{7} w^{4} \quad 5h^{5} \quad 12096 + 2704 h^{6} w^{4} \quad \mathrm{Cos} \quad {}^{7\,2} \frac{hw}{}$$

$$3628800 h^{6} \mathrm{Sin}[2hw] \mathrm{Sin}\frac{7hw}{2}$$

$$+ \frac{1}{464486400 h^{6} w^{4}} \; {}^{+}{}_{n} \mathrm{Csc} \quad \frac{hw}{2} \; {}^{7} \mathrm{Csc}[hw] \quad 10 h^{2} \quad 342704 h^{6} w^{4} + h^{4} \quad 616896 \; {}^{hw} \quad 526352 h^{4} w^{4} \quad +18 h^{4}$$

$$108 h^{3} \quad 48110 h^{7} w^{4} + h^{3} \quad 67200 \quad 57456 h^{4} w^{4} \; \mathrm{Cos}\frac{3}{2}$$

$$3628800 h^{6} \mathrm{Cos}[2hw] \mathrm{Cos}\frac{9hw}{2}$$

$$+6h^{5} \quad 82660 h^{7} w^{4} \mathrm{Cos} \quad {}^{5\,2} \frac{hw}{} \quad +9592 h^{7} w^{4} \mathrm{Cos} \quad {}^{7\,2} \frac{hw}{} \quad 60480 h \mathrm{Cos} \quad {}^{9\,2} \frac{hw}{} \Big)$$

$$+ 3628800 h^{6} \mathrm{Sin}[2hw] \mathrm{Sin}\frac{9hw}{2}$$

$$+\frac{1}{116121600h^{9}w^{4}}f_{n+6}\mathrm{Csc}\left[\frac{hw}{2}\right]^{7}\mathrm{Csc}[hw]\ 6350400h^{6}\mathrm{Cos}[2hw]\ 3\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$+\mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$+\frac{1}{2}\ 14h\ 1663112h^{9}w^{4}+360h^{5}\ 8064\ 3152h^{4}w^{4}+2016h^{5}\ 3420\ 784h^{4}w^{4}+392h^{5}\ 25920+526$$

$$+2h\ 14h\ 125454h^{8}w^{4}+4h^{4}\ 423360\ 142928h^{4}w^{4}+16h^{4}\ 196560+47312h^{4}w^{4}\ \mathrm{Cos}\left[\frac{3hw}{2}\right]+1$$

$$+6350400h^{6}\mathrm{Sin}[2hw]\ 3\mathrm{Sin}\left[\frac{7hw}{2}\right]+\mathrm{Sin}\left[\frac{9hw}{2}\right]$$

$$464486400h^{9}w^{4}\ f_{n+7}\mathrm{Csc}[hw]\ 4h\ 4102244h^{5}w^{4}\ 40320h^{5}\ 132+80h^{4}w^{4}\ 4032h$$

$$\frac{1}{\ }\ \frac{hw}{\ }^{7}$$

$$+8h^{2}\ 124246h^{6}w^{4}+28h^{4}\ 112320\ 61808h^{4}w^{4}+112h^{4}\ 47520+19312h^{4}w^{4}\ \mathrm{Cos}\left[\frac{3hw}{2}\right]$$

$$+6h^{3}\ 150040h^{7}w^{4}\mathrm{Cos}[5hw]\ 7\mathrm{Cos}\left[\frac{7hw}{2}\right]+\mathrm{Cos}\left[\frac{9hw}{2}\right]\ 58792h^{7}w^{4}+15h^{3}\ 28224+3280h^{4}w^{4}\ \mathrm{Cos}[7hw]+13536h^{7}w^{4}$$

$$+3628800h^{6}\mathrm{Sin}[2hw]\ 7\mathrm{Sin}\left[\frac{7hw}{2}\right]+\mathrm{Sin}\left[\frac{9hw}{2}\right]\Big)$$

$$+\frac{1}{232243200h^{6}w^{4}}f_{n+2}\ \mathrm{Csc}\ \frac{hw}{2}^{7}\ \mathrm{Csc}[hw]\ \ 14h\ 1208912h^{9}w^{4}+360h^{5}\ 10752\ \ 4016h^{4}w^{4}\ \ 22176h^{5}\ \ 660+112h$$

$$+12700800h^{6}\mathrm{Cos}[2hw]\ \ \mathrm{Cos}\ \frac{7hw}{2}^{2}+3\mathrm{Cos}\ \frac{9hw}{2}^{2}$$

$$+2h\ \ 14h\ \ 389418h^{8}w^{4}+4h^{4}\ 544320\ \ 171568h^{4}w^{4}+16h^{4}\ \ 317520+61552h^{4}w^{4}\ \ \mathrm{Cos}\ \frac{3hw}{2}\ \ +225060h^{9}w^{4}\mathrm{Cos}$$

$$+12700800h^{6}\mathrm{Sin}[2hw]\ \ \mathrm{Sin}\ \frac{7hw}{2}^{2}+3\mathrm{Sin}\ \frac{9hw}{2}^{2}$$

$$\frac{1}{464486400h^{6}w^{4}}f_{n+5}\mathrm{Csc}\ \frac{hw}{2}^{7}\ \mathrm{Csc}[hw]\ \ 140h\ 536036h^{6}w^{4}+1152h^{5}\ \ \frac{hw}{}\ \ 1092\ \ 272h^{4}w^{4}$$

$$4032h^{5}\ \ 876+112h^{4}w^{4}\ \ +112h^{5}\ \ 41472+5200h^{4}w^{4}\ \ \mathrm{Cos}\ \frac{}{}$$

$$+25401600h^{6}\mathrm{Cos}[2hw]\ \ 5\mathrm{Cos}\ \frac{7hw}{2}+3\mathrm{Cos}\ \frac{9hw}{2}$$

$$+2h\ \ 28h\ \ 389418h^{8}w^{4}+4h^{4}\ 907200\ \ 171568h^{4}w^{4}+16h^{4}\ \ 453600+61552h^{4}w^{4}\ \ \mathrm{Cos}\ \frac{3hw}{2}$$

$$6177480h^{9}w^{4}\mathrm{Cos}\ \frac{5hw}{2}$$

$$+3h^{2}\ \ 732864h^{7}w^{4}+5h^{3}\ \ 60480+12464h^{4}w^{4}\ \ \mathrm{Cos}\ \frac{7hw}{2}\ \ +5\ \ 55384h^{7}w^{4}\ \ h^{3}\ 254016+55504h^{4}w^{4}\ \ \mathrm{Cos}\ \frac{9h}{}$$

$$+25401600h^{6}\mathrm{Sin}[2hw]\ \ 5\mathrm{Sin}\ \frac{7hw}{2}^{2}+3\mathrm{Sin}\ \frac{9hw}{2}^{2}$$

The derivative formulae are;

$$y_{n}^{0} = \left(\frac{60h}{47y\_n} + \frac{3h}{10y\_n+3} + \frac{4h}{15y\_n+4} + \frac{5h}{6y\_n+5} \right.$$

$$+ \frac{1}{464486400h^{6}w^{4}} \frac{hw}{2}^{7} \left(\frac{f\_n+8Csc}{Csc[hw]} \right. \left. 25h\ 60480h^{4} + 170424h^{6}w^{4}\ Cos \frac{hw}{2} \right. \frac{hw}{2}$$

$$108h^{2} \quad 40320h^{3} \quad 32240h^{7}w^{4} \quad Cos \frac{3}{2} + 2842560h^{5}Cos \frac{7}{2}$$

$$+ 60h^{3} \quad 21660h^{6}w^{4}Cos \frac{5hw}{2} \quad + 498h^{6}w^{4}Cos \frac{7hw}{2}$$

$$+ 3628800h^{6}wSin \frac{7hw}{2}$$

$$+ \frac{1}{46448640h^{6}w^{4}} f\_n+4Csc \frac{hw}{2}^{8} \quad 47628000h^{5} + 3175200h^{5}Cos[4hw]$$

$$+4 \quad 72056320 \quad +587970 \quad Cos[\quad hw]+2526930h^{8}w^{4}Cos[2hw]+1288170h^{8}w^{4}Cos[3hw]+1 \quad + 12700800h^{6}wSin[4hw]$$

$$1 \quad \frac{1}{464486400h^{6}w^{4}} \frac{f\_n+6Csc}{Csc[hw]} \frac{hw}{2}^{7} \quad 15h^{4} \quad 616896h^{8} + 21960h^{4}w^{4} \quad Cos \frac{hw}{2}$$

$$180h^{2} \quad 67200h^{3} \quad 24096h^{7}w^{4} \quad Cos \frac{3}{2}$$

$$+ 60h^{3} \quad 48300h^{6}w^{4}Cos \frac{5hw}{2} \quad + 25122h^{6}w^{4}Cos \frac{7hw}{2}$$

$$+ 2842560h^{5}Cos \frac{9hw}{2} \quad + 3628800h^{6}wSin \frac{9hw}{2}$$

$$\frac{1}{116121600h^{6}w^{4}} \frac{f\_n+6Csc}{hw} \frac{hw}{2}^{7} \quad \frac{1}{Csc[hw]} \frac{hw}{2} \quad 7\ 6894720h^{5} + 6763320h^{5}w^{4} \quad Cos \frac{hw}{2}$$

$$+4h \quad 15h^{2} \quad 635040h^{3}Cos \frac{7hw}{2} \quad 211680h^{3}Cos \frac{9hw}{2} \quad 8h \quad 635040h^{3}Co$$

$$7\ 3144960h^{4} + 1744020h^{6}w^{4} \quad Cos \frac{3hw}{2} \quad +6466500h^{6}w^{4}Cos \frac{5hw}{2}$$

$$+ 6350400h^{6}w \ 3Sin \frac{hw}{7} \ 2 + Sin \frac{hw}{9} \ 2$$

$$\frac{1}{464486400h^{6}w^{4}}\,f\_n+7\mathrm{Csc}\left[\frac{hw}{2}\right]^{7}\mathrm{Csc}[hw]^{2}\left(9918720h^{5}\right.$$

$$15285960h^{9}w^{4}\,\mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$+8h\left(5322240h^{7}+3484260h^{5}w^{2}\,\mathrm{Cos}\left[\frac{3hw}{2}\right]\right.$$

$$47h^{2}\left(423360h^{3}\mathrm{Cos}\left[\frac{7hw}{2}\right]60480h^{3}\mathrm{Cos}\left[\frac{9hw}{2}\right]\right.$$

$$60h^{3}\left(209880h^{6}w^{4}\mathrm{Cos}\left[\frac{5hw}{2}\right]\right.$$

$$+25122h^{5}w^{2}\mathrm{Cos}\left[\frac{7hw}{2}\right]+498h^{5}w^{2}\mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$+3628800h^{6}w\left(7\mathrm{Sin}\left[\frac{7hw}{2}\right]+\mathrm{Sin}\left[\frac{9hw}{2}\right]\right)$$

$$+\frac{1}{232243200h^{6}w^{4}}\,f\_n+2\mathrm{Csc}\left[\frac{hw}{2}\right]^{7}\mathrm{Csc}[hw]^{7}\left(14636160h^{5}\right.$$

$$243720h^{9}w^{4}\,\mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$15h^{2}\left(211680h^{3}\mathrm{Cos}\left[\frac{7hw}{2}\right]635040h^{3}\mathrm{Cos}\left[\frac{9hw}{2}\right]\right.$$

$$+4h\left(7\left(5080320h^{4}+3495780h^{8}w^{4}\,\mathrm{Cos}\left[\frac{3hw}{2}\right]\right.\right.$$

$$1574100h^{8}w^{4}\mathrm{Cos}\left[\frac{5hw}{2}\right]$$

$$8h\left(211680h^{3}\mathrm{Cos}\left[\frac{7hw}{2}\right]635040h^{3}\mathrm{Cos}\left[\frac{9hw}{2}\right]\right.$$

$$+15h^{2}\left(432831h^{6}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right]23901h^{6}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right]\right.$$

$$+12700800h^{6}w\left(\mathrm{Sin}\left[\frac{7hw}{2}\right]+3\mathrm{Sin}\left[\frac{9hw}{2}\right]\right.$$

$$\frac{1}{464486400h^{6}w^{4}}\Big(f\_n+5\mathrm{Csc}\frac{hw}{2}\Big)^{7}\mathrm{Csc}[hw]\,70\,\big(3532032h^{5}+2351880h^{9}w^{4}\big)\mathrm{Cos}\frac{hw}{2}$$

$$15h^{2}\quad 2116800h^{3}\mathrm{Cos}\frac{7\,2}{hw}\quad 1270080h^{3}\mathrm{Cos}\frac{9\,2}{hw}$$

$$+4h\quad 14\quad 7257600h^{4}+3495780h^{6}w^{4}\quad \mathrm{Cos}\frac{hw}{3\,2}\quad +29655000h^{8}w^{4}\mathrm{Cos}\frac{hw}{5\,2}\quad 8h\quad 2116800h^{3}C$$

$$+\,25401600h^{6}w\quad 5\mathrm{Sin}\frac{7}{2}+3\mathrm{Sin}\frac{9}{2}$$

$$\frac{1}{464486400h^{6}w^{4}}\Big(f\_n+5\mathrm{Csc}\frac{hw}{2}\Big)^{7}\mathrm{Csc}[hw]\,70\,\big(5080320h^{5}+2351880h^{9}w^{4}\big)\mathrm{Cos}\frac{hw}{2}$$

$$15h^{2}\quad 1270080h^{3}\mathrm{Cos}\frac{7}{2}\quad 2116800h^{3}\mathrm{Cos}\frac{9}{2}$$

$$+4h\quad 14\quad 9192960h^{4}+1744020h^{6}w^{4}\quad \mathrm{Cos}\frac{hw}{3\,2}\quad +54905400h^{6}w^{4}\mathrm{Cos}\frac{hw}{5\,2}\quad 8h\quad 1270080h^{3}C$$

$$+\,25401600h^{6}w\quad 3\mathrm{Sin}\frac{7\,2}{hw}+5\mathrm{Sin}\frac{9\,2}{hw}\Big)$$

$$\frac{1}{928972800h^{9}w^{4}}\, f\_n{+}1\,\mathrm{Csc}\,\frac{hw}{2}\,\mathrm{Sec}\,\frac{hw}{2}\,\left(33143040h^{5}\,\mathrm{Cos}\,\frac{3hw}{2}\right.$$

$$+\,13834440h^{9}w^{4}\,\mathrm{Cos}\,\frac{hw}{2} + 8h\,11128320h^{4}\,1045260h^{9}w^{4}$$

$$47h^{2} + 60h^{9}\,60480h^{3}\mathrm{Cos}\,\frac{hw}{2} + 862200h^{9}w^{4}\mathrm{Cos}\,\frac{5hw}{2}\,423360h^{3}\mathrm{Cos}\,\frac{hw}{2} + 498h^{6}w^{4}\mathrm{Cos}\,\frac{7hw}{2}$$

$$+\,25122h^{6}w^{4}\mathrm{Cos}\,\frac{9hw}{2} + 3628800h^{6}w\,\mathrm{Sin}\,\frac{7hw}{2} + 7\mathrm{Sin}\,\frac{9hw}{2}$$

$$y_{n+1}^{0} = \left(30h\,\underline{13y\_n} + 6h\,\underline{5y\_n{+}3} + 2h\,\underline{y\_n{+}4} + 10h\,\underline{y\_n{+}5}\right.$$

$$+\frac{1}{46448640h^{6}w^{4}}\, f\_n{+}4\,\mathrm{Csc}\,\frac{hw}{2}\,\left(6350400h^{5}\right.$$

$$7h\,382612h^{8}w^{4} + 420h^{4}\,1152 + 7h^{4}w^{4}\,280h^{4}\,7344 + 23h^{4}w^{4}\,\mathrm{Cos}[hw]$$

$$1209824h^{9}w^{4}\mathrm{Cos}[2hw]\,383976h^{9}w^{4}\mathrm{Cos}[3hw]$$

$$16h^{2}\,546h^{7}w^{4}\,20h^{3}\,5292 + 71h^{4}w^{4}\,\mathrm{Cos}[4hw]$$

$$+3h\,7h\,206810h^{7}w^{4} + 420h^{3}\,1152 + 7h^{4}w^{4}\,\mathrm{Cos}[hw] + 7\,382612h^{8}w^{4} + 420h^{4}\,1152 + 7h^{4}w^{4}$$

$$+\frac{1}{464486400h^{9}w^{4}}\, f\_n{+}8\,\mathrm{Csc}\,\frac{hw}{2}\,\mathrm{Csc}[hw]\,\left(20h^{2}\,65507h^{7}w^{4}\right.$$

$$+\,3h^{3}\,20160 + 569h^{4}w^{4}\,\mathrm{Cos}\,\frac{\ }{2} + 15h\,104090h^{8}w^{4}$$

$$+\,h^{4}\,60480\,3281h^{4}w^{4} + 3h^{4}\,20160 + 569h^{4}w^{4}\,\mathrm{Cos}\,\frac{hw}{2}$$

$$9h^{2}\,23559h^{7}w^{4} + 35h^{3}\,559440h^{9}w^{4}\mathrm{Cos}\,\frac{3hw}{2}$$

$$1152\,35h^{4}w^{4}\,\mathrm{Cos}\,\frac{3hw}{2}$$

$$+\,24h^{3}\,1365h^{6}w^{4}\mathrm{Cos}\,\frac{5hw}{2}\,15h^{6}w^{4}\mathrm{Cos}\,\frac{7hw}{2}$$

$$26h^{2}\,10040h^{7}w^{4}\mathrm{Cos}\,\frac{5hw}{2}$$

$$+\,811h^{7}w^{4}\,5h^{3}\,12096 + 169h^{4}w^{4}\,\mathrm{Cos}\,\frac{7hw}{2}$$

$$\frac{hw}{7}\qquad\qquad \frac{7hw}{7}$$

$$3628800h^{6}w\mathrm{Cos}\,\frac{\ }{2}\,\mathrm{Sin}[hw] + 3628800h^{6}w\mathrm{Cos}[hw]\mathrm{Sin}\,\frac{\ }{2}$$

$$+\frac{1}{464486400h^{8}w^{4}}f_{n}\mathrm{Csc}\frac{hw}{2}\mathrm{Csc}[hw]\ 10h^{2}\ 21925h^{7}w^{4}+9h^{3}\ 12096+725h^{4}w^{4}\ \mathrm{Cos}\frac{hw}{2}$$

$$+5h\ 65588h^{8}w^{4}+h^{4}\ 616896\ 32897h^{4}w^{4}+9h^{4}\ 12096+725h^{4}w^{4}\ \mathrm{Cos}\frac{hw}{2}$$

$$451440h^{7}w^{7}\mathrm{Cos}\ 45h^{2}\ 11573h^{7}w^{7}+h^{7}\ 67200\ 3591h^{7}w^{7}\ \mathrm{Cos}\ 3\ 2$$

$$3hw\ +1233h^{6}w^{4}\mathrm{Cos}\frac{hw}{2}$$

$$+24h^{3}\ 46965h^{6}w^{4}\mathrm{Cos}\ 5\ 2\ \frac{7hw}{2}$$

$$26h^{2}\ 56870h^{7}w^{4}\mathrm{Cos}\ 5\ 2+9752h^{7}w^{4}\mathrm{Cos}\ \frac{7hw}{2}\ 60480h^{3}\mathrm{Cos}\ \frac{9hw}{2}$$

$$3628800h^{6}w\mathrm{Cos}\ \frac{9hw}{2}\mathrm{Sin}[hw]+3628800h^{6}w\mathrm{Cos}[hw]\mathrm{Sin}\ \frac{9hw}{2}$$

$$+\frac{1}{116121600h^{8}w^{4}}f\_n+6\mathrm{Csc}\frac{hw}{2}\mathrm{Csc}[hw]\ \frac{1}{2}\ 7h\ 2875702h^{8}w^{4}+180h^{4}\ 8064\ 197h^{4}w^{4}+196h^{4}\ 25920+329h^{4}w^{4}\ \mathrm{Cos}\frac{hw}{2}$$

$$+7\ 3316678h^{9}w^{4}+90h^{5}\ 8064\ 197h^{4}w^{4}+2016h^{5}\ 3420\ 49h^{4}w^{4}+196h^{5}\ 25920+329h^{4}w^{4}\ \mathrm{Cos}\frac{hw}{2}$$

$$7h\ 917282h^{8}w^{4}+2h^{4}\ 423360\ 8933h^{4}w^{4}+16h^{4}\ 196560+2957h^{4}w^{4}\ \mathrm{Cos}\ 3\ 2$$

$$2412680h^{9}w^{4}\mathrm{Cos}\frac{hw}{2}$$

$$8h^{2}\ 7\ 4844h^{7}w^{4}+5h^{3}\ 18144+25h^{4}w^{4}\ \mathrm{Cos}\ 2+2786h^{7}w^{4}\ 10h^{3}\ 21168+293h^{4}w^{4}\ \mathrm{Cos}\frac{hw}{2}$$

$$+3h\ 7h\ 745868h^{7}w^{4}+2h^{3}\ 423360\ 8933h^{4}w^{4}\ \mathrm{Cos}\frac{hw}{2}+7\ 917282h^{8}w^{4}+2h^{4}\ 423360\ 8933h^{4}w^{4}+16h^{4}\ 196560+2957h^{4}w$$

$$6350400h^{6}w\ 3\mathrm{Cos}\ 2+\mathrm{Cos}\ 9hw\ 2\ \mathrm{Sin}[hw]+6350400h^{5}w\mathrm{Cos}[hw]\ 3\mathrm{Sin}\ 7hw\ 2\ +\mathrm{Sin}\ 9hw\ 2$$

$$\frac{1}{464486400h^{o}w^{4}}\, f_{n+7}\, \mathrm{Csc}\, h\, \frac{hw}{2}\, i \quad \mathrm{Csc}[hw]\, 2h\, 6503698h^{8}w^{4} \quad 20160h^{4} \quad 132$$

$$5h^{4}w^{4} \quad + 56h^{4} \quad 155520 + 3193h^{4}w^{4} \quad \mathrm{Cos}\, \frac{hhw\, i}{2} + 2\, 7508224h^{9}w^{4} \quad 10080h^{5} \quad 132$$

$$5h^{4}w^{4}\, 4032h^{5} \quad 2460 + 67h^{4}w^{4} \quad + 56h^{5} \quad 155520 + 3193h^{4}w^{4} \quad \mathrm{Cos}\, \frac{hhw\, i}{2} + 6h^{2}$$

$$1527756h^{7}w^{4} + 14h^{3}\, 112320\, 3863h^{4}w^{4}\, \mathrm{Cos}\, \frac{h3hw\, i}{} + 4h\, 1757050h^{8}w^{4} + 14h^{4}\, 112320\, 2$$

$$3863h^{4}w^{4} \quad + 112h^{4}\, 47520 + 1207h^{4}w^{4} \quad \mathrm{Cos}\, \frac{h3hw\, i}{} + 24h^{3} \quad 56220h^{6}w^{4}\, \mathrm{Cos}\, \frac{h5hw\, i}{}$$

$$+ 1233h^{6}w^{4}\, \mathrm{Cos}\, h\, \frac{7hw}{2}\, i \quad 15h^{6}w^{4}\, \mathrm{Cos}\, h\, \frac{9hw}{2}\, i \quad 26h^{2} \quad 126470h^{7}w^{4}\, \mathrm{Cos}\, h\, \frac{5hw}{2}\, i + \quad 12827h^{7}w^{4}$$

$$15h^{3}\, 28224 + 205h^{4}w^{4} \quad \mathrm{Cos}\, \frac{h7hw\, i}{} + 811h^{7}w^{4} \quad 5h^{3}\, 12096 + 169h^{4}w^{4} \quad \mathrm{Cos}\, \frac{h9hw\, i}{}$$

$$3628800h^{o}w\, 7\mathrm{Cos}\, h\, \frac{2}{7hw}\, 1' + \mathrm{Cos}\, h\, \frac{2}{9hw}\, i \quad \mathrm{Sin}[hw]\, 7 + 3628800h^{o}w\mathrm{Cos}[hw] \quad 7\mathrm{Sin}\, h\, \frac{2}{7hw}\, i$$

$$+ \mathrm{Sin}'' \frac{2}{} \quad 232243200h^{6}w^{4}\, \mathrm{Csch}\, \frac{2}{} \quad \mathrm{Csc}[hw] \quad 7h\, 250406h^{o}w^{4} + 180h^{4}\, 10752$$

$$251h^{4}w^{4} + 28h^{4}\, 285120 + 3323h^{4}w^{4} \quad \mathrm{Cos}'' \frac{2}{} \quad 332482h^{9}w^{4} + 90h^{5}\, hw \quad 10752$$

$$251h^{4}w^{4} \quad 22176h^{5} \quad 660 + 7h^{4}w^{4} \quad + 28h^{5} \quad 285120 + 3323h^{4}w^{4} \quad \mathrm{Cos}\, h\, \frac{}{2}\, i \quad 3hw\, i$$

$$+2h^{4} \quad 544320 \quad 10723h^{4}w^{4} \quad +16h^{4} \quad 317520 + 3847h^{4}w^{4} \quad \mathrm{Cos}\, \frac{}{2}$$

$$505880h^{9}w^{4}\, \mathrm{Cos}\, h\, \frac{}{2}\, i \quad 8h^{2} \quad 7 \quad 41068h^{7}w^{4} + 105h^{3} \quad 288 + 19h^{4}w^{4} \quad \mathrm{Cos}\, h\, \frac{7}{2}\, i$$

$$21422h^{7}w^{4} + 10h^{3}\, 63504 + 703h^{4}w^{4}\, \mathrm{Cos}\, \frac{h9hw\, i}{} + 3h$$

$$7h\, 1455388h^{7}w^{4} \quad 2$$

$$2h^{3}\, 544320 \quad 10723h^{4}w^{4} \quad \mathrm{Cos}\, \frac{h3hw\, i}{2} + 7\, 1912270h^{8}w^{4} + 2h^{4}\, 544320$$

$$10723h^{4}w^{4} \quad + 16h^{4}\, 317520 + 3847h^{4}w^{4} \quad \mathrm{Cos}\, \frac{h3hw\, i}{} \quad 154530h^{8}w^{4}\, \mathrm{Cos}\, \frac{h5hw\, i}{}$$

$$+ 8h^{2} \quad 177324h^{6}w^{4}\, \mathrm{Cos}\, \frac{h7hw\, i}{2} \quad 23490h^{6}w^{4}\, \mathrm{Cos}\, \frac{h9hw\, i}{2} \quad 6h \quad 7\, 41068h^{7}w^{4} + 105h^{3}$$

$$288 + 19h^{4}w^{4} \quad \mathrm{Cos}\, \frac{h7hw\, i}{} \quad 21422h^{7}w^{4} + 10h^{3} \quad 63504 + 703h^{4}w^{4} \quad \mathrm{Cos}\, \frac{h9hw\, i}{}$$

$h^6 w \quad 7hw \quad 3\mathrm{Cos}\ 9hw \quad \mathrm{Sin}[hw]$

$12700800 \quad \mathrm{Cos}\ h\ \frac{}{2}\ i + 7hw\ h\ \frac{}{2}\ i \quad 9hw$

$12700800 h^{\circ} w \mathrm{Cos}[hw]\ \mathrm{Sin}\ \frac{}{} + 3\mathrm{Sin}\ \frac{}{}$

$\frac{}{464486400 h^{\circ} w^4} f_{n+5} \mathrm{Csc}\ h\ \frac{}{2}\ i\ \mathrm{Csc}[hw]\ 70h \quad 999130h^8 w^4 + 576h^4 \quad 1092$

$17h^4 w^4 + 56h^4 41472 + 325h^4 w^4 \quad \mathrm{Cos}\ \frac{hhw}{2}\ i + 70\ 1152256h^9 w^4$

$+ 288h^5 \quad 1092 \quad 17h^4 w^4 \quad 4032h^5 \quad 876 + 7h^4 w^4 + 56h^5 \quad 41472$

$325h^4 w^4 \quad \mathrm{Cos}\ \frac{hhw}{2}\ i \quad 14h\ 1912270h^8 w^4 + 2h^4 \quad 907200 \quad 10723h^4 w^4$

$16h^4 453600 + 3847h^4 w^4 \quad \mathrm{Cos}\ \frac{h3hw}{22}\ i \quad 12182960h^9 w^4 \mathrm{Cos}\ \frac{h5hw}{}\ i$

$h^2\ 7\ 26331h^7 w^4 + 5h^3 60480 + 779h^4 w^4 \quad \mathrm{Cos}\ \frac{h7hw}{2}\ i$

$+ 5 \quad 3241h^7 w^4\ h^3 \quad 254016 + 3469h^4 w^4\ \mathrm{Cos}\ h\ \frac{9hw}{2}\ i + 3h \quad 14h$

$1455388h^7 w^4 + 2h^3\ 907200\ 10723h^4 w^4\ \mathrm{Cos}\ \frac{h3hw}{} + 14\ 1912270h^8 w^4\ 2$

$2h^4\ 907200\ 10723h^4 w^4 +$

$16h^4 453600 + 3847h^4 w^4$

$\mathrm{Cos}\ \frac{h3hw}{2}\ i$

$958500h^8 w^4 \mathrm{Cos}\ \frac{h5hw}{222}\ i + 8h^2\ 29463h^6 w^4 \mathrm{Cos}\ \frac{h7hw}{}\ i \quad 585h^6 w^4 \mathrm{Cos}\ \frac{h9hw}{}\ i$

$h\ 7\ 26331h^7 w^4 + 5h^3 60480 + 779h^4 w^4\ \mathrm{Cos}\ \frac{h7hw}{2}\ i + 5\ 3241h^7 w^4\ 2$

$h^{\circ} \quad 254016 + 3469h^4 w^4\ \mathrm{Cos}\ \frac{9hw}{9hw}\ h\ 2\ i \quad 25401600h^{\circ} w\ 5\mathrm{Cos}\ \frac{7hw}{7hw}\ h\ 2\ 9hw'$

$+ 3\mathrm{Cos}\ \frac{h\ 1}{2}\ i \quad \mathrm{Sin}[hw] + 25401600h^6 w \mathrm{Cos}[hw]\ 5\mathrm{Sin}\ \frac{h}{2}\ i + 3\mathrm{Sin}\ \frac{h\ 2}{}\ i$

$\frac{}{464486400 h^{\circ} w^4} f_{n+3} \mathrm{Csc}\ h\ \frac{}{2}\ i\ \mathrm{Csc}[hw]\ 70h \quad 999130h^8 w^4 + 576h^4 \quad 1260$

$17h^4 w^4 + 280h^4 10368 + 65h^4 w^4 \quad \mathrm{Cos}\ \frac{hhw}{2}\ i + 70\ 1152256h^9 w^4$

$$+ 288h^5 \quad 1260 \quad 17h^4w^4 \quad 28224h^5 \quad 180 + h^4w^4 \quad + 280h^5 \quad 10368$$

$$65h^4w^4 \ \mathrm{Cos}\frac{hhw \quad i}{2} \quad 14h \ 917282h^8w^4 + 2h^4 \ 1028160 \quad 8933h^4w^4$$

$$16h^4 574560 + 2957h^4w^4 \quad \mathrm{Cos}\frac{h3hw \ i}{22} \quad 28482320h^9w^4 \mathrm{Cos}\frac{h5hw \ i}{\quad}$$

$$36288 + 1829h^4w^4 \ \mathrm{Cos}\frac{h7hw \ i}{2} \quad \frac{\quad}{\quad}$$

+5 19001h7w4 7h3 60480 + 773h4w4 Cos h9hw/2 i

+ 3h 14h 745868h7w4 + 2h3 1028160

8933h4w4 Cos h3hw/2 i + 14 917282h8w4

2h4 1028160 8933h4w4 + 16h4 574560

2957h4w4 Cos h3hw/22 i + 5364180h8w4Cos h5hw/ i + 8h2

541065h6w4Cos h7hw/2 i + 83415h6w4Cos h9hw/2 i 6h 7

28437h7w4 + 5h336288 + 1829h4w4 Cos h7hw/2 i

5 19001h7w4 7h3 60480 + 773h4w4 Cos h9hw/2 i

25401600h6w 3Cos h7hw/22 i + 5Cos h9hw/ i Sin[hw]

+ 25401600h6wCos[hw] 3Sin 1 hw'' 8 i hw h i
7hw 9hw
2 + 5Sin 2

928972800h6w4 8 4 f4 h i h4 4 i 4 2h
n+1 Csc 2 Sec 2

60480h 68 + 3h w + 56h 311040

5591794h w 4 4 hs hw i 4 4 5 68
+ 7993h4w4 Cos 2 + 2 4723456h3w4 30240h3

+ 3h w 4032h 8220 + 259h w + 56h 311040

+ 7993h4w4 Cos h i + 6h2 667684h7w4 + 14h3 164160
2

7393h4w4 Cos h3hw/ i + 4h 755574h8w4 + 14h4 164160 2

7393h4w4 + 336h433120 + 1159h4w4 Cos h3hw/2 i

3 6 w4Cos 5hw 6 4 h 7hw/ i
+ 24h 354780h 9hw h 2 i 15h wCos 2 hw
+ 1233h6w4Cos '' 2 i 26h2 603170h7w4Cos '' 5 2 i
h7hw/ i
811h7w4 5h3 12096 + 169h4w4 Cos 2
h9hw/ i
12827h7w4 15h3 28224 + 205h4w4 Cos 2

3628800h6w Cos h7hw/22 i + 7Cos h9hw/ i Sin[hw]

7hw 9hw
+ 3628800h6wCos[hw] Sin h i + 7Sin h 2 i
2

$$y_{n+2}^{0} = \frac{11y_{n+3}}{60h} - \frac{2y_{n+4}}{3h} + \frac{5y_{n+5}}{4h} - \frac{2y_{n+6}}{5h} + \frac{1}{46448640h^{8}w^{4}} f_{n+4} \operatorname{Csc}\left[\frac{hw}{2}\right]^{8} \left(15876000h^{5} - 14h\left(185714h^{8}w^{4} + 840h^{4}\left(1152 + 112h^{4}w^{4}\right)\right)\right)$$

$280h^{4}$ 7344 + $368h^{4}w^{4}$ Cos[hw] 510468$h^{9}w^{4}$Cos[2hw]

202932$h^{9}w^{4}$Cos[3hw] 6h^{2} 20594$h^{7}w^{4}$ 20h^{3} 5292
+ 1136 4 4 Cos[4] + 2 14 8 4244936 4 + 420 4 4
$h^{4}w^{4}$ hw h h $h^{7}w^{4}$ h^{3} 1152

+ 112h w Cos[hw] + 7 185714h w + 840h 1152 + 112h w

$280h^{4}$ 7344 + $368h^{4}w^{4}$ Cos[hw] 940814$h^{8}w^{4}$Cos[2hw]

176886$h^{8}w^{4}$Cos[3hw] 12042$h^{8}w^{4}$Cos[4hw] 8h 20594$h^{7}w^{4}$
20 1 hw 7

h^{3} 5292 + 1136$h^{4}w^{4}$ Cos[4hw] + 12700800$h^{6}w$Sin[2hw]

+ $\frac{}{464486400h^{6}w^{4}}$ f_{n+8}Csc$\left[\frac{hw}{2}\right]$ Csc[hw] 30h^{2} 82330$h^{7}w^{4}$

3h^{3}20160 + 9104$h^{4}w^{4}$ Cos$\left[\frac{hhw}{2}\right]$ + 5h 40024$h^{8}w^{4}$

h^{4} 60480 52496$h^{4}w^{4}$ + 6h^{4}20160 + 9104$h^{4}w^{4}$ Cos$\left[\frac{hhw}{2}\right]$

382788$h^{9}w^{4}$Cos$\left[\frac{3hw}{2}\right]$ + 36h^{2} 5926$h^{7}w^{4}$ + 35h^{3} 1152

560$h^{4}w^{4}$Cos$\left[\frac{3hw}{2}\right]$ + 6h^{3} 240$h^{6}w^{4}$Cos$\left[\frac{5hw}{2}\right]$

42$h^{6}w^{4}$Cos$\left[\frac{7hw}{22}\right]$ 11h^{2} 9580$h^{7}w^{4}$Cos$\left[\frac{7hw}{22}\right]$

13536$h^{7}w^{4}$ 5h^{3} 12096 + 2704$h^{4}w^{4}$ Cos$\left[\frac{7hw}{2}\right]$

3628800$h^{6}w$Cos$\left[\frac{7hw}{22}\right]$Sin[2hw] + 3628800$h^{6}w$Cos[2hw]Sin$\left[\frac{7hw}{22}\right]$

+ $\frac{1}{464486400h^{6}w^{4}}$ f_{n}Csc$\left[\frac{hw}{2}\right]$ Csc[hw]$\frac{hw}{}$10h^{2} 115238$h^{7}w^{4}$

+ 9h^{3}12096 + 11600$h^{4}w^{4}$ Cos$\left[\frac{h}{2}\right]$ 5h 342704$h^{8}w^{4}$

$$+ \qquad\qquad\qquad\qquad +hw$$

$$h^4 \quad 616896 \quad 526352h^4w^4 \quad 18h^4$$

$$12096 + 11600h^4w^4 \; Cos\frac{h}{2}i$$

$$873612h^9w^4Cos\frac{h3hwi}{} + 36h^2 \; 48110h^7w^4$$

$$h^3 \; 67200 \; 57456h^4w^4 \; Cos\frac{h3hwi}{2}$$

$$6h^3 \; 6840h^6w^4Cos\frac{h5hwi}{2} \; 210h^6w^4Cos\frac{h7hwi}{}$$

$$11h^2 \; 82660h^7w^4Cos\frac{h5hwi}{22} + 9592h^7w^4Cos\frac{h7hwi}{}$$

$$60480h^3Cos\frac{h9hwi}{22} \; 3628800h^6wCos\frac{h9hwi}{}Sin[2hw]$$

$$3628800h^6wCos[2hw]Sin\frac{9hwi}{2h}$$

$$+ \frac{1}{116121600h^6w^4 \; {}^{h+6}Csc^{4}} \; {}^{4} \; h\frac{hw}{2}i^{7} \; Csc[hw]^{4} \; 4\frac{1}{2} \; 14h^{4}$$

$$1542478h \; w + 360h \quad 8064 \quad 3152h \; w \quad + 196h$$

$$25920^{5} + 5264h^4w^4 \quad Cos^{4} \; {}^{4} \; h\frac{2}{}i \quad + 7^{5} \; 1663112h^9w^4$$

$$+ 360h \quad 8064 \quad 3152h \; w \; + 2016h \quad 3420$$

$$784h^4w^4 + 392h^5 25920 + 5264h^4w^4 \quad Cos\frac{hhw}{2}i$$

$$14h \; 125454h^8w^4 + 4h^4 \quad 423360 \quad 142928h^4w^4$$

$$16h^4 196560 + 47312h^4w^4$$

$$Cos\frac{h3hw}{2}i$$

$$1170060h^9w^4Cos\frac{h5hwi}{} \quad 3h^2 \quad 7$$

$$2814h^7w^4 + 5h^3 \quad 2$$

$$18144 + 400h^4w^4 \quad Cos\frac{h7hw}{}i + 46886h^7w^4 \quad 2$$

$$10h^3 \quad 21168 + 4688h^4w^4 \quad Cos\frac{h9hw}{2}i$$

+2h 14h 192131h^7w^4 + 2h^3 423360

142928h^4w^4 Cos h3hw i / 2 + 7 125454h^8w^4

+ 4h^4 423360 142928h^4w^4 + 16h^4

196560 + 47312h^4w^4 Cos h3hw i 2

1494200h^8w^4Cos h5hw i / 22 + 3h^2 399h^6w^4Cos h7hw i

h^6w^4 9hw / 2 h 7 2814h′w^4 + 5h^3

225 Cos 4 7hw / 2 i 9hw

18144 + 400 Cosh + 46886h′w

h^4w^4 7hw h 9hwi / 2

10h^3 21168 + 4688h^4w^4 Cos

6350400 3Cos 7hw 2 i 9hw

h^6w 1 Cos Sin[2hw]

+ 6350400h^6wCos[2hw] 3Sin hw 7 2 i h 2 i

+ Sin

464486400h^8w^4 f_{n+7}Csc h / 2 i Csc[hw] 4h^4 4

h w

3353074h w 40320h 132 + 80 hw

+ 56 h^4 155520 + 51088h^4w^4 Cos 2 4 4

+ 2 4102244h^5w^4 40320h^4 4 132 + 80h^5w^4

h 1072h w h 155520

4032 2460 + hw 112 3hw

+ 51088 Cos h 2 + 8 h^2 50119h′w^4

+ 14 h^4w^4 112320 hw h 2 i hw

h^6 61808h^4w^4 Cos

+ 6h^3 7680h^6w^4Cos h 5 / 2 i 210h^6w^4Cos h 7 / 2 i

42h^6w^4Cos h9hw i / 22 11h^2 150040h^7w^4Cos h5hw i / 2

58792h^7w^4 15h^3 28224 + 3280h^4w^4 Cos h7hw i / 2

$$13536h^7w^4\ 5h^3\ 12096 + 2704h^4w^4\ \mathrm{Cos}\ \frac{h9hw\ i}{2}$$

$$3628800h^6w\ 7\mathrm{Cos}\ \frac{h7hw\ i}{22} + \mathrm{Cos}\ \frac{h9hw\ i}{2}\ \mathrm{Sin}[2hw]$$

$$+\ \frac{1}{3628800h^{\circ}w}\mathrm{Cos}[2hw]\ 7\mathrm{Sin}\ \frac{hw''\ 7^{\circ}\ i}{7hw} + \mathrm{Sin}\ \frac{h\ 2\ i}{9hw}$$

$$+\ \frac{8\ 4}{232243200h^{\circ}w^4}\ f_{n+2}\mathrm{Csc}\ \frac{h\ i}{2}\ \mathrm{Csc}[hw]\ 14h^4\ 4$$

$$273022h\ w + 360h\ \ 10752\ \ 4016h\ w\ \ + 28h^4$$

$$285120 + 53168h^4w^4\ \mathrm{Cos}\ \frac{h\ i}{2} + 7\ \ 1208912h^9w^4$$

$$360h^5\ \ 10752\ \ 4016h^4w^4\ \ 22176h^5\ \ 660$$

$$112h^4w^4 + 56h^5 285120 + 53168h^4w^4\ \ \mathrm{Cos}\ \frac{hhw\ i}{2}$$

$$14h\ 389418h^8w^4 + 4h^4\ \ 544320\ \ 171568h^4w^4$$

$$16h^4 317520 + 61552h^4w^4\ \mathrm{Cos}\ \frac{h3hw\ i}{2}$$

$$225060h^9w^4\mathrm{Cos}\ \frac{h5hw\ i}{2}\ 3h^2\ \ 7\ 23798h^7w^4$$

$$+\ 105h^3\ \ 2$$

$$288 + 304h^4w^4\ \mathrm{Cos}\ \frac{h7hw\ i}{2}$$

$$71158h^7w^4\ \ 2$$

$$10h^3\ 63504 + 11248h^4w^4\ \mathrm{Cos}\ \frac{h9hw\ i}{2} + 2h\ 14h$$

$$653917h^7w^4 + 2h^3\ 544320\ 171568h^4w^4\ \mathrm{Cos}\ \frac{h3hw\ i}{2}$$

$$7\ 389418h^8w^4 + 4h^4\ \ 544320\ \ 171568h^4w^4$$

$$16h^4 317520 + 61552h^4w^4\ \mathrm{Cos}\ \frac{h3hw\ i}{2}$$

$$288560h^8w^4\mathrm{Cos}\ \frac{h5hw\ i}{22} + 3h^2\ 21735h^6w^4\mathrm{Cos}\ \frac{h7hw\ i}{2}$$

$$3441h^6w^4\cos\frac{h9hw\,i}{\ }\quad 4h\ 7\ 23798h^7w^4\ +$$

$$105h^3\ \ 2$$

$$288 + 304h^4w^4\cos\frac{h7hw}{i}$$

$$71158h^7w^4\ \ 2$$

$$10h^3\ 63504 + 11248h^4w^4\cos\frac{h9hw\,i}{\ }\quad 2$$

$$12700800\ \ \cos\frac{h}{2}\,i + \frac{7hw\ h}{2}\,i\ \ \frac{9hw}{\ }\,i$$

$$+\ 12700800h^{\circ}w\cos[2hw]\ \sin\frac{hw''}{\ }\ \ \frac{7^{c}\ i}{\ } + 3\sin\frac{h}{2}\,i$$

$$\frac{\ }{464486400h^{\circ}w^4}\ f_{n+5}\csc\frac{h}{2}\,i\ \ \csc[hw]\ \ 140h$$

$$550402h^8w^4 + 1152h^4\ \ 1092\ \ 272h^4w^4\ +$$

$$56h^4\ 41472 + 5200h^4w^4\cos\frac{hhw\ i}{2}$$

$$536036h^9w^4 + 1152h^4\ \ 1092\ \ 272h^4w^4$$

$$h\ \ 112h\ w\ \ + 112h\ \ 41472$$

$$4032\ \ 876 + \ \ hw$$

$$+ 5200h^4w^4\ \cos h\ \ \frac{\ }{2}'\ \ 28h\ \ 389418h^8w^4$$

$$4h^4\ \ 907200\ \ 171568h^4w^4\ \ + 16h^4\ \ 453600$$

$$61552h^4w^4\ \ \cos\frac{h3hw\,i}{22}\ \ 6177480h^9w^4\cos\frac{h5hw\,i}{\ }$$

$$3h^2\ \ 7\ \ 32864h^7w^4 + 5h^3\ \ 60480$$

$$12464h^4w^4\ \ \cos\frac{h7hw\,i}{\ }\ \ + 5\ 55384h^7w^4\ 2$$

$$h^3\ 254016 + 55504h^4w^4\cos\frac{h9hw\,i}{\ }\ \ 2$$

$$+ 2h\ 28h\ \ 653917h^7w^4 + 2h^3\ \ 907200$$

$$171568h^4w^4\cos\frac{h3hw\,i}{\ }\ \ + 14\ 389418h^8w^4\ 2$$

$$4h^4\ \ 907200\ \ 171568h^4w^4\ \ + 16h^4\ \ 453600$$

$$61552h^4w^4\ \ \cos\frac{h3hw\,i}{22}\ \ 7919120h^8w^4\cos\frac{h5hw\,i}{\ }$$

$$3h^2\ 10290h^6w^4\cos\frac{h7hw\,i}{\ }\ \ 2$$

$$2310h^{6}w^{4}\cos\left[\frac{(9hw)}{2}\right] \quad 4h\,7 \quad 32864h^{7}w^{4} + 5h^{3}$$

$$60480 + 12464\cos\left[\frac{(9hw)}{2}\right] + 5 \quad 55384h^{7}w^{4}$$

$$h^{4}w^{4} \quad 254016 + 55504h\,7hw$$

$$25401600 \quad 5\cos\left[\frac{(4w)}{2}\right] + 7hw \quad 3\cos\left[\frac{(9hw)}{2}\right] \quad \sin[2hw]$$

$$+ \quad h^{6}w$$

$$25401600h^{6}w\cos[2hw] \quad 5\sin\left[\frac{(2hw)}{2}\right]$$

$$+ 3\sin\left[\frac{(hw)}{2}\right]$$

$$464486400h^{8}w^{6} \quad f_{n+3}\csc^{4}\left[\frac{(hw)}{2}\right] \quad \csc[hw]^{4} \quad 140h^{4}$$

$$550402h\,w + 1152h \quad 1260 \quad 272h\,w \quad + 280h$$

$$10368 + 1040h^{4}w^{4} \quad \cos\left[\frac{(hw)}{2}\right] + 70 \quad 536036h^{9}w^{4}$$

$$1152h^{5} \quad 1260 \quad 272h^{4}w^{4} \quad 28224h^{5} \quad 180$$

$$16h^{4}w^{4} + 560h^{5}10368 + 1040h^{4}w^{4} \quad \cos\left[\frac{(hhw)}{2}\right]$$

$$28h\ 125454h^{8}w^{4} + 4h^{4} \quad 1028160 \quad 142928h^{4}w^{4}$$

$$16h^{4}574560 + 47312h^{4}w^{4}$$

$$\cos\left[\frac{(h3hw)}{2}\right]$$

$$19377240h^{9}w^{4}\cos\left[\frac{(3h)}{5hw}\right] \quad 3h^{2} \quad 7$$

$$266552h\,w + 5h \quad 36288$$

$$29264h^{4}w^{4} \quad \cos\left[\frac{(h7hw)}{2}\right] + 5\ 120224h^{7}w^{4}$$

$$7h^{3}\ 60480 + 12368h^{4}w^{4}\cos\left[\frac{(h9hw)}{2}\right]$$

$$+ 2h\ 28h \quad 192131h^{7}w^{4} + 2h^{3} \quad 1028160$$

$$142928h^{4}w^{4}\cos\left[\frac{(h3hw)}{2}\right] + 14\ 125454h^{8}w^{4}$$

$$h^{4} \quad 142928h^{4}w^{4} + 16h^{4} \quad 574560$$

$$+ 4 \quad 1028160$$

$$\left[\frac{(3hw)}{4}\right] \quad \left[\frac{(5hw)}{4}\right]$$

2 2

143

h^2 $h^6 w^4 \cos \frac{7hw}{}$

$+3$ 752262 $\frac{9hw\,h}{}$ 2 i 7

266552h w + 5h 36288

$29264h^4w^4 \cos \frac{h7hw\,i}{2}$ + 5 $120224h^7w^4$

$7h^3$ 60480 + $12368h^4w^4 \cos \frac{h9hw\,i}{2}$

$25401600h^6w\,3\cos \frac{h7hw\,i}{22} + 5\cos \frac{h9hw\,i}{22} \sin[2hw]$

+ 1 hwh 8 2 'hw h 2 i

$25401600h^{\upsilon} w\cos[2hw]\,3\sin \frac{7hw}{} + 5\sin \frac{9hw}{}$

928972800 8 h 4w h4 2 i h 2 4 4

$\frac{}{6\ 4} f_{n+1} \csc \frac{}{} \sec \frac{}{}$ 4h

w

2977906h w 120960h68 + 48h hw

+ 56 h 9 4 5 h 311040 + 127888h w Cos 2 i 4

+ 2 5 4 4 5

9148964h w 120960h 68 + 48h w

h 4144h w + 112h 311040

4032 8220 + hw

+ 127888 Cosh 2 h 861543h 3hw w

h4w4

+ 14 164160 hw h 2 i hw

h^{υ} $118288h^{4}w^{4}$ Cos

$+ 6h^3$ $43920h^6w^4\cos \frac{h5\ i}{2}$ $42h^6w^4\cos \frac{h7}{2}$ i

$210h^6w^4\cos \frac{h9hw\,i}{22}$ $11h^2$ $780040h^7w^4\cos \frac{h5hw\,i}{}$

$13536h^7w^4$ $5h^3$ 12096 + $2704h^4w^4 \cos \frac{h7hw\,i}{2}$

$58792h^7w^4$ $15h^3$ 28224+$3280h^4w^4 \cos \frac{h9hw\,i}{2}$

$3628800h^{\upsilon} w \cos \frac{h\ 2\ i}{7hw}$ + $7\cos \frac{hw\ h\ 2\ i}{9hw}$ hw $\sin[2hw]$

+ $3628800h^6 w\cos[2hw] \sin \frac{h\ 7}{2\ i}$ + $7\sin \frac{h\ 9}{2\ i}$

$$y_n^{0}{}_{+3} = \frac{y_n}{30h} \quad \frac{7y_{n+3}}{6h} + \frac{3y_{n+4}}{2h} \quad \frac{3y_{n+5}}{10h}$$

$$+ \frac{1}{46448640h^6w^4} f_{n+4}\mathrm{Csc}\frac{hw}{2} \quad 19051200h^5 \quad 21h \quad 192780h^8w^4$$

$$1260h^4 \quad 1152 + 567h^4w^4 \quad 280h^4 \quad 7344 + 1863h^4w^4 \quad \mathrm{Cos}[hw]$$

$$h\ 21h \quad 381270h^7w^4 + 420h^3 \quad 1152 + 567h^4w^4 \quad \mathrm{Cos}[hw]$$

$$56952h^8w^4\mathrm{Cos}[2hw] \quad 249102h^8w^4\mathrm{Cos}[3hw]$$

$$4h\ 109350h^7w^4\ 20h^3\ 5292 + 5751h^4w^4\ \mathrm{Cos}[4hw]$$

$$12700800h^6w\mathrm{Sin}[hw]$$

$$+ \frac{1}{464486400h^6w^4} f_{n+8}\mathrm{Csc}\frac{hw}{2}^{7} \quad \mathrm{Csc}[hw] \quad 30h^2 \quad 162987h^7w^4$$

$$+ 3h^3 \quad 20160 + 46089h^4w^4 \quad \mathrm{Cos}\frac{\ }{2} \quad 5h \quad 149040h^8w^4$$

$$+ h^4 \quad 60480 \quad 265761h^4w^4 \quad + 9h^4 \quad 20160 + 46089h^4w^4 \quad \mathrm{Cos}\frac{hw}{2}$$

$$+ 27h^2 \quad 93973h^7w^4 + 35h^3 \quad 1152 \quad 2835h^4w^4 \quad \mathrm{Cos}\frac{3hw}{2}$$

$$+ \quad 68193h^7w^4 \quad 5h^3 \quad 2h^2 \quad 12150h^7w^4\mathrm{Cos}\frac{5hw}{2}$$

$$12096 + 13689h^4w^4 \quad \mathrm{Cos}\frac{hw}{2}$$

$$3628800h^6w\mathrm{Cos}\frac{hw}{2}\mathrm{Sin}[3hw] + 3628800h^6w\mathrm{Cos}[3hw]\mathrm{Sin}\frac{hw}{2}$$

$$+ \frac{1}{464486400h^6w^4} f_n\mathrm{Csc}\frac{hw}{2}^{7} \quad \mathrm{Csc}[hw] \quad 15h \quad 1092942h^8w^4$$

$$+ h^4 \quad 616896 \quad 2664657h^4w^4 \quad + 27h^4 \quad 12096 + 58725h^4w^4 \quad \mathrm{Cos}\frac{hw}{2}$$

$$+ 63h^2 \quad 290871h^7w^4 + h^3 \quad 67200 \quad 290871h^4w^4 \quad 519264h^9w^4\mathrm{Cos}\frac{3hw}{2}$$

$$2h^2 \quad 80100h^7w^4\mathrm{Cos}\frac{hw}{2} \quad + 9918h^7w^4\mathrm{Cos}\frac{7hw}{2}$$

$$60480h^3\mathrm{Cos}\frac{9hw}{2} \quad 3628800h^6w\mathrm{Cos}\frac{9hw}{2}\mathrm{Sin}[3hw]$$

$$145$$

$$3628800h^6w\mathrm{Cos}[3hw]\mathrm{Sin}\frac{9hw}{2}$$

$$+\frac{1}{116121600h^{6}w^{4}}f_{n+6}\mathrm{Csc}\frac{hw}{2}^{7}\ \mathrm{Csc}[hw]\ \frac{1}{2}\ 21h\ 2989614h^{8}w^{4}+540h^{4}\ 8064\ 15957h^{4}w^{4}+196$$

$$h^{9}w^{4}\qquad h^{5}\qquad h^{4}w^{4}\qquad h^{5}\qquad h^{4}w^{4}\qquad h^{5}\qquad 25920+26649h$$

$$+7\ 5257062\quad +810\quad 8064\ 15957\quad +2016\ 3420\ 3969\quad +588\quad \mathrm{Cos}\frac{hw}{2}^{3}$$

$$21h\ 509166h^{8}w^{4}+6h^{4}\ 423360\ 723573h^{4}w^{4}+16h^{4}\ 196560+239517h^{4}w^{4}$$

$$+h\ 21h\ 1286724h^{7}w^{4}+2h^{3}\ 423360\ 723573h^{4}w^{4}\ \mathrm{Cos}\frac{hw}{2}^{3}\ +7\ 509166h^{8}w^{4}+6h^{4}\ 423360\ 7$$

$$6350400h^{6}w\ 3\mathrm{Cos}\frac{hw}{2}^{7}+\mathrm{Cos}\frac{hw}{2}^{9}\ \mathrm{Sin}[3hw]$$

$$+\ 6350400h^{9}w\mathrm{Cos}[3hw]\ 3\mathrm{Sin}\qquad +\ \mathrm{Sin}\ 9\ 2$$

$$\frac{1}{464486400h^{6}w^{4}\,f_{n+7}\,\mathrm{Csc}\,\frac{hw}{2}\,\mathrm{Csc}[hw]\,6h\,9112278h^{8}w^{4}\,\mathrm{Cos}\,\frac{hw}{2}}$$

$$60480h^{\cdot\cdot}\quad 132+405h^{\cdot\cdot}w^{\cdot\cdot}\quad +56h^{\cdot\cdot}\quad 155520+258633h^{\cdot\cdot}w^{\cdot\cdot}$$

$$+\,2\,15172920h^{5}w^{9}\,4\quad 90720h^{5}\,132+405h^{4}w^{4}\quad hw$$

$$4032h\quad 2460+5427h\,w\quad +168h\quad 155520+258633h\,w\quad \mathrm{Cos}\,\frac{hw}{2}$$

$$+\,6h^{2}\quad 4126308h^{7}w^{4}+14h^{3}\quad 112320\quad 312903h^{4}w^{4}\quad \mathrm{Cos}\,\frac{3}{2}$$

$$h\,2192022h\,w+42h^{8}\,4\quad 4\quad 4\quad 112320\quad 312903h\,w\quad 3hw\quad 4\quad 4\quad 7\,4\quad 5hw$$

$$+\,112h\quad 47520+97767h\,w\quad \mathrm{Cos}\,\frac{}{2}\quad 2h\quad 160110h\,w\,\mathrm{Cos}\,\frac{}{2}$$

$$+\quad 258993h^{\prime}w^{\cdot\cdot}\quad 15h^{\circ}\quad 28224+16605h^{\cdot\cdot}w^{\cdot\cdot}\quad \mathrm{Cos}\,\frac{7}{2}hw$$

$$+\quad 68193h^{\prime}w^{\cdot\cdot}\quad 5h^{\circ}\quad 12096+13689h^{\cdot\cdot}w^{\cdot\cdot}\quad \mathrm{Cos}\,\frac{9}{2}hw$$

$$3628800h^{6}w\;7\mathrm{Cos}\,\frac{7}{2}hw+\mathrm{Cos}\,\frac{9}{2}hw\;\mathrm{Sin}[3hw]$$

$$+\,3628800h^{\circ}w\mathrm{Cos}[3hw]\;7\mathrm{Sin}\,\frac{7}{2}hw+\mathrm{Sin}\,\frac{9}{2}hw$$

$$y_{n+5}^{0} = \frac{y_n}{30h} + \frac{5y_{n+3}}{6h} \quad \frac{5y_{n+4}}{2h} + \frac{17y_{n+5}}{10h}$$

$$+ \frac{1}{46448640h^6w^4} f_{n+4}\mathrm{Csc}\left[\frac{hw}{2}\right]^8 \quad 31752000h^5 \quad 35h \quad 5162500h^8w^4$$

$$2100h^4 \quad 1152 + 4375h^4w^4 \quad 280h^4 \quad 7344 + 14375h^4w^4 \quad \mathrm{Cos}[hw]\ h$$

$$35h \quad 1660114h^7w^4 + 420h^3 \quad 1152 + 4375h^4w^4 \quad \mathrm{Cos}[hw]$$

$$1498280h^8w^4\mathrm{Cos}[2hw] + 654930h^8w^4\mathrm{Cos}[3hw]$$

$$4h\ 869210h^7w^4 \quad 20h^3 \quad 5292 + 44375h^4w^4 \quad \mathrm{Cos}[4hw]$$

$$12700800h^6w\mathrm{Sin}[hw]$$

$$+ \frac{1}{464486400h^6w^4} f_{n+8}\mathrm{Csc}\left[\frac{hw}{2}\right]^7 \quad \mathrm{Csc}[hw] \quad 25h \quad 3231746h^8w^4 \quad \frac{hw}{\mathrm{Cos}\left[\frac{hw}{2}\right]}$$

$$+ h^{\text{[illegible]}} \quad 60480 \quad 2050625h^{\text{[illegible]}}w^{\text{[illegible]}} \quad + 15h^{\text{[illegible]}} \quad 20160 + 355625h^{\text{[illegible]}}w^{\text{[illegible]}} \quad \mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$153h^2 \quad 765625h^7w^4 + 35h^3 \quad 191520h^9w^4\mathrm{Cos}\left[\frac{3}{2}\right]$$

$$1152 \quad 21875h^{\text{[illegible]}}w^{\text{[illegible]}} \quad \mathrm{Cos}\left[\frac{3}{2}\right]$$

$$\frac{hw}{}$$

$$+ 528015h^7w^4 \quad 5h^3 \quad 2h^2 \quad 10300h^{\prime}w^4\mathrm{Cos}\left[\frac{5}{2}\right]$$

$$12096 + 105625h^{\text{[illegible]}}w^{\text{[illegible]}} \quad \mathrm{Cos}\left[\frac{7}{2}\right]$$

$$\frac{hw}{}$$

$$+ 3628800h^6w\mathrm{Cos}[5hw]\mathrm{Sin}\left[\frac{7hw}{2}\right] \quad 3628800h^6w\mathrm{Cos}\left[\frac{hw}{2}\right] \quad \mathrm{Sin}[5hw]$$

$$+ \frac{1}{464486400h^6w^4} f_{n+8}\mathrm{Csc}\left[\frac{hw}{2}\right]^{\prime} \quad \mathrm{Csc}[hw] \quad 50h^2 \quad \frac{hw}{} \quad 4062797h^{\prime}w^{\text{[illegible]}}$$

$$+ 9h^3 \quad 12096 + 453125h^4w^4 \quad \mathrm{Cos}\left[\frac{}{2}\right] \quad 35h\ 170000h^8w^4$$

$$+ h^{\text{[illegible]}} \quad 616896 \quad 20560625h^{\text{[illegible]}}w^{\text{[illegible]}} \quad + 45h^{\text{[illegible]}} \quad 12096 + 453125h^{\text{[illegible]}}w^{\text{[illegible]}} \quad \mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$45h^2 \quad 2257019h^{\prime}w^4 + h^3 \quad 67200 \quad 2244375h^4w^4 \quad \mathrm{Cos}\left[\frac{3hw}{2}\right]$$

$$2h^2 \quad 78250h^7w^4\mathrm{Cos}\left[\frac{5}{2}\right] + 10060h^7w^4\mathrm{Cos}\left[\frac{7}{2}\right]$$

$$60480h^3\mathrm{Cos}\left[\frac{9hw}{2}\right] + 3628800h^6w\mathrm{Cos}[5hw]\mathrm{Sin}\left[\frac{9hw}{2}\right]$$

$$148$$

$$3628800h^6w\mathrm{Cos}\left[\frac{9hw}{}\right]\mathrm{Sin}[5hw]\ 2$$

$$\frac{1}{+\ 116121600h^{6}w^{4}\ f_{n+6}\ \mathrm{Csc}\ \frac{h\ \frac{hw}{2}\ i}{123125h^{4}w^{5}}\ \mathrm{Csc}[hw]\ \frac{1}{2}\ \frac{35h\ 70043426h^{8}w^{4}}{25920 + 205625h^{4}w^{4}}\ \mathrm{Cos}\ \frac{h\ \frac{hw}{2}\ i}{+\ 900h^{4}\ 8064}}$$

$$+\ 7\ \ 137258750h\ w + 2250h\ \ 8064\ \ 123125h\ w\ \ + 2016h\ \ 3420$$

$$30625h^{4}w^{4} + 980h^{5}\ 25920 + 205625h^{4}w^{4}\ \ \mathrm{Cos}\ \frac{hhw\ \ i}{2}$$

$$35h\ 26261250h^{8}w^{4} + 10h^{4}\ \ 423360\ \ 5583125h^{4}w^{4} + 16h^{4}\ \ 196560$$

$$1848125h^{4}w^{4}\ \mathrm{Cos}\ \frac{h3hw\ i}{h\ 35h\ 11280796h^{7}w^{4} + 2h^{3}\ 423360\ 2}$$

$$5583125h^{4}w^{4}\ \mathrm{Cos}\ \frac{h3hw\ i}{+\ 7\ 26261250h^{8}w^{4} + 10h^{4}\ 423360\ 2}$$

$$125h^{4}w^{4} + 16h^{4}\ 196560 + 1848125h^{4}w^{4}\ \ \mathrm{Cos}\ \frac{h3hw\ i}{2}$$

$$+\ 759250h^{8}w^{4}\mathrm{Cos}\ \frac{5hw}{7hw}\ \ + 2h\ 7\ \ \ 73140h^{7}w^{4} + 5h^{3}\ \ \ 18144$$

$$+\ 15625h^{4}w^{4}\ \ \ \mathrm{Cos}\ \frac{h^{2}\ 9hw}{2}\ \ + 1831130h^{7}w^{4}\ \ \ 10h^{3}\ \ 21168$$

$$+\ \ 183125h^{4}w^{4}\ \ \ \mathrm{Cos}\ \frac{h\ \ 2\ \ i}{}\ \ \ + 6350400h^{9}w\mathrm{Cos}[5hw]\ \ 3\mathrm{Sin}\ \frac{h\ \ 2\ \ i}{}$$

$$+\ \mathrm{Sin}\ \frac{2\ 1}{}\ \ \ 6350400\ \ h^{9}w\ 3\mathrm{Cos}\ \frac{h\ \ 2\ i}{}\ \ + \mathrm{Cos}\ \frac{h\ \ 2\ \ i}{}\ \ \ \mathrm{Sin}[5hw]$$

$$\frac{h\ \ i}{464486400h^{9}w^{4}\ f_{n+7}\ \mathrm{Csc}\ \frac{2}{}\ \mathrm{Csc}[hw]\ \ 10h\ \ 202258598h^{8}w^{4}\ \ 100800h^{4}}$$

$$132 + 3125h^{4}w^{4}\ \ + 56h^{4}\ \ \ 155520 + 1995625h^{4}w^{4}\ \ \mathrm{Cos}\ \frac{hhw\ \ i}{2}$$

$$+\ 2\ 397565000h^{9}w^{4}\ \ 252000h^{5}\ \ \ 132 + 3125h^{4}w^{4}\ \ \ 4032h^{5}$$

$$2460 + 41875h^{4}w^{4} + 280h^{5}\ 155520 + 1995625h^{4}w^{4}\ \mathrm{Cos}\ \frac{hhw\ \ i}{2}\ \ 10h^{2}$$

$$33992828h^{7}w^{4} + 14h^{3}\ 112320\ 2414375h^{4}w^{4}\ \mathrm{Cos}\ \frac{h3hw\ i}{2}$$

$$12h\ 84516250h^{8}w^{4} + 70h^{4}\ \ 112320\ \ \ 2414375h^{4}w^{4} + 112h^{4}$$

$$47520 + 754375h^{4}w^{4}\ \ \mathrm{Cos}\ \frac{h3hw\ i}{22}\ \ \ \ 2h^{2}\ 148150h^{7}w^{4}\mathrm{Cos}\ \frac{h5hw\ i}{}$$

$$1931935h^{7}w^{4}\ 15h^{3}\ 28224 + 128125h^{4}w^{4}\ \mathrm{Cos}\ \frac{h7hw\ i}{2}$$

$$528015h^{7}w^{4}\ 5h^{3}\ 12096 + 105625h^{4}w^{4}\ \mathrm{Cos}\ \frac{h9hw\ i}{2}$$

$$+\ 3628800h^{6}w\mathrm{Cos}[5hw]\ 7\mathrm{Sin}\ \frac{7hw}{2\ i}\ \ 149\ \ \ \frac{9hw}{h\ \ 2\ i}\ + \mathrm{Sin}\ \frac{h}{}$$

$h^6 w \quad 7hw \quad + \mathrm{Cos} \quad 9hw \quad \mathrm{Sin}[5hw]$

$3628800 \; 1 \quad 7\mathrm{Cos} \; h \; \frac{}{2} \; i \; hw \; {}_7 \; h \; \frac{}{2} \; i$

$+ \; \frac{}{232243200h^6 w^4} \; f_{n+2} \mathrm{Csc} \; h \; \frac{}{2} i \quad \mathrm{Csc}[hw] \; 35h \; 82706414h^8 w^4 + 900h^4 \quad 10752$

$156875h^4 w^4 + 28h^4 \quad 285120 + 2076875h^4 w^4 \quad \mathrm{Cos} \; \frac{hhw}{2} \; i$

$+ 7 \; 159226250h^9 w^4 + 2250h^5 \quad 10752 \; 156875h^4 w^4 \quad 22176h^5$

$660 + 4375h^4 w^4 + 140h^5 \quad 285120 + 2076875h^4 w^4 \quad \mathrm{Cos} \; \frac{hhw}{2} \; i$

$35h \; 28548750h^8 w^4 + 10h^4 \; 544320 \quad 6701875h^4 w^4 + 16h^4 \quad 317520$

$2404375h^4 w^4 \; \mathrm{Cos} \; h3hw \; i \quad h \; 35h \; 13656284h^7 w^4 + 2h^3 \; 544320 \; {}^2$

$6701875h^4 w^4 \; \mathrm{Cos} \; \frac{h3hw \; i}{} + 7 \; 28548750h^8 w^4 + 10h^4 \; 544320 \; {}^2$

$375h^4 w^4 + 16h^4 317520 + 2404375h^4 w^4 \; \mathrm{Cos} \; h3hw \; i \; {}_2$

$+ 148150h^8 w^4 \mathrm{Cos} \; h \; \frac{5hw}{7hw} \quad + 2h \; 7 \quad 1192660h^7 w^4 + 105h^3 \quad 288$

$+ 11875h^4 w^4 \quad \mathrm{Cos} \; h \; \frac{9hw}{2} \quad 4354570h^7 w^4 + 10h^3 \; 63504$

$+ 439375h^4 w^4 \quad \mathrm{Cos} \; \frac{9hw}{2} \quad 12700800h^6 w \mathrm{Cos}[5hw] \; \mathrm{Sin} \; h \; \frac{}{2} \; i$

$+ 3\mathrm{Sin} \; h \; \frac{1}{2} \; i \quad 12700800h^6 w \; \mathrm{Cos} \; hw \; {}_7 \; h \; \frac{}{2} \; i \quad + 3\mathrm{Cos} \; h \; \frac{}{2} \; i \quad \mathrm{Sin}[5hw]$

$\frac{}{464486400h^6 w^4} \; f_{n+5} \mathrm{Csc} \; h \; \frac{}{2} \; i \quad \mathrm{Csc}[hw] \; 350h \; 19047614h^8 w^4$

$+ 2880h^4 \; 1092 \quad 10625h^4 w^4 \quad {}_5 + 56 \quad h^4 \quad 41472 + 203125h^4 w^4 \quad \mathrm{Cos} \; h \; \frac{}{2} \; i$

$37265000h \; w + 7200h \quad h \; w$

$+ 70 \quad h^5 \quad 4375h^4 w^4 \quad 1092 \quad h^5 \; 10625 \quad 41472$

$4032 \quad 876 + \frac{hw}{2} \; i \quad + 280$

$+ 203125h^4 w^4 \quad \mathrm{Cos} \; h \; \frac{}{2} \; i \quad 70h \quad 28548750h^8 w^4 + 10h^4 \quad 907200$

$375h^4 w^4 + 16h^4 453600 + 2404375h^4 w^4 \; \mathrm{Cos} \; h3hw \; i \; {}_2$

$h \; 70h \; 13656284h^7 w^4 + 2h^3 \; 907200 \; 6701875h^4 w^4 \; \mathrm{Cos} \; h3hw \; i \; {}_2$

hw hw

+14 28548750h°w⁴ + 10h⁴ 907200 6701875h⁴w⁴ + 16h⁴

453600 + 2404375h⁴w⁴ Cosh 3/2 i + 4703500h⁸w⁴Cos h 5/2 i

2h 7 2422145h⁷w⁴ + 5h³ 60480 + 486875h⁴w⁴ Cos h7hw
i 2

h⁷w⁴ h³ + 2168125h⁴w⁴ Cos 9hw

+ 5 2171605 2540167hw i 9hw h 9hw h 2 i
+ 25401600 Cos[5 7]hw h 2 i
h°w hw 5Sin + 3Sin
1 h 2 hw 7 h 2 i
25401600h°w 5Cos + 3Cos Sin[5hw]

h i
464486400h⁶w⁴ f n+3 Csc 2 Csc[hw] 350h 19047614h⁸w⁴ h hw/2 i
+ 2880h⁴ 1260 9 10625h⁴w⁴ 5 + 280 h⁴ 10368 + 40625h⁴w⁴ 5 Cos

+ 70 37265000h w + 7200h1260 10625h w28224h

180 + 625h⁴w⁴ + 1400h⁵ 10368 + 40625h⁴w⁴ Cos hhw/2 i
h h°w⁴ h⁴ 1028160 5583125h⁴w⁴ + 16h⁴

70 26261250 + 10 3hw
574560 + 1848125h⁴w⁴ Cosh 2 i h 70h 11280796h⁷w⁴
2h³ 1028160 5583125h⁴w⁴ Cos h3hw i/2 + 14 26261250h⁸w⁴
h⁴ 5583125h⁴w⁴ + 16h⁴ 574560

+ 10 1028160 3hw 5hw
+ 1848125h⁴w⁴ Cosh 2 + 17280100h⁸w⁴Cos 2 + 2h 7

9905h⁷w⁴ + 5h³336288 + 1143125h⁴w⁴ Cos h7hw i 2

h⁷w⁴ h³ 483125h⁴w⁴ Cos 9hw

+ 5 3438725 7 60480 +hw hw h 2 i
7 9

+ 25401600h⁶wCos[5hw] 3Sin h 2 i + 5Sin h 2 i

$$\frac{1}{25401600h^{6}w}\,3\mathrm{Cos}\left[\frac{7hw}{2}\right]+5\mathrm{Cos}\left[\frac{9hw}{2}\right]\mathrm{Sin}[5hw]$$

$$\frac{}{928972800h^{6}w^{4}}\,f_{n+1}\mathrm{Csc}\left[\frac{h}{2}\right]\mathrm{Sec}\left[\frac{h}{2}\right]\;10h\;286550342h^{8}w^{4}$$

$$302400h^{4}\;68+1875h^{4}w^{4}+56h^{4}\;311040+4995625h^{4}w^{4}\;\mathrm{Cos}\left[\frac{hhw}{2}\right]$$

$$+\,2\;671405000h^{9}w^{4}\;756000h^{5}\;68+1875h^{4}w^{4}\;4032h^{5}\;\mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$10h\;\frac{65049852}{2}\;8220+161875h^{4}w^{4}\;h^{8}w^{4}+14\;280h^{5}\;h^{4}\;164160\;311040+4995625h^{4}w^{4}\;h^{4}w^{4}\;\mathrm{Cos}[h]$$

$$12h\;80053750h\;w+70h\;164160\;4620625h\;w+336h\;33120$$

$$+\,724375h^{4}w^{4}\;\mathrm{Cos}\left[\frac{3}{2}\right]\;2h^{2}\;759250h^{7}w^{4}\mathrm{Cos}\left[\frac{hw}{2}\right]+528015h^{7}w^{4}$$

$$5h^{3}\;12096+105625h^{4}w^{4}\;\mathrm{Cos}[7hw]+1931935h^{7}w^{4}\;15h^{3}\;28224^{2}$$

$$+\,128125h^{4}w^{4}\;\mathrm{Cos}\left[\frac{9hw}{2}\right]+3628800h^{6}w\mathrm{Cos}[5hw]\;\mathrm{Sin}\left[\frac{7hw}{2}\right]$$

$$+\,7\mathrm{Sin}\left[\frac{h}{2}\right]\;3628800h^{6}w\mathrm{Cos}\left[\frac{h}{2}\right]+7\mathrm{Cos}\left[\frac{h}{2}\right]\mathrm{Sin}[5hw]$$

$$y_n^{0+6} = \frac{11y_n}{60h} + \frac{10y_{n+3}}{3h} - \frac{27y_{n+4}}{4h} + \frac{18y_{n+5}}{5h}$$

$$+ \frac{1}{46448640h^6w^4} f_{n+4}\mathrm{Csc}\left[\frac{hw}{2}\right]^8 \Big(85730400h^5 \quad 42h \quad 14245038h^8w^4$$

$$2520h^4 \quad 1152 + 9072h^4w^4 \quad 280h^4 \quad 7344 + 29808h^4w^4 \quad \mathrm{Cos}[hw]$$

$$2843316h^9w^4\mathrm{Cos}[2hw] \quad 1153116h^9w^4\mathrm{Cos}[3hw]$$

$$6h^2 \; 1818486h^7w^4 \; 20h^3 \; 5292 + 92016h^4w^4 \; \mathrm{Cos}[4hw]$$

$$2h \; 42h \quad 3453000h^7w^4 + 420h^3 \quad 1152 + 9072h^4w^4 \quad \mathrm{Cos}[hw]$$

$$+7 \quad 14245038h^8w^4 + 2520h^4 \quad 1152 + 9072h^4w^4 \qquad 280h^4$$

$$7344 + 29808h^4w^4 \qquad \mathrm{Cos}[hw]$$

$$4051278h^8w^4\mathrm{Cos}[2hw] + 1631178h^8w^4\mathrm{Cos}[3hw] \quad 12042h^8w^4\mathrm{Cos}[4hw]$$

$$8h \; 1818486h^7w^4 \quad 20h^3 \quad 5292 + 92016h^4w^4 \quad \mathrm{Cos}[4hw]$$

$$12700800h^6w\mathrm{Sin}[2hw]$$

$$\frac{1}{464486400h^6w^4} f_{n+8}\mathrm{Csc}\left[\frac{hw}{2}\right]^7 \Big(\mathrm{Csc}[hw]30h^2 \qquad \mathrm{Cos}\left[\frac{hw}{2}\right] \qquad 2109762h^7w^4$$

$$+ 3h^3 \quad 20160 + 737424h^4w^4 \quad \mathrm{Cos}\left[\frac{hw}{2}\right] \quad 35h \quad 8879040h^8w^4$$

$$+ h^4 \quad 60480 \quad 4252176h^4w^4 \quad + 18h^4 \quad 20160 + 737424h^4w^4 \quad \mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$1576260h^9w^4\mathrm{Cos}\left[\frac{3}{2}\right]$$

$$324h^2 \quad 1591066h^7w^4 + 35h^3 \quad 1152 \; 45360h^4w^4 \quad \mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$6h^3 \quad 6840h^6w^4\mathrm{Cos}\left[\frac{5}{2}\right] \quad 210h^6w^4\mathrm{Cos}\left[\frac{7}{2}\right]$$

$$11h^2 \qquad 7740h^7w^4\mathrm{Cos}[5/2]$$

$$+ 1095336h^7w^4 \quad 5h^3 \quad 12096 + 219024h^4w^4 \quad \mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$\frac{hw}{7} \qquad \frac{hw}{7}$$

$$+ 3628800h^6w\mathrm{Cos}[6hw]\mathrm{Sin}\left[\frac{hw}{2}\right] \quad 3628800h^6w\mathrm{Cos}\left[\frac{hw}{2}\right] \quad \mathrm{Sin}[6hw]$$

$$+ \frac{1}{464486400h^{6}w^{4}\,f_{n+7}} \quad \text{hw} \quad \text{Csc}\left[\frac{h\,hw}{2}\right] \quad \text{Csc[hw]} \quad 90h^{2} \quad 8398350h^{7}w^{4} + 9h^{3} \quad 12096$$

$$+ 939600h^{4}w^{4}\ \text{Cos}\left[\frac{h}{2}\right] 45h8067672h^{8}w^{4} + h^{4}\ \text{Cos}\left[\frac{h3hw}{2}\right] 616896\ 42634512h^{4}w^{4} + 54h^{4}$$

$$12096 + 939600h^{4}w^{4}\ \text{Cos}\left[\frac{hhw}{2}\right] 300780h^{9}w^{4}\text{Cos}\left[\frac{h}{2}\right] 180h^{2}\ 4669122h^{7}w^{4}$$

$$+ h^{3}\ 67200\ 4653936h^{4}w^{4}\ \text{Cos}\left[\frac{h5hw}{2}\right] 6h^{3}\ 240h^{6}w^{4}\text{Cos}\left[\frac{h7hw}{2}\right] 42h^{6}w^{4}\text{Cos}\left[\frac{h9hw}{2}\right]$$

$$11h^{2}\ 80820h^{7}w^{4}\text{Cos}\left[\frac{h}{2}\right] + 9792h^{7}w^{4}\text{Cos}\left[\frac{h}{2}\right] 60480h^{3}\text{Cos}\left[\frac{h}{2}\right]$$

$$+ 3628800h^{6}w\text{Cos}[6hw]\text{Sin}\left[\frac{hhw}{2}\right] \quad 3628800h^{6}w\text{Cos}\left[\frac{h9hw}{2}\right] \text{Sin}[6hw]$$

$$+ \frac{}{116121600h^{6}w^{4}\,f_{n+6}}\text{Csc}\left[\frac{h}{2}\right] \text{Csc[hw]}\ \frac{}{2}\ 42h\ 189806298h^{8}w^{4} + 1080h^{4}\ 8064$$

$$255312h^{4}w^{4} + 196h^{4}\ 25920 + 426384h^{4}w^{4}\ \text{Cos}\left[\frac{hhw}{2}\right] + 7\ 452465568h^{9}w^{4}$$

$$+ 3240h^{5}\ 8064\ 255312h^{4}w^{4} + 2016h^{5}\ 3420\ 63504h^{4}w^{4} + 1176h^{5}\ 25920$$

$$426384h^{4}w^{4}\ \text{Cos}\left[\frac{hhw}{2}\right] 42h\ 77816322h^{8}w^{4} + 12h^{4}\ 423360\ 11577168h^{4}w^{4}$$

$$16h^{4}196560 + 3832272h^{4}w^{4}\ \text{Cos}\left[\frac{h3hw}{2}\right] 1153980h^{9}w^{4}\text{Cos}\left[\frac{h5hw}{2}\right] 3h^{2}\ 7$$

$$157806h^{7}w^{4} + 5h^{3}18144 + 32400h^{4}w^{4}\ \text{Cos}\left[\frac{h7hw}{2}\right] + 3798306h^{7}w^{4}\ 2$$

$$10h^{3}\ 21168 + 379728h^{4}w^{4}\ \text{Cos}\left[\frac{h9hw}{2}\right] 2h\ 42h\ 23483517h^{7}w^{4} + 2h^{3}\ 423360\ 2$$

$$11577168h^{4}w^{4}\ \text{Cos}\left[\frac{h3hw}{2}\right] + 7\ 77816322h^{8}w^{4} + 12h^{4}\ 423360\ 11577168h^{4}w^{4}\ 2$$

$$+ 16h^{4}\ 196560 + 3832272h^{4}w^{4}\ \text{Cos}\left[\frac{3hw}{2}\right] + 1550160h^{8}w^{4}\text{Cos}\left[\frac{5hw}{2}\right]$$

$$+3h^{2}\ 21735h^{6}w^{4}\text{Cos}\left[\frac{7hw}{2}\right] 3441h^{6}w^{4}\text{Cos}\left[\frac{9hw}{2}\right] +4h\ 7\ 157806h^{7}w^{4} +5h^{3}\ 18144$$

$$+ 32400h^{4}w^{4}\ \text{Cos}\left[\frac{7hw}{2}\right] + 3798306h^{7}w^{4}\ 10h^{3}\ 21168 + 379728h^{4}w^{4}\ \text{Cos}\left[\frac{7hw}{2}\right]$$

$$+ 6350400h^{6}w\text{Cos}[6hw]\ 3\text{Sin}\left[\frac{hhw}{2}\right] + \text{Sin}\left[\frac{hhw}{2}\right] 6350400h^{6}w\ 3\text{Cos}\left[\frac{hhw}{2}\right]$$

$$+ \text{Cos}\left[\frac{h9hw}{2}\right] \text{Sin[6hw]} \quad \frac{}{464486400h^{6}w^{4}\,f_{n+7}}\text{Csc}\left[\frac{h}{2}\right] \text{Csc[hw]}\ 12h547272462h^{8}w^{4}$$

$$120960h^{4}\ 132 + 6480h^{4}w^{4} + 56h^{4}\ 155520 + 4138128h^{4}w^{4}\ \text{Cos}\left[\frac{hhw}{2}\right]$$

$$+ 2\ 1308392604h^{9}w^{4}\ 362880h^{5}\ 132 + 6480h^{4}w^{4}\ 4032h^{5}$$

$$2460 + 86832h^{4}w^{4} + 336h^{5}\ 155520 + 4138128h^{4}w^{4}\ \text{Cos}\left[\frac{hhw}{2}\right]$$

$$+336h^{5} \quad 155520 + 4138128h^{4}w^{4}\mathrm{Cos}\left[\frac{hw}{2}\right] \quad 24h^{2} \quad 245737530h^{8}w^{4}$$

$$84h^{4} \quad 112320 \quad 5006448h^{4}w^{4} \quad + 112h^{4} \quad 47520$$

$$1564272h^{4}w^{4} \quad \mathrm{Cos}\left[\frac{h3hw}{22}\right] \quad 6h^{3} \quad 139320h^{7}w^{4}\mathrm{Cos}\left[\frac{h5hw}{}\right]$$

$$3994992h^{7}w^{4} \quad 15h^{3} \quad 28224 + 265680h^{4}w^{4}\mathrm{Cos}\left[\frac{h7hw}{}\right] + 1095336h^{7}w^{4} \quad 2$$

$$5 \quad h^{5} \quad 12096 + 219024h^{4}w^{4} \quad \mathrm{Cos}\left[\frac{9hw}{2}\right] + 3628800h^{5} \quad 7\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$+ \mathrm{Cos}\left[\frac{21}{}\right] \quad \mathrm{Cos}[6 \quad] + \quad hw \quad 3628800h^{5} \quad 7\mathrm{Sin}\left[\frac{h}{2}\right] + \mathrm{Sin}\left[\frac{h}{2}\right] \quad \mathrm{Sin}[6hw]$$

$$+ \frac{5}{232243200h^{5}w^{4}} f_{n+2}\mathrm{Csc}\left[\frac{h4}{2}\right] \quad \mathrm{Csc}[hw] \quad 42h \quad 530539272h^{5}w^{4}$$

$$+ 3240h \quad 10752 \quad 325296h \quad w \quad 22176h \quad 660 + 9072h \quad w$$

$$+ 168h^{5} \quad 285120 + 4306608h^{4}w^{4} \quad \mathrm{Cos}\left[\frac{hw}{2}\right] \quad 2h \quad 42h \quad 87325110h^{8}w^{4}$$

$$12h^{4} \quad 544320 \quad 13897008h^{4}w^{4} \quad + 16h^{4} \quad 317520$$

$$4985712h^{4}w^{4} \quad \mathrm{Cos}\left[\frac{h3hw}{22}\right] + 208980h^{9}w^{4}\mathrm{Cos}\left[\frac{}{}\right] + 3h^{2} \quad 7$$

$$2530422h^{7}w^{4} + 105h^{3} \quad 288 + 24624h^{4}w^{4} \quad \mathrm{Cos}\left[\frac{h7hw}{2}\right] \quad 9070578h^{7}w^{4}$$

$$+ 10 \quad h^{5} \quad 63504 + 911088h^{4}w^{4} \quad \mathrm{Cos}\left[\frac{h}{2}\right] + 12700800h^{5} \quad \mathrm{Cos}\left[\frac{h}{2}\right]$$

$$+ 3\mathrm{Cos}\left[\frac{h}{2}\right] \quad \mathrm{Cos}[6hw] + 12700800h^{6} \quad \mathrm{Sin}\left[\frac{h}{2}\right] + 3\mathrm{Sin}\left[\frac{h}{2}\right] \quad \mathrm{Sin}[6hw]$$

$$\frac{5}{464486400h^{6}w^{4}} f_{n+5}\mathrm{Csc}\left[\frac{h4}{2}\right] \quad \mathrm{Csc}[hw] \quad 420h \quad 122942556h^{9}w^{4}$$

$$+ 10368h \quad 1092 \quad 22032h \quad w \quad 4032h \quad 876 + 9072h \quad w \quad + 336h$$

$$41472 + 421200h^{4}w^{4}\mathrm{Cos}\left[\frac{hw}{2}\right] \quad 2h \quad 84h \quad 87325110h^{8}w^{4} + 12h^{4} \quad 907200$$

$$7008h^{4}w^{4} + 16h^{4}453600 + 4985712h^{4}w^{4} \quad \mathrm{Cos}\left[\frac{h3hw}{2}\right] \quad \frac{}{}$$

12297960h^9w^4Cos h5hw i 3h^2 7 5124024h^7w^4 + 5h^3 60480 2

1009584h^4w^4 Cos h7hw i + 5 4517136h^7w^4 h^3 254016 2

+ 4495824h^4w^4 Cos h 9 hw 2 i 2h 84h 28212387h^7w^4 + 2h^3 907200

13897008h^4w^4 Cos h3hw i + 14 87325110h^8w^4 + 12h^4 907200 2

7008h^4w^4 + 16h^4453600 + 4985712h^4w^4 Cos h3hw i 2

+ 23508000h^8w^4Cos h 5hw hw2 i + 3h^2 752262h^6w^4Cos h 7hw 2 i

+ 106290h^6w^4Cos h 9 2 i 7 hw + 4h^7 5124024h^7w^4 + 5h^3 60480

+ 1009584h^4w^4 Cos h 9hw 2 i + 5 4517136h^7w^4 h^3 254016 7hw

+ 4495824h^4w^4 9hw Cos h 2 i + 25401600h^6wCos[6hw] 5Sin 9hw 2 i

h 1 i hw 7 h i h 2 i

+ 3Sin 2 25401600h^6w 5Cos 2 + 3Cos Sin[6hw]

464486400h^6w^4 f_{n+3}Csc h 2 i Csc[hw] 420h 51570750h^8w^4 h hw i

+ 3456 1260 9 4 4 5 5 2

h^4 22032h^4w^4 + 280h^4 10368 + 84240h^4w^4 Cos

+ 70 122942556h w + 10368h 1260 22032h w28224h

180 + 1296h^4w^4 + 1680h^5 10368 + 84240h^4w^4 Cos hhw 2 i

h h^3w^4 h^4 1028160 11577168h^4w^4 + 16h^4

84 77816322 + 12 hw hw

574560 + 3832272h^4w^4Cos h 3 2 i 25497720h^9w^4Cos h 5 2 i

288 + 2370384h^4w^4 Cos h7hw i 2

5 7067736h^7w^4 7h^3 60480 + 1001808h^4w^4 Cos h9hw i 2

2h 84h 23483517h^7w^4 + 2h^3 1028160 11577168h^4w^4 Cos h3hw i 2

+14 77816322h^8w^4 + 12h^4 1028160 11577168h^4w^4 + 16h^4

574560 + 3832272h^4w^4 Cos h3hw i / 22 + 34314480h^8w^4 Cos h5hw i

+3h^2 10290h^6w^4Cos h7hw i / 2 2310h^6w^4Cos h9hw i / 2 +4h 7 12077712h^7w^4

5h^3 36288 + 2370384h^4w^4 Cos h7hw i + 5 7067736h^7w^4 7h^3

60480 2

+ 1001808h^4w^4 Cos 9hw h 2 i / 9hw + 7hw 25401600h^6wCos[6hw] 9hw 3Sin h 2 i / 7hw

+ 5Sin h 1 i / 2 25401600h^6w 3Cos hw 8 / 2 hw i + 5Cos h i / 2 Sin[6hw]

928972800h^6w^4 f_{n+1}Csc h i / 2 Sec h / 2 i 12h 826369806h^8w^4 362880h^4

68 + 3888h^4w^4 + 56h^4 311040 + 10358928h^4w^4 Cos hhw i / 2

+ 2 2103103836h^9w^4 1088640h^5 68 + 3888h^4w^4 4032h^5 hw i / 2

8220 + 335664h^4w^4 + 336h^5 311040 + 10358928h^4w^4 Cos

24 134706279 8 w 4 + 14 4 164160 9581328 4 w 4 Cos h 4 2 i

h h + 84h 164160 9581328h + 336h 33120

16 300594654 hw

+1502064h^4w^4 Cos h 3 / 2 6h^3 7680h^6w^4Cos h5hw i / 5 / 2 210h^6w^4Cos 7hw / 2 i

42h^6w^4Cos h9hw i / 22 11h^2 769320h^7w^4Cos + 1095336h^7w^4

5h^3 12096 + 219024h^4w^4 Cos h7hw i + 3994992h^7w^4 15h^3 28224 2

h^4w^4 Cos 9hw 3628800h^6wCos[6hw] Sin 7hw

+ 265680 hw / 9 h 2 i + 7hw / 7 9hw / 9 h 2 i

+ 7Sin h i / 2 3628800h^6wCos h / 2 i + 7Cos h i / 2 Sin[6hw]

$$y_n^{0+7} = \frac{13y_n}{30h} + \frac{41y_{n+3}}{6h} - \frac{25y_{n+4}}{2h} + \frac{61y_{n+5}}{10h}$$

$$+ \frac{1}{46448640h^6w^4} f_{n+4}\mathrm{Csc}\left[\frac{hw}{2}\right]^8 \Big(158760000h^5 \quad 49h \quad 33252116h^8w^4$$

$$2940h^4 \quad 1152 + 16807h^4w^4 \quad 280h^4 \quad 7344 + 55223h^4w^4 \quad \mathrm{Cos}[hw]$$

$$7430752h^9w^4\mathrm{Cos}[2hw] \quad 3232152h^9w^4\mathrm{Cos}[3hw]$$

$$16h^2 \quad 3386334h^7w^4 \quad 20h^3 \quad 5292 + 170471h^4w^4 \quad \mathrm{Cos}[4hw]$$

$$3h \quad 49h \quad 6569486h^7w^4 + 420h^3 \quad 1152 + 16807h^4w^4 \quad \mathrm{Cos}[hw]$$

$$+7 \quad 33252116h^8w^4 + 2940h^4 \quad 1152 + 16807h^4w^4 \quad 280h^4$$

$$7344 + 55223h^4w^4 \quad \mathrm{Cos}[hw]$$

$$4702824h^8w^4\mathrm{Cos}[2hw] + 2586474h^8w^4\mathrm{Cos}[3hw] \quad 16032h^8w^4\mathrm{Cos}[4hw]$$

$$12h \quad 3386334h^7w^4 \quad 20h^3 \quad 5292 + 170471h^4w^4 \quad \mathrm{Cos}[4hw]$$

$$12700800h^6w\mathrm{Sin}[3hw]$$

$$\frac{1}{464486400h^6w^4} f_{n+8}\mathrm{Csc}\left[\frac{hw}{2}\right]^7 \mathrm{Csc}[hw] \quad 70h^2 \quad 3980435h^7w^4$$

$$+ 3h^3 \quad 20160 + 1366169h^4w^4 \quad \mathrm{Cos}\left[\frac{hw}{2}\right] \quad 45h \quad 20558956h^8w^4$$

$$+ h^4 \quad 60480 \quad 7877681h^4w^4 \quad + 21h^4 \quad 20160 + 1366169h^4w^4 \quad \mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$4466448h^9w^4\mathrm{Cos}\left[\frac{3}{2}\right]$$

$$549h^2 \quad 2950341h^7w^4 + 35h^3 \quad 115284035h^4w^4 \quad \mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$24h^3 \quad 46965h^6w^4\mathrm{Cos}\left[\frac{5}{2}\right] + 1233h^6w^4\mathrm{Cos}\left[\frac{7}{2}\right]$$

$$26h^2 \quad 33530h^7w^4\mathrm{Cos}[5/2]$$

$$+ \quad 2028901h^7w^4 \quad 5h^3 \quad 12096 + 405769h^4w^4 \quad \mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$+ 3628800h^6w\mathrm{Cos}[7hw]\mathrm{Sin}\left[\frac{7hw}{2}\right] \quad 3628800h^6w\mathrm{Cos}\left[\frac{7hw}{2}\right] \quad \mathrm{Sin}[7hw]$$

$\mp \frac{1}{464486400h^6w^4}$ f_nCsc $\frac{hw}{2}^7$ Csc[hw] $\frac{hw}{140h^2}$ 15569653h^{7}w^4

8 4 4 4 4

+ 9h^3 12096 + 1740725h^{4}w^4 Cos $\overline{2}$

55h ^{4} 30565366h w + h ^{4} ^{4} 616896 ^{hw} 78985697h w ^{9} ^{4} 3hw

hw

+ 63h 12096 + 1740725h w Cos $\overline{2}$ 1883952h w Cos $\overline{2}$

369h^2 8641015h^{7}w^4 + h^3 67200 8621991h^{4}w^4 Cos $\frac{3}{2}$

24h^3 1365h^{6}w^{4}Cos $\frac{5hw}{2}$ 15h^{6}w^{4}Cos $\frac{7hw}{2}$

26h^2 80360h^{7}w^{4}Cos 5 2 + 9842h^{7}w^{4}Cos 7 2

hw hw

60480h^{3}Cos 9 2 + 3628800h^{6}wCos[7hw]Sin 9 2

hw hw

3628800h^{6}wCos $\frac{9hw}{2}$ Sin[7hw] + $\frac{1}{464486400h^6w^4}$ f_nCsc h$\frac{hw}{2}$ i^7 Csc[hw]

4597880h^{8}w^{4}Cos h$\frac{hw}{2}$ i 110h 15569653h^{7}w^4 + 9h^3 12096

h^{4}w^4 hw h^{8}w^4 h^4 616896

+ 1740725 Cosh 2 10 30565366h^{4}w^4 + hw

78985697 + hw 12096 + 1740725 h 2i

4477284h^{8}w^{4}Cos h$\frac{3}{2}$i 216h 8641015h^{7}w^4 + h^3 67200

h3hw i h5hw i

8621991h^{4}w^4 Cos 24h^3 10130h^{5}w^{4}Cos

22

h7hw i h5hw i h7hw i

406h^{5}w^{4}Cos 52h^2 1365h^{6}w^{4}Cos 15h^{6}w^{4}Cos

222

h h^{7}w^{4}Cos $\frac{5hw}{}$ + 9842h^{7}w^{4}Cos $\frac{7hw}{}$ 60480h^{3}Cos $\frac{9hw}{}$

18 80360 h$\frac{9hw}{2}$ 2 i 7 1 h 2 h$\frac{9hw}{2}$i h 2 i

3628800h^{6}w^{2}Cos Cos[7hw] 3628800h^{6}w^{2}Sin Sin[7hw]

$\mp \frac{1}{116121600h^6w^4}$ f_{n+6}Csch $\frac{}{2}$ Csc[hw] $\frac{hw}{2}$ 49 h 82810438 h^{7}w^4

+ 180h^3 8064 472997h^{4}w^{4}Cosh 2 i

$h^8w^4 \quad h^4 \quad 8064 \quad 472997h^4w^4 \quad h^4$

$+14 \quad 436464422 \quad + 1260 \quad hw \quad + 196$

$25920 + 789929h^4w^4 \operatorname{Cos}\frac{h}{2}i \quad 98h \quad 43609084h^7w^4$

$2h^3 \; 423360 \; 21448133h^4w^4 \operatorname{Cos}\frac{h3hwi}{2}$

$14 \; 187395374h^8w^4 + 14h^4 \quad 423360 \quad 21448133h^4w^4$

$16h^4 196560 + 7099757h^4w^4$

$\operatorname{Cos}\frac{h3hwi}{2}$

$5207100h^8w^4\operatorname{Cos}\frac{h5hwi}{22} \quad 16h^2 \; 177324h^6w^4\operatorname{Cos}\frac{h7hwi}{}$

$23490h^6w^4\operatorname{Cos}\frac{h9hwi}{} \quad 12h \; 7 \; 283276h^7w^4$

$+ 5h^3 \; 2$

$18144 + 60025h^4w^4 \operatorname{Cos}\frac{h7hwi}{} \quad + 7023086h^7w^4 \; 2$

$9hw$

$10h^3 \quad 21168 + 703493h^4w^4 \operatorname{Cos}\frac{h}{2} \; i$

$$y_{n+8}^{0} = 60h + \frac{47y_n}{56h} + \frac{3h}{68725630h^8w^4 + 3360h^4 1152 + 28672h^4w^4} + \frac{34y_{n+3}}{280h^4 7344 + 94208h^4w^4} + \frac{4h}{\text{Cos}[hw]} + \frac{79y_{n+4}}{11664240h^9w^4\text{Cos}[2hw] 6780240h^9w^4\text{Cos}[3hw]} + \frac{5h}{30h^2 5792360h^7w^4 20h^3 5292 + 290816h^4w^4 \text{Cos}[4hw] 4h} + \frac{46y_{n+5}}{56h 11464840h^7w^4 + 420h^3 1152 + 28672h^4w^4 \text{Cos}[hw]} + \frac{1}{7 68725630h^8w^4 + 3360h^4 1152 + 28672h^4w^4 280h^4 7344 + 94208h^4w^4 \text{Cos}[hw]} + \frac{46448640h^6w^4 f_{n+4}\text{Csc}}{3693998h^8w^4\text{Cos}[2hw] + 4904298h^8w^4\text{Cos}[3hw] + 7110h^8w^4\text{Cos}[4hw] 16h 5792360h^7w^4 20h^3 5292 + 290816h^4w^4 \text{Cos}[4hw] 12700800h^6w\text{Sin}[4hw]}$$

$$+ \frac{1}{464486400h^6w^4 f_{n+8}\text{Csc}\frac{hw}{2}\text{Csc}[hw] 120h^2 6866602h^7w^4 + 3h^3 20160 + 2330624h^4w^4 \text{Cos}\frac{hw}{2}} \frac{250840800h^5}{55h 42119000h^8w^4 + h^4 60480 13438976h^4w^4 + 24h^4 20160 + 2330624h^4w^4 \text{Cos}\frac{hw}{2}\text{Cos}\frac{3hw}{2}}$$

$$47h^2 \quad 90400h^7w^4\text{Cos}\frac{hw}{2} \quad 60h^3 \quad 48300h^6w^4\text{Cos}\frac{hw}{2} + 25122h^6w^4\text{Cos}\frac{7hw}{2}$$

$$+ 3628800h^6w\text{Cos}[8hw]\text{Sin}\frac{hw}{2} \quad 3628800h^6w\text{Cos}\frac{7hw}{2}\text{Sin}[8hw]$$

$$+ \frac{1}{464486400h^6w^4 f_n\text{Csc}\frac{hw}{2}\text{Csc}[hw] 200h^2 26601422h^7w^4 + 9h^3 12096}$$

$$2969600h^4w^4\text{Cos}\frac{hhwi}{2} \quad 65h 78838888h^8w^4 + h^4 616896 134746112h^4w^4 + 72h^4 2$$

$$12096 + 2969600h^4w^4\text{Cos}\frac{hhwi}{2} \quad 5081184h^9w^4\text{Cos}\frac{h3hwi}{2} \quad 612h^2 14733416h^7w^4$$

$$+ h^3 \quad 67200 14708736h^4w^4\text{Cos}\frac{h3hwi}{2} \quad 60h^3 \quad 21660h^6w^4\text{Cos}\frac{h5hwi}{2} + 498h^6w^4\text{Cos}\frac{h7hwi}{2}$$

h^2 $90400h^7w^4$ $h\frac{5hw}{2}$ + $9808h^7w^4\mathrm{Cos}\frac{7hw}{2}$ $60480h^3\mathrm{Cos}\frac{9hw}{2}$

47 Cos

+ $3628800h^{10}w\mathrm{Cos}[8hw]\mathrm{Sin}$ $3628800h^{10}w\mathrm{Cos}$ Sin[8hw]

+ $\frac{}{116121600h^{10}w^4}$ $f_{n+6}\mathrm{Csc}$ $\frac{}{2}$ Csc[hw] $\frac{}{2}$ 56h 890809454h^8w^4

h 806912h w + 196h

+ 1440 8064 $\frac{hw}{2}$ 25920 h^5

+ $1347584h^4w^4$ Cos + 7 2928715112h^9w^4 + $5760h^9$ 8064

806912h w + 2016 hw 3420 200704 + 1568 25920

+ $1347584h^4w^4$ Cos $\frac{}{2}$ 56h $393278076h^8w^4$ + $16h^4$ 423360

$568h^4w^4$ + $16h^4$196560 + $12111872h^4w^4$ Cos $\frac{}{2}$

$6895200h^9w^4\mathrm{Cos}\frac{5hw}{}$ $15h^2$ 7 $452088h^7w^4$ + $5h^3$ 18144 2

$102400h^4w^4$ Cos + $11960984h^7w^4$ $10h^3$ 21168 2

+ $1200128h^4w^4$ Cos $\frac{hw}{2}$ 4h 56h $74283709h^7w^4$ + $2h^3$ 423360

$36589568h^4w^4$ Cos + 7 $393278076h^8w^4$ + $16h^4$ 423360 2

$568h^4w^4$ + $16h^4$196560 + $12111872h^4w^4$ Cos $\frac{}{2}$

+ $5251540h^8w^4\mathrm{Cos}\frac{5hw}{2}$ + $15h^2$ $432831h^6w^4\mathrm{Cos}\frac{7hw}{2}$

$23901h^6w^4\mathrm{Cos}$ + 8h $7452088h^7w^4$ + $5h^3$

18144 2

$102400h^4w^4$ Cos + $11960984h^7w^4$ $10h^3$ 21168 2

+ $1200128h^4w^4$ $\frac{9hw}{2}$ 6350400h^6wCos[8hw] 3Sin $\frac{7hw}{2}$

+ Sin $\frac{9hw}{2}$ Cos + $\frac{7hw}{2}$ + Cos $\frac{9hw}{2}$ Sin[8hw]

$\frac{}{464486400h^{10}w^4}$ $f_{n+7}\mathrm{Csc}$ $\frac{}{2}$ Csc[hw] 16h $2553657038h^8w^4$

161280h 132 + 20480h w

$$+56 \quad {}_{5}155520 + \quad {}_{13078528h^{4}w^{4}} \quad \text{Cos} \frac{h}{\frac{hw}{2}} \quad {}_{5} \quad i + 2 \quad {}_{8436251960h^{4}w^{4}} \quad {}_{5}$$

$$h^{4}$$

$$645120h \quad 132 + 20480h\,w \quad 4032h \quad 2460 + 274432h\,w \quad + 448h$$

$$155520+13078528h^{4}w \quad 3hw\text{Cos}'' \quad \frac{2\ i}{4} \quad 64h^{2} \quad 222372473hw \quad + 14h \quad 112320$$

$$15822848h^{4}w^{4} \quad \text{Cos}\frac{h}{\frac{2}{5hw}}' \quad 24h \quad 1220853292h^{8}w^{4} \quad + 112\frac{h^{4}}{3hw}\frac{i}{112320}$$

$$15822848h^{4}w^{4} \quad + 112h^{4} \quad 47520 + 4943872h^{4}w^{4} \quad \text{Cos}\frac{h}{\frac{2}{7hw}}$$

$$h^{3} \quad h^{6}w^{4}\text{Cos}\frac{\quad}{\quad} \quad + 498h^{6}w^{4}\text{Cos}\frac{\quad}{\quad}i$$

$$60 \quad 862200 \quad \frac{hw}{\frac{2}{9}}'' \quad \frac{h\ 2\ i}{i} \quad 47h^{2} \quad 919360h^{7}w^{4}\text{Cos}\frac{h\ 2\ hw}{\frac{2}{5}}'' \quad + \quad 12605008h^{7}w^{4}$$

$$+ 25122h^{6}w^{4}\text{Cos}$$

$$15h^{3} \quad 28224 + 839680h^{4}w^{4}\text{Cos}\frac{h7hw\ i}{} \quad + 3470928h^{7}w^{4} \quad 5h^{3} \quad 12096^{2}$$

$$692224h^{4}w^{4} \quad \text{Cos}\frac{h\ 2\ i}{9hw} \quad + 3628800h^{9}w\text{Cos}[8hw] \quad 7\text{Sin}\frac{h\ 2\ i}{7hw}$$

$$+ \quad 9hw$$

$$+ \text{Sin}\frac{2\ 1}{\quad}' \quad 3628800 \quad \frac{hw}{h^{9}w} \quad 7\text{Cos}\frac{h\ 2\ i}{7} \quad + \text{Cos}\frac{h\ 2\ i}{\quad} \quad \text{Sin}[8hw]$$

$$+ \frac{4}{232243200h^{9}w^{4}} \quad f_{n+2}\text{Csc}\frac{h\ 4\ i}{2} \quad \text{Csc}[hw] \quad 56h \quad 1091456774h^{8}w^{4}$$

$$+ 1440 \quad h \quad 1028096h\,w \quad + 28h \quad 285120$$

$$+ 13611008_{h^{4}w^{4}} \quad {}_{4}\text{Cos}\frac{hw}{2}'' \quad {}_{5}i + 7 \quad 3498091880_{4} \quad {}_{4} \quad + 5760_{5} \quad 10752$$

$$660 + 28672h\,w \quad + 224h$$

$$1028096 \quad 22176 \quad \frac{hw}{2}' \quad 56h \quad 452260476h^{8}w^{4}$$

$$285120 + 13611008h^{4}w^{4}\text{Cos}''$$

$$16h^{4} \quad 544320 \quad 43921408h^{4}w^{4} \quad + 16h^{4} \quad 317520$$

$$15757312h^{4}w^{4} \quad \text{Cos}\frac{h3hw\ i}{22} \quad 6895200h^{9}w^{4}\text{Cos}\frac{h5hw\ i}{\quad} \quad 15h^{2} \quad 7$$

$$111608h^{7}w^{4} + 105h^{3} \quad 288 + 77824h^{4}w^{4} \quad \text{Cos}\frac{h7hw\ i}{\quad} \quad 2$$

$$28754584h^{7}w^{4} + 10h^{3} \quad 63504 + 2879488h^{4}w^{4}\text{Cos}\frac{h9hw\ i}{\quad} \quad 2$$

$$4h \quad 56h \quad 88728419h^{7}w^{4} + 2h^{3} \quad 544320 \quad 43921408h^{4}w^{4}\text{Cos}\frac{h3hw\ i}{\quad} \quad 2$$

$$+ 7 \quad 452260476h^{8}w^{4} + 16h^{4} \quad 544320 \quad 43921408h^{4}w^{4} \quad + 16h^{4} \quad \frac{3hw}{\quad} \quad \frac{5hw}{\quad}$$

$$317520 + 15757312h\ w\ \text{Cos}\frac{h}{2\ i} + 10143940h\ w\text{Cos}\frac{h}{2\ i}$$

+15h 81039h w Cos 7hw i +1731 7hw Cos 9hw i +8h 7 8111608h w

+ 105 288 + 778249hw Cosh 2 i 28754584 7hw

h h w + 10h 63504

+ 9hw h 2 i 7hw 9hw h 2 i

2879488h w Cos + 12700800h wCos[8hw] Sin

h 1 i hw 7 h i h i

+ 3Sin 12700800h w Cos + 3Cos Sin[8hw]

464486400h w f Csc h i Csc[hw] 560h 242773406h w

+ 4608 1092 9 4 5 4 4 5 h hw i

h 69632h w + 56h 41472 + 1331200h w Cos

+ 70 797748728h w + 18432h1092 69632h w4032h

876 + 28672h w + 448h 41472 + 1331200h w Cos hhw i 2

112h 452260476h w + 16h 907200 43921408h w

16h4453600 + 15757312h4w4

h3hw i

Cos 2

158376000h9w4Cos h5hw i 15h2 7

16150256h7w4 + 5h3 2

60480 + 3190784h w Cos h7hw i + 5 14263984h w h 254016

2

+ 14209024h w Cos h hw 9 2 i 4h 112h 88728419h w + 2h 907200

h3hw i

43921408h4w4 Cos + 14 452260476h8w4 + 16h4 907200 2

408h4w4 + 16h4453600 + 15757312h4w4 Cos h3hw i 2

h w Cos 5hw + 15h 359478h w Cos 7hw

+ 139372600 Cos 9hw 2 i h 2

+ 75750 h6w4 + 8h 7 16150256h w + 5h 60480

7

+ 3190784h w Cosh 2hw i + 5 14263984h w h 254016 h 7hw

2 2

164

$$+3\mathrm{Sin}\left[\frac{9hw}{2}\right] \quad h^6 w\, 5\mathrm{Cos}\left[\frac{7hw}{2}\right] \quad \mathrm{Sin}[8hw] + 3\mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$\frac{25401600\, hw\, 7}{464486400h^6 w^4 f_{n+3}\mathrm{Csc}\left[\frac{hw}{2}\right]} \quad \mathrm{Csc}[hw]\, 560h\, 242773406h^8 w^4$$

$$+ 4608h^{\cdot\cdot}\ 1260\ 69632h^{\cdot\cdot}w^{\cdot\cdot} + 280h^{\cdot\cdot}\ 10368 + 266240h^{\cdot\cdot}w^{\cdot\cdot}\ \mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$+ 70\ 797748728h\, w + 18432h\ 1260\ 69632h\, w28224h$$

$$180 + 4096h^4 w^4 + 2240h^5\ 10368 + 266240h^4 w^4\ \mathrm{Cos}\left[\frac{hhw}{2}\right]$$

$$112h\ 393278076h^{\circ} w^{\cdot\cdot} + 16h^{\cdot\cdot}\ 1028160\ 36589568h^{\cdot\cdot}w^{\cdot\cdot} + 16h^{\cdot\cdot}\ 574560$$

$$+12111872h^4 w^4\ \mathrm{Cos}\left[\frac{3}{2}\right] 47h^2\ 919360h^7 w^4 \mathrm{Cos}\left[\frac{5hw}{2}\right] + 3470928h^7 w^4$$

$$5h^3\ 12096 + 692224h^4 w^4\ \mathrm{Cos}\left[\frac{7hw}{2}\right] + 12605008h^7 w^4\ 15h^3\ 28224$$

$$h^4 w^4 \quad \frac{9hw}{2} \quad 3628800h^6 w\mathrm{Cos}[8hw]\ \mathrm{Sin}\left[\frac{7hw}{2}\right]$$

$$+ 839680\frac{hw}{9}\ \mathrm{Cos}\left[\frac{9hw}{2}\right] + \frac{7hw}{9}\ \frac{9hw}{2}$$

$$+ 7\mathrm{Sin}\left[\frac{h}{2}\right] 3628800h^6 w\mathrm{Cos}\left[\frac{h}{2}\right] + 7\mathrm{Cos}\left[\frac{h}{2}\right] \mathrm{Sin}[8hw]$$

$$y_n'' = \frac{2y_n}{5h^2} \quad \frac{3y_{n+3}}{h^2} + \frac{4y_{n+4}}{h^2} \quad \frac{7y_{n+5}}{5h^2} + \frac{1}{46448640h^6w^4} f_{n+4} \quad 50803200h^4$$

$$14 \quad 2056320h^4 + 587970h^8w^4 \quad \mathrm{Cos}[hw] \quad 5053860h^8w^4\mathrm{Cos}[2hw]$$

$$2576340h^8w^4\mathrm{Cos}[3hw] \quad 3386880h^4\mathrm{Cos}[4hw]$$

$$12700800h^6w^2\mathrm{Cos}[4hw] \quad 14220h^8w^4\mathrm{Cos}[4hw]$$

$$4h \; 14 \; 483840h^3 \quad 217224h^7w^4 \quad \mathrm{Cos}[hw] \quad 2566032h^7w^4\mathrm{Cos}[2hw]$$

$$2672208h^7w^4\mathrm{Cos}[3hw] \quad 423360h^3\mathrm{Cos}[4hw]$$

$$187824h^7w^4\mathrm{Cos}[4hw] \quad \mathrm{Csc}\frac{hw}{2}$$

$$\frac{1}{464486400h^6w^4} f_{n+8} \quad 50h \quad 60480h^{\circ} \quad 73434h'w^{\cdot\cdot} \quad \mathrm{Cos}\frac{hw}{2} \quad hw$$

$$10 \quad 60480h^4 + 170424h^8w^4 \quad \mathrm{Cos}\frac{}{2} \quad 905688h^8w^4\mathrm{Cos}\frac{3}{2}$$

$$+ 126h \; 40320h^{\circ} \quad 32240h'w^{\cdot\cdot} \quad \mathrm{Cos}\frac{3hw}{2} \quad 1451520h^{\cdot\cdot}\mathrm{Cos}\frac{7hw}{2}$$

$$+ 60h^3 \quad 26590h^5w^4\mathrm{Cos}\frac{5}{2} \quad 3628800h^6w^2\mathrm{Cos}\frac{7hw}{2}$$

$$3178h^5w^4\mathrm{Cos}\frac{7}{2}$$

$$\frac{hw}{94h^2} \quad 21660h^6w^4\mathrm{Cos}\frac{hw}{52}$$

$$+ 498h^6w^4\mathrm{Cos}\frac{7}{2} \quad \mathrm{Csc}\frac{hw}{2} \quad \mathrm{Csc}[hw]$$

$$\frac{hw}{2} \quad 7$$

$$\frac{1}{464486400h^6w^4} f_n \quad 30h \quad 108864h^{\circ} + 94338h'w^{\cdot\cdot} \quad \mathrm{Cos}\frac{hw}{2} \quad hw$$

$$10 \; 616896h^4 + 21960h^8w^4 \quad \mathrm{Cos}\frac{}{2} \quad 7456680h^8w^4\mathrm{Cos}\frac{3hw}{2}$$

$$+ 162h \; 67200h^3 \quad 24096h^7w^4 \quad \mathrm{Cos}\frac{3}{2}$$

$$+ 60h^3 \quad 72990h^5w^4\mathrm{Cos}\frac{5hw}{52} \quad 49014h^5w^4\mathrm{Cos}\frac{hw}{72}$$

$$94h^2 \quad 48300h^{\circ}w^{\cdot\cdot}\mathrm{Cos} \quad + 25122h^{\circ}w^{\cdot\cdot}\mathrm{Cos}\frac{hw}{}$$

$$1451520h^4\mathrm{Cos}\frac{9hw}{2}$$

$$3628800166h^6w^2\mathrm{Cos}\frac{9hw}{22} \quad \mathrm{Csc}\frac{hw}{} \quad \mathrm{Csc}[hw] \quad 7$$

$$\frac{1}{464486400h^6w^4}\ {}^{n+7}\ 48709120h^4$$

$$9586482h^8w^4\ \mathrm{Cos}\ \frac{hw}{2}\ i + 16h\ 1572480h^3h$$

$$1596807h^7w^4\ \mathrm{Cos}\ h\ \frac{3hw}{2}\ i\ 4$$

$$5322240h^4 + 3484260h^8w^4\ \mathrm{Cos}\ \frac{3hw\ i}{2h}$$

$$3628800h^{\circ}w^{2}\ \ 7\mathrm{Cos}\ \frac{hw\ h\ \ 2\ i}{7hw} + \mathrm{Cos}\ \frac{h\ \ 2\ hw\ i}{9hw} + 24h$$

$$423360h^3\mathrm{Cos}\ h\ \frac{7}{2}\ i\ \ 60480h^3\mathrm{Cos}\ h\ \frac{9}{2}\ i$$

$$60h^3 176700h^5w^4\mathrm{Cos}\ \frac{h5}{hw\ i\ \ 2}$$

$$49014h^5w^4\mathrm{Cos}\ \frac{h7hw\ i}{2}\ \ 3178h^5w^4\mathrm{Cos}\ \frac{h9hw\ i}{2}$$

$$94h^2\ 209880h^6w^4\mathrm{Cos}\ \frac{h5hw\ i}{22} + 25122h^6w^4\mathrm{Cos}\ \frac{h7hw\ i}{}$$

$$+ 498h^6w_1\mathrm{Cos}\ \frac{9hw}{} \ \mathrm{Csc}\ \frac{hw}{}\ i\ 7\mathrm{Csc}[hw]$$

$$+ \frac{}{232243200h^6w^4}\ f\ {}^{+2}\ 14\ \ 7983360h^4$$

$$+ 575574h^{\circ}w^{4}\ \ \mathrm{Cos}\ \frac{hw}{2}\ \ 14\ \ 5080320h^{4}\ \ 5hw$$

$$+ 3495780h^8w^4\ \ \mathrm{Cos}\ \frac{h\ \ 7hw}{2}\ \ 3148200h^8w^4\mathrm{Cos}\ \frac{9hw\ \ h\ \ i}{2}$$

$$h^6w^2\ \ \mathrm{Cos}\ \frac{}{}\ + 3\mathrm{Cos}\ \frac{}{}\ + 16h$$

$$12700800\ \ 211680h^3\mathrm{Cos}\ \frac{hw\ h\ 2\ \ i}{7\ \ 2}\ \ 635040h^3\mathrm{Cos}\ \frac{h\ 2\ hw}{h\ \ 9\ \ 2}\ i$$

$$30h^2\ 432831h^6w^4\mathrm{Cos}\ \frac{h7hw\ i}{2}$$

$$23901h^6w^4\mathrm{Cos}\ \frac{h9\ hw\ i}{2} + 4h\ \ 14\ \ 1088640h^3$$

$$1547421h^7w^4\ \mathrm{Cos}\ \frac{h3hw\ i}{22}\ 3004290h^7w^4\mathrm{Cos}\ \frac{h5hw\ i}{}$$

$$h^3\mathrm{Cos}\ \frac{7hw}{2}$$

$$+ 2\ \ 211680\ \ \frac{hw}{9}\ \ i$$

$$635040h^3\mathrm{Cos}\ \frac{h\ \ 2\ hw}{7}\ i + 15h^2\ \ \frac{hw}{9}\ i$$

2

16h 432831h^6 w^4 Cos h 7hw / 2 i

23901h^6 w^4 Cos h9hw i / 22 Csc hhw i^7 Csc[hw]

1 / 464486400h^o w^4 f n+5hw 140 2322432h^4

1456866h^8 w^4 Cos h / 2 3hw 28 7257600h^4 5hw

+ 3495780 h^o w^4 Cos h / 2 5Cos 7 hw / 2 593100000 9hw Cos h / 2 i + 16h

25401600 h^o w^2 hw h 2 i 2116800h^3 Cosh 7 / 2 h 2 9 hw / 2 i

30h^2 473382h^6 w^4 Cos h7hw i / 2 1270080h^3 Cos

+ 10230h^6 w^4 Cos h9 hw / 2 i + 4h 28 1814400h^3

1547421h^7 w^4 Cos h3hw i / 22 43014900h^7 w^4 Cos h5hw i / 22

+ 2 2116800h^3 Cos h 7hw / 2 i 1270080h^3 Cos h 9hw / 2 i

15h^2 hw i / 2 2664174h^5 w^4 Cos h7 / 2

68050h^5 w^4 Cos h 9hw / 2 i 16h 473382h^6 w^4 Cos h 7hw / 2 i

+ 10230h^6 w^4 Cos h 9 / 2 i Csc h / 2 i Csc[hw]

1 / 464486400h^o w^4 f n+3hw 140 2903040h^4

1456866h^8 w^4 Cos h / 2 i 28 3hw

h^4 1744020h^o w^4 Cos

9192960 + h 5hw / 2 h 2 i 9hw

109810800h^o w^4 Cos 7hw

25401600h^o w^2 3Cos hw h 2 i + 5Cos h 2 hw' + 16h

1270080h^3 Cos h 7 / 2 i 2116800h^3 Cos h 9 / 2 i

2 2

168

$30h^2$ $359478h^6w^4\cos\frac{7hw}{}$ $h^6w^4\cos\frac{9hw}{}$

$+ 4h\ 28$ $2056320h^5$ $787779h^4w^4\cos\frac{3hw}{2}$ $58174860h^4w^4\cos\frac{5hw}{2}$

$+ 2$ $1270080h^3\cos\frac{7hw}{2}$ $2116800h^3\cos\frac{9hw}{2}$

$+ 15h^2$ $h^5w^4\cos\frac{}{}$ $98510h^5w^4\cos\frac{}{}$

$16h$ $359478h^5w^4\cos\frac{7hw}{2}$ 1092658 $+ 75750h^5w^4\cos\frac{9hw}{2}$ $\csc\frac{hw}{2}$ $\csc[hw]$

$+ \frac{1}{116121600h^5w^4}$ $6350400h^6w^2\ 3\cos\frac{hw}{2}$ $+ \cos\frac{hw}{2}$

$+ \frac{}{2}$ 14 $5080320h^4$ $4214754h^8w^4\cos\frac{hw}{2}$ 14 $3144960h^4$

$+ 1744020h^8w^4\cos\frac{3hw}{2}$ $12933000h^8w^4\cos\frac{5hw}{2}$ $+ 16h$

$635040h^3\cos\frac{7hw}{2}$ $211680h^3\cos\frac{9hw}{2}$ $30h^2$ $81039h^6w^4\cos\frac{7hw}{2}$

h^6w^4 $\cos\frac{9hw}{2}$ $+ 4h\ 14$ $846720h^3$ $787779h^4w^4\cos\frac{3hw}{2}$

$+ 1731$ $10617150h^4w^4\cos\frac{5hw}{2}$ $+2$ $635040h^5\cos\frac{7hw}{2}$ $211680h^5\cos\frac{9hw}{2}$

$+ 15$ 126434 h^2 $h^5w^4\cos\frac{}{}$ $11212h^5w^4\cos\frac{}{}$

16 1 $h\ 81039h^5w^4\cos\frac{}{}$ $+ 1731h^5w^4\cos\frac{}{}$ $\csc\frac{2hw}{}$ $\csc[hw]$

$\frac{}{928972800h^5w^4}$ $4\ 17418240h^4$ $15333618h^8w^4\cos\frac{}{2}$

$16h\ 2298240h^3 + 3737127h^7w^4\cos\frac{3hw}{}$ $4\ 11128320h^4\ 2$

$1045260h^8w^4\cos\frac{3hw}{2}$ $3628800h^6w^2\cos\frac{7hw}{}$

$+ 7\cos\frac{9hw}{}$ h $60480h^3\cos\frac{7hw}{}$ $423360h^3\cos\frac{9hw}{}$

$+ 60h^3$ $495940h^5w^4\cos\frac{5hw}{2}$ $+ 24$ $3178h^5w^4\cos\frac{7hw}{2}$

$49014h^5w^4\cos\frac{9hw}{2}$ $94h^2$ $862200h^6w^4\cos\frac{}{}$

$+ 498h^6w^4\cos\frac{7hw}{2}$ $+ 25122h^6w^4\cos\frac{9hw}{2}$ $\csc\frac{hw}{2}$ $\sec\frac{hw}{2}$

$$y''_{n+1} = \frac{10h}{3y_n^{2}} \frac{2y_{n+3}^{7}}{h^{2}} + \frac{5y_{n+4}^{3}}{2h^{2}} \frac{4y_{n+5}}{5h^{2}} + \frac{1}{46448640h^{9}w^{4}} f_{n+4} \quad 31752000h^{8}w^{4} \quad 14h$$

$$206810h^{4}w^{4} + 420h^{4}w^{4} \quad 1152 + 7h^{4}w^{4} \quad \mathrm{Cos}[hw] \quad 14 \quad 382612h^{8}w^{4}$$

$$+ 420h^{8}w^{4} \quad 1152 + 7h^{6}w^{2} \quad 280h^{8}w^{4} \quad 7344 + 23h^{6}w^{2} \quad \mathrm{Cos}[hw]$$

$$+ 74256h^{8}w^{4}\mathrm{Cos}[2hw] \quad 12700800h^{8}w^{4}\mathrm{Cos}[3hw] + 251244h^{8}w^{4}\mathrm{Cos}[3hw]$$

$$+ 32064h^{8}w^{4}\mathrm{Cos}[4hw] + 24h\,546h^{7}w^{4} \quad 20h^{3}\,5292 + 71h^{4}w^{4}\mathrm{Cos}[4hw]$$

$$+ 3h \quad 19614h^{7}w^{4}\mathrm{Cos}[hw] + 14 \quad 206810h^{7}w^{4} + 420h^{3} \quad 1152 + 7h^{4}w^{4} \quad \mathrm{Cos}[hw]$$

$$2103640h^{7}w^{4}\mathrm{Cos}[2hw] \quad 379530h^{7}w^{4}\mathrm{Cos}[3hw] + 29232h^{7}w^{4}\mathrm{Cos}[4hw]$$

$$+ 4 \quad 546h^{7}w^{4} \quad 20h^{3} \quad 5292 \quad hw + 71h^{4}w^{4} \quad \mathrm{Cos}[4hw] \quad \mathrm{Csc}\left[\frac{hw}{2}\right]$$

$$+ \frac{}{464486400h^{9}w^{4}} f_{n+8}\mathrm{Csc}\left[\frac{h}{2}\right] \mathrm{Csc}[hw] \quad 111240h^{8}w^{4}\mathrm{Cos}\left[\frac{h}{2}\right] + 30h$$

$$65507h^{7}w^{4} + 3h^{3}\,20160 + 569h^{4}w^{4}\,\mathrm{Cos}\left[\frac{hw}{2}\right] \quad 10\,104090h^{8}w^{4} +$$

$$h^{4}\,60480\,3281h^{4}w^{4} + 3h^{4}\,20160 + 569h^{4}w^{4}\,\mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$404964h^{8}w^{4}\mathrm{Cos}\left[\frac{3hw}{2}\right] + 72h\,23559h^{7}w^{4} + 35h^{3} \quad 1152 \quad 35h^{4}w^{4} \quad \mathrm{Cos}\left[\frac{3hw}{2}\right]$$

$$3628800h^{6}w^{2}\mathrm{Cos}[hw]\mathrm{Cos}\left[\frac{7hw}{2}\right] + 24h^{3}\,10130h^{5}w^{4}\mathrm{Cos}\left[\frac{5hw}{2}\right]$$

$$+ 406h^{9}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right] \quad 5hw \quad 52h^{2} \quad 1365h^{9}w^{4}\mathrm{Cos}\left[\frac{5hw}{2}\right] \quad 15h^{9}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right] \quad 7hw$$

$$+ 18h\,10040h^{7}w^{4}\mathrm{Cos}\left[\frac{h}{2}\right] \quad 2hw + 811h^{7}w^{4} \quad 5h^{3} \quad 12096 + 169hw \quad 7h \quad h^{4}w^{4} \quad \mathrm{Cos}\left[\frac{2}{}\right]$$

$$3628800h^{6}w^{2}\mathrm{Sin}[hw]\mathrm{Sin}\left[\frac{h}{2}\right] + \frac{}{464486400h^{9}w^{4}} f_{n}\mathrm{Csc}\left[\frac{h}{2}\right] \mathrm{Csc}[hw]$$

$$109700h^{8}w^{4}\mathrm{Cos}\left[\frac{hw}{2}\right] + 10h\,21925h^{7}w^{4} + 9h^{3}\,12096 + 725h^{4}w^{4}\,\mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$10\,65588h^{8}w^{4} + h^{4}\,616896\,32897h^{4}w^{4} + 9h^{4}\,12096 + 725h^{4}w^{4}\,\mathrm{Cos}\left[\frac{hw}{2}\right]$$

379008h^8 w^4 Cos h3hw i / 67200 2 + 108h 11573h^7 w^4 + h^3

3591h^4 w^4 Cos h3hw i / 22 + 24h^3 41170h^5 w^4 Cos h5hw i

7210h^5 w^4 Cos h7hw i / 22 52h^2 46965h^6 w^4 Cos h5hw i

+ 1233h^6 w^4 Cos h 7hw / 2 ' 5hw 3628800h^6 w^2 Cos[hw] Cos 7hw h 9hw i / 2

+ 18 56870 9hw h 2 i h 2 9'hw

h h' w^4 Cos / 2 + 9752h' w^4 Cos

60480 1 Cos'' / 2 i hw 7 h 2 i

h^3 3628800h^6 w^2 Sin[hw] Sin

+ f Csc Csc[hw]

116121600h^6 w^4 n+6 h 2' 7hw 9hw

1 6350400h^6 w^2 Cos[hw] 3Cos / + Cos / i hw

+ 7h 1230766h^7 w^4 + 180h^3 8064 197h^4 w^4 Cos h 2 i

14 2875702h^8 w^4 + 180h^4 8064 197h^4 w^4

196h^4 25920 + 329h^4 w^4 Cos hhw i / 2 14h

745868h^7 w^4 + 2h^3 423360 8933h^4 w^4 Cos h3hw i 2

14 917282h^8w^4 + 2h^4 423360 8933h^4w^4 + 16h^4

196560 + 2957h^4w^4 Cos h3hwi/22 + 780780h^8w^4Cos h5hwi

16h^2 4620h^6w^4Cos h7hwi/22 66h^6w^4Cos h9hwi/22 + 12h 7 4844h^7w^4

5h^3 18144 + 25h^4w^4 Cos h7hwi/21168 + 2786h^7w^4 10h^3

293h^4w^4 Cos h9hwi/22 + 3h 1195586h^7w^4Cos h3hwi/22 + 14745868h^7w^4

+ 2h^3 h^4w^4 h 3hw/2 i 3295590h^7w^4Cos 5hw

423360 8933 hwCos h 7/2 i hw h 9/2 i

+ 8h^2 31192h^5w^4Cos + 1460h^5w^4Cos

12h 4620h^6w^4Cos h7hwi/22 66h^6w^4Cos h9hwi/22 + 2 7 4844h^7w^4

5h^3 18144 + 25h^4w^4 Cos h7hwi/21168 + 2786h^7w^4 10h^3

+ 293h^4w^4 Cos 9hw h^6w^2Sin[hw] 3Sin 7hw

hw h 2 i 1 6350400 hw 7 h 2 i

+ Sin h 9/2 i / 464486400h^6w^4 f_{n+7}Csc h/2 i Csc[hw] 2h 2795914h^7w^4

20160h^3 132 + 5h^4w^4 Cos hhw/2 i + 4 6503698h^8w^4 20160h^4 132

+5h^4w^4 +56h^4 155520+3193h^4w^4 Cos hhw i/2 +2387172h^8w^4Cos h3hw i/2

8h^2 1527756h^7w^4 + 14h^3 112320 3863h^4w^4 Cos h3hw i

4 1757050h^6w^4 + 14h^4 112320 3863h^4w^4 + 112h^4 47520 7hw

+ 1207 9hw Cos'' 3hw/2 ' 3628800 5hw h 2 i 7hw

h^4w^4 h^6w^2Cos[hw] 7Cos

h i 9hw h 5'hw h i

+ Cos 2 + 24h^3 103960h^5w^4Cos 2 7210h^5w^4Cos 2 hw

+ 406h w 5 4 Cos'' 2hw' 52 56220hhw Cos'' 2 i

h^2 6w^4

$$+ 1233h^{6}w^{4}\mathrm{Cos}\left[\frac{7}{2}\right] \quad 15h^{6}w^{4}\mathrm{Cos}\left[\frac{9}{2}\right] \quad + 18h \quad 126470h^{7}w^{4}\mathrm{Cos}\left[\frac{5}{2}\right]$$

$$12827h^{7}w^{4} \quad 15h^{3} \quad 28224 + 205h^{4}w^{4}\mathrm{Cos}\left[\frac{}{2}\right]$$

+140 999130h^8w^4 + 576h^4 1092 17h^4w^4 + 56h^4 41472 + 325h^4w^4 Cos hhw i / 2

28h 1455388h^7w^4 + 2h^3 907200 10723h^4w^4 Cos h3hw i / 28 1912270h^8w^4 2

2h^4 907200 10723h^4w^4 +

16h^4453600 + 3847h^4w^4

Cos h3hw i / 2

1917000h^8w^4Cos h5hw i / 222 25401600h^6w^2Cos[hw] 5Cos h7hw i / + 3Cos h9hw i /

16h^2 29463h^6w^4Cos h7hw i / 2 585h^6w^4Cos h9hw i / 2 + 12h 7 26331h^7w^4 + 5h^3

60480 + 779h^4w^4 Cos h7hw i / 222 + 5 3241h^7w^4 h^3 254016 + 3469h^4w^4 Cos h9hw i /

3h 4009964h^7w^4Cos h3hw i / + 28 1455388h^7w^4 + 2h^3 907200 2

10723h^4w^4 Cos h3hw i / 222 17527300h^7w^4Cos h5hw i / + 8h^2200550h^5w^4Cos h7hw i /

+ 9050h^5w^4Cos h 9hw i / 2 12h 29463h^6w^4Cos h 7hw i / 2 585h^6w^4Cos h 9hw / 2 i

2 7 26331h^7w^4 + 5h^360480 + 779h^4w^4 Cos h7hw i / + 5

3241h^7w^4 h^3 254016 2 /

+ 3469h^4w^4 1 Cos h 9hw i / 2 hw 7 25401600h^6w^2Sin[hw] h 5Sin 7hw i / 2 + 3Sin h 9hw i / 2

464486400h^6w^4 f_{n+3}Csc h / 2 i Csc[hw] 70h 425938h^7w^4 + 576h^3 1260

17h^4w^4 Cos hhw i / 2 + 140 999130h^8w^4 + 576h^4 1260 17h^4w^4 + 280h^4 10368

+ 65h^4w^4 Cos h hw i / 2 28h745868h^7w^4 + 2h^3 1028160 8933h^4w^4 Cos h 3hw / 2 i

28 917282h^8w^4 + 2h^4 1028160 8933h^4w^4 + 16h^4574560 + 2957h^4w^4 Cos h3hw i / 2 /

8 4 Cosh 5hw / 6 2Cos[hw] 3Cos 7hw / + 5Cos 9hw / 16h^2

10728360h w hw 2 i 25401600h w 9hw h 2 i h 2 i

541065h^6w^4Cos 7 / 2 + 83415h^6w^4Cos / 2 + 12h 7 28437h^7w^4 + 5h^3

36288 + 1829h^4w^4 Cos h7hw i / + 5 19001h^7w^4 7h^3 60480 + 773h^4w^4 Cos h9hw i /

3h 2391172h^7w^4Cos h3hw i / + 28 745868h^7w^4 + 2h^3 1028160 2

8933h^4w^4 Cos h3hw i / 222 38400940h^7w^4Cos h5hw i / + 8h^2584934h^5w^4Cos h7hw i /

5 4 h 9hw i h 7hw i 6 4 h 9hw i

2 2 2

173

+2 7 28437h^7w^4 + 5h^3 36288 + 1829h^4w^4 9hw Cos h 7hw i

+ 5 19001h^7w^4 7h^3 60480 + 773h^4w^4 Cos 7hw 9hw 2 i

1 hw 8 2 hw h 2 i

25401600h^6w^2Sin[hw] 3Sin + 5Sin

928972800h^6w^4 n hw h 2 i h 2 i

f +1 Csc Sec 2h 6691882h^7w^4 60480h^5

2 + 4 hw hw

68 + 3h^4w^4 Cosh 5591794h^8w^4 60480h^4 68 + 3h^4w^4

+ 56h^4 311040 + 7993h^4w^4 Cos h i 14928564h^8w^4Cos h 3 i

2 2

8h 667684h^7w^4 + 14h^3 164160 7393h^4w^4 Cos h3hw i

2

4 755574h^6w^4 + 14h^4 164160 7393h^4w^4 + 336h^4 33120 7hw

+ 1159h hw Cos 3hw 2 i 3628800 Cos[hw] Cos 2 i hw

4w^4 h^6w^2 hw

+ 7Cos h 9 i + 24h^3 442520h^5w^4Cos h 5 i + 406h^5w^4Cos h 7 i

2 2 2

7210h^5w^4Cos h9hw i 52h^2 354780h^6w^4Cos h5hw i 15h^6w^4Cos h7hw i

222

+ 1233h^6w^4Cos h 9hw i + 18h 603170h^7w^4Cos h 5hw i + 811h^7w^4

2 h7hw i 2

5h^3 12096 + 169h^4w^4 Cos + 12827h^7w^4 15h^3 28224

2

9hw 7hw 9hw

+ 205h^4w^4 Cos h 3628800h^6w^2Sin[hw] Sin h + 7Sin h

2 i 2 i 2 i

$$y''_{n+2} = \frac{y_n}{5h^2} - \frac{y_{n+3}}{h^2} + \frac{y_{n+4}}{h^2} - \frac{y_{n+5}}{5h^2} + \frac{1}{46448640h^6w^4} f_{n+4}\, h^4\, 28h\, 12700800\, 244936h\, w + 420h\, 1152 + 112h\, w\, 280h\, 7344 + 368h\, w\, \mathrm{Cos}[hw]\, 14\, 185714h\, w\, 8\, 4$$

$$+ 840h\, 1152 + 112h\, w\, 280h\, 7344 + 368h\, w\, \mathrm{Cos}[hw]$$

$$12700800h^6w^2\mathrm{Cos}[2hw] + 1881628h^8w^4\mathrm{Cos}[2hw]$$

$$+ 353772h^3w^4\mathrm{Cos}[3hw] + 24084h^4w^4\mathrm{Cos}[4hw] + 16h\, 20594h^7w^4$$

$$20\, h\, 5292 + 1136h^7w^4\, \mathrm{Cos}[4hw] + 2h^3\, 288960h^4w^4\, \mathrm{Cos}[hw]$$

$$+ 14\, 244936h^7w^4 + 420h\, 1152 + 112h^7w^4\, \mathrm{Cos}[hw]$$

$$+ 341488h^7w^4\, \mathrm{Cos}[2hw] + 14832h^7w^4\, \mathrm{Cos}[3hw] + 16968h^7w^4\, \mathrm{Cos}[4hw]$$

$$+ 4\, \frac{1}{20594h^7w^4}\, 20h^3\, 5292 + 1136h^7w^4\, \mathrm{Cos}[4hw]\, \mathrm{Csc}\left[\frac{hw}{2}\right] i\, 8$$

$$+ \frac{1}{464486400h^6w^4} f_{n+8}\mathrm{Csc}\left[\frac{h}{2}\right] i\, \mathrm{Csc}[hw]\, 504540h^8w^4\mathrm{Cos}\left[\frac{hw}{2}\right] i$$

$$+ 10h\, 82330h^7w^4 + 3h^3\, 20160 + 9104h^4w^4\, \mathrm{Cos}\left[\frac{h}{2}\right] i$$

$$10\, 40024h^8w^4 + h^4\, 60480\, 52496h^4w^4 +$$

$$6h^4\, 20160 + 9104h^4w^4\, \mathrm{Cos}\left[\frac{hhw}{2}\right] i$$

$$878976h^8w^4\mathrm{Cos}\left[\frac{h3hw}{2}\right] i + 18h\, 5926h^7w^4 + 35h^3\, 1152$$

$$560h^4w^4\, \mathrm{Cos}\left[\frac{h3hw}{2}\right] i\, 3628800h^6w^2\mathrm{Cos}[2hw]\mathrm{Cos}\left[\frac{h7hw}{2}\right] i$$

$$+ 6h^3\, 4770h^5w^4\mathrm{Cos}\left[\frac{h5hw}{2}\right] i\, 450h^5w^4\mathrm{Cos}\left[\frac{h7hw}{2}\right] i$$

$$22h^2\, 240h^6w^4\mathrm{Cos}\left[\frac{h5hw}{2}\right] i\, 42h^6w^4\mathrm{Cos}\left[\frac{h7hw}{2}\right] i$$

$$12h\, 9580h^7w^4\mathrm{Cos}\left[\frac{h5hw}{2}\right] i + 13536h^7w^4\, 5h^3\, 12096^2$$

$$2704h^4w^4\, \mathrm{Cos}\left[\frac{h7hw}{2}\right] i\, 3628800h^6w^2\mathrm{Sin}[2hw]\mathrm{Sin}\left[\frac{h7hw}{2}\right] i$$

$$+ \frac{1}{464486400h^6w^4} f_n\mathrm{Csc}\left[\frac{hw}{2}\right] i\, \mathrm{Csc}[hw]\, 40420h^8w^4\mathrm{Cos}\left[\frac{hhw}{2}\right] i$$

$$10h\, 115238h^7w^4 + 9h^3\, 12096 + 11600h^4w^4\, \mathrm{Cos}\left[\frac{hhw}{2}\right] i$$

$$10\, 342704h^8w^4 + h^4\, 616896\, 526352h^4w^4 + 18h^4$$

$$12096 + 11600h\, w\, \mathrm{Cos}\left[\frac{h}{2}\right] i + 118656h\, w\, \mathrm{Cos}\left[\frac{h}{2}\right] i$$

+54h 48110h7w4 + h3 67200 57456h4w4 Cos h3hw i 2 + 6h3

29570h5w4Cos h5hw i 22 2 + 1234h5w4Cos h7hw i 22 h2 6840h6w4Cos h5hw i

210h6w4Cos h7hw i 22 3628800h6w2Cos[2hw]Cos h9hw i

+ 12h 82660h7w4Cos h5hw i 2 + 9592h7w4Cos h7hw i 2

60480h3Cos h9hw i 22 3628800h6w2Sin[2hw]Sin h9hw i

' 1 116121600h6w4 f n+6Csc '' hw 2 i 7 Csc[hw]

6350400h6w2Cos[2hw] 3Cos h 7 2hw i 2 + Cos h 9 2 i + 1 2 14h 1831178h7w4

+ 180h 8064 3152 h w Cosh 1542478h w + 360h 8064 hw

3152h4w4 + 196h4 25920 + 5264h4w4 Cosh i 2 28h

192131h7w4 + 2h3 423360 142928h4w4 Cos h3hw i 14 125454h8w4 2

4h4 423360 142928h4w4 +

16h4196560 + 47312h4w4

Cos h3hw i 2

2988400h8w4Cos h5hw i 22 2 6h2 399h6w4Cos h7hw i 225h6w4Cos h9hw i

8h 7 2814h7w4 + 5h318144 + 400h4w4

Cos h7hw i + 46886h7w4 2

10h3 21168 + 4688h4w4 Cos h9hw i 22 + 2h 4900224h7w4Cos h3hw i

14192131h7w4 + 2h3 423360 142928h4w4 Cos h3hw i

2

+338050h7w4Cos h 5hw 2 i +3h2 11970h5w4Cos h 7hw i 2 1768h5w4Cos h 9hw 2 i

8h 399h6w4Cos h7hw i 22 225h6w4Cos h9hw i + 2 7 2814h7w4

$$+ 5h^{3} \quad 18144 \quad 400h^{4}w^{4} \quad \mathrm{Cos}\,\frac{7hw}{} \quad + \quad 46886h^{\prime}w^{4} \quad 10h^{3} \quad 21168$$

$$+ 4688h^{*}w^{*}{}^{hw} \quad \mathrm{Cos}\,\frac{+h\,9hw}{2\;i\;1} \quad h \quad \frac{2\;\;i}{6350400} \quad {}^{hw}\;h^{\cup}w^{\angle}\,\mathrm{Sin}[2hw] \quad 3\mathrm{Sin}\,\frac{h\;7hw}{2} \quad {}^{i}$$

$$+ \mathrm{Sin}\,\frac{h\,9}{2} \quad \frac{}{i464486400h^{6}w^{4}} \quad f_{n+7}\mathrm{Csc}\,\frac{h}{2} \quad i \quad \mathrm{Csc}[hw]4h \quad 4481792h^{7}w^{4}$$

$$20160h^{3}132 + 80h^{4}w^{4} \quad \mathrm{Cos}\,\frac{hhw\;i}{1762}$$

+4 3353074h^8w^4 40320h^4 132 + 80h^4w^4

56h^4155520 + 51088h^4w^4 Cos hhw i 2

6314496h^8w^4Cos h3hw i 4 124246h^8w^4 + 28h^4 112320 2

61808h^4w^4 + 112h^447520 + 19312h^4w^4 Cos h3hw i 2

h^6w^2 hw 7Cos 7hw + Cos 9hw

3628800 Cos[2] hw h 2 i h 2hw

+ 6h^3 5180h^5w^4Cos h 5 2 i + 1234h^5w^4Cos h 7 2 i

450h^5w^4Cos h9hw i 22h^2 7680h^6w^4Cos h5hw i

210h^6w^4Cos h7hw i 22 42h^6w^4Cos h9hw i

12h 150040h^7w^4Cos h5hw i 2 + 58792h^7w^4

15h^3 28224 + 3280h^4w^4 Cos 7hw i + 13536h^7w^4

5h^3 12096 + 2704h^4w^4 Cos h hw2 i 7

3628800h^6w^2Sin[2hw] 7Sin h 2 i

+Sin h 9hw/2 i + 1/(232243200h^6w^4) f_{n+2}Csc h hw/2 i Csc[hw] 14h 426326h^7w^4

180h^3 10752 4016h^4w^4 Cos hhw/2 i + 14 273022h^8w^4 + 360h^4 10752

4016h^4w^4 + 28h^4 285120 + 53168h^4w^4 Cos hhw/2 i 28h

653917h^7w^4 + 2h^3 544320 171568h^4w^4 Cos h3hw/2 i 14 389418h^8w^4 2

h^4 171568h^4w^4 + 16h^4 317520 + 61552h^4w^4 Cos 3hw/2 h 2 i

+ 4 544320 h^5w^4 hw Cos 5hw/2 12700800 h^5w^4 Cos[2hw] Cos hw h 7hw/2 i hw

+ 577120 + 3Cos h 9/2 i 6h^2 21735h^6w^4Cos h 7/2 i 3441h^6w^4Cos h 9/2 i

8h 7 23798h^7w^4 +

105h^3 288 + 304h^4w^4

Cos h7hw/2 i

71158h^7w^4 + 10h^3 63504 + 11248h^4w^4 Cos h9hw/2 i

2h 8555904h^7w^4Cos h3hw/2 i + 14 653917h^7w^4 + 2h^3 544320 2

171568h^4w^4 Cos h3hw/2 i + 111550h^7w^4Cos h5hw/2 i + 3h^2

90538h^5w^4Cos h7hw/2 i + 14560h^5w^4Cos h9hw/2 i 8h 21735h^6w^4Cos h7hw/2 i

3441h^6w^4Cos h9hw/2 i + 2 7 23798h^7w^4 +

105h^3 288 2

304h^4w^4 Cos h7hw/2 i 71158h^7w^4 + 10h^3

63504 2

11248h^4w^4 Cos h9hw/2 i 12700800h^6w^2Sin[2hw] Sin h7hw/2 i

+ 3Sin h 9hw/2 i 46448640hwhw 1/(4) f_{n+5}Csc hw/2 i 7 Csc[hw] 140h 593744h^7w^4

+ 576h^3 1092 272h^4w^4 Cos h/2 i + 140 550402h^8w^4

$$+ 1152h^4 \quad 1092 \quad 272h^4w^4 \quad + 56h^4 41472 + 5200h^4w^4 \qquad \text{Cos}\left(\frac{hw}{2}\right)$$

$$7200\ 171568h^4w^4\ \text{Cos}\left(\frac{h3hw}{2}\right)$$

28 389418h^{8}w^{4} + 4h^{4} 907200 171568h^{4}w^{4} + 16h^{4}

453600 + 61552h^{4}w^{4} Cos $\frac{h3hw\ i}{22}$ + 15838240h^{8}w^{4}Cos $\frac{h5hw\ i}{22}$

25401600h^{6}w^{2}Cos[2hw] 5Cos $\frac{h7hw\ i}{22}$ + 3Cos $\frac{h9hw\ i}{22}$ 6h^{2} 10290h^{6}w^{4}Cos $\frac{h7hw\ i}{22}$

2310h^{6}w^{4}Cos $\frac{h9hw\ i}{22}$ + 8h 732864h^{7}w^{4} + 5h^{3} 60480 + 12464h^{4}w^{4} Cos $\frac{h7hw\ i}{22}$

+ 5 55384h^{7}w^{4} h^{3} 254016 + 55504h^{4}w^{4} Cos $\frac{h\ 9hw}{2}$ i + 2h 17111808h^{7}w^{4}Cos $\frac{h\ 3hw}{2}$ i

28653917h^{7}w^{4} + 2h^{3} 907200 171568h^{4}w^{4} Cos $\frac{h3hw\ i}{22}$

1145740h^{7}w^{4}Cos $\frac{h5hw\ i}{22}$ + 3h^{2} 99274h^{5}w^{4}Cos $\frac{h7hw\ i}{22}$ 12730h^{5}w^{4}Cos $\frac{h9hw\ i}{22}$

8h 10290h^{6}w^{4}Cos $\frac{h7hw\ i}{22}$ 2310h^{6}w^{4}Cos $\frac{h9hw\ i}{22}$ + 2 732864h^{7}w^{4}

5h^{3}60480 + 12464h^{4}w^{4} Cos $\frac{h7hw\ i}{22}$ + 5 55384h^{7}w^{4} h^{3} 254016

+ 55504h^{4}w^{4} Cos $\frac{h\ 9hw\ i}{2}$ hw 25401600h^{6}w^{2}Sin[2hw] 5Sin $\frac{h7hw\ i}{2}$ + 3Sin $\frac{h9hw\ i}{2}$

f_{n+3}Csc Csc[hw] 140h 593744h^{7}w^{4} + 576h^{3} 1260

464486400h^{6}w^{4} 272h w Cosh $\frac{hw}{2}$ h 2 i

550402h^{6}w^{4} + 1152h^{4} 1260 272h^{4}w^{4}

+ 280h 10368 + 3hw 1040h^{4}w^{4} Cos $\frac{h\ 2\ i}{2}$ 56h 192131h^{7}w^{4} + 2h^{5} 1028160

142928h w Cosh 125454hw 5hw

h^{8}w^{4} + 4h^{4} 1028160 142928h^{4}w^{4}

+ 16h^{4} 574560 + 47312h^{4}w^{4} Cosh $\frac{3\ i}{2}$ + 37451200h^{8}w^{4}Cos $\frac{h\ i}{2}$

25401600h^{6}w^{2}Cos[2hw] 3Cos $\frac{h7hw\ i}{22}$ + 5Cos $\frac{h9hw\ i}{22}$ 6h^{2}752262h^{6}w^{4}Cos $\frac{h7hw\ i}{22}$

+106290h^{6}w^{4}Cos $\frac{h\ 2\ i}{9hw}$ +8h 7 266552h^{7}w^{4} +5h^{3} 36288+29264h^{4}w^{4} Cos $\frac{h\ hw\ i}{7hw\ 2}$

+ 5 120224h^{7}w^{4} 7h^{3} 60480 + 12368h^{4}w^{4} Cos $\frac{h\ i}{2}$ + 2h 9800448h^{7}w^{4}Cos $\frac{h\ 3}{2\ i}$

28192131h^{7}w^{4} + 2h^{3} 1028160 142928h^{4}w^{4} Cos $\frac{h3hw\ i}{22}$

9625100h^{7}w^{4}Cos $\frac{h5hw\ i}{22}$ + 3h^{2} 124390h^{5}w^{4}Cos $\frac{h7hw\ i}{22}$ 12950h^{5}w^{4}Cos $\frac{h9hw\ i}{22}$

8h 752262h^6 w^4 Cos h 7hw / 2 i + 106290h^6 w^4 Cos h 9hw / 2 i

2 7266552h^7 w^4 + 5h^3 36288 + 29264h^4 w^4 Cos h7hw i

2

h^7 w^4 h^3 12368h^4 w^4 Cos 9hw

+ 5 120224 7 60480 + 7hw hw 9hw h 2 i

25401600h^υ w^ζ Sin[2hw] 3Sin + 5Sin

928972800h^υ w^ϟ n+1 hw h 2 i h 2 i

f Csc Sec 4h 4673696h′ w^ϟ 60480h^υ

68 + 48 Cos + 4 hw 3hw

h^ϟ w^ϟ 2977906h^υ w^ϟ 120960h^ϟ 68 + 48h^ϟ w^ϟ

+ 56 2 h 2 i

h^ϟ 311040 + 127888h^ϟ w^ϟ Cos 2655744h^υ w^ϟ Cos

h w + 28h 164160 118288h w + 336h 33120

4 2175422 3hw 5hw h 7hw 7hw

+ 18544 9hw Cos^n 2 i 3628800 h^υ w^ζ Cos[2hw] Cos i

h^ϟ w^ϟ

h i hw h i hw h i

+ 7Cos 2 + 6h^3 177540h^5 w^4 Cos 2 450h^5 w^4 Cos 2

9 i 5

+ 1234h^5 w^4 Cosh 2 22h^2 43920h^6 w^4 Cos h 2 i

h7hw i h9hw i h5hw i

42h^6 w^4 Cos 210h^6 w^4 Cos + 12h 780040h^7 w^4 Cos

222

h7hw i

13536h^7 w^4 5h^3 12096 + 2704h^4 w^4 Cos 2

h9hw i

58792h^7 w^4 15h^3 28224 + 3280h^4 w^4 Cos 2

7hw 9hw

3628800h^6 w^2 Sin[2hw] Sin h 2 i + 7Sin h 2 i

n

$y_n^{00}+3$ − $\frac{y_n}{10h^2}$ $\frac{2h_6}{y_{n+4}}$ $\frac{2\ 5h}{2y_{n+5}}$ + $\frac{46448640h\ w}{1\ 6\ 4}$ ' $_{n+4}$ 7 4 6350400h 3 4

12700800h 4 w 4 Cos[hw] 42h 381270h 8 w 4 + 420h 4 1152

+ 567 h 4 w 4 Cos[hw 4] 14 192780h 4 w 4 + 1260h 1152

+ 567h w280h 7344 + 1863h w Cos[hw]

113904h 8 w 4 Cos[2hw] + 498204h 8 w 4 Cos[3hw] + 8h 109350h 7 w 4

20 h 5292 + 5751h 7 w 4 4 Cos[4hw] + h 1983786h 3 w 4 4 Cos[hw]

+ 14 381270h 7 w 4 + 420h 1152 + 567h 7 w 4 Cos[hw] 7 4

+ 1144920h w Cos[2hw] 85230h w Cos[3hw] + 41424h w Cos[4hw]

+ 4 $\frac{1}{109350h^7w^4}$ 20h 5292 + 5751h 7 w 4 hw Cos[4hw] Csc hw 2 i 8

+ $\frac{1}{464486400h^9w^4}$ f_{n+8}Csc h 2 i Csc[hw] 1307940h 8 w 4 Cos h 2 i

10h 162987h 7 w 4 + 3h 3 20160 + 46089h 4 w 4 Cos hhw 2 i

10 149040h 8 w 4 + h 4 60480 265761h 4 w 4 +

9h 4 20160 + 46089h 4 w 4 Cos hhw 2 i

316008h 8 w 4 Cos h3hw 2 i 36h 93973h 7 w 4 + 35h 3 1152 2

2835h 4 w 4 Cos h3hw 2 i 3628800h 6 w 2 Cos[3hw]Cos h7hw 2 i

4h 2 4125h 6 w 4 Cos h5hw 2 i 423h 6 w 4 Cos h7hw 2 i

6h 12150h 7 w 4 Cos h5hw 2 i + 68193h 7 w 4 5h 3 12096 2

13689h 4 w 4 Cos h7hw 2 i 3628800h 6 w 2 Sin[3hw]Sin h7hw 2 i

+ $\frac{1}{464486400h^9w^4}$ f_nCsc h hw 2 i 7 Csc[hw]30h

541917h 7 w 4 + 9h 3 12096 + 58725h 4 w 4 Cos hhw 2 i

10 1092942h 8 w 4 + h 4 616896 2664657h 4 w 4 + 27h 4

12096 + 58725h 4 w 4 Cos hhw 2 i + 1288404h 8 w 4 Cos h3hw 2 i

$4h^2$ $6915h^6w^4\mathrm{Cos}\left[\frac{5hw}{2}\right] + 633h^6w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]$ $3628800h^6w^2\mathrm{Cos}[3hw]\mathrm{Cos}\left[\frac{9hw}{2}\right]$

$+ 6h\ 80100h^7w^4\mathrm{Cos}\left[\frac{hw}{2}\right] + 9918h^7w^4\mathrm{Cos}\left[\frac{12hw}{2}\right]$ $60480h^3\mathrm{Cos}\left[\frac{7hw}{2}\right]$

$3628800h^6w^2\mathrm{Sin}[3hw]\mathrm{Sin}\left[\frac{hw}{2}\right] + \frac{116121600h^6w^4}{1}f_{n+6}\mathrm{Csc}\left[\frac{hw}{2}\right]\mathrm{Csc}[hw]$

$6350400h^6w^2\mathrm{Cos}[3hw]\ 3\mathrm{Cos}\left[\frac{7hw}{2}\right] + \mathrm{Cos}\left[\frac{9hw}{2}\right] + 21h\ 3963190h^7w^4$

$180h^3\ 8064\ 15957h^4w^4\ \mathrm{Cos}\left[\frac{hhw}{2}\right] + 14\ 2989614h^8w^4 + 540h^4\ 8064\ 15957h^4w^4$

$196h^4\ 25920 + 26649h^4w^4\ \mathrm{Cos}\left[\frac{hhw}{2}\right]\ 42h\ 1286724h^7w^4 + 2h^3\ 423360$

$723573h^4w^4\ \mathrm{Cos}\left[\frac{3hw}{2}\right]\ 14\ 509166h^8w^4 + 6h^4\ 423360\ 723573h^4w^4 + 16h^4$

$196560+239517h^4w^4\ \mathrm{Cos}\left[\frac{3hw}{2}\right] + 1542420h^8w^4\mathrm{Cos}\left[\frac{5hw}{2}\right] + 4h\ 7\ 4428h^7w^4 + 5h^3$

$18144+2025h^4w^4\ \mathrm{Cos}\left[\frac{7hw}{2}\right] + 236214h^7w^4\ 10h^3\ 21168+23733h^4w^4\ \mathrm{Cos}\left[\frac{9hw}{2}\right]$

$7\ 4\mathrm{Cos}\left[\frac{3hw}{2}\right] + 14\ 1286724h^7w^4 + 2h^3\ 423360\ 723573h^4w^4\ \mathrm{Cos}\left[\frac{3hw}{2}\right]$

$+ h\ 5312874hw\left[\frac{5hw}{2}\right] + 834450h^7w^4\mathrm{Cos}\left[\frac{5hw}{2}\right] + 4h\ 9996h^6w^4\mathrm{Cos}\left[\frac{7hw}{2}\right] + 1746h^6w^4\mathrm{Cos}\left[\frac{9hw}{2}\right] + 2\ 7$

$4428h^7w^4 + 5h^3 18144 + 2025h^4w^4\ \mathrm{Cos}\left[\frac{7hw}{2}\right] + 236214h^7w^4\ 10h^3\ 21168$

$+ 23733h^4w^4\ \mathrm{Cos}\left[\frac{9hw}{2}\right]\ 6350400h^6w^2\mathrm{Sin}[3hw]\ 3\mathrm{Sin}\left[\frac{7hw}{2}\right] + \mathrm{Sin}\left[\frac{9hw}{2}\right]$

$\frac{1}{464486400h^6w^4}f_{n+7}\mathrm{Csc}\left[\frac{hw}{2}\right]\mathrm{Csc}[hw]\ 6h\ 10622526h^7w^4\ 20160h^3\ 132$

$405h^4w^4\ \mathrm{Cos}\left[\frac{hhw}{2}\right] + 4\ 9112278h^8w^4\ 60480h^4\ 132 + 405h^4w^4 + 56h^4\ 155520$

$258633h^4w^4\ \mathrm{Cos}\left[\frac{hhw}{2}\right] + 2784252h^8w^4\mathrm{Cos}\left[\frac{h3hw}{2}\right]\ 8h\ 4126308h^7w^4 + 14h^3\ 112320$

$312903h^4w^4\ \mathrm{Cos}\left[\frac{h3hw}{2}\right]\ 4\ 2192022h^8w^4 + 42h^4\ 112320\ 312903h^4w^4 + 112h^4\ 2$

$47520 + 97767h^4w^4\ \mathrm{Cos}\left[\frac{h3hw}{2}\right]\ 3628800h^6w^2\mathrm{Cos}[3hw]\ 7\mathrm{Cos}\left[\frac{h7hw}{2}\right] + \mathrm{Cos}\left[\frac{h9hw}{2}\right]$

$4h^2\ 13500h^6w^4\mathrm{Cos}\frac{5hw}{5hw'} + 633h^6w^4\mathrm{Cos}\frac{7hw}{2} \quad 423h^6w^4\mathrm{Cos}\frac{9hw}{2}$

$h \quad h'w^4\mathrm{Cos}\frac{}{} \quad h'w^4 \quad h^3$

$+ 6 \quad 160110 \quad 7\ hw^2 + 258993 \quad 15 \quad 28224\ 7hw$

$+ 16605h^4w^4 \quad \mathrm{Cos}\frac{}{2} + 68193h'w^4 \quad 5h^3 \quad 12096$

$+ 13689h^4w^4 \quad \mathrm{Cos}\frac{9hw}{2} \quad 3628800 \quad h^3w^2\mathrm{Sin}[3hw]\ 7\mathrm{Sin}\frac{h}{2}$

$+ \mathrm{Sin}\frac{h}{2} + \frac{232243200h^3w^4}{} \quad f_{+2}\mathrm{Csc}\frac{hw}{2} \quad \mathrm{Csc}[hw]\ 21h\ 3996978h'w^4$

$+ 180h^3 \quad 10752\ 20331h^4w^4\ \mathrm{Cos}\frac{h}{2} + 143268194h^8w^4 \quad \frac{hw}{}$

$+ 540h^4 \quad 10752\ 20331h^4w^4 + 28h^4\ 285120 + 269163h^4w^4 \quad \mathrm{Cos}\frac{h}{2}$

$42h\ 1346724h^7w^4 + 2h^3\ 544320\ 868563h^4w^4\ \mathrm{Cos}\frac{h3hw}{2}$

$14\ 225666h^8w^4 + 6h^4 \quad 544320 \quad 868563h^4w^4 + 16h^4$

$317520 + 311607h^4w^4\ \mathrm{Cos}\frac{h3hw}{22} + 320220h^8w^4\mathrm{Cos}\frac{h5hw}{}$

$12700800h^6w^2\mathrm{Cos}[3hw]\ \mathrm{Cos}\frac{h7hw}{22} + 3\mathrm{Cos}\frac{h9hw}{} + 4h\ 7106668h^7w^4$

$105h^3 288 + 1539h^4w^4 \quad \mathrm{Cos}\frac{h7hw}{2} \quad 529254h^7w^4 + 10h^3 \quad 63504$

$56943h^4w^4\ \mathrm{Cos}\frac{h9hw}{22} + h\ 11477886h^7w^4\mathrm{Cos}\frac{h3hw}{} + 14\ 1346724h^7w^4$

$+ 2h^3 \quad 544320 \quad 868563h^4w^4 \quad \mathrm{Cos}\frac{3hw}{2} \quad 133110h'w^4\mathrm{Cos}\frac{5hw}{}$

$4h \quad 17556h^6w^4\mathrm{Cos}\frac{hw}{2} + 3246h^6w^4\mathrm{Cos}\frac{hw}{2} + 2\ 7$

$106668h^7w^4 + 105h^3 288 + 1539h^4w^4 \quad \mathrm{Cos}\frac{h}{2}$

$529254h^7w^4 + 10h^3\ 63504 + 56943h^4w^4\ \mathrm{Cos}\frac{h9hw}{2}$

$12700800h^6w^2\mathrm{Sin}[3hw]\ \mathrm{Sin}\frac{7hw}{2} + 3\mathrm{Sin}\frac{9hw}{2}$

1 hw 7 f +5Csc Csc[hw] 210h 1173878h7w4 + 576h3 1092

464486400h6w4 n hw h 2 i

1377h‘w‘ Cos h 2 i + 140 hw 762126h’w‘ + 1728h‘ 1092 1377h‘w‘

41472 + 3hw Cosh 2 ' 84 1346724 + 2

+ 56h4 26325h4w4 h h′w4 h3 907200

Cosh 2 ' 28 2256663hw + 6 907200 5hw

868563h4w4 h8w4 h4 868563h4w4

+ 16h‘ 453600 + 311607h‘w‘ Cos hw h 2 i + hw h 2 i 7673400h’w‘Cos

25401600h6w2Cos[3hw] 5Cos h 7 2 i + 3Cos h 9 2 i + 4h 7

7551h7w4 + 5h360480 + 63099h4w4 Cos h7hw i 2

h9hw i 2

5 279099h7w4 h3 254016 + 280989h4w4 Cos

h 22955772h7w4Cos h3hw i 2 + 28 1346724h7w4 + 2h3 907200

868563h4w4 Cos h3hw i 22 + 4914060h7w4Cos h5hw i

4h 92463h6w4Cos h7hw i 2 13425h6w4Cos h9hw i 2 + 2 7

7551h7w4 + 5h360480 + 63099h4w4 Cos h7hw i 2

h7w4 h3 280989h‘w‘ Cos 9hw

+ 5 279099 254016 + 7hw i 9hw h i 2 i

1 hw h7 2 h 2

25401600h’w2Sin[3hw] 5Sin + 3Sin

f +3Csc Csc[hw] 210h 1173878h7w4 + 576h3 1260

464486400h6w4 n hw h 2 i hw

1377h‘w‘ Cos h 2 ' + 140 762126h’w‘ + 1728h‘ 1260 1377h‘w‘

+280 3hw h 2 i

h‘ 10368+5265h‘w‘ Cos 84h 1286724h′w‘ +2h’ 1028160

723573 Cos'' 2 ' 28 5091663hw + 6 1028160 5hw

h4w4 h8w4 h4 723573h4w4

hw h 2 i hw h 2 i

+ 16h‘ 574560 + 239517h‘w‘ Cos + 32826600h’w‘Cos

25401600h6w2Cos[3hw] 3Cos h 7 2 i + 5Cos h 9 2 i + 4h 7

949311h7w4 + 5h336288 + 148149h4w4 Cos h7hw i

1842

+5 489771h⁷w⁴ 7h³ 60480 + 62613h⁴w⁴ Cos h9hw i / 2

h 10625748h⁷w⁴Cos h3hw i / 2 + 28 1286724h⁷w⁴ + 2h³ 1028160

723573h⁴w⁴ Cos 3hw / n i h′w⁴Cos 5hw 4h

433713h⁶w⁴Cos 7hw / 2 i + 63855h⁶w⁴Cos 9hw / 2 i + 2 7

311h⁷w⁴ + 5h³36288 + 148149h⁴w⁴ Cos h7hw i / 2

+ 5 489771h⁷w⁴ h³ 62613h⁴w⁴ Cos 9hw / 2 i

1 / 25401600h⁵w²Sin[3hw] 7 60480 + 7hw / 3Sin hw / 2 + 5Sin 9hw / 2 i

928972800h⁵w⁴ / 68 + 243 f n+1 Csc hw / 2 Sec h / 2 i 6h 15670750h′w⁴ 60480h⁵

Cosh h / 2 i + 4 7421238hw 181440 68 + 3hw

h⁴w⁴ h⁸w⁴ / 2 + 3633588 h⁴ 3hw h / 2 i 243h⁴w⁴

+ 56h⁴ 311040 + 647433h⁴w⁴ h⁵w⁴Cos h / 2 i

8h 7846788h′w⁴ + 14h⁵ / 3 164160 598833h⁴w⁴ Cos h / 7 4 4

6392358h w + 42h 164160 598833h w + 336h 33120

+ 93879 9hw Cos hw / 2 i 3628800 5hw h hw / 2 i 7hw

h / 2 i hw h⁵w²Cos[3hw] Cos h / 2 i hw h / 2 i

+ 7Cos 4h² 31620h⁶w⁴Cos 423h⁶w⁴Cos

+ 633h⁶w⁴Cos h9 / 2 i + 6h 771210h⁷w⁴Cos h5 / 2 i

68193h⁷w⁴ 5h³ 12096 + 13689h⁴w⁴ Cos h7hw i / 2

258993h⁷w⁴ 15h³ 28224 + 16605h⁴w⁴ Cos h9hw i / 2

3628800h⁶w²Sin[3hw] Sin h 7hw / 2 i + 7Sin h 9hw / 2 i

$$y_n^{00} + 4 = \frac{h^2}{y_{n+3}} \quad \frac{h^2}{2y_{n+4}} + \frac{h^2}{y_{n+5}} + \frac{46448640h^6w^4}{1} f_{n+4} \quad 25401600h^4$$

$$12700800h^6w^2 \quad 56h \quad 761480h^7w^4 + 420h^3 \quad 1152$$

$$+ 1792h^4w^4 \quad \mathrm{Cos}[hw] \quad 14 \quad 1444190h^8w^4$$

$$+ 1680 \quad h^4 \quad 1152 + 1792h^4w^4 \quad 280h^8 \quad 7344$$

$$+ 5888h\,w \quad \mathrm{Cos}[hw] \quad 782628h\,w\,\mathrm{Cos}[2hw] \quad h^8w^4 \quad hw \quad 27660h^6w^4\mathrm{Cos}[4hw]\,\mathrm{Csc}\,\frac{hw}{\ }^{8}$$

$$+ 221868 \frac{1}{464486400h^6w^4} f_{n+8}\mathrm{Csc}\,h\,\frac{hw}{2}\,i \quad \mathrm{Csc}[hw] \quad 1086840h^8w^4\mathrm{Cos}\,h\,\frac{hw}{2}\,i$$

$$30h \quad 410450h^7w^4 + 3h^3 \quad 20160 + 145664h^4w^4 \quad \mathrm{Cos}\,\frac{hhw\,i}{2}$$

$$h^6w^4 \quad h^4 \quad 60480 \quad 839936h^4w^4$$

$$10 \quad 908032 \quad + \quad \mathrm{Cos}\,\frac{hw\,i}{2}$$

$$+ 12h^4 \quad 20160 + 3hw \quad 145664h^4w^4$$

$$136584h^8w^4\mathrm{Cos}\,\frac{h\,3hw}{2} \quad 90h\ 311940h^7w^4 + 35h^3 \quad 1152 \quad 5hw$$

$$8960h^4w^4 \quad \mathrm{Cos}\,\frac{7hw\,h}{2}\,i + 2h^2 \quad 540h^6w^4\mathrm{Cos}\,\frac{7hw\,h}{2}\,i$$

$$+ 42h^6w^4\mathrm{Cos}\,\frac{h}{2} \quad 3628800h^6w^2\mathrm{Cos}\,\frac{h}{2} \quad \mathrm{Cos}[4hw]$$

$$3628800h^6w^2\mathrm{Sin}\,\frac{h}{2} \quad \mathrm{Sin}[4hw]$$

$$+ \frac{1}{464486400h^6w^4} f_n\mathrm{Csc}\,\frac{h}{2}\,i \quad \mathrm{Csc}[hw]$$

$$267320h^8w^4\mathrm{Cos}\,\frac{hhw\,i}{2} \quad 50h \quad 1681558h^7w^4 + 9h^3 \quad 12096$$

$$+ 185600h^4w^4\mathrm{Cos}\,h\,\frac{hw\,i}{2} \quad 10 \quad 1740032h^8w^4 + h^4 \quad 616896$$

$$8421632h^4w^4 + 36h^4 \quad 12096 + 185600h^4w^4 \quad \mathrm{Cos}\,\frac{hhw\,i}{2}$$

$$456264h^8w^4\mathrm{Cos}\,\frac{h3hw\,i}{\ } \quad 54h\ 927508h^7w^4 + h^3\ 67200\ 2$$

$$919296h^4w^4 \quad \mathrm{Cos}\,\frac{h3hw\,i}{22} + 2h^2 540h^6w^4\mathrm{Cos}\,\frac{h5hw\,i}{\ }$$

$$+42h^6w^4\mathrm{Cos}\left[\frac{7hw}{2}\right] \quad 3628800h^6w^2\mathrm{Cos}[4hw]\mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$3628800h^6w^2\mathrm{Sin}[4hw]\mathrm{Sin}\left[\frac{9hw}{2}\right]$$

$$+\frac{1}{116121600h^6w^4}f_{n+6}\mathrm{Csc}\left[\frac{hw}{2}\right]^7\mathrm{Csc}[hw]$$

$$6350400h^6w^2\mathrm{Cos}[4hw]\ 3\mathrm{Cos}\left[\frac{7}{2}hw\right] + \mathrm{Cos}\left[\frac{9}{2}hw\right] + \frac{1}{2}\ 28h\ 9110410h^7w^4$$

$$+180\ 8064\ 50432\ h^3\ h^4w^4\ \mathrm{Cos}^n\ 20028998h^5w^4 + 720h^4\ 8064$$

$$50432h^4w^4 + 196h^4\ 25920 + 84224h^4w^4\ \mathrm{Cos}\left[\frac{3hw}{2}\right]\ h\ 4586173h^7w^4$$

$$h^5\ h^4w^4\ 8w^4\ h^4$$

$$+2\ 4233602286848\ \mathrm{Cos}\left[\frac{h}{2}\right]^{14}\ 6121352hw + 8\ 423360$$

$$2286848h^4w^4 + 16h^4\ 196560 + 756992h^4w^4\ \mathrm{Cos}\left[\frac{3}{2}h\right]$$

$$3480h^8w^4\mathrm{Cos}\left[\frac{5hw}{22}\right] + 2h^2 1449h^6w^4\mathrm{Cos}\left[\frac{7hw}{22}\right]$$

$$+291h^5w^4\mathrm{Cos}\left[\frac{9hw}{2}\right]\ \frac{1}{464486400h^5w^4}\ 6350400h^5w^2\mathrm{Sin}[4hw]\ 3\mathrm{Sin}\left[\frac{7hw}{2}\right]^7$$

$$+\mathrm{Sin}\left[\frac{h}{2}\right]\ f_{n+7}\mathrm{Csc}\left[\frac{hw}{2}\right]\mathrm{Csc}[hw]\ 8h\ 25849792h^7w^4$$

$$20160h^3\ 132 + 1280h^4w^4\ \mathrm{Cos}\left[\frac{h}{2}\right] + 457935822h^8w^4\ 80640h^4$$

$$132 + 1280h^4w^4 + 56h^4\ 155520 + 817408h^4w^4\ \mathrm{Cos}\left[\frac{hhw}{2}\right]\ 16h$$

$$13861497h^7w^4 + 14h^3\ 112320\ 988928h^4w^4\ \mathrm{Cos}\left[\frac{3hw}{2}\right]$$

$$4\ 20681800h^5w^4 + 56h^4\ 112320\ 988928h^4w^4 + 112h^4\ 47520$$

$$+308992h^4w^4\ \mathrm{Cos}\left[\frac{9hw}{2}\right]\ 3628800h^6w^2\mathrm{Cos}[4hw]\ 7\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$+\mathrm{Cos}\left[\frac{h}{2}\right]\ + 2h^2\ 3480h^6w^4\mathrm{Cos}\left[\frac{hw}{2}\right] + 42h^6w^4\mathrm{Cos}\left[\frac{h}{2}\right]$$

$$+42h^6w^4\mathrm{Cos}\left[\frac{9}{2}h\right]\ 3628800h^6w^2\mathrm{Sin}[4hw]\ 7\mathrm{Sin}\left[\frac{7}{2}h\right]$$

+Sin h 9hw/2 i + 1/(232243200h^6w^4) f n hw +2Csc hw/2 i 7 Csc[hw] 28h 11646166h′w^4

+ 180h^3 10752 64256h^4w^4 Cos h/2 i + 14 22576046h^8w^4 hw

+ 720h^4 10752 64256h^4w^4 + 28h^4 285120 + 850688h^4w^4 Cos h/2 i

56h 5526371h^7w^4 + 2h^3 544320 2745088h^4w^4 Cos h3hw i/2

14 6046960h^8w^4 + 8h^4 544320 2745088h^4w^4 + 16h^4

317520 + 984832h^4w^4 Cos h3hw i/22 3480h^8w^4Cos h5hw i

12700800h^6w^2Cos[4hw] Cos 7hw h 2 i/7hw + 3Cos h9hw i/9hw/2

+ 2h^2 1449h^6w^4Cos h/2 i 7hw + 291h^6w^4Cos 9hw/2 i

1 12700800h^6w^2Sin[4hw] Sin hw 72/7 + 3Sin h 2 i/2

f +5Csc Csc[hw] 280h 2524432h^7w^4 + 576h^3 1092

464486400h^6w^4 n hw h 2i

4352h^4w^4 Cos h/2 i + 140 5450078h^8w^4 + 2304h^4 1092 4352h^4w^4

56h^4 41472 + 83200h^4w^4 Cos hhw/2 i 112h 5526371h^7w^4

2h^3 907200 2745088h^4w^4 Cos h3hw/2 i 28 6046960h^8w^4 + 8h^4 907200

088h^4w^4 + 16h^4453600 + 984832h^4w^4 Cos h3hw i/2

+ 219120h^8w^4Cos 5hw/2 i h^6w^2Cos[4hw] 5Cos 7hw

+ 3Cos h 9hw/2 i h 2 i 25401600 46578 hw/h^6w^4 Cos^n 7hw/2 i hw + 6990h^6w^4Cos h 2 i h 9hw/2 i

25401600h^6w^2Sin[4hw] 5Sin h 7/2 i + 3Sin h 9/2 i

$$\frac{1}{464486400h^6w^4}\, f_{n+3}\, \mathrm{Csc}\left[\frac{hw}{2}\right]^7 \mathrm{Csc}[hw]\, 280h\, 2524432h^7w^4 + 576h^5\, 1260$$

$$4352\, \mathrm{Cos}\left[\frac{hw}{2}\right] + 140\, 5450078hw + 2304\, 1260\, 4352$$

$$+ 280h^4\, 10368 + 16640h^4w^4\, \mathrm{Cos}\left[\frac{h3hw}{2}\right] 112h\, 4586173h^7w^4$$

$$2h^3\, 1028160\, 2286848h^4w^4\, \mathrm{Cos}[28]\, 6121352h^8w^4\, 2$$

$$+8\, 1028160\, 5hw\, 2286848h^4w^4 + 16h^4\, 574560 + 756992h^4w^4\, 7hw\, \mathrm{Cos}\left[\frac{3hw}{2}\right]$$

$$+ 219120h^8w^4\, \mathrm{Cos}[\ldots]\, h^6w^2\, \mathrm{Cos}[4hw]\, 3\mathrm{Cos}[\ldots]$$

$$+ 5\mathrm{Cos}\left[\frac{9hw}{2}\right] + 2h^2\, 25401600\, 46578h^5w^4\, \mathrm{Cos}\left[\frac{7hw}{2}\right] 9hw + 6990h^5w^4\, \mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$25401600h^5w^2\, \mathrm{Sin}[4hw]\, 3\mathrm{Sin}\left[\frac{hw}{2}\right] + 5\mathrm{Sin}\left[\frac{hw}{2}\right]$$

$$\frac{1}{928972800h^6w^4}\, f_{n+1}\, \mathrm{Csc}\left[\frac{h}{2}\right] \mathrm{Sec}\left[\frac{hw}{2}\right] 8h\, 46599520h^7w^4$$

$$60480h^3\, 68 + 768h^4w^4\, \mathrm{Cos}\left[\frac{h}{2}\right] + 471486606h^8w^4\, 241920h^4$$

$$68 + 768h^4w^4 + 56h^4\, 311040 + 2046208h^4w^4\, \mathrm{Cos}\left[\frac{hhw}{2}\right] 16h$$

$$26547751h^7w^4 + 14h^3\, 164160\, 1892608h^4w^4\, \mathrm{Cos}\left[\frac{h3hw}{2}\right]$$

$$4\, 6063504h^5w^4 + 56h^4\, 164160\, 1892608h^4w^4 + 336h^4\, 33120$$

$$+ 296704h^4w^4\, \mathrm{Cos}\left[\frac{9hw}{2}\right] 3628800\, 5hw\, h^5w^2\, \mathrm{Cos}[4hw]\, \mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$+ 7\mathrm{Cos}\left[\frac{h}{2}\right] + 2h^2\, 3480h^6w^4\, \mathrm{Cos}\left[\frac{h}{2}\right] + 42h^6w^4\, \mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$+ 42h^6w^4\, \mathrm{Cos}\left[\frac{9}{2}\right] 3628800h^6w^2\, \mathrm{Sin}[4hw]\, \mathrm{Sin}\left[\frac{7}{2}\right] + 7\mathrm{Sin}\left[\frac{9}{2}\right]$$

$$y_n^{00} + 5 = \frac{y_n}{10h^2} + \frac{2y_{n+3}}{h^2} \quad \frac{7y_{n+4}}{2h^2} + \frac{8y_{n+5}}{5h^2} + \frac{1}{46448640h^6 w^4}\,{}_{n+4} \quad 44452800h^4$$

$$12700800h^6 w^2 \mathrm{Cos}[hw] \;\; 70h \;\; 1660114h^7 w^4 + 420h^3 \;\; 1152$$

$$+ 4375h^4 w^4 \quad \mathrm{Cos}[hw] \;\; 14\; 5162500h^8 w^4 + 2100h^4 \quad 1152 + 4375h^4 w^4$$

$280h^4$ 7344 + $14375h^4w^4$ Cos[hw] + $2996560h^8w^4$Cos[2hw]

$1309860h^8w^4$Cos[3hw] 8h $869210h^7w^4$ $20h^3$ 5292

+ $44375h^4w^4$ Cos[4hw] h $6750870h^7w^4$Cos[hw] + 14 $1660114h^7w^4$

+ $420h^3$ 1152 + $4375h^4w^4$ Cos[hw] $2700152h^7w^4$Cos[2hw]

w^4Cos[4hw] + 4 $869210h^7w^4$ $20h^3$ 5292

+ $989262h^7w^4$Cos[3hw] $41424h^8$

+ $44375h^4w^4$ Cos[4hw] Csc $\frac{hw}{2i}$ $\frac{1}{+464486400h^6w^4}$ f_{n+8}Csc $\frac{hw}{2i}^7$ Csc[hw]

50h 992059h^7 w^4 + 3h^3 20160 + 355625h^4 w^4 Cos [hhw/2 i]

8 4 + h^4 60480 2050625h^4 w^4 + 15h^4 20160

10 3231746hw [hw/2] 843948h^3 w^4 Cos [h 3hw/2 i] 144h 765625h^5 w^4

+ 35h^3 1152 21875h^4 w^4 Cos [hw/2] 4h^2 6915h^6 w^4 Cos [h 5hw/2 i]

+ 633h^6 w^4 Cos [h 7/2 i] 6h 10300h^7 w^4 Cos [h/2 i] + 528015h^7 w^4

5h^3 12096 + 105625h^4 w^4 Cos [h7hw/22 i] 3628800h^6 w^2 Cos [h7hw/2 i] Cos[5hw]

3628800h^6 w^2 Sin [h 7hw/2 i] Sin[5hw] + [1/464486400h^6 w^4] f_n Csc [h hw/2 i] Csc[hw]^7

1895900h^8 w^4 Cos [hhw/2 i] 70h 4062797h^7 w^4 + 9h^3 12096

+ 453125h^4 w^4 [hw/2 i] h^8 w^4 + h^4 616896 20560625h^4 w^4

Cos h 2 10 170000 [hw/2] 261720h^8 w^4 Cos [3hw/2 i]

+ 45h^4 12096 + 453125h^4 w^4 Cos

108h 2257019h^7 w^4 + h^3 67200 2244375h^4 w^4 Cos [h3hw/2 i]

$$4h^{2}\quad 4125h^{6}w^{4}\mathrm{Cos}\frac{h5hw\ i}{2}\quad 423h^{6}w^{4}\mathrm{Cos}\frac{h7hw\ i}{2}$$

$$6h\ 78250h^{7}w^{4}\mathrm{Cos}\frac{h5hw\ i}{2}+10060h^{7}w^{4}\mathrm{Cos}\frac{h7hw\ i}{2}\quad 60480h^{3}\mathrm{Cos}\frac{h9hw\ i}{2}$$

$$3628800h^{6}w^{2}\mathrm{Cos}\frac{h\ 2\ i}{9hw}\ \mathrm{Cos}[5hw]\quad 3628800h^{6}w^{2}\mathrm{Sin}\frac{h\ 2\ i}{9hw}\ \mathrm{Sin}[5hw]$$

$$+\frac{}{116121600h^{6}w^{4}}f_{n+6}\mathrm{Csc}''\frac{}{2}'\ \mathrm{Csc}[hw\ ]\ \frac{}{2}35\quad h\quad h^{7}w^{4}$$

$$+180h^{3}\quad 8064\ 123125h^{4}w^{4}\ \mathrm{Cos}h\frac{}{2}i+14\quad 70043426h^{8}w^{4}$$

$$+900h^{4}\quad 8064\ 123125h^{4}w^{4}+196h^{4}\ 25920+205625h^{4}w^{4}\quad \mathrm{Cos}\ h\frac{hw}{2}i$$

$$70h\ 11280796h^{7}w^{4}+2h^{3}\ 423360\ 5583125h^{4}w^{4}\ \mathrm{Cos}\ h3hw\ i\ 2$$

$$14\ 26261250h^{8}w^{4}+10h^{4}\quad 423360\quad 5583125h^{4}w^{4}+16h^{4}$$

$$196560+1848125h^{4}w^{4}\ \mathrm{Cos}\frac{h3hw\ i}{2}\quad 1518500h^{8}w^{4}\mathrm{Cos}\frac{h5hw\ i}{2}$$

$$144+15625h^{4}w^{4}\ \mathrm{Cos}\ h7hw\ i\ 2$$

$$1831130h^{7}w^{4}\ 10h^{3}\ 21168+183125h^{4}w^{4}\ \mathrm{Cos}\frac{h9hw\ i}{2}$$

$$h\ 4591650h^{7}w^{4}\mathrm{Cos}\frac{h3hw\ i}{2}+14\ 11280796h^{7}w^{4}+2h^{3}\ 423360$$

$$5583125h^{4}w^{4}\ \mathrm{Cos}\ h\frac{3hw}{2}i+786250h^{7}w^{4}\mathrm{Cos}\ h\frac{5hw}{2}i+4h$$

$$17556h^{6}w^{4}\mathrm{Cos}\frac{h7hw\ i}{2}+3246h^{6}w^{4}\mathrm{Cos}\frac{h9hw\ i}{2}+2\ 773140h^{7}w^{4}$$

$$5h^{3}18144+15625h^{4}w^{4}\quad \mathrm{Cos}\ h7hw\ i\ 2\quad 1831130h^{7}w^{4}\quad 10h^{3}\quad 21168$$

$$+\ 183125h^{4}w^{4}\quad \mathrm{Cos}\frac{h\ 2\ i}{9hw}\quad 6350400h^{6}w^{2}\ 3\mathrm{Cos}\frac{h\ 2\ i}{7hw}$$

$$+\mathrm{Cos}\ h\frac{9}{2}i\ \mathrm{Cos}[5hw]\quad 6350400h^{6}w^{2}\ 3\mathrm{Sin}\ h\frac{7}{2}i+\mathrm{Sin}\ h\frac{9}{2}i\ \mathrm{Sin}[5hw]$$

$$\frac{1}{464486400h^{6}w^{4}}f_{n+7}\mathrm{Csc}\left[\frac{hw}{2}\right]\mathrm{Csc}[hw]\ 10h\ 60112466h^{7}w^{4}$$

$$20160h^{3}132 + 3125h^{4}w^{4}\ \mathrm{Cos}\left[\frac{hhw}{2}\right]$$

$$h^{6}w^{4}\qquad h^{4}\qquad 3125h^{4}w^{4}$$

$$+ 4\ \ 202258598\qquad 100800\qquad 132 + hw$$

$$+ 56\qquad 155520 + 3hw$$

$$h^{4}\qquad 1995625h^{4}w^{4}\ \ \mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$2884620h^{8}w^{4}\mathrm{Cos}\left[\frac{h}{2}\right]\ 24h\ 33992828h^{7}w^{4} + 14h^{3}\ 112320$$

$$2414375h^{4}w^{4}\mathrm{Cos}\left[\frac{3h}{2}\right]\ 4\ \ 84516250h^{8}w^{4}$$

$$70h^{4}\ 112320\ 2414375h^{4}w^{4} + 112h^{4}\ 47520$$

$$754375h^{4}w^{4}\ \mathrm{Cos}\left[\frac{3hw}{2}\right]\ 4h^{2}31620h^{6}w^{4}\mathrm{Cos}\left[\frac{5hw}{2}\right]$$

$$423h^{6}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right] + 633h^{6}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right]$$

6h 148150h^7w^4Cos 5hw/2 + 1931935h^7w^4 15h^3 28224 + 128125h^4w^4 Cos 7hw/2

+ 528015h^9w^4 5h^3 12096 + 105625h^4w^4 Cos 7hw/2 3628800h^6w^2 7Cos 9hw/2

+ Cos h/2 Cos[5hw] hw 7 3628800h^6w^2 7Sin h/2 + Sin h/2 Sin[5hw]

+ f +2Csc Csc[hw] 35h 27149018h^7w^4 + 180h^3 10752

232243200h^6w^4 156875h n h hw/2 h/2 + 14 Cos

+ 28h 285120 + 3hw Cos h/2 h^6w^4 + 900h^4 10752 156875h^4w^4

2076875h^4w^4 70h 13656284h^7w^4 + 2h^5 544320

6701875h^4w^4 h/2 Cos 14 28548750h^6w^4 + 10h^4 544320 6701875h^4w^4 hw 5hw

+ 16h^4 317520 + 2404375h^4w^4 Cos h 3/2 i 296300h^8w^4Cos h/2 i 4h 7

1192660h^7w^4 + 105h^3 288 + 11875h^4w^4 Cos h7hw i 4354570h^7w^4 2

3 4 4 9hw 7w^4Cos 3hw

+ 10h 63504 + 439375h w Cos 2 i h 5605110h 3hw h 2 i h 5hw i

+ 14 13656284h w 7hw + 2h^5 544320 6701875h^4w^4 Cos h/2 + 84910h^7w^4Cos 2 i

h 7hw h 9hw

+ 4h 9996h^6w^4Cos 2 1746h^6w^4Cos 2 + 2 7 1192660h^7w^4 + 105h^3 288

+11875h w 4 Cos h 2 7hw 9hw 4354570h^7w^4 + 10h^5 63504 + 439375h^4w^4 Cos 7hw 2 i

hw h i 1 i hw 7 h i

12700800h^6w^2 Cos 2 + 3Cos 2 Cos[5hw] 12700800h^6w^2 Sin 2

+ 3Sin h 9/2 i Sin[5hw] 464486400h^6w^4 f n+5 Csc h/2 i Csc[hw] 350h 5572346h^7w^4

576h^3 1092 10625h^4w^4 Cos hhw/2 i + 140 19047614h^8w^4

2880h^4 1092 10625h^4w^4 + 56h^4 41472 + 203125h^4w^4 Cos hhw/2 i

140h 13656284h^7w^4 + 2h^3 907200 6701875h^4w^4 Cos h3hw i 2

8 4 h^4 907200 $6701875h^4w^4$ + $16h^4$ 453600

28 $28548750hw$ + 10 hw

+ $2404375h^4w^4$ Cos $\frac{3}{2}$ $9407000h^8w^4$Cos $\frac{5hw}{2}$ $4h$ 7 $2422145h^7w^4$

$5h^3$60480 + $486875h^4w^4$ Cos $\frac{h7hw\ i}{2}$ + 5 $2171605h^7w^4$ h^3 254016

$2168125h^4w^4$ Cos $\frac{h9hw\ i}{22}$ h $11210220h^7w^4$Cos $\frac{h3hw\ i}{}$ + 28 $13656284h^7w^4$

+ $2h^3$ 907200 $6701875h^4w^4$ Cos $\frac{h\ 3hw\ i}{2}$ + $9190300h^7w^4$Cos $\frac{h\ 5hw\ i}{2}$ + 4h

$433713h^6w^4$Cos $\frac{h7hw\ i}{22}$ + $63855h^6w^4$Cos $\frac{h9hw\ i}{}$ + 2 $72422145h^7w^4$ + $5h^3$ 60480

+$486875h$ w 4 Cos 4 h $\frac{27hw'}{7hw}$ +5 $2171605h'w^4$ h^3 254016+$2168125h^4w^4$ Cos $\frac{7''hw2\ i}{9hw}$

9hw h i 1 h i hw 7 h i

$25401600h^6w^2$ 5Cos $\frac{}{2}$ + 3Cos $\frac{}{2}$ Cos[5hw] $25401600h^6w^2$ 5Sin $\frac{}{2}$

h 2 i hw h i

+ 3Sin $\frac{}{}$ Sin[5hw] $\frac{}{464486400h^6w^4}$ f_{n+3}Csc $\frac{}{2}$ Csc[hw] 350h $5572346h'w^4$

+ $576h^3$ 1260 $10625h^4w^4$ Cos h $\frac{i}{2}$ + 140 $19047614h^8w^4$ + $2880h^4$ 1260

$10625h^4w^4$ + $280h^4$ 10368 + $40625h^4w^4$ Cos $\frac{hhw\ i}{2}$ 140h $11280796h^7w^4$

$2h^3$ 1028160 $5583125h^4w^4$ Cos $\frac{h3hw\ i}{}$ 28 $26261250h^8w^4$ + $10h^4$ 1028160 2

$5583125h^4w^4$ +$16h^4$ 574560+$1848125h^4w^4$ Cos $\frac{h3hw\ i}{}$ $34560200h^8w^4$Cos $\frac{5hw\ i}{}$

22h

7 4 3 36288 + $1143125h^4w^4$ Cos $\frac{7hw}{}$ + 5 7 4

4h 7 $5949905hw$ + 5h $\frac{9hw}{}$ h $\frac{2hw}{3}$ 3438725hw

$7h^3$60480 + $483125h^4w^4$ Cos $\frac{}{2}$ i h $9183300h^7w^4$Cos h $\frac{}{2}$ i

28 $11280796h^7w^4$ + $2h^3$ 1028160 $5583125h^4w^4$ Cos $\frac{h3hw\ i}{2}$

$h^7 w^4 \mathrm{Cos}\ \frac{5hw}{2} + 4h\ 92463h^6 w^4 \mathrm{Cos}\ \frac{7hw}{2}$

$+16202740\ \ h\ \frac{9hw}{2}\ \ 13425h^6 w^4 \mathrm{Cos}\ \ + 2\ \ 7\ \ 5949905h^7 w^4 + 5h^5$

$36288 + 1143125h^4 w^4\ \ \mathrm{Cos}\ \frac{\ }{\ } + 5\ 3438725h^7 w^4$

$7h\ \ 60480 + 483125h^4 w^4\ \ \mathrm{Cos}\ \frac{7hw}{2}\ \ \mathrm{Cos}\ \frac{9hw}{2}$

$25401600h^6 w^2\ \ 3\mathrm{Cos}\ \frac{7hw}{2} + 5\mathrm{Cos}\ \frac{9hw}{2}\ \ \mathrm{Cos}[5hw]$

$25401600h^6 w^2\ \ 3\mathrm{Sin}\ \frac{\ }{\ } + 5\mathrm{Sin}\ \frac{\ }{\ }\ \ \mathrm{Sin}[5hw]$

$928972800h^6 w^4\ \ \mathrm{Csc}\ \frac{\ }{\ } + 4\ \ \mathrm{Sec}\ \frac{\ }{\ }\ \ 10h\ 111091442h^7 w^4\ \ 60480h^5$

$68 + 1875h^4 w^4\ \ \mathrm{Cos}^h\ \ 286550342h\ w^4\ \ 302400h^4 w^4$

$68 + 1875h^4 w^4\ \ 56h^4\ \ 311040\ \ h^4 w^4\ \ \mathrm{Cos}\ \frac{\ }{\ }$

$1925700h^8 w^4 \mathrm{Cos}\ \frac{\ }{\ }\ \ 24h\ 65049852h^7 w^4 + 14h^3\ \ 164160$

$4620625\ \ w^4\ \ \mathrm{Cos}\ \frac{\ }{\ }\ \ 80053750\ \ + 70\ \ 3hw$

$4h^2\ \ 13500h^6 w^4 \mathrm{Cos}\ \frac{5}{2} + 633h^6 w^4 \mathrm{Cos}\ \frac{7}{2}$

$423h^6 w^4 \mathrm{Cos}\ \frac{9hw}{2}\ \ 6h\ 759250h^7 w^4 \mathrm{Cos}\ \frac{5hw}{2} + 528015h^7 w^4$

$5h^3\ 12096 + 105625h^4 w^4\ \mathrm{Cos}\ \frac{7hw}{2} + 1931935h^7 w^4$

$15\ \ h^5\ \ 28224 + 128125h^4 w^4\ \ \mathrm{Cos}\ \frac{9hw}{2}\ \ 3628800h^5 w^2\ \mathrm{Cos}\ \frac{7hw}{2}$

$+ 7\mathrm{Cos}\ \frac{9}{2}\ \ \mathrm{Cos}[5hw]\ \ 3628800h^6 w^2\ \mathrm{Sin}\ \frac{7}{2} + 7\mathrm{Sin}\ \frac{9}{2}\ \ \mathrm{Sin}[5hw]$

$$\frac{1}{232243200h^6w^4}f_{n+2}\left(hw\right)\left(\frac{h}{2}\right)\left(\frac{i}{2}\right)^7\mathrm{Csc}[hw]\quad 35h\ 27149018h^7w^4 + 180h^3\quad 10752$$

$$156875h^4w^4\ \mathrm{Cos}\left[\frac{h}{2}\right]i + 14\ 82706414h^8w^4 + 900h^4\quad 10752$$

$$156875h^4w^4 + 28h^4\ 285120 + 2076875h^4w^4\ \mathrm{Cos}\left[\frac{hhw}{2}\right]i\ 70h$$

$$13656284h^7w^4 + 2h^3\ 544320\ 6701875h^4w^4\ \mathrm{Cos}\left[\frac{h3hw}{2}\right]i$$

$$14\ 28548750h^6w^4 + 10h^4\quad 544320\ 6701875h^4w^4\quad + 16h^4$$

$$317520 + 2404375h^4w^4\ \mathrm{Cos}\left[h\frac{3}{2}\right]i\quad 296300h^8w^4\mathrm{Cos}\left[h\frac{5hw}{2}\right]i\quad 4h\quad 7$$

$$1192660h^7w^4 + 105h^3\quad 288 + 11875h^4w^4\quad \mathrm{Cos}\left[\frac{h7hw}{2}\right]i\quad 4354570h^7w^4$$

$$+ 10h^3\quad 63504 + 439375h^4w^4\ \mathrm{Cos}\left[h\frac{9hw}{2}\right]i\quad h\quad 5605110h^7w^4\mathrm{Cos}\left[h\frac{3hw}{2}\right]i$$

$$14\ 13656284h^7w^4 + 2h^3\ 544320\ 6701875h^4w^4\ \mathrm{Cos}\left[\frac{h3hw}{2}\right]i$$

$$84910h^7w^4\mathrm{Cos}\left[\frac{h5hw}{222}\right]i + 4h\ 9996h^6w^4\mathrm{Cos}\left[\frac{h7hw}{2}\right]i\quad 1746h^6w^4\mathrm{Cos}\left[\frac{h9hw}{2}\right]i$$

$$2\ 7\ 1192660h^7w^4 + 105h^3\ 288 + 11875h^4w^4\ \mathrm{Cos}\left[\frac{h7hw}{2}\right]i$$

$$4354570h^7w^4 + 10h^3\ 63504 + 439375h^4w^4\ \mathrm{Cos}\left[\frac{h9hw}{2}\right]i$$

$$12700800h^6w^2\ \mathrm{Cos}\left[\frac{7hw}{2}\right]i + 3\mathrm{Cos}\left[\frac{9hw}{2}\right]i\ \mathrm{Cos}[5hw]$$

$$12700800h^6w^2\ \mathrm{Sin}\left[\frac{h}{2}\right]hw + 3\mathrm{Sin}\left[\frac{h}{2}\right]i\ \mathrm{Sin}[5hw]$$

$$\frac{}{464486400h^6w^4}f_{n+5}\left(h\right)\left(\frac{}{2}\right)i\ \mathrm{Csc}[hw]\ 350h\ 5572346h^7w^4$$

$$+ 576h^3\quad 1092\ 10625h^4w^4\quad \mathrm{Cos}\left[h\frac{}{2}\right]i + 140\ 19047614h^8w^4\quad hw$$

$$+ 2880h^4\quad 1092\ 10625h^4w^4\quad + 56h^4\ 41472 + 203125h^4w^4\ \mathrm{Cos}\left[h\frac{}{2}\right]i$$

$$140h\ 13656284h^7w^4 + 2h^3\ 907200\ 6701875h^4w^4\ \mathrm{Cos}\left[h3hw\right]i$$

$$8 \quad 4 \quad 10h^4 \quad 907200 \quad 6701875h^4w^4 \quad + 16h^4 \quad 453600$$

$$28 \quad 28548750hw \quad + 3hw$$

$$+ 2404375h^4w^4 \ \mathrm{Cos}\left(\frac{3hw}{2}\right) \ 9407000h^8w^4\mathrm{Cos}\left(\frac{5hw}{2}\right) \ 4h \quad 7 \quad 2422145h^7w^4$$

$$5h^3 60480 + 486875h^4w^4 \quad \mathrm{Cos}\frac{h7hw\ i}{2} + 5\ 2171605h^7w^4 \quad h^3 \quad 254016$$

$$2168125h^4w^4 \ \mathrm{Cos}\frac{h9hw\ i}{22} \ h\ 11210220h^7w^4\mathrm{Cos}\frac{h3hw\ i}{2} + 28\ 13656284h^7w^4$$

$$+ 2h^3\ 907200 \ 6701875h^4w^4 \quad \mathrm{Cos}\left(\frac{3hw}{2}\right) + 9190300h^7w^4\mathrm{Cos}\left(\frac{5hw}{2}\right) + 4h$$

$$433713h^6w^4\mathrm{Cos}\frac{h7hw\ i}{22} + 63855h^6w^4\mathrm{Cos}\frac{h9hw\ i}{22} + 2\ 72422145h^7w^4 \ + 5h^3 \quad 60480$$

$$+486875h^4w^4 \quad \mathrm{Cos}\frac{7hw}{2} \quad +5\ 2171605h^7w^4 \quad h \quad 4 \quad 4 \quad \mathrm{Cos}$$

$$h\ 27\ hw\ i \quad 1\ 9hw \quad i \quad 254016 + 2168125hw \quad h\ 7\ hw\ i$$

$$25401600h^6w^2 \ 5\mathrm{Cos}\left(\frac{9hw}{2}\right) + 3\mathrm{Cos}\left(\frac{hw}{2}\right) \ \mathrm{Cos}[5hw] \ 25401600h^6w^2 \ 5\mathrm{Sin}\left(\frac{hw}{2}\right)$$

$$+ 3\mathrm{Sin}\left(\frac{hw}{2}\right) \ \mathrm{Sin}[5hw] \ \frac{hw}{464486400h^6w^4} \ f_{n+3}\mathrm{Csc}\left(\frac{hw}{2}\right) \ \mathrm{Csc}[hw]\ 350h\ 5572346h^7w^4$$

$$+576h^5 \ 1260 \quad 10625h^4w^4 \ \mathrm{Cos}\left(\frac{hw}{2}\right) + 140\ 19047614h^6w^4 + 2880h^4 \quad 1260 \quad 10625h^4w^4$$

$$+ 280h^4 \quad 10368 \quad 40625h^4w^4 \quad \mathrm{Cos}\left(\frac{hw}{2}\right) + 2h^5\ 1028160$$

$$5583125h^4w^4 \ \mathrm{Cos}\left(\frac{hw}{2}\right) \quad 28 \quad 26261250h^6w^4 + 10h^4 \quad 1028160 \quad 5583125h^4w^4$$

$$+ 16h^4 \quad 574560 + 1848125h^4w^4 \ \mathrm{Cos}\left(\frac{hw}{2}\right) \ 34560200h^8w^4\mathrm{Cos}\left(\frac{5hw}{2}\right) \ 4h\ 7$$

$$5949905h^7w^4 + 5h^3\ 36288 + 1143125h^4w^4 \ \mathrm{Cos}\frac{h7hw\ i}{2} + 5\ 3438725h^7w^4 \ 7h^3\ 60480\ 2$$

$$+ 483125h^4w^4 \ \mathrm{Cos}\left(\frac{9hw}{2}\right) \ h \quad 9183300h^7w^4\mathrm{Cos}\left(\frac{3hw}{2}\right) + 28 \quad 11280796h^7w^4$$

$$2h^3\ 1028160 \ 5583125h^4w^4 \ \mathrm{Cos}\frac{h3hw\ i}{22} + 16202740h^7w^4\mathrm{Cos}\frac{h5hw\ i}{2}$$

$$+4h\ 92463h^6w^4\mathrm{Cos}\left(\frac{7hw}{2}\right) \quad 13425h^6w^4\mathrm{Cos}\left(\frac{9hw}{2}\right) \quad +2\ 7 \quad 5949905h^7w^4 + 5h^5 \quad 36288$$

$$+1143125h^4w^4 \ \mathrm{Cos}\left(\frac{7hw}{2}\right) + 5 \quad 3438725h^7w^4 \quad 7h^3 \ 60480 + 483125h^4w^4 \ \mathrm{Cos}\left(\frac{7hw}{2}\right)$$

$$25401600h^6w^2 \ 3\mathrm{Cos}\left(\frac{hw}{2}\right) + 5\mathrm{Cos}\left(\frac{hw}{2}\right) \ \mathrm{Cos}[5hw] \ 25401600h^6w^2 \ 3\mathrm{Sin}\left(\frac{hw}{2}\right)$$

$$+ 5\mathrm{Sin}\left(\frac{9hw}{2}\right) \ \mathrm{Sin}[5hw] \quad \frac{}{928972800h^6w^4} \ f_{n+1}\mathrm{Csc}\left(\frac{hw}{2}\right) \ \mathrm{Sec}\left(\frac{hw}{2}\right)$$

$$+16202740h^{7}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right] + 4h\ 92463h^{6}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$13425h^{6}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right] + 2\ 7\ 5949905h^{7}w^{4} + 5h^{3}$$

$$36288 + 1143125h^{4}w^{4}\ \mathrm{Cos}\left[\frac{h}{2}\right] + 5\ 3438725h^{7}w^{4}$$

$$7h^{3}\ 60480 + 483125h^{4}w^{4}\ \mathrm{Cos}\left[\frac{h9}{2}\right]$$

$$25401600h^{6}w^{2}\ 3\mathrm{Cos}\left[\frac{h7hw}{2}\right] + 5\mathrm{Cos}\left[\frac{h9hw}{2}\right]\mathrm{Cos}[5hw]$$

$$25401600h^{6}w^{2}\ 3\mathrm{Sin}\left[\frac{h7hw}{2}\right] + 5\mathrm{Sin}\left[\frac{hhw}{2}\right]\mathrm{Sin}[5hw]$$

$$928972800h^{6}w^{4}f_{n+1}\mathrm{Csc}\left[\frac{h}{2}\right]\mathrm{Sec}\left[\frac{h}{2}\right]\ 10h\ 111091442h^{7}w^{4}$$

$$60480h^{3}\ 68 + 1875h^{4}w^{4}\ \mathrm{Cos}\left[\frac{hhw}{2}\right] + 4\ 286550342h^{8}w^{4}$$

$$302400h^{4}\ 68 + 1875h^{4}w^{4} + 56h^{4}\ 311040$$

$$+ 4995625h^{4}w^{4}\ \mathrm{Cos}\left[\frac{hw}{2}\right]\ 7\ 24h\ 65049852h^{4}w^{4}h + 14h^{3}\ 164160\ 1925700h^{5}w^{4}\mathrm{Cos}\left[\frac{3hw}{2}\right]$$

$$4620625h^{4}w^{4}\mathrm{Cos}\left[\frac{3hw}{2}\right] 4\ 80053750h^{8}w^{4} + 70h^{4}\ 164160$$

$$4620625h^{4}w^{4} + 336\ 5hw33120 + 724375\mathrm{Cos}\left[\frac{3hw}{2}\right]$$

$$4h^{2}\ 13500h^{6}w^{4}\mathrm{Cos}\left[\frac{h9hw}{2}\right] + 633h^{6}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right]$$

$$423h^{6}w^{4}\mathrm{Cos}\left[\frac{h5hw}{2}\right]6h\ 759250h^{7}w^{4}\mathrm{Cos}$$

$$528015h^{7}w^{4}\ 5h^{3}\ 12096 + 105625h^{4}w^{4}\ \mathrm{Cos}\left[\frac{h7hw}{2}\right]$$

$$1931935h^{7}w^{4}\ 15h^{3}\ 28224 + 128125h^{4}w^{4}\ \mathrm{Cos}\left[\frac{h9hw}{2}\right]$$

$$3628800h^{6}w^{2}\ \mathrm{Cos}\left[\frac{h7hw}{2}\right] + 7\mathrm{Cos}\left[\frac{h9hw}{2}\right]\mathrm{Cos}[5hw]$$

$$3628800h^{6}w^{2}\ \mathrm{Sin}\left[\frac{h7hw}{2}\right] + 7\mathrm{Sin}\left[\frac{h9hw}{2}\right]\mathrm{Sin}[5hw]$$

$$y_{n+6}'' = \frac{y_n}{5h^2} + \frac{3y_{n+3}}{h^2} \quad \frac{5y_{n+4}}{h^2} + \frac{11y_{n+5}}{2} + \frac{1}{46448640h^4 w^4 6\ 4} f_{n+4} \quad 63504000h^4$$

$$84h \quad 3453000h\ w + 420h \qquad 1152 + 9072h\ w \quad Cos[hw]$$

$$14 \quad 14245038h^8 w^4 + 2520h^4 \quad 1152 + 9072h^4 w^4$$

$$280h^4 \quad 7344 + 29808h^4 w^4 \quad Cos[hw] \quad 12700800h^6 w^2 Cos[2hw]$$

$$+ 8102556h^{\circ} w^{4} Cos[2hw] \quad 3262356h^{\circ} w^{4} Cos[3hw]$$

$$+ 24084h^8 w^4 \ {}_4 Cos[4hw] \quad 16h \quad 1818486h^7 w^4 \ {}_7\ {}_4 \quad 20 \quad h^3 \qquad 5292$$

$$+ 92016h\ w \quad Cos[4hw] \quad 2h \qquad 6694464h\ w\ Cos[hw] + 14$$

$3453000h^7w^4 + 420h^3\ 1152 + 9072h^4w^4\ \mathrm{Cos}[hw]$

$1896720h^7w^4\mathrm{Cos}[2hw] + 889200h^7w^4\mathrm{Cos}[3hw]$

$16968h^7w^4\mathrm{Cos}[4hw] + 4\ 1818486h^7w^4$

$20h^3\ 5292 + 92016h^4w^4\ \mathrm{Cos}[4hw]\ \overline{\ }\mathrm{Csc}\ \frac{hhw}{2}\ i_8$

$\frac{1}{+464486400h^8w^4}\ f_{n+8}\mathrm{Csc}\ h\ \frac{hw}{2}\ i\ \mathrm{Csc}[hw]\ 550860h^8w^4\mathrm{Cos}\ h\ \frac{hw}{2}\ i$

$70h\ \ 2109762h^7w^4 + 3h^3\ \ 20160 + 737424h^4w^4\ \ \mathrm{Cos}\ \frac{hhw}{2}\ i$

$10\ \ 8879040h^8w^4 + h^4\ \ 60480\ 4252176h^4w^4 + 18h^4$

$20160 + 737424h^4w^4\ \mathrm{Cos}\ \frac{hhw}{2}\ i\ \ 3964464h^8w^4\mathrm{Cos}\ \frac{h3hw}{2}\ i$

$1591066h^7w^4 + 35h^3\ 1152\ 45360h^4w^4\ \mathrm{Cos}\ \frac{h3hw}{2}\ i$

$6h^3\ \ 29570h^5w^4\mathrm{Cos}\ \frac{h5hw}{2}\ i\ \ 1234h^5w^4\mathrm{Cos}\ \frac{h7hw}{2}\ i$

$22h^2\ 6840h^6w^4\mathrm{Cos}\ \frac{h5hw}{2}\ i\ \ 210h^6w^4\mathrm{Cos}\ \frac{h7hw}{2}\ i$

$12h\ 7740h^7w^4\mathrm{Cos}\ \frac{h5hw}{2}\ i + 1095336h^7w^4\ 5h^3\ 12096\ 2$

$+ 219024h^4w^4\ \mathrm{Cos}\ h\ \frac{7hw}{2}\ i\ \ 3628800h^6w^2\mathrm{Cos}\ h\ \frac{7hw}{2}\ i\ \mathrm{Cos}[6hw]$

$3628800h^6w^2\mathrm{Sin}\ \frac{h7hw}{2}\ i\ \mathrm{Sin}[6hw]$

$$\frac{1}{+\ 464486400h^{6}w^{4}}\ f_{n}\mathrm{Csc}\ h\frac{hw}{2}\ i\ \ \mathrm{Csc}[hw]\ \ 3928500h^{8}w^{4}\mathrm{Cos}\ h\frac{hw}{2}\ i$$

$$h\ w + h\ \ h\ w\ \ h$$

$$10\ \ 8067672\ \ \frac{hw}{2}\ i\ \ 616896\ 426345123hw\ \ h\ \ 2\ \ i\ \ +\ 54\ \ 12096$$

$$+\ 939600h^{4}w^{4}\ \ \mathrm{Cos}\ h\ \ 1171728h^{3}w^{4}\mathrm{Cos}\ \ 162h\ 4669122h^{2}w^{4}$$

$$+\ h^{3}\ \ 67200\ \ 4653936h^{4}w^{4}\ \ \mathrm{Cos}\ \ 6h^{3}\ \ 4770h^{5}w^{4}\mathrm{Cos}$$

$$+\ 450h^{5}w^{4}\mathrm{Cos}\ \ 22h^{2}\ \ 240h^{6}w^{4}\mathrm{Cos}\ \ 42h^{6}w^{4}\mathrm{Cos}$$

$$12\ \ 80820\ \ h\ \ h^{2}w^{4}\mathrm{Cos}\ \ +\ 9792h^{2}w^{4}\mathrm{Cos}\ \ 60480h^{3}\mathrm{Cos}$$

$$3628800h^{6}w^{2}\mathrm{Cos}\ \ \mathrm{Cos}[6hw]\ \ 3628800h^{6}w^{2}\mathrm{Sin}\ \ \mathrm{Sin}[6hw]$$

$$\frac{}{116121600h^{6}w^{4}}\ f_{n+6}\mathrm{Csc}\ \ \frac{hw}{2}\ \ \mathrm{Csc}[hw]\ \ 42\ \ 43712266\ \ h\ \ h^{7}w^{4}$$

$$+\ 180h^{3}\ \ 8064\ \ 255312h^{4}w^{4}\mathrm{Cos}\ h\ \ \frac{}{2}\ \ +\ 14\ \ 189806298h^{8}w^{4}\ \ hw$$

$$+\ 1080h^{4}\ \ 8064\ \ 255312h^{4}w^{4}\ +\ 196h^{4}\ \ 25920\ +\ 426384h^{4}w^{4}\ \ \mathrm{Cos}\ h\ \ \frac{}{2}\ i$$

$$84h\ 23483517h^{7}w^{4} + 2h^{3}\ 423360\ 11577168h^{4}w^{4}\ \mathrm{Cos}\ h3hw\ i\ \ 2$$

$$14\ 77816322h^{8}w^{4} + 12h^{4}\ \ 423360\ \ 11577168h^{4}w^{4} + 16h^{4}$$

$$196560 + 3832272h^{4}w^{4}\ \ \mathrm{Cos}\ \ h3hw\ i\ \ 22\ \ 3100320h^{8}w^{4}\mathrm{Cos}\ \ h5hw\ i$$

$$6h^{2}\ \ 21735h^{6}w^{4}\mathrm{Cos}\ \ h7hw\ i\ \ 2\ \ 3441h^{6}w^{4}\mathrm{Cos}\ \ h9hw\ i\ \ 2\ \ 8h\ \ 7$$

$$7806h^{7}w^{4} + 5h^{3}18144 + 32400h^{4}w^{4}\ \ \mathrm{Cos}\ h7hw\ i\ \ +\ 3798306h^{7}w^{4}\ \ 2$$

$$10h^{3}\ \ 21168 + 379728h^{4}w^{4}\mathrm{Cos}\ h\ \ 9hw\ \ 2\ \ i\ \ 2h\ \ 13001856h^{7}w^{4}\mathrm{Cos}\ h\ \ 3hw\ \ 2\ \ i$$

$h^7w^4 + 2h^3$ h^4w^4 $\frac{3hw}{2}$

+14 23483517 hw 423360 11577168 hw Cos h i

+ 807810h^{7}w^{4}Cos h$\frac{5}{2}$i + 3h^2 90538h^{5}w^{4}Cos h$\frac{7}{2}$i

14560h^{5}w^{4}Cos $\frac{h9hwi}{22}$ + 8h 21735h^{6}w^{4}Cos $\frac{h7hwi}{2}$

3441h^{6}w^{4}Cos $\frac{h9hwi}{18144}$ + 2 7157806h^{7}w^4 + 5h^3

32400h^{4}w^4 Cos $\frac{h7hwi}{2}$ + 3798306h^{7}w^4 10h^3 21168

+ 379728h^{4}w^4 Cos $\frac{h\ 2\ i}{9hw}$ 6350400h^{6}w^2 3Cos $\frac{h\ 2}{7hw}$ 9hw

+ Cos $\frac{21}{}$ Cos[6] hw 6350400h^{6}w^2 3Sin h 2 + Sin h 2 i Sin[6hw]

464486400h^{6}w^4 f Csc h i Csc[hw] 12h 126121344h^{7}w^4 20160h^3

132 + 6480h^{4}w^4 Cosh 2 i + 4 547272462h^{8}w^4 120960h^4

132 + 6480h^{4}w^4 + 56h^4 155520 + 4138128h^{4}w^4 Cos $\frac{hhw\ i}{2}$

14089728h^{8}w^{4}Cos $\frac{h3hwi}{2}$ 32h 70739769h^{7}w^4 + 14h^3 112320

5006448h^{4}w^4 Cos $\frac{h3hwi}{}$ 4 245737530h^{8}w^4 + 84h^4 112320 2

448h^{4}w^4 + 112h^{4}47520 + 1564272h^{4}w^4 Cos h3hw i 2

6h^3 177540h^{5}w^{4}Cos $\frac{h5hwi}{22}$ + 450h^{5}w^{4}Cos $\frac{h7hwi}{2}$ 1234h^{5}w^{4}Cos $\frac{h9hwi}{}$

22h^2 43920h^{6}w^{4}Cos $\frac{h5hwi}{22}$ 42h^{6}w^{4}Cos $\frac{h7hwi}{2}$ 210h^{6}w^{4}Cos $\frac{h9hwi}{}$

12h 139320h^{7}w^{4}Cos $\frac{h5hwi}{}$ + 3994992h^{7}w^4 15h^3 28224 2

265680h^{4}w^4 Cos $\frac{h7hwi}{2}$ + 1095336h^{7}w^4 5h^3 12096

+ 2190240h^{7}w^4 hw 7Cos 7hw

hw Cos h $\frac{9hw}{2}$ I 3628800 hw h $\frac{7hw}{2}$ i hw

+ Cos h$\frac{9}{2}$i Cos[6hw] 3628800h^{6}w^2 7Sin h$\frac{7}{2}$i + Sin h$\frac{9}{2}$i Sin[6hw]

$$\frac{1}{232243200h^6w^4}f_{n+2}\left(hw\right)\left(\frac{hw}{2}\right)\operatorname{Csc}\left[\frac{hw}{2}\right]^7\operatorname{Csc}[hw]\quad 42h\ 56256342h^7w^4 + 180h^3\quad 10752$$

$$325296h^4w^4\ \operatorname{Cos}\left[\frac{h}{2}\right] + 14\ 228679386h^8w^4 + 1080h^4\quad 10752$$

$$325296h^4w^4 + 28h^4\quad 285120 + 4306608h^4w^4\ \operatorname{Cos}\left[\frac{hhw}{2}\right]$$

$$84h\ 28212387h^7w^4 + 2h^3\ 544320\ 13897008h^4w^4\ \operatorname{Cos}\left[\frac{h3hw}{2}\right]$$

$$14\ 87325110h^8w^4 + 12h^4\ 544320\ 13897008h^4w^4 + 16h^4$$

$$317520 + 4985712h^4w^4\ \operatorname{Cos}\left[\frac{h3hw}{22}\right]\quad 689040h^8w^4\operatorname{Cos}\left[\frac{h5hw}{22}\right]$$

$$6h^2\ 399h^6w^4\operatorname{Cos}\left[\frac{h7hw}{22}\right]\quad 225h^6w^4\operatorname{Cos}\left[\frac{h9hw}{22}\right]\quad 8h\ 72530422h^7w^4$$

$$105h^3 288 + 24624h^4w^4\ \operatorname{Cos}\left[\frac{h7hw}{2}\right]$$

$$9070578h^7w^4$$

$$+ 10h^3\ 63504 + 911088h^4w^4\ \operatorname{Cosh}\left[\frac{9hw}{2}\right]\quad 8141952h^3hw\operatorname{Cosh}\left[\frac{3hw}{2}\right]$$

$$+ 14\ 28212387\quad h^7w^4 + 2h^3\ 544320\ 13897008h^4w^4\ \operatorname{Cos}\left[\frac{7hw}{2}\right]$$

$$h^7w^4\operatorname{Cosh}\left[\frac{h}{2}\right]\quad h^2\quad h^5w^4\operatorname{Cos}\left[\frac{h}{2}\right]$$

$$+ 581310\quad \frac{hw^2}{2} + 3\quad 11970\ \operatorname{Cos}\left[\frac{7hw}{2}\right]\quad 225h^6w^4\operatorname{Cos}\left[\frac{9hw}{2}\right]$$

$$+ 1768h^5w^4\operatorname{Cos}\left[\frac{9}{2}\right] + 8h\ 399h^6w^4\operatorname{Cos}\left[\frac{7}{2}\right]$$

$$2\ 72530422h^7w^4 + 105h^3\ 288 + 24624h^4w^4\ \operatorname{Cos}\left[\frac{h7hw}{2}\right]$$

$$9070578h^7w^4 + 10h^3\ 63504 + 911088h^4w^4\ \operatorname{Cos}\left[\frac{h9hw}{2}\right]$$

$$12700800h^6w^2\ \operatorname{Cos}\left[\frac{7hw}{2}\right] + 3\operatorname{Cos}\left[\frac{9hw}{2}\right]\quad \operatorname{Cos}[6hw]$$

$$12700800h^6w^2\ \operatorname{Sin}\left[\frac{h}{2}\right] + 3\operatorname{Sin}\left[\frac{h}{2}\right]\quad \operatorname{Sin}[6hw]$$

$$\frac{}{464486400h^6w^4}f_{n+5}\left(h\right)\left(\frac{h}{2}\right)\operatorname{Csc}\left[\frac{hw}{2}\right]\operatorname{Csc}[hw]\ 420h\ 11657168h^7w^4$$

$$+ 576h^3\ 1092\ 22032h^4w^4\ \operatorname{Cos}\left[\frac{h}{2}\right] + 140\ 51570750h^8w^4$$

$$+ 3456h^4\ 1092\ 22032h^4w^4 + 56h^4\ 41472 + 421200h^4w^4\ \operatorname{Cos}\left[\frac{hw}{2}\right]$$

168h 28212387h^{7}w^{4} + 2h^{3} 907200 13897008h^{4}w^{4} Cos$^{h3hw\ i}$ 2

28 87325110h^{8}w^{4} + 12h^{4} 907200 13897008h^{4}w^{4}
202

h^4 h^4w^4 $3hw$ $47016000h^8w^4\mathrm{Cos}\ 5hw$

$+16$ $453600 + 4985712hw\ \mathrm{Cos}\ h\ \frac{}{2}\ i$ hw $h\ \frac{}{2}\ i$

$6h^2$ $752262h^6w^4\mathrm{Cos}\ h\ \frac{7}{2}\ i + 106290h^6w^4\mathrm{Cos}\ h\ \frac{9}{2}\ i$ $8h$ 7

$4024h^7w^4 + 5h^3 60480 + 1009584h^4w^4\ \mathrm{Cos}\ h7hw\ i\ 2$

$5\ 4517136h^7w^4\ h^3\ 254016 + 4495824h^4w^4\ \mathrm{Cos}\ h9hw\ i\ 2$

$2h\ 16283904h^7w^4\mathrm{Cos}\ h3hw\ i\ 2 + 28\ 28212387h^7w^4 + 2h^3\ 907200$

$13897008h^4w^4\ \mathrm{Cos}\ h\ 2\ 7hw\ 3hw + 30741900h^7w^4\mathrm{Cos}\ 9hw\ i\ 5hw\ 2$

$+ 3h^2$ $124390h^5w^4\mathrm{Cos}\ hw\ 2 + 12950$ $h^5w^4\mathrm{Cos}\ hw\ h\ 2\ i$ $+ 8h$

$752262h^6w^4\mathrm{Cos}\ h\ \frac{7}{2}\ i + 106290h^6w^4\mathrm{Cos}\ h\ \frac{9}{2}\ i + 2\ 7$

$4024h^7w^4 + 5h^3 60480 + 1009584h^4w^4\ \mathrm{Cos}\ h7hw\ i\ 2$

$+ 5$ 4517136 $7hw$ $9hw$ $\mathrm{Cos}\ h\ 9hw\ 2\ i$

h^7w^4 h^3 $254016 + 4495824h^4w^4$

$25401600h^6w^2\ 5\mathrm{Cos}\ 7hw\ i\ 2 + 3\mathrm{Cos}\ 9hw\ 2\ \mathrm{Cos}[6hw]$

$25401600h^6w^2\ 5\mathrm{Sin}\ 2\ hw\ 2 + 3\mathrm{Sin}\ 7\ h\ 2\ i\ \mathrm{Sin}[6hw]$

$464486400h^6w^4\ f\ n+3\mathrm{Csc}\ 2\ i\ \mathrm{Csc}[hw]\ 420h\ 11657168h^7w^4$

$+ 576h^3$ $1260\ 22032h^4w^4\ \mathrm{Cos}\ h\ 2\ i + 140\ 51570750h^8w^4$

$+ 3456h^4$ $1260\ 22032h^4w^4 + 280h^4\ 10368 + 84240h^4w^4\ \mathrm{Cos}\ h\ hw\ 2i$

$168h\ 23483517h^7w^4 + 2h^3\ 1028160\ 11577168h^4w^4\ \mathrm{Cos}\ h3hw\ i\ 2$

$28\ 77816322h^6w^4 + 12h^4$ hw $1028160\ 11577168h^4w^4$ $+ 16h^4$

$574560 + 3832272h^4w^4\ \mathrm{Cos}\ h\ 3\ 2\ i$ $68628960h^8w^4\mathrm{Cos}\ h\ 5hw\ 2\ i$

$$6h^2 \quad 10290h^6w^4\mathrm{Cos}\frac{h7hw\ i}{2} \qquad 2310h^6w^4\mathrm{Cos}\frac{h9hw\ i}{2} \qquad 8h\ 7\ 12077712h^7w^4$$

$$5h^3 36288 + 2370384h^4w^4\ \mathrm{Cos}\frac{h7hw\ i}{\ } + 5$$

$$7067736h^7w^4\ \ 2$$

$$7h^3\ 60480 + 1001808h^4w^4\ \ \mathrm{Cos}\frac{h9hw\ i}{22} \qquad 2h\ 26003712h^7w^4\mathrm{Cos}\frac{h3hw\ i}{\ }$$

$$28\ 23483517h^7w^4 + 2h^3\ 1028160\ 11577168h^4w^4\ \mathrm{Cos}\frac{h3hw\ i}{2}$$

$$19971060h^7w^4\mathrm{Cos}\frac{h5hw\ i}{22} + 3h^2 99274h^5w^4\mathrm{Cos}\frac{h7hw\ i}{\ } + 12730h^5w^4\mathrm{Cos}\frac{h9hw\ i}{\ }$$

$$+ 8h\ 10290h^6w^4\mathrm{Cos}\frac{h7hw\ i}{2} \qquad 2310h^6w^4\mathrm{Cos}\frac{h9hw\ i}{2} \qquad + 2\ 7\ \ 12077712h^7w^4$$

$$5h^3 36288 + 2370384h^4w^4\ \mathrm{Cos}\frac{h7hw\ i}{\ } + 5$$

$$7067736h^7w^4\ \ 2$$

$$7h^3\ 60480 + 1001808h^4w^4\ \ \mathrm{Cos}\frac{h9hw\ i}{22} \qquad 25401600h^6w^2\ \underline{3\mathrm{Cos}}\frac{h7hw\ i}{\ }$$

$$+ 5\mathrm{Cos}\frac{h\ 1\ \ i}{\frac{9hw}{2}}\ \mathrm{Cos}[6hw] \qquad \frac{hw\ \ 8}{25401600h^6w^2} \qquad 3\mathrm{Sin}\frac{h\ 7hw\ i}{2} \qquad + 5\mathrm{Sin}\frac{h\ \ \ i}{\frac{9hw}{2}}\ \mathrm{Sin}[6hw]$$

$$\frac{928972800h^6w^4}{\ } \qquad \frac{hw\ \ h\ \ 2i}{f_{n+1}\mathrm{Csc}\ \ \ \mathrm{Sec}\frac{h\ 2\ i}{\ }} \qquad 12h\ 230163232h^7w^4 \qquad 60480h^6 \qquad 68$$

$$+ 3888h\ \ w\ \ \mathrm{Cos}h \qquad \frac{2i}{\ } + 4\ \ 826369806h \qquad hw \qquad hw$$

$$_{4}\ \ _{4} \qquad {}_{8}w^4 \qquad 362880h^4 \qquad 68 + 3888h^4w^4$$

$$+ 56h^4 \qquad 311040 + 10358928h^4w^4 \quad \mathrm{Cos}\frac{h\ \ \ \ }{2\ \ i}\ 6741504h^8w^4\mathrm{Cos}\frac{h\ 3\ \ \ \ }{2\ \ i}$$

$$32h\ 134706279h^7w^4 + 14h^3\ 164160\ 9581328h^4w^4\ \mathrm{Cos}\frac{h3hw\ i}{4}\ 300594654h^8w^4\ 2$$

$$84h^4\ 164160\ 9581328h^4w^4 + 336h^4\ 33120 + 1502064h^4w^4\ \mathrm{Cos}\frac{h3hw\ i}{2}$$

$$6h^3\ 5180h^5w^4\mathrm{Cos}\frac{h5hw\ i}{22} \qquad 1234h^5w^4\mathrm{Cos}\frac{h7hw\ i}{\ } \qquad + 450h^5w^4\mathrm{Cos}\frac{h9hw\ i}{\ }$$

$$22h^2\ 7680h^6w^4\mathrm{Cos}\frac{h5hw\ i}{22} \qquad 210h^6w^4\mathrm{Cos}\frac{h7hw\ i}{\ } \qquad 42h^6w^4\mathrm{Cos}\frac{h9hw\ i}{\ }$$

$$12h\ 769320h^7w^4\mathrm{Cos}\frac{h5hw\ i}{22} + \ 1095336h^7w^4 \quad 5h^3\ 12096 + 219024h^4w^4\ \ \mathrm{Cos}\frac{h7hw\ i}{\ }$$

$$+ \ \ 3994992h^7w^4 \qquad 15h^6\ \ 28224 + 265680h^4w^4 \qquad 7hw\ \ \mathrm{Cos}\frac{h\ 9hw\ i}{2} \qquad 9hw \qquad 3628800h^6w^2 \qquad \mathrm{Cos}\frac{h\ 7hw\ i}{2}$$

$$+ 7\mathrm{Cos}\frac{h\ \ 9\ \ i}{2}\ \mathrm{Cos}[6hw] \quad 3628800h^6w^2\ \ \mathrm{Sin}\frac{h\ \ \ \ i}{2} + 7\mathrm{Sin}\frac{h\ \ \ \ i}{2}\ \mathrm{Sin}[6hw]$$

$$y_n^{00} + 7$$

$$= \frac{3y_n}{10h^2} + \frac{4y_{n+3}}{h^2} - \frac{13y_{n+4}}{2h^2} + \frac{14y_{n+5}}{5h^2} + \frac{1}{46448640h^6w^4} f_{n+4} \quad 82555200h^4$$

$$98h \quad 6569486h^7w^4 + 420h^3 \quad 1152 + 16807h^4w^4 \quad \mathrm{Cos}[hw]$$

$$14 \quad 33252116h^8w^4 + 2940h^4 \quad 1152 + 16807h^4w^4 \quad 280h^4 \quad 7344$$

$$+ 55223h^4w^4 \quad \mathrm{Cos}[hw] + 9405648h^8w^4\mathrm{Cos}[2hw]$$

$$12700800h^6w^2\mathrm{Cos}[3hw] \quad 5172948h^8w^4\mathrm{Cos}[3hw] + 32064h^8w^4\mathrm{Cos}[4hw]$$

$$24h \; 3386334h^7w^4 \quad 20h^3 \quad 5292 + 170471h^4w^4 \quad \mathrm{Cos}[4hw]$$

$$3h \; 5196450h^7w^4\mathrm{Cos}[hw]$$

$$14 \quad 6569486h^7w^4 + 420h^3 \quad 1152 + 16807h^4w^4 \quad \mathrm{Cos}[hw]$$

$$548408h^7w^4\mathrm{Cos}[2hw] + 1283562h^7w^4\mathrm{Cos}[3hw] \quad 29232h^7w^4\mathrm{Cos}[4hw]$$

$$4 \; 3386334h^7w^4 \; 20h^3 \; 5292 + 170471h^4w^4 \; \mathrm{Cos}[4hw] \; \mathrm{Csc}\frac{hw}{2}$$

$$+ \frac{1}{464486400h^6w^4} f_{n+8} \mathrm{Csc}\frac{hw}{2} \quad \mathrm{Csc}[hw] \quad 1072260h^8w^4\mathrm{Cos}\frac{hw}{2}$$

$$90h \quad 3980435h^7w^4 + 3h^3 \quad 20160 + 1366169h^4w^4 \qquad \mathrm{Cos}\frac{hw}{2}$$

$$4 \qquad 10 \quad 4 \; 4 \qquad 8 \; 4 \qquad 4 \qquad 8 \; 4 \qquad 4 \; 4$$

$$20558956h\,w + h \quad 60480 \qquad 787768\,1h\,w \qquad 3hw$$

$$+ 21h \quad 20160 + 1366169h\,w \quad \mathrm{Cos}\frac{hw}{2} \quad 6116544h\,w\,\mathrm{Cos}\frac{hw}{2}$$

$$252h \; 2950341h^7w^4 + 35h^3 \quad 115284035h^4w^4 \quad \mathrm{Cos}\frac{hw}{2}$$

$$24h^3 \quad 41170h^5w^4\mathrm{Cos}\frac{5}{2} + 7210h^5w^4\mathrm{Cos}\frac{7}{2}$$

$$52h^2 \qquad 46965h^5w^4\mathrm{Cos}\frac{5}{2} \qquad + 1233h^6w^4\mathrm{Cos}\frac{7}{2}$$

$$18h \quad 33530h^7w^4\mathrm{Cos}\frac{hw}{2}$$

$$+ \; 2028901h^7w^4 \quad 5h^3 \quad 12096 + 405769h^4w^4 \quad \mathrm{Cos}\frac{hw}{2}$$

$$3628800h^6w^2\mathrm{Cos}\frac{hw}{2} \quad \mathrm{Cos}[7hw]$$

$$3628800h^6w^2\mathrm{Sin}\frac{7hw}{2} \quad \mathrm{Sin}[7hw]$$

$$\frac{1}{464486400h^6w^4}\left(\frac{hw}{2}\right)^7 \operatorname{Csc}\left[\frac{hw}{2}\right] \quad 4597880\,h^5w^4\operatorname{Cos}\left[\frac{hw}{2}\right] \quad 110h$$

$$h\,w \quad h \quad 616896 \quad 78985697h\,w \quad + 63h \quad 12096$$

$$30565366 \quad + \quad \operatorname{Cos}\left[\frac{hw}{2}\right] \quad 4477284h^5w^4\operatorname{Cos}\left[\frac{3hw}{2}\right] \quad h\,8641015h^7w^4$$

$$+ h^3 \quad 67200 \quad 8621991h^4w^4\operatorname{Cos}\left[\frac{3}{2}h\right] \quad 24h^3 \quad 10130h^5w^4\operatorname{Cos}\left[\frac{5}{2}h\right]$$

$$406h^5w^4\operatorname{Cos}[7hw] \quad 52h^2 \quad 1365h^6w^4\operatorname{Cos}[5hw] \quad 15h^6w^4\operatorname{Cos}[7hw]$$

$$h \quad h^7w^4\operatorname{Cos}\left[\frac{5hw}{2}\right] \quad + 9842h^7w^4\operatorname{Cos}\left[\frac{7hw}{2}\right] \quad 60480h^3\operatorname{Cos}\left[\frac{9hw}{2}\right]$$

$$18 \quad 80360 \quad \left(\frac{9hw}{2}\right) \quad 7 \quad 1 \quad h \quad 2 \quad h \quad \frac{9hw}{2} \quad h \quad 2$$

$$3628800h^6w^2\operatorname{Cos}[7hw] \quad 3628800h^6w^2\operatorname{Sin}[7hw]$$

$$\frac{}{116121600h^6w^4}\operatorname{Csc}\left[\frac{hw}{2}\right] \quad \operatorname{Csc}[hw] \quad 2 \quad 49 \quad 82810438$$

$$+ 180h^3 \quad 8064 \quad 472997h^4w^4\operatorname{Cos}\left[\frac{}{2}\right] \quad + 14 \quad 436464422h^8w^4 \quad hw$$

$$+ 1260h^4 \quad 8064 \quad 472997h^4w^4 + 196h^4 \quad 25920 + 789929h^4w^4\operatorname{Cos}\left[\frac{}{2}\right]$$

$$14\,187395374h^8w^4 + 14h^4 \quad 423360 \quad 21448133h^4w^4 + 16h^4$$

$$196560 + 7099757h^4w^4 \quad \operatorname{Cos}[3hw] \quad 5207100h^8w^4\operatorname{Cos}[5hw]$$

$$16h^2 \quad 177324h^6w^4\operatorname{Cos}\left[\frac{7hw}{2}\right] \quad 23490h^6w^4\operatorname{Cos}\left[\frac{9hw}{2}\right] \quad 12h \quad 7$$

$$3276h^7w^4 + 5h^3\,18144 + 60025h^4w^4 \quad \operatorname{Cos}[7hw]$$

$$7023086h^7w^4\,10h^3\,21168 + 703493h^4w^4\operatorname{Cos}[9hw]$$

$$3h\,20898794h^7w^4\operatorname{Cos}[3hw] \quad + 14\,43609084h^7w^4 + 2h^3\,423360$$

$$21448133h^4w^4 \quad \operatorname{Cos}\left[\frac{3hw}{2}\right] \quad + 1546050h^7w^4\operatorname{Cos}\left[\frac{5hw}{2}\right]$$

+8h2 219800h5w4Cos 7hw h hw 2 20788h5w4Cos 9hw h hw 2

h 7 i h 9
+ 12h 177324h6w4Cos 2 23490h6w4Cos 2 i + 2 7

h7hw i
3276h7w4 + 5h318144 + 60025h4w4 Cos + 7023086h7w4 2

9hw h 9hw i 7hw 9hw h 7hw i
10h3 21168 + 703493h4w4 Cos 2 3Sin 6350400h6w2 3Cos 2
h 21 Cos[7] hw 7 2 i h 2 i
+ Cos hw 6350400h6w2 + Sin Sin[7hw]

fn+7Csc 2 Csc[hw] 14h 236310022h7w4 20160h3
h8w4
464486400 4 4 h hw i 8 4 h4

132 + 12005h w Cosh 2 i + 4 1254983006h w 141120hw
hw 155520 + 7666393 h 2 i
132 + 12005h4w4 + 56h4 h4w4 Cos

h 3 i
9868404h8w4Cos 2 40h 130992636h7w4 + 14h3 112320
h3hw i
9275063h4w4 Cos 4 585698326h8w4 + 98h4 112320 2

h3hw i
063h4w4 + 112h447520 + 2898007h4w4 Cos 2

h5hw i h7hw i h9hw i
24h3 442520h5w4Cos 406h5w4Cos + 7210h5w4Cos
222
h5hw i h7hw i h9hw i
52h2 354780h6w4Cos 15h6w4Cos + 1233h6w4Cos
222

h5hw i
18h 316190h7w4Cos + 7392917h7w4 15h3 28224 2
h7hw i
492205h4w4 Cos + 2028901h7w4 5h3 12096
2
h9hw i h7hw i
405769h4w4 Cos 3628800h6w2 7Cos
22

9hw 7hw 9hw
h i h h i
+ Cos 2 Cos[7hw] 3628800h6w2 7Sin 2 i + Sin 2 Sin[7hw]

$$\frac{1}{232243200h^6w^4}\,f_{n+2}\,\mathrm{Csc}\left[\frac{hw}{2i}\right]^7\,\mathrm{Csc}[hw]^{49}\,105515266\,h^7w^4$$

$$+180h^3\,10752\,602651h^4w^4\,\mathrm{Cos}\left[\frac{hw}{2}\right]i+14\,531212618h^8w^4$$

$$+1260h^4\,10752\,602651h^4w^4+28h^4\,285120+7978523h^4w^4\,\mathrm{Cos}\left[\frac{hw}{2}\right]i$$

$$98h\,52142348h^7w^4+2h^3\,544320\,25745923h^4w^4\,\mathrm{Cos}\left[\frac{3hw}{2}\right]i$$

$$14\,213517858h^8w^4+14h^4\,544320\,25745923h^4w^4+16h^4$$

$$317520+9236647h^4w^4\,\mathrm{Cos}\left[\frac{h3hwi}{22}\right]\,4735380h^8w^4\,\mathrm{Cos}\left[\frac{h5hwi}{22}\right]$$

$$16h^2\,4620h^6w^4\,\mathrm{Cos}\left[\frac{h7hwi}{22}\right]\,66h^6w^4\,\mathrm{Cos}\left[\frac{h9hwi}{22}\right]\,12h\,74735052h^7w^4$$

$$105h^3\,288+45619h^4w^4\,\mathrm{Cos}\left[\frac{h7hwi}{2}\right]$$

$$16838878h^7w^4$$

$$10h^3\,63504+1687903h^4w^4\,\mathrm{Cos}\left[\frac{h9hwi}{22}\right]\,3h\,12597214h^7w^4\,\mathrm{Cos}\left[\frac{h3hwi}{22}\right]$$

$$+14\,52142348\,h^7w^4+2h^3\,544320\,25745923\,h^4w^4\,\mathrm{Cos}\left[\frac{3hw}{2}\right]i$$

$$+4214950h^7w^4\,\mathrm{Cos}\left[\frac{h5hw}{2}\right]i+8h^2\,31192h^5w^4\,\mathrm{Cos}\left[\frac{h7hw}{2}\right]i$$

$$1460h^5w^4\,\mathrm{Cos}\left[\frac{h9hwi}{222}\right]+12h\,4620h^6w^4\,\mathrm{Cos}\left[\frac{h7hwi}{22}\right]\,66h^6w^4\,\mathrm{Cos}\left[\frac{h9hwi}{22}\right]$$

$$2\,74735052h^7w^4+105h^3\,288+45619h^4w^4\,\mathrm{Cos}\left[\frac{h7hwi}{2}\right]$$

$$16838878h^7w^4+10h^3\,63504+1687903h^4w^4\,\mathrm{Cos}\left[\frac{h9hwi}{2}\right]$$

$$12700800h^6w^2\,\mathrm{Cos}\left[\frac{h\,hwi}{7hw/2}\right]+3\mathrm{Cos}\left[\frac{h\,hwi}{9hw/2}\right]\,\mathrm{Cos}[7hw]$$

$$12700800h^6w^2\,\mathrm{Sin}\left[\frac{h\,7}{2i}\right]+3\mathrm{Sin}\left[\frac{h\,9}{2}\right]i\,\mathrm{Sin}[7hw]$$

1 f +5Csc hw 7Csc[hw] 490h 22213534h^{7}w^4 + 576h^3 1092

464486400h^{6}w^4 n hw h 2 i

40817h w Cos h 2 + 140 hw 118709558h^{8}w^4 + 4032h^4 1092 40817h^{4}w^4

41472 + 3hw Cos h 2

+ 56h^4 780325h^{4}w^4 196h 52142348h^{7}w^4 + 2h^3 907200

25745923h w h 2 i hw 907200 5hw

4 4 Cos 28 213517858h^{8}w^4 + 14h^4 25745923h^{4}w^4

+ 16h^4 453600 + 9236647h^{4}w^4 Cos h 3 i 2 137429160h^{8}w^{4}Cos h i 2 16h^2

h7hw i h9hw i

541065h^{6}w^{4}Cos 2 2 +83415h^{6}w^{4}Cos 12h 79528939h^{7}w^4 +5h^3 60480

h 7hw i h 9hw

+ 1870379h^{4}w^4 Cos 2 + 5 8370439h^{7}w^4 h^3 254016 + 8329069h^{4}w^4 Cos 2 i

h3hw i

3h 25194428h^{7}w^{4}Cos + 28 52142348h^{7}w^4 + 2h^3 907200 2

h3hw i h5hw i h7hw i

25745923h^{4}w^4 Cos 2 2 2 + 59517740h^{7}w^{4}Cos + 8h^2 584934h^{5}w^{4}Cos

h9hw i h7hw i h9hw i

48730h^{5}w^{4}Cos 2 2 2 + 12h541065h^{6}w^{4}Cos + 83415h^{6}w^{4}Cos

h7hw i

2 79528939h^{7}w^4 + 5h^3 60480 + 1870379h^{4}w^4 Cos + 5 8370439h^{7}w^4 2

h9hw i h7hw i

h^3 254016 + 8329069h^{4}w^4 Cos 2 2 25401600h^{6}w^2 5Cos

h 1 i hw 7 h 7hw i h 2 i

+ 3Cos 9hw 2 Cos[7hw] 25401600h^{6}w^2 5Sin 2 + 3Sin 9hw Sin[7hw]

f +3Csc Csc[hw] 490h 22213534h^{7}w^4 + 576h^3 1260

464486400h^{6}w^4 n hw h 2 i

Cos h 2 + 140 hw

40817h^{4}w^4 118709558h^{8}w^4 + 4032h^4 1260 40817h^{4}w^4

4 156065h^{4}w^4 7 4 3

+ 280h 10368 + hw Cos h 2 i 196h 43609084h w + 2h 1028160

3

h i

21448133h^{4}w^4 Cos 2 28 187395374h^{8}w^4 + 14h^4 1028160 21448133h^{4}w^4

$$+16h^4\ 574560 + 7099757h^4w^4\ \mathrm{Cos}\left[\frac{3hw}{2}\right]\ 124783800h^8w^4\mathrm{Cos}\left[\frac{5hw}{2}\right]$$

$$16h^2\ 29463h^6w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]\ 585h^6w^4\mathrm{Cos}\left[\frac{9hw}{2}\right]\ 12h\ 722183707h^7w^4$$

$$5h^3 36288 + 4391429h^4w^4\ \mathrm{Cos}\left[\frac{7hw}{2}\right] + 5\ 13046999h^7w^4\ 7h^3\ 60480^2$$

$$1855973h^4w^4\ \mathrm{Cos}\left[\frac{9hw}{2}\right]\ 3h\ 41797588h^7w^4\mathrm{Cos}\left[\frac{3hw}{2}\right] + 28\ 43609084h^7w^4$$

$$+ 2h^3\ 21448133h^4w^4\ \frac{3hw}{\ } + 38644100h^7w^4\mathrm{Cos}\ \frac{5hw}{\ }$$

$$1028160\ \mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$+ 8h^2 200550h^5w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]\ 9050h^5w^4\mathrm{Cos}\left[\frac{9hw}{2}\right] + 12h\ 29463h^6w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$585h^6w^4\mathrm{Cos}\left[\frac{9hw}{2}\right] + 2\ 722183707h^7w^4 + 5h^3\ 36288 + 4391429h^4w^4\ \mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$5\ 13046999h^7w^4\ 7h^3\ 60480 + 1855973h^4w^4\ \mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$25401600h^6w^2\ 3\mathrm{Cos}\left[\frac{7hw}{2}\right] + 5\mathrm{Cos}\left[\frac{9hw}{2}\right]\ \mathrm{Cos}[7hw]\ 25401600h^6w^2\ 3\mathrm{Sin}\left[\frac{7hw}{2}\right]$$

$$+ 5\mathrm{Sin}\left[\frac{9}{2}\right]\ \mathrm{Sin}[7hw]\ \frac{}{928972800h^6w^4}\ f_{n+1}\mathrm{Csc}\left[\frac{}{2}\right]\ \mathrm{Sec}\left[\frac{}{2}\right]\ 14h\ 429177766h^7w^4$$

$$60480h^3\ 68 + 7203h^4w^4\ \mathrm{Cos}\left[\frac{hw}{2}\right] + 4\ 1964524094h^8w^4\ 423360h^4\ 68$$

$$+ 7203h^4w^4 + 56h^4\ 311040 + 19191193h^4w^4\ \mathrm{Cos}\left[\frac{hw}{2}\right]\ 20423004h^8w^4\mathrm{Cos}\left[\frac{3hw}{2}\right]$$

$$40h\ 249473548h^7w^4 + 14h^3\ 164160\ 17750593h^4w^4\ \mathrm{Cos}\left[\frac{3hw}{4}\right]\ 805756218h^8w^4\ 2$$

$$98h^4\ 164160\ 17750593h^4w^4 + 336h^4\ 33120 + 2782759h^4w^4\ \mathrm{Cos}\left[\frac{3hw}{2}\right]$$

$$24h^3\ 103960h^5w^4\mathrm{Cos}\left[\frac{5hw}{2}\right] + 7210h^5w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]\ 406h^5w^4\mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$52h^2\ 56220h^6w^4\mathrm{Cos}\left[\frac{5hw}{2}\right] + 1233h^6w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]\ 15h^6w^4\mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$18h\ 792890h^7w^4\mathrm{Cos}\left[\frac{5hw}{2}\right] + 2028901h^7w^4\ 5h^3\ 12096 + 405769h^4w^4\ \mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$+ 7392917h^7w^4\ 15h^3\ 28224 + 492205h^4w^4\ \mathrm{Cos}\left[\frac{9hw}{2}\right]\ 3628800h^6w^2\ \mathrm{Cos}\left[\frac{7hw}{2}\right]]$$

$$7\mathrm{Cos}\left[\frac{9hw}{2}\right]\ \mathrm{Cos}[7hw]$$

$$3628800h^6w^2\ \mathrm{Sin}\left[\frac{7hw}{2}\right]$$

$$+ 7\mathrm{Sin}\left[\frac{9}{2}\right]\ \mathrm{Sin}[7hw]$$

$$y_n^{00} + 8$$

$$= \frac{2y_n}{5h^2} + \frac{5y_{n+3}}{h^2} - \frac{8y_{n+4}}{h^2} + \frac{17y_{n+5}}{5h^2} + \frac{1}{46448640h^6w^4} f_{n+4} \quad 101606400h^4$$

$$112h \quad 11464840h^7w^4 + 420h^3 \quad 1152 + 28672h^4w^4 \quad \mathrm{Cos}[hw]$$

$$14 \quad 68725630h^8w^4 + 3360h^4 \quad 1152 + 28672h^4w^4 \; 280h^4 \quad 7344$$

$$+ 94208h^4w^4 \quad \mathrm{Cos}[hw] + 7387996h^8w^4\mathrm{Cos}[2hw]$$

$$9808596h^8w^4\mathrm{Cos}[3hw] \quad 12700800h^6w^2\mathrm{Cos}[4hw]$$

$$14220h^8w^4\mathrm{Cos}[4hw] \; 32h \; 5792360h^7w^4 \quad 20h^3 \quad 5292 +$$

$$290816h^4w^4 \quad \mathrm{Cos}[4hw] \; 4h \; 4171776h^7w^4\mathrm{Cos}[hw]$$

$$14 \; 11464840h^7w^4 + 420h^3 \quad 1152 + 28672h^4w^4 \quad \mathrm{Cos}[hw] +$$

$$1010800h^7w^4\mathrm{Cos}[2hw] + 3576240h^7w^4\mathrm{Cos}[3hw] + 187824h^7w^4\mathrm{Cos}[4hw]$$

$$4 \; 5792360h^7w^4 \; 20h^3 \; 5292 + 290816h^4w^4 \; \mathrm{Cos}[4hw] \; \mathrm{Csc}\frac{hw}{2}$$

$$+ \frac{1}{464486400h^6w^4} f_{n+8}\mathrm{Csc}\frac{hw}{2} \quad \mathrm{Csc}[hw] \quad 2064240h^8w^4\mathrm{Cos}\frac{hw}{2}$$

$$110h \quad 6866602h^7w^4 + 3h^3 \qquad 20160 + 2330624h^4w^4 \qquad \mathrm{Cos}$$

$$4 \qquad 10 \quad 4 \; 4 \qquad 8 \quad 4 \quad 4 \qquad 60480 \qquad 4 \quad 4$$

$$+ 24h \quad 20160 + 2330624h\,w \quad \mathrm{Cos}\frac{hw}{2} \quad 15358392h\,w\,\mathrm{Cos}\frac{hw}{2}$$

$$306h \; 5032408h^7w^4 + 35h^3 \quad 1152 \; 143360h^4w^4 \quad \mathrm{Cos}\frac{hw}{2}$$

$$60h^3 \quad 72990h^5w^4\mathrm{Cos}\frac{5hw}{2} + 49014h^5w^4\mathrm{Cos}\frac{7hw}{2}$$

$$24h \quad 90400h^7w^4\mathrm{Cos}\frac{5hw}{2}$$

$$+ \; 3470928h^7w^4 \quad 5h^3 \quad 12096 + 692224h^4w^4 \quad \mathrm{Cos}\frac{7hw}{2}$$

$$3628800h^6w^2\mathrm{Cos}\frac{7hw}{2} \quad \mathrm{Cos}[8hw]$$

$$3628800h^6w^2\mathrm{Sin}\frac{7hw}{2} \quad \mathrm{Sin}[8hw]$$

$$+\frac{1}{464486400h^{6}w^{4}}f_{n}\mathrm{Csc}\left[\frac{hw}{2}\right]^{7}\mathrm{Csc}[hw]$$

$$4962800h^{8}w^{4}\mathrm{Cos}\left[\frac{hw}{2}\right]130h\ 26601422h^{7}w^{4}$$

$$+9h^{3}12096+2969600h^{4}w^{4}\ \mathrm{Cos}\left[\frac{hw}{2}\right]10$$

$$78838888h^{8}w^{4}+h^{4}\ 616896\ 134746112h^{4}w^{4}+$$

$$72h^{4}\ 12096+2969600h^{4}w^{4}\mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$6833736h^{8}w^{4}\mathrm{Cos}\left[\frac{h3hw}{2}\right]270h\ 14733416h^{7}w^{4}\ 2$$

$$h^{3}\ 67200\ 14708736h^{4}w^{4}\mathrm{Cos}\left[\frac{h3hw}{2}\right]$$

$$60h^{3}\ 26590h^{5}w^{4}\mathrm{Cos}\left[\frac{5hw}{2}\right]+3178h^{5}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$94h^{2}\ 21660h^{6}w^{4}\mathrm{Cos}\left[\frac{h5}{2}\right]+498h^{6}w^{4}\mathrm{Cos}\left[\frac{h7}{2}\right]$$

$$24h\ 90400h^{7}w^{4}\mathrm{Cos}\left[\frac{h5hw}{2}\right]+9808h^{7}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right]$$

$$60480h^{3}\mathrm{Cos}\left[\frac{h9hw}{2}\right]3628800h^{6}w^{2}\mathrm{Cos}\left[\frac{h9hw}{2}\right]\mathrm{Cos}[8hw]$$

$$3628800h^{6}w^{2}\mathrm{Sin}\left[\frac{h9hw}{2}\right]\mathrm{Sin}[8hw]$$

$$+ \frac{1}{116121600h^6w^4} f_{n+6}\mathrm{Csc}\left[\frac{hw}{2}\right]^7 \left(\mathrm{Csc}[hw] \frac{1}{256} \left(\frac{h}{2}\right)^7 w^4 \left(142860170 + 180h^3 \left(8064 + 806912h^4w^4 \mathrm{Cos}\left[\frac{h}{2}\right] + 14 \left(890809454h^8w^4 + 1440h^4 \left(8064 + 806912h^4w^4 + 196h^4 \left(25920 + 1347584h^4w^4 \mathrm{Cos}\left[\frac{hw}{2}\right]\right)\right)\right)\right)\right)\right)$$

$$112h\,(74283709h^7w^4 + 2h^3\,(423360 + 36589568h^4w^4 \mathrm{Cos}\left[\frac{h3hw}{2}\right]$$

$$14\,(393278076h^8w^4 + 16h^4\,(423360 + 36589568h^4w^4 + 16h^4\,(196560 + 12111872h^4w^4 \mathrm{Cos}\left[\frac{h3hw}{2}\right] + 10503080h^8w^4 \mathrm{Cos}\left[\frac{h5hw}{2}\right]$$

$$30h^2\,(432831h^6w^4 \mathrm{Cos}\left[\frac{h7hw}{2}\right] + 23901h^6w^4 \mathrm{Cos}\left[\frac{h9hw}{2}\right] + 16h\,(7\,(452088h^7w^4 + 5h^3\,(18144 + 102400h^4w^4 \mathrm{Cos}\left[\frac{h7hw}{2}\right] + 11960984h^7w^4\,(2\,(10\,(h^3\,(21168 + 1200128h^4w^4 \mathrm{Cos}\left[\frac{h\,(2\,(9hw)}{2}\right] + 4\,(h\,(18407424h^4w^4 \mathrm{Cos}\left[\frac{h\,(2\,(3hw)}{2}\right] + 14\,(74283709\,(5hw\,(h^4w^4 + 2h^3\,(423360 + 36589568h^4w^4 \mathrm{Cos}\left[\frac{h\,(2\,(7hw)}{2}\right] + 3923650h^4w^4 \mathrm{Cos}\left[\frac{hw}{2}\right] + 15h^2\,(259182h^5w^4 \mathrm{Cos}\left[\frac{h\,(2\,(hw)}{2}\right] + 38084h^5w^4 \mathrm{Cos}\left[\frac{h\,(9)}{2}\right] + 16h\,(432831h^6w^4 \mathrm{Cos}\left[\frac{h\,(7)}{2}\right] + 23901h^6w^4 \mathrm{Cos}\left[\frac{h9hw}{2}\right] + 2\,(7452088h^7w^4 + 5h^3\,(18144 + 102400h^4w^4 \mathrm{Cos}\left[\frac{h7hw}{2}\right] + 11960984h^7w^4\,(10h^3\,(21168 + 1200128h^4w^4 \mathrm{Cos}\left[\frac{h\,(2\,(9hw)}{2}\right] + 6350400h^6w^2\,(3\mathrm{Cos}\left[\frac{h\,(2\,(7hw)}{2}\right] + \mathrm{Cos}\left[\frac{h\,(9)}{2}\right]\,\mathrm{Cos}[8hw] + 6350400h^6w^2\,(3\mathrm{Sin}\left[\frac{h\,(7)}{2}\right] + \mathrm{Sin}\left[\frac{h\,(9)}{2}\right]\,\mathrm{Sin}[8hw]$$

$\frac{1}{464486400 \; h^{6} w^{7}} f_{n+7} \mathrm{Csc}\left[\frac{hw}{2}\right]^{7} \mathrm{Csc}[hw]$ 16h 404029376$h^{7}w^{4}$ 20160h^{3}

h hw

132 + 20480$h^{4}w^{4}$ Cos$\frac{h}{2}$ i + 4 2553657038$h^{8}w^{4}$ 161280h^{4}

132 + 20480$h^{4}w^{4}$ + 56h^{4} 155520 + 13078528$h^{4}w^{4}$ Cos $\frac{hhw}{2}$ i

52887552$h^{8}w^{4}$Cos $\frac{h3hw}{}$ i 48h 222372473$h^{7}w^{4}$ + 14h^{3} 112320 2

15822848$h^{4}w^{4}$ Cos $\frac{h3hw}{}$ i 4 1220853292$h^{8}w^{4}$ + 112h^{4} 112320 2

+ 112h^{4} 47520 + 4943872$h^{4}w^{4}$ Cos $\frac{h3hw}{2}$ i

60h^{3} 495940$h^{5}w^{4}$Cos $\frac{h5hw}{22}$ i + 3178$h^{5}w^{4}$Cos h7hw i

+ 49014h w 9hw 94h 862200hw Cos 5hw i hw

$h^{6}w^{4}$

+ 498$h^{6}w^{4}$Cos $\frac{h7}{2}$ i + 25122$h^{6}w^{4}$Cos $\frac{h9}{2}$ i 24h 919360$h^{7}w^{4}$Cos $\frac{h5}{2}$ i

12605008$h^{7}w^{4}$ 15h^{3} 28224 + 839680$h^{4}w^{4}$ Cos $\frac{h7hw}{2}$ i + 3470928$h^{7}w^{4}$

5h^{3} 12096 + 692224$h^{4}w^{4}$ Cos $\frac{h9hw}{22}$ i 3628800$h^{6}w^{2}$ 7Cos h7hw i

+ Cos 2 1 Cos[8] hw 7 h 2 i h 2 i

9hw hw 3628800$h^{6}w^{2}$ 7Sin $\frac{7hw}{}$ + Sin $\frac{9hw}{}$ Sin[8hw]

+ $\frac{1}{232243200 h^{6} w^{4}}$ f_{n+2}Csch $\frac{hw}{2}$ Csc[hw] 56 181694678 $h^{7}w^{4}$

+ 180h^{3} 10752 1028096$h^{4}w^{4}$ Cos $\frac{h}{2}$ i + 14 1091456774$h^{8}w^{4}$

+ 1440h 10752 1028096h w + 28h285120

+ 13611008$h^{4}w^{4}$ Cos $\frac{hhw}{2}$ 112h 88728419$h^{7}w^{4}$ + 2h^{3} 544320

43921408$h^{4}w^{4}$ Cos$\frac{h3}{2}$ i 14 452260476$h^{8}w^{4}$ + 16h^{4} 544320

43921408$h^{4}w^{4}$ h3hw i

+ 16h^{4}317520 + 15757312$h^{4}w^{4}$ Cos 2

$$28754584h^{\prime}w^{4} \quad {}^{+7hw}_{10h^{5}} \quad 63504 + 2879488h^{4}w^{4} \quad \mathrm{Cos}\frac{h\ 2\ i}{9hw}$$

$$12700800h^{6}w^{2} \quad \mathrm{Cos}\frac{{}^{\prime\prime}\ {}^{7hw}\ {}^{\prime}}{2} + 3\mathrm{Cos}\frac{{}^{h9hw\ i}}{2} \quad \mathrm{Cos}[8hw]$$

$$12700800h^{6}w^{2} \quad \frac{\mathrm{Sin}^{\prime\prime}\ {}^{2}\ {}^{hw}}{} + 3\mathrm{Sin}\frac{{}^{7}\ h\ {}^{2}\ i}{} \quad \mathrm{Sin}[8hw]$$

$$\frac{}{464486400h^{6}w^{4}} f_{n+5} \quad \mathrm{Csc}\frac{}{h\ 2\ i} \quad \mathrm{Csc}[hw]\ 560h\ 38655248h^{7}w^{4}$$

$$+ 576h^{3} \quad 1092 \quad 69632h^{4}w^{4} \quad \mathrm{Cos}\frac{h}{2}i + 140 \quad 242773406h^{8}w^{4}$$

$$+ 4608h^{4} \quad 1092 \quad 69632h^{4}w^{4} + 56h^{4}\ 41472 + 1331200h^{4}w^{4} \quad \mathrm{Cos}\frac{n}{2}\frac{hw}{i}$$

$$224h\ 88728419h^{7}w^{4} + 2h^{3}\ 907200\ 43921408h^{4}w^{4}\ \mathrm{Cos}\frac{h3hw\ i}{2}$$

$$28\ 452260476h^{8}w^{4} + 16h^{4} \quad 907200 \quad 43921408h^{4}w^{4} + 16h^{4}$$

$$453600 + 15757312h^{4}w^{4} \quad \mathrm{Cos}\frac{3hw\ i}{} \quad 278745200h^{8}w^{4}\mathrm{Cos}\frac{h5hw\ i}{}$$

$$30h^{2} \quad 359478h^{6}w^{4}\mathrm{Cos}\frac{{}^{22h}\ h\ {}^{7hw}}{2}i + 75750h^{6}w^{4}\mathrm{Cos}\frac{h\ 9hw\ i}{2} \quad 16h \quad {}^{7}$$

$$50256h^{7}w^{4} + 5h^{3}60480 + 3190784h^{4}w^{4}\ \mathrm{Cos}\frac{h7hw\ i}{2}$$

$$5\ 14263984h^{7}w^{4}\ h^{3}\ 254016 + 14209024h^{4}w^{4}\ \mathrm{Cos}\frac{h9hw\ i}{2}$$

$$4h\ 21676032h^{\prime}w^{4}\mathrm{Cos}\frac{{}^{\prime\prime}3hw\ \ i}{2} + 28\ 88728419h^{\prime}w^{4} + 2h^{3} \quad 907200$$

$$43921408h^{4}w^{4} \quad \mathrm{Cos}\frac{h}{2} \quad {}^{7hw\ \prime} + 79291660h^{\prime}w^{4}\mathrm{Cos}\frac{{}^{9\ \prime\prime}hw\ \ i}{2}$$

$$+ 15h^{2} \quad 1092658h^{5}w^{4}\mathrm{Cos}\frac{h}{hw2}i + 98510h^{5}w^{4}\mathrm{Cos}\frac{h}{hw2}i$$

$$+ 16h \quad 359478h^{6}w^{4}\mathrm{Cos}\frac{{}^{\prime\prime}\ {}^{7}}{2}i + 75750h^{6}w^{4}\mathrm{Cos}\frac{{}^{\prime\prime}\ {}^{9}}{2}i + 2 \quad 7$$

$$50256h^{7}w^{4} + 5h^{3}60480 + 3190784h^{4}w^{4}\ \mathrm{Cos}\frac{h7hw\ i}{2}$$

$$+ 5 \quad 14263984 \quad {}^{7hw} \quad {}^{9hw} \quad {}^{h} \quad {}^{2\ i}_{9hw}$$

$$h^{7}w^{4} \quad h^{3}\ 254016 + 14209024h^{4}w^{4} \quad \mathrm{Cos}$$

$$25401600h^{6}w^{2} \quad 5\mathrm{Cos}\frac{h\ {}^{hw\ \prime}}{2} + 3\mathrm{Cos}\frac{h\,hw\ i}{2} \quad \mathrm{Cos}[8hw]$$

$$25401600h^{6}w^{2}5\mathrm{Sin}\frac{h\ {}^{7}}{2\ i} + 3\mathrm{Sin}\frac{h\ {}^{9}}{2}i \quad \mathrm{Sin}[8hw]$$

$$\frac{1}{464486400h^6w^4}\ f_{n+3}\ \mathrm{Csc}\ \frac{hw}{h\ 2\ i}\ ^7\ \mathrm{Csc}[hw]\quad 560h\ 38655248h^7w^4$$

$$+\ 576h^3\quad 1260\ \ 69632h^4w^4\mathrm{Cos}\frac{h}{2}\ i\ +\ 140\ \ 242773406h^8w^4\qquad hw$$

$$+\ 4608h^4\quad 1260\ \ 69632h^4w^4 + 280h^4\quad 10368 + 266240h^4w^4\mathrm{Cos}\frac{h}{2}\ i$$

$$224h\ 74283709h^7w^4 + 2h^3\ 1028160\ 36589568h^4w^4\ \mathrm{Cos}\frac{h3hw\ i}{2}$$

$$28\ 393278076h^8w^4 + 16h^4\ \ 1028160\quad 36589568h^4w^4 + 16h^4$$

$$574560 + 12111872h^4w^4\ \mathrm{Cos}\frac{h3hw\ i}{2}\quad 228244400h^8w^4\mathrm{Cos}\frac{5hw\ i}{2}$$

$$30h^2\ 473382h^6w^4\mathrm{Cos}\frac{h7hw\ i}{2}\quad 20287880h^8w^4\mathrm{Cos}\frac{h5hw\ i}{2}$$

$$30h^2\quad 81039h^6w^4\mathrm{Cos}\frac{h\ 7hw\ i}{2}\ +\ 1731h^6w^4\mathrm{Cos}\frac{h\ 9hw\ i}{2}\quad 16h\ 7$$

$$8111608h^7w^4 + 105h^3\quad 288 + 77824h^4w^4\quad \mathrm{Cos}\frac{h\ 9hw\ i}{2}$$

$$28754584\quad h^7w^4\quad + 10\quad h^3\quad 63504 + 2879488h^4w^4\quad \mathrm{Cos}\frac{h\ 3hw\ i}{2}$$

$$4h\ 10838016h^7w^4\mathrm{Cos}\frac{3hw\ i}{2}\ +\ 14\ \ 88728419h^7w^4 + 2h^3\quad 544320$$

$$43921408h^4w^4\quad \mathrm{Cos}\frac{h\ 7hw\ i}{2}\ +\ 11536510h^7w^4\mathrm{Cos}\frac{9hw\ i}{2}$$

$$126434\qquad \mathrm{Cosh}\frac{hw}{2}\ +\ 11212\qquad \mathrm{Cos}\frac{hw\ 2\ i}{2}$$

$$+\ 15h^2\qquad h^5w^4\qquad h^5w^4\mathrm{Cos}$$

$$+\ 16h\ \ 81039h^6w^4\mathrm{Cos}\frac{h\ 7\ i}{2}\ +\ 1731h^6w^4\mathrm{Cos}\frac{h\ 9\ i}{2}\ +\ 2\ \ 7$$

$$111608h^7w^4 + 105h^3 288 + 77824h^4w^4\ \mathrm{Cos}\frac{h7hw\ i}{2}$$

$$+10230h^{6}w^{4}\mathrm{Cos}\frac{h9hwi}{} \qquad 16h\ 7$$

$$\frac{}{2} \qquad 37654256h^{7}w^{4} + 5h^{3} \qquad 36288$$

$$7491584h^{4}w^{4}\ \mathrm{Cos}\frac{h7hwi}{} + 5\ 22218416h^{7}w^{4}\ 7h^{3}\ 60480\ {}^{2}$$

$$+ 3166208h^{4}w^{4}\ \mathrm{Cos}\frac{h\ \frac{hw}{9}\ i}{2} \quad 4h \quad 36814848h^{7}w^{4}\mathrm{Cos}\frac{h\ \frac{hw}{3}\ i}{2}$$

$$h'w^{4} + 2h^{3} \qquad h^{4}w^{4}\ \mathrm{Cos}\frac{3hw}{2}i$$

$$+ 64131700 \qquad \frac{h\ \frac{h}{2}\ i}{} \qquad 664174 \qquad \frac{\mathrm{Cos}\ \frac{hw}{2}\ i}{}$$

$$+ 68050h^{5}w^{4}\mathrm{Cos}\frac{h\ \frac{9}{2}\ i}{} + 16h \quad 473382h^{6}w^{4}\mathrm{Cos}\frac{h\ \frac{7}{2}\ i}{}$$

$$10230h^{6}w^{4}\mathrm{Cos}\frac{h9hwi}{} + 2$$

$$\frac{}{} \qquad 737654256h^{7}w^{4} + 5h^{3}\ {}^{2}$$

$$36288 + 7491584 \qquad \frac{h\ \frac{}{2}\ i}{} \qquad \frac{9hw}{}$$

$$h^{3} \qquad \frac{h^{4}w^{4}}{} \qquad \frac{}{}$$

$$25401600h^{6}w^{2}\ 3\mathrm{Cos}\frac{''7hw\ i}{2} + 5\mathrm{Cos}\frac{9hw\ i}{2} \quad \mathrm{Cos}[8hw]$$

$$1 \qquad h \qquad 2\ hw' \quad 8 \qquad ''hw\ i$$

$$25401600h^{6}w^{2}\ 3\mathrm{Sin}\frac{}{} + 5\mathrm{Sin}\frac{}{2} \quad \mathrm{Sin}[8hw]$$

$$\frac{}{928972800h^{6}w^{4}}\ f_{n+1}\mathrm{Csc}\frac{h\ \frac{}{2}\ i}{} \ \mathrm{Sec}\frac{h\ \frac{}{2}\ i}{} \quad 16h\ 735812960h^{7}w^{4}$$

$$60480h^{3} \quad 68 + 12288h^{4}w^{4}\ \mathrm{Cos}\frac{hhw\ i}{2} + 4\ 4094882318h^{8}w^{4}$$

$$h^{4} \qquad 12288h^{4}w^{4} + 56h^{4} \qquad 311040$$

$$483840 \quad 68 + \qquad \frac{hw}{2}' \qquad 23396352h^{8}w^{4}\mathrm{Cos}''\frac{3hw}{2}i$$

$$+ 32739328h^{4}w^{4}\ \mathrm{Cos}h$$

$$48h\ 425362983h^{7}w^{4} + 14h^{3}\ 164160\ 30281728h^{4}w^{4}\ \mathrm{Cos}^{h3hw\ i}\ {}_{2}$$

$$\frac{}{}$$

$$4\ 1798880812h^{8}w^{4} + 112h^{4} \quad 164160 \quad 30281728h^{4}w^{4} + 336h^{4}$$

$$33120 + 4747264h^{4}w^{4}\ \mathrm{Cos}\frac{h3hwi}{22} \quad 60h^{3}\ 176700h^{5}w^{4}\mathrm{Cos}\frac{h5hwi}{}$$

$$+ 49014h^{5}w^{4}\mathrm{Cos}\frac{h7hwi}{2} + 3178h^{5}w^{4}\mathrm{Cos}\frac{h9hwi}{2} \quad hw]$$

$$y_n^{000} = \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} - \frac{3y_{n+4}}{2h^3} + \frac{3y_{n+5}}{5h^3}$$

$$+ \frac{1}{464486400h^6w^4} f_{n+8} \operatorname{Csch}\left[\frac{hw}{2}\right] \left(7\operatorname{Csc}[hw] + 926250\operatorname{Cos}[3hw] - 2\operatorname{Cos}[7hw]\right)$$

$$30 \quad 60480h^3 \quad 73434h^7w^4 \operatorname{Cos}\left[\frac{3hw}{2}\right] \quad 288414h^7w^4 \operatorname{Cos}\left[\frac{7hw}{2}\right]$$

$$54 \quad 40320 \quad 32240 \quad 5hw \quad h^7w^4 \operatorname{Cos}\left[\frac{5hw}{2}\right] + 362880h^3 \operatorname{Cos}\left[\frac{7hw}{2}\right]$$

$$+ 60h^3 \quad 1800h^4w^4 \operatorname{Cos}\left[\frac{5hw}{2}\right] + 10332h^4w^4 \operatorname{Cos}\left[\frac{7hw}{2}\right]$$

$$141 \quad 26590 \quad h^5w^4 \operatorname{Cos}\left[\frac{5hw}{2}\right] \quad 3178h^5w^4 \operatorname{Cos}\left[\frac{7hw}{2}\right]$$

$$+ 72 \quad 21660 \quad h^6w^4 \operatorname{Cos}\left[\frac{hw}{2}\right] + 498h^6w^4 \operatorname{Cos}\left[\frac{hw}{2}\right]$$

$$3628800h^6w^3 \operatorname{Sin}\left[\frac{7h}{2}\right] + \frac{}{46448640h^6w^4} f_{n+4} \operatorname{Csc}\left[\frac{h}{2}\right] \quad 19051200h^3$$

$$42\,483840h^3 \quad 217224h^7w^4 \operatorname{Cos}[hw] + 7698096h^7w^4 \operatorname{Cos}[2hw]$$

$$+ 8016624h^7w^4 \operatorname{Cos}[3hw] + 1270080h^3 \operatorname{Cos}[4hw] + 563472h^7w^4 \operatorname{Cos}[4hw]$$

$$+ 4 \quad 413952h^6w^4 \operatorname{Cos}[hw] \quad 545664h^6w^4 \operatorname{Cos}[2hw]$$

$$+ 2467584h^6w^3 \operatorname{Cos}[3hw] + 839328h \quad 1 \operatorname{Cos}[4 \quad]$$

$$12700800h^6w^3 \operatorname{Sin}[4 \quad hw] + \frac{}{464486400h^6w^4} f_n \operatorname{Csch}\left[\frac{hw}{2}\right] \operatorname{Csc}[hw] \quad hw]$$

$$12902490h^7w^4 \operatorname{Cos}\left[\frac{3hw}{2}\right] \quad 30 \quad 108864h^3 + 94338h^7w^4 \operatorname{Cos}\left[\frac{7hw}{2}\right]$$

$$+ 33268158h^7w^4 \operatorname{Cos}\left[\frac{5hw}{2}\right] \quad 54 \quad 67200h^3 \quad 24096h^7w^4 \operatorname{Cos}\left[\frac{7hw}{2}\right]$$

$$+ 60h^3 \quad 235080 \quad h^4w^4 \operatorname{Cos}\left[\frac{5hw}{2}\right] + 70812h^4w^4 \operatorname{Cos}\left[\frac{7hw}{2}\right]$$

$$141h^2 \quad 72990 \quad h^5w^4 \operatorname{Cos}\left[\frac{5hw}{2}\right] \quad 49014h^5w^4 \operatorname{Cos}\left[\frac{7hw}{2}\right]$$

$$+ 72 \quad h \quad 48300 \quad h^6w^4 \operatorname{Cos}\left[\frac{hw}{2}\right] + 25122h^6w^4 \operatorname{Cos}\left[\frac{hw}{2}\right]$$

$$+ 362880h^3 \operatorname{Cos}\left[\frac{h}{2}\right] \quad 3628800h^6w^3 \operatorname{Sin}\left[\frac{h}{2}\right]$$

$$\frac{1}{116121600h^6w^4}\left(\frac{\mathrm{Csc}\left[\frac{hw}{2}\right]^7}{2}\right)\left(21\,1451520h^{7hw}\mathrm{Cos}\left[\frac{3hw}{2}\right]+9hw\right)h^3$$

$$+1531786h^7w^4\mathrm{Cos}\left[\frac{h}{2}\right]+42\,846720h^5\,787779h^7w^4\mathrm{Cos}\left[\frac{h}{2}\right]+6\,635040h^3\mathrm{Cos}\left[\frac{9hw}{2}\right]+211680h^3\mathrm{Cos}\left[\frac{h}{2}\right]$$

$$+31851450h^7w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]+45h^2\,126434h^5w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]+11212\,h^5w^4\mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$+48\,h\,81039\,h^6w^4\mathrm{Cosh}\left[\frac{3hw}{2}\right]+1731h^6w^4\mathrm{Cos}\left[\frac{h}{2}\right]+4\,h\,81039\,h^6w^4\mathrm{Cos}\left[\frac{5hw}{2}\right]$$

$$+4\,1397760\,\mathrm{Csc}\left[\frac{hw}{2}\right]^7+6804480\,\mathrm{Csc}\left[\frac{9hw}{2}\right]+15h^2\,102186h^4w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]+36306h^4w^4\mathrm{Cos}\left[\frac{9hw}{2}\right]+24h$$

$$126434h^5w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]+11212h^5w^4\mathrm{Cos}\left[\frac{9hw}{2}\right]+6\,81039h^6w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$+1731h^6w^4\mathrm{Cos}\left[\frac{9hw}{2}\right]+\mathrm{Sin}\left[\frac{9hw}{2}\right]\left(\frac{1\,6350400}{464486400h^6w^4\,\mathrm{Csc}[hw]^{7+n}}\right)h^6w^3\,3\mathrm{Sin}\left[\frac{7hw}{2}\right]+\mathrm{Csc}[hw]^7\,6\,2661120h^3$$

$$+3604416h^7w^4\,\mathrm{Cos}\left[\frac{h}{2}\right]+5332992h^7w^4\mathrm{Cos}\left[\frac{3}{2}h\right]+12\,1572480h^3$$

$$1596807h^7w^4\,\mathrm{Cos}\left[\frac{3hw}{2}\right]+6423360h^3\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$60480h^3\mathrm{Cos}\left[\frac{9hw}{2}\right]+60h^3\,41040h^4w^4\mathrm{Cos}\left[\frac{5hw}{2}\right]$$

$$+70812h^4w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]+10332h^4w^4\mathrm{Cos}\left[\frac{9hw}{2}\right]+141h^2$$

$$176700h^5w^4\mathrm{Cos}\left[\frac{5hw}{2}\right]+49014h^5w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]+3178h^5w^4\mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$+72h\,209880\,h^6w^4\mathrm{Cos}\left[\frac{hw}{2}\right]+25122h^6w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]+498h^6w^4\mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$+3628800h^6w^3\,7\mathrm{Sin}\left[\frac{h}{2}\right]+\mathrm{Sin}\left[\frac{h}{2}\right]$$

$$+ \frac{1}{232243200h^6w^4} f_{n+2}\mathrm{Csc}\left[\frac{hw}{2}\right]^7 \mathrm{Csc}[hw]\, 21\, 1935360h^3$$

$$3882h^7w^4 \mathrm{Cos}\left[\frac{hw}{2}\right] 42\, 1088640h^5\, 1547421h^7w^4 \mathrm{Cos}\left[\frac{3hw}{2}\right] 9hw$$

$$+ 9012870h^7w^4\mathrm{Cos}\left[\frac{7hw}{2}\right] 6\, 211680h^3\mathrm{Cos}\left[\frac{9hw}{2}\right] 635040h^3\mathrm{Cos}\left[\frac{h}{2}\right]$$

$$45\, 259182h^2 h^5w^4\mathrm{Cos}\left[\frac{3hw}{2}\right] 38084h^5w^4\mathrm{Cos}\left[\frac{5hw}{2}\right]$$

$$+ 48h\, 432831h^6w^4\mathrm{Cos}\left[\frac{hw}{2}\right] 23901h^6w^4\mathrm{Cos}\left[\frac{hw}{2}\right] + 4h$$

$$1311744\, h^6w^4\mathrm{Cos}\left[\frac{hw}{2}\right] + 2442240\, h^6w^4\mathrm{Cos}\left[\frac{hw}{2}\right] + 15h^2$$

$$79254h^4w^4\mathrm{Cos}\left[\frac{7hw}{2}\right] + 122706h^4w^4\mathrm{Cos}\left[\frac{9hw}{2}\right] 24h$$

$$259182h^5w^4\mathrm{Cos}\left[\frac{7hw}{2}\right] 38084h^5w^4\mathrm{Cos}\left[\frac{9hw}{2}\right] + 6\, 432831h^6w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$23901h^6w^4\mathrm{Cos}\left[\frac{9hw}{2}\right] 12700800h^6w^5\, \mathrm{Sin}\left[\frac{7hw}{2}\right] + 3\mathrm{Sin}\left[\frac{9hw}{2}\right]$$

$$\frac{1}{464486400h^6w^4} f_{n+5}\mathrm{Csc}\left[\frac{hw}{2}\right] \mathrm{Csc}[hw]\, 210\, 628992h^3$$

$$+ 513296h^7w^4 \mathrm{Cos}\left[\frac{hw}{2}\right] 84\, 1814400h^5\, 1547421h^7w^4 \mathrm{Cos}\left[\frac{3hw}{2}\right]$$

$$+ 129044700h^7w^4\mathrm{Cos}\left[\frac{5hw}{2}\right] 6\, 2116800h^3\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$1270080h^3\mathrm{Cos}\left[\frac{9hw}{2}\right] 45h^2\, 664174h^5w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$68050h^5w^4\mathrm{Cos}\left[\frac{9hw}{2}\right] + 48h\, 473382h^6w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$+ 10230h^6w^4\mathrm{Cos}\left[\frac{9hw}{2}\right] + 4h\, 2623488h^6w^4\mathrm{Cos}\left[\frac{3hw}{2}\right]$$

$$+ 20797440h^6w^4\mathrm{Cos}\left[\frac{hw}{2}\right] + 15h^2\, 315252h^4w^4\mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$+ 219060h^4w^4\mathrm{Cos}\left[\frac{9hw}{2}\right] 24h\, 664174h^5w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$68050h^5w^4\mathrm{Cos}\left[\frac{9hw}{2}\right] + 6\, 473382h^6w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$+ 10230h^6w^4\mathrm{Cos}\left[\frac{9hw}{2}\right] 25401600h^6w^3\, 5\mathrm{Sin}\left[\frac{7hw}{2}\right]$$

$$+3\sin\left(\frac{9hw}{\ }\right) \quad \frac{1}{464486400h^{6}w^{4}\ {}^{n+3}\csc\left(\frac{hw}{2}\right)} \quad \csc[hw]^{7} \quad 210^{3hw} \quad 725760h^{3}$$

$$+ 513296h^{7}w^{4}\cos\left(\frac{hw}{2}\right) \quad 84 \quad 2056320h^{3} \quad 787779h^{7}w^{4}\cos\left(\frac{hw}{2}\right)$$

$$+ 174524580h^{7}w^{4}\cos\left(\frac{5}{2}\right) \quad 61270080h^{3}\cos\left(\frac{h^{7}}{2}\right)$$

$$2116800h^{3}\cos\left(\frac{9hw}{22}\right) \quad 45h^{2} \quad 1092658h^{5}w^{4}\cos\left(\frac{7hw}{\ }\right)$$

$$98510h^{5}w^{4}\cos\left(\frac{9hw}{\ }\right) \quad + 48h \quad 359478h^{6}w^{4}\cos\left(\frac{7hw}{2}\right)$$

$$+ 75750h^{6}w^{4}\cos\left(\frac{5hw}{2}\right) \quad + 4h \quad 2795520h^{6}w^{4}\cos\left(\frac{7hw}{2}\right)$$

$$+ 12318720 \quad h^{6}w^{4}\cos\left(\frac{9hw}{2}\right) \quad + 15h^{2} \quad 12852h^{4}w^{4}\cos\left(\frac{7hw}{2}\right)$$

$$+ 383220h^{4}w^{4}\cos\left(\frac{9hw}{2}\right) \quad 24h \quad 1092658h^{5}w^{4}\cos\left(\frac{7hw}{2}\right)$$

$$98510h^{5}w^{4}\cos\left(\frac{9hw}{2}\right) \quad + 6 \quad 359478h^{6}w^{4}\cos\left(\frac{7hw}{2}\right)$$

$$+ 75750h^{6}w^{4}\cos\left(\frac{hw}{2}\right) \quad 25401600h^{6}w^{3} \quad 3\sin\left(\frac{hw}{2}\right) \quad + 5\sin\left(\frac{hw}{2}\right)$$

$$928972800h^{6}w^{4} \quad f \quad \csc(\ ) \quad \sec(\ ) \quad 4112640h^{3}$$

$$+ 12426592 \quad \cos\left(\frac{hw}{2}\right)^{n+1} \quad 74833920 \quad \cos\left(\frac{3hw}{2}\right)^{6}$$

$$12 \quad 2298240h^{3} + 3737127h^{7}w^{4}\cos\left(\frac{3hw}{2}\right)^{6} \quad 60480h^{3}\cos\left(\frac{7hw}{2}\right)$$

$$423360h^{3}\cos\left(\frac{9hw}{22}\right) + 60h^{3} \quad 231120h^{4}w^{4}\cos\left(\frac{5hw}{\ }\right)$$

$$+ 10332h^{4}w^{4}\cos\left(\frac{7hw}{5hw}\right) \quad + 70812h^{4}w^{4}\cos\left(\frac{9hw}{7hw}\right) \quad 141h^{2}$$

$$495940h^{5}w^{4}\cos\left(\frac{5hw}{2}\right) \quad 3178h^{5}w^{4}\cos\left(\frac{7hw}{2}\right) \quad 49014h^{5}w^{4}\cos\left(\frac{9hw}{2}\right)$$

$$+ 72h \quad 862200h^{6}w^{4}\cos\left(\frac{hw}{2}\right) \quad + 498h^{6}w^{4}\cos\left(\frac{hw}{2}\right)$$

$$+ 25122h^{6}w^{4}\cos\left(\frac{9}{2}\right) \quad 3628800h^{6}w^{3}\sin\left(\frac{7}{2}\right) \quad + 7\sin\left(\frac{9}{2}\right)$$

$$y_{n+1}^{000} = \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} - \frac{3y_{n+4}}{2h^3} + \frac{3y_{n+5}}{5h^3}$$

$$+ \frac{1}{46448640h^6w^4} f_{n+4}\mathrm{Csc}\left[\frac{hw}{2}\right]^8 \left(19051200h^3\right.$$

$$58842h^7w^4\mathrm{Cos}[hw] - 42 - 206810h^7w^4 + 420h^3 - 1152$$

$$+ 7h^4w^4 - \mathrm{Cos}[hw] + 6310920h^7w^4\mathrm{Cos}[2hw] + 1138590h^7w^4\mathrm{Cos}[3hw]$$

$$87696h^7w^4\mathrm{Cos}[4hw] - 12 - 546h^7w^4 - 20h^3 - 5292$$

$+ 71h^4w^4 \quad \mathrm{Cos}[4hw] + 3h \quad 44982h^6w^4\mathrm{Cos}[hw]$

$+ 1915704h^6w^4\mathrm{Cos}[2hw] + 1348326h^6w^4\mathrm{Cos}[3hw]$

$43848h^6w^4\mathrm{Cos}[4hw] \quad 12700800h^6w^3\mathrm{Sin}[3hw]$

$+ \frac{1}{464486400h^{\upsilon}w^4}\, f_{n+8}\mathrm{Csc}\left[\frac{hw}{2}\right]^7 \mathrm{Csc}[\quad hw] \quad 177690 \quad h'w^{*}\mathrm{Cos}\left[\frac{hw}{2}\right]$

$30 \quad 65507h^7w^4 + 3h^3 \quad 20160 + 569h^4w^4\,\mathrm{Cos}\left[\frac{\quad}{2}\right]$

$2523150h^7w^4\mathrm{Cos}\left[\frac{h3hw}{\quad}\right] \quad 54\ 23559h^7w^4 + 35h^3\ 1152\ 2$

$35h^4w^4\ \mathrm{Cos}\left[\frac{h3hw}{2\ 2}\right] + 24h^3\ 22590h^4w^4\mathrm{Cos}\left[\frac{h5hw}{\quad}\right]$

$378h^4w^4\mathrm{Cos}\left[\frac{h7hw}{2\ 2}\right] \quad 78h^2 10130h^5w^4\mathrm{Cos}\left[\frac{h5hw}{\quad}\right]$

$+ 406h^5w^4\mathrm{Cos}\left[h\frac{7hw}{2}\right] + 54h \quad 1365h^6w^4\mathrm{Cos}\left[h\frac{5hw}{2}\right]$

$15h^6w^4\mathrm{Cos}\left[\frac{h7hw}{2\ 2}\right] \quad 6\ 10040h^7w^4\mathrm{Cos}\left[\frac{h5hw}{\quad}\right]$

$811h^7w^4\ 5h^3\ 12096 + 169h^4w^4\ \mathrm{Cos}\left[\frac{h7hw}{2}\right]$

$3628800h^6w^3\mathrm{Cos}\left[\frac{h7hw}{2\ 2}\right]\mathrm{Sin}[hw] \quad 3628800h^6w^3\mathrm{Cos}[hw]\mathrm{Sin}\left[\frac{h7hw}{\quad}\right]$

$+ \frac{1}{464486400h^{7}\ {}^{6}w^4}\, f_n\mathrm{Csc}\left[\frac{hw}{2}\right]^7 \mathrm{Csc}[hw] \quad 672390h'w^{4}\mathrm{Cos}\left[h\frac{hw}{2}\right]$

$h\ w + 9h \quad 12096$

$30 \quad 21925$

$+ 725h^4w^4 \quad \mathrm{Cosh}\left[\frac{hw}{2}\right] \quad 514134h^7w^4\mathrm{Cos}\left[\frac{3hw}{2}\right] \quad 54$

$11573h^7w^4 + h^3\ 67200\ 3591h^4w^4\ \mathrm{Cos}\left[\frac{h3hw}{2}\right]$

+24h^3 26370h^4w^4Cos 5hw + 19782h^4w^4Cos 7hw

78h 41170h w^4Cos hw 2 i 7hw 2 i 7210h^5w^4Cos

+ 54h 46965h^6w^4Cos h5hw i 2 i + 1233h^6w^4Cos h7hw i 2 i

6 56870h^7w^4Cos h5hw i 22 + 9752h^7w^4Cos h7hw i

60480h^3Cos h9hw i 22 + 3628800h^6w^3Cos h9hw i Sin[hw]

3628800h^6w^3Cos[hw]Sin h hw 9hw 7i 2 1

+ f +6Csc Csc[hw] –

116121600h^6w^4 n hw h 2 i 2

Cos h I

1272180h w hw 2 + 21 1230766hw 3hw 2 I 42

197h^4w^4 Cos 2 i 3586758h^7w^4Cos 2 I

745868h^7w^4 + 2h^3 423360 8933h^4w^4 Cos h3hw i 2

7 4Cos 5hw 24h^2 31192h^5w^4Cos 7hw

+ 9886770h w 9hw 2 i h 7hw 2 i h 2 i

+ 1460h w 9hw 2 i + 36h 4620h^5w^4Cos

66h w 6 4 Cos 2 6 7hw

7 4844h^7w^4 + 5h^3

18144 + 25h^4w^4 Cos h 2 i 9hw + 2786h^7w^4

10h^3 21168 + 293h^4w 3hw Cos 2 i 5hw

n 4

7hw i 9hw 2 i

+ 3h 5776596h^6w^4Cos + 5981580h^6w^4Cos

h 7hw h 9hw

+ 8h^2 75096h^4w^4Cos 2 1404h^4w^4Cos 2

7hw h 2 + 1460h 9hw h 2 i

18h 31192h^5w^4Cos 5w^4Cos

h 7hw 9hw i

+ 6 4620h^6w^4Cos 2 66h^6w^4Cos 2

3Cos 2 i + hw 2 i 9hw

+ 6350400h^5w^3 Cos Sin[hw]

6350400h^6w^3Cos[hw] 3Sin h 7 2 i + Sin h 2 i

$$\frac{1}{464486400h^{6}w^{4}}\left(f_{n}\left(\frac{hw}{2}\right)+7\operatorname{Csc}\left[\frac{hw}{2}\right]^{7}\operatorname{Csc}[hw]\right.$$

$$973308\left(\operatorname{Cosh}\left[\frac{hw}{2}\right]+6\left(132+5h^{4}w^{4}\operatorname{Cosh}\left[\frac{hw}{2}\right]+9510756h^{7}w^{4}\operatorname{Cos}\left[\frac{3hw}{2}\right]\right)\right.$$

$$1527756h^{7}w^{4}+14h^{3}\left(112320\right.3863h^{4}w^{4}\operatorname{Cos}\left[\frac{h3hw}{2}\right]$$

$$+24h^{3}\left(124200h^{4}w^{4}\operatorname{Cos}\left[\frac{h5hw}{2}\right]+19782h^{4}w^{4}\operatorname{Cos}\left[\frac{h7hw}{2}\right]\right.$$

$$378h^{4}w^{4}\operatorname{Cos}\left[\frac{h9hw}{22}\right]78h^{2}\left(103960h^{5}w^{4}\operatorname{Cos}\left[\frac{h5hw}{2}\right]\right.$$

$$7210h^{5}w^{4}\left(\operatorname{Cos}\left[\frac{7hw}{2}\right]+406h^{5}w^{4}\operatorname{Cos}\left[\frac{9hw}{2}\right]\right.$$

$$+54\left(56220h^{5}w^{4}\operatorname{Cos}\left[\frac{7hw}{2}\right]+1233\left(15h^{6}w^{4}\operatorname{Cos}\left[\frac{9hw}{2}\right]+6\left(126470h^{7}w^{4}\operatorname{Cos}\left[\frac{5hw}{2}\right]\right.\right.\right.$$

$$12827h^{7}w^{4}\left(15h^{3}\left(28224+205h^{4}w^{4}\operatorname{Cos}\left[\frac{h7hw}{2}\right]\right.\right.$$

$$811h^{7}w^{4}\left(5h^{3}\left(12096+169h^{4}w^{4}\operatorname{Cos}\left[\frac{h9hw}{2}\right]\right.\right.$$

$$+3628800h^{6}w^{3}\left(7\operatorname{Cos}\left[\frac{7hw}{2}\right]+\operatorname{Cos}\left[\frac{9hw}{2}\right]\right)\operatorname{Sin}[hw]$$

$$3628800h^{6}w^{3}\operatorname{Cos}[hw]\left(7\operatorname{Sin}\left[\frac{h}{2}\right]+\operatorname{Sin}\left[\frac{h}{2}\right]\right.$$

$$\frac{1}{232243200h^6w^4}\,f_{n+2}\,\frac{\operatorname{Csc}\left[\frac{hw}{2}\right]}{\operatorname{Csc}[hw]}$$

$$2336964h^7w^4\operatorname{Cos}\left[\frac{hw}{2}\right] + 21\left(532598hw\operatorname{Cos}\left[\frac{hw}{2}\right] + 6014946h^7w^4\operatorname{Cos}\left[\frac{3hw}{2}\right] + 42\left(251h^4w^4\operatorname{Cos}[hw] + 1455388h^7w^4 + 2h^3\left(544320 + 10723h^4w^4\operatorname{Cos}[3hw]\right)\right)\right)$$

$$+ 1880070h^7w^4\operatorname{Cos}\left[\frac{5hw}{2}\right] + 24h^2\left(219800h^5w^4\operatorname{Cos}\left[\frac{7hw}{2}\right] + 20788h^5w^4\operatorname{Cos}\left[\frac{9hw}{2}\right] + 36h\left(177324h^5w^4\operatorname{Cos}\left[\frac{7hw}{2}\right] + 23490h^6w^4\operatorname{Cos}\left[\frac{9hw}{2}\right]\right)\right)$$

$$288 + 19h^4w^4\operatorname{Cos}[7hw] + 21422h^7w^4 + 10h^3\left(41068h^7w^4 + 105h^3\left(63504 + 703h^4w^4\operatorname{Cos}\left[\frac{9hw}{2}\right] + 3h\left(9369108h^6w^4\operatorname{Cos}\left[\frac{3hw}{2}\right] + 1601100h^6w^4\operatorname{Cos}\left[\frac{9hw}{2}\right] + 8h^2\left(115416h^4w^4\operatorname{Cos}\left[\frac{7hw}{2}\right] + 12996h^4w^4\operatorname{Cos}\left[\frac{hw}{2}\right] + 18h\left(219800h^5w^4\operatorname{Cos}\left[\frac{7hw}{2}\right] + 20788h^5w^4\operatorname{Cos}\left[\frac{9hw}{2}\right] + 6\left(177324h^6w^4\operatorname{Cos}\left[\frac{7hw}{2}\right] + 23490h^6w^4\operatorname{Cos}\left[\frac{9hw}{2}\right]\right)\right)\right)\right)\right)$$

$$+ 12700800h^6w^3\operatorname{Cos}[7hw] + 3\operatorname{Cos}\left[\frac{9hw}{2}\right]\operatorname{Sin}[hw] + 3\operatorname{Sin}\left[\frac{9hw}{2}\right]\operatorname{Sin}\left[\frac{7hw}{2}\right]$$

$$\frac{1}{464486400h^6w^4}\,f_{n+5}\,\frac{\operatorname{Csc}\left[\frac{hw}{2}\right]}{\operatorname{Csc}[hw]}$$

$$3703140h^7w^4\operatorname{Cos}\left[\frac{hw}{2}\right] + 210\left(425938h^7w^4 + 576h^3\left(1092 + 17h^4w^4\operatorname{Cos}[hw] + 12029892h^7w^4\operatorname{Cos}\left[\frac{3hw}{2}\right]\right)\right)$$

$$84\left(1455388h^7w^4 + 2h^3\left(907200 + 10723h^4w^4\operatorname{Cos}[3hw] + 52581900h^7w^4\operatorname{Cos}[5hw]\right)\right)$$

h^2 $^5w^4$ $\frac{7hw}{}$

24 200550h 9hw Cos 2 i

+ 9050h^{5}w^{4}Cos $\frac{9hw}{2}$ i + 36h 29463h^{6}w^{4}Cos $\frac{7hw}{2}$ i

585 Cos 2 6 7hw + 5

h^6w^4 7 26331h$'$w^4 h^3

60480 + 779 2 i hw

h^4w^4 Cosh + 5 3241h$'$w^4

h^3 254016 + 3469h^4w^4 Cosh $\frac{9}{2}$ i

+ 3h 18738216h^{6}w^{4}Cos $\frac{h3hw\ i}{2}$

+ 25232040h^{6}w^{4}Cos $\frac{h5hw\ i}{2}$

8h^2 467082h^{4}w^{4}Cos $\frac{h7hw\ i}{2}$

9270h^4w^4Cos h $\frac{hw}{9}$ i

h 27 hw 18h

200550h^{5}w^{4}Cos h $\frac{7hw}{2}$ + 9050h^{5}w^{4}Cos h $\frac{9hw}{2}$ i

+ 6 29463h^6w^4Cos h $\frac{7hw}{2}$ 585h^6w^4Cos $\frac{9hw}{2}$ i

h^6w^3 3Cos Sin[hw]

+ 25401600 5Cos 2 + 7hw h 2 i 9hw i

25401600h^{5}w^{3}Cos[hw] 5Sin 7 2 + 3Sin h 2

h

464486400h^6w^4 f_{n+3}Csc 2 i Csc[hw]

3703140h^7w^4Cos $\frac{hhw\ i}{2}$ + 210 425938h^7w^4

+ 576h^3 1260 17h^4w^4 Cos $\frac{hhw\ i}{2}$

7173516h^7w^4Cos $\frac{h3hw\ i}{}$ 84 745868h^7w^4 2

+ 2h^3 1028160 8933h^4w^4 Cos $\frac{h3hw\ i}{2}$

+ 115202820h^{7}w^{4}Cos $\frac{h5hw\ i}{2}$ 24h^2

584934h^{5}w^{4}Cos $\frac{h7hw\ i}{2}$ + 48730h^{5}w^{4}Cos $\frac{h9hw\ i}{2}$

36h541065h6w4Cos h7
hw I
226 2

$$+83415h^{6}w^{4}\mathrm{Cos}\frac{h9hw\,i}{2}\quad 6\ 7\quad 36288\quad 28437h^{7}w^{4}+5h^{3}$$

$$1829h^{4}w^{4}\mathrm{Cos}\frac{h7hw\,i}{h9hw\,i}+5\ 19001h^{7}w^{4}\ 7h^{3}\ 60480^{2}\ h3hw\,i$$

$$773h^{4}w^{4}\ \mathrm{Cos}\frac{}{22}+3h\ 11553192h^{6}w^{4}\mathrm{Cos}\frac{}{}$$

$$+26735400h^{6}w^{4}\mathrm{Cos}\frac{h\,5hw\,i}{h9hw\,i}+8h^{2}\ 769482h^{4}w^{4}\mathrm{Cos}\frac{h\,7hw\,i}{h7hw\,i}$$

$$12150h^{4}w^{4}\mathrm{Cos}\frac{}{22}18h\ 584934h^{5}w^{4}\mathrm{Cos}\frac{}{}$$

$$+48730h^{5}w^{4}\mathrm{Cos}\frac{9hw}{h\ 9hw\ i}+6\ 541065h^{6}w^{4}\mathrm{Cos}\frac{7hw}{h\ 72hw\ i}$$

$$+83415\ h^{6}w^{4}\mathrm{Cos}\frac{h\ 2\ i}{2}+25401600h^{6}w^{3}\ 3\mathrm{Cos}\frac{h\ 2\ 7hw}{\pi}$$

$$+5\mathrm{Cos}\frac{9hw\ i}{2}\ \mathrm{Sin}[hw]\ \frac{1}{25401600h^{6}w^{3}\mathrm{Cos}[hw]}\ 3\mathrm{Sin}\frac{hw\ i}{2}$$

$$+5\mathrm{Sin}\frac{h\ 2\ i}{hw}\ \frac{}{928972800h^{6}w^{4}}\ f_{n+1}\mathrm{Csc}\frac{2\ i}{}\ \mathrm{Sec}\frac{h\ 2\ i}{}$$

$$14397180h^{7}w^{4}\mathrm{Cos}\frac{h\ i}{hhw\,i}+6\ 6691882h^{7}w^{4}\ 60480h^{3}\ h3hw\,i$$

$$68+3h^{4}w^{4}\ \mathrm{Cos}\frac{}{22}16501380h^{7}w^{4}\mathrm{Cos}\frac{}{}$$

$$12\ 667684h^{7}w^{4}+14h^{3}\ 164160\ 7393h^{4}w^{4}\ \mathrm{Cos}\ h3hw\,i\ 2\ \frac{}{}$$

$$+24h^{3}\ 233640h^{4}w^{4}\mathrm{Cos}\frac{5hw}{9hw\ h\ 2\ i}\ 378h^{4}w^{4}\mathrm{Cos}\frac{7hw}{9hw\ h\ 2\ i\ 5hw\,i}$$

$$+19782h\ w^{4}\ 78h^{2}\ 442520h^{5}w^{4}\mathrm{Cos}$$

$$+406h^{5}w^{4}\ \mathrm{Cos}\frac{}{2\ i}\ 5hw\ 7210h^{5}w^{4}\mathrm{Cos}\frac{h\ 2\ 7hw}{}$$

$$+54h\ 354780h^{6}w^{4}\mathrm{Cos}\frac{9hw\ h\ 2\ i}{h\ i}\ 15h^{6}w^{4}\mathrm{Cos}\frac{h\,5hw}{2}$$

$$+1233h^{6}w^{4}\mathrm{Cos}\frac{}{2}\ 6\ 603170h^{7}w^{4}\mathrm{Cos}\frac{hw\ i}{2}$$

$$+811h^{7}w^{4}\ 5h^{3}\ 12096+169h^{4}w^{4}\mathrm{Cos}\frac{7}{2}\ h\ 9\ hw\,i$$

$$+12827h^{7}w^{4}\ 15h^{3}\ 28224+205h^{4}w^{4}\ \mathrm{Cos}\frac{}{}$$

$$+3628800h^{6}w^{3}\ \mathrm{Cos}\frac{n}{2}\ i+\frac{hw}{7}\ 7\mathrm{Cos}\frac{}{}\ h\ 2\ 9hw\ \mathrm{Sin}[hw]$$

$$3628800h^{6}w^{3}\mathrm{Cos}[hw]\ \mathrm{Sin}\frac{h\ i}{2}+7\mathrm{Sin}\frac{h}{2\ i}$$

$$y_{n+2}^{000} = \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} - \frac{3y_{n+4}}{2h^3} + \frac{3y_{n+5}}{5h^3}$$

$$+ \frac{1}{46448640h^6w^4} f_{n+4}\mathrm{Csc}\left[\frac{hw}{2}\right]^8 \left(19051200h^3\right.$$

$$58842h^7w^4\mathrm{Cos}[hw] - 42 - 206810h^7w^4 + 420h^3 - 1152$$

$$+ 7h^4w^4\mathrm{Cos}[hw] + 6310920h^7w^4\mathrm{Cos}[2hw] + 1138590h^7w^4\mathrm{Cos}[3hw]$$

$$87696h^7w^4\mathrm{Cos}[4hw] - 12546h^7w^4 - 20h^3 - 5292$$

$+ 71h^4w^4 \quad Cos[4hw] + 3h \quad 44982h^6w^4Cos[hw]$

$+ 1915704h^6w^4Cos[2hw] + 1348326h^6w^4Cos[3hw]$

$43848h^6w^4Cos[4hw] \quad 12700800h^6w^3Sin[3hw]$

$+ \frac{1}{464486400h^{\circ}w^4}{}^{f}{}_{n+8}Csc \quad \frac{hw}{2} \quad {}^{7} \quad Csc[\quad hw \quad] \quad 177690 \quad h'w^{\bullet}Cos \quad hw \quad \frac{hw}{2}i$

$30 \quad 65507h^7w^4 + 3h^3 \quad 20160 + 569h^4w^4 \; Cos\,h \; \frac{}{2}i$

$2523150h^7w^4Cos\frac{h3hw\,i}{} \quad 54\;23559h^7w^4 + 35h^3\;1152\;2$

$35h^4w^4 \; Cos \; \frac{h3hw\,i}{2^2} + 24h^3 \; 22590h^4w^4Cos\frac{h5hw\,i}{}$

$378h^4w^4Cos \; \frac{h7hw\,i}{2^2} \; 78h^2 10130h^5w^4Cos \; \frac{h5hw\;i}{}$

$+ 406h^5w^4Cos \; h\frac{7hw}{2}i \quad + 54h \quad 1365h^6w^4Cos \; h\frac{5hw}{2}i$

$15h^6w^4Cos \; \frac{h7hw\,i}{2^2} \; 6\;10040h^7w^4Cos \; \frac{h5hw\,i}{}$

$811h^7w^4\;5h^3\;12096 + 169h^4w^4 \; Cos \; \frac{h7hw\,i}{2}$

$3628800h^6w^3Cos \; \frac{h7hw\,i}{2^2}Sin[hw] \quad 3628800h^6w^3Cos[hw]Sin \; \frac{h7hw\,i}{}$

$$+ \frac{1}{464486400h^{6}w^{4}} f_{n}\mathrm{Csc}\left[\frac{hw}{2}\right]^{7}\mathrm{Csc}[hw]\ 672390h^{7}w^{4}\mathrm{Cos}\left[\frac{hw}{2}\right]\ 30\ \ 21925h^{7}w^{4}$$

$$+ 9h^{3}12096 + 725h^{4}w^{4}\ \ \mathrm{Cos}\left[\frac{hw}{2}\right]\ 514134h^{7}w^{4}\mathrm{Cos}\left[\frac{3hw}{2}\right]\ 54$$

$$11573h^{7}w^{4} + h^{3}\ 67200\ \ 3591h^{4}w^{4}\ \ \mathrm{Cos}[h3hw] + 24h^{3}26370h^{4}w^{4}\mathrm{Cos}\left[\frac{h5hw}{22}\right]$$

$$+ 19782h^{4}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right]\ 78h^{2}\ \ 41170h^{5}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right]\ 7210h^{5}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$+ 54h\ 46965h^{5}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right] + 1233h\ 9hw\ \ h^{5}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right]\ 6\ 56870h^{7}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$+ 9752h^{7}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right]\ 60480h^{5}\mathrm{Cos}\left[\frac{hw}{2}\right]\ 3628800h^{6}w^{3}\mathrm{Cos}[hw]\mathrm{Sin}\left[\frac{hw}{2}\right]\ 1 + 116121600h^{6}w^{4}f_{n+6}\mathrm{Csc}\left[\frac{hw}{2}\right]\mathrm{Csc}[hw]\ \frac{1}{2}$$

$$1272180h^{7}w^{4}\mathrm{Cos}\left[\frac{hhw}{2}\right] + 21\ 1230766h^{7}w^{4} + 180h^{3}\ \ 8064\ \ 197h^{4}w^{4}\ \ \mathrm{Cos}[h3hw]\left[\frac{hhw}{2}\right]$$

$$3586758h^{7}w^{4}\mathrm{Cos}[h3hw]\ 42745868h^{7}w^{4} + 2h^{3}\ \ 423360\ \ 8933h^{4}w^{4}\ \ \mathrm{Cos}\left[\frac{h5hw}{22}\right]$$

$$+ 9886770h^{7}w^{4}\mathrm{Cos}\left[\frac{h5hw}{7hw}\right]\ 24h^{2}\ \ 31192h^{5}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right] + 1460h^{5}w^{4}\mathrm{Cos}\left[\frac{h9hw}{2}\right]$$

$$+ 36h^{6}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right]\ 4620h\ \ 66h^{6}w^{4}\mathrm{Cos}\left[\frac{h}{2}\right]\ 6\ 7\ \ 4844h^{7}w^{4} + 5h^{3}\ \ 18144$$

$$+ 25h^{4}w^{4}\mathrm{Cos}\left[\frac{h}{2}\right] + 3hw\ \ 2786h^{7}w^{4}\ \ 10h^{3}\ \ 21168 + 293h^{4}w^{4}\ \ \mathrm{Cos}\left[\frac{h}{2}\right]$$

$$+ 3h\ 5776596h^{6}w^{4}\mathrm{Cos}\left[\frac{h}{2}\right] + 5hw\ \ 5981580h^{6}w^{4}\mathrm{Cos}\left[\frac{h}{2}\right] + 7hw\ \ 8h^{2}\ \ 75096h^{4}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$1404h^{4}w^{4}\mathrm{Cos}\left[\frac{h}{2}\right]\ 7hw\ \ 18h\ \ 31192h^{5}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right] + 1460h^{5}w^{4}\mathrm{Cos}\left[\frac{h}{2}\right]\ 7hw$$

$$+ 6\ 4620h^{5}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right]\ 66h^{5}w^{4}\mathrm{Cos}\left[\frac{h}{2}\right] + 7hw\ \ 6350400h^{5}w^{3}\ 3\mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$+ \mathrm{Cos}\left[\frac{h}{2}\right]\ 1\ \ \mathrm{Sin}[hw]\ 6350400h^{5}w^{3}\mathrm{Cos}[hw]\ 3\mathrm{Sin}\left[\frac{hw}{2}\right] + \mathrm{Sin}\left[\frac{hw}{2}\right]$$

$$464486400h^{6}w^{4}f_{n+7}\mathrm{Csc}\left[\frac{h}{2}\right]\mathrm{Csc}[hw]\ \ 973308h^{7}w^{4}\mathrm{Cos}\left[\frac{h}{2}\right]$$

$$+9510756h^7w^4\mathrm{Cos}\frac{h3hw\,i}{2} \quad 12 \quad 1527756h^7w^4$$

$$14h^3\ 112320\ 3863h^4w^4\ \mathrm{Cos}\frac{h3hw\,i}{2}$$

$$24h^3\ 124200h^4w^4\mathrm{Cos}\frac{h5hw\,i}{22} + 19782h^4w^4\mathrm{Cos}\frac{h7hw\,i}{}$$

$$378h^4w^4\mathrm{Cos}\frac{h9hw\,i}{22}78h^2 \quad 103960h^5w^4\mathrm{Cos}\frac{h5hw\,i}{}$$

$$h^5w^4 \quad \mathrm{Cos}\frac{7hw}{2} + 406h^5w^4\mathrm{Cos}\frac{9hw}{}$$

$$7210 \quad h^5w^4\mathrm{Cos} \quad h\ 2\ 7hw$$

$$+ 54 \quad 56220 \quad h\ 2\ i \quad + 1233h^5w^4\mathrm{Cos}$$

$$15h^6w^4\mathrm{Cos}\frac{9hw}{2}\ i\ 6 \quad 126470h^7w^4\mathrm{Cos}\frac{hw^2\,i}{2}$$

$$12827h^7w^4\ 15h^3\ 28224 + 205h^4w^4\ \mathrm{Cos}\frac{h7hw\,i}{2}$$

$$811h^7w^4\ 5h^3\ 12096 + 169h^4w^4\ \mathrm{Cos}\frac{h9hw\,i}{2}$$

$$h^6w^3 \quad 7\mathrm{Cos}\frac{7hw}{2} \quad \mathrm{Cos}\frac{9hw}{} \quad \mathrm{Sin}[hw]$$

$$+ 3628800 \quad 3628800h^5w^5\mathrm{Cos}[hw]\ 7\mathrm{Sin} \quad + \mathrm{Sin}\frac{9hw}{2}$$

$$+ \frac{}{232243200h^5w^4}\ f_{n+2}\mathrm{Csc}\frac{h}{2}\ i\ \mathrm{Csc}[hw]$$

$$2336964h^7w^4\mathrm{Cos}\frac{hhw}{2}\ i + 21\ 532598h^7w^4$$

$$+ 180h^3\ 10752\ 251h^4w^4\ \mathrm{Cos}\frac{hhw}{2}\ i$$

$$6014946h^7w^4\mathrm{Cos}\frac{h\ 3hw}{2}\ 42$$

$$1455388h^7w^4 + 2h^3\ 544320$$

$$10723h^4w^4\ \mathrm{Cos}\frac{h3hw\,i}{22} + 1880070h^7w^4\mathrm{Cos}\frac{h5hw\,i}{}$$

$$24h^2 219800h^5w^4\mathrm{Cos}\frac{h7hw\,i}{2}$$

$$+20788h^5w^4\cos\left(h\frac{9hw}{2}i\right) + 36h\ 177324h^6w^4\cos\left(h\frac{7hw}{2}i\right)$$

$$23490h^6w^4\cos\left(h9hwi\right) \quad 6\ 7\ 41068h^7w^4 +$$

$$105h^3\ 2$$

$$288 + 19h^4w^4\cos\left(h7hwi\right)$$

$$21422h^7w^4\ 2$$

$$10h^3\ 63504 + 703h^4w^4\cos\left(h9hwi\right)$$

$$3h\ 9369108h^6w^4\cos\left(\frac{h3hwi}{22}\right) + 1601100h^6w^4\cos\left(h5hwi\right)$$

$$+ 8h^2\ 115416h^4w^4\cos\left(\frac{7hw}{}\right) + 12996h^4w^4\cos\left(\frac{9hw}{}\right)$$

$$h \qquad h^9w^4\cos\left(h\ 2\frac{7hw}{}\right) + 20788h^9w^4\cos\left(h\ 2\frac{9hw}{}\right)$$

$$18\ \ 219800 \qquad \frac{7hw}{2}\ i \qquad \frac{9hw}{2}\ i$$

$$+ 6\ \ 177324h^9w^4\cos \qquad 23490h^9w^4\cos$$

$$+ 12700800h^9w^9 \quad \cos\left(h\frac{7hw}{}\right) \quad 3\cos\left(\frac{9hw}{}\right) \quad \sin[hw]\ \ h\ 2\ i$$

$$12700800h^9w^9\cos[hw]\sin\left(h\ 2\right) + 7hw\left(h\ 2\ i\right) + 3\sin\left(\frac{9hw}{}\right)$$

$$\frac{1}{464486400h^6w^4}\ f_{n+5}\csc\left(\frac{hw}{}\right)\csc[hw]\ \ h\ 2\ i$$

$$3703140h^7w^4\cos\left(\frac{hw}{n}\right) + 210\ \ 425938h^7w^4 \quad + 576h^9 \quad 1092$$

$$17h^4w^4\ \cos\left(\frac{hw}{2}\right) \quad 12029892h^7w^4\cos\left(\frac{hw}{2}\right)i$$

$$84\ \ 1455388h^7w^4 + 2h^3\ \ 907200$$

$$10723h^4w^4\ \cos\left(\frac{h3hwi}{22}\right) + 52581900h^7w^4\cos\left(h5hwi\right)$$

$$24h^2 200550h^5w^4\cos\left(h7hw\ i\ 2\right)$$

$$9050h^5w^4\mathrm{Cos}\frac{h9hw\ i}{2} + 36h\ 29463h^6w^4\mathrm{Cos}\frac{h7hw\ i}{2}$$

$$585h^6w^4\mathrm{Cos}\frac{h9hw\ i}{\ } \quad 6\ 7\ 26331h^7w^4$$

$$+ 5h^3\ \ 2$$

$$60480 + 779h^4 + w^4\ \ \mathrm{Cos}\frac{h7hw\ i}{\ } + 5\ 3241h^7w^4\ \ 2$$

$$h^3\ \ 254016 + 3469h^4w^4\ \ \mathrm{Cos}\frac{h9hw\ i}{2}$$

$$+ 3h\ \ 18738216h^6w^4\mathrm{Cos}\frac{h3hw\ i}{2}$$

$$+ 25232040h^6w^4\mathrm{Cos}\frac{h5hw\ i}{2}$$

$$8h^2\ 467082h^4w^4\mathrm{Cos}\frac{h7hw\ i}{2}$$

$$9270h^4w^4\mathrm{Cos}\frac{h\ \frac{hw}{9}\ \ i}{2}\ \ 18h$$

$$200550h^5w^4\mathrm{Cos}\frac{h\ \frac{7hw}{7hw}}{2} + 9050h^5w^4\mathrm{Cos}\frac{h\ \frac{9hw}{9hw}\ \ i}{2}$$

$$+ 6\ \ 29463h^6w^4\mathrm{Cos}\frac{h\ \ 7hw}{2}\ \ 585h^6w^4\mathrm{Cos}\frac{9hw\ \ i}{2}$$

$$+ 25401600\ \ \frac{h^6w^3}{5\mathrm{Cos}\ \ 2}\ \ \frac{3\mathrm{Cos}}{\ }\ \ \mathrm{Sin}[hw]$$

$$25401600h^5w^5\mathrm{Cos}[hw]\ 5\mathrm{Sin}\ \ \frac{7hw}{7\ 2}\ \ \frac{h\ \ 2\ \ i}{h}\ \ \frac{9hw}{2}\ \ i + 3\mathrm{Sin}$$

$$464486400h^5w^4\ f_{n+3}\mathrm{Csc}\frac{h}{2\ i}\ \ \mathrm{Csc}[hw]$$

$$3703140h^7w^4\mathrm{Cos}\frac{hhw\ \ i}{2} + 210\ 425938h^7w^4$$

$$+ 576h^3\ 1260\ 17h^4w^4\ \mathrm{Cos}\frac{hhw\ \ i}{2}$$

$$7173516h^7w^4\mathrm{Cos}\frac{h3hw\ i}{\ }\ \ 84\ \ 745868h^7w^4\ 2$$

$$+ 2h^3\ \ 1028160\ 8933h^4w^4\ \ \mathrm{Cos}\frac{h3hw\ i}{2}$$

$$+ 115202820h^7w^4\mathrm{Cos}\frac{h5hw\ i}{2}$$

$$24h^2 584934h^5w^4\mathrm{Cos}\frac{h7hw\ i}{\ }\ \ 2$$

$$+ 48730h^5w^4\mathrm{Cos}\frac{9hw}{h\ \ 7hw} + 36h$$

$$h^5w^4\ \mathrm{Cos}\frac{h}{\ } + 83415h^5w^4\mathrm{Cos}\ \ \frac{h\ \ 9hw}{\ }\ \ i$$

541065

i
232

7 28437h⁷w⁴ + 5h³ 36288 + 1829h⁴w⁴ Cos h7hw i + 5 19001h⁷w⁴ 2

7h³ 60480 + 773h⁴w⁴ Cos h9hw i + 3h 11553192h⁶w⁴Cos h3hw i

22 22

+ 26735400h⁶w⁴Cos 5hw + 8h² 769482h⁴w⁴Cos h 7hw

9hw 2 i 2 7hw i

h 9hw 7hw i

12150h⁴w⁴Cos 2 18h 584934h⁵w⁴Cos 2

h hw i h hw

+ 48730h⁵w⁴Cos 2 + 6 541065h⁶w⁴Cos 2

h 9 i h 7

+ 83415h⁶w⁴Cos 2 + 25401600h⁶w³ 3Cos 2 i

9hw 7hw y_n

+ 5Cos 2 Sin[hw] 25401600h⁶w³Cos[hw] 3Sin 8² i 10h³

y_{n+3} 3y_{n+4} 3y_{n+5} 1 hw

+ h³ 2h³ + 5h³ + 46448640h⁶w⁴ f_{n+4}Csc h 2 i19051200h³

866880h⁷w⁴Cos[hw] 42 244936h⁷w⁴ + 420h³ 1152

+ 112h⁴w⁴ Cos[hw] 1024464h⁷w⁴Cos[2hw] 44496h⁷w⁴Cos[3hw]

50904h⁷w⁴Cos[4hw] 12 20594h⁷w⁴ 20h³ 5292

+ 1136h⁴w⁴ Cos[4hw] + 2h 1161888h⁶w⁴Cos[hw] + 2192064h⁶w⁴Cos[2hw]

h⁶w⁴ hw 20832h⁶w⁴Cos[4hw] 12700800h⁶w³Sin[2hw]

199584 1 hw 7 hw

+ 464486400h⁶w⁴ f_{n+8}Csc 2 Csc[ ] 978570 h′w⁴Cos hw 2 i

30 82330h⁷w⁴ + 3h³ 20160 + 9104h⁴w⁴ Cos h 2 i

h3hw i

429030h⁷w⁴Cos 54 5926h⁷w⁴ + 35h³ 1152 2

h3hw i h5hw i

560h⁴w⁴ Cos 22 + 6h³ 4680h⁴w⁴Cos 22

h 7hw i hw hw 5hw i h 7hw

468h⁴w⁴Cos 2 33h² 4770h⁵w⁴Cos 2 450h⁵w⁴Cos 2

5 i h 7 i 6 h 5 i

+ 36h 240h⁶w⁴Cos h 2 42h⁶w⁴Cos 2 9580h⁷w⁴Cos 2

h7hw i

13536h⁷w⁴ 5h³ 12096 + 2704h⁴w⁴ Cos 2

h7hw i h7hw i

3628800h⁶w³Cos 22 Sin[2hw] 3628800h⁶w³Cos[2hw]Sin 22

$$\frac{1}{464486400h^{6}w^{4}}\ h\frac{hw}{2}i^{7}\qquad \frac{hw}{2}i$$

$$30\quad 115238h^{7}w^{4}+9h^{3}\qquad 12096+11600h^{4}w^{4}\mathrm{Cos}\,h\frac{}{2}i$$

$$2208978h^{7}w^{4}\mathrm{Cos}\frac{h3hw\,i}{}\ 54\ 48110h^{7}w^{4}+h^{3}\ 67200\ 2\ h5hw\,i$$

$$57456h^{4}w^{4}\ \mathrm{Cos}\frac{}{22}+6h^{3}\ 32040h^{4}w^{4}\mathrm{Cos}$$

$$+972h^{4}w^{4}\mathrm{Cos}\frac{7hw}{}\qquad 33h^{2}\qquad 29570h^{5}w^{4}\mathrm{Cos}\frac{5hw}{2}i$$

$$+1234h^{5}w^{4}\mathrm{Cos}\frac{}{}\ +36h\ 6840h^{6}w^{4}\mathrm{Cos}\frac{}{}\qquad h^{6}w^{4}\mathrm{Cos}\frac{}{}\qquad 9hw\frac{7hw}{2}i$$

$$6\ \ 82660h^{7}w^{4}\mathrm{Cos}\frac{}{2}+9592h^{7}w^{4}\mathrm{Cos}\frac{}{}\qquad 60480h^{3}\mathrm{Cos}\frac{}{2}$$

$$+3628800h^{6}w^{3}\mathrm{Cos}\frac{}{}\ \mathrm{Sin}[2hw]\qquad 3628800h^{6}w^{3}\mathrm{Cos}[2hw]\mathrm{Sin}\frac{}{}$$

$$\frac{}{116121600h^{6}w^{4}}f\qquad \mathrm{Csc}\frac{}{2}\qquad \mathrm{Csc}[hw]\ \frac{}{2}\ 1951488h\frac{hw}{2}\ \mathrm{Cos}\frac{}{2}i$$

$$h^{7}w^{4}\quad 180h^{3}\qquad h^{4}w^{4}$$

$$14700672h^{7}w^{4}\mathrm{Cos}\frac{}{2}\qquad 42\qquad 192131h^{7}w^{4}+2h^{3}\ 423360\ 5hw$$

$$142928h^{4}w^{4}\ \mathrm{Cos}\frac{}{}\qquad h^{7}w^{4}\mathrm{Cos}\frac{}{2}i$$

$$9\qquad h^{2}\ 11970h^{5}w^{4}\mathrm{Cos}\frac{}{}\qquad 1768\qquad h^{5}w^{4}\mathrm{Cos}\frac{}{}$$

$$+24h\ \ 399h^{6}w^{4}\mathrm{Cos}\frac{h7}{2}i\ 225h^{6}w^{4}\mathrm{Cos}\frac{h9}{2}i$$

$$7\ 2814h^{7}w^{4}+5h^{3}18144+400h^{4}w^{4}\ \mathrm{Cos}\frac{h7hw\,i}{2}$$

$$+ 46886h^{7}w^{4} \quad 10h^{3} \quad 21168 + 4688h^{4}w^{4} \quad \cos\frac{9hw}{5hw}$$

$$+ 2h\ 7643328h^{6}w^{4}\cos\frac{hhw\ i}{2} + 1569600h^{6}w^{4}\cos\frac{hw''}{2}$$

$$+ 3h^{2} \quad 11466h^{4}w^{4}\cos\frac{h\frac{7}{2}i}{h7hwi} \quad 1854h^{4}w^{4}\cos\frac{h\frac{9}{2}i}{h9hwi}$$

$$12h\ 11970h^{5}w^{4}\cos\frac{h7hwi}{222} \quad 1768h^{5}w^{4}\cos\frac{h9hwi}{222} + 6\ 399h^{6}w^{4}\cos\frac{h7hwi}{222}$$

$$225h^{6}w^{4}\cos\frac{h9hwi}{22} + 6350400h^{6}w^{3} \quad 3\cos\frac{h7hw\ i}{22}$$

$$+ \cos\frac{h\ 2_1'}{9hw} \quad \sin[2hw] \quad 6350400h^{7}w^{3}\cos[2hw] \quad 3\sin\frac{h\ 2\ 'hw}{7hw} + \sin\frac{h\ 2\ i}{9hw}$$

$$464486400h^{6}w^{4}\frac{f \quad \csc\frac{i}{h\ 2}}{n+7\ 3hw} \quad \csc[\quad] \quad 1432704\frac{hw}{h\ hw''\ i\ 2\ i}$$

$$+ 6\ 4481792h^{7}w^{4} \quad 20160h^{3} \quad 132 + 80h^{4}w^{4} \quad \cos\frac{h\ 3hw'}{2}$$

$$391680h^{7}w^{4}\cos\frac{h\ 3hw'}{hw''\ 2\ i} \quad 12 \quad 50119h^{7}w^{4} + 14h^{3} \quad 112320$$

$$61808h^{4}w^{4} \quad \cos\frac{5hw}{hw\ h\ 2\ i} + 6h^{3}\ 55440h^{4}w^{4}\cos\frac{5hw}{h\ 2\ i}$$

$$+ 972h^{4}w^{4}\cos\frac{h\frac{7}{2hw}i}{''\frac{7}{2}'} \quad 468h^{4}w^{4}\cos\frac{h\frac{9}{2hw}i}{''\frac{9}{2}'} \quad 33h^{2}\ 5180h^{5}w^{4}\cos\frac{h}{hw\ 2\ i}$$

$$+ 1234h^{5}w^{4}\cos\frac{h7hwi}{222} \quad 450h^{5}w^{4}\cos\frac{h9hwi}{222} \quad + 36h \quad 7680h^{6}w^{4}\cos\frac{h5hw\ i}{222}$$

$$210h^{6}w^{4}\cos\frac{h7hwi}{222} \quad 42h^{6}w^{4}\cos\frac{h9hwi}{222} \quad 6\ 150040h^{7}w^{4}\cos\frac{h7hwi}{222}$$

$$58792h^{7}w^{4}\ 15h^{3}\ 28224 + 3280h^{4}w^{4}\cos\frac{h7hwi}{h9hwi} + 13536h^{7}w^{4}\ 2$$

$$5h^{3}\ 12096 + 2704h^{4}w^{4}\ \cos\frac{h7hwi}{22} + 3628800h^{6}w^{3}\ 7\cos\frac{h7hwi}{22}$$

$$+ \cos\frac{h\frac{9hw}{2}i}{} \quad \sin[2hw] \quad 3628800h^{6}w^{3}\cos[2hw]\ 7\sin\frac{h\frac{7hw}{2}i}{} + \sin\frac{h\frac{9hw}{2}i}{}$$

$$\frac{1}{232243200h^{6}w^{4}}\Big[\frac{hw}{2}\Big]^{7}\,\mathrm{Csc}\Big[\frac{hw}{2}\Big]\,10090752\,h^{7}w^{4}\mathrm{Cos}\Big[\frac{hw}{2}\Big]$$

$$h^{7}w^{4}\;180h^{3}\;h^{4}w^{4}$$

$$25667712h^{7}w^{4}\mathrm{Cos}\Big[\frac{3hw}{2}\Big]\;42\;653917h^{7}w^{4}+2h^{3}\;544320$$

$$171568h^{4}w^{4}\;\mathrm{Cos}\Big[\frac{7hw}{2}\Big]\;334650h^{7}w^{4}\mathrm{Cos}\Big[\frac{9hw}{2}\Big]$$

$$90538\,h^{5}w^{4}\mathrm{Cos}\Big[\frac{7hw}{2}\Big]+14560h^{5}w^{4}\mathrm{Cos}\Big[\frac{7hw}{2}\Big]$$

$$21735h^{6}w^{4}\mathrm{Cos}\Big[\frac{9hw}{2}\Big]\;3441h^{6}w^{4}\mathrm{Cos}\Big[\frac{9hw}{2}\Big]\;23798h^{7}w^{4}$$

$$304h^{4}w^{4}\;\mathrm{Cos}\Big[\frac{3hw}{2}\Big]\;71158h^{7}w^{4}+10h^{5}\;63504$$

$$11248h^{4}w^{4}\;\mathrm{Cos}\Big[\frac{9hw}{2}\Big]+2h\;14540736h^{6}w^{4}\mathrm{Cos}\Big[\frac{7hw}{2}\Big]$$

$$137280h^{6}w^{4}\mathrm{Cos}\Big[\frac{9hw}{2}\Big]+3h^{2}\;122346h^{4}w^{4}\mathrm{Cos}\Big[\frac{7hw}{2}\Big]$$

$$16254h^{4}w^{4}\mathrm{Cos}\Big[\frac{9hw}{2}\Big]\;12h\;90538h^{5}w^{4}\mathrm{Cos}\Big[\frac{9hw}{2}\Big]$$

$$14560\,h^{5}w^{4}\;\mathrm{Cos}\Big[\frac{hw}{2}\Big]+6\;21735h^{6}w^{4}\mathrm{Cos}\Big[\frac{hw}{2}\Big]\;3441h^{6}w^{4}\mathrm{Cos}\Big[\frac{hw}{2}\Big]$$

$$12700800h^{6}w^{3}\;\mathrm{Cos}\Big[\frac{7hw}{2}\Big]+3\mathrm{Cos}\Big[\frac{9hw}{2}\Big]\mathrm{Sin}[2hw]$$

$$12700800h^{6}w^{3}\mathrm{Cos}[2hw]\;\mathrm{Sin}\Big[\frac{7hw}{2}\Big]+3\mathrm{Sin}\Big[\frac{9hw}{2}\Big]$$

$$\frac{1}{464486400h^{6}w^{4}}\Big[\frac{hw}{2}\Big]^{7}\mathrm{Csc}[hw]\,5900160h^{7}w^{4}\,\mathrm{Cos}\Big[\frac{hw}{2}\Big]$$

$$210\;593744h^{7}w^{4}+576h^{3}\;1092\;272h^{4}w^{4}\mathrm{Cos}\Big[\frac{hw}{2}\Big]$$

$$51335424h^{7}w^{4}\mathrm{Cos}\Big[\frac{3hw}{2}\Big]\;84\;653917h^{7}w^{4}$$

$$2h^{3}\;907200\;171568h^{4}w^{4}\mathrm{Cos}\Big[\frac{3hw}{2}\Big]$$

$$3437220h^{7}w^{4}\mathrm{Cos}\Big[\frac{5hw}{2}\Big]\;9h^{2}\;99274h^{5}w^{4}\mathrm{Cos}\Big[\frac{7hw}{2}\Big]$$

$$12730h^{5}w^{4}\mathrm{Cos}\Big[\frac{9hw}{2}\Big]+24h\;10290h^{6}w^{4}\mathrm{Cos}\Big[\frac{7hw}{2}\Big]$$

$$2310h^{6}w^{4}\cos\frac{h\,9hw}{2} \quad 6 \quad 7\ 32864h^{7}w^{4} + 5h^{5} \quad 60480 \quad 9hw$$

$$+ 12464h^{4}w^{4}\cos\frac{h\,2\,3hw}{} + 5\ 55384h^{7}w^{4}\ h^{5}\ 254016 + 55504h^{4}w^{4}\cos\frac{h\,2\,7hw}{5hw}$$

$$+ 2h\ 29081472h^{6}w^{4}\cos\frac{9hw\ h\ 2\ i}{} + 10838400h^{6}w^{4}\cos\frac{7hw\ 2}{} + 3h^{2}\ 98532h^{4}w^{4}\cos\frac{9hw\ h\ 2\ i}{}$$

$$13500h^{4}w^{4}\cos\frac{7hw}{5} \quad 99274h^{4}w^{4}\cos\frac{9hw\ 2}{5} \quad 12730h^{4}w^{4}\cos\frac{2\ 7hw}{}$$

$$+ 6\ 10290h^{6}w^{4}\cos\frac{9hw\ h\ 2\ i}{} \quad 2310h^{6}w^{4}\cos\frac{h\ 2\ i}{} \quad + 25401600h^{6}w^{5}\ 5\cos\frac{7hw\ 9hw\ 2\ i}{}$$

$$+ 3\cos\frac{h\ 1\ i}{2}\ \sin[2hw] \quad 25401600h^{6}w^{5}\cos[2hw]\ 5\sin\frac{h\ hw}{2} + 3\sin\frac{h\ i}{2}$$

$$464486400h^{6}w^{4}\ f_{n+3}\csc\frac{h}{2}\ \csc[hw]\ 5900160h^{7}w^{4}\cos\frac{3hw\ i}{2} + 210\ 593744h^{7}w^{4}$$

$$+ 576h^{3}\ 1260\ 272h^{4}w^{4}\cos\frac{h\ i}{2}\ 29401344h^{7}w^{4}\cos\frac{h\ i}{2}\ 84$$

$$192131h^{7}w^{4} + 2h^{3}\ 1028160\ 142928h^{4}w^{4}\cos\frac{h3hw\ i}{22} + 28875300h^{7}w^{4}\cos\frac{h5hw\ i}{}$$

$$9h^{2}\ 124390h^{5}w^{4}\cos\frac{h7hw\ i}{22} \quad 12950h^{5}w^{4}\cos\frac{h9hw\ i}{} + 24h752262h^{6}w^{4}\cos\frac{h7hw\ i}{}$$

$$106290h^{6}w^{4}\cos\frac{h9hw\ i}{} \quad 6\ 7\ 266552h^{7}w^{4} + 5h^{3}$$

$$36288\ 2$$

$$29264h^{4}w^{4}\cos\frac{h7hw\ i}{22} + 5\ 120224h^{7}w^{4}\ 7h^{3}\ 60480 + 12368h^{4}w^{4}\cos\frac{h9hw\ i}{}$$

$$w^{4}\cos\frac{3hw}{} + 27680640h^{6}w^{4}\cos\frac{5hw}{n} + 3h^{2}\ 552132h^{4}w^{4}\cos\frac{7hw}{}$$

$$+ 2h\ 15286656h\ \frac{9hw\ h\ 2\ i}{2\ i}\ 75420h^{4}w^{4}\cos \quad 12h\ 124390h^{5}w^{4}\cos\frac{7hw\ 2\ i}{2\ i}\ 12950h^{5}w^{4}\cos\frac{9hw\ h\ 2\ i}{2\ i} + 6$$

$$752262h^{6}w^{4}\cos\frac{h7hw\ i}{22} + 106290h^{6}w^{4}\cos\frac{h9hw\ i}{} + 25401600h^{6}w^{3}\ 3\cos\frac{h7hw\ i}{}$$

$$+ 5\cos\frac{h\ 9hw\ i}{2}\ \sin[2hw]\ 25401600h^{6}w^{3}\cos[2hw]\ 3\sin\frac{h\ 7hw\ i}{2} + 5\sin\frac{h\ 9hw}{2\ i}$$

$$\frac{1}{928972800\, h^{6} w^{4}} f_{n+1} \operatorname{Csc}\left[\frac{h w}{2 i}\right]^{8} \operatorname{Sech}\left[\frac{h w}{2 i}\right]$$

$$8717184 h^{7} w^{4} \operatorname{Cos}\left[\frac{h h w}{2 i}\right] + 6\, 4673696 h^{7} w^{4}$$

$$60480 h^{3}\, 68 + 48 h^{4} w^{4} \operatorname{Cos}\left[\frac{h h w}{2 i}\right]$$

$$13429248 h^{7} w^{4} \operatorname{Cos}\left[\frac{h 3 h w}{2 i}\right] 12\, 861543 h^{7} w^{4}$$

$$14 h^{3}\, 164160\, 118288 h^{4} w^{4} \operatorname{Cos}\left[\frac{h 3 h w}{2 i}\right]$$

$$6 h^{3}\, 248400 h^{4} w^{4} \operatorname{Cos}\left[\frac{h 5 h w}{2 i}\right] 468 h^{4} w^{4} \operatorname{Cos}\left[\frac{h 7 h w}{2 i}\right]$$

$$+ 972 h^{4} w^{4} \operatorname{Cos}\left[\frac{9 h w}{2 i}\right] 33 h^{2} 177540 h^{5} w^{4} \operatorname{Cos}\left[\frac{5 h w}{2 i}\right]$$

$$450 h^{5} w^{4} \operatorname{Cos}\left[\frac{7 h w}{2 i}\right] + 5 h w\, 1234 h^{5} w^{4} \operatorname{Cos}\left[\frac{9 h w}{2 i}\right] 7 h w$$

$$h\, h^{6} w^{4} \operatorname{Cos}\left[\frac{\ }{\ }\right] 42 h^{6} w^{4} \operatorname{Cos}\left[\frac{\ }{\ }\right]$$

$$+ 36\, 43920\, h w\, h\, 2 i\, h\, 2 h w$$

$$210 h^{6} w^{4} \operatorname{Cos}\left[\frac{9}{2 i}\right] 6\, 780040 h^{7} w^{4} \operatorname{Cos}\left[\frac{5}{2 i}\right]$$

$$13536 h^{7} w^{4}\, 5 h^{3}\, 12096 + 2704 h^{4} w^{4} \operatorname{Cos}\left[\frac{h 7 h w}{2 i}\right]$$

$$58792 h^{7} w^{4}\, 15 h^{3}\, 28224 + 3280 h^{4} w^{4} \operatorname{Cos}\left[\frac{h 9 h w}{2 i}\right]$$

$$+ 3628800\, h^{6} w^{3} \operatorname{Cos}\left[\frac{h 7 h w}{2 i}\right] 7 \operatorname{Cos}\left[\frac{9 h w}{2 i}\right] \operatorname{Sin}[2 h w]$$

$$3628800 h^{6} w^{3} \operatorname{Cos}[2 h w] \operatorname{Sin}\left[\frac{h}{2 i}\right] + 7 \operatorname{Sin}\left[\frac{h}{2 i}\right]$$

$$y_{n+3}^{\prime\prime\prime} = \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} - \frac{3y_{n+4}}{2h^3} + \frac{3y_{n+5}}{5h^3} + \frac{1}{46448640h^6w^4} f_{n+4}\operatorname{Csch}\left[\frac{hw}{2}\right] \left(19051200h^5 - 58842h^7w^4\operatorname{Cos}[hw] - 42 - 206810h^7w^4 + 420h^3 - 1152 + 7h^4w^4 - 7\operatorname{Cos}[hw] + 6310920h^7w^4\operatorname{Cos}[2hw] + 1138590h^3w^4\operatorname{Cos}[3hw] - 87696h^6w^4\operatorname{Cos}[4hw] - 12 - 546h^6w^4 - 20h^3 - 5292 + 71h^6w^4\operatorname{Cos}[4hw] + 3h^6w^4\operatorname{Cos}[hw] + 44982h^6w^4\operatorname{Cos}[hw] + 1915704h^6w^4\operatorname{Cos}[2hw] + 1348326h^6w^3\operatorname{Cos}[3hw] - 43848h^7\operatorname{Cos}[4hw] - 12700800h^6w^3\operatorname{Sin}[3hw] + \frac{1}{464486400h^6w^4} f_{n+8}\operatorname{Csch}\left[\frac{hw}{2}\right]\left(177690h^7w^4\operatorname{Cos}[hw] - 30 - 65507h^7w^4 + 3h^3 - 20160 + 569h^4w^4\operatorname{Cos}\left[\frac{h(3hw)}{2}\right] + 2523150h^7w^4\operatorname{Cosh}\left[\frac{h(5hw)}{2}\right] - 54 - 23559h^7w^4 + 35h^3 - 1152 - 35h^4w^4\operatorname{Cos}\left[\frac{h(7hw)}{2}\right] + 24h^3 - 22590h^4w^4\operatorname{Cos}\left[\frac{h(5hw)}{2}\right] - 378h^4w^4\operatorname{Cos}\left[\frac{h(7hw)}{2}\right] - 78h^2 - 10130h^5w^4\operatorname{Cos}\left[\frac{h(5hw)}{2}\right] + 54h^5 - 1365h^6w^4\operatorname{Cos}\left[\frac{5hw}{2}\right] - 15h^6w^4\operatorname{Cos}\left[\frac{7hw}{2}\right] + 406h^6w^4\operatorname{Cos}\left[\frac{7hw}{2}\right] - 10040h^7w^4\operatorname{Cos}\left[\frac{5hw}{2}\right] + 811h^5w^4 - 12096 + 169h^4w^4\operatorname{Cos}\left[\frac{7hw}{2}\right] + 3628800h^6w^3\operatorname{Cos}[hw]\operatorname{Sin}\left[\frac{h(hw)}{2}\right] + 3628800h^6w^3\operatorname{Cos}[hw]\operatorname{Sin}\left[\frac{h(hw)}{2}\right]\right) + \frac{1}{464486400h^6w^4} f_n\operatorname{Csc}\left[\frac{hw}{2}\right]\left(672390h^7w^4\operatorname{Cos}[hw] - 30 - 21925h^7w^4 + 9h^3 - 12096 + 725h^4w^4\operatorname{Cos}[hw] - 514134h^7w^4\operatorname{Cos}\left[\frac{h(3hw)}{2}\right] - 11573h^7w^4 + h^3 - 67200 - 3591h^4w^4\operatorname{Cos}\left[\frac{h(3hw)}{2}\right] + 24h^3 - 26370h^4w^4\operatorname{Cos}\left[\frac{h(5hw)}{2}\right] + 19782h^4w^4\operatorname{Cos}\left[\frac{h(7hw)}{2}\right] - 78h^2 - 41170h^5w^4\operatorname{Cos}\left[\frac{h(5hw)}{2}\right] - 7210h^5w^4\operatorname{Cos}\left[\frac{h(7hw)}{2}\right] + 54h^5 - 46965h^6w^4\operatorname{Cos}\left[\frac{h(5hw)}{2}\right]\right)$$

$$+1233h^{6}w^{4}\cos\left(\frac{7hw}{2}\right) \quad 56870h^{7}w^{4}\cos\left(\frac{5hw}{2}\right)$$

$$+9752h^{7}w^{4}\cos\left(\frac{7hw}{2}\right) \quad 60480h^{3}\cos\left(\frac{9hw}{2}\right)$$

$$3628800h^{6}w^{3}\cos\left(\frac{9hw}{2}\right)\sin[hw]$$

$$3628800h^{6}w^{3}\cos[hw]\sin\left(\frac{9hw}{2}\right)$$

$$+\frac{}{116121600h^{6}w^{4}}\left(f_{n}(hw)+6\csc\left(\frac{h}{2}\right)\right)\csc[hw]$$

$$1272180h^{7}w^{4}\cos\left(\frac{hw}{2}\right) + 21 \quad 1230766hw^{7}\left(\frac{3hw}{2}\right)$$

$$197h^{4}w^{4}\cos\left(\frac{3hw}{2}\right) \quad 3586758h^{7}w^{4}\cos\left(\frac{3hw}{2}\right) \quad 42$$

$$745868h^{7}w^{4} + 2h^{3}\ 423360\ 8933h^{4}w^{4}\cos$$

$$+9886770h^{7}w^{4}\cos\left(\frac{5hw}{2}\right) \quad 24h^{2} \quad 5w^{4}\cos\left(\frac{7hw}{2}\right)$$

$$+1460h^{4}w^{4}\cos\left(\frac{9hw}{2}\right) \quad 31192h^{5}w^{4}\cos\left(\frac{7hw}{2}\right)$$

$$+36h\ 4620h^{6}w^{4}\cos$$

$$66h^{6}w^{4}\cos\left(\frac{7hw}{2}\right)$$

$$18144 + 25h^{6}w^{4}\cos\left(\frac{7}{2}\right) \quad 4844h^{7}w^{4} + 5h^{3}$$

$$10h^{3} \quad 21168 + 293h^{4}w^{4}\cos\left(\frac{9hw}{2}\right) + 2786h^{7}w^{4}$$

$$+3h\ 5776596h^{6}w^{4}\cos\left(\frac{7hw}{2}\right) + 5981580h^{6}w^{4}\cos\left(\frac{9hw}{2}\right)$$

$$+8h^{2} \quad 75096h^{4}w^{4}\cos\left(\frac{7hw}{2}\right) \quad 1404h^{4}w^{4}\cos\left(\frac{9hw}{2}\right)$$

$$18h \quad 31192h^{5}w^{4}\cos\left(\frac{7hw}{2}\right) + 1460h^{5}w^{4}\cos\left(\frac{9hw}{2}\right)$$

$$+6\ 4620h^{6}w^{4}\cos\left(\frac{7hw}{2}\right) \quad 66h^{6}w^{4}\cos\left(\frac{9hw}{2}\right)$$

$$+6350400h^{6}w^{3}\ 3\cos\left(\frac{7hw}{2}\right) + \cos\left(\frac{9hw}{2}\right)\sin[hw]$$

$$6350400h^{6}w^{3}\cos[hw]\ 3\sin\left(\frac{7}{2}\right) + \sin\left(\frac{}{2}\right)$$

1 f +7Csc hw 7Csc[hw]

464486400h 6 w 4 n hw h 2 i

7 4 + 6 2795914h7w4 20160h3

973308h wCosh 2 hw 3hw

132 + 5h4w4 Cos 2 + 9510756h 7 w 4 Cos 2 12

1527756h7w4 + 14h3 112320 3863h4w4 Cos h3hw i 2

h5hw i h7hw i

+ 24h3 124200h4w4Cos + 19782h4w4Cos

2 2

h9hw i h5hw i

378h4w4Cos 78h2 103960h5w4Cos

22

5 4 h 7hw 406h 5 w 4 Cos 9hw

7210h wCos 2 i hw h 2 i 7hw

+ 54h 56220h 6 w 4 Cos 5 2 + 1233h6w4Cos 2 i

h9hw i h5hw i

15h6w4Cos 6 126470h7w4Cos + 12827h7w4

22

h7hw i

15h3 28224 + 205h4w4 Cos + 811h7w4 5h3 12096 2

4w4 Cos 9hw + 3628800h 6 w 3 7Cos 7hw

+ 169h 9hw h 2 i hw h 2 7 hw i

+ Cos n 9hw i Sin[hw] 1 3628800h7w4Cos[hw] 7 h 7Sin

2 2

h 2 i hw h i

+ Sin + 232243200h 6 w 4 fn+2Csc 2 Csc[hw]

7 4 7w4 + 180h3 10752

Cos h

2336964h w hw 2 + 21 532598h 3hw

251h4w4 Cos 2 i 6014946h 7 w 4 Cos 2 42

1455388h7w4 + 2h3 544320 10723h4w4 Cos h3hw i 2

5hw 7hw

+ 1880070h 7 w 4 Cos h i h

2 24h2 219800h5w4Cos 2 i

$$+20788h^5w^4\cos\left(h\frac{9hw}{2}i\right) + 36h\ 177324h^6w^4\cos\left(h\frac{7hw}{2}i\right)$$

$$23490h^6w^4\cos\left(h9hwi\right) \quad 6\ 7\ 41068h^7w^4 +$$

$$105h^3\ 2$$

$$288 + 19h^4w^4\cos\left(h7hwi\right)$$

$$21422h^7w^4\ 2$$

$$10h^3\ 63504 + 703h^4w^4\cos\left(h9hwi\right)$$

$$3h\ 9369108h^6w^4\cos\left(\frac{h3hwi}{22}\right) + 1601100h^6w^4\cos\left(h5hwi\right)$$

$$+ 8h^2\ 115416h^4w^4\cos\left(\frac{7hw}{}\right) + 12996h^4w^4\cos\left(\frac{9hw}{}\right)$$

$$h \qquad h^5w^4\cos\left(\frac{h\ 2\ 7hw}{2\ i}\right) + 20788h^5w^4\cos\left(\frac{h\ 2\ 9hw}{2\ i}\right)$$

$$18\ 219800 \qquad \frac{7hw}{2} \qquad 23490h^5w^4\cos \qquad \frac{9hw}{}$$

$$+ 6\ 177324h^5w^4\cos$$

$$+ 12700800h^5w^5 \qquad \cos\left(\frac{h\ 7hw}{}\right) \qquad 3\cos\left(\frac{9hw}{}\right) \qquad \sin[hw]\left(\frac{h\ 2}{}\right) \quad i$$

$$12700800h^5w^5\cos[hw]\sin\left(\frac{h\ 2}{}\right) + \left(\frac{7hw}{}\right) + 3\sin\left(\frac{h\ 2}{}\right) \quad i\left(\frac{9hw}{}\right)$$

$$\frac{1}{464486400h^6w^4}\ f_{n+5}\csc\left(\frac{hw}{n}\right)\ 7\ 2\ i \qquad \csc[hw]\left(\frac{h\ 2}{}\right) \quad i$$

$$3703140h^7w^4\cos\left(\frac{hw}{n}\right) \quad + 210\ 425938h^7w^4 + 576h^5 \qquad 1092$$

$$17h^4w^4\ \cos\left(\frac{hw}{2}\ 2\ i\right) \qquad 12029892h^7w^4\cos\left(\frac{hw}{2}\ i\right)$$

$$84 \quad 1455388h^7w^4 + 2h^3\ 907200$$

$$10723h^4w^4\ \cos\left(\frac{h3hwi}{22}\right) + 52581900h^7w^4\cos\left(h5hwi\right)$$

$$24h^2 200550h^5w^4\cos\left(h7hw\ i\ 2\right)$$

$$+9050h^{5}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right] + 36h\ 29463h^{6}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$585h^{6}w^{4}\mathrm{Cos}[9hw] \quad 6\ 7\ 26331h^{7}w^{4} + 5h^{3}$$

$$\frac{}{60480}\ 2$$

$$779h^{4}w^{4}\ \mathrm{Cos}[7hw] + 5\ 3241h^{7}w^{4}\ h^{3}\ 254016^{2}\ [3hw]$$

$$3469h^{4}w^{4}\ \mathrm{Cos}\left[\frac{}{2}\right] + 3h\ 18738216h^{6}w^{4}\mathrm{Cos}\left[\frac{}{2}\right]$$

$$+ 25232040h^{6}w^{4}\mathrm{Cos}\left[\frac{5hw}{2}\right] + 8h^{2}\ 467082h^{4}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$9270h^{4}w^{4}\mathrm{Cos}[9hw] \quad 18h\ 200550h^{5}w^{4}\mathrm{Cos}[7hw]$$

$$+ 9050h^{5}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right] + 6\ 29463h^{6}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$585\ h^{6}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right] + 25401600h^{6}w^{3}\ 5\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$+ 3\mathrm{Cos}\left[\frac{hw}{2}\right]\ \mathrm{Sin}[hw]\ 25401600h^{6}w^{3}\mathrm{Cos}[hw]\ 5\mathrm{Sin}\left[\frac{}{2}\right]$$

$$+ 3\mathrm{Sin}\left[\frac{9}{2}\right]\ \frac{}{464486400h^{6}w^{4}}\ f_{n+3}\mathrm{Csc}\left[\frac{}{2}\right]\ \mathrm{Csc}[hw]$$

$$3703140h^{7}w^{4}\mathrm{Cos}\left[\frac{hw}{2}\right]$$

+210 425938h^7w^4 + 576h^3 1260 17h^4w^4 Cos h hw i 2 7173516h^7w^4Cos h 3hw i 2 84

745868h^7w^4 + 2h^3 1028160 8933h^4w^4 Cos h3hw i 2 + 115202820h^7w^4Cos h5hw i 2

24h^2 584934h^5w^4Cos h7hw i 2 + 48730h^5w^4Cos h9hw i 2 + 36h

541065h^6w^4Cos h7hw i 2 + 83415h^6w^4Cos h9hw i 2 6 7 28437h^7w^4 + 5h^3 36288

4 4 7hw 2 + 5 19001h^7w^4 7h^3 60480 + 773h^4w^4 9hw 2

+ 1829h w Cosh 9hw h 3hw 2 i + 26735400h^6w^4Cos 5hw 2 + 8h^2 769482h^4w^4Cos 9hw 2 i 7hw 2 i

12150h^4w^4Cos h 2 i hw 18h 584934h^5w^4Cos 9hw 2 i + 48730h^5w^4Cos h 7hw 2

+ 6 541065h^6w^4Cos h 7 2 i + 83415h^6w^4Cos h 2 i + 25401600h^6w^3 3Cos h 2 i

+ 5Cos 9hw 2 Sin[hw] 25401600h^6w^8 3Cos[hw] 3Sin 7hw 2 h y_n 10h^3 + y_{n+3} h^3 3y_{n+4} 2h^3

+ 3y_{n+5} 5h^3 + 1 46448640h^6w^4 f_{n+4}Csc hw 2 i 19051200h^3 866880h^7w^4Cos[hw]

42 244936h^7w^4 + 420h^3 1152 + 112h^4w^4 Cos[hw] 1024464h^7w^4Cos[2hw]

44496h^7w^4Cos[3hw] 50904h^7w^4Cos[4hw] 12 20594h^7w^4 20h^3 5292

$+ 1136h^4w^4 \quad Cos[4hw] + 2h\ 1161888h^6w^4Cos[hw] + 2192064h^6w^4Cos[2hw]$

$h^6w^4Cos[3hw] + 20832h^6w^4Cos[4hw] \quad 12700800h^6w^3Sin[2hw]$

$199584 + \frac{1}{464486400h^6w^4} f_{n+8}Csc\left[\frac{hw}{2}\right]^7 Csc[hw]978570h^7w^4Cos\left[\frac{hw}{2}\right]$

$30 82330h^7w^4 + 3h^3 \quad 20160 + 9104h^4w^4 \quad Cos\left[\frac{hhw}{2}\right]$

$+ 429030h^7w^4Cos\left[\frac{3hw}{2}\right] \quad 54 5926h^7w^4 + 35h^3 \quad 1152 \quad 560h^4w^4 \quad Cos\left[\frac{3hw}{2}\right]$

$+ 6h^3\ 4680h^4w^4Cos\left[\frac{h}{2}\right] \quad 468h^4w^4Cos\left[\frac{h}{2}\right] \quad 33h^2 \quad 4770h^5w^4Cos\left[\frac{h}{2}\right]$

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$$450h^5w^4\mathrm{Cos}\left[\frac{7hw}{2}\right] + 36h\ 240h^6w^4\mathrm{Cos}\left[\frac{5hw}{2}\right] 42h^6w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$6\ 9580h^7w^4\mathrm{Cos}\left[\frac{5hw}{2}\right] + 13536h^7w^4\ 5h^3\ 12096$$

$$+ 2704h^4w^4\ \mathrm{Cos}\left[\frac{7hw}{2}\right] + 3628800h^6w^3\mathrm{Cos}\left[\frac{7hw}{2}\right]\mathrm{Sin}[2hw]$$

$$3628800h^6w^3\mathrm{Cos}[2hw]\mathrm{Sin}\left[\frac{h}{2}\right] + \frac{}{464486400h^6w^4 f_n\mathrm{Csc}\left[\frac{h}{2}\right]\mathrm{Csc}[hw]}$$

$$228330h^7w^4\mathrm{Cos}\left[\frac{hhw}{2}\right] 30\ 115238h^7w^4 + 9h^3\ 12096$$

$$+ 11600h^4w^4\ \mathrm{Cos}\left[\frac{2}{hw}\right] + 2208978h^7w^4\mathrm{Cos}\left[\frac{3hw}{2}\right] 54\ 48110h^7w^4$$

$$67200\ 7hw\ \mathrm{Cos}\left[\frac{h}{2}\right] + h^3\ 57456h^4w^4 + 6h^3\ 32040h^4w^4\mathrm{Cos}\left[\frac{5hw}{2}\right]$$

$$+ 972h^4w^4\mathrm{Cos}\left[\frac{h\ 7hw}{2}\right] 33h^2\ 29570h^5w^4\mathrm{Cos}\left[\frac{5hw}{2}\right] 7hw$$

$$+ 1234\ \mathrm{Cos}\left[\frac{7hw}{2}\right] + 36\ 6840\ 9hw\ 2\ i$$

$$h^5w^4\ \frac{}{\pi}\ h\ h^6w^4\mathrm{Cos}\left[\frac{}{\pi}\right] 210h^6w^4\mathrm{Cos}\left[\frac{}{}\right]$$

$$+ \frac{6}{1}\ 82660h^7w^4\mathrm{Cos}\left[\frac{9hw}{2}\right] + 9592h^7w^4\mathrm{Cos}\left[\frac{h\ 2}{}\right] 60480h^3\mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$3628800h^6w^3\mathrm{Cos}\left[\frac{h\ 2}{}\right]\mathrm{Sin}[2hw]\ \frac{1}{7}\ 3628800h^6w^3\mathrm{Cos}[2hw]\mathrm{Sin}\left[\frac{}{}\right]$$

$$\frac{}{116121600h^6w^4 f_{n+6}\mathrm{Csc}\left[\frac{}{2}\right]\mathrm{Csc}[hw]}\ \frac{}{2}\ 1951488h^7w^4\ \mathrm{Cos}\left[\frac{h\ 2}{}\right]$$

$$142928h^4w^4\ \frac{h^7w^4}{}\ 180h^3\ h^4w^4\ \mathrm{Cos}\left[\frac{}{}\right]$$

$$14700672h^7w^4\mathrm{Cos}\left[\frac{h\ 3hw}{2}\right] 42\ 192131h^7w^4 + 2h^3\ 423360$$

$$\mathrm{Cos}\left[\frac{h}{}\right] i\ 1014150\ \frac{}{}$$

$$h^2\ 11970h^3w^4\mathrm{Cos}\left[\frac{72hw}{}\right]\ h^3w^4\mathrm{Cos}\left[\frac{h\ 2}{9hw}\right]$$

$$+ 24h\ 399h^6w^4\mathrm{Cos}\left[\frac{h\ 7}{2}\right] 225h^6w^4\mathrm{Cos}\left[\frac{h\ 9}{2}\right]$$

$$7\ 2814h^7w^4 + 5h^3 18144 + 400h^4w^4\ \mathrm{Cos}\left[\frac{h7hw}{2}\right]$$

$$+ 46886h^{7}w^{4} \quad 10h^{3} \quad 21168 + 4688h^{4}w^{4} \quad \cos\frac{9hw}{5hw}$$

$$+ 2h\ 7643328h^{6}w^{4}\cos\frac{hhw\ i}{2} + 1569600h^{6}w^{4}\cos\frac{hw''\ '}{2}$$

$$+ 3h^{2} \quad 11466h^{4}w^{4}\cos\frac{h\ 7}{2\ i} \quad 1854h^{4}w^{4}\cos\frac{h\ 9}{2\ i}$$

$$12h\ 11970h^{5}w^{4}\cos\frac{h7hw\ i}{222} \quad 1768h^{5}w^{4}\cos\frac{h9hw\ i}{222} + 6\ 399h^{6}w^{4}\cos\frac{h7hw\ i}{222}$$

$$225h^{6}w^{4}\cos\frac{h9hw\ i}{22} + 6350400h^{6}w^{3} \quad 3\cos\frac{h7hw\ i}{22}$$

$$+ \cos\frac{h\ 2_{1}\ '}{9hw} \quad \sin[2hw] \quad 6350400h^{7}w^{3}\cos[2hw] \quad 3\sin\frac{h\ 2\ 'hw}{7hw} + \sin\frac{h\ 2\ i}{9hw}$$

$$464486400h^{6}w^{4}\frac{f \quad \csc\ i}{h\ 2} \quad \csc[\ ] \quad 1432704\frac{hw\ h^{7}w^{4}\cos}{h\ hw''\ 2\ i}$$

$$+ 6\ 4481792h^{7}w^{4} \quad 20160h^{3} \quad 132 + 80h^{4}w^{4} \quad \cos\frac{h\ 3hw'}{2}$$

$$391680h^{7}w^{4}\cos\frac{5hw}{hw''\ 2\ i} \quad 12 \quad 50119h^{7}w^{4} + 14h^{3} \quad 112320$$

$$61808h^{4}w^{4} \quad \cos\frac{hw\ h\ 2\ i}{} + 6h^{3}\ 55440h^{4}w^{4}\cos\frac{5hw}{}$$

$$+ 972h^{4}w^{4}\cos\frac{h\ 7\ i}{2hw} \quad 468h^{4}w^{4}\cos\frac{h\ 9\ i}{2hw} \quad 33h^{2}\ 5180h^{5}w^{4}\cos\frac{h}{hw\ 2\ i}$$

$$+ 1234h^{5}w^{4}\cos\frac{h7hw\ i}{2} \quad 450h^{5}w^{4}\cos\frac{h9hw\ i}{2} + 36h \quad 7680h^{6}w^{4}\cos\frac{h5hw\ i}{2\ i}$$

$$210h^{6}w^{4}\cos\frac{h7hw\ i}{222} \quad 42h^{6}w^{4}\cos\frac{h9hw\ i}{222} \quad 6\ 150040h^{7}w^{4}\cos\frac{h7hw\ i}{}$$

$$58792h^{7}w^{4}\ 15h^{3}\ 28224 + 3280h^{4}w^{4}\cos\frac{h7hw\ i}{h7hw\ i} + 13536h^{7}w^{4}\ 2$$

$$5h^{3}\ 12096 + 2704h^{4}w^{4}\ \cos\frac{h9hw\ i}{22} + 3628800h^{6}w^{3}\ 7\cos\frac{h7hw\ i}{22}$$

$$+ \cos\frac{9hw}{} \quad \sin[2hw] \quad 3628800h^{6}w^{3}\cos[2hw] \quad 7\sin\frac{7hw}{} + \sin\frac{9hw}{}$$

$$+ \frac{h\ 21\ '}{232243200h^{6}w^{4}}\frac{f \quad \csc\ ''}{n+2\ 2\ '} \quad \csc[\ ] \quad 10090752\frac{hw\ h\ 2\ i\ hw\ h\ 2\ i}{h'w^{4}\cos\ hw\ h\ 2i}$$

$$+ 21\ 426326h^{7}w^{4} + 180h^{3}\ 10752\ 4016h^{4}w^{4}\ \cos\frac{h\ 2\ i}{3hw\ i}$$

$$25667712h^{7}w^{4}\cos\frac{42}{2^{h}}\ 653917h^{7}w^{4}$$

$$+ 2h^{3}\ 544320\ 171568h^{4}w^{4}\cos\frac{h\ \frac{3hw}{2\ i}}{246}$$

$334650h^7w^4\cos\left(\frac{5hw}{2}\right) \quad 9h^2 \quad 90538h^5w^4\cos\left(\frac{7hw}{2}\right)$

$+ 14560h^5w^4\cos\left(\frac{9hw}{2}\right) + 24h \quad 21735h^6w^4\cos\left(\frac{}{2}\right)$

$3441h^6w^4\cos\left(\frac{9hw}{2}\right) \quad 6\ 7\ 23798h^7w^4 + 105h^3$

$304h^4w^4\cos\left(\frac{7hw}{2}\right) \quad 71158h^7w^4 + 10h^3$

$11248h^4w^4\cos\left(\frac{9hw}{2}\right) + 2h\ 14540736h^6w^4\cos\left(\frac{3hw}{2}\right)$

$+ 137280h^6w^4\cos\left(\frac{5hw}{2}\right) + 3h^2\ 122346h^4w^4\cos\left(\frac{7hw}{2}\right)$

$16254h^4w^4\cos\left(\frac{9hw}{2}\right) \quad 12h \quad 90538h^5w^4\cos\left(\frac{7hw}{2}\right)$

$+ 14560h^5w^4\cos\left(\frac{9hw}{2}\right) + 6 \quad 21735h^6w^4\cos\left(\frac{7hw}{2}\right)$

$3441h^6w^4\cos\left(\frac{9hw}{2}\right) + 12700800h^6w^3\cos\left(\frac{7hw}{2}\right)$

$399330h^7w^4\cos\left(\frac{5hw}{2}\right) + 12h\ 17556h^6w^4\cos\left(\frac{7hw}{2}\right)$

$3246h^6w^4\cos\left(\frac{9hw}{2}\right) \quad 6\ 7$

$106668h^7w^4 \quad 2$

$105h^3\ 288 + 1539h^4w^4$

$\cos\left(\frac{7hw}{2}\right)$

$529254h^6w^4 + 10h^3\ 63504 + 56943h^4w^4\cos\left(\frac{5hw}{2}\right)$

$+ h\ 7121940h^6w^4\cos\left(\frac{3hw}{2}\right) + 34380h^6w^4\cos\left(\frac{9hw}{2}\right)$

$6h\ 504h^5w^4\cos\left(\frac{7hw}{2}\right) \quad 68h^5w^4\cos\left(\frac{9hw}{2}\right) + 6$

$17556h^6w^4\cos\left(\frac{7hw}{2}\right) + 3246h^6w^4\cos\left(\frac{9hw}{2}\right)$

$+ 12700800h^6w^3\cos\left(\frac{7hw}{2}\right) + 3\cos\left(\frac{9hw}{2}\right)\sin[3hw]$

$12700800h^6w^3\cos[3hw]\sin\left(\frac{7hw}{2}\right) + 3\sin\left(\frac{9hw}{2}\right)$

$$\frac{1}{464486400h^{6}w^{4}}\, f_{n+5}\, \mathrm{Csc}\left[\frac{h}{2}\right]^{7} \mathrm{Csc}[hw]$$

$$41651820h^{7}w^{4}\mathrm{Cos}\left[\frac{hw}{2}\right] + 210\ 1173878h^{7}w^{4}$$

$$+ 576h^{3}\ 1092\ 1377h^{4}w^{4}\mathrm{Cos}\left[\frac{hhw}{2}\right]$$

$$68867316h^{7}w^{4}\mathrm{Cos}\left[\frac{h3hw}{}\right] \quad 84\ 1346724h^{7}w^{4}\ 2^{h}$$

$$2h^{3}\ 907200\ 868563h^{4}w^{4}\mathrm{Cos}\left[\frac{h3hw}{2}\right]$$

$$14742180h^{7}w^{4}\mathrm{Cos}\left[\frac{h5hw}{2^{2}}\right] + 12h\ 92463h^{6}w^{4}\mathrm{Cos}\left[\frac{h7hw}{}\right]$$

$$13425h^{6}w^{4}\mathrm{Cos}\left[\frac{9hw}{}\right] \quad 7 \quad 277551h^{7}w^{4} + 5h^{3}$$

$$h\ \ 2\ \ i\ \ \mathrm{Cosh}\ \ 6\ 7hw\ \ 2\ \ i\ \ 9hw$$

$$h\ \ 60480 + 63099h^{4}w^{4}\ \ 3\ \ 254016 + 280989h\ 3hw\ \mathrm{Cos}\left[\frac{}{2}\right] + 5\ 279099h^{7}w^{4}\ \ i\ \ 5hw$$

$$h\ \ h^{6}w^{4}\mathrm{Cos}\left[\frac{4w^{4}}{}\right] \quad 1582680h^{6}w^{4}\mathrm{Cos}\left[\frac{}{}\right]$$

$$+\ \ 14243880\ \ 7hw\ \ 2\ \ i\ \ h\ \ 9hw\ \ h\ \ 2\ i$$

$$6h\ 6398h^{5}w^{4}\mathrm{Cos}\left[\frac{}{2}\right] \quad 1090h^{5}w^{4}\mathrm{Cos}\left[\frac{}{2}\right]$$

$$+ 6\ \ 92463h^{6}w^{4}\mathrm{Cos}\left[\frac{h\ \ 7hw}{2}\right] \quad 13425h^{6}w^{4}\mathrm{Cos}\left[\frac{9hw\ \ h}{2}\right]\ \ i$$

$$h^{6}w^{3}\ \ 3\mathrm{Cos}\ \ \mathrm{Sin}[3hw]$$

$$+ 25401600\ \ 5\mathrm{Cos}\ \ 2\ \ i\ + \frac{7hw}{}\ \ 2\ \ i\ \ \frac{hw}{9}$$

$$25401600h^{6}w^{3}\mathrm{Cos}[3hw]\ \ 5\mathrm{Sin}\ \ h\ \ 2\ \ i\ + 3\mathrm{Sin}\ \ h\ \ 2\ \ i$$

$$\frac{1}{464486400h^{o}w^{4}}\, f_{n+3}\mathrm{Csc}\left[\frac{hw}{2}\right]^{7} \mathrm{Csc}[hw]$$

$$41651820h^{7}w^{4}\mathrm{Cos}\left[\frac{hw}{2}\right] + 210\ 1173878h^{7}w^{4}$$

$$+ 576h^{3}\ 1260\ 1377h^{4}w^{4}\mathrm{Cos}\left[\frac{hhw}{2}\right]$$

$$31877244h^{7}w^{4}\mathrm{Cos}\left[\frac{h3hw}{2}\right] \quad 84\ 1286724h^{7}w^{4}\ 2^{h}$$

$$2h^{3}\ 1028160\ 723573h^{4}w^{4}\mathrm{Cos}\left[\frac{h3hw}{2}\right]$$

$$35779500h^{7}w^{4}\mathrm{Cos}\left[\frac{h5hw}{2}\right] + 12h$$

$$433713h^{6}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right] + 63855h^{6}w^{4}\mathrm{Cos}\left[\frac{h9hw}{2}\right] \quad 6\ 7$$

$$949311h^{7}w^{4} + 5h^{3}36288 + 148149h^{4}w^{4}\ \mathrm{Cos}\left[\frac{h7hw}{2}\right]$$

$$5\ 489771h^{7}w^{4}\ 7h^{3}\ 60480 + 62613h^{4}w^{4}\mathrm{Cos}\left[\frac{h9hw}{2}\right]$$

$$h\ 3516072h^{6}w^{4}\mathrm{Cos}\left[\frac{h3hw}{2}\right] + 14009640h^{6}w^{4}\mathrm{Cos}\left[\frac{h5hw}{2}\right]$$

$$6h\ 428414h^{5}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right] \quad 61250h^{5}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$+ 6\ \ 433713h^{o}w^{4}\mathrm{Cos}\left[h\left[\frac{7hw}{2}\right]\right] + 63855h^{o}w^{4}\mathrm{Cos}\left[h\left[\frac{9hw}{2}\right]\right]$$

$$+ 25401600h^{6}w^{3}\ 3\mathrm{Cos}\left[\left[\frac{7hw}{2}\right]\right] \quad 5\mathrm{Cos}\left[\frac{9hw}{2}\right] \quad h\ \mathrm{Sin}[3hw]$$

$$25401600h^{o}w^{3}\mathrm{Cos}[3hw]\ 3\mathrm{Sin}\left[\left[\frac{7hw}{2}\right]\right] + 5\mathrm{Sin}\left[\frac{9hw}{2}\right]$$

$$\frac{1}{928972800h^{6}w^{4}}\, f_{n+1}\mathrm{Csc}\left[\frac{hw}{2}\right]^{8} \mathrm{Sec}\left[\frac{hw}{2}\right]$$

$$1295028h^{7}w^{4}\mathrm{Cos}\left[\left[\frac{hw}{2}\right]\right] + 6\ \ 15670750h^{7}w^{4}\ \ 60480h^{3}$$

$$68 + 243h^{4}w^{4}\ \ \mathrm{Cos}\left[\frac{h3hw}{2}\right] \quad 5733252h^{7}w^{4}\mathrm{Cos}$$

$$12\ 7846788h^{7}w^{4} + 14h^{3}\ \ 164160$$

$$598833h^{4}w^{4}\ \ \mathrm{Cos}\left[\frac{h3hw}{2}\right] \quad 6h^{2}\ 1400h^{5}w^{4}\mathrm{Cos}\left[\frac{h5hw}{2}\right]$$

$$46h^{5}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right] + 18h^{5}w^{4}\mathrm{Cos}\left[\frac{h9hw}{2}\right] + 18h$$

$$31620h^{6}w^{4}\mathrm{Cos}\left[\frac{h5hw}{2}\right] \quad 423h^{6}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right]$$

$$+ 633h^{6}w^{4}\mathrm{Cos}\left[h\left[\frac{9hw}{2}\right]\right] \quad 6 \quad 771210h^{7}w^{4}\mathrm{Cos}\left[h\left[\frac{5hw}{2}\right]\right]$$

$$68193h^{7}w^{4}\ 5h^{3}\ 12096 + 13689h^{4}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right]$$

$$258993h^{7}w^{4}\ 15h^{3}\ 28224 + 16605h^{4}w^{4}\mathrm{Cos}\left[\frac{h9hw}{2}\right]$$

$$+ 3628800h^{o}w^{3}\ \mathrm{Cos}\left[h\left[\frac{7hw}{2}\right]\right] + 7\mathrm{Cos}\left[h\left[\frac{9hw}{2}\right]\right] \quad \mathrm{Sin}[3hw]$$

$$3628800h^{6}w^{3}\mathrm{Cos}[3hw]\ \mathrm{Sin}\left[h\left[\frac{7}{2}\right]\right] + 7\mathrm{Sin}\left[h\left[\frac{}{2}\right]\right]$$

$$y_{n+4}^{000} = \frac{y_n}{h^{3}} + \frac{y_{n+3}}{h^{3}} 4 \quad \frac{3y_{n+4}}{h^{3}} + \frac{3y_{n+5}}{h^{3}} + \frac{1}{46448640h^{6}w^{4}} \quad 3 \quad 19051200h^{5}$$

$$14192640h^{4}w^{4}\mathrm{Cos}[hw] \quad 42 \quad 761480h^{4}w^{4} + 420h \quad 1152$$

$$1356048h^{7}w^{4}\mathrm{Cos}[3hw] \quad 12\ 351540h^{7}w^{4}$$

$$20h^{3}\ 5292 + 18176h^{4}w^{4}\ \mathrm{Cos}[4hw]\ \mathrm{Csc}\left[\frac{hhw}{2}\right]^{8}$$

$$+\frac{1}{464486400h^{6}w^{4}} f_{n+8}\mathrm{Csc}\left[\frac{hw}{2}\right]^{7}\mathrm{Csc}[hw]$$

$$2517990h^{7}w^{4}\mathrm{Cos}\left[\frac{hw}{2}\right] \quad 30 \quad 410450h^{7}w^{4} + 3h^{3}$$

$$20160 + 145664h^{4}w^{4}\ \mathrm{Cos}\left[\frac{hhw}{22}\right] \quad 295974h^{7}w^{4}\mathrm{Cos}\left[\frac{h3hw}{22}\right]$$

$$54\ 311940h^{7}w^{4} + 35h^{3}\ 1152\ 8960h^{4}w^{4}\ \mathrm{Cos}\left[\frac{h3hw}{2}\right]$$

$$+ 3h^{2} \quad 8510h^{5}w^{4}\mathrm{Cos}\left[\frac{h5hw}{2}\right] + 838h^{5}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right]$$

$$6\ 14720h^{7}w^{4}\mathrm{Cos}\left[\frac{h5hw}{2}\right] + 215800h^{7}w^{4}\ 5h^{3}\ 12096$$

$$+ 43264h^{4}w^{4}\ \mathrm{Cos}\left[\frac{7hw}{2}\right] \quad 3628800h^{6}w^{3}\mathrm{Cos}[4hw]\mathrm{Sin}\left[\frac{7hw}{2}\right]$$

$$3628800h^{6}w^{3}\mathrm{Cos}\left[\frac{7hw}{2}\right]\mathrm{Sin}[4hw]$$

$$+ \frac{1}{464486400h^{6}w^{4}} f_{n+8}\mathrm{Csc}\left[\frac{hw}{2}\right]^{7}\mathrm{Csc}[hw]$$

$$1544490h^{7}w^{4}\mathrm{Cos}\left[\frac{hw}{2}\right] \quad 30 \quad 1681558h^{7}w^{4} + 9h^{3}$$

$$12096 + 185600h^{4}w^{4}\ \mathrm{Cos}\left[\frac{hhw}{22}\right] \quad 1482570h^{7}w^{4}\mathrm{Cos}\left[\frac{h3hw}{22}\right]$$

$$54\ 927508h^{7}w^{4} + h^{3}\ 67200\ 919296h^{4}w^{4}\ \mathrm{Cos}\left[\frac{h3hw}{2}\right]$$

$$+ 3h^{2}\ 8510h^{5}w^{4}\mathrm{Cos}\left[\frac{h5hw}{2}\right] \quad 838h^{5}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right]$$

$$6\ 75680h^{7}w^{4}\mathrm{Cos}\left[\frac{h5hw}{22}\right] + 10328h^{7}w^{4}\mathrm{Cos}\left[\frac{h7hw}{22}\right]$$

$$60480h^{3}\mathrm{Cos}\left[\frac{h9hw}{22}\right] + 3628800h^{6}w^{3}\mathrm{Cos}\left[\frac{h9hw}{22}\right]\mathrm{Sin}[4hw]$$

$$3628800h^{6}w^{3}\mathrm{Cos}[4hw]\mathrm{Sin}\left[\frac{h9hw}{2}\right]$$

$$\frac{1}{116121600h^6w^4}\,f_{n+6}\,\frac{\operatorname{Csc}\left[\frac{hw}{2}\right]^7\operatorname{Csc}[hw]}{\frac{1}{2}}$$

$$39373824h^7w^4\operatorname{Cos}\left[\frac{hw}{2}\right] + 21\,9110410h^7w^4 + 180h^3\,3hw\,8064$$

$$50432h^4w^4\operatorname{Cos}\left[\frac{5hw}{2}\right] \quad 9074688h^7w^4\operatorname{Cos}\left[\frac{hw}{2}\right] \quad 7hw\,3hw$$

$$42\,4586173h^7w^4 + 2h^3 \quad 423360 \quad 2286848h^4w^4 \quad \operatorname{Cos}\left[\frac{3hw}{2}\right]$$

$$2201670h^7w^4\operatorname{Cos}\left[\frac{hw}{2}\right] + 3h^2 \quad 20986h^5w^4\operatorname{Cos}\left[\frac{hw}{2}\right]$$

$$+ 3836h^5w^4\operatorname{Cos}\left[\frac{9hw}{2}\right] \quad 27hw\,6\,7\,25420h^7w^4 + 5h^3 \quad 18144$$

$$+ 6400h^4w^4 \quad \operatorname{Cos}\left[\frac{7hw}{2}\right] + 747884h^7w^4 \quad 10h^3 \quad 21168$$

$$+ 9hw\,75008h^4w^4 \quad \operatorname{Cos}\left[\frac{hw}{2}\right] + 6350400h^6w^3\,3\operatorname{Cos}\left[\frac{2hw}{7}\right]$$

$$+ \operatorname{Cos}\left[\frac{9hw}{2}\right] \quad \operatorname{Sin}[4hw] \quad 1 \quad 6350400h^6w^3\operatorname{Cos}[4hw]\,3\operatorname{Sin}\left[\frac{hw}{2}\right]$$

$$+ \operatorname{Sin}\left[\frac{h2}{2}\right] \quad \frac{hw}{464486400h^6w^4}\,f_{n+7}\operatorname{Csc}\left[\frac{h}{2}\right]\operatorname{Csc}[hw]$$

$$24600576h^7w^4\operatorname{Cos}\left[\frac{hw}{2}\right] + 6\,25849792h^7w^4 \quad 9hw\,20160h^3$$

$$132 + 1280h^4w^4\operatorname{Cos}\left[\frac{h5hw}{2}\right] + 838h^5w^4\operatorname{Cos}\left[\frac{h}{2}\right]$$

$$6\,168320h^7w^4\operatorname{Cos}\left[\frac{h7hw}{2}\right] + 797528h^7w^4\,15h^3\,28224^2$$

$$52480h^4w^4\operatorname{Cos}\left[\frac{h9hw}{2}\right] + 215800h^7w^4\,5h^3\,12096^2$$

$$43264h^4w^4\operatorname{Cos}\left[\frac{h7hw}{2}\right] + 3628800h^6w^3\,7\operatorname{Cos}\left[\frac{hw}{2}\right]$$

$$+ \operatorname{Cos}\left[\frac{h\,hw\,9hw}{2}\right] \quad \operatorname{Sin}[4hw] \quad 1 \quad 3628800h^6w^3\operatorname{Cos}[4hw]\,7\operatorname{Sin}\left[\frac{h7hw}{2}\right]$$

$$+ \operatorname{Sin}\left[\frac{h9}{2}\right] + \frac{}{232243200h^6w^4}\,f_{n+2}\operatorname{Csc}\left[\frac{h}{2}\right]\operatorname{Csc}[hw]$$

$$22944768h^7w^4\operatorname{Cos}\left[\frac{hw}{2}\right]$$

h^7w^4 $180h^3$ 10752

+21 11646166 +hw hw

$64256h^4w^4$ Cos $h\frac{i}{2}$ $25503744h^7w^4$Cos $h\frac{3}{2}i$

42 5526371h^7w^4 + $2h^3$ 544320

$2745088h^4w^4$ Cos $\frac{h3hw\,i}{22}$ $556410h^7w^4$Cos $\frac{h5hw\,i}{}$

+ $3h^2$ $20986h^5w^4$Cos $n\frac{7hw}{2}i$ $3836h^5w^4$Cos $n\frac{9hw}{2}i$

6 7 $457388h^7w^4$ + $105h^3$ 288

$4864h^4w^4$ Cos $\frac{h7hw\,i}{}$ $1761580h^7w^4$ 2

$10h^3$ 63504 + $179968h^4w^4$ Cos $\frac{h9hw\,i}{2}$

$12700800h^6w^3$ Cos $\frac{h7hw\,i}{22}$ + 3Cos $\frac{h9hw\,i}{}$ Sin[4hw]

$12700800h^6w^3$Cos[4hw] Sin $\frac{h7hw\,i}{22}$ + 3Sin $\frac{h9hw\,i}{}$

$\frac{1}{464486400h^6w^4}$ f_{n+5}Csc $h\frac{hw}{2}i$ 7 Csc[hw]

$141926400h^7w^4$Cos $\frac{hw\,i}{2}$ + 210 $2524432h^7w^4$

+ $576h^3$ 1092 $4352h^4w^4$ Cos $\frac{hhw\,i}{2}$

$51007488h^7w^4$Cos $\frac{h3hw\,i}{}$ 84 5526371h^7w^4

$2h^3$ 907200 $2745088h^4w^4$ Cos $\frac{h3hw\,i}{2}$ $2h$

$8427300h^7w^4$Cos $\frac{h5hw\,i}{2}$ + $3h^2$

$289758h^5w^4$Cos $\frac{h7hw\,i}{22}$ + $43070h^5w^4$Cos $\frac{h9hw\,i}{}$

6 7 $952296h^7w^4$ + $5h^3$

60480 + $199424h^4w^4$ Cos $\frac{h7hw\,i}{2}$

$$+5\quad 884776\qquad 7hw + 888064\quad 9hw\,\mathrm{Cos}\,\frac{h}{9hw}i$$

$$h^{7}w^{4}\qquad h^{5}\ 254016\qquad h^{4}w^{4}$$

$$+ 25401600h^{5}w^{5}\ 5\mathrm{Cos}\,\frac{h}{2}i + 7hw\,h\,\frac{2}{3}\mathrm{Cos}\,i9hw\,\mathrm{Sin}[4hw]$$

$$25401600h^{5}w^{5}\mathrm{Cos}[4hw]\ 5\mathrm{Sin}\,\frac{hw}{2}i + 3\mathrm{Sin}\,\frac{h}{2}i$$

$$\frac{1}{464486400h^{5}w^{4}}f_{n+3}\mathrm{Csc}\,\frac{h}{2}i\ \mathrm{Csc}[hw]$$

$$141926400h^{7}w^{4}\mathrm{Cos}\,\frac{hw}{2}i + 210\ 2524432h^{7}w^{4}$$

$$+ 576h^{3}\ 1260\ 4352h^{4}w^{4}\mathrm{Cos}\,\frac{hhw}{2}i$$

$$18149376h^{7}w^{4}\mathrm{Cos}\,\frac{h3hw}{4586173h^{7}w^{4}}i\ 84\ \frac{h}{2}$$

$$2h^{3}\ 1028160\ 2286848h^{4}w^{4}\mathrm{Cos}\,\frac{h3hw}{2}i$$

$$54923100h^{7}w^{4}\mathrm{Cos}\,\frac{h5hw}{22}i + 3h^{2}\ 289758h^{5}w^{4}\mathrm{Cos}\,\frac{h7hw}{}i$$

$$43070h^{5}w^{4}\mathrm{Cos}\,\frac{h\,9hw}{2}i\qquad 67\ 2582280h^{7}w^{4} + 5h^{3}$$

$$36288 + 468224\qquad \mathrm{Cos}\,\frac{7hw}{2}\ \frac{9hw}{2}$$

$$h^{6}w^{3}\qquad \frac{4w^{4}}{}\qquad 5\mathrm{Cos}\,\frac{}{}\ \mathrm{Sin}[4hw]$$

$$+ 25401600\qquad 3\mathrm{Cos}\,h\ 2\ i + \frac{7hw}{}\,h\ 2\ i\ \frac{hw}{9}$$

$$25401600h^{6}w^{3}\mathrm{Cos}[4hw]\ 3\mathrm{Sin}\,\frac{h}{2}i + 5\mathrm{Sin}\,\frac{h}{2}i$$

$$y_{n+5}^{000} = \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} - \frac{3y_{n+4}}{2h^3} + \frac{3y_{n+5}}{5h^3}$$

$$+ \frac{1}{464486400h^6w^4} f_{n+4}\mathrm{Csc}\left[\frac{hw}{2}\right] \quad 19051200h^3$$

$$20252610h^7w^4\mathrm{Cos}[hw] \quad 42 \quad 1660114h^7w^4 + 420h^3 \quad 1152$$

$$+ 4375h^4w^4 \quad \mathrm{Cos}[hw] + 8100456h^7w^4\mathrm{Cos}[2hw]$$

$$2967786h^7w^4\mathrm{Cos}[3hw] + 124272h^7w^4\mathrm{Cos}[4hw]$$

$$12\ 869210h^7w^4 \quad 20h^3 \quad 5292 + 44375h^4w^4 \quad \mathrm{Cos}[4hw]$$

$$h \quad 3769962h^6w^4\mathrm{Cos}[hw] \quad 774984h^6w^4\mathrm{Cos}[2hw]$$

$$+ 205254h^6w^4\mathrm{Cos}[3hw] \quad 25032h^6w^4\mathrm{Cos}[4hw] \quad \frac{1}{464486400h^6w^4} f_{n+8}\mathrm{Csc}\left[\frac{hw}{2}\right] \mathrm{Csc}[hw]$$

$$+ 12700800h^6w^3\mathrm{Sin}[hw]$$

$$2913750h^7w^4\mathrm{Cos}\left[\frac{hw}{2}\right] \quad 30 \quad 992059h^7w^4 + 3h^3$$

$$20160 + 355625h^4w^4 \quad \mathrm{Cos}\left[\frac{hw}{2}\right] \quad 2151954h^7w^4\mathrm{Cos}\left[\frac{3hw}{2}\right]$$

$$765625h^7w^4 + 35h^3 \quad 1152 \quad 21875h^4w^4 \quad \mathrm{Cos}\left[\frac{3hw}{2}\right]$$

$$6h^2 \quad 90h^5w^4\mathrm{Cos}\left[\frac{5hw}{2}\right] \quad 18h^5w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$18h6915h^6w^4\mathrm{Cos}\left[\frac{5hw}{2}\right] + 633h^6w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$6\ 10300h^7w^4\mathrm{Cos}\left[\frac{5hw}{2}\right] + 528015h^7w^4 \quad 5h^3 \quad 12096^2$$

$$+ 105625h^4w^4 \quad \mathrm{Cos}\left[\frac{7hw}{2}\right] \quad 3628800h^6w^3\mathrm{Cos}[5hw]\mathrm{Sin}\left[\frac{7hw}{2}\right]$$

$$3628800h^6w^3\mathrm{Cos}\left[\frac{7hw}{2}\right]\mathrm{Sin}[5hw]$$

$$+ \frac{1}{464486400h^6w^4} f_n\mathrm{Csc}\left[\frac{hw}{2}\right] \mathrm{Csc}[hw]$$

$$4849290h^7w^4\mathrm{Cos}\left[\frac{hw}{2}\right] \quad 30 \quad 4062797h^7w^4 + 9h^3$$

$$12096 + 453125h^4w^4 \quad \mathrm{Cos}\left[\frac{hw}{2}\right] \quad 671382h^7w^4\mathrm{Cos}\left[\frac{3hw}{2}\right]$$

$$h^7w^4 \quad h^3 \quad 67200$$

$$54 \quad 2257019 \quad + \quad 3hw \quad 7hw$$

$$410h^5w^4\mathrm{Cos}\frac{h}{2} \quad 46h^5w^4\mathrm{Cos}\frac{h}{2} \quad ihw$$

$$18h \quad 4125h^6w^4\mathrm{Cos}\frac{h5}{2} \quad 423h^6w^4\mathrm{Cos}\frac{h7}{2}$$

$$6\ 78250h^7w^4\mathrm{Cos}\frac{h5hw}{22} + 10060h^7w^4\mathrm{Cos}\frac{h7hw}{22}$$

$$60480h^3\mathrm{Cos}\frac{9hw}{2} \quad 3628800h^6w^3\mathrm{Cos}[5hw]\mathrm{Sin}\frac{h9hw}{2}$$

$$3628800h^6w^3\mathrm{Cos}\frac{9hw}{2}\mathrm{Sin}[5hw]$$

$$+\frac{1}{116121600h^6w^4}f_{n+6}\mathrm{Csc}\frac{hw}{2}\mathrm{Csc}[hw]^7 \quad \frac{1}{2}$$

$$44295300h^7w^4\mathrm{Cos}\frac{hhw}{2} + 21\ 20769086h^7w^4$$

$$+ 180h^3 \quad 8064 \quad 123125h^4w^4\mathrm{Cos}\frac{hhw}{2}$$

$$13774950h^7w^4\mathrm{Cos}\frac{3hw}{2} \quad 42\ 11280796h^7w^4$$

$$2h^3\ 423360\ 5583125h^4w^4\mathrm{Cos}\frac{h3hw}{2}$$

$$2358750h^7w^4\mathrm{Cos}\frac{h5hw}{22} \quad 12h17556h^6w^4\mathrm{Cos}\frac{h7hw}{22}$$

$$3246h^6w^4\mathrm{Cos}\frac{h9hw}{2} \quad 6\ 7\ 73140h^7w^4 +$$

$$5h^3$$

$$18144 + 15625h^4w^4 \quad \mathrm{Cos}\frac{h7hw}{2} + 1831130h^7w^4$$

$$10h^3 \quad 21168 + 183125h^4w^4 \quad \mathrm{Cos}\frac{h9hw}{2}$$

$$h\ 7121940h^6w^4\mathrm{Cos}\frac{h3hw}{22} + 34380h^6w^4\mathrm{Cos}\frac{h5hw}{22}$$

+6h 504h^{5}w^{4}Cos h 7hw/2 i + 68h^{5}w^{4}Cos h 9hw/2 i + 6

17556h^{6}w^{4}Cos h7hw i/22 + 3246h^{6}w^{4}Cos h9hw i/

6350400h^{6}w^{3}Cos[5hw] 3Sin h 7hw 2 / 9hw h 2 i

+ 1 h 2 hw 7 h 2 i

6350400h^{6}w^3 3Cos / + Cos / Sin[5hw]

464486400h^{6}w^4 f$_{n+7}$Csc h i/2 Csc[hw]

24240300h^{7}w^{4}Cos hhw i/2 + 6 60112466h^{7}w^4

20160h^3 132 + 3125h^{4}w^4 Cos hhw i/2

13821372h^{7}w^{4}Cos h3hw i 12 33992828h^{7}w^4/2

14h^3 112320 2414375h^{4}w^4 Cos h3hw i/2

6h^{2}1400h^{5}w^{4}Cos h5hw i/2

+ 46h^{5}w^{4}Cos h7hw i/2 18h^{5}w^{4}Cos h9hw i/2 18h

31620h^{6}w^{4}Cos h5hw i/22 423h^{6}w^{4}Cos h7hw i/

+ 633h^{6}w^{4}Cos h 9hw 2 i 6 148150h^{7}w^{4}Cos h 5hw 2 i

1931935h^{7}w^4 15h^3 28224

128125h^{4}w^4 Cos h7hw i/2 + 528015h^{7}w^4

12096 + 105625 h/2

3628800h^{6}w^{3}Cos[5hw] 7Sin h 7hw 2 i hw

+ Sin h 9/2 i + 3628800h^{6}w^3 7Cos h 7/2 i

hw

h 9 i

+Cos 2 1 Sin[5hw]

+ f +2Csc hw 7

232243200h6w4 n hw h 2 i Csc[hw]

h 2

47226900h′w4Cos + 21 27149018h′w4 + 180h3 3hw 10752

h i

156875h4w4 Cos 2 16815330h7w4Cos h 2 i

5hw hw Cos

42 13656284h′w4 + 2h3 544320 6701875h 4w4 3hw i

2

h i h 7

254730h7w4Cos 2 12h 9996h6w4Cos 2 i

h9hw i

1746h6w4Cos 6 7

1192660h7w4 2

105h3288 + 11875h4w4

h7hw i

Cos 2

4354570h7w4 + 10h3 63504 + 439375h4w4 Cos h9hw i 2

h3hw i h5hw i

h 1758036h6w4Cos 99060h6w4Cos

22

+ 6h 952h5w4Cos 7hw + 196h5w4Cos 9hw i

+ 6 9996h w Cos 2 7hw h 2 9hw

6 4 1746h6w4Cos

6 3 hw] Sin + 3Sin

12700800h w Cos[5 7hw h 2 i 9hw h 2 i

+ 12700800h5w3 h Cos + 3Cos h Sin[5hw]

f Csc Csc[hw]

464486400h6w4 n+5 hw h 2 i

n

199720500h w Cos 2i + 210 hw i

+ 576h 3hw h 2

3 1092 10625h4w4 Cos

h 3hw hw

33630660h′w4Cos 2 84 13656284h′w4 + 2h3 907200

h 5 h

6701875h4w4 Cos 2 27570900h7w4Cos 2 i

12h 433713h6w4Cos h7hw i / 2

63855h6w4Cos h9hw i / 5h3 6 7 2422145h7w4 +

60480 + 486875h4w4 Cos h7hw i / 2 + 5 2171605h7w4 2

h3 254016 + 2168125h4w4 Cos h9hw i / 2

h 3516072h6w4Cos h3hw i / 2 + 14009640h6w4Cos h5hw i / 2

+ 6h 428414h6w4Cos 7hw / 2 + 61250h6w4Cos 9hw / 2

+ 6 433713h6w4Cos h 2 i / 2 + 63855h6w4Cos h9hw i / 2

h6w3 hw 5Sin + 3Sin

25401600 Cos[5 h 7]hw / 2 + 3Cos h 2 i / 2 Sin[5hw]

25401600h6w3 5Cos

464486400h6w4 fn+3Csc h 2 i / 2 Csc[hw]

199720500h7w4Cos hw i / 2 + 210 5572346h7w4

+ 576h3 1260 10625h4w4 Cos hhw i / 2

27549900h7w4Cos h3hw i 84 11280796h7w4 2 h

2h3 1028160 5583125h4w4 Cos h3hw i / 2

48608220h7w4Cos h5hw i / 2 12h 92463h6w4Cos h7hw i

13425h6w4Cos h9hw i / 5949905h7w4 6 7 2

5h3 36288 + 1143125h4w4 Cos h7hw i / 2

+5 3438725h⁷w⁴ 7h³ 60480 + 483125h⁴w⁴ Cos h9hw i / 2

h 14243880h⁶w⁴Cos h3hw i / 22 1582680h⁶w⁴Cos h5hw i

+ 6h 6398h⁵w⁴Cos 7hw / 2 i + 1090h⁵w⁴Cos 9hw / 2 i

+ 6 92463h⁵w⁴Cos h / 2 i 7hw 13425h⁵w⁴Cos h 9hw / 2 i

6 3 hw] 3Sin + 5Sin

25401600h w Cos[5 h 7hw h 2 i 9hw h 2 i

+ 25401600h⁵w⁵ 3Cos + 5Cos Sin[5hw]

f +1Csc Sec

928972800h⁶w⁴ n hw h 2 i h 2 i

29213100h⁷w⁴Cos hhw i / 2 i + 6 111091442h⁷w⁴ 60480h³

68 + 1875h⁴w⁴ Cos h3hw i 5813220h⁷w⁴Cos / 22

12 65049852h⁷w⁴ + 14h³ 4620625h⁴w⁴ Cos 3hw

9hw h 5hw 164160 h 7hw h 2 i

6h² 2040h⁵w⁴Cos 18h⁵w⁴Cos 2

+ 46h w Cos 2hw 18h 13500h 9hw h 2 i

5 4 w⁴Cos

+ 633h⁶w⁴Cos h 7 / 2 i 423h⁶w⁴Cos h / 2 i

6 759250h⁷w⁴Cos h5hw i + 528015h⁷w⁴ 5h³ 12096 2

105625h⁴w⁴ Cos h7hw i + 1931935h⁷w⁴ 2

15h³ 28224 + 128125h⁴w⁴ Cos 9hw

hw h 2 i 9hw

3628800h⁶w³Cos[5hw] Sin 7 / 2 i + 7Sin / 2 i

3628800h⁶w³ Cos h7hw i / 22 + 7Cos h9hw i Sin[5hw]

$$y_n^{000}{}_{+6} = \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} - \frac{3y_{n+4}}{2h^3} + \frac{3y_{n+5}}{5h^3} + \frac{1}{46448640h^6w^4} f_{n+4} \operatorname{Csch}\left[\frac{hw}{2}\right]^8 \Big(19051200h^3$$

$$20083392h^7w^4 \operatorname{Cos}[hw] \; 42 \quad 3453000h^7w^4 + 420h^3 \; 1152$$

$$+ 9072h^4w^4 \quad \operatorname{Cos}[hw] + 5690160h^7w^4 \operatorname{Cos}[2hw] \quad 2667600h^7w^4 \operatorname{Cos}[3hw]$$

$$+ 50904h^7w^4 \operatorname{Cos}[4hw] \; 12 \; 1818486h^7w^4 \quad 20h^3 \; 5292$$

+ 92016h^4 w^4 Cos[4hw] 2h 1161888h^6 w^4 Cos[hw] + 2192064h^6 w^4 Cos[2hw]

h^6 w^4 Cos[3hw] + 20832h^6 w^4 Cos[4hw] + 12700800h^6 w^3 Sin[2hw]

199584 1 hw 7 4 Co hw

+ 464486400h^6 w^4 f_{n+8} Csch 2 i Csc[hw] 1545510h w 7 s h hw 2 i

30 2109762h 7 hw 4 + 3h 20160 + 737424h^4 w^4 Cos

4658850h^7 w^4 Cos h 3 2 i 54 1591066h^7 w^4 + 35h^3 1152

h3hw i h5hw i h7hw i

45360h^4 w^4 Cos 6h^3 32040h^4 w^4 Cos + 972h^4 w^4 Cos

22 22

h5hw i h7hw i h5hw i

33h^2 29570h^5 w^4 Cos 1234h^5 w^4 Cos 36h 6840h^6 w^4 Cos

22 22

h7hw i h5hw i

210h^6 w^4 Cos 6 7740h^7 w^4 Cos + 1095336h^7 w^4

5h^3 219024h^4 w^4 Cos 7hw 3628800h^6 w^3 Cos[6hw] Sin 7hw

12096 + 7hw h 2 i 1 hw 7 h 2 i

+ 3628800h^6 w^3 Cos 2 Sin[6hw] + 464486400h^6 w^4 f_n Csc h 2 i Csc[hw]

hhw i

5411610h^7 w^4 Cos 2 30 8398350h^7 w^4 + 9h^3 12096

+ 939600h^4 w^4 hw h′ w^4 Cos 3hw i 54 4669122h′ w^4

Cos h 2 i 2479734 hw h 2 5hw

3

h i

+ h^3 67200 4653936h^4 w^4 Cos 2 6h^3 4680h^4 w^4 Cos h 2 i

260

468h^4 w^4 Cos h^7 hw/2 i hw 33h^2 4770h^5 w^4 Cos h^5 hw/2 i

+ 450h^5 w^4 Cos h^7 hw/2 i 36h 240h^6 w^4 Cos h^5 hw/2 i

42h^6 w^4 Cos h7hw i/22 6 80820h^7 w^4 Cos h5hw i/

+ 9792h^7 w^4 Cos h7hw i/2 60480h^3 Cos h9hw i/2

3628800h^6 w^3 Cos[6hw] Sin 9hw i/2h

3628800h^6 w^3 Cos 9hw i/Sin[6hw] 2 h

1/(+116121600h^6 w^4 f_{n+6} Csc h hw/2 i 7 Csc[hw] 1/2)

16692480h^7 w^4 Cos hw/2 i + 21 43712266h^7 w^4 h

+ 180h^3 8064 255312h^4 w^4 Cos hhw/2 i

39005568h^7 w^4 Cos h3hw i/2 42 23483517h^7 w^4

2h^3 423360 11577168h^4 w^4 Cos h3hw i/2

2423430h^7 w^4 Cos h5hw i/22 9h^2 90538h^5 w^4 Cos h7hw i/

14560h^5 w^4 Cos h9hw i/22 24h 21735h^6 w^4 Cos h7hw i/

3441h^6 w^4 Cos h9hw i/5h^3 6 7 157806h^7 w^4 +

2

18144 + 32400h^4 w^4 Cos h7hw i/ + 3798306h^7 w^4 2

10h^3 21168 + 379728h^4 w^4 Cos h9hw i/2

2h 14540736h^6 w^4 Cos h3hw i/2

+ 137280h^6 w^4 Cos h 5hw/2 hw i + 3h^2 122346h^4 w^4 Cos h 7hw/2 hw i

16254h^4 w^4 Cos h 9/2 i + 12h 90538h^5 w^4 Cos h 7/2 i

$$14560h^5w^4\cos\left(\frac{9hw}{2}\right) + 6\ 21735h^6w^4\cos\left(\frac{7hw}{2}\right)$$

$$3441h^6w^4\cos\left(\frac{9hw}{2}\right)$$

$$6350400h^6w^3\cos[6hw]\ 3\sin\left(\frac{7hw}{2}\right) + \sin\left(\frac{9hw}{2}\right)$$

$$+\ 6350400h^6w^3\ 3\cos\left(\frac{7hw}{2}\right) + \cos\left(\frac{9hw}{2}\right)\ \sin[6hw]$$

$$464486400h^6w^4 f_{n+7}\csc\left(\frac{h}{2}\right)\csc[hw]$$

$$11829888h^7w^4\cos\left(\frac{hw}{2}\right) + 6\ 126121344h^7w^4$$

$$20160h^3\ 132 + 6480h^4w^4\cos\left(\frac{hhw}{2}\right)$$

$$30580224h^7w^4\cos\left(\frac{h3hw}{2}\right)\ 12\ 70739769h^7w^4\ 2^h$$

$$14h^3\ 112320\ 5006448h^4w^4\cos\left(\frac{h3hw}{2}\right)$$

$$6h^3\ 248400h^4w^4\cos\left(\frac{h5hw}{2}\right)\ 468h^4w^4\cos\left(\frac{h7hw}{2}\right)$$

$$+\ 972h^4w^4\cos\left(\frac{9hw}{2}\right)\ 33h^2\ 177540h^5w^4\cos\left(\frac{5hw}{2}\right)$$

$$+\ 450h^5w^4\cos\left(\frac{h7}{2}\right)\ 1234h^5w^4\cos\left(\frac{h9}{2}\right)$$

$$36h\ 43920h^6w^4\cos\left(\frac{h5hw}{2}\right)\ 42h^6w^4\cos\left(\frac{h7hw}{2}\right)$$

$$210h^6w^4\cos\left(\frac{h9hw}{2}\right)\ 6\ 139320h^7w^4\cos\left(\frac{h5hw}{2}\right)$$

$$3994992h^7w^4\ 15h^3\ 28224$$

$$265680h^4w^4\cos\left(\frac{h7hw}{2}\right) + 1095336h^7w^4$$

$$5\ h^3\ 12096 + 219024\ h^4w^4\cos\left(\frac{9hw}{2}\right)$$

$$3628800\ h^6w^3\ 7\sin\left(\frac{7hw}{2}\right) + \sin\left(\frac{9hw}{2}\right)$$

$$\cos[6hw]$$

$$+\ 3628800h^6w^3 7\cos\left(\frac{h}{2}\right) + \cos\left(\frac{h}{2}\right)\ \sin[6hw]$$

$$+232243200h^{6}w^{4}f_{n+2}\mathrm{Csc}\left[\frac{hw}{2}\right]^{7}\mathrm{Csc}[hw]$$

$$40207104h^{7}w^{4}\mathrm{Cos}\left[\frac{hhw}{2}\right] + 21\ 56256342h^{7}w^{4}$$

$$+ 180h^{3}\ 10752\ 325296h^{4}w^{4}\ \mathrm{Cos}\left[\frac{hhw}{2}\right]$$

$$24425856h^{7}w^{4}\mathrm{Cos}\left[\frac{h3hw}{2}\right]\ 42\ 28212387h^{7}w^{4}$$

$$2h^{3}\ 544320\ 13897008h^{4}w^{4}\ \mathrm{Cos}\left[\frac{h3hw}{2}\right]$$

$$1743930h^{7}w^{4}\mathrm{Cos}\left[\frac{h5hw}{22}\right]\ 9h^{2}11970h^{5}w^{4}\mathrm{Cos}\left[\frac{h7hw}{}\right]$$

$$+ 1768h^{5}w^{4}\mathrm{Cos}\left[\frac{9hw}{hhw_{2}}\right]\ 24h\ 399h^{6}w^{4}\mathrm{Cos}\left[h\frac{7hw}{2}\right]$$

$$225h^{6}w^{4}\mathrm{Cos}\left[h\frac{9}{2}\right]\ 6\ 7\ 2530422h^{7}w^{4} + 105h^{3}$$

$$288 + 24624h^{4}w^{4}\ \mathrm{Cos}\left[\frac{h7hw}{}\right]$$

$$9070578h^{7}w^{4}\ 2$$

$$10h^{3}\ 63504 + 911088h^{4}w^{4}\ \mathrm{Cos}\left[\frac{h9hw}{2}\right]$$

$$2h\ 7643328h^{6}w^{4}\mathrm{Cos}\left[\frac{h3hw}{22}\right] + 1569600h^{6}w^{4}\mathrm{Cos}\left[\frac{h5hw}{}\right]$$

$$+ 3h^{2}\ 11466h^{4}w^{4}\mathrm{Cos}\left[h\frac{7hw}{2}\right]\ 1854h^{4}w^{4}\mathrm{Cos}\left[h\frac{9hw}{2}\right]$$

$$+ 12\ 11970\ 7hw\ h\ 2\ i\ 9hw\ h\ 2\ i$$

$$h\ h^{5}w^{4}\mathrm{Cos}\left[\frac{}{}\right] + 1768h^{5}w^{4}\mathrm{Cos}\left[\frac{}{}\right]$$

$$+ 6\ 399\ \mathrm{Cos}\ 2\ 225\ hw\ h\ 2\ i\ hw$$

$$h^{6}w^{4}\left[\frac{}{}\right]\ h^{6}w^{4}\mathrm{Cos}\left[\frac{}{}\right]$$

$$12700800h^{6}w^{3}\mathrm{Cos}[6hw]\ \mathrm{Sin}\left[h\frac{7}{2}\right] + 3\mathrm{Sin}\left[h\frac{9}{2}\right]$$

$$12700800h^{6}w^{3}\ \mathrm{Cos}\left[\frac{h7hw}{22}\right] + 3\mathrm{Cos}\left[\frac{h9hw}{}\right]\mathrm{Sin}[6hw]$$

$$\frac{1}{464486400h^{6}w^{4}}f_{n+5}\mathrm{Csc}\left[\frac{hw}{2}\right]^{7}\mathrm{Csc}[hw]$$

$$157127040h^{7}w^{4}\mathrm{Cos}\left[\frac{hw}{2}\right] + 210\ 11657168h^{7}w^{4}$$

$$+ 576h^{3}\ 1092\ 22032h^{4}w^{4}\mathrm{Cos}\left[\frac{hhw}{2}\right]$$

$$48851712h^{7}w^{4}\mathrm{Cos}\left[\frac{h3hw}{2}\right]\ 84\ 28212387h^{7}w^{4}\ 2^{h}$$

$$2h^{3}\ 907200\ 13897008h^{4}w^{4}\mathrm{Cos}\left[\frac{h3hw}{2}\right]$$

$$92225700h^{7}w^{4}\mathrm{Cos}\left[\frac{h5hw}{2}\right]\ 9h^{2}$$

$$124390h^{5}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right] + 12950h^{5}w^{4}\mathrm{Cos}\left[\frac{h9hw}{2}\right]$$

$$24h\ 752262h^{6}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right]$$

$$106290h^{6}w^{4}\mathrm{Cos}\left[\frac{h9hw}{2}\right]\ 6\ 7\ 5124024h^{7}w^{4}$$

$$+ 5h^{3}\ 2^{h^{4}w^{4}}$$

$$60480 + 1009584\ \mathrm{Cos}\left[\frac{7hw}{2}\right]\ 5\ 4517136h^{7}w^{4}\ ihw$$

$$h^{3}\ 254016 + 4495824h^{4}w^{4}\ \mathrm{Cos}\left[\frac{9}{2}\right]\ i$$

$$h\ h^{6}w^{4}\mathrm{Cos}\left[\frac{3hw}{2}\right]$$

$$2\ 15286656\ \mathrm{Cos}\left[\frac{5hw}{2}\right]\ i$$

$$+27680640h^{6}w^{4}\mathrm{Cos}\left[\frac{hw}{2}\right]\ i + 3h^{2}\ 552132h^{4}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right]\ i$$

$$75420h^{4}w^{4}\mathrm{Cos}\left[\frac{h^{9}}{2}\right]\ i\ hw + 12h$$

$$124390h^{5}w^{4}\mathrm{Cos}\left[\frac{7}{2}\right]\ i + 12950h^{5}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right]\ i + 6$$

$$752262h^{6}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right] + 106290h^{6}w^{4}\mathrm{Cos}\left[\frac{h9hw}{2}\right]$$

$$25401600h^{6}w^{3}\mathrm{Cos}[6hw]\ 5\mathrm{Sin}\left[\frac{7hw}{2}\right]\ h$$

$$+ 3\mathrm{Sin}\left[\frac{h^{9}hw}{2}\right]\ i + 25401600h^{6}w^{3}\ 5\mathrm{Cos}\left[\frac{h^{7}hw}{2}\right]\ i$$

$$+3\cos\left(\frac{h\left(\frac{9hw}{2}\right)}{1}\right)\sin[6hw]$$

$$464486400h^{6}w^{4}\,f_{n+3}\csc\left(\frac{hw}{2}\right)^{7}\csc[hw]$$

$$157127040h^{7}w^{4}\cos\left(\frac{hw}{2}\right) + 210\ 11657168h^{7}w^{4}$$

$$+ 576h^{3}\ 1260\ 22032h^{4}w^{4}\cos\left(\frac{hhw}{2}\right)$$

$$78011136h^{7}w^{4}\cos\left(h3hw\right)\ 84\ 23483\cancel{517h^{7}w^{4}}$$

$$2h^{3}\ 1028160\ 11577168h^{4}w^{4}\cos\left(\frac{h3hw}{2}\right)$$

$$h^{7}w^{4}\cos\left(\frac{5hw}{2}\right)\ 9h^{2}$$

$$59913180\ h^{6}w^{4}\ h\left(\frac{7hw}{2}\right)\ + 12730h^{6}w^{4}\cos\left(\frac{9hw}{2}\right)$$

$$99274\ \cos\left(\frac{hw}{2}\right)\ h\ 2$$

$$24h\ 10290h^{6}w^{4}\cos\left(\frac{hw}{2}\right)^{7}\ 2310h^{6}w^{4}\cos\left(\frac{hw}{2}\right)^{9}$$

$$6\ 7\ 12077712h^{7}w^{4} + 5h^{3}\ 36288$$

$$2370384h^{4}w^{4}\cos\left(h7hw\right) + 5\ 7067736h^{7}w^{4}\ 2$$

$$7h^{3}\ 60480 + 1001808h^{4}w^{4}\cos\left(\frac{h9hw}{2}\right)$$

$$2h\ 29081472h^{6}w^{4}\cos\left(h3hw\right)\ 2$$

$$10838400h^{6}w^{4}\cos\left(\frac{5hw}{2}\right)$$

$$3h^{2}\ 98532h^{4}w^{4}\cos\left(\frac{h7hw}{2}\right)\ h$$

$$13500h^{4}w^{4}\cos\left(\frac{h\ 7hw}{2}\right)^{9hw}\ + 12h\ 9hw$$

$$99274h^{6}w^{4}\cos\left(\frac{h\ 2\ 7hw}{2}\right) + 12730h^{6}w^{4}\cos\left(\frac{h\ 9hw}{2}\right)$$

$$+ 6\ 10290h^{6}w^{4}\cos\left(\frac{h\ 2}{2}\right)\ 2310h^{6}w^{4}\cos\left(\frac{h\ 2\ 9hw}{2}\right)$$

$$25401600\ h^{6}w^{3}\ hw\ 3\sin\left(\frac{7hw}{2}\right) + 5\sin\left(\frac{9hw}{2}\right)$$

$$+25401600h^{6}w^{3}\ 3\cos[6hw]\ +5\cos[6hw]\ \sin[6hw]$$

$$928972800h^{6}w^{4}\,f_{n+1}\csc\left(\frac{h}{2}\right)\ 265\ \sec\left(\frac{h}{2}\right)$$

$$25764480h^7w^4\mathrm{Cos}\left[\frac{hw}{2}\right] + 6\,230163232h^7w^4$$

$$60480h^3\,68 + 3888h^4w^4\,\mathrm{Cos}\left[\frac{hhw}{2}\right]$$

$$19192320h^7w^4\mathrm{Cos}\left[\frac{h3hw}{2}\right] \quad 12\,134706279h^7w^4\,2^h$$

$$14h^3\,164160\,9581328h^4w^4\,\mathrm{Cos}\left[\frac{h3hw}{2}\right]$$

$$6h^3\,55440h^4w^4\mathrm{Cos}\left[\frac{h5hw}{2}\right] + 972h^4w^4\mathrm{Cos}\left[\frac{h7hw}{2}\right]$$

$$468h^4w^4\mathrm{Cos}\left[\frac{h9hw}{2}\right] \quad 33h^2 \quad 5180h^5w^4\mathrm{Cos}\left[\frac{h5hw}{2}\right]$$

$$1234h^5w^4\mathrm{Cos}\left[\frac{h7hw}{2}\right] + 450h^5w^4\mathrm{Cos}\left[\frac{h9hw}{2}\right]$$

$$36h\,7680h^6w^4\mathrm{Cos}\left[\frac{h5hw}{2}\right] \quad 210h^6w^4\mathrm{Cos}\left[\frac{h7hw}{2}\right]$$

$$42h^6w^4\mathrm{Cos}\left[h\frac{9hw}{2}\right] \quad 6 \quad 769320h^7w^4\mathrm{Cos}\left[h\frac{5hw}{2}\right] \quad h7hw$$

$$1095336h^7w^4\,5h^3\,12096 + 219024h^4w^4\,\mathrm{Cos}\left[\frac{h7hw}{2}\right]$$

$$3994992h^7w^4\,15h^3\,28224 + 265680h^4w^4\,\mathrm{Cos}\left[\frac{h9hw}{2}\right]$$

$$3628800\,h^6w^3\,\mathrm{Cos}[6hw]\,\mathrm{Sin}\left[h\frac{7hw}{2}\right] + 7\mathrm{Sin}\left[\frac{9hw}{2}\right]$$

$$+\,3628800h^6w^3\mathrm{Cos}\left[\frac{h}{2}\right] + 7\mathrm{Cos}\left[\frac{h}{2}\right]\mathrm{Sin}[6hw]$$

$$y_{n+7}^{000} = \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} \quad \frac{3y_{n+4}}{2h^3} + \frac{3y_{n+5}}{5h^3}$$

$$+ \frac{1}{46448640h^6w^4} f_{n+4}\mathrm{Csc}\left[\frac{hw}{2}\right]^8 i\ 19051200h^3$$

$$15589350h^7w^4\mathrm{Cos}[hw]\ 42 \quad 6569486h^7w^4 + 420h^3\ 1152$$

$$+ 16807h^4w^4\ \mathrm{Cos}[hw] \quad 1645224h^7w^4\mathrm{Cos}[2hw]$$

$$3850686h^7w^4\mathrm{Cos}[3hw] + 87696h^7w^4\mathrm{Cos}[4hw]$$

$$12\ 3386334h^7w^4 \quad 20h^3\ 5292 + 170471h^4w^4\ \mathrm{Cos}[4hw]$$

$$3h \quad 44982h^6w^4\mathrm{Cos}[hw] + 1915704h^6w^4\mathrm{Cos}[2hw]$$

$$+ 1348326h^6w^4\mathrm{Cos}[3hw]\ 43848h^6w^4\mathrm{Cos}[4hw]$$

$$+ 12700800\ h^6w^3\mathrm{Sin}[3hw] \quad + \frac{1}{464486400h^6w^4} f_{n+8}\ \mathrm{Csc}\left[\frac{hw}{2}\right]^7 i\ \mathrm{Csc}[hw]$$

$$2116230h^7w^4\mathrm{Cos}\left[\frac{hw}{2}\right] i\ 30\ 3980435h^7w^4 + 3h^3$$

$$20160 + 1366169h^4w^4\ \mathrm{Cos}\left[\frac{hhw}{22}\right] i\ 1944810h^7w^4\mathrm{Cos}\left[\frac{h3hw}{2}\right] i$$

$$54\ 2950341h^7w^4 + 35h^3\ 1152\ 84035h^4w^4\ \mathrm{Cos}[h3hw]\ 2$$

$$24h^3 26370h^4w^4\mathrm{Cos}\left[\frac{h5hw}{22}\right] i + 19782h^4w^4\mathrm{Cos}\left[\frac{h7hw}{22}\right] i$$

$$78h^2\ 41170h^5w^4\mathrm{Cos}\left[\frac{h5hw}{22}\right] i + 7210h^5w^4\mathrm{Cos}\left[\frac{h7hw}{22}\right] i$$

$$54h\ 46965h^6w^4\mathrm{Cos}\left[\frac{h5hw}{22}\right] i + 1233h^6w^4\mathrm{Cos}\left[\frac{h7hw}{22}\right] i$$

$$6\ 33530h^7w^4\mathrm{Cos}[h5hw]\ i + 2028901h^7w^4\ 5h^3\ 12096\ 2$$

$$+ 405769h^4w^4\ \mathrm{Cos}\left[h\ \frac{7hw}{2}\right] i \quad 3628800h^6w^3\mathrm{Cos}[7hw]\mathrm{Sin}\left[h\ \frac{7hw}{2}\right] i$$

$$3628800h^6w^3\mathrm{Cos}\left[\frac{7hw}{2}\right] i\ \mathrm{Sin}[7hw]\ 2$$

$$+ \frac{1}{464486400h^6w^4} f_n\mathrm{Csc}\left[h\ \frac{hw}{2}\right]^7 i\ \mathrm{Csc}[hw]\ 3709530h^7w^4\mathrm{Cos}\left[\frac{hw}{2}\right] i$$

$$30 \quad 15569653h^7w^4 + 9h^3\ 12096 + 1740725h^4w^4\ \mathrm{Cos}[h\ 2i]$$

4966542h^7w^4Cos[h3hw i / 67200 2] 54 8641015h^7w^4 + h^3

8621991h^4w^4 Cos[h3hw i / 22] 24h^3 22590h^4w^4Cos[h5hw i /]

378h^4w^4Cos[h7hw i / 22] 78h^2 10130h^5w^4Cos[h5hw i /]

406h^5w^4Cos[h 7hw / 2 ' 5hw / 2 i] 54h 1365h^5w^4Cos[7hw / 2 i] 15h^5w^4Cos[7hw / 2 i]

6 80360 Cos'' 2 i 9hw h 2 i 9hw h 2 i

h^7w^4 + 9842h^7w^4Cos[h / 2 i] 60480h^3Cos[h / w]

1 / 3628800h^5w^3Cos[7hw]Sin[h / hw 2 7 i] + 1 / 3628800h^5w^3Cos[h / 2 i] Sin[7hw]

+ f[h / 116121600h^6w^4 n+6 Csc / 2 i] Csc[hw] / 2 3428628 h^7w^4Cos[hw h 2 i]

+ 21 82810438 3hw h 2 i

h^7w^4 + 180h^3 8064 472997h^4w^4 Cos[5hw / h 2]

62696382h^7w^4Cos[h 3hw ' / 2] 42 43609084h^7w^4 + 2h^3 423360

21448133h^4w^4 Cos[h 72hw ' / h hw i / 2] 4638150h^7w^4Cos[9 ''hw i / 2]

24h^2 219800h^5w^4Cos[h hw ' / 2] 20788h^5w^4Cos[h hw ' / 2]

36h 177324h^6w^4Cos[h7 / 2 i] 23490h^6w^4Cos[h 9 / 2 i] 6 7

3276h^7w^4 + 5h^3 18144 + 60025h^4w^4 Cos[h7hw i / 2] + 7023086h^7w^4 2

10h^3 21168 + 703493h^4w^4 Cos[h9hw i / 22] 3h 9369108h^6w^4Cos[h3hw i /]

+ 1601100h^6w^4Cos[5hw / 9 hw2 '] + 8h^2 115416h^4w^4Cos[7hw / 7hw i]

+ 12996h^4w^4Cos[h hw i / 2] + 18h 219800h^5w^4Cos[''hw2 2 i]

20788h^5w^4Cos[h 9 / 2 i] + 6 177324h^6w^4Cos[h 7 / 2 i]

23490h^6w^4Cos[h9hw i / 22] 6350400h^6w^3Cos[7hw] 3Sin[h7hw i /]

+ Sin[h 9hw / 2 i] + 6350400h^6w^3 3Cos[h 7hw / 2 i] + Cos[h 9hw / 2 i] Sin[7hw]

$$\frac{1}{464486400h^{6}w^{4}}f_{n+7}\mathrm{Csc}\left[\frac{hw}{2}\right]^{7}\mathrm{Csc}[hw]$$

$$27416676h^{7}w^{4}\mathrm{Cos}\left[\frac{hw}{2}\right] + 6\,236310022h^{7}w^{4}$$

$$20160h^{3}\,132 + 12005h^{4}w^{4}\,\mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$+ 21561540h^{7}w^{4}\mathrm{Cos}\left[\frac{3hw}{2}\right]\quad 12\,130992636h^{7}w^{4}$$

$$+ 14h^{3}\;112320\;9275063h^{4}w^{4}\;\mathrm{Cos}\left[\frac{3hw}{2}\right]$$

$$24h^{3}\;233640h^{4}w^{4}\mathrm{Cos}\left[\frac{5hw}{2}\right]378h^{4}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$h^{4}w^{4}\;\mathrm{Cos}\left[\frac{9hw}{2}\right]\quad 78h^{2}\;442520h^{5}w^{4}\mathrm{Cos}\left[\frac{5hw}{2}\right]$$

$$+ 19782\;\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$406h^{5}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right] + 7210h^{5}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$54h\;354780h^{6}w^{4}\mathrm{Cos}\left[\frac{5hw}{2}\right]15h^{6}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$+ 1233h^{6}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right]\quad 6\quad 316190h^{7}w^{4}\mathrm{Cos}\left[\frac{5hw}{2}\right]$$

$$7392917h^{7}w^{4}\;15h^{3}\;28224 + 492205h^{4}w^{4}\,\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$2028901h^{7}w^{4}\;5h^{3}\;12096 + 405769h^{4}w^{4}\,\mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$h^{6}w^{3}\quad hw\quad 7\mathrm{Sin}\left[\frac{7hw}{2}\right] + \mathrm{Sin}\left[\frac{9hw}{2}\right]$$

$$3628800\quad \mathrm{Cos}[7\quad]hw\quad h\quad 2\,i\;hw\quad h\quad 2\;i$$

$$+ 3628800h^{6}w^{3}7\mathrm{Cos}\left[\frac{h}{2}\right] + \mathrm{Cos}\left[\frac{h}{2}\right]\mathrm{Sin}[7hw]$$

$$\frac{1}{+232243200h^{6}w^{4}}\, f_{n+2}\, \mathrm{Csc}\left[\frac{hw}{2}\right]^{7} i\, \mathrm{Csc}[hw]$$

$$21580188h^{7}w^{4}\mathrm{Cos}\left[\frac{hw}{2}\right] i + 21\ 105515266h^{7}w^{4}h$$

$$+ 180h^{3}\ \ 10752\ \ 602651h^{4}w^{4}\ \ \mathrm{Cos}\left[\frac{hhw}{2}\right] i$$

$$37791642h^{7}w^{4}\mathrm{Cos}\left[\frac{h3hw}{2}\right] i\ \ 42\ 52142348h^{7}w^{4}$$

$$2h^{3}\ 544320\ 25745923h^{4}w^{4}\ \mathrm{Cos}\left[\frac{h3hw}{2}\right] i$$

$$12644850h^{7}w^{4}\mathrm{Cos}\left[\frac{h5hw}{22}\right] i\ 24h^{2}\ 31192h^{5}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right] i$$

$$1460h^{5}w^{4}\mathrm{Cos}\left[\frac{h9hw}{22}\right] i\ 36h\ 4620h^{6}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right] i$$

$$66h^{6}w^{4}\mathrm{Cos}\left[\frac{h9hw}{2}\right] i\ \ 6\ 7$$

$$4735052h^{7}w^{4}\ \ 2$$

$$105h^{3}288 + 45619h^{4}w^{4}\ \mathrm{Cos}\left[\frac{h7hw}{2}\right] i$$

$$20h\ 805756218h^{8}w^{4} + 98h^{4}\ \ 164160\ \ 17750593h^{4}w^{4}$$

$$336h^{4}33120 + 2782759h^{4}w^{4}\ \mathrm{Cos}\left[\frac{h3hw}{2}\right] i$$

$$24h^{3}\ 56220h^{6}w^{4}\mathrm{Cos}\left[\frac{h5hw}{22}\right] i + 1233h^{6}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right] i$$

$$15h^{6}w^{4}\mathrm{Cos}\left[\frac{h9hw}{22}\right] i\ 26h^{2}\ \ 792890h^{7}w^{4}\mathrm{Cos}\left[\frac{h5hw}{2}\right] i$$

$$2028901h^{7}w^{4}\ \ 5h^{3}\ \ 12096$$

$$405769h^{4}w^{4}\ \mathrm{Cos}\left[\frac{h7hw}{2}\right] i + 7392917h^{7}w^{4}$$

$$h^{3}\ \ h^{4}w^{4}\ \ \mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$15\ \ 28224 + 492205\ \ \mathrm{Sin}\left[\frac{7hw}{2}\right] i + 7\mathrm{Sin}\left[\frac{9hw}{2}\right] i$$

$$+ 3628800h^{6}w\mathrm{Cos}[7hw]$$

$$3628800 h^{6} w \operatorname{Cos}\left[\frac{7}{2} h\right] + 7 \operatorname{Cos}\left[\frac{9}{2} h\right] \operatorname{Sin}[7 h w]$$

$24h^2$ $584934h^5w^4\mathrm{Cos}\frac{h7hw\ i}{2}$

$48730h^5w^4\mathrm{Cos}\frac{h9hw\ i}{2}$ $36h$

$541065h^6w^4\mathrm{Cos}\frac{h7hw}{2}i$ $+ 83415h^6w^4\mathrm{Cos}\frac{h9hw}{2}i$

$6\ 7$ $9528939h^7w^4 + 5h^3$ 60480

$1870379h^4w^4\ \mathrm{Cos}\ h7hw\ i + 5\ 8370439h^7w^4\ 2$

h^3 $254016 + 8329069h^4w^4$ $\mathrm{Cos}\frac{h9hw\ i}{2}$

$3h\ 11553192h^6w^4\mathrm{Cos}\ h3hw\ i\ 2$

$26735400h^6w^4\mathrm{Cos}\frac{5hw\ i}{2}$

$8h^2\ 769482h^4w^4\mathrm{Cos}\ h\frac{h7hw\ i}{2}2$ $12150h^4w^4\mathrm{Cos}\frac{h9hw\ i}{}$

$18h\ 584934h^5w^4\mathrm{Cos}\frac{h7hw\ i}{2}$

$48730h^5w^4\mathrm{Cos}\frac{h9hw}{2}i + 6$

$541065h^6w^4\mathrm{Cos}\frac{h7hw\ i}{2}2 + 83415h^6w^4\mathrm{Cos}\frac{h9hw\ i}{}$

h^6w^3 $hw\ 5\mathrm{Sin}\frac{7hw}{}, + 3\mathrm{Sin}\frac{9hw}{}$

25401600 $\mathrm{Cos}[7\frac{7hw}{2}]$ $h\ 7\ 2\ h\frac{9hw}{2}$ $h\ 2\ i$

$+25401600h^6w^4$ $+3\mathrm{Cos}$ $\mathrm{Sin}[7hw]$

$464486400h^6w^4 f_{n+3}\mathrm{Csc}\frac{h}{2}i\ \mathrm{Csc}[hw]$

$87732540h^7w^4\mathrm{Cos}\frac{hw\ i}{2} + 210\ 22213534h^7w^4$

$+ 576h^3\ 1260\ 40817h^4w^4\ \mathrm{Cos}\frac{hhw\ i}{2}$

$125392764h^7w^4\mathrm{Cos}\ h3hw\ i\ 84\ 43609084h^7w^4\ 2h$

$2h^3\ 1028160\ 21448133h^4w^4\ \mathrm{Cos}\frac{h3hw\ i}{2}$

$115932300h^7w^4\mathrm{Cos}\frac{h5hw}{27}i$

h^2 $h^5w^4\mathrm{Cos}\frac{h}{}$

24 200550 $\frac{hw}{9}$ $271^2\ i$ $\frac{7hw}{}$

$9050h^5w^4\mathrm{Cos}\frac{h}{2}i$ $36h$ $29463h^6w^4\mathrm{Cos}\frac{h}{2}i$

585h⁶w⁴Cos h9hw i 6 7 22183707h⁷w⁴ +

5h³ 2

36288 + 4391429h⁴w⁴ Cos h7hw i + 5 13046999h⁷w⁴ 2

7h³ 60480 + 1855973h⁴w⁴ Cos h9hw i

2

3h 18738216h⁶w⁴Cos h3hw i + 25232040h⁶w⁴Cos h5hw i

22 22

+ 8h² 467082h⁴w⁴Cos 7hw i 9270h⁴w⁴Cos 9hw i

h 7hw 2 h 9hw 2

7hw 9050h 9hw i

+ 18h 200550h⁵w⁴Cos 5 w 4 Cos

+ 6 29463h w Cos 2 585 7hw h 2 9i hw

6 4 h⁶w⁴Cos

6 3 hw] 3Sin + 5Sin

25401600h w Cos[7 h 7hw h 2 i 9hw h 2 i

+ 25401600h⁵w³ 3Cos + 5Cos Sin[7hw]

928972800h⁶w⁴ fn+1 Csc h i Sec h i

2 2

13455204h⁷w⁴Cos hw i + 6 429177766h⁷w⁴

2

60480h³ 68 + 7203h⁴w⁴ Cos hhw i

2

32320932h⁷w⁴Cos h3hw i 12 249473548h⁷w⁴ 2 h

14h³ 164160 17750593h⁴w⁴ Cos h3hw i

2

24h³ 124200h⁴w⁴Cos h5hw i + 19782h⁴w⁴Cos h7hw i

22 22

378h⁴w⁴Cos h9hw i 78h² 103960h⁵w⁴Cos h5hw i

22 22

+ 7210h⁵w⁴Cos h7hw i 406h⁵w⁴Cos h9hw i

2 2

54h 56220h⁶w⁴Cos h5hw i + 1233h⁶w⁴Cos h7hw i

22 22

15h⁶w⁴Cos h9hw i 6 792890h⁷w⁴Cos h5hw i

22 22

2028901h⁷w⁴ 5h³ 12096 + 405769h⁴w⁴ Cos h7hw i 2

7392917h⁷w⁴ 15h³ 28224 + 492205h⁴w⁴ Cos h9hw i 2

3628800h⁶w³Cos[7hw] Sin h 7hw i h 9hw i

2 2

3628800h⁶w³ Cos h7hw i + 7Cos h9hw i Sin[7hw]

22 22

$$y_{n+8}^{000} = \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} - \frac{3y_{n+4}}{2h^3} + \frac{3y_{n+5}}{5h^3}$$

$$+ \frac{1}{46448640h^6w^4} f_{n+4}\mathrm{Csc}\left[\frac{hw}{2}\right]^8 \Big(19051200h^3$$

$$12515328h^7w^4\mathrm{Cos}[hw] \quad 42 \quad 11464840h^7w^4 + 420h^3 \quad 1152$$

$$+ 28672h^4w^4 \quad \mathrm{Cos}[hw] \quad 3032400h^7w^4\mathrm{Cos}[2hw]$$

$$10728720h^7w^4\mathrm{Cos}[3hw] \quad 563472h^7w^4\mathrm{Cos}[4hw] \quad 12\ 5792360h^7w^4$$

$$20h^3 \quad 5292 + 290816h^4w^4 \quad \mathrm{Cos}[4hw] \quad 4h\ 413952h^6w^4\mathrm{Cos}[hw]$$

$$545664h^6w^4\mathrm{Cos}[2hw] + 2467584h^6w^4\mathrm{Cos}[3hw]$$

$$+ 839328\ 1 \quad \mathrm{Cos}[4 \quad] + \frac{hw}{\quad} \quad {}^7 \quad h^6w^4 \quad hw \quad 12700800h^6w^3\mathrm{Sin}[4hw]$$

$$+ \frac{1}{464486400 \quad h^7w^4} f_{n+8}\mathrm{Csc}\left[\frac{hw}{2}\right]^7 \mathrm{Csc}[hw] \quad 14966730h^7w^4\mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$30 \quad 6866602h^7w^4 + 3h^3 \quad 20160 + 2330624h^4w^4 \quad \mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$35902062h^7w^4\mathrm{Cos}\left[\frac{3hw}{2}\right] \quad 54\ 5032408h^7w^4$$

$$35h^3\ 1152\ 143360h^4w^4\ \mathrm{Cos}\left[\frac{3hw}{2}\right] \quad 60h^3\ 2$$

$$235080h^4w^4\mathrm{Cos}\left[\frac{5hw}{2}\right] + 70812h^4w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$141h^2\ 72990h^5w^4\mathrm{Cos}\left[\frac{5hw}{2}\right] + 49014h^5w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$72h \quad 48300h^6w^4\mathrm{Cos}\left[\frac{5hw}{2}\right] + 25122h^6w^4\mathrm{Cos}\left[\frac{7hw}{2}\right]$$

$$6\ 90400h^7w^4\mathrm{Cos}\left[\frac{5hw}{2}\right] + 3470928h^7w^4\ 5h^3\ 12096\ 2$$

$$+ 692224h^4w^4\ \mathrm{Cos}\left[\frac{7hw}{2}\right] \quad 3628800h^6w^3\mathrm{Cos}[8hw]\mathrm{Sin}\left[\frac{7hw}{2}\right]$$

$$3628800h^6w^3\mathrm{Cos}\left[\frac{7hw}{2}\right]\mathrm{Sin}[8hw]\ 2$$

$$+ \frac{1}{464486400h^6w^4} f_n\mathrm{Csc}\left[\frac{hw}{2}\right]^7 \mathrm{Csc}[hw]$$

$$3903930h^7w^4\mathrm{Cos}\left[\frac{hw}{2}\right] \quad 30 \quad 26601422h^7w^4 + 9h^3$$

$$12096 + 2969600h^4w^4\ \mathrm{Cos}\left[\frac{hw}{2}\right] \quad 2252178h^7w^4\mathrm{Cos}\left[\frac{3hw}{2}\right]$$

54 14733416h w +h 67200 14708736h w Cos

60h 1800h w Cos + 10332h w Cos

141h h w Cosh + 3178

72h 21660h w Cos + 498h w Cos

6 90400h w Cos + 9808h w Cos

60480h Cos 3628800h w Cos[8hw]Sin

+ 3628800h w Cos Sin[8hw]

+116121600h w f Csc Csc[hw]

9805824h w Cos + 21 142860170h w

+ 180h 8064 806912h w Cos

55222272h w Cos 42 74283709h w

2h 423360 36589568h w Cos

11770950h w Cos 45h 259182h w Cos

$$+38084h^5w^4\mathrm{Cos}\frac{9hw}{h}\quad 48h\ 432831h^6w^4\mathrm{Cos}\frac{7hw}{2}\ i$$

$$23901h^6w^4\mathrm{Cos}\frac{h\ 2\ i}{2}\quad 6\ 7\ 452088h^7w^4 + 5h^3$$

$$18144 + 102400h^4w^4\ \mathrm{Cos}\frac{7hw}{h\ 2}\ +hw\quad 11960984h'w^4$$

$$10h^3\quad 21168 + 1200128h^4w^4\ \mathrm{Cos}\frac{9}{2}\ i$$

$$4h\ 1311744h^6w^4\mathrm{Cos}\frac{h3hw\ i}{2}$$

$$2442240h^6w^4\mathrm{Cos}\frac{h5hw\ i}{2} + 15h^2\ 2$$

$$h^4w^4\quad \frac{7hw}{2}\ i + 122706h^4w^4\mathrm{Cos}\frac{9hw}{2}$$

$$79254\quad \mathrm{Cos}\ h^5w^4\mathrm{Cos}\frac{7hw}{2}\ i\quad + 38084h^5w^4\mathrm{Cos}\frac{9hw}{2}\ i$$

$$+ 24\quad 259182\quad h\frac{7hw}{2}\ i\quad 7hw\quad h\frac{9hw}{2}\ i$$

$$+6\quad 432831h^5w^4\mathrm{Cos}\frac{7hw}{2}\quad 23901h^5w^4\mathrm{Cos}\frac{9hw}{2}$$

$$h^6w^3\quad hw\quad 3\mathrm{Sin}\frac{h\ 2\ i}{2} + \mathrm{Sin}\frac{h\ 2\ i}{2}$$

$$6350400\quad \mathrm{Cos}[8\ h\ 7]hw\quad h\ 2\ 9hw\ i\quad h\ 2\ i$$

$$+\ 6350400h^5w^1\ 3\mathrm{Cos}\quad + \mathrm{Cos}\quad \mathrm{Sin}[8hw]$$

$$464486400h^5w^4\ f_{n+7}\mathrm{Csc}\ h\ i\ \mathrm{Csc}[hw]$$

$$58490880h^7w^4\mathrm{Cos}\frac{hw\ i}{2} + 6\ 404029376h^7w^4$$

$$20160h^3\ 132 + 20480h^4w^4\ \mathrm{Cos}\frac{hhw\ ih}{2}$$

$$+ 114499584h^7w^4\mathrm{Cos}\frac{h3hw\ i}{2}\quad 12\ 222372473h^7w^4$$

$$+ 14h^3\ 112320\quad 15822848h^4w^4\quad \mathrm{Cos}\frac{h3hw\ i}{2}$$

$$60h^3\ 3231120h^4w^4\mathrm{Cos}\frac{h5hw\ i}{2}$$

$$+ 10332h^4w^4\mathrm{Cos}\frac{h7hw\ i}{2}\quad + 70812h^4w^4\mathrm{Cos}\frac{h9hw\ i}{2}$$

$$141h^2\ 495940h^5w^4\mathrm{Cos}\frac{h5hw\ i}{2}$$

$$+3178h^5w^4\mathrm{Cos}\frac{h7hwi}{2} + 49014h^5w^4\mathrm{Cos}\frac{h9hwi}{2}$$

$$72h\ 862200h^6w^4\mathrm{Cos}\frac{h5hwi}{22} + 498h^6w^4\mathrm{Cos}\frac{h7hwi}{}$$

$$+ 25122h^6w^4\mathrm{Cos}\ h\frac{9hw}{2}\ i\ \ 6\ \ 919360h^7w^4\mathrm{Cos}\ h\frac{5hw}{2}\ i\ \frac{h7hwi}{}$$

$$12605008h^7w^4\ 15h^3\ 28224 + 839680h^4w^4\ \mathrm{Cos}\frac{h9hwi}{2}$$

$$3470928h^7w^4\ 5h^3\ 12096 + 692224h^4w^4\ \mathrm{Cos}$$

$$hw]\ \ 7\mathrm{Sin}\frac{7hw}{}\ + \mathrm{Sin}\frac{9hw}{}$$

$$3628800h^6w^3\mathrm{Cos}[8\ \ \frac{7hw}{2}\ \ h\ \ 2\ \ \frac{9hw}{}\ \ h\ \ 2\ \ i$$

$$+ 3628800\ 1h^6w^3\ 7\mathrm{Cos}\ \ \frac{hw}{2}\ \ \frac{hw}{}\ \ h\ \ 2\ \ \mathrm{Sin}[8hw]$$

$$+ \frac{}{232243200h^6w^4}\ f\ \ {}_{+2}\mathrm{Csc}\ \ \frac{hw}{}\ \ h\ \ 2\ i\ \ \mathrm{Csc}[hw]$$

$$21504000h^7w^4\ \mathrm{Cos}\ \ \frac{hw}{2}\ \ \frac{hw}{}\ + 21\ \ 181694678h^7w^4\ \ \frac{3hw}{2}\ i\ \ 180h^3\ \ 10752$$

$$1028096h^4w^4\ \mathrm{Cos}\ \ \frac{hw}{2}\ \ 32514048h^7w^4\mathrm{Cos}\ \ \frac{3hw}{2}\ i\ \ \frac{3hw}{2}\ i$$

$$42\ 88728419h^7w^4 + 2h^5\ \ 544320\ \ 43921408h^4w^4\ \ \mathrm{Cos}\ \frac{7hwh}{}$$

$$34609530h^7w^4\mathrm{Cos}\ \frac{9hw}{2}\ i\ \ 45h^2\ \ 126434h^5w^4\mathrm{Cos}\ \frac{7hw}{2}\ i$$

$$+ 11212h^5w^4\mathrm{Cos}\ \frac{9hw}{2}\ i\ \ 48h\ \ 81039h^6w^4\mathrm{Cos}\ \frac{7hw}{2}\ i$$

$$1731h^6w^4\mathrm{Cos}\frac{h9hwi}{}\ \ 6\ 7\ 8111608h^7w^4\ + 105h^3$$

$$288\ \ 2$$

$$77824h^4w^4\ \mathrm{Cos}\frac{h7hwi}{}\ \ 28754584h^7w^4\ + 10h^3$$

$$63504\ \ 2$$

$$2879488h^4w^4\ \ \mathrm{Cos}\ \frac{h9hwi}{22}\ 4h\ \ 1397760h^6w^4\mathrm{Cos}\ \ \frac{h3hwi}{}$$

$$+6804480h^{6}w^{4}\mathrm{Cos}\left[\frac{h^{5}hw}{2}\right] + 15h^{2}\ 102186h^{4}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right] + 36306h^{4}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$h\ h^{5}w^{4}\mathrm{Cos}\left[\frac{\ }{2}\right] + 11212h^{5}w^{4}\mathrm{Cos}\left[\frac{\ }{2}\right]$$

$$+ 24\ 126434\ \mathrm{Cos}\left[\frac{7hw}{2}\right] + 6\ 81039h^{5}w^{4}\ \ 7hw\ 1731h^{5}w^{4}\mathrm{Cos}\ h\left[\frac{9hw}{2}\right]$$

$$12700800\ h^{6}w^{3}\ \mathrm{Cos}[8\ \ hw]\ \mathrm{Cos}\left[\frac{7hw}{2}\right]\mathrm{Sin}\left[\frac{\ }{2}\right] + 12700800h^{5}w^{5}\ \ 1\ \ 9hw\ \ 3\mathrm{Cos}\ h\ \ 2\ \ i\ \ \mathrm{Sin}[8hw] + 3\mathrm{Sin}\left[\frac{\ }{2}\right]$$

$$464486400h^{6}w^{4}\ f_{n+5}\mathrm{Csc}\left[\frac{h}{2}\right]\ \mathrm{Csc}[hw]$$

$$79134720h^{7}w^{4}\mathrm{Cos}\left[\frac{hw}{2}\right] + 210\ 38655248h^{7}w^{4}$$

$$+ 576h^{3}\ 1092\ 69632h^{4}w^{4}\ \mathrm{Cos}\left[\frac{hhw}{2}\right]$$

$$65028096h^{7}w^{4}\mathrm{Cos}\left[\frac{h3hw}{\ }\right]\ 84\ 88728419h^{7}w^{4}\ 2^{h}$$

$$2h^{3}\ 907200\ 43921408h^{4}w^{4}\ \mathrm{Cos}\left[\frac{h3hw}{2}\right]$$

$$237874980h^{7}w^{4}\mathrm{Cos}\left[\frac{5hw}{2}\right]$$

$$45h^{2}\ 1092658h^{5}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right]^{h}$$

$$+ 98510h^{5}w^{4}\mathrm{Cos}\left[\frac{h9hw}{2}\right]\ 48h\ \ 359478h^{6}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right]$$

$$75750h^{6}w^{4}\mathrm{Cos}\left[\frac{h9hw}{\ }\right]\ 6\ 7\ 16150256h^{7}w^{4}$$

$$+ 5h^{3}\ \ 2$$

$$60480 + 3190784h^{4}w^{4}\ \mathrm{Cos}\left[\frac{7hw}{2}\right]\ \ 9hw\ + 5\ 14263984h^{4}w^{4}$$

$$h^{3}\ 254016 + 14209024h^{4}w^{4}\ \mathrm{Cos}\left[\frac{hw}{2}\right]$$

$$4h\ \ 2795520h^{6}w^{4}\mathrm{Cos}\left[\frac{h}{2}\right]$$

$$+12318720h^{6}w^{4}\mathrm{Cos}\left[\frac{5hw}{2}\right]$$

$$+\ 15h^{2}\ 12852h^{4}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right]+383220h^{4}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$+\ 24\ \ 1092658\ h^{5}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right]+98510h^{5}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$+\ 6\ \ 359478h^{5}w^{4}\mathrm{Cos}\left[\frac{7hw}{2}\right]\ \ 75750h^{5}w^{4}\mathrm{Cos}\left[\frac{9hw}{2}\right]$$

$$25401600\ \ h^{6}w^{3}\ \ \mathrm{Cos}[8hw]\ \ 5\mathrm{Sin}\left[\frac{7hw}{2}\right]+3\mathrm{Sin}\left[\frac{9hw}{2}\right]$$

$$+\ 25401600h^{5}w^{3}\ \ 5\mathrm{Cos}\left[\frac{7hw}{2}\right]+3\mathrm{Cos}\left[\frac{9hw}{2}\right]\ \ \mathrm{Sin}[8hw]$$

$$464486400h^{5}w^{4}\ f_{n+3}\mathrm{Csc}\left[\frac{h}{2}\right]\mathrm{Csc}[hw]$$

$$79134720h^{7}w^{4}\mathrm{Cos}\left[\frac{hw}{2}\right]+210\ 38655248h^{7}w^{4}$$

$$+\ 576h^{3}\ 1260\ 69632h^{4}w^{4}\ \mathrm{Cos}\left[\frac{hhw}{2}\right]$$

$$110444544h^{7}w^{4}\mathrm{Cos}\left[\frac{h3hw}{2}\right]\ \ 84\ 74283709h^{7}w^{4}\ 2^{h}$$

$$2h^{3}\ 1028160\ 36589568h^{4}w^{4}\ \mathrm{Cos}\left[\frac{h3hw}{2}\right]$$

$$192395100h^{7}w^{4}\mathrm{Cos}\left[\frac{5hw}{2}\right]$$

$$45h^{2}\ 664174h^{5}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right]+68050h^{5}w^{4}\mathrm{Cos}\left[\frac{h9hw}{2}\right]$$

$$48h\ \ 473382h^{6}w^{4}\mathrm{Cos}\left[\frac{h7hw}{2}\right]+10230h^{6}w^{4}\mathrm{Cos}\left[\frac{h9hw}{2}\right]$$

$$6\ 7\ \ 37654256h^{7}w^{4}+5h^{3}\ \ 36288$$

$$7491584h^{4}w^{4}\ \mathrm{Cos}\left[\frac{h7hw}{2}\right]+5\ 22218416h^{7}w^{4}$$

$$7h^{3}\ \ 60480+3166208h^{4}w^{4}\ \mathrm{Cos}\left[\frac{h9hw}{2}\right]\ 4h$$

$$2623488h^{6}w^{4}\mathrm{Cos}\left[\frac{h3hw}{2}\right]+20797440h^{6}w^{4}\mathrm{Cos}\left[\frac{h5hw}{2}\right]$$

$$+15h^{2}\ 315252h^{4}w^{4}\cos\left(\frac{7hw}{2}\right) + 219060h^{4}w^{4}\cos\left(\frac{9hw}{2}\right)$$

$$+ 24h\ 664174h^{5}w^{4}\cos\left(\frac{7hw}{2}\right) + 68050h^{5}w^{4}\cos\left(\frac{9hw}{2}\right)$$

$$+ 6\ 473382h^{6}w^{4}\cos\left(\frac{7hw}{2}\right) + 10230h^{6}w^{4}\cos\left(\frac{9hw}{2}\right)$$

$$25401600h^{6}w^{3}\cos[8hw]\ 3\sin\left(\frac{7hw}{2}\right) + 5\sin\left(\frac{9hw}{2}\right)$$

$$+ 25401600h^{6}w^{3}\ 3\cos\left(\frac{7hw}{2}\right) + 5\cos\left(\frac{9hw}{2}\right)\sin[8hw]$$

$$928972800h^{6}w^{4}\ \frac{f_{n+1}\csc\left(\frac{hw}{2}\right)\sec\left(\frac{hw}{2}\right)}{\left(\frac{hw}{2}\right)} + 6\ 735812960h^{4}w^{4}\ 60480h^{5}$$

$$14450688h^{7}w^{4}\cos\left(\frac{hw}{2}\right)$$

$$68 + 12288h^{4}w^{4}\cos\left(\frac{hw}{2}\right)\ 12214272h^{7}w^{4}\cos\left(\frac{3hw}{2}\right)$$

$$60h^{5}\ 41040h^{4}w^{4}\cos\left(\frac{9hw}{2}\right) + 70812h^{4}w^{4}\cos\left(\frac{5hw}{2}\right)$$

$$+ 10332h^{4}w^{4}\cos\left(\frac{hw}{2}\right)\ 141h^{2}\ 176700h^{5}w^{4}\cos\left(\frac{9hw}{2}\right)$$

$$+ 49014h^{5}w^{4}\cos\left(\frac{7hw}{2}\right) + 3178h^{5}w^{4}\cos\left(\frac{hw}{2}\right)$$

$$72h\ 209880h^{6}w^{4}\cos\left(\frac{5hw}{2}\right) + 25122h^{6}w^{4}\cos\left(\frac{7hw}{2}\right)$$

$$+ 498h^{6}w^{4}\cos\left(\frac{9hw}{2}\right)\ 6\ 919360h^{7}w^{4}\cos\left(\frac{5hw}{2}\right)$$

$$3470928h^{7}w^{4}\ 5h^{3}\ 12096 + 692224h^{4}w^{4}\cos\left(\frac{7hw}{2}\right)$$

$$12605008h^{7}w^{4}\ 15h^{3}\ 28224 + 839680h^{4}w^{4}\cos\left(\frac{9hw}{2}\right)$$

$$3628800h^{6}w^{3}\cos[8hw]\ \sin\left(\frac{7hw}{2}\right) + 7\sin\left(\frac{9hw}{2}\right)$$

$$+ 3628800h^{6}w^{3}\cos\left(\frac{7hw}{2}\right) + 7\cos\left(\frac{9hw}{2}\right)\sin[8hw]$$

3.5 Block Form of Methods

The methods obtained in sections 3.3 and 3.4 are presented in block form as

$$A_{1}Y_{+1} = A_{0}Y + B_{1}F_{+1} + B_{0}F \tag{3.25}$$

where

$$Y_{n+1} = [y_{n+1}; y_{n+2}; \quad y_{n+k}; hy_{n+1}^{0}; h^{0}_{n+2}; hy_{n+k}^{0};$$
$$h^2 y_{n+1}^{00}; h^2 y_{n+2}^{00}; \quad h^2 y_{n+k}^{00}; h^3 y_{n+1}^{000}; h^3 y_{n+2}^{000};$$
$$h^3 y_{n+4}^{000}]^T$$

$$Y_{n} = [y_{n}; y_{n+1}; \quad y_{n+k+1}; hy_{n}^{0}; hy_{n+1}^{0}; hy_{n+k+1}^{0};$$
$$h^2 y_{n}^{00}; h^2 y_{n+1}^{00}; \quad h^2 y_{n+k+1}^{00}; h^3 y_{n}^{000}; h^3 y_{n+1}^{000}; \quad h^3 y_{n+k+1}^{000}$$
$$]^T$$

$$F_{n+1} = [f_{n+1}; f_{n+2}; \quad f_{n+k}; hf_{n+1}^{0}; hf_{n+2}^{0}; \quad hf_{n+k}^{0}$$
$$h^2 f_{n+1}^{00}; h^2 f_{n+2}^{00}; \quad h^2 f_{n+k}^{00}; h^3 f_{n+1}^{000}; h^3 f_{n+2}^{000}; h^3 f_{n+k}^{000}]^T$$

$$F_{n} = [f_{n}; f_{n+1}; \quad f_{n+k+1}; hf_{n}^{0}; hf_{n+1}^{0}; \quad hf_{n+k+1}^{0}$$
$$h^2 f_{n}^{00}; h^2 f_{n+1}^{00}; \quad h^2 f_{n+k+1}^{00}; h^3 f_{n+1}^{000}; h^3 f_{n+k+1}^{000}]^T$$

and the coe cients A_i, B_i are 4k 4k matrices. Where k is the step length.

3.5.1 Case k = 4

$$A_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & & & & & & & & & & & & & & & \\ \frac{11}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & & & & & & & & & \\ & & & & & & & & & & & & 2000000010000000 & & & \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & & & & & & & & & & & 000000000000000 & & & \\ 1 & & & & & & & & & & & & 00000000000000 & & & \\ & & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & & \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$
A_1 = \begin{bmatrix}
0 & 3 & 2^{3} & 3^{1} & 0000000000000 & 1 \\
 & 46 & 41 & & 0000000000000 & \\
 & & - & - & & \\
\frac{1}{2} & 1 & \frac{1}{6} & & 0100000000000 & \\
 & & 1 & \frac{1}{3} & & \\
1 & & \frac{2}{3} & & 0010000000000 & \\
3 & & 11 & & & \\
2 & 3 & 6 & & & \\
 & & & & & \\
7 & \frac{19}{} & \frac{13}{} & & 0000100000000 & \\
5 & 4 & 1 & & 0000000000000 & \\
 & & & & & \\
2 & 1 & 0 & & 0000010000000 & \\
 & & & & & \\
 & 12 & & & 10000001000000 & \\
4 & 5 & & & 200000000100000 & \\
 & & & & & \\
7 & 8 & & & 300000000010000 & \\
 & & & & & \\
3 & 3 & & & 100000000000000 & \\
 & & & & & \\
3 & 3 & & & 10000000001000 & \\
 & & & & & \\
3 & 3 & & & 100000000000100 & \\
 & & & & & \\
3 & 3 & & & 100000000000010 & \\
 & & & & & \\
3 & 3 & & & 100000000000001 &
\end{bmatrix}
$$

@ A

0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1

20160 193

720

B 31 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

60480

B 11 C

B 12096 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 23 C

B 20160 C

B C

B 151 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 4463 C

B 60480 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 157 C

B 60480 C

B 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B = B 60480 C

0 B 11 C

B 60480 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 73 C

B 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 409 C

B 60480 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 60480 C

B 53 C

B 160 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B C

B 5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 288 C

B 13 C

B 1440 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 1 C

B 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 160 C

B 29 C

B 1440 C

0 31 79 31 1 0 0 0 0 0 0 0 0 0 0 0 0 1

180 120 180 720

B 181 209 1 1

15120 10080 3024 60480 0 0 0 0 0 0 0 0 0 0 0 0 0 C

727 397 13 41

B 15120 1440 15120 60480 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 5040 672 5040 20160 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 493 79 59 41 C

B C

B 679 12793 7489 85 0 0 0 0 0 0 0 0 0 0 0 0 C

B 311 121 47 1 C

B C

B 2160 10080 15120 12096 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 10531 1327 283 241 C

B 1187 17 11 11 C

B = 1 B 15120 10080 15120 60480 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 53 773 53 11 C

B 2593 6619 1417 241 C

B 15120 10080 15120 60480 0 0 0 0 0 0 0 0 0 0 0 0 C

B 23 1 1 3 C

B 20 16 10 160 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 5491 14599 15739 859 C

B C

B 91 73 17 11 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 41 79 1 11 C

B 180 240 18 1440 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 1 77 17 3 C

B 360 240 360 1440 C

B 288 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 49 143 119 95 C

3.5.2 Case k = 6

$$A_0 = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{11}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{11}{24} \\
-\frac{13}{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{3}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{2} \\
\frac{3}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{4} \\
\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

$$A_1 = \begin{bmatrix} 0 & 1 & 1 & 0000000000000000000000 \\ 0 & \frac{9}{2} & 16 & 9 & 0100000000000000000000 \\ 0 & \frac{5}{2} & 5 & \frac{15}{4} & 1000000000000000000000 \\ 0 & & 8 & \frac{3}{2} & 0000000000000000000000 \\ 0 & \frac{1}{4} & \frac{1}{3} & \frac{1}{8} & 0010000000000000000000 \\ 0 & 1 & 4 & \frac{1}{4} & 0001000000000000000000 \\ 0 & \frac{3}{4} & \frac{1}{3} & \frac{3}{8} & 0000100000000000000000 \\ 0 & 1 & \frac{8}{3} & \frac{7}{4} & 0000010000000000000000 \\ 0 & \frac{17}{2} & \frac{23}{2} & \frac{31}{2} & 0000001000000000000000 \\ 0 & 9 & \frac{44}{3} & \frac{27}{4} & 0000000100000000000000 \\ 0 & \frac{7}{2} & 4 & 5 & 0000000000000000000000 \\ 0 & 2 & 2 & \frac{1}{4} & 0000000010000000000000 \\ 0 & 1 & 0 & \frac{1}{2} & 0000000001000000000000 \\ 0 & 1 & 2 & 1 & 0000000000100000000000 \\ 0 & \frac{5}{2} & 4 & \frac{7}{4} & 0000000000010000000000 \\ 0 & 4 & 6 & \frac{5}{2} & 0000000000001000000000 \\ 0 & \frac{11}{2} & 8 & \frac{13}{4} & 0000000000000100000000 \\ 0 & \frac{3}{2} & 2 & \frac{3}{4} & 0000000000000000000000 \\ 0 & \frac{3}{2} & 2 & \frac{3}{4} & 0000000000000000100000 \\ 0 & \frac{3}{2} & 2 & \frac{3}{4} & 0000000000000000010000 \\ 0 & \frac{3}{2} & 2 & \frac{3}{4} & 0000000000000000001000 \\ 0 & \frac{3}{2} & 2 & \frac{3}{4} & 0000000000000000000100 \\ 0 & \frac{3}{2} & 2 & \frac{3}{4} & 0000000000000000000010 \\ 0 & \frac{3}{2} & 2 & \frac{3}{4} & 0000000000000000000001 \end{bmatrix}$$

0 1

1/1680 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

1/3780 0 0 0C C

1/15120 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

137/16800 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

5/36288 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

83/907200 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

1/6300 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

11/64800 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

47/907200 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

11/20160 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

19763/302400 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

421/151200 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

2/4725 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

13/— 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

—/302400 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

47/75600 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

53/21600 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

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36791/120960 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

461/40320 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

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49/17280 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

199/120960 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

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359 @ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

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13 2311 617 2311 13 1

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

84 1680 210 1680 84 1680

143 3343 313 1 1 1

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

0 3360 20160 7560 1120 2016 12096 0 0C C

B 79 461 131 359 13 5

B 2016 1344 189 2016 3360 12096 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C

B 87 5561 617 13 11 1 C

B 280 10080 1260 3360 12600 10080 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0CC

B 3359 3103 173 11 5 401

B 50400 24192 18144 2016 3024 1814400 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C

B 977 421 1199 277 3 109 C

B 75600 4032 22680 60480 2800 907200 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

B 67 827 617 17 19 3 C

B 12600 13440 10080 4032 16800 22400 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C

B 149 103 311 103 1 19 C

B 12600 864 1620 10080 37800 453600 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C

B 439 25201 121301 15187 3 149 C

B 6048 40320 90720 30240 700 362880 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

B 4247 5993 577 82332411887 C

B 25200 4032 168 4032 5040 100800 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C

B 71683 7543 499 4127 229 877 C

1 B 75600 8640 1512 60480 10800 302400 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

B 479 2021 37 53 643 7 C

B = B 100800 6048 480 10080 302400 21600 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C

B C

B 675 70 45 945 1575 9450 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C

B 97 233 461 233 97 1 C

B 302400 60480 6048 60480 302400 43200 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

B 991 4147 1739 97 547 173 C

B 25200 12096 2520 960 75600 302400 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C

B 3251 499 1039 3863 8279 47 C

B 43200 720 672 3780 100800 18900 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0CC

B 5189 5827 9577 57599 5747 9949

B 37800 6048 3780 30240 5400 151200 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C

B 42967 13729 29 31999 2519 277 C

B 30240 120960 48 120960 30240 24192 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

B 10411 106651 241 8423 91 341 C

B 30240 120960 15120 120960 4320 120960 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C

B 2153 12673 3823 1235 407 67 C

B 30240 120960 15120 24192 30240 40320 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C

B 773 1399 617 7367 397 181 C

B 30240 3456 1680 30240 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C C

B 1577 35423 14929 7753 983 361 C

B 30240 120960 15120 17280 30240 120960 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C

533 50021 2173 148249 115813 C

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

30240 120960 3024 120960 30240 1152 0C A

527 9599 2249 55327 44137 36809

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

@ 4320 120960 1680 120960 30240 120960

3.5.3 Case k = 8

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5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

B 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C
B 10 10^1 0000000100000000000000000000000 c

2
5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

B C
13
B 30 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C
B 11 C
B 60 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

B 1
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B 1 C
B 60 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C
B 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C
B 11 C
B 60 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C
B 13 C
B 30 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

B 47
60 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C
B 2 C
B 5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C
B 3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C
A_0 = B 1 C
B 5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C
B 1 C
B 10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

B 10 C
B C
B C

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C
B 1
10^3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C
B 10 0000000000000000000000000000000 c
B 1
1
5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 C
B 1 C
B 10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C
B 1
10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C
B 1 C
B 10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C
B 1
10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C
B 1 C
B 10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C
B 1
10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C
B 1 C
B 10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C
B 1 C
B 10 0000000000000000000000000000000 C

$$A_1 = \begin{bmatrix} 0 & 0 & 16 & 30 & 16 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 7 & 14 & & \frac{42}{5} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & \frac{9}{2} & & \frac{18}{5} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & \frac{3}{2} & & \frac{2}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 15 & & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 5 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 6 & 2 & 10 & & & & & & & & & & & & & & & & & & & & & & & & & & & \\ 0 & 0 & & 3 & 4 & 5 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & & 3 & 3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 2 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 5 & 5 & 17 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 6 & 2 & 10 & & & & & & & & & & & & & & & & & & & & & & & & & & & \\ 0 & 0 & & \frac{3}{41} & \frac{4}{25} & \frac{5}{61} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 6 & 2 & 10 & & & & & & & & & & & & & & & & & & & & & & & & & & & \\ 0 & 0 & & \frac{34}{} & \frac{79}{} & \frac{46}{} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 3 & 4 & 7 & & & & & & & & & & & & & & & & & & & & & & & & & & & \\ 0 & 0 & & 3 & 4 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 2 & 5 & \frac{4}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 1 & 2 & \frac{1}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 0 & \frac{1}{2} & \frac{2}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 2 & \frac{7}{2} & \frac{8}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 3 & 5 & \frac{11}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 4 & \frac{13}{2} & \frac{14}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 5 & 8 & \frac{17}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 1 & \frac{3}{2} & \frac{3}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 1 & \frac{3}{2} & \frac{3}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 1 & \frac{3}{2} & \frac{3}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 1 & \frac{3}{2} & \frac{3}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 1 & \frac{3}{2} & \frac{3}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 1 & \frac{3}{2} & \frac{3}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 1 & \frac{3}{2} & \frac{3}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 1 & \frac{3}{2} & \frac{3}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & & 1 & \frac{3}{2} & \frac{3}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & & 1 & \frac{3}{2} & \frac{3}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & & 1 & \frac{3}{2} & \frac{3}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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0 731 12121 102889 143921 102889 12121 731 23
4536 9072 22680 18144 22680 9072 4536 181440

2753 4729 158381 88531 4093 1 7
2592 5184 2592 51840 64800 25920 12960 103680
B 979 16097 27589 174301 53777 107 43 5
60480 120960 60480 241920 302400 24192 60480 96768
B 1171 68317 40679 42701 4463 613 67 11
B 18144 181440 90720 362880 453600 181440
B 559 50089 10009 52781 2279 193 13 1
B 36288 362880 36288 725760 907200 362880 181440 207360
715549 3834191 175961 323963 3571 9553 1597 229
B 1995840 3991680 181440 1596672 1995840 3991680 1995840
B 3648797 2388599 842207 18569 590089 43361 1619 1379
B 59875200 23950080 11975040 47900160 59875200 10886400 1710720 13685760
B 86147 7216061 991511 391621 6529 2213 1597 71
2993760 29937600 2993760 5987520 14968800 5987520 14968800 5987520
B 2017 352447 2829559 143873 17213 14599 1447 5513
B 362880 7983360 19958400 2280960 2851200 7983360 3991680 159667200
B 90821 1231687 2414651 143921 151093 101623 10411 8077
B 29937600 59875200 29937600 2177280 29937600 59875200 29937600 239500800
68669 1022851 1883141 1928743 113111 9307 29 2137
B 11975040 23950080 11975040 9580032 11975040 23950080 155520 95800320
B 24481 9067 8641 1847911 46247 1627 8 1
B 831600 36960 10395 1330560 92400 415800 10395 14080
B 8475976259559 23659021 164664193 121916051 2295565 532523 571
11975040 10886400 11975040 47900160 59875200 4790016 59875200 13685760
B 152129 2500891 106440791 153673759 12310981 24035687 414643 281791
B 1197504 2395008 29937600 23950080 2721600 11975040 855360 34214400
B 62033963 21312751 89751797 2245499 1556051 9253511 1143419 1004621
B 59875200 17107200 59875200 47900160 8553600 119750400 59875200 479001600
1881827 10668601 36165539 8572363 1655431 1267351 147871 125893
B 59875200 17107200 59875200 47900160 59875200 119750400 59875200 479001600
$B_1 = B$ 2202121 21378899 6113297 243689 55871 52109 46367 41021
59875200 119750400 59875200 9580032 8553600 17107200 59875200 479001600
B 904891 16978481 3012997 3655853 157391 54233 2543 563
B 59875200 119750400 8553600 47900160 59875200 119750400 59875200 479001600
B 3133 50641 242899 3638891 242899 50641 3133 1747
B 59875200 119750400 59875200 47900160 59875200 119750400 59875200 479001600
B 138209 15945487 27320087 4905613 6071717 139823 60029 34091
8553600 119750400 59875200 6842880 59875200 17107200 59875200 479001600
B 1976207 31634677 54716489 11029819 60438689 10620541 272281 137443
B 59875200 119750400 59875200 6842880 59875200 119750400 59875200 479001600
B 2746889 7038073 79832657 24511559 16807661 122679367 4776587 988451
B 59875200 17107200 59875200 9580032 8553600 119750400 59875200 479001600
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B 57077 177481 540979 55327 652483 7201 65783 2047
B 36288 1814400 302400 120960 907200 24192 907200 259200
B 620303 44267 89639 126943 89401 3769 3553 353
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B 1607 502423 662233 319 54659 41611 1597 143
B 43200 1814400 907200 24192 907200 1814400 302400 259200
B 17681 3959 407 78527 89611 2453 1243 1231
1814400 22680 86400 362880 1814400 151200 362880 3628800
B 17939 69571 473191 143921 587 33707 3319 211
B 907200 604800 907200 362880 8640 1814400 907200 604800
B 307 16991 733501 122123 831659 743 3853 281
24192 113400 1814400 120960 1814400 18144 604800 518400
B 2773 28687 31081 283057 1073789 248281 19127 533
B 129600 259200 60480 362880 907200 604800 907200 362880
@ 4369 7249 554909 11851 423769 591809 649543 1363
1814400 37800 1814400 10368 604800 453600 1814400 172800
26801 59531 1064039 29509 2034493 9277 13729 520697
302400 362880 907200 120960 907200 259200 8640 1814400

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0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
C
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C
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C
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C
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C
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C
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0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0C
A
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Chapter 4

ANALYSIS OF METHODS

4.1 Introduction

The properties and behaviour of a numerical scheme is of central concern in numerical analysis since most methods could only handle certain class of problems. It is therefore important to be able to analyse the basic properties of a numerical method in order to know the merits and demerits of such method viz-a-viz other existing ones. In this chapter, the basic properties of the proposed BVMs are discussed; these include; zero stability, consistency and convergence of the methods.

4.1.1 Order of the Methods

Let L be the linear di erence operator associated with (1.1) and de ned as

$$\text{[illegible]} \quad {}_{k} \quad h_{j}y(x + jh) \;\; h^{4} \;\; {}_{j}y^{iv}(x + jh)^{\prime} \qquad (4.1)$$

X

and y(x) is continuously di erentiable in the interval [a; b]. Obtaining the Taylor's expansion of $_j$; $_j$; y(x + jh) and $y^{iv}(x + jh)$ about the point x_n in h and y we have

$$L[y(x); h] = c_0y(x) + c_1hy^{0}(x) + c_2h^2y^{00}(x) + c_3h^3y^{000}(x) + c_{p+1}h^{p+1}y^{(p+1)}(x) + c_{p+2}h^{p+2}(x)y^{(p+2)}(x) + c_{p+3}h^{p+3}y^{(p+3)}(x) + c_{p+4}h^{p+4}y^{(p+4)}(x) + \qquad (4.2)$$

De nition 4.1.1

The di erence operator L associated with (4.1) is said to be of order p if

$$C_0 = C_1 = \quad = C_p = C_{p+1} = C_{p+2} = C_{p+3} = 0$$

but C_{p+4} 6= 0

De nition 4.1.2

The term

$$`_n = C_{p+4}h^{(p+4)}y^{(p+4)}(x)$$

is called the local truncation error and the constant $C_{P\ +4}$ is called error constant.

4.1.2 Consistency of Methods

A method (1.1) is said to be consistent if each numerical scheme that forms the block satis es conditions ;

(i) it has order p > 4

$$\sum_{r=0}^{k} x_r = 0$$

4.1.3 Zero Stability

Zero stability is concerned with the stability of the di erence system 4.1 as h tends to zero. Thus, as h ! 0, the block method (3.25) tends to the di erence scheme

$$A_0 Y \quad A_1 Y_{\ 1} = 0 \tag{4.3}$$

where A_0 and A_1 are 4k by 4k constant matrices.

De nition 4.1.4

The method (1.1) is said to be zero stable if the roots of the rst characteristic polynomial

$$\sum_{i}^{1} X$$

has modulus less than or equal to one, and if every root with modulus one has multiplicity that does not exceed four.

Theorem 4.1:

Consistency and zero stability are neccessary and su cient for a linear mul-tistep method to be convergent

4.1.4 Convergence

According to fundamental theorem of Dahquist (Henrici,1962), a method is convergence if it is consistent and zero stable.

4.2 Analysis of Boundary Value Methods

In this section, the order, error constant, consistency, zero stability and con-vergence of the proposed BVMs with polynomial basis are obtained for the various step length.

4.2.1 Order and Error Constants of Methods

Case k = 4

By taking the Taylor's series expansion of the terms of the four-step BV M and denoting the principal term of the local truncation error as LTEi, we obtain

$$LTE1 = \frac{h^{10}y^{(10)}(x_n)}{3024} \qquad LTE2 = \frac{h^{9}y^{(9)}(x_n)}{3600} \qquad LTE3 = \frac{11h^{9}y^{(9)}(x_n)}{33600}$$

$$LTE4 = \frac{53h^{9}y^{(9)}(x_n)}{151200} \qquad LTE5 = \frac{11h^{9}y^{(9)}(x_n)}{33600} \qquad LTE6 = \frac{h^{9}y^{(9)}(x_n)}{3600}$$

$$LTE7 = \frac{23h^{9}y^{(9)}(x_n)}{6048} \qquad LTE8 = \frac{11h^{9}y^{(9)}(x_n)}{60480} \qquad LTE9 = \frac{h^{10}y^{(10)}(x_n)}{43200}$$

$$LTE10 = \frac{11h^{9}y^{(9)}(x_n)}{60480} \qquad LTE11 = \frac{23h^{9}y^{(9)}(x_n)}{6048} \qquad LTE12 = \frac{883h^{9}y^{(9)}(x_n)}{60480}$$

$$LTE13 = \frac{251h^{9}y^{(9)}(x_n)}{60480} \qquad LTE14 = \frac{211h^{9}y^{(9)}(x_n)}{60480} \qquad LTE15 = \frac{251h^{9}y^{(9)}(x_n)}{60480}$$

$$LTE16 = \frac{883h^{9}y^{(9)}(x_n)}{60480}$$

Thus, the four-step BVM has order P = (6; 5; 5; 5; 5; 5; 5; 5; 6; 5; 5; 5; 5; 5; 5; 5)^T and error constant

$$C_{p+4} = \left(\frac{1}{3024}; \frac{1}{3600}; \frac{11}{33600}; \frac{53}{151200}; \frac{11}{33600}; \frac{1}{3600}; \frac{23}{6048}; \frac{11}{60480}; \frac{1}{43200}; \frac{11}{60480}; \frac{23}{6048}; \frac{883}{60480}; \frac{251}{60480}; \frac{211}{60480}; \frac{251}{60480}; \frac{883}{6048}\right)^T$$

Case k = 6

By expanding the terms of the six-step BVM in Taylor's series and denoting the principal term of the local truncation error as LTEi we obtain,

$$LTE1 = \frac{h^{12}y^{(12)}(x_n)}{120960}; \quad LTE2 = \frac{h^{11}y^{(11)}(x_n)}{12096}; \quad LTE3 = \frac{h^{11}y^{(11)}(x_n)}{12096};$$

$$LTE4 = \frac{h^{11}y^{(11)}(x_n)}{10395}; \quad LTE5 = \frac{593h^{11}y^{(11)}(x_n)}{3991680}; \quad LTE6 = \frac{h^{11}y^{(11)}(x_n)}{22275};$$

$$LTE7 = \frac{409h^{11}y^{(11)}(x_n)}{6652800}; \quad LTE8 = \frac{h^{11}y^{(11)}(x_n)}{22275}; \quad LTE9 = \frac{593h^{11}y^{(11)}(x_n)}{3991680};$$

$$LTE10 = \frac{h^{11}y^{(11)}(x_n)}{10395}; \quad LTE11 = \frac{23h^{11}y^{(11)}(x_n)}{9450}; \quad LTE12 = \frac{13h^{11}y^{(11)}(x_n)}{43200};$$

$$LTE13 = \frac{h^{11}y^{(11)}(x_n)}{9450}; \quad LTE14 = \frac{1747h^{12}y^{(12)}(x_n)}{479001600}; \quad LTE15 = \frac{h^{11}y^{(11)}(x_n)}{9450};$$

$$LTE16 = \frac{13h^{11}y^{(11)}(x_n)}{43200}; \quad LTE17 = \frac{23h^{11}y^{(11)}(x_n)}{9450}; \quad LTE18 = \frac{19h^{11}y^{(11)}(x_n)}{2025};$$

$$LTE19 = \frac{3601h^{11}y^{(11)}(x_n)}{1814400}; \quad LTE20 = \frac{13h^{11}y^{(11)}(x_n)}{14175} \quad LTE21 = \frac{1201h^{11}y^{(11)}(x_n)}{1814400};$$

$$LTE22 = \frac{13h^{11}y^{(11)}(x_n)}{14175}; \quad LTE23 = \frac{2301h^{11}y^{(11)}(x_n)}{1814400}; \quad LTE24 = \frac{19h^{11}y^{(11)}(x_n)}{2025}$$

Thus,the six-step BVM has order

$$= (8; 7; 7; 7; 7; 7; 7; 7; 7; 7; 7; 7; 7; 8; 7; 7; 7; 7; 7; 7; 7; 7; 7; 7; 7)^T$$

and error constant

$$C_{p+4} = \frac{11}{120960}; \frac{1}{12096}; \frac{1}{12096}; \frac{1}{10395}; \frac{593}{3991680}; \frac{1}{22275}; \frac{409}{6652800}; \frac{1}{2275}; \frac{593}{3991680}; \frac{1}{10395}$$

$$= \frac{23}{9450}; \frac{13}{43200}; \frac{1}{9450}; \frac{1747}{479001600}; \frac{1}{9450}; \frac{13}{43200}; \frac{23}{9450}; \frac{19}{2025}; \frac{3601}{1814400}; \frac{13}{14175}; \frac{1201}{1814400}$$

$$\frac{13}{14175}; \frac{2301}{1814400}; \frac{19}{2025}^T$$

Case k = 8

By expanding the terms of the eight-step BVM in Taylor's series and denoting

the principal term of the local truncation error as LTEi, we obtain;

$$LT\ E1 = \frac{1003y^{(14)}(x_n)h^{14}}{11975040};\quad LT\ E2 = \frac{193y^{(13)}(x_n)h^{13}}{3628800};\quad LT\ E3 = \frac{y^{(13)}(x_n)h^{13}}{1209600};$$

$$LT\ E4 = \frac{193y^{(13)}(x_n)h^{13}}{3628800};\quad LT\ E5 = \frac{y^{(13)}(x_n)h^{13}}{1209600};\quad LT\ E6 = \frac{23893y^{(13)}(x_n)h^{13}}{181621440};$$

$$LT\ E7 = \frac{3130901y^{(13)}(x_n)h^{13}}{43589145600};\quad LT\ E8 = \frac{43369y^{(13)}(x_n)h^{13}}{7264857600};\quad LT\ E9 = \frac{15469y^{(13)}(x_n)h^{13}}{968647680};$$

$$LT\ E10 = \frac{90107y^{(13)}(x_n)h^{13}}{5448643200};\quad LT\ E11 = \frac{15469y^{(13)}(x_n)h^{13}}{968647680};\quad LT\ E12 = \frac{43369y^{(13)}(x_n)h^{13}}{7264857600};$$

$$LT\ E13 = \frac{3130901y^{(13)}(x_n)h^{13}}{43589145600};\quad LT\ E14 = \frac{23893y^{(13)}(x_n)h^{13}}{181621440};\quad LT\ E15 = \frac{223907y^{(13)}(x_n)h^{13}}{119750400};$$

$$LT\ E16 = \frac{33119y^{(13)}(x_n)h^{13}}{159667200};\quad LT\ E17 = \frac{16931y^{(13)}(x_n)h^{13}}{239500800};\quad LT\ E18 = \frac{2143y^{(13)}(x_n)h^{13}}{479001600};$$

$$LT\ E19 = \frac{119803y^{(14)}(x_n)h^{14}}{186810624000};\quad LT\ E20 = \frac{2143y^{(13)}(x_n)h^{13}}{479001600};\quad LT\ E21 = \frac{16931y^{(13)}(x_n)h^{13}}{239500800};$$

$$LT\ E22 = \frac{33119y^{(13)}(x_n)h^{13}}{159667200};\quad LT\ E23 = \frac{223907y^{(13)}(x_n)h^{13}}{119750400};\quad LT\ E24 = \frac{3254947y^{(13)}(x_n)h^{13}}{479001600};$$

$$LT\ E25 = \frac{525599y^{(13)}(x_n)h^{13}}{479001600};\quad LT\ E26 = \frac{25093y^{(13)}(x_n)h^{13}}{68428800};\quad LT\ E27 = \frac{12329y^{(13)}(x_n)h^{13}}{68428800};$$

$$LT\ E28 = \frac{78499y^{(13)}(x_n)h^{13}}{479001600};\quad LT\ E29 = \frac{12329y^{(13)}(x_n)h^{13}}{68428800};\quad LT\ E30 = \frac{25093y^{(13)}(x_n)h^{13}}{68428800};$$

$$LT\ E31 = \frac{525599y^{(13)}(x_n)h^{13}}{479001600};\quad LT\ E32 = \frac{3254947y^{(13)}(x_n)h^{13}}{479001600}$$

Thus, the eight-step BVM has order

$$= (10; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 10; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9)^T$$

and the error constant

$$C_{p+4} = \frac{1003}{11975040}; \frac{193}{3628800}; \frac{193}{1209600}; \frac{193}{3628800}; \frac{193}{1209600}; \frac{23893}{181621440}; \frac{3130901}{43589145600}; \frac{43369}{7264857600};$$

$$\frac{15469}{968647680}; \frac{90107}{5448643200}; \frac{15469}{968647680}; \frac{43369}{7264857600}; \frac{3130901}{43589145600}; \frac{23893}{181621440}; \frac{223907}{119750400};$$

$$\frac{33119}{159667200}; \frac{16931}{239500800}; \frac{2143}{479001600}; \frac{119803}{186810624000}; \frac{2143}{479001600}; \frac{16931}{239500800}; \frac{33119}{159667200}; \frac{223907}{119750400};$$

$$\frac{3254947}{68428800}; \frac{525599}{479001600}; \frac{25093}{68428800}; \frac{12329}{68428800}; \frac{78499}{479001600}; \frac{12329}{68428800}; \frac{25093}{68428800}; \frac{525599}{479001600}; \frac{479001600}{479001600}; \frac{68428800}{3254947}$$

$$\frac{3254947}{479001600}^T$$

4.2.2 Consistency of Methods

Case k = 4

Since each numerical method in the four-step BVM has order p > 4, then by De nition 4.1.2, the four step BVM is consistent.

Case k = 6

Since each numerical scheme in the Six-Step BVM has order p > 4,then by de nition 4.1.2, the six-step BVM is consistent.

Case k = 8

Since each numerical scheme in the eight-Step BVM has order p > 4,then by de nition 4.1.2, the eight-step BVM is consistent.

4.2.3 Zero Stability of Methods

Case k = 4

As h ! 0, (3.25) reduces to (4.3) where A_0 and A_1 are 16 16 matrices given by;

$$A_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & & & & & & & & & & & & & & & \\ 3 & & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & & \\ \frac{11}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & & \\ & & & & & & & & & 2000000010000000 & & & & & & \\ & & & & & & & & & & & & & & & \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & & & & & & & & & \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & & & & & & & & & \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & & & & & & & & & \end{bmatrix} \quad (4.5)$$

1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0A
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

$$A_1 = \begin{bmatrix} 0 & 3 & 23 & 31 & 0000000000000 & 1 \\ & 46 & 41 & 0000000000000 & & \\ & - & - & & & \\ \frac{1}{2} & 1 & \frac{1}{6} & 0100000000000 & & \\ & 1 & \frac{1}{3} & & & \\ 1 & \frac{1}{2} & & 0010000000000 & & \\ 3 & & 11 & & & \\ 2 & 3 & 6 & & & \\ - & & - & & & \\ 7 & \frac{19}{4} & \frac{13}{1} & 0000100000000 & & \\ 5 & & & 0000000000000 & & \\ 2 & 1 & 0 & 0000010000000 & & \\ 1 & 2 & & 10000001000000 & & \\ 4 & 5 & & 2000000010000 & & \\ 7 & 8 & & 3000000000100 & & \\ 3 & 3 & & 1000000000000 & & \\ 3 & 3 & & 10000000001000 & & \\ 3 & 3 & & 1000000000100 & & \\ 3 & 3 & & 1000000000010 & & \\ 3 & 3 & & 10000000000001 & & \end{bmatrix} \qquad (4.6)$$

Using (4.4), the rst characteristics polynomial is

$$(R) = R^{12}(1 + R^4) = 0$$

This gives

$R_1 = R_2 = R_3 = R_5 = R_6 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = 0$; $R_{13} = R_{14} = R_{15} = R_{16} = 1$

Thus, by de nition (4.1.3), the four step BVM is zero stable.

Case k = 6

As h ! 0, (3.25) reduces to (4.3) where A_0 and A_1 are 24x24 matrices given

by :

0 1

1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

$\frac{1}{4}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 1 4 . 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B $\frac{11}{24}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B $\frac{1}{12}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B $\frac{1}{24}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B $\frac{1}{12}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 11 24 C

B $\frac{13}{12}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

A_0 B $\frac{3}{4}$ 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 C

= B $\frac{1}{2}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C (4.7)

B $\frac{1}{0}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B $\frac{1}{4}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 1 2 C

B $\frac{3}{4}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B $\frac{1}{4}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 C

B 1 4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B $\frac{1}{4}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 1 4 C

B $\frac{1}{4}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B $\frac{1}{4}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

B 1 4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C

@ A

$$A_1 = \begin{bmatrix} 0 & 1 & 1 & 1 & 0000000000000000000000 \\ 0 & \frac{9}{2} & 16 & \frac{9}{2} & 0100000000000000000000 \\ 0 & \frac{5}{2} & 5 & \frac{15}{4} & 1000000000000000000000 \\ 0 & \frac{1}{4} & \frac{1}{3} & \frac{1}{8} & 0010000000000000000000 \\ 0 & 1 & \frac{4}{3} & \frac{1}{4} & 0001000000000000000000 \\ 0 & \frac{3}{4} & \frac{1}{3} & \frac{3}{8} & 0000100000000000000000 \\ 0 & 1 & \frac{8}{3} & \frac{7}{4} & 0000010000000000000000 \\ 0 & \frac{17}{23} & & \frac{31}{} & \\ 0 & 9 & \frac{44}{3} & \frac{27}{4} & 0000000100000000000000 \\ 0 & \frac{7}{2} & 4 & \frac{5}{4} & 0000000000000000000000 \\ 0 & 2 & 2 & \frac{1}{2} & 0000000010000000000000 \\ 1 & & & 1 & 0000000000010000000000 \\ 0 & 1 & 2 & 1 & 0000000000001000000000 \\ 0 & \frac{5}{2} & 4 & \frac{7}{4} & 0000000000000100000000 \\ 0 & 4 & 6 & \frac{5}{} & 0000000000000010000000 \\ 0 & \frac{11}{2} & 8 & \frac{13}{4} & 0000000000000001000000 \\ 0 & \frac{3}{2} & 2 & \frac{3}{4} & 0000000000000000000000 \\ 0 & \frac{3}{2} & 2 & \frac{3}{4} & 0000000000000000100000 \\ 0 & \frac{3}{2} & 2 & \frac{3}{4} & 0000000000000000010000 \\ 0 & \frac{3}{2} & 2 & \frac{3}{4} & 0000000000000000001000 \\ 0 & \frac{3}{2} & 2 & \frac{3}{4} & 0000000000000000000100 \\ 0 & \frac{3}{2} & 2 & \frac{3}{4} & 0000000000000000000010 \\ 0 & \frac{3}{2} & 2 & \frac{3}{4} & 0000000000000000000001 \end{bmatrix} \qquad (4.8)$$

Using (4.4), the rst characteristic polynomial is 1 _

This gives

$R_1 = R_2 = R_3 = R_5 = R_6 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12}\ R_{14} = R_{13} =$

$= R_{15} = R_{16} = R_{17} = R_{18} = R_{19} = R_{20} = 0,$

$R_{21} = R_{22} = R_{23} = R_{24} = 1$

This, by De nition 4.1.3, the step six-step BVM is zero

stable Case k = 8

As h ! 0 (3.25) reduces to (4.3) where A_0 and A_1 are 32 32 matrices

given by

$$A_0 = \begin{pmatrix}
5^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{2}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{47}{60} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{13}{30} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{11}{60} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{30} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{60} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{30} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{11}{60} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{13}{30} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{47}{60} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{2}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{3}{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{3}{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{2}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \qquad (4.9)$$

$$A_1 = \left(\begin{array}{ccccccccccccccccccccccccccccccccc}
0&0&7&14&\frac{42}{5}&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&16&30&16&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&2&\frac{9}{2}&\frac{18}{5}&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&2&2&\frac{3}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&2&\frac{3}{2}&\frac{2}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&\frac{10}{3}&\frac{15}{4}&\frac{6}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&\frac{5}{6}&\frac{1}{2}&\frac{1}{10}&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&\frac{2}{3}&\frac{5}{4}&\frac{2}{5}&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&\frac{7}{6}&\frac{3}{2}&\frac{3}{10}&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&\frac{2}{3}&\frac{1}{4}&\frac{2}{5}&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&\frac{5}{6}&\frac{5}{2}&\frac{17}{10}&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&\frac{10}{3}&\frac{27}{4}&\frac{18}{5}&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&\frac{41}{6}&\frac{25}{2}&\frac{61}{10}&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&\frac{34}{3}&\frac{79}{4}&\frac{46}{5}&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&2&\frac{5}{2}&\frac{4}{5}&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&1&\frac{1}{5}&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&\frac{1}{2}&\frac{2}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&2&\frac{7}{2}&\frac{8}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&3&5&\frac{11}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&4&\frac{13}{2}&\frac{14}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&5&8&\frac{17}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&1&\frac{3}{2}&\frac{3}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&\frac{3}{2}&\frac{3}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&1&\frac{3}{2}&\frac{3}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&1&\frac{3}{2}&\frac{3}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&1&\frac{3}{2}&\frac{3}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&1&\frac{3}{2}&\frac{3}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&1&\frac{3}{2}&\frac{3}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&1&\frac{3}{2}&\frac{3}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&1&\frac{3}{2}&\frac{3}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{array}\right) \tag{4.10}$$

using (4.4), the first characteristics polynomial (R) is

$$R^{28}(1 + R)^4 = 0$$

this gives

$$R_1 = R_2 = R_3 = R_5 = R_6 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13} = R_{14}$$

$$R_{15} = R_{16} = R_{17} = R_{18} = R_{19} = R_{20} = R_{21} = R_{22} = R_{23} = R_{24} = R_{25}$$

$$R_{26} = R_{27} = R_{28} = 0;\ R_{29} = R_{30} = R_{31} = R_{32} = 1$$

Thus, by de nition (4.1.3), the eight step BVM is zero stable.

4.2.4 Convergence of Methods

Following Theorem (4.1), the four, six and eight steps BVMs are convergent since they satisfy the necessary and su cient condition of consistency and zero stability.

4.3 Analysis of Trignometrically- tted Block Methods

In this section, the order, error constant, consistency, zero stability and con-vergence of the TFBMs shall be obtained for various step length.

4.3.1 Order and Error constants of Methods

Case k = 4

Expanding the coe cients of the four-step TFBM and substituting u = !h

we have;

$$y_{n+4} = y_n + 4y_{n+1} \quad 6y_{n+2} + 4y_{n+3}$$

$$\frac{79h^4}{120} + T_{n+2} \quad \frac{h^4u^2}{504} \quad \frac{h^4u^4}{5760} \quad \frac{h^4u^6}{95040} \quad \frac{691h^4u^8}{1320883200} \quad \frac{h^4u^{10}}{43545600}$$

$$T_{n+3} \quad \frac{31h^4}{180} + \frac{h^4u^2}{756} + \frac{h^4u^4}{8640} + \frac{h^4u^6}{142560} + \frac{691h^4u^8}{1981324800} + \frac{h^4u^{10}}{65318400}$$

$$+ f_n \quad \frac{h^4}{720} \quad \frac{h^4u^2}{3024} \quad \frac{h^4u^4}{34560} \quad \frac{h^4u^6}{570240} \quad \frac{691h^4u^8}{7925299200} \quad \frac{h^4u^{10}}{261273600}$$

$$T_{n+1} \quad \frac{31h^4}{180} + \frac{h^4u^2}{756} + \frac{h^4u^4}{8640} + \frac{h^4u^6}{142560} + \frac{691h^4u^8}{1981324800} + \frac{h^4u^{10}}{65318400}$$

$$+ f_{n+4} \quad \frac{h^4}{720} \quad \frac{h^4u^2}{3024} \quad \frac{h^4u^4}{34560} \quad \frac{h^4u^6}{570240} \quad \frac{691h^4u^8}{7925299200} \quad \frac{h^4u^{10}}{261273600}$$

$$y^0_n = \frac{11y_n}{6h} + \frac{3y_{n+1}}{h} \quad \frac{3y_{n+2}}{2h} + \frac{y_{n+3}}{3h}$$

$$\frac{h^3}{20160} + T_{n+4} \quad \frac{h^3u^2}{14400} \quad \frac{2381h^3u^4}{159667200} \quad \frac{14221h^3u^6}{6810804000} \quad \frac{1277257h^3u^8}{5230697472000} \quad \frac{35099011h^3u^{10}}{1333827855360000}$$

$$+ f_n \quad \frac{193h^3}{20160} \quad \frac{h^3u^2}{2880} \quad +\frac{167h^3u^4}{31933440} \quad +\frac{4567h^3u^6}{2179457280} \quad + \frac{270859h^3u^8}{1046139494400} + \frac{1041043h^3u^{10}}{38109367296000}$$

$$+ T_{n+3} \quad \frac{h^3}{720} + \frac{h^3u^2}{1800} + \frac{1577h^3u^4}{39916800} + \frac{32447h^3u^6}{7783776000} + \frac{619369h^3u^8}{1307674368000} + \frac{391253h^3u^{10}}{7578567360000}$$

$$\frac{181h^3}{1008} + T_{n+1} \quad \frac{h^3u^2}{900} \quad \frac{31h^3u^4}{39916800} \quad \frac{228757h^3u^6}{54486432000} \quad \frac{696407h^3u^8}{1307674368000} \quad \frac{9276313h^3u^{10}}{166728481920000}$$

$$+ f_{n+2} \quad \frac{209h^3}{3360} \quad \frac{h^3u^2}{800} \quad \frac{773h^3u^4}{26611200} \quad + \frac{37h^3u^6}{1651104000} \quad + \frac{2963h^3u^8}{67060224000} + \frac{668747h^3u^{10}}{222304642560000}$$

$$y^0_{n+1} = \frac{y_n}{3h} \quad \frac{y_{n+1}}{2h} + \frac{y_{n+2}}{h} \quad \frac{y_{n+3}}{6h}$$

$$\frac{397h^3}{10080} + T_{n+2} \quad \frac{29h^3u^2}{302400} \quad \frac{h^3u^4}{84480} \quad \frac{57881h^3u^6}{72648576000} \quad \frac{7237h^3u^8}{174356582400} \quad \frac{830143h^3u^{10}}{444609285120000}$$

$$+ f_n \quad \frac{31h^3}{60480} \quad \frac{163h^3u^2}{907200} \quad \frac{1307h^3u^4}{53222400} \quad \frac{6619h^3u^6}{2476656000} \quad \frac{1429277h^3u^8}{5230697472000} \quad \frac{36800711h^3u^{10}}{1333827855360000}$$

$$+ f_{n+4} \quad \frac{41h^3}{60480} \quad \frac{67h^3u^2}{453600} \quad \frac{1097h^3u^4}{53222400} \quad \frac{524591h^3u^6}{217945728000} \quad \frac{1356907h^3u^8}{5230697472000} \quad \frac{4496321h^3u^{10}}{166728481920000}$$

$$\frac{13h^3}{3024} + T_{n+3} \quad \frac{239h^3u^2}{907200} \quad \frac{31h^3u^4}{831600} \quad \frac{991301h^3u^6}{217945728000} \quad \frac{50797h^3u^8}{100590336000} \quad \frac{71110993h^3u^{10}}{1333827855360000}$$

$$+ f_{n+1} \quad \frac{31h^3}{60480} \quad \frac{163h^3u^2}{907200} \quad \frac{1307h^3u^4}{53222400} \quad \frac{6619h^3u^6}{2476656000} \quad \frac{1429277h^3u^8}{5230697472000} \quad \frac{36800711h^3u^{10}}{1333827855360000}$$

$$y_{n+2}^{0} = \frac{y_n}{6h} + \frac{y_{n+1}}{h} + \frac{2y_{n+2}}{3h} + \frac{y_{n+3}}{} + f_{n+2} \frac{79h^3}{1440} + \frac{h^3u^2}{6048} + \frac{h^3u^4}{69120} + \frac{h^3u^6}{1140480}$$

$$\frac{691h^3u^8}{52254720} + \frac{h^3u^{10}}{15850598400}$$

$$+ f_{n+3} \frac{59h^3}{15120} + \frac{109h^3u^2}{453600} + \frac{1453h^3u^4}{39916800} + \frac{54749h^3u^6}{12108096000} + \frac{16909h^3u^8}{33530112000} + \frac{11848381h^3u^{10}}{222304642560000}$$

$$y_{n+3}^{0} + f_n \frac{11h^3}{12096} + \frac{23h^3u^2}{113400} + \frac{4061h^3u^4}{159667200} + \frac{17827h^3u^6}{6604416000} + \frac{53071h^3u^8}{193729536000} + \frac{6136903h^3u^{10}}{222304642560000}$$

$$+ f_{n+1} \frac{493h^3}{15120} \frac{209h^3u^2}{453600} \frac{247h^3u^4}{4435200} \frac{620141h^3u^6}{108972864000} \frac{735461h^3u^8}{1307674368000} \frac{37246843h^3u^{10}}{666913927680000}$$

$$+ f_{n+4} \frac{41h^3}{60480} \frac{67h^3u^2}{453600} \frac{1097h^3u^4}{53222400} \frac{524591h^3u^6}{217945728000} \frac{1356907h^3u^8}{5230697472000} \frac{4496321h^3u^{10}}{166728481920000}$$

$$= \frac{y_n}{3h} + \frac{3y_{n+1}}{2h} \frac{3y_{n+2}}{h} + \frac{11y_{n+3}}{6h}$$

$$+ f_{n+2} \frac{121h^3}{672} \frac{19h^3u^2}{33600} \frac{7h^3u^4}{152064} \frac{196919h^3u^6}{72648576000} \frac{23167h^3u^8}{174356582400} \frac{2573257h^3u^{10}}{444609285120000}$$

$$+ f_{n+3} \frac{47h^3}{5040} + \frac{h^3u^2}{20160} \frac{289h^3u^4}{19958400} \frac{28529h^3u^6}{8717829120} \frac{580711h^3u^8}{1307674368000} \frac{1040381h^3u^{10}}{20520428544000}$$

$$y_{n+4}^{0} + f_n \frac{23h^3}{20160} \frac{13h^3u^2}{50400} \frac{4831h^3u^4}{159667200} \frac{651991h^3u^6}{217945728000} \frac{215561h^3u^8}{747242496000} \frac{9418067h^3u^{10}}{333456963840000}$$

$$+ f_{n+1} \frac{311h^3}{5040} + \frac{71h^3u^2}{100800} + \frac{757h^3u^4}{9979200} + \frac{1500901h^3u^6}{217945728000} + \frac{812381h^3u^8}{1307674368000} + \frac{77917793h^3u^{10}}{1333827855360000}$$

$$+ f_{n+4} \frac{h^3}{20160} + \frac{h^3u^2}{14400} + \frac{2381h^3u^4}{159667200} + \frac{14221h^3u^6}{6810804000} + \frac{1277257h^3u^8}{5230697472000} + \frac{35099011h^3u^{10}}{1333827855360000}$$

$$= \frac{11y_n}{6h} + \frac{7y_{n+1}}{h} \frac{19y_{n+2}}{2h} + \frac{13y_{n+3}}{3h}$$

$$+ f_{n+2} \frac{12793h^3}{10080} \frac{361h^3u^2}{151200} \frac{7697h^3u^4}{26611200} \frac{1181h^3u^6}{61152000} \frac{13883h^3u^8}{13837824000} \frac{371411h^3u^{10}}{8233505280000}$$

$$+ f_{n+3} \frac{7489h^3}{15120} + \frac{149h^3u^2}{113400} + \frac{8501h^3u^4}{39916800} + \frac{103273h^3u^6}{6054048000} + \frac{72977h^3u^8}{62270208000} + \frac{1162999h^3u^{10}}{13894040160000}$$

$$\frac{151h^3}{} \frac{487h^3u^2}{} \frac{6089h^3u^4}{} \frac{6823h^3u^6}{} + \frac{3001h^3u^8}{} + \frac{8579887h^3u^{10}}{}$$

$$+ I_{n+1} \frac{60480}{679h^3} + f_{n+4} \frac{907200}{53h^3u^2} \frac{159667200}{6893h^3u^4} \frac{6054048000}{7517h^3u^6} \frac{35582976000}{10321h^3u^8} \frac{444609285120000}{119027h^3u^{10}}$$

$$\frac{12096}{2160} + \frac{181440}{28350} + \frac{31933440}{39916800} + \frac{121080960}{864864000} + \frac{49816166400}{62270208000} \frac{12703122432000}{5052378240000}$$

$$85h^3 \qquad 47h^3u^2 \qquad 1861h^3u^4 \qquad 643h^3u^6 \qquad 20861h^3u^8 \qquad 436151h^3u^{10}$$

$$y''_n = \frac{2y_n}{h^2} - \frac{5y_{n+1}}{h^2} + \frac{4y_{n+2}}{h^2} - \frac{y_{n+3}}{h^2}$$

$$+ T_{n+2} \quad \frac{1327h^2}{10080} + \frac{61h^2u^2}{7200} + \frac{2701h^2u^4}{11404800} + \frac{551h^2u^6}{141523200} + \frac{9901h^2u^8}{201180672000} + \frac{486191h^2u^{10}}{60628538880000}$$

$$+ T_{n+4} \quad \frac{241h^2}{60480} + \frac{37h^2u^2}{75600} + \frac{3373h^2u^4}{95800320} + \frac{36433h^2u^6}{18681062400} + \frac{5267h^2u^8}{57062154240} + \frac{7867763h^2u^{10}}{2000741783040000}$$

$$+ f_n \quad \frac{4463h^2}{60480} + \frac{167h^2u^2}{50400} + \frac{54679h^2u^4}{479001600} + \frac{60677h^2u^6}{18681062400} + \frac{1190999h^2u^8}{15692092416000} + \frac{1259831h^2u^{10}}{1000370891520000}$$

$$+ T_{n+3} \quad \frac{283h^2}{15120} + \frac{31h^2u^2}{16800} + \frac{1021h^2u^4}{119750400} + \frac{3473h^2u^6}{1334361600} + \frac{788569h^2u^8}{3923023104000} + \frac{21083627h^2u^{10}}{2000741783040000}$$

$$f_{n+1} \quad \frac{10531h^2}{15120} \quad \frac{1429h^2u^2}{151200} \quad \frac{36793h^2u^4}{119750400} \quad \frac{72799h^2u^6}{9340531200} \quad \frac{531143h^2u^8}{3923023104000} \quad \frac{44111h^2u^{10}}{285820254720000}$$

$$y''_{n+1} = \frac{y_n}{h^2} - \frac{2y_{n+1}}{h^2} + \frac{y_{n+2}}{h^2} + T_{n+2} \quad \frac{17h^2}{10080} \quad \frac{41h^2u^2}{100800} \quad \frac{1091h^2u^4}{79833600} \quad \frac{1847h^2u^6}{6227020800} \quad \frac{3001h^2u^8}{2615348736000} \quad \frac{8197h^2u^{10}}{27220976640000}$$

$$+ f_n \quad \frac{157h^2}{60480} \quad \frac{h^2u^2}{6300} \quad \frac{601h^2u^4}{95800320} \quad \frac{3673h^2u^6}{18681062400} \quad \frac{1459h^2u^8}{285310771200} \quad \frac{210113h^2u^{10}}{2000741783040000}$$

$$+ T_{n+4} \quad \frac{11h^2}{60480} + \frac{h^2u^2}{43200} + \frac{823h^2u^4}{479001600} + \frac{83h^2u^6}{849139200} + \frac{5711h^2u^8}{1207084032000} + \frac{821879h^2u^{10}}{4001483566080000}$$

$$+ T_{n+3} \quad \frac{11h^2}{15120} + \frac{h^2u^2}{11200} + \frac{67h^2u^4}{59875200} \quad \frac{361h^2u^6}{3736212480} \quad \frac{35621h^2u^8}{3923023104000} \quad \frac{2045411h^2u^{10}}{4001483566080000}$$

$$+ f_{n+1} \quad \frac{1187h^2}{15120} + \frac{137h^2u^2}{302400} + \frac{8h^2u^4}{467775} + \frac{9193h^2u^6}{18681062400} + \frac{41623h^2u^8}{3923023104000} + \frac{438799h^2u^{10}}{4001483566080000}$$

$$y''_{n+2} = \frac{y_{n+1}}{h^2} - \frac{2y_{n+2}}{h^2} + \frac{y_{n+3}}{h^2} + f_{n+2} - \frac{773h^2}{10080} + \frac{h^2u^2}{7200} - \frac{823h^2u^4}{79833600} + \frac{83h^2u^6}{141523200} - \frac{5711h^2u^8}{201180672000} + \frac{821879h^2u^{10}}{666913927680000}$$

$$+ f_{n+3} - \frac{53h^2}{15120} + \frac{h^2u^2}{10800} - \frac{823h^2u^4}{119750400} + \frac{83h^2u^6}{212284800} - \frac{5711h^2u^8}{301771008000} + \frac{821879h^2u^{10}}{1000370891520000}$$

$$+ f_{n} - \frac{11h^2}{60480} + \frac{h^2u^2}{43200} - \frac{823h^2u^4}{479001600} + \frac{83h^2u^6}{849139200} - \frac{5711h^2u^8}{1207084032000} + \frac{821879h^2u^{10}}{4001483566080000}$$

$$+ f_{n+1} - \frac{53h^2}{15120} + \frac{h^2u^2}{10800} - \frac{823h^2u^4}{119750400} + \frac{83h^2u^6}{212284800} - \frac{5711h^2u^8}{301771008000} + \frac{821879h^2u^{10}}{1000370891520000}$$

$$+ f_{n+4} - \frac{11h^2}{60480} + \frac{h^2u^2}{43200} - \frac{823h^2u^4}{479001600} + \frac{83h^2u^6}{849139200} - \frac{5711h^2u^8}{1207084032000} + \frac{821879h^2u^{10}}{4001483566080000}$$

$$y'''_{n+3} = \frac{y_n}{h^2} + \frac{4y_{n+1}}{h^2} - \frac{5y_{n+2}}{h^2} + \frac{2y_{n+3}}{h^2}$$

$$+ f_{n+2} - \frac{6619h^2}{10080} + \frac{241h^2u^2}{100800} - \frac{14951h^2u^4}{79833600} + \frac{67367h^2u^6}{6227020800} - \frac{195883h^2u^8}{373621248000} + \frac{4318421h^2u^{10}}{190546836480000}$$

$$+ f_{n+3} - \frac{1417h^2}{15120} + \frac{179h^2u^2}{100800} - \frac{3977h^2u^4}{29937600} + \frac{140233h^2u^6}{18681062400} - \frac{1409803h^2u^8}{3923023104000} + \frac{61699999h^2u^{10}}{4001483566080000}$$

$$+ f_{n} - \frac{73h^2}{60480} + \frac{31h^2u^2}{100800} - \frac{13037h^2u^4}{479001600} + \frac{15467h^2u^6}{9340531200} - \frac{1293937h^2u^8}{15692092416000} + \frac{14493421h^2u^{10}}{4001483566080000}$$

$$+ f_{n+1} - \frac{2593h^2}{15120} + \frac{61h^2u^2}{43200} - \frac{6997h^2u^4}{59875200} + \frac{25847h^2u^6}{3736212480} - \frac{1332559h^2u^8}{3923023104000} + \frac{59215789h^2u^{10}}{4001483566080000}$$

$$+ f_{n+4} - \frac{241h^2}{60480} + \frac{37h^2u^2}{75600} - \frac{3373h^2u^4}{95800320} + \frac{36433h^2u^6}{18681062400} - \frac{5267h^2u^8}{57062154240} + \frac{7867763h^2u^{10}}{2000741783040000}$$

$$y''_{n+4} = \frac{2y_n}{h^2} + \frac{7y_{n+1}}{h^2} + \frac{8y_{n+2}}{h^2} + \frac{3y_{n+3}}{h^2}$$

$$+ f_{n+2}\left(\frac{14599h^2}{10080} + \frac{227h^2u^2}{50400} + \frac{1259h^2u^4}{11404800} + \frac{26699h^2u^6}{1556755200} + \frac{2865073h^2u^8}{261534873600} + \frac{3270791h^2u^{10}}{60628538880000}\right)$$

$$+ f_{n+3}\left(\frac{15739h^2}{15120} + \frac{49h^2u^2}{7200} + \frac{9073h^2u^4}{119750400} + \frac{58241h^2u^6}{9340531200} + \frac{315031h^2u^8}{560431872000} + \frac{8795711h^2u^{10}}{285820254720000}\right)$$

$$+ f_n\left(\frac{409h^2}{60480} + \frac{29h^2u^2}{25200} + \frac{8917h^2u^4}{95800320} + \frac{101953h^2u^6}{18681062400} + \frac{76087h^2u^8}{285310771200} + \frac{23183063h^2u^{10}}{2000741783040000}\right)$$

$$+ f_{n+1}\left(\frac{5491h^2}{15120} + \frac{121h^2u^2}{151200} + \frac{26699h^2u^4}{119750400} + \frac{22193h^2u^6}{1334361600} + \frac{3524929h^2u^8}{3923023104000} + \frac{82344827h^2u^{10}}{2000741783040000}\right)$$

$$+ f_{n+4}\left(\frac{859h^2}{12096} + \frac{401h^2u^2}{151200} + \frac{26959h^2u^4}{479001600} + \frac{4843h^2u^6}{18681062400} + \frac{1545361h^2u^8}{15692092416000} + \frac{6397819h^2u^{10}}{1000370891520000}\right)$$

$$y_n''' = \frac{y_n}{h^3} + \frac{3y_{n+1}}{h^3} + \frac{3y_{n+2}}{h^3} + \frac{y_{n+3}}{h^3} + f_{n+4}\left(\frac{3h}{160} + \frac{13hu^2}{4480} + \frac{41hu^4}{134400} + \frac{1039hu^6}{35481600} + \frac{18197hu^8}{6457651200} + \frac{321977hu^{10}}{1162377216000}\right)$$

$$+ f_n\left(\frac{53h}{160} + \frac{283hu^2}{24192} + \frac{11hu^4}{20160} + \frac{27hu^6}{788480} + \frac{213533hu^8}{74724249600} + \frac{191111hu^{10}}{697426329600}\right)$$

$$+ f_{n+1}\left(\frac{23h}{20} + \frac{649hu^2}{20160} + \frac{179hu^4}{134400} + \frac{1303hu^6}{17740800} + \frac{250853hu^8}{43589145600} + \frac{28799hu^{10}}{52835328000}\right)$$

$$+ f_{n+3}\left(\frac{h}{10} + \frac{181hu^2}{60480} + \frac{149hu^4}{403200} + \frac{317hu^6}{5913600} + \frac{146357hu^8}{26153487360} + \frac{971119hu^{10}}{1743565824000}\right)$$

$$+ f_{n+2}\left(\frac{h}{16} + \frac{19hu^2}{720} + \frac{97hu^4}{134400} + \frac{hu^6}{67200} + \frac{799hu^8}{6706022400} + \frac{1297hu^{10}}{145297152000}\right)$$

$$y_{n+1}''' = \frac{y_n}{h^3} + \frac{3y_{n+1}}{h^3} + \frac{3y_{n+2}}{h^3} + \frac{y_{n+3}}{h^3} + f_{n+2}\left(\frac{73h}{240} + \frac{hu^2}{336} + \frac{11hu^4}{57600} + \frac{13hu^6}{1330560} + \frac{691hu^8}{1585059840} + \frac{17hu^{10}}{958003200}\right)$$

$$+ f_n\left(\frac{5h}{288} + \frac{311hu^2}{120960} + \frac{167hu^4}{604800} + \frac{8791hu^6}{319334400} + \frac{476117hu^8}{174356582400} + \frac{2857753hu^{10}}{10461394944000}\right)$$

$$+ f_{n+4}\left(\frac{11h}{1440} + \frac{191hu^2}{120960} + \frac{257hu^4}{1209600} + \frac{7751hu^6}{319334400} + \frac{1352341hu^8}{523069747200} + \frac{2795873hu^{10}}{10461394944000}\right)$$

$$+ f_{n+3}\left(\frac{17h}{360} + \frac{131hu^2}{60480} + \frac{437hu^4}{1209600} + \frac{1033hu^6}{22809600} + \frac{13691hu^8}{2724321600} + \frac{2764933hu^{10}}{5230697472000}\right)$$

$$+ f_{n+1}\left(\frac{91h}{360} + \frac{53hu^2}{8640} + \frac{149hu^4}{241920} + \frac{9311hu^6}{159667200} + \frac{366589hu^8}{65383718400} + \frac{2888693hu^{10}}{5230697472000}\right)$$

$$y_{n+2}^{000} = \frac{y_n}{h^3} + \frac{3y_{n+1}}{h^3} + \frac{3y_{n+2}}{h^3} + \frac{y_{n+3}}{h^3} + f_{n+2} + \frac{79h}{240} + \frac{hu^2}{1008} + \frac{hu^4}{11520} + \frac{hu^6}{190080} + \frac{691hu^8}{2641766400} + \frac{hu^{10}}{87091200}$$

$$+ f_{n+3} \frac{h}{18} + \frac{19hu^2}{6720} + \frac{479hu^4}{1209600} + \frac{7471hu^6}{159667200} + \frac{94967hu^8}{18681062400} + \frac{2775853hu^{10}}{5230697472000}$$

$$+ f_n \frac{13h}{1440} + \frac{11hu^2}{5760} + \frac{73hu^4}{302400} + \frac{8311hu^6}{319334400} + \frac{1397947hu^8}{523069747200} + \frac{2835913hu^{10}}{10461394944000}$$

$$+ f_{n+1} \frac{41h}{180} + \frac{251hu^2}{60480} + \frac{619hu^4}{1209600} + \frac{781hu^6}{14515200} + \frac{28415hu^8}{5230697472} + \frac{2855933hu^{10}}{5230697472000}$$

$$+ f_{n+4} \frac{11h}{1440} + \frac{191hu^2}{120960} + \frac{257hu^4}{1209600} + \frac{7751hu^6}{319334400} + \frac{1352341hu^8}{523069747200} + \frac{2795873hu^{10}}{10461394944000}$$

$$y_{n+3}^{000} = \frac{y_n}{h^3} + \frac{3y_{n+1}}{h^3} + \frac{3y_{n+2}}{h^3} + \frac{y_{n+3}}{h^3} + f_{n+2} \frac{77h}{80} + \frac{5hu^2}{1008} + \frac{7hu^4}{19200} + \frac{hu^6}{49280} + \frac{7601hu^8}{7925299200} + \frac{13hu^{10}}{319334400}$$

$$+ f_{n+3} \frac{17h}{40} + \frac{451hu^2}{60480} + \frac{59hu^4}{80640} + \frac{1159hu^6}{17740800} + \frac{24337hu^8}{4086482400} + \frac{989591hu^{10}}{1743565824000}$$

$$+ f_n \frac{h}{160} + \frac{151hu^2}{120960} + \frac{37hu^4}{201600} + \frac{799hu^6}{35481600} + \frac{261347hu^8}{104613949440} + \frac{918611hu^{10}}{3487131648000}$$

$$+ f_{n+1} \frac{h}{8} + \frac{17hu^2}{20160} + \frac{11hu^4}{44800} + \frac{97hu^6}{2534400} + \frac{14561hu^8}{3113510400} + \frac{99439hu^{10}}{193729536000}$$

$$+ f_{n+4} \frac{3h}{160} + \frac{13hu^2}{4480} + \frac{41hu^4}{134400} + \frac{1039hu^6}{35481600} + \frac{18197hu^8}{6457651200} + \frac{321977hu^{10}}{1162377216000}$$

$$y_{n+4}^{000} = \frac{y_n}{h^3} + \frac{3y_{n+1}}{h^3} + \frac{3y_{n+2}}{h^3} + \frac{y_{n+3}}{h^3}$$

$$+ f_{n+2} \frac{143h}{240} + \frac{41hu^2}{1680} + \frac{221hu^4}{403200} + \frac{29hu^6}{6652800} + \frac{35219hu^8}{87178291200} + \frac{13901hu^{10}}{435891456000}$$

$$+ f_{n+3} \frac{119h}{90} + \frac{1867hu^2}{60480} + \frac{1471hu^4}{1209600} + \frac{10607hu^6}{159667200} + \frac{18127hu^8}{3353011200} + \frac{251911hu^{10}}{475517952000}$$

$$+ f_n \frac{29h}{1440} + \frac{391hu^2}{120960} + \frac{101hu^4}{302400} + \frac{901hu^6}{29030400} + \frac{506521hu^8}{174356582400} + \frac{2937833hu^{10}}{10461394944000}$$

$$+ f_{n+1} \frac{49h}{180} + \frac{101hu^2}{60480} + \frac{587hu^4}{1209600} + \frac{9679hu^6}{159667200} + \frac{777391hu^8}{130767436800} + \frac{2993437hu^{10}}{5230697472000}$$

$$+ f_{n+4} \frac{95h}{288} + \frac{275hu^2}{24192} + \frac{25hu^4}{48384} + \frac{415hu^6}{12773376} + \frac{57965hu^8}{20922789888} + \frac{22613hu^{10}}{83691159552}$$

Denoting the principal term of the local truncation error as LTEi, we obtain

$$\text{LT E1} = \frac{h^{10}}{3024}\left(!^2 y^{(6)}(x_n) + y^{(10)}(x_n)\right); \qquad \text{LT E2} = \frac{11^9}{3600}\left(!^2 y^{(7)}(x_n) + y^{(9)}(x_n)\right);$$

$$\text{LT E3} = \frac{11}{33600}\left(!^2 y^{(7)}(x_n) + y^{(9)}(x_n)\right); \qquad \text{LT E4} = \frac{53}{151200}\left(!^2 y^{(7)}(x_n) + y^{(9)}(x_n)\right);$$

$$\text{LT E5} = \frac{11^9}{33600}\left(!^2 y^{(7)}(x_n) + y^{(9)}(x_n)\right); \qquad \text{LT E6} = \frac{11^9}{3600}\left(!^2 y^{(7)}(x_n) + y^{(9)}(x_n)\right);$$

$$\text{LT E7} = \frac{23h^9}{6048}\left(!^2 y^{(7)}(x_n) + y^{(9)}(x_n)\right); \qquad \text{LT E8} = \frac{11h^9}{60480}\left(!^2 y^{(7)}(x_n) + y^{(9)}(x_n)\right);$$

$$\text{LT E9} = \frac{h^{10}}{43300}\left(!^2 y^{(6)}(x_n) + y^{(10)}(x_n)\right); \qquad \text{LT E10} = \frac{11h^9}{60480}\left(!^2 y^{(7)}(x_n) + y^{(9)}(x_n)\right);$$

$$\text{LT E11} = \frac{23h^9}{6048}\left(!^2 y^{(7)}(x_n) + y^{(9)}(x_n)\right); \qquad \text{LT E12} = \frac{883h^9}{60480}\left(!^2 y^{(7)}(x_n) + y^{(9)}(x_n)\right);$$

$$\text{LT E13} = \frac{251h^9}{60480}\left(!^2 y^{(7)}(x_n) + y^{(9)}(x_n)\right); \qquad \text{LT E14} = \frac{211h^9}{60480}\left(!^2 y^{(7)}(x_n) + y^{(9)}(x_n)\right);$$

$$\text{LT E15} = \frac{251h^9}{60480}\left(!^2 y^{(7)}(x_n) + y^{(9)}(x_n)\right); \qquad \text{LT E16} = \frac{883}{60480}\left(!^2 y^{(7)}(x_n) + y^{(9)}(x_n)\right)$$

Thus, the four step TFBM has order $P = (6; 5; 5; 5; 5; 5; 5; 5; 6; 5; 5; 5; 5; 5; 5; 5)^T$ and error constant

$$C_{p+4} = \frac{1}{3024}; \frac{1}{3600}; \frac{11}{33600}; \frac{53}{151200}; \frac{11}{33600}; \frac{1}{3600}; \frac{23}{6048}; \frac{11}{60480}; \frac{1}{43200}; \frac{11}{60480}; \frac{23}{6048}; \frac{883}{60480}$$

$$\frac{}{6048}, \frac{211}{60480}, \frac{251}{60480}, \frac{883}{60480}\Big)^T 6048$$

Case k = 6

Expanding the coe cients of the six-step TFBM and substituting u = !h

we have;

$$y_{n+6} - y_n + 9y_{n+2} - 16y_{n+3} + 9y_{n+4} + y_{n+3} = \frac{617h^4}{210} + \frac{h^4u^2}{6048} + \frac{23h^4u^4}{266112} + \frac{60829h^4u^6}{4358914560} + \frac{5527h^4u^8}{4358914560}$$

$$+ \frac{255421h^4u^{10}}{2964061900800} + f_n \quad !$$

$$\frac{h^4}{1680} + \frac{h^4u^2}{120960} + \frac{23h^4u^4}{5322240} + \frac{60829h^4u^6}{87178291200} + \frac{5527h^4u^8}{87178291200} + \frac{255421h^4u^{10}}{59281238016000} \quad !$$

$$+ f_{n+6} \quad \frac{h^4}{1680} + \frac{h^4u^2}{120960} + \frac{23h^4u^4}{5322240} + \frac{60829h^4u^6}{87178291200} + \frac{5527h^4u^8}{87178291200} + \frac{255421h^4u^{10}}{59281238016000} \quad !$$

$$+ f_{n+5} \quad \frac{13h^4}{84} + \frac{h^4u^2}{20160} + \frac{23h^4u^4}{887040} + \frac{60829h^4u^6}{14529715200} + \frac{5527h^4u^8}{14529715200} + \frac{255421h^4u^{10}}{9880206336000} \quad !$$

$$+ f_{n+4} \quad \frac{2311h^4}{1680} + \frac{h^4u^2}{8064} + \frac{23h^4u^4}{354816} + \frac{60829h^4u^6}{5811886080} + \frac{5527h^4u^8}{5811886080} + \frac{255421h^4u^{10}}{3952082534400} \quad !$$

$$+ f_{n+2} \quad \frac{13h^4}{1680} + \frac{h^4u^2}{8064} + \frac{23h^4u^4}{354816} + \frac{60829h^4u^6}{5811886080} + \frac{5527h^4u^8}{5811886080} + \frac{255421h^4u^{10}}{3952082534400} \quad !$$

$$+ f_{n+1} \quad 84 \quad 20160 \quad 887040 \quad 14529715200 \quad 14529715200 \quad 9880206336000 \quad !$$

$$y_{n+1} = \frac{y_n}{4} + \frac{3y_{n+2}}{2} + y_{n+3} + \frac{y_{n+4}}{4} + f_{n+3}\frac{313h^4}{7560} - \frac{79h^4u^2}{145152} - \frac{95h^4u^4}{2128896} - \frac{244931h^4u^6}{104613949440} - \frac{215h^4u^8}{2324754432} - \frac{595097h^4u^{10}}{213412456857600}!$$

$$+f_n\frac{h^4}{3780} + \frac{199h^4u^2}{2903040} + \frac{83h^4u^4}{14192640} + \frac{63961h^4u^6}{190207180800} + \frac{5413h^4u^8}{348713164800} + \frac{2637137h^4u^{10}}{4268249137152000}!$$

$$+T_{n+6}\frac{h^4}{12096} - \frac{41h^4u^2}{2903040} - \frac{59h^4u^4}{42577920} - \frac{213709h^4u^6}{2092278988800} - \frac{547h^4u^8}{87178291200} - \frac{1446943h^4u^{10}}{4268249137152000}!$$

$$+T_{n+1}\frac{143h^4}{3360} - \frac{53h^4u^2}{161280} - \frac{593h^4u^4}{21288960} - \frac{550691h^4u^6}{348713164800} - \frac{24877h^4u^8}{348713164800} - \frac{1956457h^4u^{10}}{711374856192000}!$$

$$+T_{n+4}\frac{h^4}{1120} + \frac{13h^4u^2}{64512} + \frac{131h^4u^4}{8515584} + \frac{92051h^4u^6}{139485265920} + \frac{1037h^4u^8}{69742632960} - \frac{85583h^4u^{10}}{284549942476800}!$$

$$+T_{n+5}\frac{h^4}{2016} + \frac{h^4u^2}{483840} + \frac{23h^4u^4}{21288960} + \frac{60829h^4u^6}{348713164800} + \frac{5527h^4u^8}{348713164800} + \frac{255421h^4u^{10}}{237124952064000}!$$

$$+T_{n+2}\frac{3343h^4}{20160} + \frac{17h^4u^2}{27648} + \frac{439h^4u^4}{8515584} + \frac{397811h^4u^6}{139485265920} + \frac{617h^4u^8}{4981616640} + \frac{47251h^4u^{10}}{10538886758400}!$$

$$y_{n+5} = \frac{y_n}{4} + \frac{3y_{n+2}}{2} + 5y_{n+3} + \frac{15y_{n+4}}{4} + T_{n+3}\frac{131h^4}{189} - \frac{85h^4u^2}{145152} - \frac{47h^4u^4}{709632} - \frac{121981h^4u^6}{20922789888} - \frac{14279h^4u^8}{34871316480} - \frac{207707h^4u^{10}}{8536498274304}!$$

$$T_n\frac{h^4}{15120} - \frac{h^4u^2}{82944} - \frac{13h^4u^4}{42577920} + \frac{30253h^4u^6}{418455797760} + \frac{53h^4u^8}{5535129600} + \frac{630127h^4u^{10}}{853649827430400}!$$

$$+T_{n+6}\frac{5h^4}{12096} + \frac{41h^4u^2}{580608} + \frac{59h^4u^4}{8515584} + \frac{213709h^4u^6}{418455797760} + \frac{547h^4u^8}{17435658240} + \frac{1446943h^4u^{10}}{853649827430400}!$$

$$+T_{n+5}\frac{13h^4}{3360} - \frac{11h^4u^2}{32256} - \frac{731h^4u^4}{21288960} - \frac{183133h^4u^6}{69742632960} - \frac{58039h^4u^8}{348713164800} - \frac{1310807h^4u^{10}}{142274971238400}!$$

$$+\frac{1}{3870720h^4w^4}f_{n+4}\csc\frac{hw^{\#5}}{2} - \frac{359h^4}{2016} - \frac{125h^4u^2}{193536} - \frac{577h^4u^4}{8515584} + \frac{152557h^4u^6}{27897053184} + \frac{25219h^4u^8}{69742632960} + \frac{130519h^4u^{10}}{6323332055040}! +$$

$$f_{n+2} = \frac{461h^4}{1344} + \frac{5h^4u^2}{21504} + \frac{269h^4u^4}{8515584} + \frac{91405h^4u^6}{27897053184} + \frac{8809h^4u^8}{34871316480} + \frac{902399h^4u^{10}}{56909988495360} \quad !$$

$$+f_{n+1} = \frac{79h^4}{2016} + \frac{h^4u^2}{96768} + \frac{23h^4u^4}{4257792} + \frac{60829h^4u^6}{69742632960} + \frac{5527h^4u^8}{69742632960} + \frac{255421h^4u^{10}}{47424990412800} \quad !$$

$$y_n^0 = \frac{13y_n}{12h} + \frac{3y_{n+2}}{h} + \frac{8y_{n+3}}{3h} + \frac{3y_{n+4}}{4h} \quad !$$

$$+T_{n+6} = \frac{h^3}{10080} + \frac{149h^3u^2}{7983360} + \frac{76159h^3u^4}{14529715200} + \frac{368419h^3u^6}{523069747200} + \frac{3348053h^3u^8}{44460928512000} + \frac{353257967h^3u^{10}}{47306427936768000}$$

$$+T_{n+2} = \frac{5561h^3}{10080} + \frac{661h^3u^2}{532224} + \frac{46379h^3u^4}{322882560} + \frac{14591h^3u^6}{996323328} + \frac{1718669h^3u^8}{1778437140480} + \frac{6121901h^3u^{10}}{99592479866880} \quad !$$

$$+T_{n+1} = \frac{87h^3}{280} + \frac{263h^3u^2}{443520} + \frac{192961h^3u^4}{2421619200} + \frac{243487h^3u^6}{29059430400} + \frac{13252721h^3u^8}{22230464256000} + \frac{991919219h^3u^{10}}{23653213968384000} \quad !$$

$$+T_{n+5} = \frac{11h^3}{12600} + \frac{277h^3u^2}{1330560} + \frac{1489h^3u^4}{161441280} + \frac{13513h^3u^6}{7925299200} + \frac{5384783h^3u^8}{22230464256000} + \frac{649435277h^3u^{10}}{23653213968384000} \quad !$$

$$+f_{n+4} = \frac{13h^3}{3360} + \frac{15h^3u^2}{19712} + \frac{31489h^3u^4}{968647680} + \frac{23711h^3u^6}{11623772160} + \frac{42671h^3u^8}{523069747200} + \frac{14064509h^3u^{10}}{556546211020800} \quad !$$

$$+f_{n+3} = \frac{167h^3}{1260} + \frac{533h^3u^2}{399168} + \frac{85313h^3u^4}{726485760} + \frac{290909h^3u^6}{26153487360} + \frac{100871h^3u^8}{171003571200} + \frac{57080657h^3u^{10}}{2365321396838400} \quad !$$

$$+f_n = \frac{137h^3}{16800} + \frac{131h^3u^2}{1140480} + \frac{641h^3u^4}{37739520} + \frac{950237h^3u^6}{523069747200} + \frac{4513h^3u^8}{33606144000} + \frac{3179723h^3u^{10}}{321812434944000} \quad !$$

$$y^{0}_{n+1} = \frac{11y_n}{24h} + \frac{y_{n+2}}{4h} + \frac{y_{n+3}}{3h} \quad \frac{y_{n+4}}{8h}$$

$$+\tau_{n+6} \frac{401h^3}{1814400} + \frac{8927h^3u^2}{191600640} + \frac{716081h^3u^4}{116237721600} + \frac{8524721h^3u^6}{12553673932800} + \frac{3691h^3u^8}{52929676800} + \frac{23879070359h^3u^{10}}{3406062811447296000}$$

$$+f_{n+3} \frac{173h^3}{18144} + \frac{5305h^3u^2}{9580032} + \frac{36457h^3u^4}{645765120} + \frac{191357h^3u^6}{57062154240} + \frac{55183h^3u^8}{362946355200} + \frac{974826577h^3u^{10}}{170303140572364800}$$

$$\frac{5h^3}{3359h^3} \quad \frac{2791h^3u^2}{4931h^3u^2} \quad \frac{1372307h^3u^4}{438961h^3u^4} \quad \frac{72769h^3u^6}{9191359h^3u^6} \quad \frac{165997h^3u^8}{72145h^3u^8} \quad \frac{25828723513h^3u^{10}}{192792211h^3u^{10}}$$

$$+f_n \frac{36288}{50400} + \frac{27371520}{10644480} + \frac{116237721600}{8302694400} + \frac{71735279616}{2092278988800} + \frac{1954326528000}{203249958912} + \frac{3406062811447296000}{6238210277376000}$$

$$+f_{n+1} \frac{11h^3}{2016} + \frac{17h^3u^2}{387072} + \frac{11971h^3u^4}{4649508864} + \frac{1438289h^3u^6}{836911595520} + \frac{1385399h^3u^8}{5081248972800} + \frac{1461961147h^3u^{10}}{45414170819297280}$$

$$+\tau_{n+5} \frac{5h^3}{3024} + \frac{4183h^3u^2}{31933440} + \frac{1104049h^3u^4}{58118860800} + \frac{996301h^3u^6}{418455797760} + \frac{6700439h^3u^8}{25406244864000} + \frac{15594438047h^3u^{10}}{567677135241216000}$$

$$+f_{n+2} \frac{3103h^3}{24192} + \frac{10049h^3u^2}{12773376} + \frac{156041h^3u^4}{1788272640} + \frac{5648143h^3u^6}{836911595520} + \frac{1272121h^3u^8}{2540624486400} + \frac{9259458889h^3u^{10}}{227070854096486400}$$

$$y^{0}_{n+2} = \frac{y_n}{12h} \quad \frac{y_{n+2}}{h} + \frac{4y_{n+3}}{3h} \quad \frac{y_{n+4}}{4h} + f_{n+3} \frac{1199h^3}{22680} + \frac{1381h^3u^2}{11975040} + \frac{11041h^3u^4}{1452971520} + \frac{159667h^3u^6}{784604620800} \quad \frac{109h^3}{4}$$

$$+\tau_{n+6} \frac{6757h^3u^2}{907200} + \frac{124321h^3u^4}{239500800} + \frac{8414899h^3u^6}{29059430400} + \frac{5405203h^3u^8}{15692092416000} + \frac{27761778397h^3u^{10}}{88921857024000\ 4257578514309120000}$$

$$+\tau_{n+1} \frac{977h^3}{75600} + \frac{2203h^3u^2}{39916800} + \frac{21493h^3u^4}{1614412800} + \frac{763403h^3u^6}{373621248000} + \frac{1216879h^3u^8}{4940103168000} + \frac{18914976163h^3u^{10}}{709596419051520000}$$

$$+\tau_{n+5} \frac{3h^3}{2800} + \frac{331h^3u^2}{2661120} + \frac{86561h^3u^4}{4843238400} + \frac{1132631h^3u^6}{523069747200} + \frac{718807h^3u^8}{2964061900800} + \frac{1228428499h^3u^{10}}{47306427936768000}$$

$$+\tau_{n+4} \frac{277h^3}{60480} + \frac{3173h^3u^2}{15966720} + \frac{16267h^3u^4}{645765120} + \frac{2911411h^3u^6}{1046139494400} + \frac{594289h^3u^8}{1976041267200} + \frac{9091076573h^3u^{10}}{283838567620608000}$$

$$+f_{n+2} \frac{421h^3}{4032} + \frac{137h^3u^2}{5322240} + \frac{347h^3u^4}{25159680} + \frac{2592077h^3u^6}{1046139494400} + \frac{1839469h^3u^8}{5928123801600} + \frac{456172631h^3u^{10}}{13516122267648000}$$

$$y^{0}_{n+3} = \frac{y_n}{24h} - \frac{3y_{n+2}}{4h} + \frac{y_{n+3}}{3h} + \frac{3y_{n+4}}{8h} + f_{n+3}\left(\frac{617h^3}{10080} + \frac{h^3u^2}{290304} + \frac{23h^3u^4}{12773376} + \frac{60829h^3u^6}{209227898880}\right.$$

$$\left. + \frac{5527h^3u^8}{209227898880} + \frac{255421h^3u^{10}}{142274971238400}\right) !$$

$$+f_n\left(\frac{h^3}{6300} - \frac{9871h^3u^2}{319334400} + \frac{545059h^3u^4}{116237721600} - \frac{1107667h^3u^6}{1902071808000} + \frac{11466181h^3u^8}{177843714048000} - \frac{12785191117h^3u^{10}}{1892257117470720000}\right) !$$

$$+f_{n+6}\left(\frac{3h^3}{22400} + \frac{9761h^3u^2}{319334400} + \frac{524129h^3u^4}{116237721600} + \frac{1653721h^3u^6}{2988969984000} + \frac{1832731h^3u^8}{29640619008000} + \frac{1382831243h^3u^{10}}{210250790830080000}\right) !$$

$$+f_{n+5}\left(\frac{19h^3}{16800} - \frac{103h^3u^2}{844800} + \frac{345931h^3u^4}{19372953600} - \frac{2538661h^3u^6}{1162377216000} + \frac{43515749h^3u^8}{177843714048000} - \frac{24721107409h^3u^{10}}{946128558735360000}\right) !$$

$$+f_{n+4}\left(\frac{17h^3}{4032} + \frac{3217h^3u^2}{21288960} + \frac{18637h^3u^4}{861020160} + \frac{3655919h^3u^6}{1394852659200} + \frac{10526591h^3u^8}{35568742809600} + \frac{12105771257h^3u^{10}}{378451423494144000}\right) !$$

$$+f_{n+2}\left(\frac{827h^3}{13440} - \frac{1109h^3u^2}{7096320} + \frac{188663h^3u^4}{7749181440} - \frac{473801h^3u^6}{154983628800} + \frac{114769h^3u^8}{342007142400} - \frac{13124901047h^3u^{10}}{378451423494144000}\right) !$$

$$+f_{n+1}\left(\frac{67h^3}{12600} + \frac{6599h^3u^2}{53222400} + \frac{11117h^3u^4}{587059200} + \frac{8224273h^3u^6}{3487131648000} + \frac{601747h^3u^8}{2309658624000} + \frac{25740237199h^3u^{10}}{946128558735360000}\right) !$$

$$y^{0}_{n+4} = \frac{y_n}{12h} + \frac{y_{n+2}}{h} - \frac{8y_{n+3}}{3h} + \frac{7y_{n+4}}{4h}$$

$$+f_n\left(\frac{11h^5}{64800} + \frac{3461h^5u^2}{119750400} + \frac{67393h^5u^4}{14529715200} - \frac{4663667h^5u^6}{7846046208000} + \frac{2937499h^5u^8}{44460928512000} + \frac{1126556657h^5u^{10}}{163753019781120000}\right) !$$

$$+f_{n+3}\left(\frac{311h^5}{1620} - \frac{773h^5u^2}{5987520} + \frac{10753h^5u^4}{726485760} - \frac{536051h^5u^6}{392302310400} + \frac{220747h^5u^8}{2223046425600} - \frac{3434279h^5u^{10}}{569194988544000}\right) !$$

$$+f_{n+6}\left(\frac{19h^3}{453600} - \frac{383h^3u^2}{23950080} + \frac{45887h^3u^4}{14529715200} - \frac{718313h^3u^6}{1569209241600} + \frac{499201h^3u^8}{8892185702400} - \frac{2672163239h^3u^{10}}{425757851430912000}\right) !$$

$$+T_{n+5} \quad \frac{h^3}{37800} + \frac{1019h^3u^2}{19958400} + \frac{27007h^3u^4}{2421619200} + \frac{2215693h^3u^6}{1307674368000} + \frac{227203h^3u^8}{1058593536000} + \frac{8693140739h^3u^{10}}{354798209525760000} \quad !$$

$$+T_{n+4} \quad \frac{103h^3}{10080} + \frac{41h^3u^2}{2661120} + \frac{43h^3u^4}{5125120} + \frac{839821h^3u^6}{523069747200} + \frac{76093h^3u^8}{329340211200} + \frac{191688823h^3u^{10}}{6758061133824000} \quad !$$

$$+T_{n+2} \quad \frac{103h^3}{864} + \frac{1669h^3u^2}{7983360} + \frac{29633h^3u^4}{968647680} + \frac{147071h^3u^6}{40236134400} + \frac{1126331h^3u^8}{2964061900800} + \frac{5309885629h^3u^{10}}{141919283810304000} \quad !$$

$$+f_{n+1} \quad \frac{149h^3}{12600} + \frac{19h^3u^2}{147840} + \frac{16171h^3u^4}{807206400} + \frac{93937h^3u^6}{37362124800} + \frac{135461h^3u^8}{494010316800} + \frac{665170739h^3u^{10}}{23653213968384000} \quad !$$

$$4h\,3h + 8h = \frac{11y_n}{} + \frac{17y_{n+2}}{} + \frac{23y_{n+3}}{} + \frac{31y_{n+4}}{} + \frac{y_{n+5}}{24h}$$

$$+f_n \quad \frac{47h^3}{907200} + \frac{8201h^3u^2}{191600640} + \frac{485851h^3u^4}{116237721600} + \frac{4510007h^3u^6}{12553673932800} + \frac{1446803h^3u^8}{35568742809600} + \frac{3430562749h^3u^{10}}{681212562289459200} \quad !$$

$$+T_{n+3} \quad \frac{121301h^3}{90720} + \frac{6031h^3u^2}{9580032} + \frac{558343h^3u^4}{5811886080} + \frac{556331h^3u^6}{57062154240} + \frac{13039457h^3u^8}{17784371404800} + \frac{7701083191h^3u^{10}}{170303140572364800} \quad !$$

$$+T_{n+6} \quad \frac{149h^3}{362880} + \frac{20263h^3u^2}{191600640} + \frac{534179h^3u^4}{38745907200} + \frac{16749289h^3u^6}{12553673932800} + \frac{35197h^3u^8}{308756448000} + \frac{1914998831h^3u^{10}}{200356635967488000} \quad !$$

$$+T_{n+5} \quad \frac{3h^3}{700} + \frac{739h^3u^2}{1520640} + \frac{537631h^3u^4}{8302694400} + \frac{13206073h^3u^6}{2092278988800} + \frac{18826669h^3u^8}{35568742809600} + \frac{693438509h^3u^{10}}{16219346721177600} \quad !$$

$$+T_{n+4} \quad \frac{15187h^3}{30240} + \frac{10775h^3u^2}{12773376} + \frac{209171h^3u^4}{1788272640} + \frac{9662857h^3u^6}{836911595520} + \frac{2562533h^3u^8}{2736057139200} + \frac{15985715503h^3u^{10}}{227070854096486400} \quad !$$

$$+T_{n+2} \quad \frac{25201h^3}{40320} + \frac{13h^3u^2}{129024} + \frac{126167h^3u^4}{4649508864} + \frac{515285h^3u^6}{167382319104} + \frac{2902721h^3u^8}{17784371404800} + \frac{583549121h^3u^{10}}{227070854096486400} \quad !$$

$$+T_{n+1} \quad \frac{439h^3}{6048} + \frac{3457h^3u^2}{31933440} + \frac{413359h^3u^4}{58118860800} + \frac{138113h^3u^6}{298896998400} + \frac{15896603h^3u^8}{177843714048000} + \frac{8868181433h^3u^{10}}{567677135241216000} \quad !$$

$$y^{0}_{n+6} = \frac{13y_n}{12h} + \frac{9y_{n+2}}{h} - \frac{44y_{n+3}}{3h} + \frac{27y_{n+4}}{4h} + f_n\left(\frac{11h^3}{20160} + \frac{7h^3u^2}{253440} + \frac{96121h^3u^4}{9686476800} + \frac{33947h^3u^6}{23247544320} - \frac{1829063h^3u^8}{12703122432000} - \frac{67537579h^3u^{10}}{+0 + 5565462110208000}\right)!$$

$$+f_{n+3}\left(\frac{557h^3}{168} - \frac{403h^3u^2}{266112} + \frac{11527h^3u^4}{484323840} - \frac{69653h^3u^6}{17435658240} + \frac{89351h^3u^8}{114002380800} - \frac{21830773h^3u^{10}}{315376186245120}\right)!$$

$$+f_{n+6}\left(\frac{887h^3}{100800} + \frac{659h^3u^2}{5322240} - \frac{227h^3u^4}{18450432} + \frac{369899h^3u^6}{348713164800} - \frac{5834063h^3u^8}{88921857024000} + \frac{70459379h^3u^{10}}{13516122267648000}\right)!$$

$$+f_{n+5}\left(\frac{2411h^3}{5040} - \frac{1721h^3u^2}{2661120} + \frac{249877h^3u^4}{4843238400} - \frac{19147h^3u^6}{4981616640} + \frac{8183437h^3u^8}{44460928512000} - \frac{658969711h^3u^{10}}{47306427936768000}\right)!$$

$$+f_{n+4}\left(\frac{8233h^3}{4032} + \frac{1465h^3u^2}{1064448} - \frac{142229h^3u^4}{1937295360} + \frac{20963h^3u^6}{6340239360} - \frac{227063h^3u^8}{3556874280960} + \frac{8510923h^3u^{10}}{995924798668800}\right)!$$

$$+f_{n+2}\left(\frac{5993h^3}{4032} + \frac{953h^3u^2}{1064448} + \frac{73067h^3u^4}{1937295360} + \frac{648511h^3u^6}{69742632960} + \frac{105737h^3u^8}{95103590400} + \frac{1803062033h^3u^{10}}{18922571174707200}\right)!$$

$$+f_{n+1}\left(\frac{4247h^3}{25200} - \frac{697h^3u^2}{2661120} - \frac{36143h^3u^4}{968647680} - \frac{1088063h^3u^6}{174356582400} - \frac{29091571h^3u^8}{44460928512000} - \frac{2623739281h^3u^{10}}{47306427936768000}\right)!$$

$$y^{00}_{n} = \frac{3y_n}{4h^2} - \frac{7y_{n+2}}{2h^2} + \frac{4y_{n+3}}{h^2} - \frac{5y_{n+4}}{4h^2}$$

$$+f_{n+3}\left(\frac{499h^2}{1512} - \frac{214897h^2u^2}{11975040} - \frac{162389h^2u^4}{3113510400} + \frac{3946949h^2u^6}{156920924160} + \frac{9041803h^2u^8}{5130107136000} + \frac{5921032189h^2u^{10}}{70959641905152000}\right)!$$

$$+f_n\left(\frac{19763h^2}{302400} + \frac{506353h^2u^2}{239500800} + \frac{1547669h^2u^4}{62270208000} - \frac{5196869h^2u^6}{3138418483200} - \frac{198013279h^2u^8}{1333827855360000} - \frac{650998253h^2u^{10}}{83481931653120000}\right)!$$

$$+f_{n+6}\left(\frac{877h^2}{302400} - \frac{10937h^2u^2}{34214400} - \frac{1222891h^2u^4}{62270208000} - \frac{2697029h^2u^6}{3138418483200} - \frac{37073599h^2u^8}{1333827855360000} - \frac{775094077h^2u^{10}}{141919283810304000}\right)$$

$$\tau_{n+2}\ \frac{7543h^2}{8640} + \frac{312049h^2u^2}{15966720} + \frac{624149h^2u^4}{4151347200} \quad \frac{4363589h^2u^6}{209227898880} \quad \frac{20623817h^2u^8}{12703122432000} \quad \frac{7636344893h^2u^{10}}{94612855873536000}\ !$$

$$\tau_{n+1}\ \frac{71683h^2}{75600} \quad \frac{409201h^2u^2}{39916800} \quad \frac{98719h^2u^4}{943488000} + \frac{4780229h^2u^6}{523069747200} + \frac{19021111h^2u^8}{24700515840000} + \frac{3117219199h^2u^{10}}{78844046561280000}\ !$$

$$\tau_{n+5}\ \frac{229h^2}{10800} \quad \frac{20593h^2u^2}{39916800} + \frac{108733h^2u^4}{1482624000} + \frac{239513h^2u^6}{40236134400} + \frac{63896879h^2u^8}{222304642560000} + \frac{2490406781h^2u^{10}}{236532139683840000}\ !$$

$$+\tau_{n+4}\ \frac{4127h^2}{60480} + \frac{23549h^2u^2}{3193344} \quad \frac{299371h^2u^4}{4151347200} \quad \frac{3530309h^2u^6}{209227898880} \quad \frac{30240053h^2u^8}{29640619008000} \quad \frac{8496403h^2u^{10}}{191137082572800}\ !$$

$$y''_{n+1} = \frac{y_n}{2h^2} \quad \frac{2y_{n+2}}{h^2} + \frac{2y_{n+3}}{h^2} \quad \frac{y_{n+4}}{2h^2} + f_{n+3} \quad \frac{37h^2}{480} + \frac{49201h^2u^2}{23950080} + \frac{60677h^2u^4}{566092800} + \frac{1316131h^2u^6}{313841848320}$$

$$+\frac{129853h^2u^8}{1058593536000} + \frac{64712573h^2u^{10}}{32750603956224000} \quad \text{Csc}\ \frac{\overset{\#6}{hw}}{2} + f_n \quad \frac{421h^2}{151200}$$

$$\frac{121273h^2u^2}{124540416000} \quad \frac{1852007h^2u^4}{6276836966400} \quad \frac{4378267h^2u^6}{1333827855360000} \quad \frac{37509649h^2u^8}{2838385676206080000} \quad \frac{2853881419h^2u^{10}}{479001600}\ !$$

$$\tau_{n+1}\ \frac{479h^2}{100800} + \frac{97249h^2u^2}{79833600} + \frac{4371461h^2u^4}{62270208000} + \frac{223837h^2u^6}{69742632960} + \frac{83200037h^2u^8}{666913927680000} + \frac{17964551963h^2u^{10}}{4257578514309120000}\ !$$

$$\tau_{n+4}\ \frac{53h^2}{10080} \quad \frac{25177h^2u^2}{31933440} \quad \frac{817781h^2u^4}{24908083200} \quad \frac{98473h^2u^6}{139485265920} + \frac{4786693h^2u^8}{266765571072000} + \frac{5196590461h^2u^{10}}{1703031405723648000}\ !$$

$$\tau_{n+5}\ \frac{643h^2}{302400} + \frac{1153h^2u^2}{79833600} \quad \frac{52397h^2u^4}{8895744000} \quad \frac{725293h^2u^6}{1046139494400} \quad \frac{34115603h^2u^8}{666913927680000} \quad \frac{12916971269h^2u^{10}}{4257578514309120000}\ !$$

$$+\tau_{n+2}\ \frac{2021h^2}{6048} \quad \frac{14645h^2u^2}{6386688} \quad \frac{3186901h^2u^4}{24908083200} \quad \frac{2336843h^2u^6}{418455797760} \quad \frac{53871127h^2u^8}{266765571072000} \quad \frac{2048834231h^2u^{10}}{340606281144729600}\ !$$

$$y''_{n+2} = \frac{y_n}{4h^2} - \frac{y_{n+2}}{2h^2} + \frac{y_{n+4}}{4h^2} + f_{n+3}\left(\frac{2h^2}{45} - \frac{263h^2u^2}{374220} + \frac{487h^2u^4}{8845200} - \frac{6967h^2u^6}{2451889440} + \frac{309269h^2u^8}{2778808032000} - \frac{22164907h^2u^{10}}{6652466428608000}\right)$$

$$+f_n\left(\frac{2h^2}{4725} + \frac{659h^2u^2}{7484400} + \frac{521h^2u^4}{72072000} + \frac{20113h^2u^6}{49037788800} + \frac{3139337h^2u^8}{166728481920000} + \frac{5851933h^2u^{10}}{7826431092480000}\right)!$$

$$+f_{n+6}\left(\frac{h^2}{9450} - \frac{19h^2u^2}{1069200} + \frac{479h^2u^4}{277992000} - \frac{6179h^2u^6}{49037788800} + \frac{183389h^2u^8}{23818354560000} - \frac{18384349h^2u^{10}}{44349776190720000}\right)!$$

$$+f_{n+1}\left(\frac{31h^2}{675} - \frac{527h^2u^2}{1247400} + \frac{33491h^2u^4}{972972000} - \frac{15731h^2u^6}{8172964800} + \frac{7206481h^2u^8}{83364240960000} - \frac{221130629h^2u^{10}}{66524664286080000}\right)!$$

$$f_{n+4}\left(\frac{h^2}{945} + \frac{131h^2u^2}{498960} + \frac{7361h^2u^4}{389188800} + \frac{47h^2u^6}{59439744} + \frac{571891h^2u^8}{33345696384000} - \frac{10823233h^2u^{10}}{26609865714432000}\right)!$$

$$h^2 f_{n+5}\left(\frac{h^2}{1575} + \frac{h^2u^2}{1247400} + \frac{1349h^2u^4}{972972000} - \frac{599h^2u^6}{2724321600} + \frac{1639639h^2u^8}{83364240960000} + \frac{88141187h^2u^{10}}{66524664286080000}\right)!$$

$$+f_{n+2}\left(\frac{17h^2}{70} + \frac{79h^2u^2}{99792} + \frac{24781h^2u^4}{389188800} + \frac{97h^2u^6}{27941760} + \frac{384227h^2u^8}{2565053568000} + \frac{5752507h^2u^{10}}{1064394628577280}\right)!$$

$$y''_{n+3} = \frac{y_{n+2}}{h^2} - \frac{2y_{n+3}}{h^2} + \frac{y_{n+4}}{h^2} + f_{n+3}\left(\frac{461h^2}{6048} + \frac{1747h^2u^2}{23950080} + \frac{42017h^2u^4}{6227020800} + \frac{29957h^2u^6}{62768369664} + \frac{475171h^2u^8}{16672848192000} + \frac{214381241h^2u^{10}}{141919283810304000}\right)!$$

$$+f_n\left(\frac{h^2}{43200} - \frac{1747h^2u^2}{479001600} + \frac{42017h^2u^4}{124540416000} - \frac{29957h^2u^6}{1255367393280} + \frac{475171h^2u^8}{333456963840000} - \frac{214381241h^2u^{10}}{2838385676206080000}\right)!$$

$$+f_{n+6}\left(\frac{97h^2}{302400} - \frac{1747h^2u^2}{79833600} + \frac{42017h^2u^4}{20756736000} - \frac{29957h^2u^6}{209227898880} + \frac{475171h^2u^8}{55576160640000} - \frac{214381241h^2u^{10}}{473064279367680000}\right)!$$

$$+f_{n+5}\frac{h^2}{43200}\frac{1747h^2u^2}{479001600}\frac{42017h^2u^4}{124540416000}\frac{29957h^2u^6}{1255367393280}\frac{475171h^2u^8}{333456963840000}$$

$$\frac{214381241h^2u^{10}}{2838385676206080000}!$$

$$+f_{n+4}\frac{233h^2}{60480}\frac{1747h^2u^2}{31933440}\frac{42017h^2u^4}{8302694400}\frac{29957h^2u^6}{83691159552}\frac{475171h^2u^8}{22230464256000}\frac{214381241h^2u^{10}}{189225711747072000}!$$

$$+f_{n+2}\frac{233h^2}{60480}\frac{1747h^2u^2}{31933440}\frac{42017h^2u^4}{8302694400}\frac{29957h^2u^6}{83691159552}\frac{475171h^2u^8}{22230464256000}\frac{214381241h^2u^{10}}{189225711747072000}!$$

$$+f_{n+1}\frac{97h^2}{302400}+\frac{1747h^2u^2}{79833600}+\frac{42017h^2u^4}{20756736000}+\frac{29957h^2u^6}{209227898880}+\frac{475171h^2u^8}{55576160640000}+\frac{214381241h^2u^{10}}{473064279367680000}!$$

$$y''_{n+4}=\frac{y_n}{4h^2}+\frac{5y_{n+2}}{2h^2}-\frac{4y_{n+3}}{h^2}+\frac{7y_{n+4}}{4h^2}$$

$$+f_{n+3}\frac{1739h^2}{2520}\frac{1273h^2u^2}{1710720}\frac{238699h^2u^4}{3113510400}\frac{141907h^2u^6}{22417274880}\frac{9521077h^2u^8}{22230464256000}\frac{5295361079h^2u^{10}}{212878925715456000}!$$

$$+f_n\frac{13h^2}{302400}\frac{3761h^2u^2}{239500800}\frac{40021h^2u^4}{62270208000}+\frac{4343h^2u^6}{89669099520}+\frac{10870991h^2u^8}{1333827855360000}+\frac{940395517h^2u^{10}}{1419192838103040000}!$$

$$+f_{n+6}\frac{173h^2}{302400}+\frac{21583h^2u^2}{239500800}+\frac{8213h^2u^4}{988416000}+\frac{262099h^2u^6}{448345497600}+\frac{46255471h^2u^8}{1333827855360000}+\frac{7769535607h^2u^{10}}{4257578514309120000}!$$

$$+f_{n+5}\frac{547h^2}{75600}\frac{17359h^2u^2}{39916800}\frac{1273537h^2u^4}{31135104000}\frac{4037h^2u^6}{1358622720}\frac{11006743h^2u^8}{60628538880000}\frac{20834432293h^2u^{10}}{2128789257154560000}!$$

$$+f_{n+4}\frac{97h^2}{960}+\frac{2627h^2u^2}{3193344}+\frac{994817h^2u^4}{12454041600}+\frac{20219h^2u^6}{3321077760}+\frac{103381933h^2u^8}{266765571072000}+\frac{3672051553h^2u^{10}}{170303140572364800}!$$

$$+f_{n+2}\frac{4147h^2}{12096}+\frac{4687h^2u^2}{15966720}+\frac{437377h^2u^4}{12454041600}+\frac{14549h^2u^6}{4269957120}+\frac{67997453h^2u^8}{266765571072000}+\frac{13411908709h^2u^{10}}{851515702861824000}!$$

$$+f_{n+1}\frac{991h^2}{25200}\frac{463h^2u^2}{39916800}\frac{158657h^2u^4}{31135104000}\frac{20593h^2u^6}{24908083200}\frac{1026637h^2u^8}{13610488320000}\frac{994339471h^2u^{10}}{193526296104960000}!$$

$$y''_{n+5} = \frac{y_n}{2h^2} + \frac{4y_{n+2}}{h^2} - \frac{6y_{n+3}}{h^2} + \frac{5y_{n+4}}{2h^2}$$

$$+T_{n+3} \frac{1039h^2}{672} + \frac{47221h^2u^2}{23950080} + \frac{398347h^2u^4}{6227020800} - \frac{873713h^2u^6}{313841848320} - \frac{1262993h^2u^8}{2470051584000} - \frac{17503072771h^2u^{10}}{425757851430912000} !$$

$$+f_n \frac{47h^2}{75600} + \frac{24851h^2u^2}{479001600} + \frac{29119h^2u^4}{4612608000} + \frac{3935849h^2u^6}{6276836966400} + \frac{63429721h^2u^8}{1333827855360000} + \frac{25223453579h^2u^{10}}{8515157028618240000} !$$

$$+T_{n+6} \frac{47h^2}{18900} - \frac{119293h^2u^2}{479001600} - \frac{1582907h^2u^4}{124540416000} - \frac{2188423h^2u^6}{6276836966400} + \frac{4771901h^2u^8}{1333827855360000}$$

$$+ \frac{296445211h^2u^{10}}{258035061473280000} !$$

$$+T_{n+5} \frac{8279h^2}{100800} + \frac{95269h^2u^2}{79833600} + \frac{3564161h^2u^4}{62270208000} + \frac{389237h^2u^6}{348713164800} - \frac{43644613h^2u^8}{666913927680000} - \frac{37068456697h^2u^{10}}{4257578514309120000} !$$

$$+T_{n+4} \frac{3863h^2}{3780} - \frac{14249h^2u^2}{6386688} - \frac{339943h^2u^4}{3558297600} - \frac{146999h^2u^6}{418455797760} + \frac{10424789h^2u^8}{38109367296000} + \frac{8957767501h^2u^{10}}{340606281144729600} !$$

$$+T_{n+2} \frac{499h^2}{720} - \frac{23197h^2u^2}{31933440} - \frac{10481h^2u^4}{24908083200} + \frac{9715h^2u^6}{2145927168} + \frac{131631343h^2u^8}{266765571072000} + \frac{60229599121h^2u^{10}}{1703031405723648000} !$$

$$+T_{n+1} \frac{3251h^2}{43200} - \frac{827h^2u^2}{79833600} - \frac{1174079h^2u^4}{62270208000} - \frac{2915137h^2u^6}{1046139494400} - \frac{160960253h^2u^8}{666913927680000} - \frac{5226921533h^2u^{10}}{327506039562240000} !$$

$$y''_{n+6} = \frac{3y_n}{4h^2} + \frac{11y_{n+2}}{2h^2} - \frac{8y_{n+3}}{h^2} + \frac{13y_{n+4}}{4h^2}$$

$$\frac{9577h^2}{5987520} - \frac{108191h^2u^2}{5987520}$$

$$+f_{n+3} \frac{3780}{0} !$$

$$\frac{182107h^2u^4}{1556755200} + \frac{104753h^2u^6}{7132769280} + \frac{27060557h^2u^8}{33345696384000} + \frac{51344159h^2u^{10}}{2729216996352000}$$

$$+f_n \frac{53h^2}{21600} - \frac{37537h^2u^2}{119750400} - \frac{510533h^2u^4}{31135104000} - \frac{527323h^2u^6}{1569209241600} + \frac{13174363h^2u^8}{666913927680000} + \frac{1905494989h^2u^{10}}{709596419051520000} !$$

$$+T_{n+6} \frac{9949h^2}{151200} + \frac{253919h^2u^2}{119750400} + \frac{874747h^2u^4}{31135104000} - \frac{136711h^2u^6}{120708403200} - \frac{67295477h^2u^8}{666913927680000} - \frac{3240443123h^2u^{10}}{709596419051520000} !$$

$$+f_{n+5}\frac{5747h^2}{5400}+\frac{205343h^2u^2}{19958400}+\frac{91981h^2u^4}{741312000}+\frac{1568923h^2u^6}{261534873600}+\frac{855299h^2u^8}{1764322560000}+\frac{794262257h^2u^{10}}{39422023280640000}\,!$$

$$+f_{n+4}\frac{57599h^2}{30240}+\frac{156767h^2u^2}{7983360}+\frac{412987h^2u^4}{2075673600}+\frac{1360603h^2u^6}{104613949440}+\frac{40472197h^2u^8}{44460928512000}+\frac{1525130419h^2u^{10}}{47306427936768000}\,!$$

$$+f_{n+2}\frac{5827h^2}{6048}+\frac{11923h^2u^2}{1596672}+\frac{48773h^2u^4}{2075673600}+\frac{943963h^2u^6}{104613949440}+\frac{4549639h^2u^8}{14820309504000}+\frac{4226273h^2u^{10}}{1051253954150400}\,!$$

$$+f_{n+1}\frac{5189h^2}{37800}+\frac{1577h^2u^2}{2851200}+\frac{25423h^2u^4}{471744000}+\frac{735643h^2u^6}{261534873600}+\frac{237277h^2u^8}{111152321280000}+\frac{1047838637h^2u^{10}}{118266069841920000}\,!$$

$$y^{000}_{n}=\frac{y_n}{4h^3}+\frac{3y_{n+2}}{2h^3}-\frac{2y_{n+3}}{h^3}+\frac{3y_{n+4}}{4h^3}+f_{n+6}\frac{277h}{24192}+\frac{613hu^2}{414720}+\frac{15877hu^4}{127733760}+\frac{2873137hu^6}{298896998400}$$

$$+\frac{4983359hu^8}{6276836966400}+\frac{23698361hu^{10}}{328326856704000}\,!$$

$$+f_{n+2}\frac{13729h}{120960}+\frac{29717hu^2}{414720}+\frac{5191hu^4}{3649536}+\frac{33084647hu^6}{2092278988800}+\frac{1098563hu^8}{1255367393280}+\frac{855520789hu^{10}}{4268249137152000}\,!$$

$$+f_{n+1}\frac{42967h}{30240}+\frac{7889hu^2}{207360}+\frac{57329hu^4}{63866880}+\frac{23355131hu^6}{1046139494400}+\frac{5110723hu^8}{3138418483200}+\frac{444939217hu^{10}}{2134124568576000}\,!$$

$$+f_{n+5}\frac{2519h}{30240}+\frac{533hu^2}{1036800}+\frac{26639hu^4}{63866880}+\frac{217988377hu^6}{5230697472000}+\frac{10937917hu^8}{3138418483200}+\frac{66367451hu^{10}}{217767813120000}\,!$$

$$+f_{n+4}\frac{31999h}{120960}+\frac{10261hu^2}{414720}+\frac{5647hu^4}{25546752}+\frac{134297369hu^6}{2092278988800}+\frac{6925757hu^8}{1255367393280}+\frac{1882829803hu^{10}}{4268249137152000}\,!$$

$$+f_{n+3}\frac{29h}{48}+\frac{2221hu^2}{34560}+\frac{155hu^4}{193536}+\frac{432533hu^6}{13412044800}+\frac{107911hu^8}{34871316480}+\frac{1119073hu^{10}}{6974263296000}\,!$$

$$+f_{n+2}\frac{36791h}{120960}+\frac{16391hu^2}{2073600}+\frac{26107hu^4}{127733760}+\frac{66822221hu^6}{10461394944000}+\frac{39493hu^8}{81517363200}+\frac{1197957127hu^{10}}{21341245685760000}\,!$$

$$y^{000}_{n+1}=\frac{y_n}{4h^3}+\frac{3y_{n+2}}{2h^3}-\frac{2y_{n+3}}{h^3}+\frac{3y_{n+4}}{4h^3}+f_{n+6}\frac{341h}{120960}+\frac{7733hu^2}{14515200}+\frac{557hu^4}{8515584}+\frac{72255409hu^6}{10461394944000}$$

$$\frac{484567hu^{8}\ 1479052483hu^{10}}{697426329600\ 21341245685760000}\ !$$

$$T_{n+3} \quad \frac{241h}{15120} \quad \frac{953hu^{2}}{103680} \quad \frac{3653hu^{4}}{6386688} \quad \frac{4665361hu^{6}}{174356582400} \quad \frac{321487hu^{8}}{313841848320} \quad \frac{34929721hu^{10}}{1067062284288000}\ !$$

$$+f_{n} \quad \frac{461h}{40320} + \frac{281hu^{2}}{193536} + \frac{15661hu^{4}}{127733760} + \frac{4009903hu^{6}}{418455797760} + \frac{384929hu^{8}}{482833612800} + \frac{625823hu^{10}}{8622725529600}\ !$$

$$T_{n+1} \quad \frac{10411h}{30240} \quad \frac{48821hu^{2}}{7257600} \quad \frac{6995hu^{4}}{12773376} \quad \frac{19499203hu^{6}}{475517952000} \quad \frac{210809hu^{8}}{64049356800} \quad \frac{3132753571hu^{10}}{10670622842880000}\ !$$

$$T_{n+5} \quad \frac{91h}{4320} + \frac{1759hu^{2}}{1451520} + \frac{1187hu^{4}}{5806080} + \frac{2007919hu^{6}}{80472268800} + \frac{8400719hu^{8}}{3138418483200} + \frac{34390297hu^{10}}{125536739328000}\ !$$

$$T_{n+4} \quad \frac{8423h}{120960} + \frac{5609hu^{2}}{2903040} \quad \frac{1049hu^{4}}{25546752} \quad \frac{44263243hu^{6}}{2092278988800} \quad \frac{3718129hu^{8}}{1255367393280} \quad \frac{1409193041hu^{10}}{4268249137152000}\ !$$

$$+T_{n+2} \quad \frac{106651h}{120960} + \frac{34417hu^{2}}{2903040} + \frac{3281hu^{4}}{3649536} + \frac{18319963hu^{6}}{298896998400} + \frac{5647051hu^{8}}{1255367393280} + \frac{1618771367hu^{10}}{4268249137152000}\ !$$

$$y^{000}_{n+2} = \frac{y_{n}}{4h^{3}} + \frac{3y_{n+2}}{2h^{3}} \cdot \frac{2y_{n+3}}{h^{3}} + \frac{3y_{n+4}}{4h^{3}} \cdot + f_{n+3} \quad \frac{3823h}{15120} + \frac{1457hu^{2}}{725760} + \frac{5479hu^{4}}{31933440} + \frac{627749hu^{6}}{58118860800}$$

$$\frac{179233hu^{8}}{313841848320} + \frac{28767367hu^{10}}{1067062284288000}\ !$$

$$+f_{n} \quad \frac{49h}{17280} \quad \frac{1159hu^{2}}{2073600} \quad \frac{2857hu^{4}}{42577920} \quad \frac{72567629hu^{6}}{10461394944000} \quad \frac{1531hu^{8}}{2214051840} \quad \frac{210076289hu^{10}}{3048749383680000}\ !$$

$$T_{n+6} \quad \frac{67h}{40320} + \frac{1733hu^{2}}{4838400} + \frac{31897hu^{4}}{638668800} + \frac{61268147hu^{6}}{10461394944000} + \frac{3981919hu^{8}}{6276836966400} + \frac{4025639hu^{10}}{60801269760000}\ !$$

$$T_{n+5} \quad \frac{407h}{30240} \quad \frac{8941hu^{2}}{7257600} \quad \frac{11663hu^{4}}{63866880} \quad \frac{16698079hu^{6}}{747242496000} \quad \frac{1556921hu^{8}}{627683696640} \quad \frac{2797231211hu^{10}}{10670622842880000}\ !$$

$$+T_{n+1} \quad \frac{2153h}{30240} + \frac{17683hu^{2}}{7257600} + \frac{13027hu^{4}}{45619200} + \frac{150784999hu^{6}}{5230697472000} + \frac{8860003hu^{8}}{3138418483200} + \frac{2969835413hu^{10}}{10670622842880000}\ !$$

$$T_{n+4} \quad \frac{1235h}{24192} + \frac{457hu^{2}}{580608} + \frac{20939hu^{4}}{127733760} + \frac{9993733hu^{6}}{418455797760} + \frac{3623453hu^{8}}{1255367393280} + \frac{271092911hu^{10}}{853649827430400}\ !$$

$$+T_{n+2} \quad \frac{12673h}{120960} \quad \frac{11027hu^{2}}{2903040} \quad \frac{53813hu^{4}}{127733760} \quad \frac{83867111hu^{6}}{2092278988800} \quad \frac{4698851hu^{8}}{1255367393280} \quad \frac{1528068757hu^{10}}{4268249137152000}\ !$$

$$y^{000}_{n+3} = \frac{y_n}{4h^3} + \frac{3y_{n+2}}{2h^3} - \frac{2y_{n+3}}{h^3} + \frac{3y_{n+4}}{4h^3} + f_{n+3}\left(\frac{617h}{1680} - \frac{hu^2}{48384} - \frac{23hu^4}{2128896} - \frac{60829hu^6}{34871316480}\right.$$

$$+f_n\left(\frac{199h}{120960} + \frac{4819hu^2}{14515200} + \frac{30817hu^4}{638668800} + \frac{60955927hu^6}{10461394944000} + \frac{4002637hu^8}{6276836966400} + \frac{1421517749hu^{10}}{21341245685760000}\right)$$

$$+f_{n+6}\left(\frac{181h}{120960} - \frac{4789hu^2}{14515200} - \frac{30127hu^4}{638668800} - \frac{59131057hu^6}{10461394944000} - \frac{557593hu^8}{896690995200} - \frac{1398529859hu^{10}}{21341245685760000}\right)$$

$$+f_{n+5}\left(\frac{397h}{30240} + \frac{9563hu^2}{7257600} + \frac{59909hu^4}{319334400} + \frac{117349679hu^6}{5230697472000} + \frac{7756559hu^8}{3138418483200} + \frac{2785565773hu^{10}}{10670622842880000}\right)$$

$$+f_{n+4}\left(\frac{7367h}{120960} - \frac{4759hu^2}{2903040} - \frac{29437hu^4}{127733760} - \frac{57306187hu^6}{2092278988800} - \frac{760733hu^8}{251073478656} - \frac{1375541969hu^{10}}{4268249137152000}\right)$$

$$+f_{n+2}\left(\frac{1399h}{3456} + \frac{4849hu^2}{2903040} + \frac{4501hu^4}{18247680} - \frac{62780797hu^6}{2092278988800} + \frac{4102123hu^8}{1255367393280} + \frac{1444505639hu^{10}}{4268249137152000}\right)$$

$$+f_{n+1}\left(\frac{773h}{30240} - \frac{1379hu^2}{1036800} - \frac{61979hu^4}{319334400} - \frac{17546327hu^6}{747242496000} - \frac{8055017hu^8}{3138418483200} - \frac{2854529443hu^{10}}{10670622842880000}\right)$$

$$y^{000}_{n+4} = \frac{y_n}{4h^3} + \frac{3y_{n+2}}{2h^3} - \frac{2y_{n+3}}{h^3} + \frac{3y_{n+4}}{4h^3} + f_n\left(\frac{61h}{40320} - \frac{1723hu^2}{4838400} - \frac{2837hu^4}{58060800} - \frac{59443277hu^6}{10461394944000}\right.$$

$$\left. - \frac{3882433hu^8}{6276836966400} - \frac{22063673hu^{10}}{338749931520000}\right)$$

$$+f_{n+3}\left(\frac{14929h}{15120} - \frac{1487hu^2}{725760} - \frac{6169hu^4}{31933440} - \frac{2491537hu^6}{174356582400} - \frac{39817hu^8}{44834549760} - \frac{51755257hu^{10}}{1067062284288000}\right)$$

$$+f_{n+6}\left(\frac{361h}{120960} + \frac{8143hu^2}{14515200} + \frac{2903hu^4}{42577920} + \frac{74392499hu^6}{10461394944000} + \frac{493319hu^8}{697426329600} + \frac{114886301hu^{10}}{1641634283520000}\right)$$

$$+f_{n+5}\left(\frac{983h}{30240} - \frac{2539hu^2}{1036800} - \frac{93259hu^4}{319334400} - \frac{14205419hu^6}{475517952000} - \frac{704497hu^8}{241416806400} - \frac{3038799083hu^{10}}{10670622842880000}\right)$$

$$+f_{n+4}\left(\frac{7753h}{17280} + \frac{11117hu^2}{2903040} + \frac{55883hu^4}{127733760} - \frac{12763103hu^6}{298896998400} + \frac{4997309hu^8}{1255367393280} - \frac{1597032427hu^{10}}{4268249137152000}\right)$$

$$+f_{n+2}\left(\frac{35423h}{120960} - \frac{439hu^2}{580608} - \frac{18869hu^4}{127733760} - \frac{8898811hu^6}{418455797760} - \frac{664999hu^8}{251073478656} - \frac{257300177hu^{10}}{853649827430400}\right)$$

$$+f_{n+1}\left(\frac{1577h}{30240} - \frac{8851hu^2}{7257600} - \frac{1607hu^4}{9123840} - \frac{111411943hu^6}{5230697472000} - \frac{7486147hu^8}{3138418483200} - \frac{2728267541hu^{10}}{10670622842880000}\right)$$

$$y^{000}_{n+5} = \frac{y_n}{4h^3} + \frac{3y_{n+2}}{2h^3} - \frac{2y_{n+3}}{h^3} + \frac{3y_{n+4}}{4h^3} + f_n \cdot \frac{359h}{120960} + \frac{1109hu^2}{2073600} + \frac{2831hu^4}{42577920} + \frac{814069hu^6}{114960384000} + \frac{7867hu^8}{11070259200} + \frac{1502040373hu^{10}}{21341245685760000} !$$

$$+f_{n+3} \frac{2173h}{3024} + \frac{6641hu^2}{725760} + \frac{3515hu^4}{6386688} + \frac{1352357hu^6}{58118860800} + \frac{17077hu^8}{24141680640} + \frac{1085621hu^{10}}{97005662208000} !$$

$$+f_{n+6} \frac{13h}{1152} + \frac{1403hu^2}{967680} + \frac{15523hu^4}{127733760} + \frac{19684541hu^6}{2092278988800} + \frac{4904591hu^8}{6276836966400} + \frac{11303141hu^{10}}{158083301376000} !$$

$$+f_{n+5} \frac{11581h}{30240} + \frac{48731hu^2}{7257600} + \frac{34561hu^4}{63866880} + \frac{209016623hu^6}{5230697472000} + \frac{10031183hu^8}{3138418483200} + \frac{3063789901hu^{10}}{10670622842880000} !$$

$$+f_{n+4} \frac{148249h}{120960} + \frac{34327hu^2}{2903040} + \frac{22553hu^4}{25546752} + \frac{122765131hu^6}{2092278988800} + \frac{5348593hu^8}{1255367393280} + \frac{1549807697hu^{10}}{4268249137152000} !$$

$$+f_{n+2} \frac{50021h}{120960} + \frac{5519hu^2}{2903040} + \frac{19hu^4}{331776} + \frac{4521623hu^6}{190207180800} + \frac{4016587hu^8}{1255367393280} + \frac{1478156711hu^{10}}{4268249137152000} !$$

$$+f_{n+1} \frac{533h}{30240} + \frac{1777hu^2}{1451520} + \frac{13471hu^4}{63866880} + \frac{27197869hu^6}{1046139494400} + \frac{8699177hu^8}{3138418483200} + \frac{598427783hu^{10}}{2134124568576000} !$$

$$y^{000}_{n+6} = \frac{y_n}{4h^3} + \frac{3y_{n+2}}{2h^3} - \frac{2y_{n+3}}{h^3} + \frac{3y_{n+4}}{4h^3} + f_n \cdot \frac{1367h}{120960} + \frac{857hu^2}{580608} + \frac{15739hu^4}{127733760} + \frac{3949397hu^6}{418455797760} + \frac{4883873hu^8}{6276836966400} + \frac{8670889hu^{10}}{121949975347200}$$

$$+f_{n+3} \frac{2249h}{1680} + \frac{15557hu^2}{241920} + \frac{1751hu^4}{2128896} + \frac{716377hu^6}{24908083200} + \frac{96857hu^8}{34871316480} + \frac{16470031hu^{10}}{118562476032000} !$$

$$+f_{n+6} \frac{36809h}{120960} + \frac{114767hu^2}{14515200} + \frac{5249hu^4}{25546752} + \frac{68647091hu^6}{10461394944000} + \frac{3140447hu^8}{6276836966400} + \frac{1220945017hu^{10}}{21341245685760000} !$$

$$+f_{n+5} \frac{44137h}{30240} + \frac{55241hu^2}{1451520} + \frac{8249hu^4}{9123840} + \frac{24450053hu^6}{1046139494400} + \frac{5409181hu^8}{3138418483200} + \frac{458731951hu^{10}}{2134124568576000} !$$

$$+T_{n+4} \quad \frac{55327h}{120960} \quad \frac{208109hu^2}{+\,2903040} \quad \frac{3341hu^4}{+\,2322432} \quad \frac{3505387hu^6}{+\,190207180800} \quad \frac{1397021hu^8}{+\,1255367393280} \quad \frac{924484459hu^{10}}{+\,4268249137152000} \quad !$$

$$+T_{n+2} \quad \frac{9599h}{120960} \quad \frac{71917hu^2}{+\,2903040} \quad \frac{5233hu^4}{25546752} \quad \frac{9909443hu^6}{160944537600} \quad \frac{135251hu^8}{25619742720} \quad \frac{1813866133hu^{10}}{4268249137152000} \quad !$$

$$+T_{n+1} \quad \frac{527h}{4320} \quad \frac{3821hu^2}{7257600} \quad \frac{5245hu^4}{+\,12773376} \quad \frac{212513767hu^6}{+\,5230697472000} \quad \frac{10639459hu^8}{+\,3138418483200} \quad \frac{3183041429hu^{10}}{+\,10670622842880000} \quad !$$

Denoting the principal term of the local truncation term as LTEi, we obtain

LT E1 = $\frac{h^{12}}{120960}(!^2y^{(10)}(x_n) + y^{(12)}(x_n))$; LT E2 = $\frac{h^{11}}{12096}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$;

LT E3 = $\frac{h^{11}}{12096}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$; LT E4 = $\frac{h^{11}}{10395}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$;

LT E5 = $\frac{593h^{11}}{3991680}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$; LT E6 = $\frac{h^{11}}{22275}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$;

LT E7 = $\frac{409h^{11}}{6652800}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$; LT E8 = $\frac{h^{11}}{22275}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$;

LT E9 = $\frac{593h^{11}}{3991680}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$; LT E10 = $\frac{h^{11}}{10395}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$;

LT E11 = $\frac{23h^{11}}{9450}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$; LT E12 = $\frac{13h^{11}}{432400}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$;

LT E13 = $\frac{h^{11}}{9450}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$; LT E14 = $\frac{174h^{12}}{479001600}(!^2y^{(10)}(x_n) + y^{(12)}(x_n))$;

LT E15 = $\frac{h^{11}}{9450}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$; LT E16 = $\frac{13h^{11}}{43200}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$;

LT E17 = $\frac{23h^{11}}{9450}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$; LT E18 = $\frac{19h^{11}}{2025}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$;

LT E19 = $\frac{3601h^{11}}{1814400}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$; LT E20 = $\frac{13h^{11}}{14175}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$;

LT E21 = $\frac{1201h^{11}}{1814400}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$; LT E22 = $\frac{13h^{11}}{14175}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$;

LT E23 = $\frac{2301h^{11}}{1814400}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$; LT E24 = $\frac{19h^{11}}{2025}(!^2y^{(9)}(x_n) + y^{(11)}(x_n))$

Thus,the six-step TFBM has order

$$= (8; 7; 7; 7; 7; 7; 7; 7; 7; 7; 7; 7; 7; 8; 7; 7; 7; 7; 7; 7; 7; 7; 7; 7; 7)^T$$

and error constant

$$C_{p+4} = \frac{11}{120960}; \frac{1}{12096}; \frac{1}{12096}; \frac{1}{10395}; \frac{593}{3991680}; \frac{1}{22275}; \frac{409}{6652800}; \frac{1}{2275}; \frac{593}{3991680}; \frac{1}{10395}$$

$$= \frac{23}{9450}; \frac{13}{43200}; \frac{1}{9450}; \frac{1747}{479001600}; \frac{1}{9450}; \frac{13}{43200}; \frac{23}{9450}; \frac{19}{2025}; \frac{3601}{1814400}; \frac{13}{14175}; \frac{1201}{1814400}$$

$$\frac{13}{1814400}; \frac{2301}{2025}; \frac{19}{14175}^T$$

Case k = 8

Expanding the coe cients of the eight-step TFBM and substituting u = !h

we have;

$$y_{n+8} = y_n + 16y_{n+3} \quad 30y_{n+4} + 16y_{n+5}$$

$$+ f_{n+4} \frac{143921h^4}{18144} \quad \frac{1003h^4u^2}{171072} \quad \frac{175489h^4u^4}{373621248} \quad \frac{35501h^4u^6}{1601233920} \quad \frac{253789h^4u^8}{254062448640}$$

$$\frac{553698781h^4u^{10}}{9123382530662400}$$

$$+ \tau_{n+8} \frac{23h^4}{181440} \quad \frac{1003h^4u^2}{11975040} \quad \frac{175489h^4u^4}{26153487360} \quad \frac{35501h^4u^6}{112086374400} \quad \frac{253789h^4u^8}{17784371404800}$$

$$\frac{553698781h^4u^{10}}{638636777146368000}$$

$$+ f_n \frac{23h^4}{181440} \quad \frac{1003h^4u^2}{11975040} \quad \frac{175489h^4u^4}{26153487360} \quad \frac{35501h^4u^6}{112086374400} \quad \frac{253789h^4u^8}{17784371404800}$$

$$\frac{553698781h^4u^{10}}{638636777146368000}$$

$$f_{n+6} \frac{12121h^4}{9072427680934053120} \quad \frac{1003h^4u^2}{4003084800} \quad \frac{175489h^4u^4}{635156121600} \quad \frac{35501h^4u^6 \quad 253789h^4u^8}{}$$

$$\frac{553698781h^4u^{10}}{22808456326656000}$$

$$+ f_{n+7} \frac{731h^4}{4536} + \frac{1003h^4u^2}{1496880} + \frac{175489h^4u^4}{3269185920} + \frac{35501h^4u^6}{14010796800} + \frac{253789h^4u^8}{2223046425600}$$

$$\frac{553698781h^4u^{10}}{+ \quad 79829597143296000}$$

$$\tau_{n+2} \frac{12121h^4}{9072} \quad \frac{1003h^4u^2}{427680} \quad \frac{175489h^4u^4}{934053120} \quad \frac{35501h^4u^6}{4003084800} \quad \frac{253789h^4u^8}{635156121600}$$

$$\frac{553698781h^4u^{10}}{22808456326656000}$$

$$\tau_{n+5} \frac{102889h^4}{22680} + \frac{1003h^4u^2}{213840} + \frac{175489h^4u^4}{467026560} + \frac{35501h^4u^6}{2001542400} + \frac{253789h^4u^8}{317578060800}$$

$$\frac{553698781h^4u^{10}}{+ \quad 11404228163328000}$$

$$\tau_{n+3} \frac{102889h^4}{22680} + \frac{1003h^4u^2}{213840} + \frac{175489h^4u^4}{467026560} + \frac{35501h^4u^6}{2001542400} + \frac{253789h^4u^8}{317578060800}$$

$$\frac{553698781h^4u^{10}}{11404228163328000}$$

$$f_{n+1} \frac{731h^4}{4536} + \frac{1003h^4u^2}{1496880} + \frac{175489h^4u^4}{3269185920} + \frac{35501h^4u^6}{14010796800} + \frac{253789h^4u^8}{2223046425600}$$

$$\frac{553698781h^4u^{10}}{+ \quad 79829597143296000}$$

$$y_{n+7} = \frac{2y_n}{5} + 7y_{n+3} \quad 14y_{n+4} + \frac{42y_{n+5}}{5}$$

$$\tau_{n+4} \quad \frac{158381h^4}{51840} \quad \frac{1153h^4u^2}{855360} \quad \frac{141091h^4u^4}{1067489280} \quad \frac{149977h^4u^6}{16012339200} \quad \frac{3373639h^4u^8}{5081248972800}$$

$$\frac{154765379h^4u^{10}}{3258350903808000}$$

$$f_{n+8} \quad \frac{7h^4}{103680} \quad \frac{251h^4u^2}{34214400} \quad \frac{283h^4u^4}{14944849920} \quad \frac{154841h^4u^6}{2241727488000} \quad \frac{69799h^4u^8}{7258927104000}$$

$$\frac{1573358443h^4u^{10}}{1824676506132480000}$$

$$f_n \quad \frac{13h^4}{74724249600} \quad \frac{10981h^4u^2}{320246784000} \quad \frac{283597h^4u^4}{355687428096000} \quad \frac{63581h^4u^6}{3327127h^4u^8} \quad \frac{518400}{518400} \quad \frac{239500800}{239500800}$$

$$\frac{902887621h^4u^{10}}{1824676506132480000}$$

$$f_{n+6} \quad \frac{4093h^4}{1334361600} \quad \frac{571h^4u^2}{2911334400} \quad \frac{34919h^4u^4}{2540624486400} \quad \frac{8269h^4u^6}{679379h^4u^8} \quad \frac{25920}{25920} \quad \frac{3421440}{3421440}$$

$$\frac{10223569h^4u^{10}}{473941949644800}$$

$$f_{n+7} \quad \frac{h^4}{12960} \quad \frac{659h^4u^2}{119750400} \quad \frac{68423h^4u^4}{18681062400} \quad \frac{109211h^4u^6}{160123392000} \quad \frac{3408523h^4u^8}{44460928512000}$$

$$\frac{5958198361h^4u^{10}}{912338253066240000}$$

$$\tau_{n+2} \quad \frac{2753h^4}{5184} \quad \frac{15593h^4u^2}{17107200} \quad \frac{2413h^4u^4}{30326400} \quad \frac{745021h^4u^6}{160123392000} \quad \frac{3350383h^4u^8}{12703122432000}$$

$$\frac{2141010653h^4u^{10}}{130334036152320000}$$

$$f_{n+5} \quad \frac{88531h^4}{64800} \quad \frac{12079h^4u^2}{17107200} \quad \frac{210929h^4u^4}{2668723200} \quad \frac{81131h^4u^6}{12317184000} \quad \frac{3385267h^4u^8}{6351561216000}$$

$$\frac{5287727539h^4u^{10}}{130334036152320000}$$

$$\tau_{n+3} \quad \frac{4729h^4}{2592} \quad \frac{24817h^4u^2}{17107200} \quad \frac{70687h^4u^4}{533744640} \quad \frac{1344929h^4u^6}{160123392000} \quad \frac{3362011h^4u^8}{6351561216000}$$

$$\frac{4617256717h^4u^{10}}{130334036152320000}$$

$$\tau_{n+1} \quad \frac{169h^4}{2592} \quad \frac{1073h^4u^2}{3421440} \quad \frac{495941h^4u^4}{18681062400} \quad \frac{327031h^4u^6}{224172748800} \quad \frac{95393h^4u^8}{1270312243200}$$

$$\frac{60719783h^4u^{10}}{14035973124096000}$$

$$y_{n+6} = \frac{y_n}{10} + 2y_{n+3} + \frac{9y_{n+4}}{2} + \frac{18y_{n+5}}{5}$$

$$f_{n+4} \quad \frac{174301h^4}{241920} \quad \frac{1487h^4u^2}{2280960} \quad \frac{61961h^4u^4}{996323328} \quad \frac{214901h^4u^6}{49816166400} \quad \frac{927193h^4u^8}{3387499315200}$$

$$\frac{157557373h^4u^{10}}{9357315416064000}$$

$$f_{n+8} \quad \frac{5h^4}{96768} \quad \frac{1}{+464486400h^2u^4} \quad \frac{1553h^4u^2}{159667200} \quad \frac{385153h^4u^4}{348713164800} \quad \frac{336559h^4u^6}{3487131648000}$$

$$\frac{335773h^4u^8}{47424990412800} \quad \frac{352970861h^4u^{10}}{774105184419840000}$$

$$f_n \quad \frac{19h^4}{2419200} \quad \frac{203h^4u^2}{22809600} \quad \frac{234457h^4u^4}{348713164800} \quad \frac{31081h^4u^6}{1162377216000} \quad \frac{58507h^4u^8}{79041650688000}$$

$$\frac{213812227h^4u^{10}}{8515157028618240000}$$

$$T_{n+6} \quad \frac{107h^4}{24192} \quad \frac{19h^4u^2}{71280} \quad \frac{31589h^4u^4}{1132185600} \quad \frac{101h^4u^6}{45619200} \quad \frac{1591h^4u^8}{10340352000}$$

$$\frac{8721949h^4u^{10}}{894449267712000}$$

$$T_{n+7} \quad \frac{43h^4}{60480} \quad \frac{439h^4u^2}{+5702400} \quad \frac{91579h^4u^4}{+10897286400} \quad \frac{612289h^4u^6}{+871782912000} \quad \frac{1490947h^4u^8}{+29640619008000}$$

$$+ \frac{6848142131h^4u^{10}}{2128789257154560000}$$

$$f_{n+2} \quad \frac{16097h^4}{12454041600} \quad \frac{727h^4u^2}{15567552000} \quad \frac{272131h^4u^4}{8468748288000} \quad \frac{19259h^4u^6}{} \quad \frac{551357h^4u^8}{} \quad \frac{120960}{} \quad \frac{2851200}{}$$

$$\frac{565514519h^4u^{10}}{152056375511040000}$$

$$f_{n+5} \quad \frac{53777h^4}{302400} + \frac{3007h^4u^2}{5702400} + \frac{164321h^4u^4}{3113510400} + \frac{490631h^4u^6}{124540416000} + \frac{1115111h^4u^8}{4234374144000}$$

$$\frac{5013708509h^4u^{10}}{+304112751022080000}$$

$$f_{n+3} \quad \frac{27589h^4}{60480} + \frac{2941h^4u^2}{5702400} + \frac{36371h^4u^4}{778377600} + \frac{3727h^4u^6}{1257984000} + \frac{9857h^4u^8}{56458321920}$$

$$\frac{3179274887h^4u^{10}}{+304112751022080000}$$

$$T_{n+1} \quad \frac{979h^4}{60480} + \frac{115h^4u^2}{1596672} + \frac{126647h^4u^4}{+21794572800} + \frac{49463h^4u^6}{174356582400} + \frac{363439h^4u^8}{29640619008000}$$

$$\frac{268968253h^4u^{10}}{+425757851430912000}$$

$$y_{n+1} = \frac{2y_n}{5} + 2y_{n+3} \quad 2y_{n+4} + \frac{3y_{n+5}}{5}$$

$$f_{n+4} \quad \frac{42701h^4}{362880} + \frac{853h^4u^2}{855360} + \frac{83255h^4u^4}{1494484992} \quad \frac{7973h^4u^6}{16012339200} \quad \frac{1343327h^4u^8}{5081248972800}$$

$$\frac{66207359h^4u^{10}}{2851057040832000}$$

$$f_{n+8} \quad \frac{11h^4}{145152} \quad \frac{2957h^4u^2}{239500800} \quad \frac{581267h^4u^4}{523069747200} \quad \frac{161059h^4u^6}{2241727488000} \quad \frac{19951h^4u^8}{5472114278400}$$

$$\frac{1890623099h^4u^{10}}{12772735542927360000}$$

$$f_n \quad \frac{61h^4}{3628800} + \frac{9781h^4u^2}{239500800} + \frac{1413817h^4u^4}{523069747200} + \frac{129167h^4u^6}{2241727488000} \quad \frac{126349h^4u^8}{32335220736000}$$

$$\frac{6583918853h^4u^{10}}{12772735542927360000}$$

$$f_{n+6} \quad \frac{613h^4}{181440} + \frac{91h^4u^2}{3421440} \quad \frac{1289h^4u^4}{291891600} \quad \frac{35401h^4u^6}{32024678400} \quad \frac{1320071h^4u^8}{12703122432000}$$

$$\frac{245115763h^4u^{10}}{36493530122649600}$$

$$f_{n+7} \quad \frac{67h^4}{90720} \quad \frac{5459h^4u^2}{119750400} + \frac{663763h^4u^4}{130767436800} + \frac{499123h^4u^6}{1120863744000} + \frac{1308443h^4u^8}{44460928512000}$$

$$\frac{9909140273h^4u^{10}}{638636777146368000}$$

$$f_{n+2} \quad \frac{68317h^4}{181440} \quad \frac{1}{464486400h^2u^4} + \frac{13193h^4u^2}{17107200} + \frac{41593h^4u^4}{849139200} + \frac{113221h^4u^6}{160123392000}$$

$$\frac{1366583h^4u^8}{12703122432000} \quad \frac{10821189829h^4u^{10}}{912338253066240000}$$

$$f_{n+5} \quad \frac{4463h^4}{453600} \quad \frac{7279h^4u^2}{17107200} \quad \frac{333779h^4u^4}{18681062400} \quad \frac{16069h^4u^6}{12317184000} + \frac{1331699h^4u^8}{6351561216000}$$

$$\frac{14602436027h^4u^{10}}{912338253066240000}$$

$$f_{n+3} \quad \frac{40679h^4}{90720} \quad \frac{20017h^4u^2}{17107200} \quad \frac{1331321h^4u^4}{18681062400} \quad \frac{81329h^4u^6}{160123392000} \quad \frac{38713h^4u^8}{181473177600}$$

$$+ \frac{19295731781h^4u^{10}}{912338253066240000}$$

$$f_{n+1} \quad \frac{1171h^4}{23950080} \quad \frac{6551h^4u^2}{130767436800} \quad \frac{2328863h^4u^4}{224172748800} \quad \frac{74311h^4u^6}{44460928512000} + \frac{1378211h^4u^8}{18144}$$

$$+ \frac{436164137h^4u^{10}}{116115777662976000}$$

$$y_{n+2} = \frac{y_n}{10} + 2y_{n+3} \quad \frac{3y_{n+4}}{2} + \frac{2y_{n+5}}{5}$$

$$f_{n+4} \quad \frac{52781h^4}{725760} \quad \frac{449h^4u^2}{6842880} \quad \frac{227459h^4u^4}{14944849920} \quad \frac{940081h^4u^6}{448345497600} \quad \frac{1766423h^4u^8}{10162497945600}$$

$$\frac{357267493h^4u^{10}}{33175936475136000}$$

$$f_{n+8} \quad \frac{h^4}{207360} \quad \frac{251h^4u^2}{479001600} \quad \frac{283h^4u^4}{209227898880} + \frac{154841h^4u^6}{31384184832000}$$

$$+\frac{69799h^4u^8}{101624979456000} + \frac{1573358443h^4u^{10}}{25545471085854720000}$$

$$f_n \quad \frac{467h^4}{7257600} \quad \frac{647h^4u^2}{479001600} \quad \frac{453503h^4u^4}{1046139494400} \quad \frac{2035003h^4u^6}{31384184832000}$$

$$\frac{4021439h^4u^8}{711374856192000} \quad \frac{9433243289h^4u^{10}}{25545471085854720000}$$

$$f_{n+6} \quad \frac{193h^4}{37362124800} \quad \frac{7h^4u^2}{56043187200} \quad \frac{114437h^4u^4}{5081248972800} \quad \frac{19631h^4u^6}{127783h^4u^8} \quad \frac{362880}{362880} \quad \frac{342144}{}$$

$$\frac{117829199h^4u^{10}}{91233825306624000}$$

$$T_{n+7} \quad \frac{13h^4}{181440} \quad + \frac{601h^4u^2}{119750400} + \frac{2633h^4u^4}{5943974400} + \frac{237779h^4u^6}{7846046208000} + \frac{75161h^4u^8}{88921857024000}$$

$$\frac{395066453h^4u^{10}}{6386367771463680000}$$

$$T_{n+2} \quad \frac{50089h^4}{362880} \quad \frac{137h^4u^2}{4276800} \quad \frac{340481h^4u^4}{37362124800} \quad \frac{106253h^4u^6}{80061696000} \quad \frac{2893931h^4u^8}{25406244864000}$$

$$\frac{835199107h^4u^{10}}{114042281633280000}$$

$$T_{n+5} \quad \frac{2279h^4}{907200} \quad + \frac{799h^4u^2}{17107200} + \frac{42737h^4u^4}{4670265600} + \frac{1332701h^4u^6}{1120863744000} + \frac{92513h^4u^8}{977163264000}$$

$$\frac{5108234413h^4u^{10}}{+912338253066240000}$$

$$f_{n+3} \quad \frac{10009h^4}{36288} + \frac{997h^4u^2}{17107200} \quad \frac{28397h^4u^4}{+1868106240} + \frac{220693h^4u^6}{101896704000} \quad \frac{2330177h^4u^8}{+12703122432000}$$

$$\frac{10611535279h^4u^{10}}{+912338253066240000}$$

$$T_{n+1} \quad \frac{559h^4}{36288} \quad + \frac{239h^4u^2}{23950080} + \frac{6203h^4u^4}{2043241200} + \frac{54193h^4u^6}{120708403200} + \frac{1283h^4u^8}{32995123200}$$

$$\frac{3222967229h^4u^{10}}{+1277273554292736000}$$

$$y_n^{0} = \frac{47y_n}{60h} + \frac{10y_{n+3}}{3h} \quad \frac{15y_{n+4}}{4h} + \frac{6y_{n+5}}{5h}$$

$$f_{n+8} \quad \frac{229h^3}{2280960} \quad \frac{55949h^3u^2}{6227020800} \quad \frac{2884747h^3u^4}{1046139494400} \quad \frac{490447h^3u^6}{1520031744000}$$

$$\frac{1164715639h^3u^8}{40548366802944000} \quad \frac{671199101789h^3u^{10}}{281000181944401920000}$$

$$f_{n+4} \quad \frac{323963h^3}{1596672} \quad \frac{3258803h^3u^2}{622702080} \quad \frac{3243557h^3u^4}{+14944849920} \quad \frac{13801727h^3u^6}{+846874828800}$$

$$+ \frac{121100093h^3u^8}{368621516390400} \quad \frac{81539356709h^3u^{10}}{4014288313491456000}$$

$$f_n \quad \frac{129917h^3}{15966720} \quad \frac{6125963h^3u^2}{43589145600} + \frac{9371861h^3u^4}{1046139494400} + \frac{6675841h^3u^6}{8468748288000}$$

$$+ \frac{1201912391h^3u^8}{315376186245120000} \quad + \frac{508120388371h^3u^{10}}{2810001819444019200000}$$

$$f_{n+6} \quad \frac{9553h^3}{3991680} \quad \frac{1825223h^3u^2}{1556755200} \quad \frac{35881h^3u^4}{+7472424960} \quad \frac{2662853h^3u^6}{2117187072000}$$

$$\frac{45472723h^3u^8}{135161222676480} \quad \frac{28951479173h^3u^{10}}{771978521825280000}$$

$$f_{n+7} \quad \frac{1597h^3}{1995840} + \frac{1108433h^3u^2}{5448643200} + \frac{1352671h^3u^4}{130767436800} + \frac{1556449h^3u^6}{1058593536000}$$

$$\frac{5781731849h^3u^8}{+35479820952576000} \quad \frac{523784165519h^3u^{10}}{+351250227430502400000}$$

$$f_{n+2} \quad \frac{3834191h^2}{3991680} \quad \frac{4692383h^2u^2}{1556755200} \quad \frac{6307709h^2u^4}{+37362124800} \quad \frac{197819h^2u^6}{+13837824000}$$

$$\frac{6074656271h^3u^8}{+101370917007360000} \quad \frac{213290515831h^3u^{10}}{+1003572078372864000}$$

$$f_{n+5} \quad \frac{3571h^2}{1995840} + \frac{2542013h^2u^2}{778377600} \quad \frac{1711481h^2u^4}{18681062400} \quad \frac{1856479h^2u^6}{352864512000}$$

$$\frac{1039176601h^3u^8}{+5068545850368000} \quad \frac{13467899587h^3u^{10}}{+295168258344960000}$$

$$f_{n+3} \quad \frac{175961h^2}{181440} + \frac{3975593h^2u^2}{778377600} \quad \frac{4775633h^2u^4}{18681062400} \quad \frac{22034017h^2u^6}{1058593536000}$$

$$\frac{1234459549h^3u^8}{1689515283456000} \quad \frac{5988689051h^3u^{10}}{456169126533120000}$$

$$f_{n+1} \quad \frac{715549h^3}{1995840} \quad \frac{70249h^3u^2}{+70761600} \quad \frac{1567957h^3u^4}{26153487360} \quad \frac{38498597h^3u^6}{7410154752000}$$

$$\frac{241312397h^3u^8}{1013709170073600} \quad \frac{18984497479h^3u^{10}}{1848685407528960000}$$

$$y^0_{n+1} = \frac{13y_n}{30h} + \frac{5y_{n+3}}{6h} \quad \frac{y_{n+4}}{2h} + \frac{y_{n+5}}{10h}$$

$$\tau_{n+4} \quad \frac{18569h^3}{47900160} \quad \frac{205847h^3u^2}{186810624} \quad \frac{10227643h^3u^4}{89669099520} \quad \frac{11867719h^3u^6}{2078692761600}$$

$$\frac{280977017h^3u^8}{1621934672117760} \quad \frac{1936892963h^3u^{10}}{15053581175592960000} \quad !$$

$$f_{n+8} \quad \frac{1379h^3}{13685760} + \frac{5275763h^3u^2}{261534873600} + \frac{15207347h^3u^4}{6276836966400} + \frac{108272063h^3u^6}{457312407552000}$$

$$+ \frac{12262908787h^3u^8}{567677135241216000} + \frac{256556816281h^3u^{10}}{12969239166647040000} \quad !$$

$$f_n \quad \frac{46297h^3}{479001600} \quad \frac{275707h^3u^2}{5337446400} \quad \frac{35662633h^3u^4}{6276836966400} \quad \frac{1280084077h^3u^6}{3201186852864000}$$

$$\frac{102535231h^3u^8}{3861749219328000} \quad \frac{3397219186469h^3u^{10}}{168600109166641152000} \quad !$$

$$\tau_{n+6} \quad \frac{43361h^3}{10886400} + \frac{1158823h^3u^2}{18681062400} + \frac{622463h^3u^4}{56043187200} + \frac{496814623h^3u^6}{228656203776000}$$

$$+ \frac{201074513h^3u^8}{750895681536000} + \frac{3533955427h^3u^{10}}{128800694550528000} \quad !$$

$$\tau_{n+7} \quad \frac{1619f_{n+7}h^3}{1710720} + \frac{1672907f_{n+7}h^3u^2}{18681062400} + \frac{1361323f_{n+7}h^3u^4}{120708403200} + \frac{402524701f_{n+7}h^3u^6}{320118685286400}$$

$$+ \frac{74335801f_{n+7}h^3u^8}{596299511808000} + \frac{9974725547551f_{n+7}h^3u^{10}}{84300054583320576000} \quad !$$

$$f_{n+2} \quad \frac{2388599h^3}{23950080} \quad \frac{1355891h^3u^2}{1437004800} \quad \frac{11472569h^3u^4}{112086374400} \quad \frac{308234779h^3u^6}{45731240755200}$$

$$\frac{8238782021h^3u^8}{20274183401472000} \quad \frac{3428209473877h^3u^{10}}{120428649404743680000} \quad !$$

$$f_{n+5} \quad \frac{590089h^3}{59875200} + \frac{643187h^3u^2}{1698278400} + \frac{7737791h^3u^4}{224172748800} + \frac{25365013h^3u^6}{228656203776000}$$

$$\frac{182914853h^3u^8}{921553790976000} \quad \frac{3242267749429h^3u^{10}}{120428649404743680000} \quad !$$

$$\tau_{n+3} \quad \frac{842207h^3}{11975040} + \frac{25860463h^3u^2}{18681062400} + \frac{33172781h^3u^4}{224172748800} + \frac{2063353531h^3u^6}{228656203776000}$$

$$+ \frac{1607277851h^3u^8}{3379030566912000} + \frac{3490190048693h^3u^{10}}{120428649404743680000} \quad !$$

$$\tau_{n+1} \quad \frac{3648797h^3}{59875200} + \frac{44645869h^3u^2}{130767436800} + \frac{58607771h^3u^4}{1569209241600} + \frac{585906007h^3u^6}{228656203776000}$$

$$+ \frac{896594653h^3u^8}{5458433992704000} + \frac{2044529569363h^3u^{10}}{16860010916664115200 0} \quad !$$

$$y^{0}_{n+2} = \frac{11y_n}{60h} - \frac{2y_{n+3}}{3h} + \frac{5y_{n+4}}{4h} - \frac{2y_{n+5}}{5h}$$

$$+ \frac{391621h^3}{5987520} - \frac{45751h^3u^2}{467026560} + \frac{17207h^3u^4}{1601233920} - \frac{16476011h^3u^6}{5716405094400}$$

$$+ \frac{282157009h^3u^8}{1013709170073600} + \frac{56056057087h^3u^{10}}{3010716235118592000} \; !$$

$$f_{n+8} \quad \frac{71h^3}{5230697472} - \frac{22933h^3u^2}{142655385600} + \frac{125143h^3u^4}{123122571264000} - \frac{15607693h^3u^6}{5987520} \; !$$

$$- \frac{12311917h^3u^8}{806359567104000} + \frac{200764883897h^3u^{10}}{12042864940474368000}$$

$$f_n \quad \frac{1411h^3}{14968800} + \frac{18847h^3u^2}{11887948800} + \frac{1858369h^3u^4}{1569209241600} + \frac{334708097h^3u^6}{1600593426432000}$$

$$+ \frac{7041721h^3u^8}{303246332928000} + \frac{74152105759h^3u^{10}}{33720021833328230400} \; !$$

$$f_{n+6} \quad \frac{2213h^3}{5987520} - \frac{108047h^3u^2}{1334361600} + \frac{45427h^3u^4}{4483454976} - \frac{27399193h^3u^6}{22865620377600}$$

$$- \frac{38156747h^3u^8}{241359326208000} + \frac{90856150687h^3u^{10}}{4631871130951680000} \; !$$

$$f'_{n+7} \quad \frac{1597h^3}{14968800} + \frac{146383h^3u^2}{5029516800} + \frac{3888821h^3u^4}{784604620800} + \frac{542795983h^3u^6}{800296713216000}$$

$$+ \frac{2968189079h^3u^8}{35479820952576000} + \frac{234814019617h^3u^{10}}{24794133700976640000} \; !$$

$$f'_{n+2} \quad \frac{7216061h^3}{29937600} + \frac{24313h^3u^2}{9340531200} + \frac{2099267h^3u^4}{112086374400} + \frac{400612141h^3u^6}{114328101888000}$$

$$+ \frac{1929919723h^3u^8}{5068545850368000} + \frac{2078026872323h^3u^{10}}{60214324702371840000} \; !$$

$$f'_{n+5} \quad \frac{6529h^3}{14968800} + \frac{1122337h^3u^2}{9340531200} + \frac{653879h^3u^4}{112086374400} + \frac{5187877h^3u^6}{114328101888000}$$

$$+ \frac{236977669h^3u^8}{5068545850368000} + \frac{146536300447h^3u^{10}}{12042864940474368000} \; !$$

$$f_{n+3} \quad \frac{991511h^3}{2993760} - \frac{68339h^3u^2}{1868106240} + \frac{2581063h^3u^4}{112086374400} + \frac{2847167h^3u^6}{611380224000}$$

$$+ \frac{831411247h^3u^8}{1689515283456000} + \frac{2526475329019h^3u^{10}}{60214324702371840000} \; !$$

$$f'_{n+1} \quad \frac{86147h^3}{2993760} - \frac{438947h^3u^2}{65383718400} + \frac{89477h^3u^4}{12070840320} - \frac{214005667h^3u^6}{160059342643200}$$

$$- \frac{275023429h^3u^8}{1867358997504000} + \frac{826518880039h^3u^{10}}{60214324702371840000} \; !$$

$$y^0_{n+3} = \frac{y_n}{30h} \quad \frac{7y_{n+3}}{6h} + \frac{3y_{n+4}}{2h} \quad \frac{3y_{n+5}}{10h}$$

$$\frac{143873h^3}{2280960} \quad \frac{7121h^3u^2}{311351040} \quad \frac{191h^3u^4}{5977939968} \quad \frac{650717h^3u^6}{+2540624486400}$$

$$\frac{8388353h^3u^8}{+324386934423552} + \frac{1557043757h^3u^{10}}{1003572078372864000} \quad !$$

$$f_{n+8} \quad \frac{5513h^3}{159667200} \quad \frac{667621h^3u^2}{+87178291200} \quad \frac{2307623h^3u^4}{+2092278988800} \quad \frac{47644769h^3u^6}{+355687428096000}$$

$$\frac{773841947h^3u^8}{+51607012294656000} \quad \frac{47529503689h^3u^{10}}{+29578966520463360000} \quad !$$

$$f_n \quad \frac{2887h^3}{87178291200} \quad \frac{724589h^3u^2}{2092278988800} \quad \frac{2309533h^3u^4}{118562476032000} \quad \frac{15013967h^3u^6}{159667200}$$

$$!$$

$$\frac{899204863h^3u^8}{63075237249024000} \quad \frac{878147869979h^3u^{10}}{562000363888803840000}$$

$$f_{n+6} \quad \frac{14599h^3}{7983360} \quad \frac{639137h^3u^2}{+6227020800} \quad \frac{6337h^3u^4}{+410572800} \quad \frac{5438467h^3u^6}{+2822916096000}$$

$$\frac{1453661707h^3u^8}{+6758061133824000} \quad \frac{915516920147h^3u^{10}}{+40142883134914560000} \quad !$$

$$f_{n+7} \quad \frac{1447h^3}{3991680} \quad \frac{179489h^3u^2}{3962649600} \quad \frac{3460957h^3u^4}{523069747200} \quad \frac{11095057h^3u^6}{13680285696000}$$

$$!$$

$$\frac{6436623269h^3u^8}{70959641905152000} \quad \frac{2721638060329h^3u^{10}}{281000181944401920000}$$

$$f_{n+2} \quad \frac{352447h^3}{6227020800} \quad \frac{753073h^3u^2}{18681062400} \quad \frac{288811h^3u^4}{25406244864000} \quad \frac{43740467h^3u^6}{7983360}$$

$$!$$

$$\frac{3941567471h^3u^8}{20274183401472000} \quad \frac{865691519923h^3u^{10}}{40142883134914560000}$$

$$f_{n+5} \quad \frac{17213f_{n+5}h^3}{2851200} \quad \frac{83167f_{n+5}h^3u^2}{889574400} \quad \frac{1152379f_{n+5}h^3u^4}{74724249600} \quad \frac{7364153f_{n+5}h^3u^6}{3629463552000}$$

$$!$$

$$\frac{25113703f_{n+5}h^3u^8}{1113966120960002361346066759680000} \quad \frac{55319389427f_{n+5}h^3u^{10}}{}$$

$$f_{n+3} \quad \frac{2829559h^3}{19958400} \quad + \frac{810041h^3u^2}{6227020800} \quad + \frac{105109h^3u^4}{6793113600} \quad + \frac{13712533h^3u^6}{8468748288000}$$

$$\frac{621976441h^3u^8}{+3379030566912000} \quad \frac{120111259973h^3u^{10}}{+57346975907020800000} \quad !$$

$$f_{n+1} \quad \frac{2017h^3}{362880} \quad + \frac{2202251h^3u^2}{43589145600} \quad + \frac{3464777h^3u^4}{523069747200} \quad \frac{133824269h^3u^6}{+177843714048000}$$

$$\frac{6017205619h^3u^8}{+70959641905152000} \quad \frac{238362478171h^3u^{10}}{+25545471085854720000} \quad !$$

$$y^0_{n+4} = \frac{y_n}{60h}\left(\frac{2y_{n+3}}{3h} + \frac{y_{n+4}}{4h} + \frac{2y_{n+5}}{5h}\right)$$

$$\tau_{n+4} = \frac{143921h^3}{2177280} + \frac{1003h^3u^2}{20528640} + \frac{175489h^3u^4}{44834549760} + \frac{35501h^3u^6}{192148070400} + \frac{253789h^3u^8}{30487493836800} + \frac{553698781h^3u^{10}}{1094805903679488000}$$

$$f_{n+8} = \frac{8077h^3}{11887948800} + \frac{90001h^3u^2}{3138418483200} + \frac{3460727h^3u^4}{1600593426432000} + \frac{215717609h^3u^6}{329894011h^3u^8} + \frac{239500800}{1363244326573h^3u^{10}}$$

$$f_n = \frac{7571h^3}{239500800} + \frac{1172557h^3u^2}{21833735970816000} + \frac{762341h^3u^4}{130767436800} + \frac{224166847h^3u^6}{627683696640} + \frac{1452043339h^3u^8}{1600593426432000} + \frac{275085139951h^3u^{10}}{94612855873536000} + \frac{1686001091666411520000}{16860010916664115200000}$$

$$f_{n+6} = \frac{101623h^3}{4670265600} + \frac{449369h^3u^2}{10189670400} + \frac{149329h^3u^4}{11432810188800} + \frac{21149299h^3u^6}{236381567h^3u^8} + \frac{59875200}{96939545713h^3u^{10}}$$

$$\tau_{n+7} = \frac{10411h^3}{1126343522304000} + \frac{71969h^3u^2}{4301023193026560000} + \frac{2551673h^3u^4}{29937600} + \frac{3091001h^3u^6}{1634592960} + \frac{3208028123h^3u^8}{392302310400} + \frac{510455286641h^3u^{10}}{3847580352000} + \frac{5268753411457536000}{35479820952576000}$$

$$\tau_{n+2} = \frac{1231687h^3}{59875200} + \frac{126383h^3u^2}{934053120} + \frac{1993597h^3u^4}{112086374400} + \frac{114195733h^3u^6}{2194941977h^3u^8} + \frac{57164050944000}{690758193173h^3u^{10}} + \frac{10137091700736000}{30107162351185920000}$$

$$\tau_{n+5} = \frac{151093h^3}{29937600} + \frac{22381h^3u^2}{291891600} + \frac{20959h^3u^4}{1601233920} + \frac{25380469h^3u^6}{1046840083h^3u^8} + \frac{14291012736000}{3362430667h^3u^{10}} + \frac{5068545850368000}{150535811755929600}$$

$$\tau_{n+3} = \frac{2414651h^3}{29937600} + \frac{180797\,h^3u^2}{1167566400} + \frac{1084543\,h^3u^4}{56043187200} + \frac{925159\,h^3u^6}{371449319\,h^3u^8} + \frac{446594148000}{1742122199 41\,h^3u^{10}} + \frac{1689515283456000}{7526790587796480000}$$

$$f_{n+1} = \frac{90821h^3}{4086482400} + \frac{225559h^3u^2}{392302310400} + \frac{2902651h^3u^4}{20007417830400} + \frac{16918129h^3u^6}{3275535997h^3u^8} + \frac{29937600}{3228412332 7h^3u^{10}} + \frac{35479820952576000}{3292970882160960000}$$

$$y^0_{n+5} = \frac{y_n}{30h} + \frac{5y_{n+3}}{6h} \quad \frac{5y_{n+4}}{2h} + \frac{17y_{n+5}}{10h}$$

$$\tau_{n+4} \quad \frac{1928743h^3}{9580032} \quad \frac{14653h^3u^2}{84913920} \quad \frac{1401047h^3u^4}{89669099520} \quad \frac{22754929h^3u^6}{22865620377600}$$

$$\frac{479740321h^3u^8}{8109673360588800} \quad \frac{5380908931h^3u^{10}}{1505358117559296000}$$

$$f_{n+8} \quad \frac{2137h^3}{95800320} + \frac{1443583h^3u^2}{261534873600} + \frac{789241h^3u^4}{896690995200} + \frac{28583089h^3u^6}{246245142528000}$$

$$+\frac{7822812271h^3u^8}{567677135241216000} + \frac{2585718117209h^3u^{10}}{1686001091666411520000}$$

$$f_n \quad \frac{2903h^3}{261534873600} \frac{2733047h^3u^2}{6276836966400} \frac{8326781h^3u^4}{3201186852864000} \frac{462599873h^3u^6}{95800320}$$

$$\frac{2927430971h^3u^8}{189225711747072000} \quad \frac{2757907203001h^3u^{10}}{1686001091666411520000}$$

$$\tau_{n+6} \quad \frac{9307h^3}{23950080} + \frac{798851h^3u^2}{18681062400} + \frac{103091h^3u^4}{11208637440} + \frac{326070299h^3u^6}{228656203776000}$$

$$+\frac{48953813h^3u^8}{270322445352960} + \frac{2499623574313h^3u^{10}}{12042864940474368000}$$

$$f_{n+7} \quad \frac{29h^3}{130767436800} \frac{3686017h^3u^2}{1569209241600} \frac{7586507h^3u^4}{145508493312000} \frac{97202783h^3u^6}{155520}$$

$$\frac{5747174123h^3u^8}{70959641905152000} \quad \frac{697369073521h^3u^{10}}{76636413257564160000}$$

$$f_{n+2} \quad \frac{1022851h^3}{18681062400} \frac{3377779h^3u^2}{10189670400} \frac{221087h^3u^4}{228656203776000} \frac{508109731h^3u^6}{23950080}$$

$$\frac{4631016617h^3u^8}{20274183401472000} \quad \frac{218769365069h^3u^{10}}{9263742261903360000}$$

$$\tau_{n+5} \quad \frac{113111h^3}{11975040} \quad \frac{490613h^3u^2}{18681062400} \quad \frac{660773h^3u^4}{224172748800} \quad \frac{235050583h^3u^6}{228656203776000}$$

$$\frac{1595897827h^3u^8}{10137091700736000} \quad \frac{2327434488521h^3u^{10}}{12042864940474368000}$$

$$f_{n+3} \quad \frac{1883141h^3}{11975040} + \frac{666749h^3u^2}{2668723200} + \frac{6264961h^3u^4}{224172748800} + \frac{85589921h^3u^6}{32665171968000}$$

$$\frac{1931503h^3u^8}{7662200832000} + \frac{430884404527h^3u^{10}}{17204092772106240000}$$

$$\tau_{n+1} \quad \frac{68669h^3}{11975040} \quad \frac{8843873h^3u^2}{130767436800} \quad \frac{376877h^3u^4}{44834549760} + \frac{1433309477h^3u^6}{1600593426432000}$$

$$\frac{1341330953h^3u^8}{14191928381030400} + \frac{8359816151899h^3u^{10}}{84300054583320576000}$$

$$y^0_{n+6} = \frac{11y_n}{60h} + \frac{10y_{n+3}}{3h} \quad \frac{27y_{n+4}}{4h} + \frac{18y_{n+5}}{5h}$$

$$\tau_{n+4} \quad \frac{1847911h^3}{1330560} \quad \frac{101389h^3u^2}{103783680} \quad \frac{241253h^3u^4}{2490808320} \quad \frac{8824759h^3u^6}{1270312243200}$$

$$\frac{2201429h^3u^8}{4770396094464} \quad \frac{796042963h^3u^{10}}{26761922089943040}$$

$$f_{n+8} \quad \frac{h^3\ 7459h^3u^2\ 35081h^3u^4\ 9503327h^3u^6\ 14080}{440294400\ 14529715200\ 35568742809600}$$

$$\frac{733363747h^3u^8}{28383856762060800} \quad \frac{73622138851h^3u^{10}}{31222242438266880000}$$

$$f_n \quad \frac{467h^3}{13305600} \quad \frac{159409h^3u^2}{14529715200} \quad \frac{2797h^3u^4}{7925299200} + \frac{1357511h^3u^6}{19760412672000}$$

$$+ \frac{1209161h^3u^8}{95568541286400} + \frac{141262120253h^3u^{10}}{93666727314800640000}$$

$$\tau_{n+6} \quad \frac{1627h^3}{415800} \quad \frac{17957h^3u^2}{41513472} \quad \frac{26489h^3u^4}{498161664} \quad \frac{1670927h^3u^6}{325721088000} \quad \frac{76707101h^3u^8}{168951528345600}$$

$$\frac{260668564703h^3u^{10}}{6690480522485760000}$$

$$\tau_{n+7} \quad \frac{8h^3}{10395} + \frac{941219h^3u^2}{7264857600} + \frac{1504169h^3u^4}{87178291200} + \frac{160199423h^3u^6}{88921857024000}$$

$$+ \frac{62821387h^3u^8}{373471799500800} + \frac{702401397809h^3u^{10}}{46833363657400320000}$$

$$\tau_{n+2} \quad \frac{9067h^3}{36960} \quad \frac{5749h^3u^2}{16473600} \quad \frac{33643h^3u^4}{1383782400} \quad \frac{5431919h^3u^6}{12703122432000} + \frac{3071417h^3u^8}{36203898931200}$$

$$+ \frac{4831427243h^3u^{10}}{318594310594560000}$$

$$\tau_{n+5} \quad \frac{46247h^3}{92400} + \frac{284827h^3u^2}{345945600} + \frac{54511h^3u^4}{593049600} + \frac{537247h^3u^6}{67931136000} + \frac{647364071h^3u^8}{1013709170073600}$$

$$+ \frac{37808095667h^3u^{10}}{743386724720640000}$$

$$\tau_{n+3} \quad \frac{8641h^3}{10395} + \frac{767743h^3u^2}{1037836800} + \frac{785293h^3u^4}{12454041600} + \frac{2715397h^3u^6}{846874828800} + \frac{33707263h^3u^8}{337903056691200}$$

$$\frac{1681205831h^3u^{10}}{514652347883520000}$$

$$f_{n+1} \quad \frac{24481h^3}{111767040} + \frac{10477h^3u^2}{17435658240} + \frac{85171h^3u^4\ 19003279h^3u^6\ 831600}{88921857024000}$$

$$\frac{445120493h^3u^8}{7095964190515200} \quad \frac{383984212609h^3u^{10}}{46833363657400320000}$$

$$y_{n+7} = \frac{13y_n}{30h} + \frac{41y_{n+3}}{6h} + \frac{25y_{n+4}}{2h} + \frac{61y_{n+5}}{10h}$$

$$f_{n+4} \frac{23659021h^3}{11975040} + \frac{30894511h^3u^2}{18681062400} + \frac{28763921h^3u^4}{224172748800} + \frac{1732076491h^3u^6}{228656203776000}$$

$$\frac{108283781h^3u^8}{+198766503936000} + \frac{1155198674257h^3u^{10}}{24085729880948736000}!$$

$$f_{n+8} \frac{571h^2}{13685760} + \frac{573893h^2u^2}{37362124800} + \frac{17411777h^2u^4}{6276836966400} + \frac{840723701h^2u^6}{3201186852864000}$$

$$\frac{1651752787h^3u^8}{+81096733605888000} + \frac{552757556201h^3u^{10}}{337200218333282304000}!$$

$$f_n \frac{21953h^2}{479001600} \frac{2953631h^2u^2}{52306974720} \frac{33458203h^2u^4}{6276836966400} \frac{171037831h^2u^6}{457312407552000}$$

$$\frac{350518183h^3u^8}{12615047449804800} \frac{305282309009h^3u^{10}}{129692391666647040000}!$$

$$f_{n+6} \frac{2295565h^3}{4790016} \frac{104477h^3u^2}{1437004800} + \frac{2347141h^3u^4}{112086374400} + \frac{60223013h^3u^6}{20786927616000}$$

$$\frac{1576124191h^3u^8}{+6758061133824000} \frac{127138038997h^3u^{10}}{+70840382002790400000}!$$

$$f_{n+7} \frac{532523h^2}{59875200} \frac{6676301h^2u^2}{130767436800} \frac{22106059h^2u^4}{1569209241600} \frac{9566941h^2u^6}{6533034393600}$$

$$\frac{56960287h^3u^8}{496221272064000} \frac{7688922224959h^3u^{10}}{843000545833205760000}!$$

$$f_{n+2} \frac{6259559h^3}{10886400} \frac{1831237h^3u^2}{1698278400} \frac{5185177h^3u^4}{56043187200} \frac{11004283h^3u^6}{1829249630208}$$

$$\frac{89394212299h^3u^8}{20274183401472000} \frac{415555557743h^3u^{10}}{10948059036794880000}!$$

$$f_{n+5} \frac{1219160051h^3}{59875200} + \frac{2421821h^3u^2}{3736212480} + \frac{3328931h^3u^4}{224172748800} \frac{305912027h^3u^6}{228656203776000}$$

$$\frac{262284821h^3u^8}{2027418340147200} \frac{9564644268837h^3u^{10}}{120428649404743680000}!$$

$$f_{n+3} \frac{23659021h^3}{11975040} + \frac{30894511h^3u^2}{18681062400} + \frac{28763921h^3u^4}{224172748800} + \frac{1732076491h^3u^6}{228656203776000}$$

$$\frac{108283781h^3u^8}{+198766503936000} \frac{1155198674257h^3u^{10}}{+24085729880948736000}!$$

$$f_{n+1} \frac{847597h^2}{11975040} + \frac{7097131h^2u^2}{18681062400} + \frac{4169147h^2u^4}{120708403200} + \frac{3770065009h^2u^6}{1600593426432000}$$

$$\frac{1765195681h^3u^8}{+10137091700736000} \frac{12508451169407h^3u^{10}}{+84300054583320576000}!$$

$$y^{0}_{n+8} = \frac{47y_n}{60h} + \frac{34y_{n+3}}{3h} - \frac{79y_{n+4}}{4h} + \frac{46y_{n+5}}{5h}$$

$$f_{n+4} \frac{153673759h^3}{23950080} + \frac{1196747h^3u^2}{1868106240} - \frac{26226637h^3u^4}{44834549760} + \frac{153952163h^3u^6}{4573124075520}$$

$$\frac{4504971101h^3u^8}{4054836680294400} - \frac{17258235233h^3u^{10}}{6338349968670720000}$$

$$f_{n+8} \frac{281791h^3}{34214400} + \frac{9798227h^3u^2}{130767436800} - \frac{44611549h^3u^4}{3138418483200} + \frac{47395661h^3u^6}{45731240755200}$$

$$\frac{13990081597h^3u^8}{2838385676206080000} - \frac{161298900359h^3u^{10}}{64846195833323520000}$$

$$f_n \frac{571h^3}{13685760} + \frac{573893h^3u^2}{37362124800} + \frac{17411777h^3u^4}{6276836966400} + \frac{840723701h^3u^6}{3201186852864000}$$

$$+ \frac{1651752787h^3u^8}{81096733605888000} + \frac{552757556201h^3u^{10}}{33720021833328230400 0}$$

$$f_{n+6} \frac{24035687h^3}{11975040} + \frac{5497487h^3u^2}{4670265600} - \frac{35419093h^3u^4}{112086374400} + \frac{2857187h^3u^6}{134503649280}$$

$$\frac{3082508783h^3u^8}{3379030566912000} - \frac{1212396087047h^3u^{10}}{30107162351185920000}$$

$$\tau_{n+7} \frac{414643h^3}{855360} - \frac{1092551h^3u^2}{2335132800} + \frac{40015321h^3u^4}{392302310400} + \frac{287315261h^3u^6}{40014835660800}$$

$$+ \frac{1659829139h^3u^8}{5068545850368000} + \frac{1654640895857h^3u^{10}}{10537506822915072000 0}$$

$$\tau_{n+2} \frac{2500891h^3}{2395008} - \frac{3103993h^3u^2}{4670265600} - \frac{17034181h^3u^4}{112086374400} + \frac{65043431h^3u^6}{11432810188800}$$

$$+ \frac{237584147h^3u^8}{10137091700736000} + \frac{556583148193h^3u^{10}}{30107162351185920000}$$

$$\tau_{n+5} \frac{12310981h^3}{2721600} - \frac{3347117h^3u^2}{2335132800} + \frac{6164573h^3u^4}{11208637440} + \frac{198406529h^3u^6}{5716405094400}$$

$$+ \frac{275049949h^3u^8}{202741834014720} + \frac{6364886597h^3u^{10}}{124409761781760000}$$

$$\tau_{n+3} \frac{106440791h^3}{29937600} + \frac{86693h^3u^2}{212284800} + \frac{21630409h^3u^4}{56043187200} + \frac{109497797h^3u^6}{5716405094400}$$

$$+ \frac{13945709h^3u^8}{33127750656000} + \frac{114338339383h^3u^{10}}{15053581175592960000}$$

$$\tau_{n+1} \frac{152129h^3}{1197504} + \frac{5254363h^3u^2}{16345929600} + \frac{1130723h^3u^4}{35663846400} + \frac{84037h^3u^6}{163325859840}$$

$$\frac{2608861771h^3u^8}{35479820952576000} - \frac{9988279570 03h^3u^{10}}{10537506822915072000}$$

$$y''_n = \frac{2y_n}{5h^2} - \frac{3y_{n+3}}{h^2} + \frac{4y_{n+4}}{h^2} - \frac{7y_{n+5}}{5h^2}$$

$$f_{n+4} \frac{2245499h^2}{47900160} + \frac{137434123h^2u^2}{2668723200} - \frac{80423899h^2u^4}{448345497600} + \frac{5816422147h^2u^6}{76218734592000}$$

$$- \frac{156735550667h^2u^8}{40548366802944000} + \frac{1033895692409h^2u^{10}}{9263742261903360000}$$

$$f_{n+8} \frac{1004621h^2}{26687232000} - \frac{5316191h^2u^2}{2414168064000} + \frac{22257967h^2u^4}{37309870080000} - \frac{6863209h^2u^6}{479001600}$$

$$+ \frac{23616818117h^2u^8}{2838385676206080000} + \frac{9429382979143h^2u^{10}}{843000545833205760000 0}$$

$$f_n \frac{4176277h^2}{68428800} + \frac{28371053h^2u^2}{16982784000} + \frac{11682343h^2u^4}{2853107712000} - \frac{3550468469h^2u^6}{1778437140480000}$$

$$- \frac{112362639817h^2u^8}{946128558735360000} + \frac{36310670981777h^2u^{10}}{843000545833205760000 0}$$

$$f_{n+7} \frac{1143419h^2}{59875200} - \frac{403033h^2u^2}{1459458000} + \frac{237121153h^2u^4}{3923023104000} + \frac{1095092351h^2u^6}{333456963840000}$$

$$+ \frac{21471274079h^2u^8}{3547982095257600000} - \frac{927969058507h^2u^{10}}{263437670572876800000}$$

$$T_{n+2} \frac{21312751h^2}{17107200} + \frac{224757853h^2u^2}{6671808000} + \frac{24040937h^2u^4}{1120863744000} - \frac{8233913777h^2u^6}{190546836480000}$$

$$- \frac{224465213 69h^2u^8}{9215537909760000} + \frac{1463273970091h^2u^{10}}{177100955006976000 00}$$

$$T_{n+5} \frac{1556051h^2}{8553600} - \frac{46886129h^2u^2}{1667952000} + \frac{132656317h^2u^4}{560431872000} + \frac{164559869h^2u^6}{3402622080000}$$

$$- \frac{111647458471h^2u^8}{5068545850368000 0} + \frac{3861568628101h^2u^{10}}{752679058779648000 00}$$

$$T_{n+3} \frac{89751797h^2}{59875200} - \frac{45273997h^2u^2}{833976000} + \frac{28191481h^2u^4}{560431872000} + \frac{130095703h^2u^6}{1764322560000}$$

$$+ \frac{22424849207h^2u^8}{563171761152000 0} - \frac{299346105413h^2u^{10}}{2352122058686400000 0} + \frac{1}{116121600h^6w^4}$$

$$f_{n+6} \frac{9253511h^2}{119750400} + \frac{50110393h^2u^2}{6671808000} - \frac{36977747h^2u^4}{224172748800} + \frac{1132976839h^2u^6}{63515612160000}$$

$$- \frac{887458217h^2u^8}{1351612226764800} + \frac{286518644441h^2u^{10}}{4301023193026560000 0}$$

$$f_{n+1} \frac{62033963h^2}{59875200} - \frac{19172837h^2u^2}{1667952000} + \frac{15254671h^2u^4}{784604620800} + \frac{1180332449h^2u^6}{83364240960000}$$

$$+ \frac{58399965451h^2u^8}{70959641905152000} + \frac{15296582118331h^2u^{10}}{526875341145753600000 0}$$

$$y''_{n+1} = \frac{3y_n}{10h^2} - \frac{2y_{n+3}}{h^2} + \frac{5y_{n+4}}{2h^2} - \frac{4y_{n+5}}{5h^2}$$

$$f_{n+4} \frac{8572363h^2}{47900160} - \frac{3114863h^2u^2}{667180800} - \frac{13190447h^2u^4}{89669099520} + \frac{25225183h^2u^6}{12703122432000} - \frac{4711549127h^2u^8}{8109673360588800} - \frac{688325066801h^2u^{10}}{15053581175592960000}$$

$$f_{n+8} \frac{125893h^2}{479001600} + \frac{6915163h^2u^2}{186810624000} + \frac{93975911h^2u^4}{31384184832000} + \frac{320482531h^2u^6}{1778437140480000} - \frac{102854561h^2u^8}{11680599490560000} - \frac{177978171779h^2u^{10}}{49588267401953280000}$$

$$f_n \frac{1006667h^2}{479001600} - \frac{31834067h^2u^2}{186810624000} - \frac{225880381h^2u^4}{31384184832000} - \frac{73193933h^2u^6}{592812380160000} - \frac{1301284291h^2u^8}{166963863306240000} - \frac{7987572148573h^2u^{10}}{843000545833205760000}$$

$$f_{n+6} \frac{1267351h^2}{119750400} - \frac{5544289h^2u^2}{13343616000} + \frac{7005919h^2u^4}{560431872000} + \frac{123644299h^2u^6}{42343741440000} - \frac{24275701979h^2u^8}{101370917007360000} - \frac{1218889922093h^2u^{10}}{86020463860531200000}$$

$$f_{n+7} \frac{147871h^2}{59875200} - \frac{8286037h^2u^2}{93405312000} - \frac{107987749h^2u^4}{7846046208000} - \frac{1011897959h^2u^6}{889218570240000} - \frac{24634680151h^2u^8}{354798209525760000} - \frac{1458348729513 7h^2u^{10}}{421500272916602880000 0}$$

$$f_{n+2} \frac{10668601h^2}{17107200} - \frac{44293519h^2u^2}{13343616000} - \frac{36479077h^2u^4}{280215936000} - \frac{169131433h^2u^6}{127031224320000} - \frac{692114827h^2u^8}{3071845969920000} - \frac{13494172682981h^2u^{10}}{602143247023718400000}$$

$$f_{n+5} \frac{1655431h^2}{59875200} + \frac{30463193h^2u^2}{13343616000} + \frac{51940397h^2u^4}{1120863744000} - \frac{471833629h^2u^6}{127031224320000} - \frac{7972241269h^2u^8}{16895152834560000} - \frac{19545430523467h^2u^{10}}{602143247023718400000}$$

$$f'_{n+3} \frac{36165539h^2}{59875200} + \frac{69212423h^2u^2}{13343616000} + \frac{211868543h^2u^4}{1120863744000} + \frac{32819h^2u^6}{61102080000} - \frac{23198767463h^2u^8}{50685458503680000} - \frac{24507373751797h^2u^{10}}{602143247023718400000}$$

$$f'_{n+1} \frac{1881827h^2}{59875200} + \frac{107961653h^2u^2}{93405312000} + \frac{33799699h^2u^4}{713276928000} + \frac{608295031h^2u^6}{889218570240000} - \frac{1729293163h^2u^8}{27292169963520000} - \frac{29469316980127h^2u^{10}}{421500272916602880000 0}$$

$$y''_{n+2} = \frac{y_n}{5h^2} - \frac{y_{n+3}}{h^2} + \frac{y_{n+4}}{h^2} - \frac{y_{n+5}}{5h^2}$$

$$f'_{n+4} = \frac{243689h^2}{9580032} + \frac{357353h^2u^2}{242611200} + \frac{9303433h^2u^4}{89669099520} + \frac{316724293h^2u^6}{76218734592000}$$

$$+ \frac{203561971h^2u^8}{2703224453529600} + \frac{421060879277h^2u^{10}}{120428649404743680000} !$$

$$f_{n+8} = \frac{41021h^2}{186810624000} - \frac{2672207h^2u^2}{31384184832000} + \frac{43271191h^2u^4}{37309870080000} - \frac{3736739h^2u^6}{479001600} - \frac{479001600}{!}$$

$$- \frac{5746771241h^2u^8}{946128558735360000} - \frac{2733474029687h^2u^{10}}{843000545833205760000 0}$$

$$f_n = \frac{125299h^2}{479001600} + \frac{10533973h^2u^2}{186810624000} + \frac{12391411h^2u^4}{2853107712000} + \frac{129755807h^2u^6}{592812380160000}$$

$$+ \frac{152595901h^2u^8}{18551540367360000} + \frac{8558155073h^2u^{10}}{3814482107842560000 0} !$$

$$f_{n+6} = \frac{52109h^2}{17107200} + \frac{314669h^2u^2}{3335904000} + \frac{1622987h^2u^4}{1120863744000} - \frac{1295413h^2u^6}{2268414720000} - \frac{2268414720000}{!}$$

$$- \frac{87573359h^2u^8}{1251492802560000} - \frac{788633727241h^2u^{10}}{1505358117559296000 00}$$

$$f'_{n+7} = \frac{46367h^2}{59875200} + \frac{2042869h^2u^2}{46702656000} + \frac{800927h^2u^4}{150885504000} + \frac{643168369h^2u^6}{1333827855360000}$$

$$+ \frac{4055625967h^2u^8}{118266069841920000} + \frac{4310741484169h^2u^{10}}{2107501364583014400000 0} !$$

$$f'_{n+2} = \frac{21378899h^2}{119750400} + \frac{1808107h^2u^2}{1667952000} + \frac{91411343h^2u^4}{1120863744000} + \frac{371131639h^2u^6}{95273418240000}$$

$$+ \frac{4400100403h^2u^8}{33790305669120000} + \frac{13127601713h^2u^{10}}{5376278991283200000} !$$

$$f'_{n+5} = \frac{55871h^2}{8553600} - \frac{4560221h^2u^2}{6671808000} - \frac{6017519h^2u^4}{140107968000} - \frac{207909601h^2u^6}{190546836480000}$$

$$+ \frac{673335419h^2u^8}{16895152834560000} + \frac{105175175461h^2u^{10}}{15845874921676800000} !$$

$$f'_{n+3} = \frac{6113297h^2}{59875200} - \frac{11163311h^2u^2}{6671808000} - \frac{34482127h^2u^4}{280215936000} - \frac{352995857h^2u^6}{63515612160000} - \frac{63515612160000}{!}$$

$$- \frac{902985043h^2u^8}{5631717611520000} - \frac{314084816651h^2u^{10}}{3010716235118592000 0}$$

$$f'_{n+1} = \frac{2202121h^2}{59875200} - \frac{17766401h^2u^2}{46702656000} - \frac{889519h^2u^4}{30648618000} - \frac{1910065541h^2u^6}{1333827855360000} - \frac{1333827855360000}{!}$$

$$- \frac{553749607h^2u^8}{10751460894720000} - \frac{2626497967061h^2u^{10}}{2107501364583014400000 0}$$

$$y''_{n+3} = \frac{y_n}{10h^2} - \frac{y_{n+4}}{2h^2} + \frac{2y_{n+5}}{5h^2}$$

$$f_{n+4} \frac{3655853h^2}{47900160} - \frac{98993h^2u^2}{667180800} - \frac{1429957h^2u^4}{64049356800} - \frac{97271231h^2u^6}{38109367296000}$$

$$\frac{8018457877h^2u^8}{40548366802944000} - \frac{5569418237h^2u^{10}}{470424411737280000}$$

$$f_{n+8} \frac{563h^2}{479001600} + \frac{21913h^2u^2}{186810624000} + \frac{295109h^2u^4}{4483454976000} + \frac{55252903h^2u^6}{5335311421440000}$$

$$\frac{3001383739h^2u^8}{+2838385676206080000} + \frac{708674359993h^2u^{10}}{+843000545833205760000}$$

$$f_n \frac{40163h^2}{16982784000} - \frac{73987h^2u^2}{4483454976000} - \frac{3155023h^2u^4}{1778437140480000} - \frac{148112609h^2u^6}{479001600}$$

$$\frac{6346099831h^2u^8}{946128558735360000} - \frac{3560216497337h^2u^{10}}{843000545833205760000}$$

$$f_{n+6} \frac{54233h^2}{13343616000} - \frac{374059h^2u^2}{8748000} - \frac{31h^2u^4}{127031224320000} - \frac{46429853h^2u^6}{119750400}$$

$$\frac{278726341h^2u^8}{11263435223040000} - \frac{55161285283h^2u^{10}}{46318711309516800000}$$

$$f_{n+7} \frac{2543h^2}{59875200} + \frac{330233h^2u^2}{93405312000} + \frac{54463h^2u^4}{224172748800} + \frac{28783753h^2u^6}{2667655710720000}$$

$$\frac{49284667h^2u^8}{70959641905152000} - \frac{700252011307h^2u^{10}}{421500272916602880000}$$

$$f_{n+2} \frac{16978481h^2}{119750400} - \frac{1209829h^2u^2}{13343616000} - \frac{229249h^2u^4}{16012339200} - \frac{638880289h^2u^6}{381093672960000}$$

$$\frac{2705675737h^2u^8}{20274183401472000} - \frac{262420398211h^2u^{10}}{316917498433536000000}$$

$$f_{n+5} \frac{157391h^2}{59875200} + \frac{1166003h^2u^2}{13343616000} + \frac{1997381h^2u^4}{160123392000} + \frac{5806313h^2u^6}{4187842560000}$$

$$\frac{404884421h^2u^8}{+3898881423360000} + \frac{209919932119h^2u^{10}}{3542019100139520000}$$

$$f_{n+3} \frac{3012997h^2}{8553600} + \frac{2001773h^2u^2}{13343616000} + \frac{3722447h^2u^4}{160123392000} + \frac{10383487h^2u^6}{3849431040000}$$

$$\frac{3591139427h^2u^8}{+16895152834560000} + \frac{1119647100479h^2u^{10}}{86020463860531200000}$$

$$f_{n+1} \frac{904891h^2}{59875200} + \frac{2837543h^2u^2}{93405312000} + \frac{5447513h^2u^4}{1120863744000} + \frac{1527555943h^2u^6}{2667655710720000}$$

$$\frac{16283339089h^2u^8}{+354798209525760000} + \frac{12106420560683h^2u^{10}}{421500272916602880000}$$

$$y''_{n+4} = \frac{y_{n+3}}{h^2} - \frac{2y_{n+4}}{h^2} + \frac{y_{n+5}}{h^2}$$

$$f_{n+4} \frac{3638891h^2}{47900160} + \frac{119803h^2u^2}{2668723200} + \frac{2108213h^2u^4}{448345497600} + \frac{1071479h^2u^6}{2822916096000} + \frac{1051897669h^2u^8}{40548366802944000} + \frac{189466073803h^2u^{10}}{12042864940474368000}$$

$$f_{n+8} \frac{1747h^2}{479001600} + \frac{119803h^2u^2}{186810624000} + \frac{2108213h^2u^4}{31384184832000} + \frac{1071479h^2u^6}{197604126720000} + \frac{1051897669h^2u^8}{2838385676206080000} + \frac{189466073803h^2u^{10}}{843000545833205760000}$$

$$f_n \frac{1747h^2}{479001600} + \frac{119803h^2u^2}{186810624000} + \frac{2108213h^2u^4}{31384184832000} + \frac{1071479h^2u^6}{197604126720000} + \frac{1051897669h^2u^8}{2838385676206080000} + \frac{189466073803h^2u^{10}}{843000545833205760000}$$

$$f'_{n+6} \frac{50641h^2}{119750400} + \frac{119803h^2u^2}{6671808000} + \frac{2108213h^2u^4}{1120863744000} + \frac{1071479h^2u^6}{7057290240000} + \frac{1051897669h^2u^8}{101370917007360000} + \frac{189466073803h^2u^{10}}{301071623511859200000}$$

$$f_{n+7} \frac{3133h^2}{23351328000} \frac{119803h^2u^2}{3923023104000} \frac{2108213h^2u^4}{24700515840000} \frac{1071479h^2u^6}{59875200} \frac{1051897669h^2u^8}{354798209525760000} \frac{189466073803h^2u^{10}}{1053750682291507200000}$$

$$f'_{n+2} \frac{50641h^2}{119750400} + \frac{119803h^2u^2}{6671808000} + \frac{2108213h^2u^4}{1120863744000} + \frac{1071479h^2u^6}{7057290240000} + \frac{1051897669h^2u^8}{101370917007360000} + \frac{189466073803h^2u^{10}}{301071623511859200000}$$

$$f'_{n+5} \frac{242899h^2}{59875200} \frac{119803h^2u^2}{3335904000} \frac{2108213h^2u^4}{560431872000} \frac{1071479h^2u^6}{3528645120000} \frac{1051897669h^2u^8}{50685458503680000} \frac{189466073803h^2u^{10}}{150535811755929600000}$$

$$f_{n+3} \frac{242899h^2}{3335904000} \frac{119803h^2u^2}{560431872000} \frac{2108213h^2u^4}{3528645120000} \frac{1071479h^2u^6}{59875200} \frac{1051897669h^2u^8}{50685458503680000} \frac{189466073803h^2u^{10}}{150535811755929600000}$$

$$f_{n+1} \frac{3133h^2}{23351328000} \frac{119803h^2u^2}{3923023104000} \frac{2108213h^2u^4}{24700515840000} \frac{1071479h^2u^6}{59875200} \frac{1051897669h^2u^8}{354798209525760000} \frac{189466073803h^2u^{10}}{1053750682291507200000}$$

$$y''_{n+5} = \frac{y_n}{10h^2} + \frac{2y_{n+3}}{h^2} - \frac{7y_{n+4}}{2h^2} + \frac{8y_{n+5}}{5h^2}$$

$$\tau_{n+4} = \frac{4905613h^2}{6842880} - \frac{490163h^2u^2}{667180800} - \frac{31068379h^2u^4}{448345497600} - \frac{181763611h^2u^6}{38109367296000}$$

$$- \frac{12068930317h^2u^8}{40548366802944000} - \frac{269581682449h^2u^{10}}{15053581175592960000}$$

$$f_{n+8} = \frac{34091h^2}{26687232000} - \frac{339791h^2u^2}{2414168064000} - \frac{3318757h^2u^4}{5335311421440000} - \frac{613322587h^2u^6}{7696257311h^2u^8} - \frac{479001600}{4291098888257h^2u^{10}}$$

$$- \frac{946128558735360000}{843000545833205760000}$$

$$f_n = \frac{787h^2}{68428800} - \frac{1542767h^2u^2}{186810624000} - \frac{18992917h^2u^4}{31384184832000} - \frac{1404097h^2u^6}{65868042240000}$$

$$- \frac{1049088701h^2u^8}{2838385676206080000} - \frac{1168843733h^2u^{10}}{443684497806950400000}$$

$$\tau_{n+6} = \frac{139823h^2}{17107200} - \frac{4339189h^2u^2}{13343616000} - \frac{3710611h^2u^4}{112086374400} - \frac{25047431h^2u^6}{9771632640000}$$

$$- \frac{12784619h^2u^8}{73724303278080} - \frac{6447752347849h^2u^{10}}{602143247023718400000}$$

$$f_{n+7} = \frac{60029h^2}{59875200} + \frac{826933h^2u^2}{8491392000} + \frac{80249951h^2u^4}{7846046208000} + \frac{2203494983h^2u^6}{2667655710720000}$$

$$+ \frac{20333811529h^2u^8}{354798209525760000} + \frac{15029950124363h^2u^{10}}{4215002729166028800000}$$

$$f_{n+2} = \frac{15945487h^2}{119750400} - \frac{3503419h^2u^2}{13343616000} - \frac{3128831h^2u^4}{140107968000} - \frac{43387189h^2u^6}{34644879360000}$$

$$- \frac{168179731h^2u^8}{2599254282240000} - \frac{44466561031h^2u^{10}}{12288637694361600000}$$

$$\tau'_{n+5} = \frac{6071717h^2}{59875200} + \frac{8260493h^2u^2}{13343616000} + \frac{68174489h^2u^4}{1120863744000} + \frac{1703904253h^2u^6}{381093672960000}$$

$$+ \frac{1647098969h^2u^8}{5631717611520000} + \frac{10761059267033h^2u^{10}}{602143247023718400000}$$

$$\tau'_{n+3} = \frac{27320087h^2}{59875200} + \frac{7424723h^2u^2}{13343616000} + \frac{56099027h^2u^4}{1120863744000} + \frac{57348263h^2u^6}{18147317760000}$$

$$+ \frac{9313969913h^2u^8}{50685458503680000} + \frac{6492168409703h^2u^{10}}{602143247023718400000}$$

$$\tau'_{n+1} = \frac{138209h^2}{8553600} + \frac{941279h^2u^2}{13343616000} + \frac{8804713h^2u^4}{1569209241600} + \frac{704722793h^2u^6}{2667655710720000}$$

$$+ \frac{760809821h^2u^8}{70959641905152000} + \frac{2223277552373h^2u^{10}}{4215002729166028800000}$$

$$y''_{n+6} = \frac{y_n}{5h^2} + \frac{3y_{n+3}}{h^2} - \frac{5y_{n+4}}{h^2} + \frac{11y_{n+5}}{5h^2}$$

$$f'_{n+4} \frac{11029819h^2}{6842880} + \frac{801523h^2u^2}{2668723200} + \frac{879961h^2u^4}{89669099520} - \frac{21245227h^2u^6}{76218734592000}$$

$$\frac{112167007h^2u^8}{901074817843200} - \frac{1882825661117h^2u^{10}}{120428649404743680000}$$

$$f_{n+8} \frac{137443h^2}{479001600} + \frac{7404613h^2u^2}{186810624000} + \frac{94188161h^2u^4}{31384184832000} + \frac{829832743h^2u^6}{5335311421440000}$$

$$\frac{5082075991h^2u^8}{946128558735360000} + \frac{429587489293h^2u^{10}}{84300054583320576000000}$$

$$f_n \frac{28877h^2}{186810624000} - \frac{5801567h^2u^2}{31384184832000} - \frac{85388551h^2u^4}{1778437140480000} - \frac{290774399h^2u^6}{479001600}$$

$$\frac{8447086201h^2u^8}{946128558735360000} - \frac{4195238811527h^2u^{10}}{84300054583320576000000}$$

$$f'_{n+6} \frac{10620541h^2}{119750400} + \frac{1025767h^2u^2}{1667952000} + \frac{49293983h^2u^4}{1120863744000} + \frac{6125663h^2u^6}{2887073280000}$$

$$\frac{99987379h^2u^8}{1987665039360000} + \frac{90827385739h^2u^{10}}{37633952938982400000}$$

$$f''_{n+7} \frac{272281h^2}{59875200} - \frac{11507681h^2u^2}{46702656000} - \frac{344909h^2u^4}{18860688000} - \frac{1234126501h^2u^6}{1333827855360000}$$

$$\frac{1130310239h^2u^8}{39422023280640000} + \frac{297031596619h^2u^{10}}{2107501364583014400000}$$

$$f_{n+2} \frac{31634677h^2}{119750400} - \frac{1250011h^2u^2}{3335904000} - \frac{40494373h^2u^4}{1120863744000} - \frac{15956579h^2u^6}{6805244160000}$$

$$\frac{5064795653h^2u^8}{33790305669120000} - \frac{15195161181 61h^2u^{10}}{1505358117559296000 00}$$

$$f''_{n+5} \frac{60438689h^2}{59875200} - \frac{4904591h^2u^2}{6671808000} - \frac{13423447h^2u^4}{280215936000} - \frac{383048531h^2u^6}{190546836480000}$$

$$\frac{8640169h^2u^8}{16895152834560000} + \frac{2609444747029h^2u^{10}}{3010716235118592000 00}$$

$$f'_{n+3} \frac{54716489h^2}{59875200} + \frac{154409h^2u^2}{606528000} + \frac{4511821h^2u^4}{140107968000} + \frac{52003271h^2u^6}{21171870720000}$$

$$\frac{306695489h^2u^8}{1535922984960000} + \frac{703122556777h^2u^{10}}{4301023193026560000 0}$$

$$f'_{n+1} \frac{1976207h^2}{59875200} + \frac{8301589h^2u^2}{46702656000} + \frac{31470731h^2u^4}{1961511552000} + \frac{1319107409h^2u^6}{1333827855360000}$$

$$\frac{750660103h^2u^8}{13140674426880000} + \frac{7234271047849h^2u^{10}}{21075013645830144000 00}$$

$$y''_{n+7} = \frac{3y_n}{10h^2} + \frac{4y_{n+3}}{h^2} - \frac{13y_{n+4}}{2h^2} + \frac{14y_{n+5}}{5h^2}$$

$$\tau_{n+4} \frac{24511559h^2}{9580032} - \frac{4288373h^2u^2}{667180800} - \frac{5165131h^2u^4}{17933819904} - \frac{5387927h^2u^6}{1154829312000}$$

$$\frac{2281265663h^2u^8}{+8109673360588800} - \frac{207122085103h^2u^{10}}{7526790587796480000}$$

$$f_{n+8} \frac{988451h^2}{479001600} - \frac{474391h^2u^2}{2426112000} - \frac{289056421h^2u^4}{31384184832000} - \frac{388566559h^2u^6}{1778437140480000}$$

$$\frac{9970415627h^2u^8}{+2838385676206080000} - \frac{5794924975813h^2u^{10}}{84300054583320576000000}$$

$$f_n \frac{20587h^2}{68428800} + \frac{2221123h^2u^2}{186810624000} + \frac{30799871h^2u^4}{31384184832000} + \frac{50499257h^2u^6}{592812380160000}$$

$$\frac{1426915667h^2u^8}{+315376186245120000} - \frac{832981747483h^2u^{10}}{84300054583320576000000}$$

$$f_{n+6} \frac{122679367h^2}{119750400} - \frac{53681599h^2u^2}{13343616000} - \frac{52273087h^2u^4}{280215936000} - \frac{169033651h^2u^6}{42343741440000}$$

$$\frac{209575921h^2u^8}{+1987665039360000} - \frac{1301268333923h^2u^{10}}{86020463860531200000}$$

$$f_{n+7} \frac{4776587h^2}{59875200} + \frac{126737813h^2u^2}{93405312000} + \frac{498148769h^2u^4}{7846046208000} + \frac{1284234071h^2u^6}{889218570240000}$$

$$\frac{10329393799h^2u^8}{3547982095257600000} - \frac{20698728289087h^2u^{10}}{421500272916602880000000}$$

$$\tau_{n+2} \frac{7038073h^2}{17107200} - \frac{14932369h^2u^2}{13343616000} - \frac{24582101h^2u^4}{560431872000} - \frac{32963377h^2u^6}{+127031224320000}$$

$$\frac{12124284659h^2u^8}{+1013709170073600000} - \frac{4146935109131h^2u^{10}}{60214324702371840000000}$$

$$'_{n+5} \frac{16807661h^2}{8553600} + \frac{87988583h^2u^2}{13343616000} + \frac{26016971h^2u^4}{86220288000} + \frac{106309963h^2u^6}{181473177600000}$$

$$\frac{11047350143h^2u^8}{506854585036800000} - \frac{828251845303h^2u^{10}}{31691749843353600000}$$

$$'_{n+3} \frac{79832657h^2}{59875200} + \frac{49239353h^2u^2}{13343616000} + \frac{16208407h^2u^4}{101896704000} + \frac{22678379h^2u^6}{141145804800000}$$

$$\frac{356524439h^2u^8}{1535922984960000} - \frac{10774841832427h^2u^{10}}{602143247023718400000}$$

$$\tau_{n+1} \frac{2746889h^2}{59875200} - \frac{1498589h^2u^2}{13343616000} - \frac{18364331h^2u^4}{7846046208000} - \frac{335958919h^2u^6}{889218570240000}$$

$$\frac{960250987h^2u^8}{27292169963520000} - \frac{447146046469h^2u^{10}}{32423097916661760000000}$$

$$y''_{n+8} = \frac{2y_n}{5h^2} + \frac{5y_{n+3}}{h^2} - \frac{8y_{n+4}}{h^2} + \frac{17y_{n+5}}{5h^2}$$

$$f_{n+4} \frac{149735077h^2}{47900160} + \frac{131175403h^2u^2}{2668723200} - \frac{164658619h^2u^4}{448345497600} + \frac{499412399h^2u^6}{5862979584000}$$

$$\frac{172937440427h^2u^8}{40548366802944000} - \frac{16364173564997h^2u^{10}}{12042864940474368000}$$

$$f_{n+8} \frac{29258227\,h^2}{479001600} + \frac{305822863\,h^2u^2}{186810624000} + \frac{44271053\,h^2u^4}{31384184832000} - \frac{11327344447\,h^2u^6}{5335311421440000}$$

$$\frac{6927251161\,h^2u^8}{55654621102080000} - \frac{39234200545457\,h^2u^{10}}{843000545833205760000}$$

$$f_n \frac{980333h^2}{479001600} - \frac{43472057h^2u^2}{186810624000} + \frac{373588291h^2u^4}{31384184832000} - \frac{184153103h^2u^6}{592812380160000}$$

$$\frac{7414928357h^2u^8}{2838385676206080000} + \frac{6505853415463h^2u^{10}}{843000545833205760000}$$

$$f''_{n+6} \frac{213188137h^2}{119750400} + \frac{218499133h^2u^2}{6671808000} - \frac{4630291h^2u^4}{86220288000} + \frac{2969950939h^2u^6}{63515612160000}$$

$$\frac{263113624819h^2u^8}{1013709170073600000} - \frac{3971312436461h^2u^{10}}{43010231930265600000}$$

$$f''_{n+7} \frac{65893643h^2}{59875200} - \frac{11916409h^2u^2}{1061424000} + \frac{1592273h^2u^4}{784604620800} + \frac{1264824829h^2u^6}{83364240960000}$$

$$\frac{61640343403h^2u^8}{709596419051520000} + \frac{16758346900171h^2u^{10}}{526875341145753600000}$$

$$f''_{n+2} \frac{7820767h^2}{17107200} + \frac{43851673h^2u^2}{6671808000} - \frac{53824691h^2u^4}{224172748800} + \frac{370442687h^2u^6}{17322439680000}$$

$$\frac{613046341h^2u^8}{7508956815360000} - \frac{4929160074767h^2u^{10}}{301071623511859200000}$$

$$f''_{n+5} \frac{198402581h^2}{59875200} - \frac{43709317h^2u^2}{833976000} + \frac{112426201h^2u^4}{560431872000} + \frac{3850553501h^2u^6}{47636709120000}$$

$$\frac{72675177541h^2u^8}{168951528345600000} + \frac{690052509691h^2u^{10}}{4704244117372800000}$$

$$f''_{n+3} \frac{119543141h^2}{59875200} - \frac{43756769h^2u^2}{1667952000} + \frac{19717367h^2u^4}{50948352000} + \frac{62900183h^2u^6}{1134207360000}$$

$$\frac{11622668021h^2u^8}{46077689548800000} + \frac{5323333409941h^2u^{10}}{75267905877964800000}$$

$$f''_{n+1} \frac{5003099h^2}{59875200} - \frac{11863h^2u^2}{1459458000} + \frac{321355873h^2u^4}{3923023104000} + \frac{1433061871h^2u^6}{333456963840000}$$

$$\frac{37673163839h^2u^8}{354798209525760000} - \frac{197086667587h^2u^{10}}{26343767057287680000}$$

$$y'''_n = \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} - \frac{3y_{n+4}}{2h^3} + \frac{3y_{n+5}}{5h^3}$$

$$f'_{n+8} \frac{2047h}{259200} + \frac{826103hu^2}{958003200} + \frac{294768241hu^4}{5230697472000} + \frac{199776281hu^6}{62768369664000}$$

$$+ \frac{11643707hu^8}{57164050944000} + \frac{860308824013hu^{10}}{51090942171709440000} !$$

$$f_{n+4} \frac{53767h}{51840} \frac{55201hu^2}{311040} \frac{990041hu^4}{2874009600} + \frac{6059347hu^6}{32024678400} + \frac{30463243hu^8}{2690072985600}$$

$$+ \frac{4217782193hu^{10}}{9603560558592000} !$$

$$f_n \frac{12397h}{43200} \frac{1894597hu^2}{319334400} \frac{16488113hu^4}{249080832000} \frac{5168413hu^6}{+2324754432000}$$

$$+ \frac{36439hu^8}{304006348800} \frac{73068643559hu^{10}}{17030314057236480000} !$$

$$f'_{n+6} \frac{7201h}{24192} \frac{534247hu^2}{22809600} + \frac{3587029hu^4}{4981616640} \frac{123145999hu^6}{+1494484992000} \frac{389974241hu^8}{+76218734592000}$$

$$+ \frac{4325495497hu^{10}}{13367593451520000} !$$

$$f_{n+7} \frac{65783h}{907200} \frac{9893hu^2}{95800320} \frac{122651951hu^4}{373621248000} \frac{768990559hu^6}{31384184832000}$$

$$\frac{2474017907hu^8}{1600593426432000} \frac{2901477918707hu^{10}}{25545471085854720000} !$$

$$f'_{n+2} \frac{177481h}{1814400} \frac{1622527hu^2}{13685760} \frac{371991439hu^4}{373621248000} \frac{28109897hu^6}{+407586816000}$$

$$+ \frac{901577801hu^8}{228656203776000} + \frac{101345515991hu^{10}}{3649353012264960000} !$$

$$f'_{n+5} \frac{652483h}{907200} + \frac{6460429hu^2}{68428800} \frac{217545043hu^4}{373621248000} \frac{708761429hu^6}{4483454976000}$$

$$\frac{441134597hu^8}{45731240755200} \frac{1821963164017hu^{10}}{3649353012264960000} !$$

$$f_{n+3} \frac{540979h}{302400} + \frac{1441147hu^2}{7603200} \frac{47052619hu^4}{+41513472000} \frac{216177433hu^6}{1494484992000}$$

$$\frac{58706911hu^8}{6928975872000} \frac{82494267703hu^{10}}{405483668029440000} !$$

$$f_{n+1} \frac{57077h}{36288} + \frac{19480217hu^2}{479001600} \frac{212898437hu^4}{+523069747200} \frac{588303169hu^6}{31384184832000}$$

$$\frac{238426163hu^8}{228656203776000} + \frac{30642395033hu^{10}}{2322315553259520000} !$$

$$y^{000}_{n+1} = \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} - \frac{3y_{n+4}}{2h^3} + \frac{3y_{n+5}}{5h^3}$$

$$f_{n+4} \frac{126943h}{362880} + \frac{72869hu^2}{3421440} + \frac{506567hu^4}{485222400} + \frac{920947hu^6}{32024678400} - \frac{524653hu^8}{5081248972800}$$

$$\frac{12720296653hu^{10}}{182467650613248000}$$

$$f_{n+8} \frac{353h}{319334400} - \frac{78041hu^2}{209227898880} - \frac{5408867hu^4}{62768369664000} + \frac{146834557hu^6}{241920}$$

$$\frac{661320481hu^8}{3201186852864000} - \frac{1788376699hu^{10}}{94788389928960000}$$

$$f_n \frac{28669h}{3628800} + \frac{4669hu^2}{5474304} + \frac{32360479hu^4}{581188608000} + \frac{66135863hu^6}{20922789888000} + \frac{651876727hu^8}{3201186852864000}$$

$$+\frac{287390889179hu^{10}}{17030314057236480000}$$

$$f_{n+6} \frac{3769h}{64800} + \frac{57353hu^2}{68428800} - \frac{19070119hu^4}{124540416000} - \frac{40349347hu^6}{1494484992000} - \frac{333021179hu^8}{114328101888000}$$

$$\frac{2891214323hu^{10}}{10397017128960000}$$

$$f_{n+7} \frac{3553h}{259200} + \frac{58699hu^2}{68428800} + \frac{327653707hu^4}{2615348736000} + \frac{82943431hu^6}{6276836966400} + \frac{1274797hu^8}{1026021427200}$$

$$+\frac{588537261779hu^{10}}{5109094217170944000}$$

$$f_{n+2} \frac{44267h}{37800} + \frac{369517hu^2}{22809600} + \frac{369255629hu^4}{373621248000} + \frac{44838821hu^6}{896690995200}$$

$$+\frac{12943097hu^8}{4573124075520} + \frac{32451659237hu^{10}}{145974120490598400}$$

$$f_{n+5} \frac{89401h}{604800} - \frac{527hu^2}{56320} - \frac{98812279hu^4}{373621248000} + \frac{69475009hu^6}{4483454976000} + \frac{3015563hu^8}{1020786624000}$$

$$+\frac{1116578600597hu^{10}}{3649353012264960000}$$

$$f_{n+3} \frac{89639h}{362880} - \frac{153773hu^2}{6220800} - \frac{35018551hu^4}{24908083200} - \frac{30640793hu^6}{498161664000} - \frac{7246717hu^8}{2598365952000}$$

$$\frac{236509702567hu^{10}}{1216451004088320000}$$

$$f_{n+1} \frac{620303h}{479001600} - \frac{2742701hu^2}{2615348736000} - \frac{951744251hu^4}{31384184832000} - \frac{621009283hu^6}{1814400}$$

$$\frac{2390819hu^8}{1961511552000} - \frac{362233830857hu^{10}}{3649353012264960000}$$

$$y'''_{n+2} = \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} - \frac{3y_{n+4}}{2h^3} + \frac{3y_{n+5}}{5h^3}$$

$$\tau_{n+4} \frac{319h}{24192} - \frac{125hu^2}{25344} - \frac{15260549hu^4}{37362124800} - \frac{5587643hu^6}{224172748800} - \frac{2295053hu^8}{1829249630208}$$

$$\frac{9983506909hu^{10}}{182467650613248000}$$

$$f_{n+8} \frac{143h}{259200} + \frac{108151hu^2}{958003200} + \frac{11100487hu^4}{747242496000} + \frac{2957203hu^6}{1793381990400} + \frac{1084799hu^8}{6351561216000}$$

$$\frac{176440628393hu^{10}}{+10218188434341888000}$$

$$f_n \frac{1321h}{907200} - \frac{243151hu^2}{958003200} - \frac{9249707hu^4}{348713164800} - \frac{49401083hu^6}{20922789888000}$$

$$\frac{330745673hu^8}{1600593426432000} - \frac{106896799693hu^{10}}{5676771352412160000}$$

$$\tau_{n+6} \frac{41611h}{1814400} + \frac{40651hu^2}{68428800} + \frac{158863hu^4}{1257984000} + \frac{9016837hu^6}{498161664000} + \frac{489362371hu^8}{228656203776000}$$

$$\frac{280756371443hu^{10}}{+1216451004088320000}$$

$$\tau_{n+7} \frac{1597h}{302400} - \frac{85651hu^2}{159667200} - \frac{202589129hu^4}{2615348736000} - \frac{288155743hu^6}{31384184832000}$$

$$\frac{1582839763hu^8}{1600593426432000} - \frac{2606675398259hu^{10}}{25545471085854720000}$$

$$f_{n+2} \frac{502423h}{1814400} - \frac{28241hu^2}{6220800} - \frac{169266703hu^4}{373621248000} - \frac{170553821hu^6}{4483454976000}$$

$$\frac{239622557hu^8}{76218734592000} - \frac{1002005224873hu^{10}}{3649353012264960000}$$

$$f_{n+5} \frac{54659h}{907200} - \frac{94349hu^2}{68428800} + \frac{213229hu^4}{5748019200} + \frac{36450389hu^6}{4483454976000} - \frac{124869907hu^8}{76218734592000}$$

$$\frac{4077011011hu^{10}}{19515256750080000}$$

$$\tau_{n+3} \frac{662233h}{907200} + \frac{445651hu^2}{68428800} + \frac{76769633hu^4}{124540416000} + \frac{14350331hu^6}{298896998400}$$

$$\frac{833620321hu^8}{228656203776000} + \frac{542292371hu^{10}}{1829249630208000}$$

$$\tau_{n+1} \frac{1607h}{43200} + \frac{265651hu^2}{159667200} + \frac{63822559hu^4}{373621248000} + \frac{66708617hu^6}{4483454976000} + \frac{291692909hu^8}{228656203776000}$$

$$\frac{2926147619347hu^{10}}{+25545471085854720000}$$

$$y^{000}_{n+3} = \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} \quad \frac{3y_{n+4}}{2h^3} + \frac{3y_{n+5}}{5h^3}$$

$$f_{n+4} \quad \frac{78527h}{362880} + \frac{641hu^2}{684288} + \frac{350513hu^4}{3396556800} + \frac{2003557hu^6}{224172748800} + \frac{28558667hu^8}{45731240755200}$$

$$\frac{6779594771hu^{10}}{+\ 182467650613248000\ !}$$

$$f_{n+8} \quad \frac{1231h}{958003200} \quad \frac{73483hu^2}{475517952000} \quad \frac{5341537hu^4}{62768369664000} \quad \frac{86104957hu^6}{!} \quad \frac{3628800}{}$$

$$\frac{489450497hu^8}{3201186852864000} \quad \frac{829830227321hu^{10}}{51090942171709440000}$$

$$f_n \quad \frac{671h}{1209600} + \frac{33041hu^2}{319334400} + \frac{2247863hu^4}{158505984000} + \frac{3782719hu^6}{2324754432000} + \frac{6747751hu^8}{39520825344000}$$

$$\frac{294688995163hu^{10}}{+\ 17030314057236480000\ !}$$

$$f_{n+6} \quad \frac{2453h}{151200} \quad \frac{20221hu^2}{22809600} \quad \frac{1546837hu^4}{11321856000} \quad \frac{26030243hu^6}{1494484992000} \quad \frac{2194723hu^8}{1088839065600}$$

$$\frac{20582355083hu^{10}}{!\ 93573154160640000}$$

$$f_{n+7} \quad \frac{1243h}{362880} + \frac{207629hu^2}{479001600} + \frac{3064717hu^4}{47551795200} + \frac{250300643hu^6}{31384184832000}$$

$$\frac{179974103hu^8}{+\ 200074178304000} \quad \frac{2462372302879hu^{10}}{+25545471085854720000\ !}$$

$$f_{n+2} \quad \frac{3959h}{22680} + \frac{111943hu^2}{68428800} + \frac{1488923hu^4}{6793113600} + \frac{110147641hu^6}{4483454976000} + \frac{287563249hu^8}{114328101888000}$$

$$\frac{82835033143hu^{10}}{+\ 331759364751360000\ !}$$

$$f_{n+5} \quad \frac{89611h}{1814400} + \frac{35023hu^2}{68428800} + \frac{462637hu^4}{4852224000} + \frac{682003hu^6}{49268736000} + \frac{12617953hu^8}{7145506368000}$$

$$\frac{106925012867hu^{10}}{+\ 521336144609280000\ !}$$

$$f_{n+3} \quad \frac{407h}{86400} \quad \frac{15287hu^2}{7603200} \quad \frac{982963hu^4}{3773952000} \quad \frac{42058699hu^6}{1494484992000} \quad \frac{2394863hu^8}{866121984000}$$

$$\frac{1072691247 49hu^{10}}{!\ 405483668029440000}$$

$$f_{n+1} \quad \frac{17681h}{1814400} \quad \frac{28199hu^2}{43545600} \quad \frac{20931793hu^4}{237758976000} \quad \frac{314414467hu^6}{31384184832000}$$

$$\frac{20853277hu^8}{20007417830400} \quad \frac{2679319335551hu^{10}}{!\ 25545471085854720000}$$

$$y'''_{n+4} = \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} - \frac{3y_{n+4}}{2h^3} + \frac{3y_{n+5}}{5h^3}$$

$$- \frac{143921h}{362880} - \frac{1003hu^2}{3421440} - \frac{175489hu^4}{7472424960} - \frac{35501hu^6}{32024678400} - \frac{253789hu^8}{5081248972800}$$

$$- \frac{553698781hu^{10}}{182467650613248000} \quad !$$

$$f_{n+8} \quad \frac{211h}{604800} + \frac{3547hu^2}{45619200} + \frac{58771057hu^4}{5230697472000} + \frac{85485593hu^6}{62768369664000}$$

$$+ \frac{12126329hu^8}{80029671321600} + \frac{48443340797hu^{10}}{3005349539512320000} \quad !$$

$$f_n \quad \frac{61h}{87091200} \frac{7501hu^2}{581188608000} \frac{6920093hu^4}{20922789888000} \frac{29157883hu^6}{!} \frac{181440}{244810681hu^8} \frac{39426970657hu^{10}}{1600593426432000 \quad 2432902008176640000}$$

$$f'_{n+6} \quad \frac{33707h}{1814400} + \frac{2819hu^2}{2737152} + \frac{1461953hu^4}{9580032000} + \frac{5632771hu^6}{298896998400} + \frac{482769059hu^8}{228656203776000}$$

$$+ \frac{1216773331hu^{10}}{5406448907059200} \quad !$$

$$f_{n+7} \quad \frac{3319h}{479001600} \frac{219449hu^2}{2615348736000} \frac{174558281hu^4}{31384184832000} \frac{255462751hu^6}{!} \frac{907200}{484291793hu^8} \frac{352627940789hu^{10}}{533531142144000 \quad 3649353012264960000}$$

$$f_{n+2} \quad \frac{69571h}{604800} \quad \frac{28841hu^2}{22809600} \quad \frac{9147961hu^4}{53374464000} \quad \frac{88467677hu^6}{4483454976000} \quad \frac{70272209hu^8}{32665171968000}$$

$$\frac{43693746259hu^{10}}{192071211171840000} \quad !$$

$$f_{n+5} \quad \frac{587h}{8640} \quad \frac{257hu^2}{281600} \quad \frac{4864217hu^4}{33965568000} \quad \frac{7500319hu^6}{407586816000} \quad \frac{478200857hu^8}{228656203776000}$$

$$\frac{8168924081 77hu^{10}}{3649353012264960000} \quad !$$

$$f'_{n+3} \quad \frac{473191h}{907200} + \frac{94547hu^2}{68428800} + \frac{22515169hu^4}{124540416000} + \frac{10050637hu^6}{498161664000} + \frac{99294733hu^8}{45731240755200}$$

$$+ \frac{278203589723hu^{10}}{1216451004088320000} \quad !$$

$$f'_{n+1} \quad \frac{17939h}{907200} + \frac{7187hu^2}{13685760} + \frac{188597401hu^4}{2615348736000} + \frac{10536599hu^6}{1255367393280}$$

$$+ \frac{490382729hu^8}{533531142144000} + \frac{497222789303hu^{10}}{5109094217170944000} \quad !$$

$$y'''_{n+5} = \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} - \frac{3y_{n+4}}{2h^3} + \frac{3y_{n+5}}{5h^3}$$

$$f_{n+4} \frac{122123\,h}{120960} - \frac{193\,hu^2}{126720} - \frac{5610533\,hu^4}{37362124800} - \frac{2500571\,hu^6}{224172748800} - \frac{33126869\,hu^8}{45731240755200} - \frac{716999303\,hu^{10}}{16587968237568000}$$

$$f_{n+8} \frac{281h}{958003200} - \frac{107147hu^2}{5230697472000} - \frac{77689259hu^4}{62768369664000} - \frac{104121469hu^6}{518400} - \frac{8748191hu^8}{50812489728000} - \frac{888496575737hu^{10}}{51090942171709440000}$$

$$f_n \frac{1277h}{3628800} + \frac{65459hu^2}{958003200} + \frac{1416593hu^4}{134120448000} + \frac{2548997hu^6}{1902071808000} + \frac{96976459hu^8}{640237370572800} - \frac{91711181897hu^{10}}{5676771352412160000}$$

$$f'_{n+6} \frac{831659h}{1814400} + \frac{169679hu^2}{68428800} + \frac{111352457hu^4}{373621248000} + \frac{134128321hu^6}{4483454976000} - \frac{1355243hu^8}{476367091200} - \frac{983140483733hu^{10}}{3649353012264960000}$$

$$f'_{n+7} \frac{3853h}{604800} + \frac{22819hu^2}{31933440} + \frac{244288843hu^4}{2615348736000} + \frac{322366691hu^6}{31384184832000} - \frac{210816871hu^8}{200074178304000} - \frac{2697037696543hu^{10}}{25545471085854720000}$$

$$f'_{n+2} \frac{1277h}{3628800} + \frac{65459hu^2}{958003200} + \frac{1416593hu^4}{134120448000} + \frac{2548997hu^6}{1902071808000} - \frac{96976459hu^8}{640237370572800} - \frac{91711181897hu^{10}}{5676771352412160000}$$

$$f'_{n+5} \frac{831659h}{1814400} + \frac{169679hu^2}{68428800} + \frac{111352457hu^4}{373621248000} + \frac{134128321hu^6}{4483454976000} - \frac{1355243hu^8}{476367091200} - \frac{983140483733hu^{10}}{3649353012264960000}$$

$$f'_{n+3} \frac{733501h}{1814400} - \frac{2927hu^2}{68428800} - \frac{7194643hu^4}{124540416000} - \frac{2576669hu^6}{213497856000} - \frac{336977hu^8}{199874304000} - \frac{243585576359hu^{10}}{1216451004088320000}$$

$$f_{n+1} \frac{307h}{24192} - \frac{58511hu^2}{159667200} - \frac{30904063hu^4}{523069747200} - \frac{242348419hu^6}{31384184832000} - \frac{12692143hu^8}{14291012736000} - \frac{2444653941887hu^{10}}{25545471085854720000}$$

$$y^{000}_{n+6} = \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} + \frac{3y_{n+4}}{2h^3} + \frac{3y_{n+5}}{5h^3}$$

$$y'_{n+4} \quad \frac{283057h}{362880} + \frac{14869hu^2}{3421440} + \frac{13505659hu^4}{37362124800} + \frac{5090629hu^6}{224172748800} + \frac{52808123hu^8}{45731240755200}$$

$$+\frac{1268015621hu^{10}}{26066807230464000\,!}$$

$$f_{n+8} \quad \frac{533h}{362880} + \frac{235127hu^2}{958003200} + \frac{5409433hu^4}{209227898880} + \frac{146215193hu^6}{62768369664000}$$

$$+\frac{82115393hu^8}{400148356608000} + \frac{957641606989hu^{10}}{51090942171709440000\,!}$$

$$f_n \quad \frac{163h}{302400} \quad \frac{1549hu^2}{12773376} \quad \frac{27071063hu^4}{1743565824000} \quad \frac{558149hu^6}{332107776000} \quad \frac{3403129hu^8}{19760412672000}$$

$$\frac{295544244071hu^{10}}{17030314057236480000\,!}$$

$$y'_{n+6} \quad \frac{248281h}{604800} + \frac{98201hu^2}{22809600} + \frac{54082381hu^4}{124540416000} + \frac{55525903hu^6}{1494484992000}$$

$$+\frac{33796727hu^8}{10888390656000} + \frac{331048681459hu^{10}}{1216451004088320000\,!}$$

$$f_{n+7} \quad \frac{19127h}{907200} \quad \frac{764857hu^2}{479001600} \quad \frac{33286061hu^4}{201180672000} \quad \frac{1008809hu^6}{68976230400} \quad \frac{404715511hu^8}{320118685286400}$$

$$\frac{581685851671hu^{10}}{5109094217170944000\,!}$$

$$f_{n+2} \quad \frac{28687h}{259200} \quad \frac{56699hu^2}{68428800} \quad \frac{54201871hu^4}{373621248000} \quad \frac{17025529hu^6}{896690995200} \quad \frac{19939951hu^8}{9146248151040}$$

$$\frac{34045131793hu^{10}}{145974120490598400\,!}$$

$$f_{n+5} \quad \frac{1073789h}{907200} \quad \frac{82711hu^2}{13685760} \quad \frac{19660889hu^4}{33965568000} \quad \frac{207302741hu^6}{4483454976000}$$

$$\frac{815347513hu^8}{228656203776000} \quad \frac{1064154919153hu^{10}}{3649353012264960000\,!}$$

$$y'_{n+3} \quad \frac{31081h}{60480} \quad \frac{6917hu^2}{7603200} + \frac{569hu^4}{1186099200} + \frac{14800871hu^6}{1494484992000} + \frac{592583hu^8}{344881152000}$$

$$+\frac{86679935561hu^{10}}{405483668029440000\,!}$$

$$y'_{n+1} \quad \frac{2773h}{129600} + \frac{289049hu^2}{479001600} + \frac{216628249hu^4}{2615348736000} + \frac{296107967hu^6}{31384184832000}$$

$$+\frac{1601112571hu^8}{1600593426432000} + \frac{12556907939hu^{10}}{122227134382080000\,!}$$

$$y^{000}_{n+7} = \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} \quad \frac{3y_{n+4}}{2h^3} + \frac{3y_{n+5}}{5h^3}$$

$$\tau_{n+4} \quad \frac{11851h}{10368} \quad \frac{14975hu^2}{684288} \quad \frac{40760549hu^4}{37362124800} \quad \frac{991949hu^6}{32024678400} \quad \frac{683hu^8}{+203249958912}$$

$$\frac{11612899091hu^{10}}{+\ 182467650613248000\ !}$$

$$f_{n+8} \quad \frac{1363h}{172800} \quad \frac{25003hu^2}{29030400} \quad \frac{294754091hu^4}{5230697472000} \quad \frac{40079129hu^6}{12553673932800}$$

$$\frac{93777847hu^8\ 15756404687hu^{10}}{457312407552000\ \ 928926221303808000\ !}$$

$$f_n \quad \frac{763h}{518400} + \frac{226099hu^2}{958003200} + \frac{418133hu^4}{16605388800} + \frac{48282167hu^6}{20922789888000} + \frac{38632487hu^8}{188305108992000}$$

$$+ \frac{18813832363hu^{10}}{1001783179837440000\ !}$$

$$f_{n+6} \quad \frac{591809h}{453600} \quad \frac{1124599hu^2}{68428800} \quad \frac{125425063hu^4}{124540416000} \quad \frac{6914249hu^6}{135862272000}$$

$$\frac{328145627hu^8\ 333801653hu^{10}}{114328101888000\ \ 1485288161280000\ !}$$

$$\tau'_{n+7} \quad \frac{649543h}{1814400} + \frac{2774797hu^2}{479001600} + \frac{137969053hu^4}{373621248000} + \frac{628961507hu^6}{31384184832000}$$

$$\frac{82049213hu^8}{+\ 66691392768000} \quad \frac{2553355176991hu^{10}}{+\ 25545471085854720000\ !}$$

$$\tau_{n+2} \quad \frac{7249h}{37800} \quad \frac{24467hu^2}{22809600} \quad \frac{50190797hu^4}{+373621248000} \quad \frac{9005533hu^6}{+344881152000} \quad \frac{328452977hu^8}{+114328101888000}$$

$$\frac{1005957046877hu^{10}}{+\ 3649353012264960000\ !}$$

$$\tau'_{n+5} \quad \frac{423769h}{604800} + \frac{7093hu^2}{281600} + \frac{107863477hu^4}{74724249600} + \frac{283719361hu^6}{4483454976000} + \frac{20499497hu^8}{7145506368000}$$

$$\frac{727247468693hu^{10}}{+\ 3649353012264960000\ !}$$

$$\tau_{n+3} \quad \frac{554909h}{1814400} + \frac{672401hu^2}{68428800} + 124540416000 \quad \frac{37617133hu^4}{99632332800} \quad \frac{1367173hu^6}{2598365952000} \quad \frac{7468333hu^8}{}$$

$$\frac{73257349307hu^{10}}{243290200817664000\ !}$$

$$f_{n+1} \quad \frac{4369h}{1814400} \quad \frac{378797hu^2}{479001600} \quad \frac{2193109hu^4}{18289152000} \quad \frac{406764931hu^6}{31384184832000}$$

$$\frac{5864317hu^8\ 29249679479 03hu^{10}}{4763670912000\ \ 25545471085854720000\ !}$$

$$y^{000}_{n+8} = \frac{y_n}{10h^3} + \frac{y_{n+3}}{h^3} - \frac{3y_{n+4}}{2h^3} + \frac{3y_{n+5}}{5h^3}$$

$$f_{n+4} \frac{29509h}{120960} + \frac{4483hu^2}{25344} + \frac{144359hu^4}{485222400} - \frac{6130349hu^6}{32024678400} - \frac{522443333hu^8}{45731240755200}$$

$$\frac{81245259229hu^{10}}{182467650613248000}$$

$$f_{n+8} \frac{520697h}{1814400} + \frac{5675767hu^2}{958003200} + \frac{26364661hu^4}{402361344000} - \frac{12866837hu^6}{5706215424000}$$

$$\frac{490241hu^8}{4041902592000} + \frac{4383190621hu^{10}}{10426722892185600000}$$

$$f_n \frac{7153h}{907200} - \frac{119161hu^2}{136857600} - \frac{99426007hu^4}{1743565824000} - \frac{67254779hu^6}{20922789888000}$$

$$\frac{328307897hu^8}{1600593426432000} - \frac{5056949791hu^{10}}{2987774396006400000}$$

$$f_{n+6} \frac{9277h}{} + \frac{8096587hu^2}{} + \frac{13517477hu^4}{} - \frac{34798331hu^6}{}$$

$$\frac{182142841hu^8}{45731240755200} - \frac{3673489882 9hu^{10}}{12164510040883200 00}$$

$$f_{n+7} \frac{13729h}{8640} - \frac{84191hu^2}{2073600} - \frac{210090613hu^4}{523069747200} + \frac{596255393hu^6}{31384184832000}$$

$$+\frac{1687255949hu^8}{1600593426432000} - \frac{24565229567hu^{10}}{19650362373734400 00}$$

$$f_{n+2} \frac{59531h}{362880} + \frac{1586693hu^2}{68428800} - \frac{55209347hu^4}{74724249600} - \frac{373414109hu^6}{4483454976000}$$

$$\frac{393019709hu^8}{76218734592000} - \frac{1189719451177hu^{10}}{36493530122649600 00}$$

$$f_{n+5} \frac{2034493h}{907200} - \frac{12938227hu^2}{68428800} - \frac{409434451hu^4}{373621248000} + \frac{656484523hu^6}{4483454976000}$$

$$+\frac{93123851hu^8}{10888390656000} + \frac{69106070029hu^{10}}{331759364751360000}$$

$$f_{n+3} \frac{1064039h}{907200} - \frac{6428333hu^2}{68428800} + \frac{77194721hu^4}{124540416000} + \frac{238904551hu^6}{1494484992000}$$

$$+\frac{2223945793hu^8}{228656203776000} + \frac{613227175003hu^{10}}{12164510040883200 00}$$

$$f_{n+1} \frac{26801h}{302400} + \frac{27187hu^2}{159667200} + \frac{872602777hu^4}{2615348736000} + \frac{776942783hu^6}{31384184832000}$$

$$+\frac{498458143hu^8}{} + \frac{417028039957hu^{10}}{}$$

320118685286400 3649353012264960000

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Denoting the principal term of the local truncation term as LTEi, we obtain

LT E1 = $\frac{1003h^{14}}{11975040}(!^2y^{(12)}(x_n) + y^{(14)}(x_n))$; LT E2 = $\frac{193h^{13}}{3628800}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$;

LT E3 = $\frac{h^{13}}{1209600}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$; LT E4 = $\frac{193h^{13}}{3628800}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$;

LT E5 = $\frac{h^{13}}{1209600}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$; LT E6 = $\frac{23893h^{13}}{181621440}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$;

LT E7 = $\frac{3130901h^{13}}{43589145600}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$; LT E8 = $\frac{43369h^{13}}{7264857600}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$

LT E9 = $\frac{15469h^{13}}{968647680}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$; LT E10 = $\frac{90107h^{13}}{5448643200}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$

LT E11 = $\frac{15469h^{13}}{968647680}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$; LT E12 = $\frac{43369h^{13}}{7264857600}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$

LT E13 = $\frac{3130901h^{13}}{43589145600}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$ LT E14 = $\frac{23893h^{13}}{181621440}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$;

LT E15 = $\frac{223907h^{13}}{119750400}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$; LT E16 = $\frac{33119h^{13}}{159667200}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$;

LT E17 = $\frac{16931h^{13}}{239500800}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$; LT E18 = $\frac{2143h^{13}}{479001600}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$;

LT E19 = $\frac{119803h^{14}}{186810624000}(!^2y^{(12)}(x_n) + y^{(14)}(x_n))$;LT E20 = $\frac{2143h^{13}}{479001600}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$;

LT E21 = $\frac{16931h^{13}}{239500800}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$; LT E22 = $\frac{33119h^{13}}{159667200}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$;

LT E23 = $\frac{223907h^{13}}{119750400}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$; LT E24 = $\frac{3254947h^{13}}{479001600}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$;

LT E25 = $\frac{525599h^{13}}{479001600}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$; LT E26 = $\frac{25093h^{13}}{68428800}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$;

LT E27 = $\frac{12329h^{13}}{68428800}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$; LT E28 = $\frac{78499h^{13}}{479001600}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$;

LT E29 = $\frac{12329h^{13}}{68428800}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$; LT E30 = $\frac{25093h^{13}}{6842800}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$;

LT E31 = $\frac{525599h^{13}}{479001600}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$; LT E32 = $\frac{3254947h^{13}}{479001600}(!^2y^{(11)}(x_n) + y^{(13)}(x_n))$

Thus, the eight-step TFBM has order

$$= (10; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 10; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9; 9)^T$$

and the error constant

$$C_{p+4} = \left[\frac{1003}{11975040}; \frac{193}{3628800}; \frac{1}{1209600}; \frac{193}{3628800}; \frac{1}{1209600}; \frac{23893}{181621440}; \frac{3130901}{43589145600}; \frac{43369}{7264857600};\right.$$

$$\frac{15469}{968647680}; \frac{90107}{5448643200}; \frac{15469}{968647680}; \frac{43369}{7264857600}; \frac{3130901}{43589145600}; \frac{23893}{181621440}; \frac{223907}{119750400};$$

$$\frac{33119}{159667200}; \frac{16931}{239500800}; \frac{2143}{479001600}; \frac{119803}{186810624000}; \frac{2143}{479001600}; \frac{16931}{239500800}; \frac{33119}{159667200}; \frac{223907}{119750400};$$

$$\left.\frac{3254947}{68428800}, \frac{525599}{479001600}, \frac{25093}{68428800}, \frac{12329}{68428800}, \frac{78499}{479001600}, \frac{12329}{479001600}, \frac{25093}{479001600}, \frac{525599}{479001600}, \frac{3254947}{479001600}\right]^T$$

4.3.2 Consistency of Methods

Case k = 4 Since each numerical scheme in the four-step TFBM has order p > 4, then by de nition (4.1.2), the four-step TFBM is constant. Case k = 6

Since each numerical scheme in the six-step TFBM has order p > 4, then by de nition (4.1.2), the six-step TFBM is constant. Case k = 8

Since each numerical scheme in the eight-step TFBM has order p > 4, then by de nition (4.1.2), the eight-step TFBM is constant.

4.3.3 Zero Stability of Methods

Case k = 4

As h ! 0 (3.25) reduces to (4.3) where A_0 and A_1 are 16 16 matrices as de ned by (4.5) & (4.6) The rst characteristics polynomial (R) is given by $\frac{1}{-4}R^{20}(1 + R)^4 = 0$

This gives

$R_1 = R_2 = R_3 = R_5 = R_6 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = 0$; $R_{13} = R_{14} = R_{15} = R_{16} = 1$

Thus by De nition 4.1.3, the four-step TFBM is zero stable.

Case k = 6

As h ! 0, (3.25) reduces to (4.3) where A_0 and A_1 are 24 24 matrices as de ned by (4.7) & (4.8.) The rst characteristics polynomial (R) is given by

$$\frac{1}{4}R^{20}(1 + R)^4 = 0$$

This gives $R_1 = R_2 = R_3 = R_5 = R_6 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12} =$

$R_{13} = R_{14} = R_{15} = R_{16} = R_{17} = R_{18} = R_{19} = R_{20} = 0,$

$R_{21} = R_{22} = R_{23} = R_{24} = 1$

Thus, by de nition (4.1.3), the step six step TFBM is zero stable Case k = 8 As h ! 0, (3.25) reduces to (4.3) where A_0 and A_1 are 32 32 matrices as de ned by (4.9) & (4.10) The rst characteristics polynomial (R) is given by

$$\frac{1}{10}R^{28}(1 + R)^4 = 0$$

This gives

$R_1 = R_2 = R_3 = R_5 = R_6 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13} =$

$R_{14} = R_{15} = R_{16} = R_{17} = R_{18} = R_{19} = R_{20} = R_{21} = R_{22} = R_{23} = R_{24} =$

$R_{25} = R_{26} = R_{27} = R_{28} = 0,$

$R_{29} = R_{30} = 1$

Thus, by De nition 4.1.3, the eight-step TFBM is zero stable

4.3.4 Convergence of Methods

Following Theorem 4.1.1, the four, six and eight steps TFBMs are convergent since they satisfy the necessary and su cient condition of consistency and zero stability.

4.4 Numerical Examples

In this section, we shall test the performance of the developed methods on ve problems . All computations were carried out using Mathematica 8.0.

Example 4.1

Consider the BVP with variable coe cient problem:

$$y^{iv} \quad \frac{384t^4}{(2+t^2)^4} y = 24\frac{(2 \quad 11t^2)}{(2+t^2)^4} \qquad 0 \quad t \quad 1$$

$y(0) = \frac{1}{2}$; $y(1) = \frac{1}{3}$; $y^{00}(0) = \frac{1}{2}$; $y^{00}(1) = \frac{2}{27}$
whose exact solution is $y(t) = \frac{1}{2+t^2}$

Example 4.2

Consider the non-linear BVP

$y^{iv}(x) + (y^{00}(x))^2 = \sin x + \sin^2(x)$, $0 \quad x \quad 1$
$y(0) = 0$, $y^0(0) = 1$, $y(1) = \sin(1)$, $y^1(1) = \cos(1)$
whose exact solution is $y(x) = \sin x$

Example 4.3

Consider the BVP

$$y^{iv} \quad y = 4e^t \quad 0 \quad t \quad 1$$

with boundary conditions

$$y(0) = 0; \; y(1) = e; \quad y^{00}(0) = 2; \; y^{00}(1) = 3e$$

and whose exact solution is $y(t) = te^t$

Example 4.4

Consider the homogeneous linear IVP with variable coe cients

$y^{iv} = (16x^4 + 48x^2 + 12)y, \; , \; 0 \quad x \quad 1$

$y(0) = 1, \; y^0(0) = 0, \; y^{00}(0) = 2, \; y^{000}(0) = 0$ whose exact solution is $y(x) = e^{x_2}$

Example 4.5: Consider the system of ODE

$$\underline{Y}^{iv}(x) = \begin{pmatrix} 0 & e^{3x} & 0 & 0 \\ 16e^{x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 81e^{x} \\ 0 & 0 & 256e^{x} & 0 \end{pmatrix} \underline{Y}$$

where $\underline{Y} = (y; u; w; z)'$ $\quad Y(0) = (1; 1; 1; 1)'$

$\underline{Y}^0(0) = (1; 4; 3; 2)^T$; $\quad \underline{Y}^{00}(0) = (1; 16; 9; 4)^T$; $\quad \underline{Y}^{000}(0) = (1; 64$

$27; 8)^T$

whose exact solution is

$(x) = (e^{x}; e^{4x}; e^{3x}; e^{2x})^T$

Table 4.1: Error of BVM for Example 4.1

x	K = 4	K = 6	k = 8	Hollevoet, Van Daele & Berghe (2011)
0.0	0.0	0.0	0.0	0.0
0.1	7.22171E-9	2.41410E-11	8.31002E-14	9.165313E-8
0.2	1.37718E-8	4.51876E-11	1.52156E-13	8.026121E-8
0.3	1.90033E-8	6.03069E-11	1.92790E-13	5.944111E-8
0.4	2.220577E-8	6.77073E-11	2.02949E-13	3.350795E-8
0.5	2.27732E-8	6.63148E-11	1.88072E-13	1.087627E-8
0.6	2.09659E-8	5.82380E-11	1.54654E-13	3.350795E-8
0.7	1.71962E-8	4.60228E-11	1.15463E-13	5.944111E-8
0.8	1.20748E-8	3.14223E-11	7.49401E-14	8.026121E-8
0.9	6.21505E-9	1.59004E-11	3.55271E-14	9.169532E-8
1.0	0.0	0.0	0.0	2.664535E-15

Table 4.3: Error of BVM for Example 4.3

x	K = 4	K = 6	k = 8	Hollevoet, Van Daele & Berghe (2011)
0.0	0.0	0.0	0.0	0.0
0.1	1.13630E-11	5.581657E-14	1.26288E-15	1.00780242E-7
0.2	2.16664E-11	1.06914E-13	1.19349E-15	1.039538E-07
0.3	2.98944E-11	1.46827E-13	1.11022E-15	9.519132E-08
0.4	3.53514E-11	1.68754E-13	3.88578E-15	6.254245E-08
0.5	3.74507E-11	1.74305E-13	5.88418E-15	2.177964E-9
0.6	3.60059E-11	1.60538E-13	2.44249E-15	6.254244E-08
0.7	3.09652E-11	1.33893E-13	2.66454E-15	9.519131E-08
0.8	2.27727E-11	9.59233-14	6.66134E-15	1.039538E-07
0.9	1.20415E-11	4.84057E-14	7.54952E-15	1.007802E-07
1.0	0.0	0.0	0.0	2.680078E-13

Table 4.2: Error of BVM for Example 4.2

x	K = 4	K = 6	k = 8	Kasi and Raju (2012)
0.0	0.0	0.0	0.0	0.0
0.1	2.81025E-14	1.56819E-15	5.42622E-15	1.639128E-7
0.2	8.06299E-14	5.82867E-15	1.77358E-14	4.172325E-7
0.3	1.36446E-13	1.14908E-14	3.14193E-14	7.152557E-7
0.4	1.79412E-13	1.66533E-14	4.15779E-14	1.013279E-6
0.5	1.96343E-13	1.79301E-14	4.48530E-14	1.043081E-6
0.6	1.83409E-13	1.69864E-14	4.07452E-14	5.960446-7E-7
0.7	1.40443E-13	1.34337E-14	2.86438E-14	4.768372E-7
0.8	8.20455E-14	8.77076E-15	1.33227E-14	5.960464E-8
0.9	2.37588E-14	8.77076E-15	1.77636E-15	1.192083E-7
1.0	0.0	0.0	0.0	0.0

Table 4.4: Error of BVM for Example 4.4

x	K = 4	K = 6	k = 8	Kasi and Raju (2012)
0.0	0.0	0.0	0.0	0.0
0.1	6.09234E-11	1.00364E-13	4.88498E-15	1.013279-06
0.2	6.17813E-10	9.32809E-13	1.77636E-15	3.457906-06
0.3	2.21543E-9	3.32632E-12	1.43219E-14	6.496906-06
0.4	5.84745E-9	8.67806E-12	6.32827E-14	9.477139-06
0.5	1.25487E-8	1.84203E-11	1.56208E-13	1.132488E-05
0.6	2.31816E-8	3.35766E-11	3.11418E-13	1.102686E-05
0.7	3.85021E-8	5.49463E-11	5.39790E-13	9.298325E-06
0.8	5.86794E-8	8.24246E-11	8.64198E-13	5.841255E-06
0.9	8.36989E-8	1.15612E-10	1.29929E-12	2.205372E-06
1.0	1.12988E-7	1.53468E-10	1.85452E-12	0.0

Table 4.5: Error of BVM for Example 4.5

x	k = 4				k = 6				k = 8			
	Error in y	Error in z	Error in w	Error in u	Error in y	Error in z	Error in w	Error in u	Error in y	Error in z	Error in w	Error in u
0.0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	8.88178E-16	1.14186E-12	4.16891E-11	5.24978E-10	1.11022E-16	3.33067E-15	2.42917E-13	5.26135E-12	3.33067E-16	7.77156E-16	6.42819E-14	2.24176E-12
0.2	1.63203E-14	1.02304E-11	3.72175E-10	4.68150E-9	4.99660E-15	2.80886E-14	2.26807E-12	4.91203E-11	2.77556E-15	3.10862E-15	6.60805E-14	2.33105E-12
0.3	2.62346E-13	2.42306E-11	1.23174E-9	1.53569E-8	1.77636E-14	9.7366E-14	7.77728E-12	1.68084E-12	1.05471E-14	1.14353E-14	2.40702E-12	8.50527E-11
0.4	2.16338E-12	7.91494E-11	2.81470E-9	3.47644E-8	5.19584E-14	2.33535E-13	5.79698E-11	3.85704E-10	1.65423E-14	2.60902E-14	5.92054E-12	2.09328E-10
0.5	1.17243E-11	1.50218E-10	5.28098E-9	6.46704E-8	1.72307E-13	4.52138E-13	3.39032E-11	7.22297E-10	9.54792E-15	5.41234E-14	1.17039E-11	4.13986E-10
0.6	4.81002E-11	2.51959E-10	8.76318E-9	1.06557E-7	5.93858E-13	7.65554E-13	5.63667E-11	1.19265E-9	1.79190E-13	9.45910E-14	2.00203E-11	7.07509E-10
0.7	1.62434E-10	2.88336E-10	1.33749 E-8	1.61779E-7	1.88927E-12	1.18588E-12	8.60740E-11	1.81056E-9	8.12239E-13	1.48520E-13	3.11205E-11	1.09769E-9
0.8	4.75679E-10	5.62934E-10	1.921701E-8	2.31275E-7	5.41017E-12	1.71932E-12	1.23591E-10	2.58880E-9	2.69246E-12	2.16382E-13	4.52557E-11	1.59260E-9
0.9	1.25015E-9	7.79142E-10	2.63815E-8	3.17660E-7	1.40887E-11	2.37529E-12	1.69446E-10	3.53734E-9	7.57150E-12	3.01786E-13	6.266442E-11	2.19990E-9
1.0	3.01917E-9	1.04041E-9	3.49540E-8	4.21304E-7	3.38822E-11	3.16458E-12	2.24095E-10	4.67121E-9	1.09594E-11	4.08340E-13	8.34321E-11	2.92584E-9

Table 4.6: Error of TFBM for Example 4.1

x	K = 4	K = 6	k = 8	Kasi and Raju (2012)
0.0	0.0	0.0	0.0	0.0
0.1	1.14947E-8	2.57068E-11	7.87613E-13	3.42727E-7
0.2	2.24947E-8	4.99355E-11	5.74332E-13	1.04308E-7
0.3	3.24235E-8	7.10099E-11	2.52687E-13	1.13249E-6
0.4	3.98708E-8	8.57276E-11	1.30729E-13	2.59280E-6
0.5	4.34208E-8	9.11263E-11	2.60736E-13	4.02331E-6
0.6	4.24655E-8	8.67368E-11	3.66929E-13	4.85778E-6
0.7	3.66772E-8	7.29140E-11	4.01124E-13	5.30481E-6
0.8	2.68977E-8	5.21244E-11	3.98154E-13	4.58956E-6
0.9	1.41955E-9	2.70976E-11	3.78114E-13	2.96533E-6
1.0	0.0	0.0	0.0	0.0

Table 4.7: Error of TFBM for Example 4.2

x	K = 4	K = 6	k = 8	Kasi and Raju (2012)
0.0	0.0	0.0	0.0	0.0
0.1	2.86890E-12	4.16334E-15	9.86988E-14	9.958700E-8
0.2	8.2474E-12	1.33782E-14	1.41887E-13	9.293676E-7
0.3	1.39973E-11	2.57017E-14	1.28453E-13	3.31878E-7
0.4	1.85144E-11	3.63598E-14	7.38298E-13	8.66395E-7
0.5	2.03930E-11	4.50751E-14	1.55431E-15	1.84000E-8
0.6	1.92141E-11	4.52971E-14	8.20455E-14	3.9965E-7
0.7	1.48875E-11	4.13003E-14	1.54210E-13	5.4898E-8
0.8	8.74334E-12	2.53131E-14	1.65423E-13	8.2361E-8
0.9	2.56819E-12	1.12133E-14	6.71685E-14	1.1553E-8
1.0	0.0	0.0	0.0	0.0

Table 4.8: Error of TFBM for Example 4.3

x	K = 4	K = 6	k = 8	Kasi and Raju (2012)
0.0	0.0	0.0	0.0	0.0
0.1	2.44523E-10	5.79578E-13	7.99361E-15	1.013279E-6
0.2	4.66246E-10	1.09809E-12	8.88178E-15	3.4579068E-06
0.3	6.43380E-10	1.54726E-12	2.19824E-14	6.496906E-06
0.4	7.60937E-10	1.80977E-12	6.03961E-14	9.477139E-06
0.5	8.06291E-10	1.92890E-12	1.42109E-16	1.132488E-05
0.6	7.75342E-10	1.80767E-12	6.97220E-14	1.102686E-05
0.7	6.66944E-10	1.54698E-12	5.90639E-14	9.298325E-06
0.8	4.90602E-10	1.07403-12	1.00364E-13	5.841255E-06
0.9	2.59481E-10	5.71099E-13	4.44089E-14	2.205372E-06
1.0	0.0	0.0	0.0	0.0

Table 4.9: Error of TFBM for Example 4.4

x	K = 4	K = 6	k = 8	Kasi and Raju (2012)
0.0	0.0	0.0	0.0	0.0
0.1	2.84730E-10	3.15525E-13	4.25771E-14	1.639128E-07
0.2	2.88535E-9	3.19378E-12	4.90719E-14	4.172325E-07
0.3	1.03926E-8	1.14816E-11	3.04534E-13	7.152557E-07
0.4	2.77841E-8	3.06732E-11	7.34079E-13	1.013279E-06
0.5	6.03743E-8	6.63141E-11	1.18594E-12	1.043081E-06
0.6	1.13383E-7	1.23386E-10	1.89793E-12	5.960464E-07
0.7	1.91703E-7	2.06127E-10	2.72959E-12	4.768372E-07
0.8	2.97990E-7	3.15818E-10	3.76499E-12	5.960464E-08
0.9	4.34049E-7	4.52633E-10	4.94138E-12	1.192093E-07
1.0	5.98691E-7	6.13586E-10	6.13787E-12	0.0

Table 4.10: Error of Method for Example 4.5

x	k = 4				k = 6				k = 8			
	Error in y	Error in z	Error in w	Error in u	Error in y	Error in z	Error in w	Error in u	Error in y	Error in z	Error in w	Error in u
0.0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	1.52729E-9	2.02936E-8	9.27378E-8	3.25624E-7	1.77636E-15	2.56462E-14	2.619174E-12	3.75683E-11	7.77156E-15	4.38052E-13	6.79456E-13	8.94007E-12
0.2	2.73314E-9	4.01180E-8	2.98226E-7	2.07896E-6	1.55431E-15	5.31242E-13	2.65152E-11	3.54675E-10	2.55351E-14	5.43454E-13	3.24185E-13	8.07132E-12
0.3	3.45808E-9	6.13247E-8	7.80525E-7	7.32854E-6	4.21885E-15	1.85907E-12	9.11219E-11	1.21438E-9	3.66374E-14	4.79339E-13	3.20188E-12	6.03650E-11
0.4	2.28354E-8	2.68536E-7	2.28378E-7	1.39424E-5	1.38001E-13	4.53981E-12	2.11489E-10	2.78952E-9	6.32827E-14	1.07414E-13	7.92699E-12	3.89266E-11
0.5	3.12315E-8	3.94132E-7	1.06132E-6	2.96670E-5	8.55760E-13	8.75899E-12	3.99629E-10	5.22664E-9	7.21645E-14	3.99680E-14	8.45768E-12	7.91478E-11
0.6	2.57044E-8	4.71125E-7	2.70567E-6	5.35001E-5	3.68805E-12	1.48276E-11	6.65013E-10	8.63518E-9	1.77636E-14	4.75731E-13	1.58762E-11	4.06719E-11
0.7	1.772187E-8	4.88688E-7	5.27599 E-6	8.65870E-5	1.27321E-11	2.28265E-11	1.01552E-9	1.31182E-8	2.99760E-14	1.01363E-12	5.23581E-12	1.00622E-10
0.8	1.405540E-7	5.95214E-7	8.44039E-6	1.28976E-4	3.77613E-11	3.29204E-11	1.45769E-9	1.87770E-8	7.66054E-14	1.47049E-12	1.33649E-11	7.26752E-11
0.9	5.07064E-7	5.31220E-7	1.29886E-5	1.82904E-4	9.99925E-11	4.51341E-11	1.99725E-9	2.56919E-8	1.43219E-13	2.24376E-12	1.86495E-11	1.04312E-10
1.0	1.39684E-6	3.56454E-7	1.88075E-5	2.48967E-4	2.42660E-10	5.95753E-11	2.63929E-9	3.39938E-8	3.44169E-13	4.17777E-12	2.64885E-11	2.06510E-10

4.5 Graph of the Results

Figure 4.1: Graph of results for example 4.1

Figure 4.2: Graph of results for example 4.2

Figure 4.3: Graph of results for example 4.3

Figure 4.4: Graph of results for example 4.4

Chapter 5

General Conclusion

5.1 Introduction

This chapter deals with the general conclusion of the research work. The results of the examples given in the last chapter are discussed. Recommen-dations on the proposed class of methods are given. Suggestions for future research was also given

5.2 Discussion of Results

The ve examples presented in the previous chapter were solved using computer programs Mathematica 8.0. Tables 4.1 to 4.10 show the absolute errors for the given examples. All problems were solved directly without reducing them to a system of lower order ODEs. In all examples considered, the re-sults were accuarate up to 6 decimal places. This shows that both BVM and TFBM perform better than the traditional LMMS thereby making them

better numerical integrators. Also, the graphs of results presented in section 4.5 corroborated the results obtained in the tables 4.1 to 4.10.

5.3 Conclusion

A family of BVMs and TFBMs for the numerical solution of fourth order ordinary di erential equations of both initial and boundary types have been presented for the purpose of deriving and developing new integration schemes, the approaches of interpolation and collocation were adopted. These involved the evaluation of the trial solution and the di erential equations at some appropriate points. It is neccessary to state that both power series and trigonometric basis were adopted for the trial solution. The proposed scheme are advantageous in the following ways; they are self starting; they have good accuracy; no predictors are required in their implementation; and they also have the ability to solve both initial value problems and Boundary value problems. The methods were analysed for those neccessary requirements of good numerical integration schemes. They were found to be consistent, zero stable and consequently convergent.

Furthermore, the ability of the new proposed BVMs and TFBMs to handle both initial and boundary value problems in ordinary di erential equations makes them desirable in solving this class of problem than the convectional linear multistep method.

They have the advantage of being self starting, having good accuracy, requiring no predictors and has the ability to simultaneously general solutions at subintervals that do not overlap. The analysis of the methods were also

considered. All the methods are consistent and zero stable, hence convergent. The fact that BVMs and TFBMs can be used to solve both IVPs and BVPs make them more appropriate for solving ODEs than the traditional LMMs and the predictor-corrector methods.

5.4 Contribution to Knowledge

This research has contributed to knowledge in the following ways;

The development of a new class of boundary value methods with poly-nomial basis using various values of v and k for the solution of fourth order ODEs

The development a new class of block methods with trigonometric basis using various values of v and k for the solution of fourth order oscillatory ODEs.

The derived method solved both IVPs and BVPs in ODEs.

The new methods are self starting and handles fourth order ODEs directly without reducing to a system of lower order ODEs

5.5 Recommendations

The class of methods developed in this research is recommended for the solution of fourth order ODEs of both IVPs and BVPs types.

However, further research can be carried out in this area of study. This includes The development of methods with modi ed polynomial and trigono-

metric basis for the direct solution of nth order ODEs.

The use of other basis functions such as Chebychev basis functions, Legen-drres basis functions, Laguere basis functions, etc.can also be adopted. The development of methods for solution of fractional ODEs ; and the development of methods for solution of fourth and higher order partial dif-ferential equations.

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