



ON THE DYNAMIC RESPONSE OF BEAMS SUBJECTED TO NON-UNIFORM DISTRIBUTED MOVING MASSES

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Abstract

The dynamic analysis of a thin undamped simply supported beam traversed by a non-uniform distributed moving mass is presented. The governing equation of motion for this system is simplified using variables separation method in terms of modal shape and unknown function of time. A numerical iterative technique is then employed to determine the effect of linear distribution of the moving mass on the dynamic behaviour of the beam. Numerical results are presented in graphical forms to illustrate the dynamic behaviour due to this type of moving mass. It is found that the gradient's distribution of the moving mass significantly affects the dynamic behaviour of the beam and that a uniformly distributed moving mass problem produces a lower bound deflection for linearly distributed moving mass problem.

1. Introduction

The problem of determining the response of elastic structures such as beams or plates subjected to moving mass has been of practical importance in designing highway/railway bridges, machine parts and station facilities. A careful analysis of these moving mass problems (as

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they are usually referred to) became necessary when it was observed that an elastic structure subjected to a moving mass produces greater deflection and stress than the one under a static equivalent mass. Consequently, moving mass problems have been receiving attentions from various researchers [1-13, 15, 17].

A version of the the moving mass problems involves modeling the mass as a concentrated mass (i.e., the mass is assumed to act at a point as it moves). This type of moving mass does not take into consideration the mass distribution interval and hence the associated moving mass problems are thereby simplified. Moreover, the concentrated moving mass is the most common type and considerable amount of work involving this type of moving mass has been carried out. Recently, Michaltsos and Kounadis [13] accounted for the effect of the centripetal and Coriolis forces associated with the moving concentrated mass on the dynamic response of light bridges. In particular, they found that the effect of those forces on the non-dimensional dynamic displacement increases as the non-dimensional mass ($M > 0.2$) increases. Dugush and Eisenberger [1] studied the problem of determining the dynamic behaviour of multi-span non-prismatic beams under a concentrated moving mass. The corresponding exact mode shape and natural frequencies are obtained. Michaltsos [12] examined the effect of concentrated mass moving at variable speeds on the dynamic behaviour of a beam. The analysis focuses on the effect of the acceleration or deceleration on the deflection of the beam. More investigations that are recent include the work of Mahmoud and Zaid [11] who developed an iterative modal analysis technique in determining the influence of cracks on the dynamic behaviour of a beam subjected to a concentrated mass moving at a constant velocity. Although these previous studies involving moving point (concentrated) masses are impressive, the associated moving mass problems are only theoretically (mathematically) well formulated but not realistic. This is due to the fact that point masses do not exist physically.

Hence, it becomes practically imperative to consider moving masses which take into account the mass distribution interval or contact area. Such moving masses are known as *moving distributed masses*. The

distribution may be uniform or non-uniform (regularly varying or irregularly varying). Although to the authors' knowledge, there is scarcity of publications on distributed moving mass problems, one must mention the work of Esmailzadeh and Ghorashi [2] who carried out an investigation into the dynamic response of a thin beam under uniform partially distributed moving masses. They evaluated the maximum deflection of the mid-span of the thin beam and obtained critical speeds of the moving mass.

Also, by taking into consideration the effect of rotary inertia and shear deformation, Esmailzadeh and Ghorashi [3] studied the dynamic behaviour of a Timoshenko beam traversed by uniform partially distributed moving masses. Recently, Gbadeyan and Dada [5] considered the moving mass problem involving a rectangular elastic plate resting on a non-Winkler foundation and traversed by uniform partially distributed moving masses. It is found that an increase in the area of the distribution of the moving mass results in a reduction in the maximum dynamic deflection of the plate. More recently, Gbadeyan and Dada [6] presented the analysis of the dynamic behaviour of a Mindlin rectangular plate traversed by a uniformly distributed moving mass. However, in these investigations, only moving masses that are uniformly distributed were considered. For practical application, it is useful to consider moving mass problems involving regularly varying distributed moving mass problems as opposed to uniformly distributed moving masses. Clearly, the uniformly distributed mass formulation can be viewed as a special case of the regularly varying distributed mass formulation. Moreover, the case of the irregularly varying distributed mass can be treated by considering the corresponding mass diagram as being divided into narrow strips of certain length and as such treated as regularly varying distributed moving masses.

To this end, most recently, Gbadeyan and Dada [7] studied the dynamic response of a Bernoulli beam to linearly varying distributed moving force. In this investigation, the effect of centripetal and Coriolis forces which play a very important role in the study of vibration of bridges caused by moving loads (masses or forces) was neglected. As a matter of fact, the problem of assessing the response of elastic structures to moving forces is a special case of the moving mass problems. Moreover,

it has been established [8] that it may not be safe to rely on the response of the moving forces as an approximation for the response of structures to moving masses.

In this context, the present paper examines the influence of linearly varying distributed mass moving with a constant velocity on the dynamic behaviour of a thin elastic beam. The effect of the centripetal and Coriolis forces is taken into consideration. An iterative modal analysis technique is developed to solve the problem.

2. Analytical Formulation

Consider a finite elastic thin beam as shown in Figure 1 of length L , mass per unit length μ , flexural rigidity EI and having a uniform homogeneous and isotropic material.

The equation of motion describing the lateral vibration of the beam carrying the time varying force $f(x, t)$ is

$$EI \frac{\partial^4 y}{\partial x^4} + \mu \frac{\partial^2 y}{\partial t^2} = f(x, t), \quad (1)$$

where x is the length coordinate with the origin at left hand end of the beam, t is the time coordinate with the origin at the instant of the force arriving on the beam and y is the deflection of the beam measured downward from its equilibrium position when the beam is loaded with its own weight.

The beam is subjected to a force $f(x, t)$ that represents both the weight and the inertia force due to a linearly distributed mass M moving with a constant velocity v .

Hence, [14]

$$f(x, t) = \left\{ g - \frac{d^2 y}{dt^2} \right\} \left\{ M_1 \langle x - a_1 \rangle^0 + \frac{M_2 - M_1}{a_2 - a_1} \langle x - a_1 \rangle - M_2 \langle x - a_2 \rangle^0 - \frac{M_2 - M_1}{a_2 - a_1} \langle x - a_2 \rangle \right\}, \quad (2)$$

where $M_2 g$ and $M_1 g$ are forces produced by masses M_2 and M_1 acting

respectively at the left and right end points of the moving mass as depicted in Figure 1, $d = a_2 - a_1$ is the length of the distributed mass,

$a_2 = vt + \frac{d}{2}$, $a_1 = vt - \frac{d}{2}$, g is the acceleration due to gravity and the

Macauly notation is defined as

$$\langle x - a \rangle^r = \begin{cases} 0, & x < a, \\ (x - a)^r, & x \geq a, \end{cases} \quad (3)$$

where $r = 1, 2, 3, \dots$. For this problem, the acceleration of the moving mass can be written as [8]

$$\frac{d^2 y}{dt^2} = v^2 \frac{\partial^2 y}{\partial x^2} + 2v \frac{\partial^2 y}{\partial x \partial t} + \frac{\partial^2 y}{\partial t^2}. \quad (4)$$

Using equations (2) and (4), equation (1) becomes

$$EI \frac{\partial^4 y}{\partial x^4} + \mu \frac{\partial^2 y}{\partial t^2} = \left\{ g - \left(v^2 \frac{\partial^2 y}{\partial x^2} + 2v \frac{\partial^2 y}{\partial x \partial t} + \frac{\partial^2 y}{\partial t^2} \right) \right\} \left\{ M_1 \langle x - a_1 \rangle^0 + \frac{M_2 - M_1}{a_2 - a_1} \langle x - a_1 \rangle - M_2 \langle x - a_2 \rangle^0 - \frac{M_2 - M_1}{a_2 - a_1} \langle x - a_2 \rangle \right\}. \quad (5)$$

Since many highways and railway bridges consist of simply supported girders [16] useful insight into the dynamic response of bridges may be obtained by studying the response of simply supported beams under the moving masses. Hence, simply supported beams are considered. Furthermore, it is assumed that initially at time $t = 0$, the moving mass is located at the left hand support having zero deflection as well as zero velocity.

3. Method of Solution

Since the time and space functions may be separable for a modal motion, we seek for the overall response of the beam as a series solution

in terms of the normal modes in the form:

$$y(x, t) = \sum_{n=1}^{\infty} X_n(x) P_n(t), \quad (6)$$

where $X_n(x)$ is the modal shape for the n th mode of the freely vibrating beam with the corresponding generalized unknown function of time $P_n(t)$ that is to be determined.

Introducing equation (6) into equation (5), we have

$$\begin{aligned} & EI \sum_{n=1}^{\infty} X_n^{(iv)}(x) P_n(t) + m \sum_{n=1}^{\infty} X_n(x) \ddot{P}_n(t) \\ &= \left\{ g - \sum_{n=1}^{\infty} (v^2 X_n^{(II)}(x) P_n(t) + 2v X_n^{(I)}(x) \dot{P}_n(t) + X_n(x) \ddot{P}_n(t)) \right\} \\ & \cdot \left\{ M_1 \langle x - \alpha_1 \rangle^0 + \frac{M_2 - M_1}{\alpha_2 - \alpha_1} \langle x - \alpha_1 \rangle \right. \\ & \quad \left. - M_2 \langle x - \alpha_2 \rangle^0 - \frac{M_2 - M_1}{\alpha_2 - \alpha_1} \langle x - \alpha_2 \rangle \right\}, \end{aligned} \quad (7)$$

where $(\cdot)$ and $(/)$ represent differentiation with respect to time t and coordinate x , respectively. The n -th normal mode of vibration of a uniform beam satisfies

$$X_n(x) = A_n \sin \lambda_n x + B_n \cos \lambda_n x + C_n \sinh \lambda_n x + D_n \cosh \lambda_n x, \quad (8)$$

where the unknown constants A_n , B_n , C_n , D_n , and λ_n are determined by applying the end conditions of the beam. Also, free vibration of beam satisfies the equation

$$EIX_n^{(iv)}(x) = \mu \omega_n^2 X_n(x), \quad (9)$$

where the natural frequencies

$$\omega_n = \lambda_n^2 \sqrt{\frac{EI}{\mu}}, \quad n = 1, 2, 3, \dots \quad (10)$$

Introducing equation (9) into equation (7), multiplying the resultant equation by $X_s(x)$ and integrating both sides with respect to x from 0 to L , one obtains

$$\begin{aligned}
 & \ddot{P}_n(t) + \omega_n^2 P_n(t) \\
 &= \frac{1}{\kappa \mu} \int_{a_1}^L \left\{ g X_n(x) - X_n(x) \sum_{s=1}^{\infty} (v^2 X_s^{(ii)}(x) P_s(t) + 2v X_s^{(i)} \dot{P}_s(t) + X_s(x) \ddot{P}_s(t)) \right\} \\
 & \quad \left\{ M_1 + \frac{M_2 - M_1}{a_2 - a_1} (x - a_1) \right\} dx \\
 & - \frac{1}{\kappa \mu} \int_{a_2}^L \left\{ g X_n(x) - X_n(x) \sum_{s=1}^{\infty} (v^2 X_s^{(ii)}(x) P_s(t) + 2v X_s^{(i)} \dot{P}_s(t) + X_s(x) \ddot{P}_s(t)) \right\} \\
 & \quad \left\{ M_2 + \frac{M_2 - M_1}{a_2 - a_1} (x - a_2) \right\} dx, \tag{11}
 \end{aligned}$$

where

$$\int_0^L X_n(x) X_s(x) dx = \begin{cases} 0, & n \neq s, \\ \kappa, & n = s, \end{cases}$$

and κ is a constant.

Equation (11) is the desired set of generalized coupled ordinary differential equations that is applicable to any particular set of boundary conditions of the beam. As earlier mentioned in Section 2 the following conditions:

$$y(x, t)|_{x=0} = EI \frac{\partial^2 y(x, t)}{\partial x^2} \Big|_{x=0} = 0, \tag{12a}$$

$$y(x, t)|_{x=L} = EI \frac{\partial^2 y(x, t)}{\partial x^2} \Big|_{x=L} = 0, \tag{12b}$$

and initial conditions

$$y(x, t)|_{t=0} = EI \frac{\partial y(x, t)}{\partial t} \Big|_{t=0} = 0,$$

are considered.

Applying these conditions leads to

$$X_n(x) = \sin \frac{n\pi x}{L}, \quad \kappa = \frac{L}{2} \quad \text{and} \quad \omega_n^2 = \frac{(n\pi)^4 EI}{L^4 \mu}. \quad (13)$$

Substituting the set of equations (13) into equation (11), one obtains the following equation in generalized functions of time for simply supported conditions. Thus,

$$\begin{aligned} & \ddot{P}_n(t) + \omega_n^2 P_n(t) \\ &= c_3 c_1 \left\{ \left(M_1 \cos \frac{n\pi a_1}{L} - M_2 \cos \frac{n\pi a_2}{L} \right) + c_1 Gr \left(\sin \frac{n\pi a_2}{L} - \sin \frac{n\pi a_1}{L} \right) \right\} \\ &+ \frac{c_3}{2g} \sum_{s=1}^{\infty} \left[\{ c_4^2 P_s(t) - \ddot{P}_s(t) \} \left[M_1 \left(A_{ns} + c_2 \sin \frac{(n+s)\pi a_1}{L} \right) \right. \right. \\ &+ Gr \left(B_{ns} + c_2^2 \cos \frac{(n+s)\pi a_1}{L} \right) \\ &- M_2 \left(E_{ns} + c_2 \sin \frac{(n+s)\pi a_2}{L} \right) - Gr \left(F_{ns} + c_2^2 \cos \frac{(n+s)\pi a_2}{L} \right) \Big] \\ &- c_4 \dot{P}_s(t) \left[M_1 \left(c_{ns} + c_2 \cos \frac{(s+n)\pi a_1}{L} - c_2(-1)^{s+n} \right) \right. \\ &+ Gr \left(D_{ns} - c_2^2 \sin \frac{(s+n)\pi a_1}{L} \right) \\ &- M_2 \left(G_{ns} + c_2 \cos \frac{(s+n)\pi a_2}{L} - c_2(-1)^{s+n} \right) \\ &\left. \left. + Gr \left(H_{ns} - c_2^2 \sin \frac{(s+n)\pi a_2}{L} \right) - Gr c_2 d(-1)^{s+n} \right] \right], \quad (14) \end{aligned}$$

where

$$c_1 = \frac{L}{\pi(n)}, \quad c_2 = \frac{L}{\pi(n+s)}, \quad c_3 = \frac{2g}{\mu L}, \quad c_4 = \frac{s\pi v}{L}, \quad c_5 = \frac{L}{\pi(s-n)},$$

$$A_{ns} \begin{cases} -c_5 \sin \frac{(s-n)\pi a_1}{L}, & n \neq s, \\ L - a_1, & n = s, \end{cases}$$

$$E_{ns} = \begin{cases} -c_5 \sin \frac{(s-n)\pi a_2}{L}, & n \neq s, \\ L - a_2, & n = s, \end{cases}$$

$$B_{ns} = \begin{cases} c_5^2 \left\{ (-1)^{s-n} - \cos \frac{(s-n)\pi a_1}{L} \right\}, & n \neq s, \\ \frac{(L - a_1)^2}{2}, & n = s, \end{cases}$$

$$F_{ns} = \begin{cases} c_5 \left\{ (-1)^{s-n} - \cos \frac{(s-n)\pi a_2}{L} \right\}, & n \neq s, \\ \frac{(L - a_2)^2}{2}, & n = s, \end{cases}$$

$$C_{ns} = \begin{cases} c_5 \left\{ (-1)^{s-n} - \cos \frac{(s-n)\pi a_1}{L} \right\}, & n \neq s, \\ 0, & n = s, \end{cases}$$

$$D_{ns} = \begin{cases} c_5 \left\{ (-1)^{s-n} (a_1 - L) - c_5 \sin \frac{(s-n)\pi a_1}{L} \right\}, & n \neq s, \\ 0, & n = s, \end{cases}$$

$$G_{ns} = \begin{cases} c_5 \left\{ (-1)^{s-n} - \cos \frac{(s-n)\pi a_2}{L} \right\}, & n \neq s, \\ 0, & n = s, \end{cases}$$

$$H_{ns} = \begin{cases} c_5 \left\{ (a_2 - L) (-1)^{s-n} - c_5 \sin \frac{(n-s)\pi a_2}{L} \right\}, & n \neq s, \\ 0, & n = s. \end{cases}$$

It is pertinent to state here that a closed-form solution of equation (14) is not visible. Therefore, an iterative method is employed. A somehow similar iterative technique was used by Michaltsos and Kounadis [13] and Mahmoud and Zaid [11]. This technique is an excellent tool for the analysis of the moving mass problem [11].

For this purpose, equation (14) may now be written as

$$\begin{aligned} & \ddot{P}_n^k(t) + \omega_n^2 P_n^k(t) \\ &= c_3 c_1 \left\{ \left(M_1 \cos \frac{n\pi a_1}{L} - M_2 \cos \frac{n\pi a_2}{L} \right) + c_1 G_r \left(\sin \frac{n\pi a_2}{L} - \sin \frac{n\pi a_1}{L} \right) \right\} \end{aligned}$$

$$\begin{aligned}
& + \frac{c_3}{2g} \sum_{s=1}^{\infty} \left[\{c_4^2 P_s^{k-1}(t) - \ddot{P}_s^{k-1}(t)\} \left[M_1 \left(A_{ns} + c_2 \sin \frac{(n+s)\pi a_1}{L} \right) \right. \right. \\
& + Gr \left(B_{ns} + c_2^2 \cos \frac{(n+s)\pi a_1}{L} \right) - M_2 \left(E_{ns} + c_2 \sin \frac{(n+s)\pi a_2}{L} \right) \\
& - Gr \left(F_{ns} + c_2^2 \cos \frac{(n+s)\pi a_2}{L} \right) \Big] \\
& - c_4 \dot{P}_s^{k-1}(t) \left[M_1 \left(c_{ns} + c_2 \cos \frac{(s+n)\pi a_1}{L} - c_2(-1)^{s+n} \right) \right. \\
& + Gr \left(D_{ns} - c_2^2 \sin \frac{(s+n)\pi a_1}{L} \right) \\
& - M_2 \left(G_{ns} + c_2 \cos \frac{(s+n)\pi a_2}{L} - c_2(-1)^{s+n} \right) \\
& \left. \left. + Gr \left(H_{ns} - c_2^2 \sin \frac{(s+n)\pi a_2}{L} \right) - Gr c_2 d(-1)^{s+n} \right] \right]. \quad (15)
\end{aligned}$$

Let $k = 1$ represent the first iteration. $P_n^0(t)$ is set to be equal to zero in the right-hand side of equation (15) and the equation is solved to obtain $P_n^1(t)$ which is the solution for the moving force problem. The $P_n^1(t)$ and its first and second derivatives are substituted accordingly into the right-hand side of equation (15) and the resulting equation is then solved to obtain a better approximation $P_n^2(t)$. This process is repeated until the solution converges.

4. Results and Discussion

The numerical results obtained from this present problem formulation are presented in this section. For the numerical evaluation of this problem, a beam of span $L = 30\text{m}$ with flexural rigidity $EI = 2.7 \times 10^8 \text{ N m}^2$ traversed by a moving mass $M = d(M_2 + M_1)/2$ that is linearly distributed over a given length with distribution gradients $gr = 0, \pm 100, \pm 200, \pm 300, \dots, \pm 1000$ is considered. In addition, the following data are used constant velocities $v = 5\text{ms}^{-1}, 10\text{ms}^{-1}, 15\text{ms}^{-1}, 20\text{ms}^{-1}$,

25ms^{-1} , 30ms^{-1} , 35ms^{-1} , 40ms^{-1} , mass ratios $\left(M_r = \frac{M}{\mu L}\right)$ 0.3, 0.4, 0.9, 1.0, 1.2 and 1.5 and the ratios of the length of the moving mass to that of the beam $\left(db = \frac{d}{L}\right)$ 0.005, 0.05, 0.01, 0.05 or 0.06.

Figure 2 shows a plot of mid-span deflection versus time history of the moving mass on the beam for various velocities with fixed values $db = 0.05$, $M_r = 1.2$ and $gr = 200$. It shows that an increase in velocity of the moving mass produces increase in the maximum deflection.

In Figures 3 and 4, the mid-span deflection profile is presented for some various distribution gradients with the fixed values $M_r = 0.9$, $db = 0.05$ and $v = 30\text{ms}^{-1}$. It is observed from Figure 3 that the maximum mid-span deflection decreases with a decrease in positive distribution gradient while Figure 4 shows that the mid-span deflection increases as the negative distribution gradient decreases.

The maximum deflection profile for variation of velocity and mass ratio against distribution gradient with fixed value $db = 0.05$ are presented in Figures 5 and 6, respectively. It is clearly seen that the maximum deflection varies with different velocities and mass ratios. In particular, the maximum deflection increases with an increase in velocity as evident in Figure 5. The effect of mass ratio is significantly depicted in Figure 6. It is seen that the more the mass ratio, the more the maximum deflection. It is of practical interest to state here that the least maximum deflection occurs when the distribution gradient is zero in Figures 5 and 6. It implies that a uniformly distributed moving mass problem produces a lower bound deflection for linearly distributed moving mass problem. It ought to be one of the important points to note in designing moving mass.

Figure 7 depicts the time variation of deflection for some lengths of load-mass distribution with constant values $v = 20\text{ms}^{-1}$, $M_r = 1.2$ and $gr = 200$. The effect of the length of load-mass distribution is noticed before and after the mid-time of the time history of load-mass on the beam. Figure 8 shows the deflection profile at time $t = 0.6\text{s}$ for both moving force and mass cases. It can be seen that the moving mass model produces higher deflection.

5. Conclusion

The dynamic response of a finite beam traversed by a linearly distributed type of a moving mass has been investigated. The partial differential equation governing the motion of the above model of a moving mass problem is simplified by series form of separation of variables in terms of modal shapes and unknown functions of time. The simplified equation is solved using iterative method for simply supported boundary conditions. These results are carefully expressed in graphical forms. The gradient's distribution of the moving mass on a beam significantly affects the dynamic behaviour of the beam. Specifically, it is found among other results that the maximum mid-span deflection increases with an increase in absolute value of a distribution gradient and the least maximum deflection occurs when the distribution gradient is zero.

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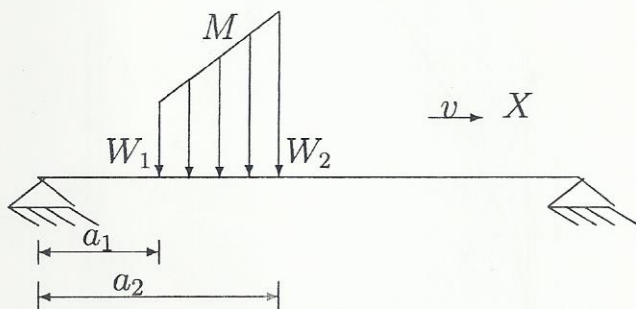


Figure 1. A beam of span L under a linearly distributed moving mass M .

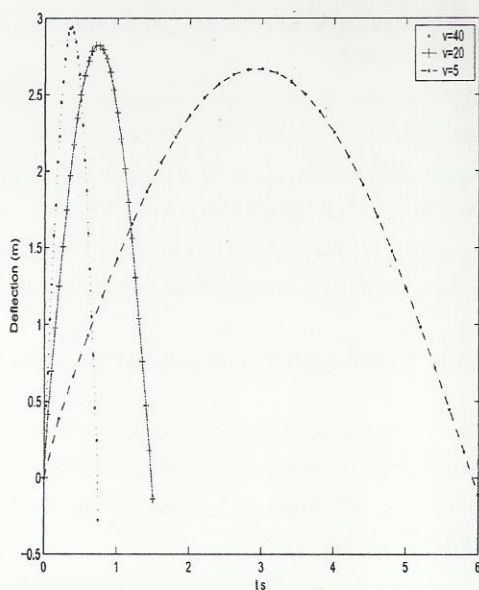


Figure 2. The mid-span deflection $1000y\left(\frac{L}{2}, t\right)$ of variation of velocity in ms^{-1} with $db = 0.05$, $Mb = 1.2$ and $gr = 200$.

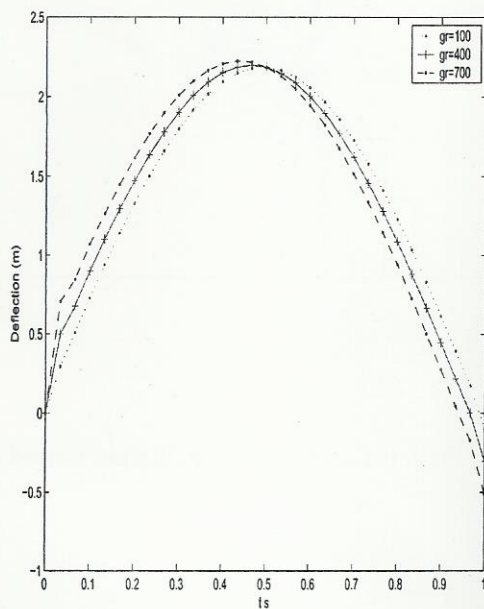


Figure 3. The mid-span deflection profile $1000y\left(\frac{L}{2}, t\right)$ for various positive distribution gradients (gr) with constant values $Mr = 0.9$, $db = 0.05$ and 30ms^{-1} .

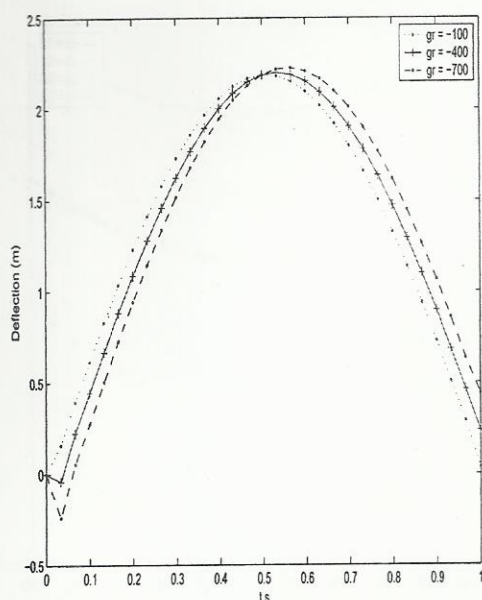


Figure 4. The mid-span deflection profile $1000y\left(\frac{L}{2}, t\right)$ for various negative distribution gradients (gr) with constant values $Mr=0.9$, $db=0.05$ and $v=30\text{ms}^{-1}$.

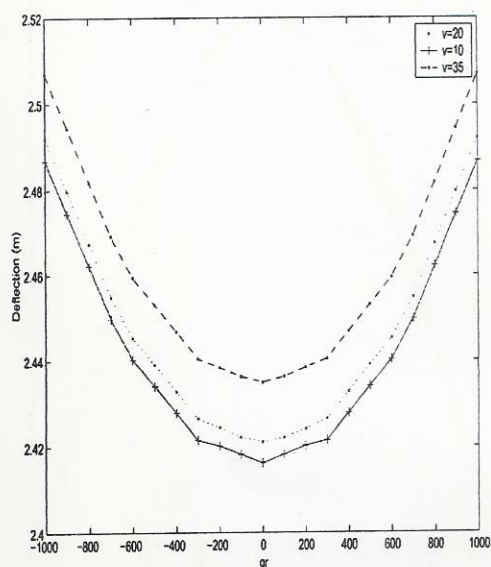


Figure 5. Maximum deflection profile $1000y\left(\frac{L}{2}, t\right)$ for variation of distribution gradient (gr) and velocity in ms^{-1} with constant values $Mr=1.0$ and $db=0.005$.

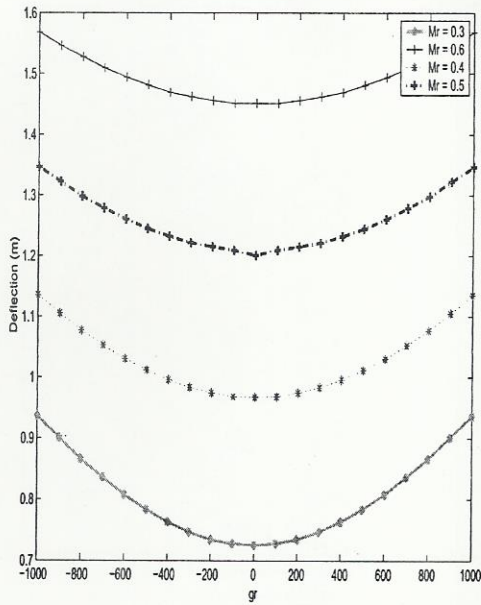


Figure 6. Maximum deflection profile $1000y\left(\frac{L}{2}, t\right)$ for variation of distribution gradient (gr) and mass ratio (Mr) with constant values $v=20\text{ms}^{-1}$ and $db=0.005$.

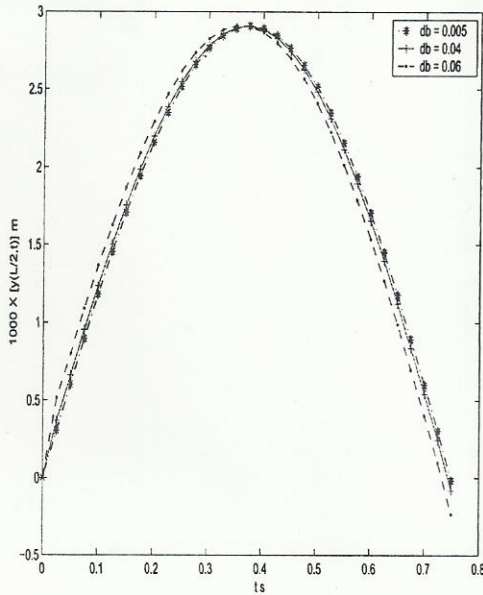


Figure 7. The time variation of mid-span deflection $1000y\left(\frac{L}{2}, t\right)$ for some lengths of load-mass distribution with constant values $v=30\text{ms}^{-1}$, $Mr=1.2$ and $gr=200$.

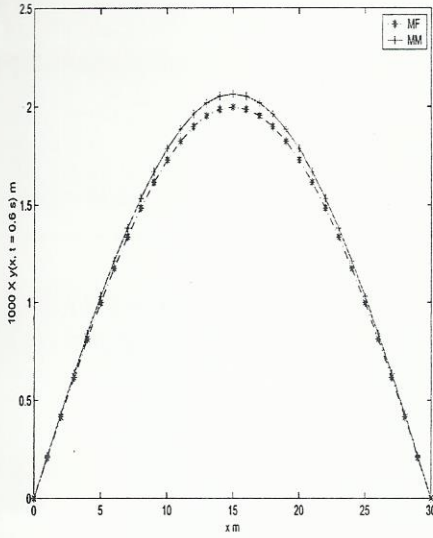


Figure 8. The deflection profile $1000y(x, 0.6s)$ for moving mass (MM) and moving force (MF) cases with constant values $v = 30\text{ms}^{-1}$, $Mr = 0.9$ and $gr = 400$.