EXACT SOLUTION OF FRACTIONAL ORDER INTEGRO-DIFFRENTIAL EQUATIONS BY COLLOCATION METHOD

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Abstract

In this paper, the application of standard collocation method on fractional integro-differential equation was carried out by assuming a modified trial solution with chebyshev polynomial basis.Equally spaced interior collocation points was adopted.In built maple 18 was used for the computation of the four illustrative examples, for the simple demonstration of the applicability,validity and reliability of the method .It is however concluded that the method is considered as one of the novel solver of the class of fractional integro-differential equation.

Keywords: Fractional integro-differential equation, Collocation points, Chebyshev polynomial

1.0. Introduction

In recent years, a growing considerable interest in the fractional integro-differential equation is simulated, due to their numerous applications in the areas of physics, chemistry, engineering, mechanics, astronomy, biology, economics and electro statistics. Differential equations involving derivatives of non-integer order have shown to be adequate models for various physical phenomena in the areas like rheology, damping laws, diffusion processes. This is more realistic and it is one reason why fractional calculus has become more and more popular (See Mittal and Nigam, 2008). In recent time, a good number of researchers have proposed and applied some efficient approximation and analytical techniques for the solution of problems of fractional calculus. Such techniques have been applied to fractional order differential equations, fractional order integral equations, and in some cases fractional order integro-differential equations. There have been attempt to solve multi-order fractional order differential equations but a complete analysis has so far not been given (Taiwo and Odetunde (2013).

Fractional differential equations have been investigated by many authors. Rawashdeh (2005) used the collocation spline method to approximate the solution of fractional equations. Taiwo and Odetunde (2013) solved multi-order fractional differential equations by an iterative decomposition method. Momani (2000) obtained local and global existence and uniqueness solutions of the integro-differential equation. Adomian Decomposition Method (ADM) is widely used by many researchers to solve the class of problems above in applied sciences (see Adomian (1994), Adomian (1989), Kaya and El-Sayed (2003)). Adomian (1989), provides an analytical approximation to linear and non linear problems in this category. In Adomian method the solution is considered as the sum of an infinite series, rapidly converging to an accurate solution. In this paper,the taditional conocation method is revisited with a little modification of the trial solution to solve fractional order integro-differential equation of the form:

$$D_*^{\alpha} y(t) = a(t)y(t) + f(t) + \int_0 k(t,s) f(y(s))ds, t[0,1]t \in [0,1] \tag{1.1}$$

Together with the initial condition

$$\sum_{i=0}^{n} a_{ij} y^{(i)}(t_j) = b_j,\ j = 0,1,2,\ldots \tag{1.2}$$

are considered and solved numerically with the proposed method. Here D_*^{α} is the Caputo's fractional derivative and α is a parameter describing the order of the fractional derivative, and $f(y(x))$ is generally a nonlinear continuous function. Such kinds of equations arise in the mathematical modeling of various physical phenomena, such as heat conduction in materials with memory. Moreover, these equations are encountered in combined conduction, convection and radiation problems (see for example Caputo, 1967; Olmstead and Handelsman, 1976; Mainardi, 1997).

2.0. Relevant Definitions

In this section, we give some basic definitions and properties of relevant terms which are useful in this work.

Definition 1: INTEGRAL EQUATION: An integral equation is an equation in which the unknown function *y(x)* appears under an integral sign. A standard integral equation is of the form:

$$y(x) = f(x) + \lambda \int_{g(x)}^{h(x)} K(x,t)y(t)dt \qquad (2.1)$$

Where *g(x)* and *h(x)* are the lower and upper limits of integration, λ is a constant parameter, and *K(x,t)* is a function of two variables x and t and is called the kernel or the nucleus of the integral equation, and $f(x)$ is a smooth continueous function.

Definition 2: INTEGRO-DIFFERENTIAL EQUATION (IDE): In a similar form, an integro-differential equation is an equation which involves both integral and derivatives of an unknown function.

A standard integro-differential equation is of the form:

$$y^{(n)}(x) = f(x) + \lambda \int_{g(x)}^{h(x)} K(x,t)y(t)dt \qquad (2.2)$$

Where *g(x), h(x), f(x)* and the kernel *K(x,t)* are defined in definition 1 above and n is the order of the IDE.

Equation (2.2) is called Fredholm Integro-differential Equation (FIDE) if both the lower and upper limits of the region of the integration are fixed numbers while it is called Volterra Integro-Differential Equation (VIDE) if the lower limit of the region of integration is a fixed number and the upper limit is not.

called An example of Fredholm Integro-differential Equation (FIDE) is given as

$$y^{(n)}(x) = f(x) + \lambda \int_{a}^{b} K(x,t)y(t)dt \qquad (2.3)$$

where a and b are are fixed numbers.

called an example of volterra Integro-differential Equation (VIDE) is given as

$$y^{(n)}(x) = f(x) + \lambda \int_{a}^{h(x)} K(x,t)y(t)dt \qquad (2.4)$$

where a is fixed number and $h(x)$ is a function of x

Definition 3: COLLOCATION METHOD: This is a method of evaluating a given differential equation or integro-differential or fractional order equation at some equally spaced interior points in order to determine the values of the unknown constants resuled from the assumed approximate solution used.

Definition 4: A real function $f(t), t > 0$, is said to be the space $C_{\mu}, \mu \in R$, if there exists a real number $p > \mu$, such that $f(t) = t^{p}h_1(t)$, where $f_1(t) \in C(0,\infty)$,and it is said to be in space C_{μ}^{n} if and only if

$$f^{(n)} \in C_{\mu}, n \in N.$$

Definition 5: The Riemann - Liouville fractional integral operator of order $\alpha > 0$, of a function $f \in C_\mu, \mu \geq -1$, is defined as

$$J^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s)ds, \alpha > 0 \tag{2.5}$$

$$J^0 f(t) = f(t) \tag{2.6}$$

Some properties of the operator J^α , are as follows:

For $f \in C_\mu, \mu \geq -1$, α, $\beta \geq 0$ and $\gamma \geq -1$

$$J^\alpha J^\beta f(t) = J^{\alpha+\beta} f(t), \tag{2.7}$$

$$J^\alpha J^\beta f(t) = J^\alpha J^\beta f(t), \tag{2.8}$$

$$J^\alpha t^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma+1+\alpha)} t^{\gamma+\alpha} \tag{2.9}$$

$$D^\alpha t^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma+1-\alpha)} t^{\gamma-\alpha} \tag{2.10}$$

Definition 6: The fractional derivative D^α of $f(t)$ in the Caputo's sense is defined as

$$D^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau)d(\tau), \tag{2.11}$$

For $n-1 < \alpha \leq n$, $n \in N$, $t > 0$, $f(t) \in C_{-1}^n$.

Definition 7: CHEBYSHEV POLYNOMIALS: Chebyshev polynomials are a sequence of orthogonal polynomials which are related to Demovire formula and are easily defined re-cursively like Fibonacci or Lucas numbers. The Chebyshev Polynomials of degree n denoted by $T_n(x)$ of the first kind and valid in the interval $-1 \leq x \leq 1$ is defined as

$$T_n(x) = \cos(n\cos^{-1}x)\ ; n > 0 \tag{2.12}$$

For $n = 0$ and 1, we obtained

$$T_o(x) = 1, \, and \ T_1(x) = x \tag{2.13}$$

And the recurrence relation is given as:

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), n \geq 1 \tag{2.14}$$

Also, Chebyshev Polynomials in the interval of $a \leq x \leq b$ is defined as

$$T_n(x) = \cos\left[n\cos^{-1}\left(\frac{2x-a-b}{b-a}\right)\right], a \leq x \leq b \tag{2.15}$$

And the recurrence relation is

$$T_{n+1}(x) = 2\left(\frac{2x-a-b}{b-a}\right)T_n(x) - T_{n-1}(x), n \geq 0 \ a \leq x \leq b \tag{2.16}$$

Few terms of the Shifted Chebyshev Polynomials valid in the interval $[0,1]$ are given below

$$T_o(x) = 1$$
$$T_1(x) = 2x - 1$$
$$T_2(x) = 8x^2 - 8x + 1$$
$$T_3(x) = 32x^3 - 48x^2 + 18x - 12.14$$

$$T_4(x) = 128x^4 - 256x^3 + 160x^2 - 32x + 1$$
$$T_5(x) = 512x^5 - 1280x^4 + 1120x^3 - 400x^2 + 50x - 1$$
$$T_6(x) = 2048x^6 - 6144x^5 + 6912x^4 - 3584x^3 + 840x^2 - 72x + 1$$
$$T_7(x) = 8192x^7 - 2867x^6 + 39424x^5 - 26990x^4 + 9408x^3 - 1568x^2 + 98x - 1$$
$$T_8(x) = 32765x^8 - 131072x^7 + 212992x^6 - 40224x^5 + 84480x^4 - 21504x^3 + 2688x^2 - 128x + 1 \quad (2.17)$$

3.0 Methodology of Chebyshev Polynomials as Basis Function on the General Fractional Order Integro-Differential Equation

In this section, we describe our form of approximation for solving the general class of fractional order integro-differential equation considered . Here, we assumed an approximate solution of the form:

$$y(t) \cong y_n = \sum_{j=0}^{n} a_j T_j(t) \quad (3.1)$$

Where n is the degree of the approximant used, $a_j (j \geq 0)$ are unknown constants to be determined and $T_j (j \geq 0)$,are chebyshev polynomial defined earlier.

For the purpose of our discussion, we consider the fractional order integro-differential of the form:

$$D_*^{\alpha} y(t) = a(t)y(t) + f(t) + \int_0 k(t,s) f(y(s))ds, \quad (3.2)$$

with the initial conduction

$$y(0) = 0 \quad (3.3)$$

pluging (3.3) on (3.1) we obtain

$$y(0) \cong y_n(0) = \sum_{j=0}^{n} a_j T_j(0) = 0 \quad (3.4)$$

simplification of (3.4) gives

$$y_n(0) = \sum_{j=0}^{n} (-1)^j a_j = 0 \quad (3.5)$$

Thus from (3.5)

$$a_0 = \sum_{j=1}^{n} (-1)^{j+1} a_j \quad (3.6)$$

substitting (3.6) into (3.1) and after simplification we have

$$y(t) \cong y_n(t) = \sum_{j=1}^{n} a_j T_j^*(t) \quad (3.7)$$

substituting (3.7) into (3.2) and after simplification we obtain

$$D_*^{\alpha} y_n(t) = a(t) \sum_{j=1}^{n} a_j T_j^*(t) + f(t) + \int_0 k(t,s) f\left(\sum_{j=1}^{n} a_j T_j^*(s) \right) ds \quad (3.8)$$

simplification of (3.8) and collecting like terms of $a_i (i \geq 0)$, gives

$$\{D^{\alpha} T_1^*(t) - p(t) T_1^*(t) - \int_0 K(t,s) T_1^*(s) ds\} a_1$$

$$+ \{D^{\alpha} T_2^*(t) - p(t) T_2^*(t) - \int_0 K(t,s) T_2^*(s) ds\} a_2 \quad (3.9)$$

$$+\ldots$$

$$+\{D^{\alpha}T_n^*(t)-p(t)T_n^*(t)-\int_0 K(t,s)T_n^*(s)ds\}a_n=f(t)$$

Thus, (3.9) is then collocated at $t=t_k$ to get

$$\{D^{\alpha}T_1^*(t_k)-p(t)T_1^*(t_k)-\int_0^k K(t_k,s)T_1^*(s)ds\}a_1$$

$$+a_2\{D^{\alpha}T_2^*(t_k)-p(t)T_2^*(t_k)-\int_0^k K(t_k,s)T_2^*(s)ds\}a_2 \tag{3.10}$$

$$+\ldots$$

$$+\{D^{\alpha}T_n^*(t_k)-p(t)T_n^*(t_k)-\int_0^k K(t_k,s)T_n^*(s)ds\}a_n=f(t_k)$$

Where,

$$t_k=a+\frac{(b-a)k}{n+1},k=1,2,3,..n$$

The collocated (3.10) resulted into a system of equations which are then put in matrix form as

$$Ax=b$$

Where,

$$A=\begin{pmatrix} A_{11} & A_{12} & A_{13} & \cdots & A_{1n} \\ A_{21} & A_{22} & A_{23} & \cdots & A_{2n} \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ A_{n1} & A_{n2} & A_{n3} & \cdots & A_{nn} \end{pmatrix},$$

$$x=(x_1,x_2,\ldots,x_n)^T,$$

$$b=(b_1,b_2,\ldots,b_n)^T$$

and

$$A_{11}=D^{\alpha}T_1^*(t_1)-p(t_1)T_1^*(t_1)-\int_0^1 K(t_1,s)T_1^*(s)_{\alpha}ds$$

$$A_{12}=D^{\alpha}T_2^*(t_1)-p(t_1)T_2^*(t_1)-\int_0^2 K(t_2,s)T_2^*(s)ds$$

$$A_{13}=D^{\alpha}T_3^*(t_1)-p(t_1)T_3^*(t_1)-\int_0^3 K(t_3,s)T_3^*(s)ds$$

$$A_{1n}=D^{\alpha}T_n^*(t_1)-p(t_1)T_n^*(t_1)-\int_0^n K(t_1,s)T_n^*(s)ds$$

$$A_{21.}=D^{\alpha}T_1^*(t_2)-p(t_2)T_1^*(t_1)-\int_0^2 K(t_2,s)T_1^*(s)ds$$

$$A_{22} = D^{\alpha}T_2^*(t_2) - p(t_2)T_2^*(t_2) - \int_0^{t_2} K(t_2,s)T_2^*(s)ds$$

$$A_{23} = D^{\alpha}T_3^*(t_2) - p(t_2)T_3^*(t_2) - \int_0^{t_2} K(t_2,s)T_3^*(s)ds$$

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(3.10)

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$$A_{2n} = D^{\alpha}T_n^*(t_2) - p(t_2)T_n^*(t_2) - \int_0^{t_2} K(t_2,s)T_n^*(s)ds$$

$$A_{31} = D^{\alpha}T_1^*(t_3) - p(t_3)T_1^*(t_3) - \int_0^{t_2} K(t_3,s)T_1^*(s)ds$$

$$A_{32} = D^{\alpha}T_2^*(t_3) - p(t_3)T_2^*(t_3) - \int_0^{t_3} K(t_3,s)T_2^*(s)ds$$

form as

$$A_{33} = D^{\alpha}T_3^*(t_3) - p(t_3)T_3^*(t_3) - \int_0^{t_3} K(t_3,s)T_3^*(s)ds$$

$$A_{3n} = D^{\alpha}T_n^*(t_3) - p(t_3)T_n^*(t_3) - \int_0^{t_3} K(t_3,s)T_n^*(s)ds$$

$$A_{n1} = D^{\alpha}T_1^*(t_n) - p(t_n)T_1^*(t_1) - \int_0^{t_1} K(t_n,s)T_1^*(s)ds$$

$$A_{n2} = D^{\alpha}T_2^*(t_n) - p(t_n)T_2^*(t_1) - \int_0^{t_1} K(t_n,s)T_2^*(s)ds$$

$$A_{n3} = D^{\alpha}T_3^*(t_n) - p(t_n)T_3^*(t_n) - \int_0^{t_1} K(t_n,s)T_3^*(s)ds$$

$$A_{nn} = D^{\alpha}T_n^*(t_n) - p(t_n)T_n^*(t_n) - \int_0^{t_n} K(t_n,s)T_n^*(s)ds \qquad (3.11)$$

After the evaluation of the integrals that appeared in the matrix, the resulting system is then solved by Mapple 18 to obtain the unknown constants that appeared in the approximate solution.

4.0 Numerical Examples

4.1 Example 1:

Consider the following fractional order integro-differential equation, for $t \in I = [0,1]$,

$$D^{0.5}y(t) = y(t) + \frac{8}{3\Gamma(0.5)}t^{1.5} - t^2 - \frac{t^3}{3} + \int_0^t y(s)ds \qquad (4.1.1)$$

$$y(0) = 0 \qquad (4.1.2)$$

The exact solution is $y(t) = t^2$

(See Fadi Awawdeh, 2011)

Description of Solution

Here we assume an approximate solution of the form:

$$y_n(t) = \sum_{j=0}^{n} a_j T_j(t) \tag{4.1.3}$$

where a_j and $T_j(t)$ are defined above

Here, we let n = 3

Thus, (4.1.3) leads to

$$y_3(t) = a_o T_o(t) + a_1 T(t)_1 + a_2 T_2(t) + a_3 T_3(t) \tag{4.1.4}$$

Substitution of $T_o(t)$, $T_1(t)$, $T_2(t)$, $T_3(t)$ from (2.14) into (4.1.4) leads to

$$y_3 = a_o + a_1(2t-1) + a_2(8t^2 - 8t + 1) + a_3(32t^3 - 48t^2 + 18t - 1) \tag{4.1.5}$$

Applying the initial condition given by (4.1.2)on (4.1.5), leads to

$$y_3(0) \equiv a_o - a_1 + a_2 - a_3 = 0 \tag{4.1.6}$$

Hence, making a_0 the subject of formula in (4.1.6) leads to

$$a_o = a_1 - a_2 + a_3 \tag{4.1.7}$$

Substituting $(6.1.7)$ into in the $(4.1.5)$ leads to

$$y_3(t) = 2ta_1 + (8t^2 - 8t)a_2 + (32t^3 - 48t^2 + 18t)a_3 \tag{4.1.8}$$

Thus, (4.1.8) is substituted into (4.1.1) to obtain

$$\begin{aligned}
&D^{0.5}\{2ta_1 + (8t^2 - 8t)a_2 + (32t^3 - 48t^2 + 18t)a_3\} \\
&- \{2ta_1 + (8t^2 - 8t)a_2 + (32t^3 - 48t^2 + 18t)a_3\} \\
&- \int_0^t \{2sa_1 + (8s^2 - 8s)a_2 + (32s^3 - 48s^2 + 18s)a_3\}ds = \frac{8t^{1.5}}{3\Gamma(0.5)} - t^2 - \frac{t^3}{3}
\end{aligned} \tag{4.1.9}$$

Hence, (4.1.9) is re-arranged to get

$$\begin{aligned}
&\{2D^{0.5}t - 2t - \int_0^t 2sds\}a_1 + \{D^{0.5}(8t^2 - 8t) - (8t^2 - 8t) - \int_0^t (8s^2 - 8s)ds\}a_2 \\
&\{D^{0.5}(32t^3 - 48t^2 + 18t) - (32t^3 - 48t^2 + 18t) - \int_0^t (32t^3 - 48t^2 + 18t)ds\}a_3 \\
&= \frac{8}{3\Gamma(0.5)}t^{1.5} - t^2 - \frac{t^3}{3}
\end{aligned} \tag{4.1.10}$$

Thus, (4.1.10) is simplified term by term to get

$$\begin{aligned}
&\left(\frac{2t^{\frac{1}{2}}}{\Gamma(3/2)} - 2t - t^2\right)a_1 + \left(\frac{16t^{\frac{3}{2}}}{\Gamma(5/2)} - \frac{8t^{\frac{1}{2}}}{\Gamma(3/2)} - 8t^2 + 8t - 8\frac{t^3}{3} + 4t^2\right)a_2 \\
&+ \left(\frac{192t^{\frac{5}{2}}}{\Gamma(7/2)} - \frac{288t^{\frac{3}{2}}}{\Gamma(5/2)} + \frac{18t^{\frac{1}{2}}}{\Gamma(3/2)} - 32t^3 + 48t^2 - 18t^4 + 16t^3 - 9t^2\right)a_3 \\
&= \frac{8t^{1.5}}{3\Gamma(0.5)} - t^2 - \frac{t^3}{3}
\end{aligned} \tag{4.1.11}$$

Therefore (4.1.11) is collocated at point $t = t_k$ to get

$$\left(\frac{2t_k^{\frac{1}{2}}}{\Gamma(3/2)}-2t_k-t_k^2\right)a_1+\left(\frac{16t_k^{\frac{3}{2}}}{\Gamma(5/2)}-\frac{8t_k^{\frac{1}{2}}}{\Gamma(3/2)}-8t_k^2+8t_k-8\frac{t_k^3}{3}+4t_k^2\right)a_2$$
$$+\left(\frac{192t_k^{\frac{5}{2}}}{\Gamma(7/2)}-\frac{288t_k^{\frac{3}{2}}}{\Gamma(5/2)}+\frac{18t_k^{\frac{1}{2}}}{\Gamma(3/2)}-32t_k^3+48t_k^2-18t_k^4+16t_k^3-9t_k^2\right)a_3 \quad (4.1.12)$$
$$=\frac{8t_k^{1.5}}{3\Gamma(0.5)}-t_k^2-\frac{t_k^3}{3}$$

Where,

$$t_k = k/4 \ , \ k=1,2,3$$

For $k=1$, $t_1=\frac{1}{4}$, thus substituting into (4.1.12) leads to

$$0.5658791670a_1-1.300677779a_2-17.46403084a_3=0.1203548612 \quad (4.1.13)$$

For $k=2$, $t_1=\frac{1}{2}$, thus substituting into (4.1.12) leads to

$$0.345769121a_1+0.538974505a_2-53.77207334a_3=0.2402563737 \quad (4.1.14)$$

For $k=3$, $t_1=\frac{3}{4}$, thus substituting into (4.1.12) leads to

$$-0.108089952a_1+2.625000000a_2-95.82807833a_3=0.2740800240 \quad (4.1.15)$$

Thus, solving (4.1.13)-(4.1.15) simultaneously using Maple 18, we obtain

$$a_1=0.50000$$
$$a_2=0.1250000$$
$$a_3=0.0000000$$

Therefore, $a_o=a_1-a_2+a_3=0.375000$

Substituting the values of $a_i(i=0,1,2,3)$ into (4.1.8) and after simplification, the exact solution is obtained.

Thatis, $y_3(x)=t^2$

4.2 Example 2

Consider the following fractional order integro-differential equation

$$y^{0.75}(t)=(\frac{-t^2e^t}{5})y(t)+\int_0 e^t sy(s)ds+\frac{6t^{2.25}}{\Gamma(3.25)} \quad (4.2.1)$$

With the initial condition

$$y(0)=0 \quad (4.2.2)$$

The exact solution is $y(t)=t^3$

(See Mittal R.C. and Ruchi Nigam, 2008)

Description of solution

Following the procedure in Example 1, we obtained

$$a_0=0.25, a_1=0.468750, a_2=0.187500, a_3=0.031250$$

Substituting the values of $a_i(i=0,1,2,3)$ into (4.1.8) and after simplification, the exact solution is obtained.

That is,

$$y_3(x)=t^3$$

4.3 Example 3

Consider the following fractional order integro-differential equation

$$D^{0.75}y(t) - (t\cos t - \sin t)y(t) - \int_0^t t\sin y(s)ds = \frac{t^{0.25}}{\Gamma(1.25)} \qquad (4.3.1)$$

$$y(0) = 0 \qquad (4.3.2)$$

The exact solution is $y(t) = t$

(See Fai Awawdeh, 2011)

Description of solution Following the procedure in Example 1

$$a_0 = 0.5, a_1 = \frac{1}{2}, a_2 = 0, a_3 = 0$$

Substituting the values of $a_i(i = 0,1,2,3)$ into (4.1.8) and after simplification, the exact solution is obtained, that is, $y_3(t) = t$

4.4 Example 4

Consider the initial value problem that consists of the multi-fractional order integro-differential equation

$$D_*^{0.5}y(t) = \frac{6}{\Gamma(3.5)}t^{2.5} - \frac{\Gamma(4)}{\Gamma(5.5)}t^{4.5} + J^{1.5}y(t),\ t \in [0,1] \qquad (4.4.1)$$

$$y(0) = 0 \qquad (4.4.2)$$

The exact solution is

$$y(t) = t^3$$

(See Taiwo and Odetunde, 2013)

Description of solution Following the procedure in Example 1

$$a_0 = 0.25, a_1 = 0.468750, a_2 = 0.187500, a_3 = 0.031250$$

Substituting the values of $a_i(i = 0,1,2,3)$ into (4.1.8) and after simplification, the exact solution is obtained. That is

$$y(t) = t^3$$

5.0 Conclusions

The traditional standard collocation method with little modification of trial solution was demonstrated on some examples of fractional integro differential equation. The method gave an exact solution for the degree of the Chebyshev polynomial for $n \geq 3$. Conclusively therefore, the method is very powerful and effective for solving fractional order integro differntial equation.

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