

A Graph-Theoretic Approach to Capacity Benefit Margin Calculation for Large Multiarea Power Systems

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Abstract—In a deregulated power market, independent system operators exchange information related to the available transfer capability (ATC) of their respective areas for efficient power system operation. Accurate ATC calculation is important for efficient market operations. Among the parameters required to calculate ATC, the capacity benefit margin (CBM) is an important one. CBM represents the interarea tie-lines capacity reserved in order to have access to external generation in an event of power supply shortage. This paper proposes a new technique for CBM evaluation using graph theory concepts. The proposed technique can be used as an effective tool to solve the CBM problem for large multiarea power systems which are not fully connected and may have multiple deficient areas, unlike previous approaches which considered only fully connected areas with only one deficient area. The IEEE 24-bus reliability test system is employed to implement the proposed method. A performance analysis of the proposed technique shows that it can find a feasible solution efficiently.

Index Terms—Capacity benefit margin (CBM), graph theory, multiarea power systems, reliability.

I. INTRODUCTION

Accurate evaluation of available transfer capability (ATC) is very important for system security and economic activities [1]. Capacity benefit margin (CBM) is one of the important parameters required to calculate ATC. CBM is the network margin reserved for the utilities to have access to external generation in case of emergency generation outages [2], [3]. Despite the extensive research works on ATC, the work done on CBM determination is limited. For efficient and sustainable power generation reliability, utilities usually have reserve capacity for unforeseen circumstances (such as sudden increase in load demand due to weather variability, decrease in supply due to faults in the system, or unplanned maintenance, etc.). This reserve capacity can be reduced without generation reliability degradation by power interchange between interconnected systems through tie-lines [4]. To minimize this reserve capacity, utilities reserve some margin in the transmission network between interconnected areas and this margin is termed as CBM. Over/under-estimation of CBM may result in inaccurate ATC results, leading to ineffective utilization of transmission system facilities and sudden generation deficiency [5], [6]. Thus, CBM is used to quantify the amount of transmission reserve capability required to meet the generation reliability. Loss of load expectation (LOLE) is commonly used as reliability metric for CBM calculation. LOLE is the average number of days or hours in a given period (usually one year) in which there

is a loss of load, i.e., the daily peak load or hourly load is expected to exceed the available generating capacity.

This paper presents a new method to evaluate the CBM for a large number of interconnected power systems consisting of multiple deficient areas and a network which is not fully connected. The main contribution of this paper is a graph-theoretic approach towards CBM evaluation which can efficiently scale to large and not fully connected power networks with multiple deficient areas. Moreover, the proposed technique can also be used as a CBM feasibility test, i.e., if the method fails to find a feasible solution then there is no feasible allocation of CBM using the available supply of power.

The rest of the paper is organized as follows. A discussion on the related study is presented in Section II. Section III describes the proposed method for the determination of CBM, and Section IV presents the simulation results. A performance analysis is given in Section V. Finally, Section VI concludes this paper.

II. LITERATURE REVIEW

Most of the previous methods to determine CBM employ optimization techniques, such as particle swarm optimization [7], evolutionary programming [8], differential evolution [9], and trial and error [10]. We can identify three major issues with the existing techniques for CBM calculation: 1) the existing techniques consider a simple scenario where there is only one deficient area; 2) these techniques assume a fully connected network of power systems; and 3) these techniques rely on complex optimization techniques and iteratively evaluate the LOLE of each area in each step which may not be feasible for large interconnected power systems. To solve these issues, we propose a technique based on graph theory which uses a graph representation of interconnected power systems. The technique is easily scalable to a large number of areas and can solve the problem of multiple deficient areas and interconnected power systems which are not fully connected.

III. PROPOSED CBM CALCULATION TECHNIQUE

Let us consider an interconnected power system with three areas. The first step in the proposed method is to create a directed graph of the interconnected power system under consideration. This is done by representing each area as a vertex of the graph and the tie-lines as edges between the vertices. Note that the capacity of all the tie-lines connecting any two areas is combined to get a single edge in each direction with the corresponding total capacity. The directed graph for a three-area interconnected power system is shown in Fig. 1. Where, $u_{i,j}$ represents the edge connecting area i and j . We observe that $u_{i,j} + u_{j,i} = C_{i,j}$, where $C_{i,j}$ denotes the capacity of the tie-line connecting area i and j . Moreover, Δ_i represents the demand (positive value) or supply (negative value) of power in area i . Δ_i is calculated using Algorithm 1.

In Algorithm 1, “Gen” represents the generation matrix of an area

$$\text{Gen} = \begin{bmatrix} G_1 & n_1 & \text{FOR}_1 \\ \vdots & \ddots & \vdots \\ G_n & n_n & \text{FOR}_n \end{bmatrix} \quad (1)$$

where the first column represents the power rating for the various generators available in an area, and the second and third columns represent the

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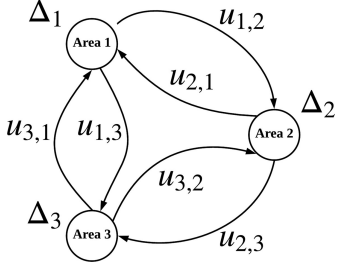


Fig. 1. Graph for a three-area power system.

Algorithm 1: Calculate Power Supply and Demand for an Area in the Power System Graph.

Input: Gen, Load, LDC, LOLE_{target}

Output: P_D

1. Calculate the Capacity Outage Probability Table (COPT) using Gen
2. Calculate LOLE using COPT and LDC
3. IF LOLE < LOLE_{target}
4. Initialize L = Load
5. WHILE (1) {
6. Calculate LOLE using L
7. IF LOLE > LOLE_{target}
8. $P_D = L - \text{Load}$
9. BREAK
10. L = L + Step}
11. ELSE
12. Initialize G = Total Generation
13. WHILE (1) {
14. Calculate LOLE using G
15. IF LOLE < LOLE_{target}
16. $P_D = \text{Total Generation} - G$
17. BREAK
18. G = G + Step}

number and forced outage rate of the corresponding generators, respectively. Moreover, **Load** denotes the total load in an area, **LDC** denotes the load duration curve, and **LOLE_{target}** denotes the maximum allowed value for LOLE. Algorithm 1 shows that the supply or demand of power in an area is calculated by respectively increasing the total load or total generation in that area in steps until the LOLE threshold is violated. The supply or demand can then be calculated respectively by taking the difference between the increased load and the original load value or increased generation and original generation value. Once the Δ_i 's have been calculated the supply and demand nodes are identified, i.e., if Δ_i is positive the node is designated as a demand node and vice versa.

The next step in the proposed technique is to add a source node and a sink node. This is done by adding edges from the source node to the supply nodes and assigning a capacity equal to the power supply of the respective nodes. Similarly, edges from the demand nodes to the sink node are added with capacities equal to the power demand of the respective nodes. Let us assume that area 1 is the demand node while areas 2 and 3 act as supply nodes in Fig. 1. Then the updated graph is shown in Fig. 2.

To find the CBM between different areas in Fig. 2, we formulate our problem as a maximum flow problem and check whether there is a feasible CBM allocation which can solve the LOLE problem in all the demand areas. If we denote the flow on the edge connecting area i and area j with $f_{i,j}$ then the maximum flow problem can be formulated as

$$\text{Maximize : } \sum_{i:(S,i) \in A} f_{S,i}. \quad (2)$$

Subject to

$$f_{i,j} \leq u_{i,j}, \quad \forall (i,j) \in A \quad (3)$$

$$u_{i,j} + u_{j,i} \leq C_{i,j}, \quad \forall (i,j) \in A \quad (4)$$

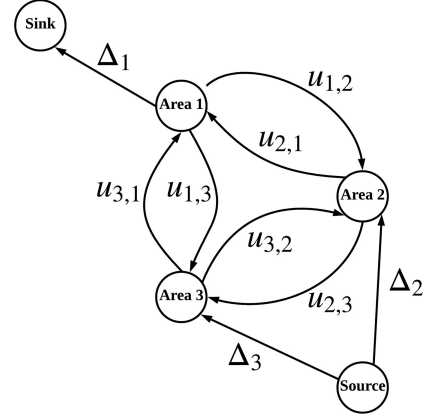


Fig. 2. Power system graph with source and sink nodes.

TABLE I
DETAILS OF THE SIX AREAS OF THE IEEE 24-BUS RTS

Areas	Buses	Generation (MW)	Load (MW)	Margin (MW)	LOLE (hr/yr)
1	15, 16, 17, 18	770	750	20	813.086
2	14, 19, 21	400	375	25	0
3	20, 22	300	128	172	0.0002
4	5, 6, 8, 9, 10, 11, 12, 13, 23	1250	1013	237	63.002
5	2, 7	492	222	270	0.0655
6	1, 3, 4, 24	592	362	230	736.170

$$\sum_{k:(i,k) \in A} f_{i,k} - \sum_{k:(k,i) \in A} f_{k,i} = 0, \quad \forall i \in V. \quad (5)$$

The steps to calculate CBM are as follows.

- 1) Use Algorithm 1 to obtain the graph representation for the inter-connected power system.
- 2) Add source and sink nodes. Add edges from the source node to supply nodes with a capacity equal to Δ_i . Add edges from demand nodes to sink node with a capacity equal to Δ_i .
- 3) Run maximum flow algorithm on the power system graph.
- 4) If the flow from the source node to sink node = total demand (D_{Total}), assign CBM from area i to area $j = f_{i,j}$, otherwise, there is no feasible allocation.

After running any of the popular algorithms for maximum flow on Fig. 2, using these steps, we can check if there is a feasible allocation of CBM, i.e., whether the total supply of power S_{Total} is sufficient to fulfill the total demand of power D_{Total} . We observe that a feasible CBM allocation is possible if the flow from the source to sink node in Fig. 2 is equal to D_{Total} . If a feasible CBM allocation is possible, then the flow value $f_{i,j}$ from area i to j is considered as the CBM between area i and j .

IV. SIMULATIONS AND RESULTS

The proposed technique was simulated in MATLAB R2015b using the IEEE 24-bus RTS [11]. The IEEE 24-bus RTS is divided into six areas as shown in Fig. 3. Moreover, the LOLE target value for all areas is set to 2.4 hours per year (h/year). Table I shows the buses, total generation, total load, and margin available in each area.

Using Algorithm 1, we have Fig. 4 identifying the supply and demand areas as well as the link capacities. Using the proposed technique for CBM calculation with the Boykov–Komogorov [12] as the maximum flow algorithm we obtain the flows shown in Fig. 5.

We observe that $D_{\text{Total}} = 1089$ MW exceeds the total flow from source to sink, i.e., 264 MW. Therefore, we can conclude that there is no feasible allocation of CBM in this power network. Note that there

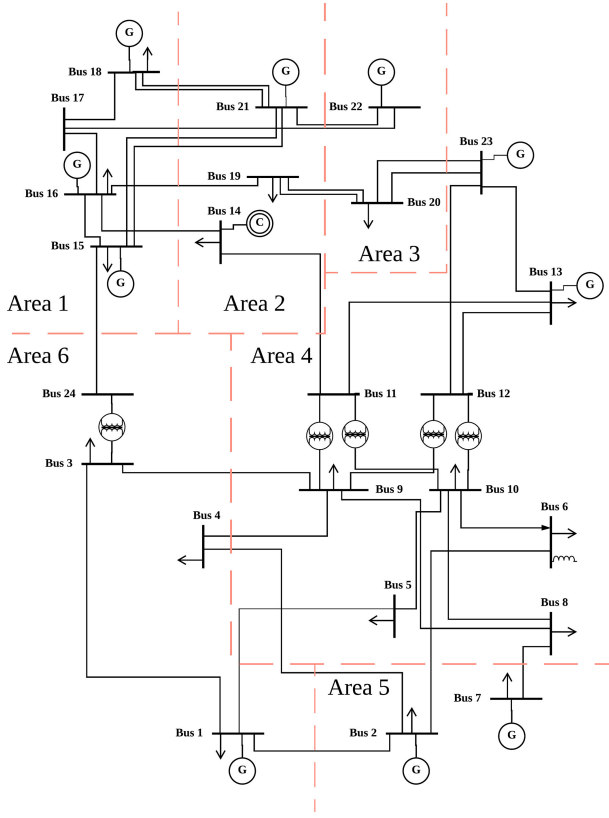


Fig. 3. IEEE 24-bus RTS divided into six areas.

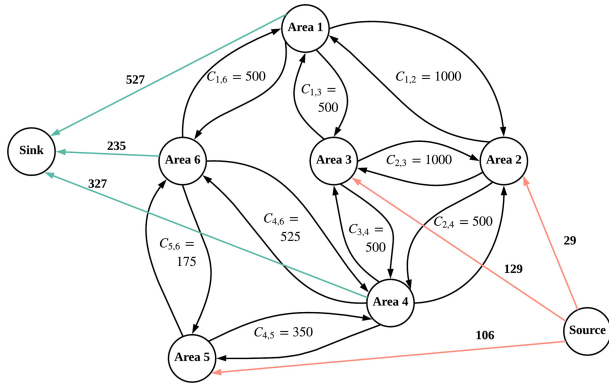


Fig. 4. IEEE 24-bus RTS graph representation. Areas 2, 3, and 5 are the supply areas while areas 1, 4, and 6 are the demand areas. Algorithm 1 using hourly load model of [11] and a step size of 1 MW.

is no power flow to area 6 and the power flow to areas 1 and 4 is not sufficient to meet the reliability requirements of these areas. To get a feasible allocation and test our proposed technique we modify the IEEE 24-bus RTS system according to Table II. Moreover, we also remove the tie-lines between area 3 and 4; and area 5 and 4. Note that we modify the tie-lines so as to satisfy the LOLE requirements of area 4, such that the proposed technique must route power from multiple areas (not directly connected) to area 4, i.e., to create a power network that is not fully connected.

The modified IEEE 24-bus RTS has four supply areas (2, 3, 5, and 6) and two demand areas (1 and 4). We can now apply the proposed CBM-calculation technique to obtain the flows shown in Fig. 6, which shows that we have a feasible CBM allocation for this power network. We observe that to meet the power demands of area 4 and 6, power is routed from areas that are not directly connected to these areas.

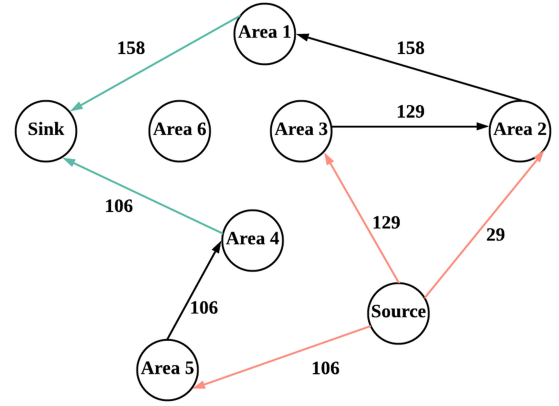


Fig. 5. CBM calculation for IEEE 24-bus RTS.

TABLE II
DETAILS OF THE SIX AREAS OF THE MODIFIED IEEE 24-BUS RTS

Area	Generation Units (Original) (MW)	LOLE (Original) (hr/yr)	Generation Units (Modified) (MW)	LOLE (Modified) (hr/yr)
1	12 x 5 155 x 1 400 x 1	813.086	12 x 5 155 x 4 400 x 2	7.396
2	400 x 1	0	--	--
3	50 x 6	0.0002	--	--
4	197 x 3 155 x 2 350 x 1	63.002	197 x 4 155 x 2 350 x 1	8.506
5	20 x 2 76 x 2 100 x 3	0.0655	20 x 2 76 x 2 100 x 4	0.0028
6	20 x 2 76 x 2 400 x 1	736.170	20 x 2 76 x 2 100 x 3 400 x 1	0.9524

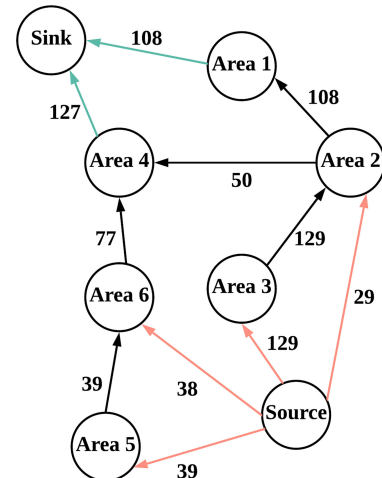


Fig. 6. CBM calculation for modified IEEE 24-bus RTS.

TABLE III
LOLE OF THE SIX AREAS BEFORE AND AFTER CBM ALLOCATION

Area	LOLE (Before CBM Allocation) (hr/yr)	LOLE (After CBM Allocation) (hr/yr)
Area 1	7.396	2.369
Area 2	0	1.76
Area 3	0.0002	2.364
Area 4	8.506	2.376
Area 5	0.0028	0.016
Area 6	0.9524	2.303

TABLE IV
RESULT OF CBM DETERMINATION USING APPROACHES IN [10] AND [7]

Area	Proposed Technique		Ref [10]		Ref [7]	
	Import / Export (MW)	LOLE (hr/yr)	Import / Export (MW)	LOLE (hr/yr)	Import / Export (MW)	LOLE (hr/yr)
1	79	2.378	79	2.378	98.46	1.931
2	-79	1.721	-40	1.049	-94.96	2.014
3	0	0.667	-39	2.048	-3.50	0.732
Total		4.766		5.475		4.677

Table III shows that the LOLE values of all areas are below the target value of 2.4 h/year. Thus, the proposed technique can be used effectively for CBM calculation in large multiarea power systems.

The techniques in [7] and [10] can only be applied to fully connected networks of power systems. Therefore, to compare the proposed technique with existing techniques for CBM calculation, we consider the modified three area 24-bus RTS [10], which consists of a fully connected network of power systems. The methods proposed in [7] and [10] are then used to determine the CBM employing the reliability data used in this paper. The results are shown in Table IV. If we take the total LOLE of all the areas as the benchmark for comparison, then the technique with lower values for total LOLE is considered better. From Table IV, we observe that the proposed technique results in a total LOLE lower than [10] but slightly higher than [7]. However, note that the proposed technique uses maximum flow algorithms to calculate CBM which have significantly lower computational complexity compared to particle swarm optimization. This shows that the proposed technique can be used as an effective tool for CBM calculation in large interconnected power systems.

V. PERFORMANCE EVALUATION

If we denote the peak load and installed capacity by $L_{\max}$ and I_c , then the worst-case time complexity to calculate LOLE is given by $C_{\text{LOLE}} = O(M(L_{\max} - I_c))$, where $M()$ represents the complexity of multiplication. Then the worst-case complexity of Algorithm 1 is given by $C_{\text{Algo1}} = O(C_{\text{LOLE}} \times \text{LOLE}_{\text{Target}})$. Moreover, if we denote the number of supply areas by n and demand areas by m then the worst-case time complexity of maximum flow algorithms can be given by $C_{\text{Flow}} = O(nm \log \frac{n^2}{m})$ [13]. Thus, the worst-case time complexity of the proposed technique is given by $C_{\text{prop}} = C_{\text{Flow}} + C_{\text{Algo1}} = O(nm \log \frac{n^2}{m} + C_{\text{LOLE}} \times \text{LOLE}_{\text{Target}})$. The existing techniques for CBM calculation based on evolutionary algorithms (EA) usually have a computational complexity of $C_{\text{EA}} = O(d \times p + C_{\text{Obj}} \times p)$ [14], where d is the dimension of the problem, p is the population size,

and C_{Obj} is the complexity of the objective function. C_{Obj} depends on LOLE calculation, and d is described by the total number of edges in a graph.

Thus, $C_{\text{EA}} = O(\frac{nm(nm-1)}{2} \times p + C_{\text{LOLE}} \times p) = O(n^2 m^2 p + C_{\text{LOLE}} \times p)$. Comparing C_{prop} and C_{EA} , we observe that the existing techniques are not scalable because they are $O(n^2 m^2)$ as opposed to the proposed technique which has a significantly lower computational complexity. This shows that the proposed technique is efficient and can scale to large multiarea power networks.

VI. CONCLUSION

This paper proposed a CBM calculation technique for large multi-area power systems. The proposed technique uses the maximum flow problem in graph theory to determine the CBM between different areas. This technique has lower complexity than existing techniques for CBM calculation and can scale large multiarea power systems. The proposed technique has the ability to evaluate CBM for interconnected power systems which have multiple deficient areas as well as areas which are not directly connected to each other. Moreover, the proposed technique can also be used as an efficient CBM feasibility test. The implementation results of the proposed technique on the IEEE 24-bus RTS showed that the proposed technique can be used as an effective tool to calculate CBM for large multiarea power systems which are not fully connected. Moreover, a comparison with existing work showed that the proposed technique can find a feasible solution for CBM allocation.

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REFERENCES

- [1] P. W. Sauer, "Technical challenges of computing available transfer capability (ATC) in electric power systems," in *Proc. 30th Int. Conf. Syst. Sci.*, 1997, vol. 5, pp. 589–593.
- [2] I. Dobson *et al.*, "Electric power transfer capability: Concepts, applications, sensitivity and uncertainty," Power Syst. Eng. Res. Centre, Cornell Univ., New York, NY, USA, Rep. no. 01-34, 2001.
- [3] N. Rep, "Available Transfer Capability Definitions and Determinations," North Amer. Elect. Rel. Council, Atlanta, GA, USA, Rep. No. RM95-8-000, 1996.
- [4] H. Kuraishi and A. Yokoyama, "Optimal CBM of tie lines between control areas in a deregulated environment," *Elect. Eng. Jpn.*, vol. 167, no. 1, pp. 35–48, 2009.
- [5] S. C. Savulescu, "A metric for quantifying the risk of blackout," in *Proc. IEEE Power Syst. Conf. Expo., PES*, 2004, pp. 1661–1664.
- [6] A. Gheorghe, M. Masera, M. Weijnen, and L. De Vries, *Critical Infrastructures at Risk- Securing European Electric Power System*. Dordrecht, Netherlands: Springer, 2006.
- [7] M. Ramezani, M.-R. Haghifam, C. Singh, H. Seifi, and M. P. Moghaddam, "Determination of capacity benefit margin in multiarea power systems using particle swarm optimization," *IEEE Trans. Power Syst.*, vol. 24, no. 2, pp. 631–641, May 2009.
- [8] M. M. Othman, N. Abd Rahman, I. Musirin, M. Fotuhi-Firuzabad, and A. Rajabi-Ghahnavieh, "A heuristic ranking approach on capacity benefit margin determination using pareto-based evolutionary programming technique," *Sci. World J.*, vol. 2015, 2015, Art. no. 731013.
- [9] R. Rajathy, R. Gnanadass, K. Manivannan, and H. Kumar, "Computation of capacity benefit margin using differential evolution," *Int. J. Comput. Sci. Math.*, vol. 3, no. 3, pp. 275–287, 2010.
- [10] Y. Ou and C. Singh, "Assessment of available transfer capability and margins," *IEEE Trans. Power Syst.*, vol. 17, no. 2, pp. 463–468, May 2002.
- [11] P. M. Subcommittee, "IEEE reliability test system," *IEEE Trans. Power App. Syst.*, vol. PAS-98, no. 6, pp. 2047–2054, Nov. 1979.
- [12] Y. Boykov and V. Kolmogorov, "An experimental comparison of min-cut/max-flow algorithms for energy minimization in vision," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 26, no. 9, pp. 1124–1137, Sep. 2004.
- [13] B. Fishbain, D. S. Hochbaum, and S. Muller, "Competitive analysis of minimum-cut maximum flow algorithms in vision problems," 2010, arXiv:1007.4531.
- [14] D. Sudholt, "Computational complexity of evolutionary algorithms, hybridizations, and swarm intelligence," Ph.D. dissertation, Dept. Comput. Sci., Techn. Univ. Dortmund, Dortmund, Germany, 2008.