



Alexandria University
Alexandria Engineering Journal

www.elsevier.com/locate/aej
www.sciencedirect.com



ORIGINAL ARTICLE

Numerical simulation of forced convective heat transfer in inclined elliptic ducts with multiple internal longitudinal fins



I.K. Adegun^{a,*}, T.S. Jolayemi^b, O.A. Olayemi^c, A.M. Adebisi^d

^a Department of Mechanical Engineering, Faculty of Engineering and Technology, University of Ilorin, Ilorin, Nigeria

^b Department of Mechanical Engineering, Institute of Technology, Kwara State Polytechnic, Ilorin, Nigeria

^c Department of Aeronautical and Astronautical Engineering, College of Engineering and Technology, Kwara State University, Malete, Nigeria

^d Department of Mechanical Engineering, School of Engineering, University of Glasgow, Glasgow G12 8QQ, United Kingdom

Received 21 July 2016; revised 28 December 2016; accepted 14 January 2017

Available online 24 February 2017

KEYWORDS

Elliptic geometry;
 Multiple internal fins;
 Fluid flow;
 Heat transfer;
 Temperature history;
 Velocity profile

Abstract The paper presents numerical study of forced convective heat transfer in internally finned elliptic geometry to determine the effect of parameters such as Reynolds number, Prandtl number, aspect ratio, fin heights, number of fins and orientations of the duct on the heat transfer coefficient, temperature history and velocity profiles. The walls were assumed to be subjected to a thermal boundary condition of constant heat flux and the effect of slip boundary is assumed negligible. Finite difference technique was adopted for the solution and results were generated using a code written in Qb-64. The study indicated that number of fins ($F = 8$) provided the optimum heat transfer augmentation. For all the geometries investigated, Nusselt number, average velocity and bulk fluid temperature were enhanced by the inclinations in the range $0^\circ \leq \theta \leq 75^\circ$. For a flow regime of $50 \leq Re \leq 200$, the effect of inclination on heat transfer is negligible, but it worth noting the existence of a critical Reynolds number, $Re = 200$, above which the effect of inclination on fluid flow becomes noticeable. Also, at $Pr \geq 5$, Nusselt number and bulk fluid temperature were independent of fluid properties. The results obtained could be applied in the prediction of parameters for the design of compact heat exchangers for industrial applications.

© 2017 Faculty of Engineering, Alexandria University. Production and hosting by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

1. Introduction

Heat transfer augmentation is of great importance to the scientist and engineers, particularly those that are interested in the design of compact heat exchangers used extensively for the cooling of heat transfer equipment. These heat exchangers are highly applicable in industries where heat loads are

* Corresponding author.

E-mail addresses: kadegun@unilorin.edu.ng (I.K. Adegun), temidayo2005sam@yahoo.com (T.S. Jolayemi), olalekan.olayemi@kwasu.edu.ng (O.A. Olayemi), 2213037A@student.gla.ac.uk (A.M. Adebisi).

Peer review under responsibility of Faculty of Engineering, Alexandria University.

<http://dx.doi.org/10.1016/j.aej.2017.01.014>

1110-0168 © 2017 Faculty of Engineering, Alexandria University. Production and hosting by Elsevier B.V.

This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

Nomenclature

a	major axis, m	T_w	wall temperature
A	cross-sectional area, m ²	U	dimensionless velocity
b	minor axis, m	W	dimensionless wall thickness
c_p	fluid specific heat, kJ/kg K	(r, ϕ, z)	modified cylindrical coordinate
D_h	hydraulic diameter, $\frac{4A}{\Lambda_c}$	z	axial direction
e	eccentricity; $\sqrt{1 - e^2}$	∂	partial differential
F	number of fins	<i>Greek symbols</i>	
H	normalized fin height	α	fin half angle ($\alpha = \gamma$, finned sectors)
k	iteration counter	β	unfinned sector
k_f	thermal conductivity of fluid, W/m K	Γ_c	perimeter
k_s	thermal conductivity of wall, W/m K	ϵ	axis ratio = $\frac{b}{a}$
n	exponent in hyper ellipse formula	η	non-dimensional coordinate $\eta = \frac{r_\phi}{a}$
Nu_m	mean Nusselt number Eq. (20)	θ	angle of channel inclinations
p	pressure, N/m ²	μ	viscosity, N S/m ²
Pe	Peclet number ($RePr$)	ρ	fluid density, kg/m ³
Pr	Prandtl number	ϕ	azimuthal direction
r	dimensional equivalent radius of the elliptic geometry	ϵ	limits
r_ϕ	dimensional radius along the azimuthal direction	<i>Subscripts</i>	
R	non-dimensional equivalent radius of the elliptic geometry	b	bulk
Re	Reynolds number	m	mean
T	dimensionless temperature	w	wall
T_b	bulk fluid temperature		

substantial for a limited available space as they offer specific characteristics such as higher heat transfer rates and less flow resistance compared to ducts of circular geometries. Laminar forced convective mechanism through ducts of different geometries had been studied by many researchers analytically, numerically and experimentally.

Hassan and Siren [1] investigated the performance of evaporative cooled heat exchangers. They determined the heat and mass transfer coefficients and presented the combined thermal-hydraulic performance in terms of an energy index. They found out that the finned tubes make higher thermal utilization of the volume with the same energy index. In their work, inclination and thermal boundary condition of peripheral uniform heat flux were not considered as considered in the current research. Ota et al. [2,3], experimentally studied heat transfer and fluid flow around elliptical cylinders of axis ratio 1:2 and 1:3. Their results showed that the heat transfer coefficient of an elliptical cylinder is higher than that of a circular one with equal circumference and the pressure drag coefficients of the former were much lower than those of the latter. Teamah et al. [4] performed a numerical investigation to determine the effect of Reynolds and Prandtl numbers on laminar forced convection in horizontal pipe partially filled with porous material. They found out that insertion of porous material inside the pipe leads to a higher average Nusselt number than that of fluid flow in clear pipe. Foong et al. [5] carried out a numerical study to determine the fluid flow and heat transfer characteristics of a square micro-channel with four internal longitudinal fins. Three dimensional numerical simulations were performed on the micro-channel with variable fin height

ratio in the presence of a hydro-dynamically developed and thermally developing laminar flow subjected to constant heat flux boundary conditions. The result of the study indicated that internal fins in a micro-channel have the potential to provide heat transfer augmentation. Bahrami et al. [6,7] used analytical method of solution and the concept of Saint-Venant principle in torsion to develop a general model for predicting pressure drop in micro-channels of arbitrary cross-section. They showed that poiseuille number, $f Re$, where f is the Fanning factor and Re is the Reynolds number, is a function of polar moment of inertia, area and perimeter of the cross-section of the channel. Their model showed good agreement with experimental and numerical data for a wide variety of cross-sections such as rectangular, trapezoidal, triangular, circular, and moon-shaped. However, they did not provide the velocity distribution in the considered micro-channel. Sakalis et al. [8] conducted a numerical investigation using Alternating Direction Implicit method (ADI) to determine the fluid flow and rate of heat transfer in straight elliptic ducts with varying aspect ratios subjected successively to constant temperature, circumferential uniform and axially linearly or exponentially varying temperature. In the thermally developed limit, their results showed that as the aspect ratio decreases, the Nusselt number increases for small axial wall temperature distributions, but decreases for large axial wall temperature values. Schubauer [9,10] conducted experiments to determine velocity distributions in the laminar boundary layer of an elliptic cylinder with axis ratio 1:3. He found out that the velocity distributions, boundary layer thickness, and its tendency to separate from the surface of the body were governed almost entirely

by the velocity distribution in the region of potential flow outside the boundary layer. Leveque solution for flow in an elliptical duct is reported by Richardson [11]. Alam and Ghoshdastidar [12] conducted a numerical study on hydrodynamically, and thermally developing flow inside circular tubes with internal longitudinal fins having tapered lateral profiles. The results showed significant heat transfer enhancement with the inclusion of internal fins. They used water and engine oil as the assumed working fluids and they concluded water to be a better coolant as compared to engine oil. Jackson [13], D'Allesio and Dennis [14,15], and D'Allesio [16], numerically investigated the flow of a viscous incompressible fluid past an inclined elliptic cylinder. They obtained solutions for Reynolds numbers up to 100 and for various inclinations. Good agreement was found with the existing result. Dong and Ebadian [17] conducted a numerical study of thermally developing flow in an elliptical duct with four longitudinal internal fins of Zero thickness. A control volume based finite difference technique was used in the numerical analysis and an optimum value of local Nusselt number was obtained as a function of fin height. Papadopoulos and Hatzikonstanti [18] studied numerically thermally developing flow in curved elliptic ducts with internal fins. They limited the number of fins used to four which were arranged along the axes.

What motivated the present authors was the discovery from literatures that the few researchers that worked on elliptic geometry using numerical method of solution had not considered the implication of using elliptic duct with multiple internal fins because of the complexity of the configuration, those that considered fins limited their works to four fins [17,18]. Secondly there was no article published that had considered studying inclined elliptic duct with multiple longitudinal fins that is subjected to a thermal boundary condition of constant heat flux. Because of this gap, the present authors seek to investigate the effects of orientation of the duct, introduction of more than four longitudinal fins into the inner surface of the duct and the use of fluids of high Prandtl numbers which is aimed at improving the design of heat exchangers in heat transfer equipment. This will enhance the life span of the heat transfer equipment and also save financial and material resources.

2. Methodology

2.1. Physical model

The physical model adopted for this study is an inclined duct of elliptic geometry with internal longitudinal fins in modified cylindrical co-ordinate system (r, ϕ, x) . The duct and all the internal longitudinal fins are shown in the schematic diagram of Fig. 1. The fins form an integral part of the duct materials and are of trapezoidal cross-section. The temperature of the fluid at inlet is assumed uniform while neglecting the axial heat fluxes in the duct and fluid. Internal longitudinal fins are arranged symmetrically along the axis inside the duct. The flow is laminar with forced convective mechanism and thermal boundary condition of constant heat flux at the outer wall of the finned duct is assumed. It is assumed to be a steady state incompressible fluid flow and the velocity profile is fully developed in the duct. There is only one non-zero velocity component which is in the flow direction U_x . Thus $U_r = U_\phi \approx 0$.

U_r, U_ϕ are insignificant when compared with U_x . The effect of gravity is neglected except in the axial direction and the assumption of no-slip condition at the walls are considered. The governing equations below were used in the mathematical modeling. (Axial direction $x = z$).

2.1.1. Coordinate system

The equations employed in this present work are expressed in modified cylindrical coordinate system to accommodate the applied boundary conditions.

Following D'Allesio [16], the mapping between Cartesian coordinate (x, y, z) and modified polar coordinates (r, ϕ, z) is

$$x = c \cosh r \cos \phi \quad y = c \sinh r \sin \phi \quad z = z \quad (1)$$

The elliptic coordinate system introduces the following metrics in the three mutually orthogonal directions.

$$h_r = h_\phi = h = c \sqrt{\sinh^2 r + \sin^2 \phi} \quad h_z = 1 \quad (2)$$

$$\frac{x^2}{(\cosh r)^2} - \frac{y^2}{(\sin \phi)^2} = 1 \quad (3)$$

$$\frac{x^2}{(\cosh r)^2} + \frac{y^2}{(\sinh r)^2} = 1 \quad (4)$$

However, hyper ellipse is described by:

$$r_\phi = \frac{a}{\{(\sin \theta)^n + (\cos \theta/\varepsilon)^n\}^{1/n}}, \quad 0 \leq \varepsilon = \frac{b}{a} \leq 1 \quad [19] \quad (5)$$

ε = Aspect ratio, a = major axes, b = minor axes.

By varying parameter n , several geometries can be created. When $n = 2$, from Eq. (5) yields an ellipse; for $a = b$, the consequent geometry is a circle.

2.2. Governing equations

2.2.1. Continuity equation

$$\frac{\partial U_r}{\partial r} + \frac{U_r}{r} + \frac{1}{r} \frac{\partial U_\phi}{\partial \phi} + \frac{\partial U_x}{\partial x} = 0 \quad (6)$$

2.2.2. Momentum transport equations

Since pressure varies only in the axial direction, the momentum transport equation for the geometry reduces to:

$$\frac{\partial^2 U_r}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 U}{\partial \phi^2} + \frac{1}{r} \frac{\partial U}{\partial r} = \frac{1}{\mu} \frac{\partial p}{\partial x} + \frac{\rho g \sin \theta}{\mu} \quad (7)$$

where ρg_x ($\rho g \sin \theta$) denotes the body force term as a result of channel inclination.

2.2.3. Energy transport equation

The energy transport equation for a steady flow in the absence of viscous dissipation term $\phi_{(i)}$ is

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} + \frac{1}{r^2} \frac{\partial^2 t}{\partial \phi^2} = \begin{cases} \frac{U_x}{\alpha} \frac{\partial t}{\partial x} & \text{Fluid - Region} \\ 0 & \text{Fins \& Solid - Region} \end{cases} \quad (8)$$

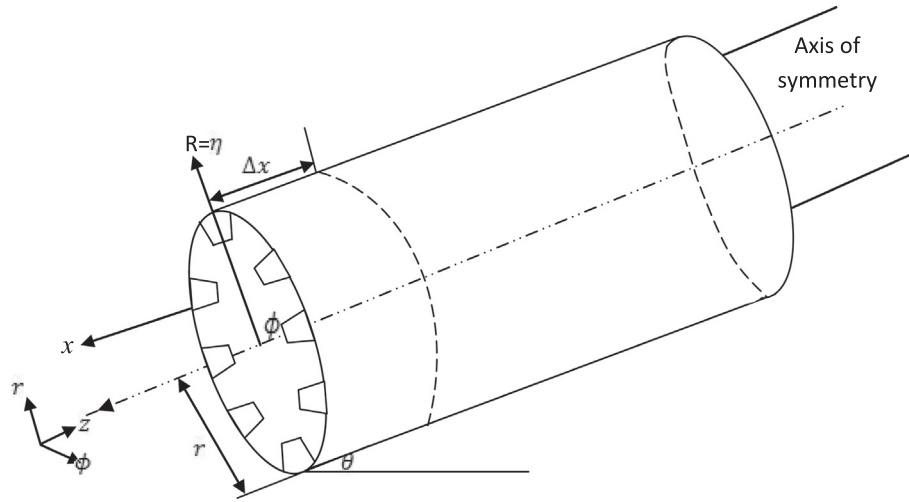


Fig. 1a Ducts of elliptic cross-section with internal longitudinal fins inclined at angle θ to the horizontal.

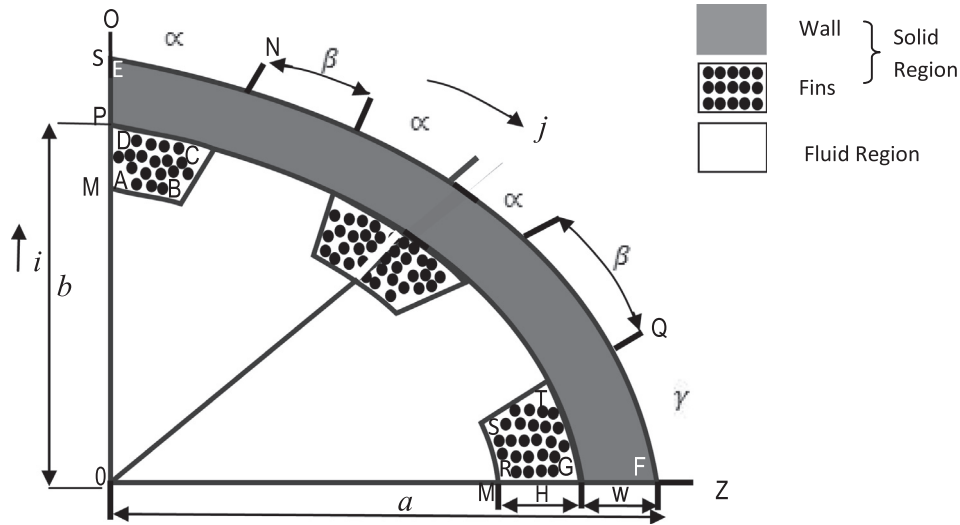


Fig. 1b Computational domain and applicable boundary conditions of the elliptic geometry.

2.3. Applicable boundary conditions

Based on the assumptions made, the boundary conditions for Eq. (2) are:

$$(a) \quad \frac{\partial u_x}{\partial \phi} = 0 \quad (9)$$

$$\frac{\partial^2 u_x}{\partial \phi^2} = 0 \quad \text{for} \quad \begin{cases} \phi = 0; & (0 \leq r \leq r_\phi - h) \\ \phi = \alpha + \beta + \gamma; & (0 \leq r \leq r_\phi - h) \end{cases} \quad (10)$$

(b) Interface between solid and fluid.

No-slip condition: $u = 0$

$$\therefore u = 0 \quad \text{for} \quad \begin{cases} 0 \leq \phi \leq \alpha; & r = r_\phi - h \\ \phi = \alpha; & r_\phi - h \leq r \leq r_\phi \\ \alpha \leq \phi \leq \alpha + \beta; & r \leq r_\phi \\ \phi = \alpha + \beta; & r_\phi - h \leq r \leq r_\phi \\ (\alpha + \beta) \leq \phi \leq (\alpha + \beta + \gamma); & r = r_\phi - h \end{cases} \quad (11)$$

Similarly, boundary condition for energy transport Eq. (8) is;

(a) Fluid region (symmetry condition)

$$\frac{\partial t_f}{\partial \phi} = 0 \quad \text{for} \quad \begin{cases} \phi = 0; & 0 \leq r \leq r_\phi - h \\ \phi = \alpha + \beta + \gamma; & 0 \leq r \leq r_\phi - h \end{cases} \quad (12)$$

(b) Solid region (symmetry condition)

$$\frac{\partial t_s}{\partial \phi} = 0 \quad \text{for} \quad \begin{cases} \phi = 0; & r_\phi - h \leq r \leq r_\phi + w \\ \phi = \alpha + \beta + \gamma; & r_\phi - h \leq r \leq r_\phi + w \end{cases} \quad (13)$$

(c) Outer wall of the channel

$$q_{r_\phi+w} = -ks \frac{\partial t}{\partial r} \bigg|_{r=r_\phi+w} \quad (14)$$

(d) Interface between solid and fluid.

$$t_f = t_s \quad \text{for} \quad \left\{ \begin{array}{ll} 0 \leq \phi \leq \alpha; & r = r_\phi - h \\ \phi = \alpha; & r_\phi - h \leq r \leq r_\phi \\ \alpha \leq \phi \leq \alpha + \beta; & r \leq r_\phi \\ \phi = \alpha + \beta; & r_\phi - h \leq r \leq r_\phi \\ (\alpha + \beta) \leq \phi \leq (\alpha + \beta + \gamma); & r = r_\phi - h \end{array} \right\} \quad (15)$$

2.4. Normalization parameters

The governing equations and the associated boundary conditions are non-dimensionalized by the following transformation parameters:

$$X = \frac{x}{aPe} \quad \eta = \frac{r_\phi}{a} \quad H = \frac{h}{a} \quad W = \frac{w}{a} \quad e = \sqrt{1 - (\varepsilon)^2}$$

$$\epsilon = \frac{b}{a} R = \frac{r}{a} \quad K^* = \frac{k_s}{k_f} \quad Pr = C_p \frac{\mu}{k}$$

$$Re = \frac{2\rho U_{\max} a}{\mu} \quad Pe = \frac{2U_{\max} a}{\alpha_t} \quad Pk = \frac{\rho g}{\partial p / a \partial x}$$

$$U = \frac{u_{\max}}{\frac{(a)^2}{\mu} \left(-\frac{\partial p}{\partial x} \right)} \quad D_h = \frac{d_h}{a} \quad T = \frac{t - t_0}{\frac{qa}{k_f}}$$

$$\eta = \sqrt{\frac{1 - e^2}{(1 - e^2 \sin \phi)}} \quad u_{\max} = \frac{1}{4} \left[\frac{a^2}{\mu} \left(-\frac{\partial p}{\partial x} \right) \right] \left[1 - \frac{r^2}{a^2} \right] \quad [19-23]$$

2.4.1. Normalized momentum transport equation

$$\frac{\partial^2 U}{\partial R^2} + \frac{1}{R} \frac{\partial U}{\partial R} + \frac{1}{R^2} \frac{\partial^2 U}{\partial \phi^2} = -1 - RePr \sin \theta \quad (16)$$

Normalized Boundary Condition for Eq. (16).

For symmetry condition

$$\frac{\partial^2 U}{\partial \phi^2} = 0 \quad @ \quad \left\{ \begin{array}{ll} \phi = 0; & 0 \leq R \leq \eta - H \\ \alpha + \beta + \gamma; & 0 \leq R \leq 1 - H \end{array} \right\} \quad (17)$$

For interface between solid and fluid

$$U = 0 \quad @ \quad \left\{ \begin{array}{ll} 0 \leq \phi \leq \alpha; & R = \eta - H \\ \phi = \alpha; & \eta - H \leq R \leq \eta \\ \alpha \leq \phi \leq \alpha + \beta; & R \leq \eta \\ \phi = \alpha + \beta; & \eta - H \leq R \leq \eta \\ \alpha + \beta \leq \phi \leq \alpha + \beta + \gamma; & R = 1 - H \end{array} \right\} \quad (18)$$

2.4.2. Normalized energy transport equation

$$\frac{\partial^2 T}{\partial R^2} + \frac{1}{R} \frac{\partial T}{\partial R} + \frac{1}{R^2} \frac{\partial^2 T}{\partial \phi^2} = \begin{cases} 2U \frac{\partial T}{\partial X} & \text{for fluid} \\ 0 & \text{for channel and fin} \end{cases} \quad (19)$$

Normalized Boundary Condition for Eq. (19)

For fluid region (Symmetry condition)

$$\frac{\partial T_f}{\partial \phi} = 0 \quad @ \quad \left\{ \begin{array}{ll} \phi = 0; & 0 \leq R \leq \eta - H \\ \phi = \alpha + \beta + \gamma; & 0 \leq R \leq 1 - H \end{array} \right\} \quad (20)$$

For Solid region (Symmetry condition)

$$\frac{\partial T_s}{\partial \phi} = 0 \quad @ \quad \left\{ \begin{array}{ll} \phi = 0; & \eta - H \leq R \leq \eta + W \\ \phi = \alpha + \beta + \gamma; & 1 - H \leq R \leq 1 + W \end{array} \right\} \quad (21)$$

For outer Wall of the Channel

$$\frac{\partial T}{\partial R} \Big|_{R=\eta+W} = \frac{-1}{k^*} \quad (22)$$

For Interface between solid and fluid

$$T_f = T_s \quad @ \quad \left\{ \begin{array}{ll} 0 \leq \phi \leq \alpha; & R = \eta - H \\ \phi = \alpha; & \eta - H \leq R \leq \eta \\ \alpha \leq \phi \leq \alpha + \beta; & R \leq \eta \\ \phi = \alpha + \beta; & \eta - H \leq R \leq \eta \\ \alpha + \beta \leq \phi \leq \alpha + \beta + \gamma; & R = 1 - H \end{array} \right\} \quad (23)$$

3. Solution techniques and computational procedure

3.1. Solution techniques

The governing equations with its associated boundary conditions are solved numerically by employing the iterative finite difference Gauss-Seidel technique. For the discretization of the governing equations, the central differences are used for the second-order derivatives, and the forward and backward differences are used for the first-order derivatives. Algebraic equations obtained for each variable are solved by simple form of Gauss-Seidel iterative procedure.

Convergence of the iteration procedures for the governing equations was achieved when the following criterion was satisfied [24].

$$\max \left\{ \frac{B(u)_{ik+1} - B(u)_{ik}}{B(u)_{ik}} \right\} \leq \varepsilon \quad (24)$$

where $B(u)$ represents the variable U or T , k is the iteration counter and $\varepsilon = 10^{-6}$.

Consequently, the Nusselt number, surface heat transfer coefficients were obtained from the numerical simulation once the convergence criterion was satisfied.

3.2. Computational procedure

Considering the symmetry of the flow, only a quarter of the whole cross-section was used in the numerical computation. The velocity fields were numerically evaluated from the momentum transport equations and its associated boundary conditions. The velocity obtained is then used for the evaluation of the energy transport equation for the fluid and solid regions.

A non-uniform grid pattern of 26×20 points was employed for the duct geometries with axis ratios 0.25, 0.33, 0.5, 0.8 and 1. All successive calculations of the present study are based on these grid sizes. To ensure the accuracy of the numerical result, numerical tests were carried out with different grid sizes to determine the effect of grid size in the numerical results, before arriving at an appropriate mesh size.

Computed values for Nusselt number, average velocity, bulk fluid temperature were evaluated by varying affecting parameters such as, Reynolds number, Prandtl number, fin height, channel inclinations and aspect ratios to determine their effects on heat transfer.

In solving the problem, a finite difference analogue was developed and results generated using a code written in Qb-64.

4. Evaluation of the mean Nusselt number

Nusselt number is a dimensionless parameter indicative of the ratio of heat flow rate by convection under unit temperature gradient to the heat flow rate by conduction process under unit temperature gradient through a thickness of D_h . The tendency of fins in providing heat transfer augmentation is critically analyzed by values of Nusselt number.

$$Nu_m = \frac{h(x)D_h}{k_f} \quad (25)$$

Hence, Nusselt number is given by:

$$\left. \frac{\partial T}{\partial R} D_h \right|_{R \leq 1} \quad (26)$$

where D_h is the hydraulic diameter of the elliptic geometry with internal longitudinal fins. It can be estimated from Eq. (14) as,

$$D_h = \frac{4 \left(\pi ab - \frac{FH\pi(2a-H)(\frac{1}{\cos \alpha} + 1)}{360} \right)}{\pi(a+b) + HF \left(\frac{2}{\cos \alpha} - \frac{\pi}{180} \left(\frac{1}{\cos \alpha} + 1 \right) \right)} \quad (27)$$

5. Results and discussion

5.1. Program validation

The program for implementing the scheme was written in Qb-64 a version of Basic programming language. In order to validate the modeling scheme, the unfinned duct results from Sakalis et al. [8] and Javery [25] were compared with a case of an unfinned duct modeled in the present study. The comparison is shown in Fig. 2 where the variation of mean Nusselt number as a function of aspect ratio is presented. As seen from Fig. 2, the comparison was found to be in good agreement. Having completed the grid independence study, and validation

of the modeling scheme by making comparison with results available in literature, the results for the finned ducts are given in Figs. 3–23.

5.2. Overall rate of heat transfer

Fig. 3 shows the plot of mean Nusselt number versus the numbers of Fin for selected aspect ratio. As can be seen from Fig. 3, the mean Nusselt number increases as the numbers of Fin increases with Fin numbers ($F = 8$) indicating the highest Nusselt number for all the geometries investigated. It is also worthy to note that there is a sharp decrease in mean Nusselt number value after Fin numbers ($F = 8$) for all the geometries investigated, indicating a critical condition. After this point, increasing the numbers of Fin reverses the rate of heat transfer.

Figs. 4 and 5 present the variation of mean Nusselt number with duct inclination for different fin heights and fin half angles respectively for $e = 0$. Fig. 4 indicates that Nusselt number steadily increases with duct inclination. Furthermore, easily noticeable in the plots of Fig. 4 is that Nusselt number increases as fin's height increases, indicating the effectiveness of fins in heat transfer augmentation.

Fig. 5 shows a significant increase in heat transfer in the range $0^\circ \leq \theta \leq 75^\circ$. While there is no significant change in Nusselt number value in the range $75^\circ \leq \theta \leq 90^\circ$. At this point, gravity no longer has effect on the flow. Worthy to note is the fact that inclination also enhances the rate of heat transfer to a certain limit until its effect becomes insignificant. Also, going by the revelation of Fig. 5, fin half angle ($\alpha = 3^\circ$) was found to give the highest value of Nusselt number among all others investigated in the geometry. This may be due to the fact that for large fin thickness, heat conduction is predominant in the flow regime. Hence, the lower the thickness of the fins the better the performance of heat transfer equipment.

Fig. 6 shows the variation of mean Nusselt number with duct inclinations for low and high Reynolds number flow regimes. For the geometry of Fig. 6, the variation of mean Nusselt number with duct inclination shows a similar trend to that experienced in Fig. 5 wherein Nusselt number increases steadily with duct inclination in the range $0^\circ \leq \theta \leq 75^\circ$ until inclination becomes insignificant in the range $75^\circ \leq \theta \leq 90^\circ$ where the Nusselt number attains its maximum value. At this point gravity no longer has an effect on the rate of heat transfer. However, from all Reynolds numbers examined, $Re = 1000$ provided the highest value of Nusselt number. Sig-

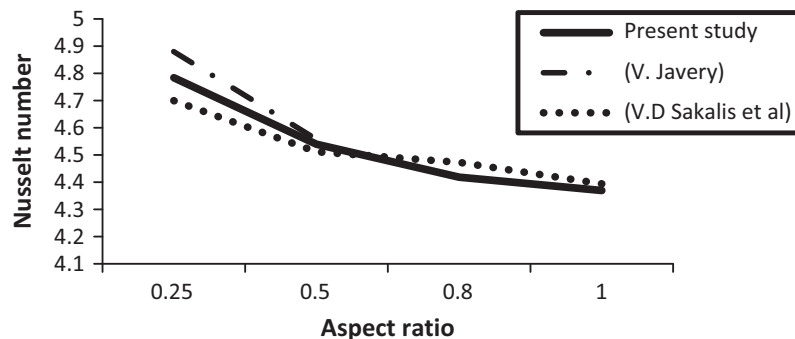


Fig. 2 Comparison of results from present work for the case of unfinned duct with Sakalis et al. [8] and Javery [25]. For $H = 0$, $Pr = 0.7$, $F = 0$, $\alpha = 0^\circ$, $W = 0.2$, $\theta = 0^\circ$.

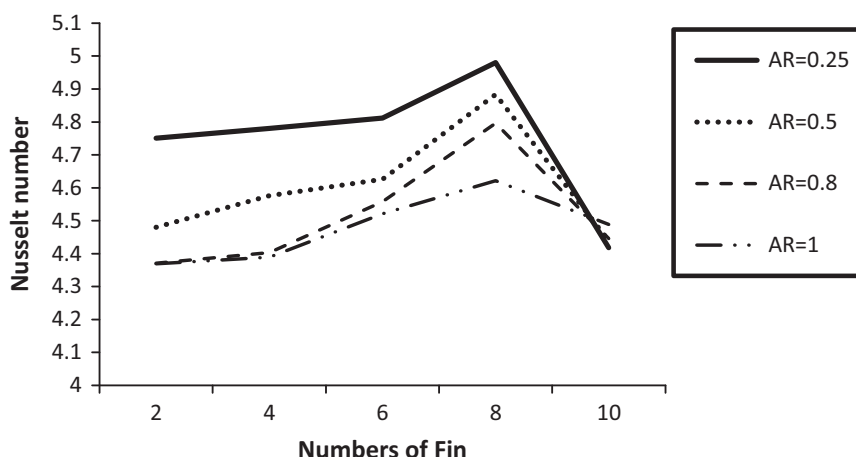


Fig. 3 Variation of Nusselt number versus numbers of fin for selected aspect ratio. For $H = 0.2$, $Pr = 0.73$, $\theta = 0^\circ$.

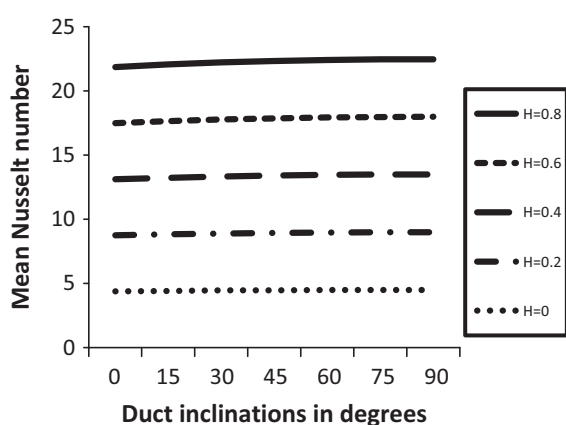


Fig. 4 Variation of mean Nusselt number with duct inclinations for different fin heights. For $e = 0$, $F = 8$, $Pr = 0.7$, $W = 0.2$, $\alpha = 3^\circ$.

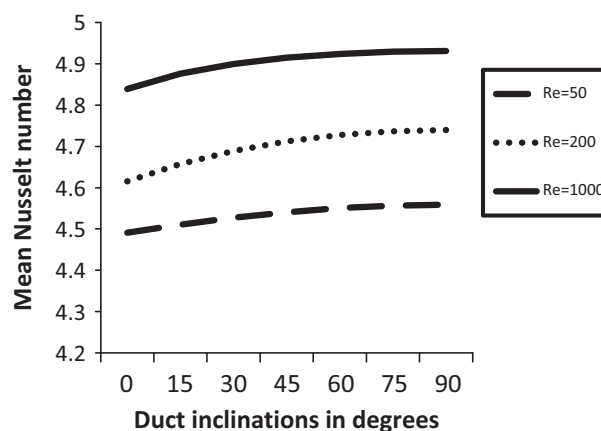


Fig. 6 Variation of mean Nusselt number with duct inclinations for low and high Reynolds number flow regimes. For $e = 0.8660$, $H = 0.2$, $Pr = 0.7$, $F = 8$, $\alpha = 3^\circ$, $W = 0.2$.

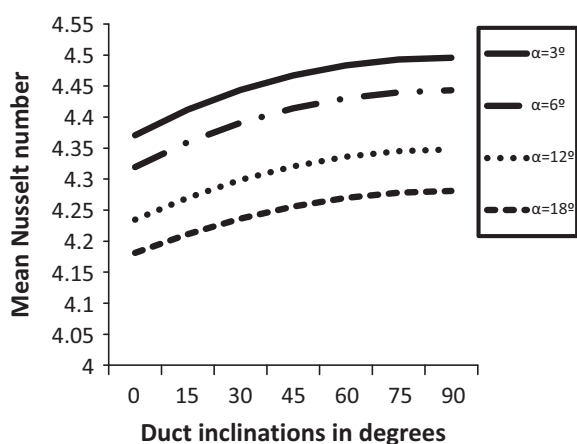


Fig. 5 Variation of mean Nusselt number with duct inclinations for different fin half angle. For $e = 0$, $F = 8$, $Pr = 0.7$, $W = 0.2$.

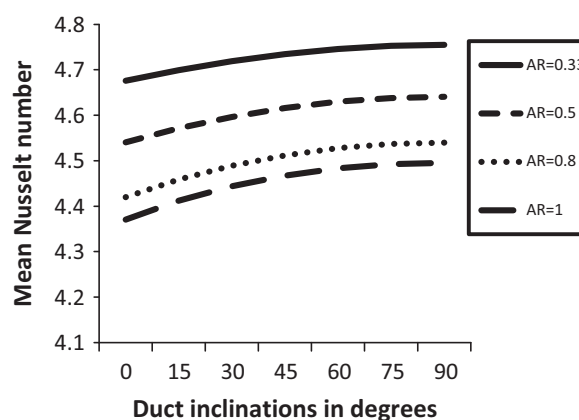


Fig. 7 Variation of mean Nusselt number with duct inclinations for various aspect ratio. For $H = 0.2$, $Pr = 0.7$, $F = 8$, $\alpha = 3^\circ$, $W = 0.2$.

nifying that the higher the Reynolds number the higher the rate of heat transfer.

The variation of mean Nusselt number against duct inclination for various geometries is considered in Fig. 7, for all

geometries investigated, Nusselt number increases steadily with duct inclination in the range $0^\circ \leq \theta \leq 75^\circ$. For $75^\circ \leq \theta \leq 90^\circ$, no significant change occurs in the value of

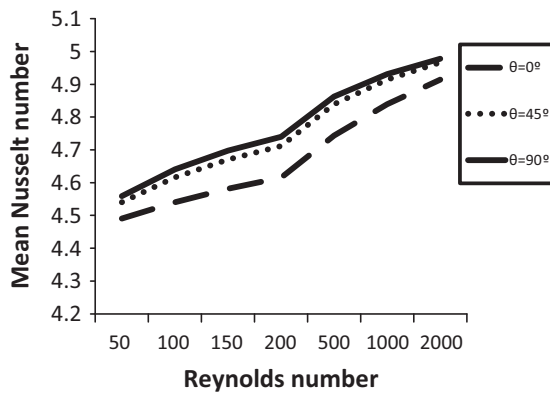


Fig. 8 Variation of mean Nusselt number as a function of Reynolds number for selected duct inclinations. For $e = 0.8660$, $H = 0.2$, $Pr = 0.7$, $F = 8$, $\alpha = 3^\circ$, $W = 0.2$.

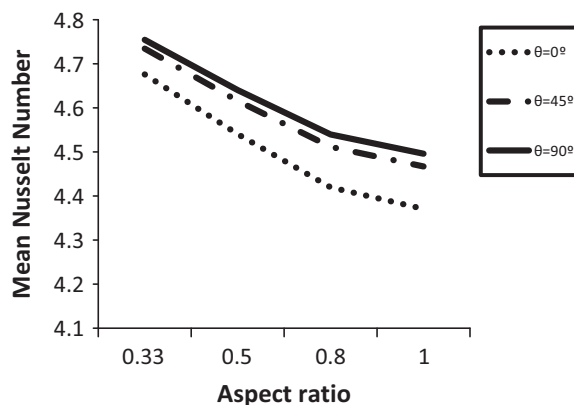


Fig. 9 Variation of mean Nusselt number as a function of aspect ratio for selected duct inclinations. For $H = 0.2$, $Pr = 0.7$, $F = 8$, $\alpha = 3^\circ$, $W = 0.2$.

Nusselt number which is at maximum at this range. A trend similar to that revealed by Figs 5 and 6 respectively. Observations from the plots clearly show that the higher the aspect ratio ϵ , the lower the value of the Nusselt number.

Fig. 8 shows the plot of mean Nusselt number as a function of Reynolds number for selected angles of inclination. A crit-

ical observation of Fig. 8 shows that although Nusselt number keeps increasing with increasing Reynolds number for all the angles of inclination considered, revelations from the plots indicate that beyond $\theta > 45^\circ$, the difference between the values of Nusselt numbers between $\theta = 45^\circ$ and $\theta = 90^\circ$ are small. This is an indication that there is little or no benefit derivable from the rate of heat transfer achieved by increasing the duct inclination above $\theta > 60^\circ$. It is also worthy to note that at $Re = 200$, for all the geometries investigated, regardless of orientation of the duct, the plot formed a slight trough indicating a critical flow condition.

However, observation from Fig. 9 shows that the lowest Nusselt number value was found at the circular geometry of the duct as compared to other geometries; indicating that the larger the aspect ratio the lesser the mean Nusselt number. This result was compared with the result of Hasan [26] within the laminar range of his experimental study.

The variation of mean Nusselt number against Prandtl number is presented in Fig. 10. It can be seen that in the range $0.7 \leq Pr \leq 5$, mean Nusselt number increases as the Prandtl number increases until it reaches a point of convergence at $Pr = 5$. At $Pr \geq 5$ Nusselt number is independent of the property of the fluid. This compares favourably with the results of Bello-Ochende and Adegun [20].

5.3. The velocity profile

Fig. 11 shows the variation of average velocity with aspect ratio for different number of Fins. It can be seen that, as the aspect ratio increases the average velocity increases, for all the numbers of Fin investigated, with exemption of Fin numbers ($F = 10$) that behaves contrary. This is indicative that flow will be impeded for numbers of Fin above ($F = 8$).

Fig. 12 presents the variation of average velocity with angle of inclination for different fin heights of the duct considered ($e = 0$). It is clear from the plots of Fig. 12, that for all finned and unfinned cases considered, average velocity increases with inclination in the range $0^\circ \leq \theta \leq 75^\circ$. At $75^\circ \leq \theta \leq 90^\circ$, the average velocities are maximum and identical. In this range, effect of inclination is insignificant. However, the un-finned duct can be seen to have the maximum value of average velocity and when internal fins are added, the region of maximum velocity shifted as observed from the plots.

Fig. 13 depicts the variation of average velocity with duct inclinations for different Reynolds number flow regimes

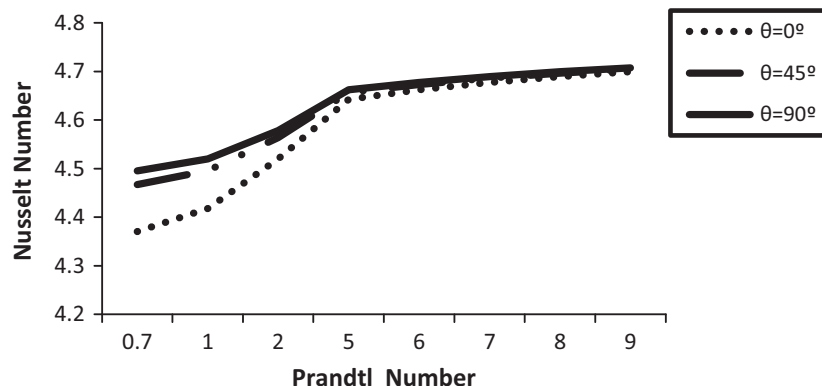


Fig. 10 Variation of Nusselt number with Prandtl number for selected duct inclinations. For $e = 0$, $H = 0.2$, $F = 8$, $\alpha = 3^\circ$, $W = 0.2$.

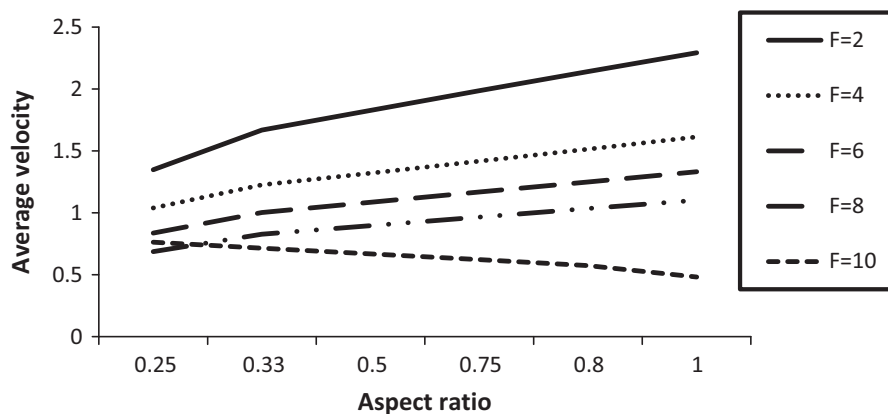


Fig. 11 Variation of average velocity as a function of aspect ratio for selected multiple fins. For $H = 0.2$, $Pr = 0.73$, $\theta = 0^\circ$.

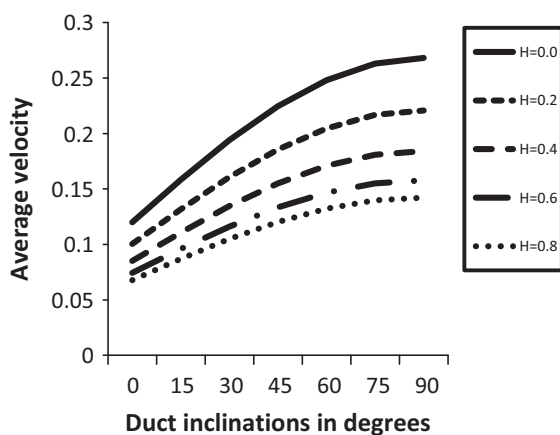


Fig. 12 Variation of average velocity with duct inclinations for different fin heights. For $e = 0$, $F = 8$, $Pr = 0.7$, $W = 0.2$, $\alpha = 3^\circ$.

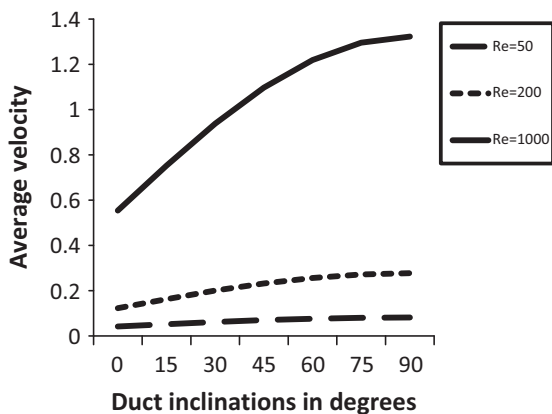


Fig. 13 Variation of average velocity with duct inclinations for low and high Reynolds number flow regimes. For $e = 0.8660$, $H = 0.2$, $Pr = 0.7$, $F = 8$, $\alpha = 3^\circ$, $W = 0.2$.

considered. It is very clear from the plots in Fig. 13 that at high Reynolds number the average velocity is distinctively higher as compared to the low Reynolds number cases. This interesting observation speaks volume about the influence, the duct incli-

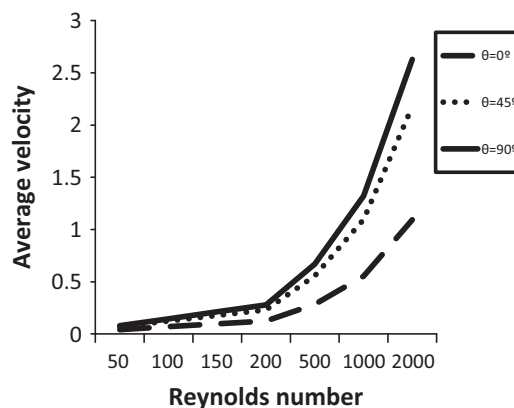


Fig. 14 Variation of average velocity as a function of Reynolds number flow regimes for selected duct inclinations. For $e = 0.8660$, $H = 0.2$, $Pr = 0.7$, $F = 8$, $\alpha = 3^\circ$, $W = 0.2$.

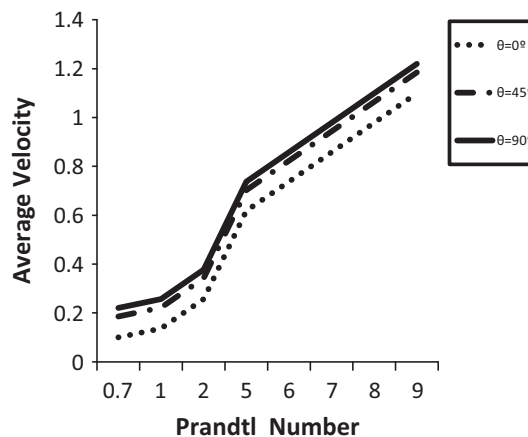


Fig. 15 Variation of average velocity with Prandtl number for selected duct inclinations. For $e = 0$, $H = 0.2$, $F = 8$, $\alpha = 3^\circ$, $W = 0.2$.

nation and Reynolds number can have on the average velocity of flow.

Fig. 14 shows the variation of average velocity as a function of Reynolds number for selected angles of duct inclination.

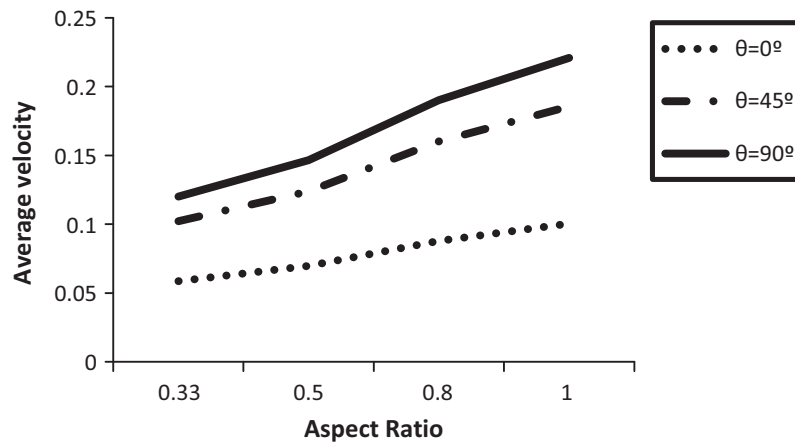


Fig. 16 Variation of average velocity with aspect ratio for selected duct inclinations. For $H = 0.2$, $Pr = 0.7$, $F = 8$, $\alpha = 3^\circ$, $W = 0.2$.

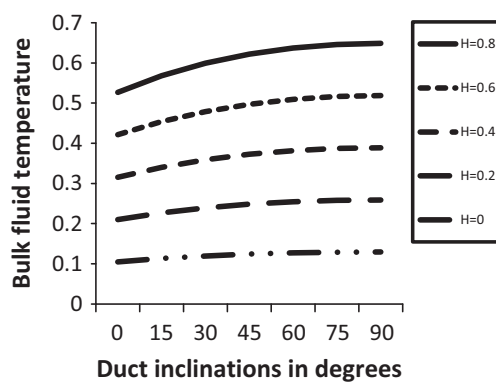


Fig. 17 Variation of bulk fluid temperature with duct inclinations for different fin height. For $e = 0$, $F = 8$, $Pr = 0.7$, $W = 0.2$, $\alpha = 3^\circ$.

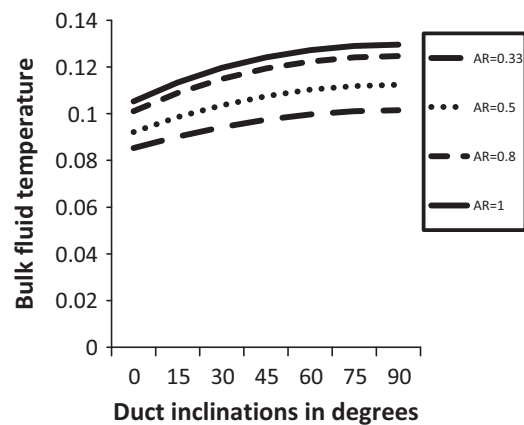


Fig. 19 Variation of bulk fluid temperature with duct inclinations for various aspect ratio. For $H = 0.2$, $Pr = 0.7$, $F = 8$, $\alpha = 3^\circ$, $W = 0.2$.

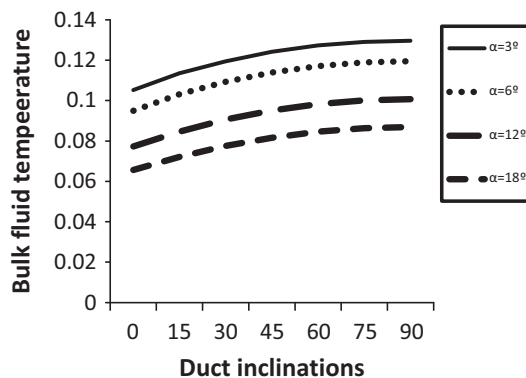


Fig. 18 Variation of bulk fluid temperature with duct inclinations for fin half angle. For $e = 0$, $F = 8$, $Pr = 0.7$, $W = 0.2$.

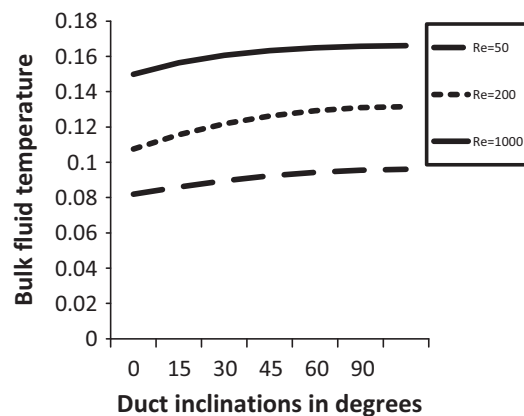


Fig. 20 Variation of bulk fluid temperature with duct inclinations for low and high Reynolds number flow regimes. For $e = 0.8660$, $H = 0.2$, $Pr = 0.7$, $F = 8$, $\alpha = 3^\circ$, $W = 0.2$.

For all the inclinations considered, it can be seen that in the range of $50 \leq Re \leq 200$, the average velocities are almost identical. Beyond this range, the effect of Reynolds number on the fluid flow becomes noticeable for all inclinations.

The variation of average velocity with Prandtl number for selected duct inclinations is presented in Fig. 15. It can be seen

that the average velocity increases slowly in the range of $0.7 \leq Pr \leq 2$, but there was a sharp increment of average velocity in the range of $2 \leq Pr \leq 5$. Interestingly, at $Pr \geq 5$,

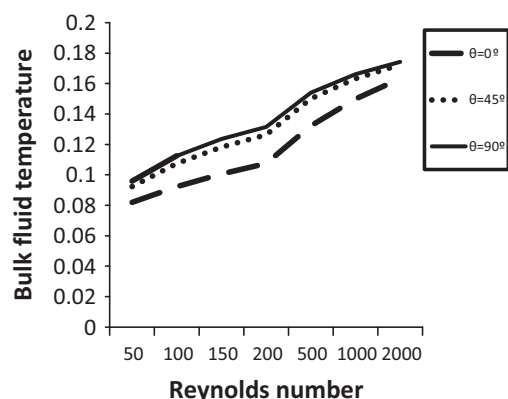


Fig. 21 Variation of bulk fluid temperature as a function of Reynolds number flow regimes for selected inclinations. For $e = 0.8660$, $H = 0.2$, $Pr = 0.7$, $F = 8$, $\alpha = 3^\circ$, $W = 0.2$.

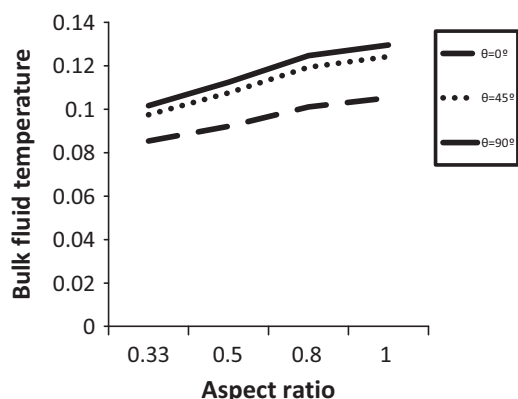


Fig. 22 Variation of bulk fluid temperature with aspect ratio for selected Duct inclinations. For $H = 0.2$, $Pr = 0.7$, $F = 8$, $\alpha = 3^\circ$, $W = 0.2$.

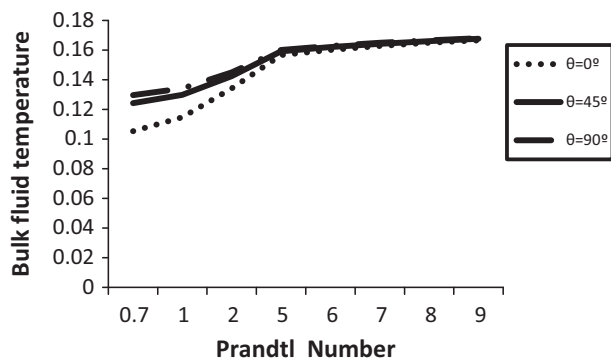


Fig. 23 Variation of bulk fluid temperature with Prandtl number for selected duct inclinations. For $e = 0$, $H = 0.2$, $F = 8$, $\alpha = 3^\circ$, $W = 0.2$.

the increment of average velocity was steady and constant. However, at $\theta > 45^\circ$, average velocity tends towards convergence with the vertical orientation signifying that inclinations are not too significant in this range.

Fig. 16 depicts the variation of average velocity with aspect ratio for different duct inclinations. Observation from Fig. 16

indicates that average velocity increases with increasing aspect ratio for all duct inclinations considered. Interestingly, the average velocity tends to converge at the higher inclination of the duct. This is an indication that there is a limit to which the duct should be inclined since no significant change would occur when increased beyond $\theta = 45^\circ$.

5.4. The temperature profile

The variations of bulk fluid temperature with duct inclinations are shown in Figs. 17–20. A close look at the figures reveals similar response of bulk fluid temperature with angle of duct inclination, as the bulk fluid temperature increases steadily with duct inclination in the range $0^\circ \leq \theta \leq 75^\circ$. Beyond this range, no significant changes could be seen particularly in the range of $75^\circ \leq \theta \leq 90^\circ$.

However, Fig. 17 indicates that with the addition of internal fins, the bulk fluid temperature increases progressively with fin height and the region of maximum bulk fluid temperature is at $H = 0.8$ due to the presence of the fins.

It can be seen clearly from Fig. 18 that fin half angle $\alpha = 3^\circ$ gives the highest value of bulk fluid temperature and the lowest value was seen at $\alpha = 18^\circ$ indicating that the higher the thickness of the fins the lesser the bulk fluid temperature. This may not be unconnected with the fact that conduction is more pronounced than convection in thicker fins as compared to thinner ones.

In Fig. 19 all the aspect ratios considered responded in a similar way towards inclination, but it can be seen that the higher the aspect ratio the higher the bulk fluid temperature.

Fig. 20 shows similar response of bulk fluid temperature with inclinations to that experienced in Figs. 17–19. It can be identified that increasing Reynolds number has a significant effect on the value of bulk fluid temperature, particularly at high Reynolds number flow regimes.

Fig. 21 shows the variation of bulk fluid temperature as a function of Reynolds number. A critical examination of Fig. 21 depicts that at $Re = 200$, a slight steep valley is observed for all conditions investigated. Prior to this point the bulk fluid temperature increases steadily with Reynolds number and after the critical point ($500 \leq Re \leq 2000$), the significant effect of Reynolds number continue to be seen. Also, it can be seen in Fig. 21 that at higher inclination angles, $\theta \geq 45^\circ$, Reynolds number effect on bulk fluid temperature becomes insignificant.

Fig. 22 shows the plot of bulk fluid temperature against aspect ratio for different duct inclinations. For all the aspect ratios considered, it can be observed that in the interval of $\theta \geq 45^\circ$, the bulk fluid temperature drifted gradually towards the vertical orientation. In the range of $\theta \geq 45^\circ$, it is worthy to note that the effect of gravity is not too significant. Fig. 23 shows that increasing the Prandtl number leads to increasing bulk fluid temperature for all duct inclinations considered until it reaches the point of convergence at $Pr = 5$. At this critical point, that is, $Pr \geq 5$ the bulk fluid temperature is independent of the properties of the fluid.

6. Conclusion

Based on this numerical study, and within the range of Reynolds numbers examined which varied from 50 to 2000 and

the angles of duct inclination which varied from 0° to 90° (at 15° interval). The following conclusions were made;

- In all the geometries investigated, optimum heat transfer augmentation occurred at Fin number ($F = 8$) and above this fin number ($F = 8$), there is a sharp drop in rate of heat transfer by convection. Hence, it is advisable not to increase the number of fin beyond ($F = 8$).
- For a given duct geometry, fin height, $H = 0.8$ is the optimum fin height that provides the best possible heat transfer augmentation. Further increment beyond this value, fin height could impede flow and slow down the heat transfer by convection.
- The enhancement of heat transfer by Reynolds number, geometries and angle of inclinations is clearly recognized.
- Over the Reynolds number range examined, the values of mean Nusselt number, average velocity and bulk fluid temperature at $75^\circ \leq \theta \leq 90^\circ$ are nearly equal.
- As the aspect ratio increases from 0.33 to 1, the mean Nusselt number decreases while the average velocity and the bulk fluid temperature increase.
- For $Pr \geq 5$, Nusselt number and bulk fluid temperature are found to be independent of the properties of the fluid.
- The exact performance of duct depends on the geometry and other operating conditions.

References

- [1] A. Hassan, K. Siren, Performance investigation of plan and finned tube evaporatively cooled heat exchanger, *Appl. Therm. Eng.* 23 (3) (2003) 325–340.
- [2] T. Ota, S. Aiba, T. Tsuruta, M. Kaga, Forced convection heat transfer from an elliptic cylinder, *Bull. Jpn. Soc. Mech. Eng.* 26 (212) (1983) 262–267.
- [3] T. Ota, H. Nishiyama, Y. Taoka, Heat transfer and flow around an elliptic cylinder, *Int. J. Heat Mass Transf.* 27 (10) (1984) 1771–1779.
- [4] M.A. Teamah, W.M. El-Maghlany, A.A. Hanafy, Effect of Reynolds and Prandtl numbers on laminar forced convection in horizontal pipe partially filled with porous material, *Eur. J. Sci. Res.* 66 (2) (2011) 171–186, ISSN 1450-216X.
- [5] A.J.L. Foong, N. Ramesh, T.T. Chandratilleke, Laminar convective heat transfer in a microchannel with internal longitudinal fins, *Int. J. Therm. Sci.* 48 (2009) 1908–1913.
- [6] M. Bahrani, M.M. Yovanovich, J.R. Culham, Pressure drop of laminar fully developed flow in microchannels of arbitrary cross-section, *ASME J. Fluid Eng.* 128 (2006) 1036–1044.
- [7] M. Bahrani, M.M. Yovanovich, J.R. Culham, A novel solution for pressure drop in singly connected microchannels of arbitrary cross-section, *Int. J. Heat Mass Transf.* 50 (2007) 2492–2502.
- [8] V.D. Sakalis, P.M. Hatzikostantinou, N. Kafou Sias, Thermally developed flow in elliptic ducts with axially variable wall temperature flow in elliptic ducts with axially variable wall temperature distribution, *Int. J. Heat Mass Transf.* 45 (2002) 25–35.
- [9] G.B. Schubauer, Air Flow in a Separating Laminar Boundary Layer, NACA TR-527, December 1934.
- [10] G.B. Schubauer, Air Flow in a Boundary Layer of an Elliptic Cylinder, NACA TR-652, August 1939.
- [11] S.M. Richardson, Leveque solution for flow in an elliptical duct, *Lett. Heat Mass Transf.* 7 (1980) 353–362.
- [12] I. Alam, P.S. Ghoshdastidar, A study of heat transfer effectiveness of circular tubes with internal longitudinal fins having tapered lateral profiles, *Int. J. Heat Mass Transf.* 45 (6) (2002) 1371–1376.
- [13] C.P. Jackson, A finite-element study of the onset of vortex shedding in flow past variously shaped bodies, *J. Fluid Mech.* 182 (1987) 23–45.
- [14] S.J.D. D'Allesio, S.C.R. Dennis, A vorticity model for viscous flow past a cylinder, *Comput. Fluids* 23 (2) (1994) 279–293.
- [15] S.J.D. D'Allesio, S.C.R. Dennis, Steady laminar forced convection from an elliptic cylinder, *J. Eng. Math.* 29 (2) (1995) 181–193.
- [16] S.J.D. D'Allesio, Steady, unsteady and linear stability of flow past an elliptic cylinder, *Can. Appl. Math. Quart.* 4 (6) (1996) 341–379.
- [17] Z.F. Dong, M.A. Ebadian, A numerical analysis of thermally developing flow in elliptical duct with internal fins, *Int. J. Heat Fluid Flow* 12 (2) (1991) 166–172.
- [18] P.R. Papadopoulos, P.M. Hatzikostanti, Numerical investigation of the thermally developing flows in a curved elliptic duct with internal fins, *J. Heat Transf.* 129 (6) (2006) 759–762.
- [19] M. Bahrani, A. Tamayol, Analytical solutions for laminar fully developed flow in microchannel with non-circular cross-section, in: ASME Fluids Engineering Division Summer Meeting August 2–5, 2009, Vail, Colorado, USA, 2009.
- [20] F.L. Bello-Ochende, I.K. Adegun, Scale analysis of laminar forced convection within the entrance of heated regular polygonal tubes, *J. Math. Assoc. Nigeria (ABACUS)* 31 (2A) (2004) 28–41.
- [21] M.N. Ozisik, Boundary Value Problems of heat Conduction, Dover Publications Inc., Mineola, New York, 2002, pp. 395–416.
- [22] Y. Peles, A. Kosar, C. Mishra, C. Kuo, B. Schneider, Forced Convective Heat Transfer across a Pin Fin Micro Heat Sink, *Int. J. Heat Mass Transf.* 48 (2005) 3615–3627.
- [23] M.F. Marinnet, S. Tardu, Convective Heat Transfer, Solved Problems. <www.wiley.com> (accessed on June 12, 2012).
- [24] J.P. Holman, Heat Transfer, Tata Mc Graw-Hill ed., Eight S.I. Metric Edition, 2002, pp. 101–120.
- [25] V. Javery, Analysis of laminar thermal entrance region of elliptical and rectangular ducts with kantorowish method, *Warme – Und Stoffubertragung A* (1976) 85–98.
- [26] A.A. Hasan, Performance Analysis of Heat Transfer Process from Wet and Dry surfaces; Cooling Towers and Heat Exchangers Dissertation from the degree of Doctor of Science in Technology, Department of Mechanical Engineering, Helsinki University of Technology, Espoo, Finland, 2005.