



Determination of temperature distribution in squeeze cast aluminium using the semi-empirical equations' method

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ARTICLE INFO

Article history:

Received 8 January 2009

Received in revised form 8 June 2009

Accepted 11 June 2009

Keywords:

Applied pressure
Pre-heat temperature
Delay time
Temperature

ABSTRACT

The effects of parameters such as applied pressure, die pre-heat temperature, delay time and retention time of applied pressure on the temperature distributions within the casting have been considered. The results using this method were compared with those of experimental and heat transfer differential equations' methods and found to be about 1.7% higher in value. The semi-empirical equations' method can be used to predict the temperature distribution of other metals and alloys at any location, if the empirical equations relating the heat transfer coefficients to applied pressures and solidifying temperatures for their three stages of solidification are known.

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1. Introduction

Squeeze casting is regarded as a combination of gravity die-casting and closed die forging in a single process. The process has been known in recent times as an advanced casting process where products with near-zero gas porosity, reduced metal wastage and better mechanical properties are obtained. The pressure applied on the solidifying molten metal plays a major role in the quality of the squeeze cast products that are produced. Application of high pressure on the solidifying metal enhances the process of heat transfer coefficients greatly, due mainly to direct contact, the liquid metal has with the die inner surface. The casting pressure applied on the melt during solidification is held on the melt, while the metal solidifies. Squeeze casting employs low die filling velocity with minimum turbulence and high-applied pressure to produce good quality products. The improved properties of squeeze cast products have been attributed to the flexibility of the process.

Several authors have reported the superiority of properties of squeeze cast products over the conventional casting process. This is brought about by the flexibility of the squeeze cast parameters. Casting parameters, as studied by different authors that have been known to have significant influence on the squeeze cast products are applied pressure, die pre-heat temperature and melt temperature. In the work of Maleki et al. (2006), squeeze cast-

ing products decrease in density at lower applied pressure and increases with higher applied pressures. They concluded that this increase becomes less significant at applied pressure of 100 Mpa. In the report, it was also observed that with increase in applied pressure, the cast specimens' grain size becomes smaller coupled with improved hardness. The melt temperatures of between 690° and 660 °C might just be enough for squeeze casting of aluminium and aluminium alloys respectively, as observed by Yang (2003). It was reported by Yang (2007) that the shorter the solidification time of casting, the higher is its density, yield strength and ultimate tensile strength. Chattopadhyay (2007) applied a fixed enthalpy formulation to model solidification for a cylindrical geometry and found that solidification time decreased asymptotically with increase in heat transfer coefficient. This phenomenon, he posited was due to higher applied pressure on the solidifying cast product. Investigating varying parameters of squeeze casting process and its application, Ghomashchi and Vikhrov (2000) noted that many advances in squeeze casting of aluminium and low melting metals are leading to the application of the process to a high temperature alloys. Santos et al. (2001) observed an increase in the heat transfer coefficients with increasing melt superheat for horizontal directional solidification and a reverse for a vertical upward directional solidification.

The aim of the present study is to determine temperature distribution using semi-empirical equations' method based on the applications of heat transfer equations. The experimental empirical equations of heat transfer coefficients as obtained by Aweda and Adeyemi (2009) related the applied pressure and solidification temperature at the three stages of solidification, namely; super-heat, solid-liquid (mushy zone) and solidified stages to heat transfer

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Nomenclature

C	specific heat
d	spatial step size
d_0	thickness of steel mould
G	grid point at the steel mould–solidified metal interface
h^*	convective heat transfer of the surrounding atmosphere around the steel mould
h_0	initial height of cast specimen metal (without pressure application)
h_c	height of solidified specimen metal after pressure application
I	grid point
i	time step
K	Thermal conductivity
L_f	latent heat
M	grid point at the solidified metal–liquid molten metal interface
MM	grid point at mushy zone–liquid metal interface
N	grid point at the centre of the solidifying cast specimen metal
P	pressure
R	internal radius of steel mould
R_0	external radius of steel mould
r	radial axis/coordinate
z	axial axis/coordinate
ρ	density of aluminium
T	temperature
T_{MM}	melting temperature of cast (aluminium) metal
T_p	pouring temperature of liquid metal (super heat temperature)
t	time
X_i	thickness of solidified metal at a specified grid point–radial direction
δ	time interval
α	thermal diffusivity

Subscripts

L	liquid metal
st	steel mould
S	solidified metal
CS	completion of solidification
SL	solid–liquid amorphous portion
a	ambient

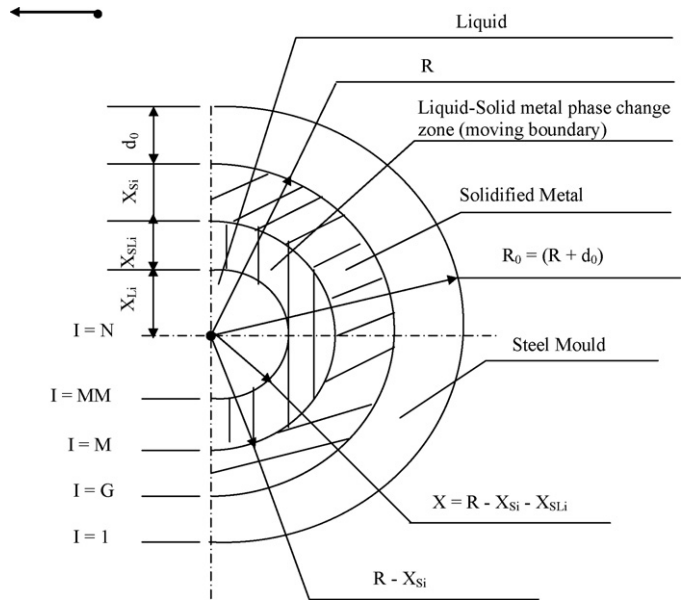


Fig. 1. Schematic representation of solidification front in one dimension.

2. Theoretical analysis

The heat transfer coefficient h , is mainly affected by the applied pressure and casting temperature as [Hu and Yu \(2002\)](#) confirmed while simulating squeeze casting of magnesium alloy. [Aweda and Adeyemi \(2009\)](#) presented empirical equations, relating heat transfer coefficient with the applied pressure and casting temperature within the solidified portion, liquid portion and the mushy zones during squeeze casting of commercial aluminium. The empirical heat transfer coefficients' equations were formulated from experimental data obtained from the solidification of aluminium in the cylindrical steel mould with varying applied pressures for three distinct stages of solidification given above. These empirical equations developed were incorporated into the heat transfer equations and solved numerically.

This model was developed based on one-dimensional heat flow in cylindrical steel mould. The one-dimensional heat flow is adequate due to the size of the cast products where the specimen's height is very small compared to its diameter. An algorithm was developed to monitor the solidifying temperature of squeeze cast aluminium in a cylindrical steel mould. Heat transfer equations generated were formulated for the steel mould, the cast metal and the interfaces. In the numerical solution effects of melt temperature, die-preheat temperature and applied pressure were considered. The heat transfer equations generated were solved using the finite difference method. The Beck's ([Beck, 1970](#)) inverse heat conduction problem based on non-linear estimation technique was adopted to determine the values of interface heat transfer coefficients, as a function of time during solidification of squeeze casting. Solidification of squeeze casting of aluminium involves phase changes and, therefore, thermal properties of aluminium are temperature dependent, making the inverse heat conduction problem non-linear.

2.1. Semi-empirical equations based heat transfer equations

The numerical simulation was based on solidification of molten metal in the cylindrical steel mould and the process is systematic. The heat transfer equations in the cast metal are due to conduction and convection. [Fig. 1](#) is the schematic diagram of the solidification front in one-dimension, showing the liquid zone, the mushy

coefficients. In that work the empirical equations for each of this distinct phase changes were determined for the mean average values of heat transfer coefficients as a function of applied pressures and solidification temperatures. This method can be applied to determine the temperature distributions of any metal when the heat transfer coefficients as a function of solidifying temperature and applied pressure are known at any of the phases. The semi-empirical method is then used to determine the temperature distributions within the squeeze cast region of aluminium under the casting parameters such as die pre-heat temperature, applied pressure, delay time and retention time of applied pressure. The results obtained by this method were then compared with the numerical (the heat transfer equations' methods) and experimental methods.

(liquid–solid metal phase change zone) and the solidified zones all as moving boundaries during the process of metal solidification. The governing heat transfer equations in one dimension obtained for squeeze casting of commercially pure aluminium in the interface and in the cast region are represented by heat transfer equations.

2.1.1. In the steel mould–atmosphere interface, $I = 1$

The heat conducted from the molten metal to the steel mould is convected to the ambient through outer surface of the mould at $I = 1$ represented mathematically:

$$\frac{\partial T_{st}}{\partial t} = \frac{2K_{st}}{\rho_{st}C_{st}d_{st}} \frac{\partial T_{st}}{\partial r} - \frac{2h^*}{\rho_{st}C_{st}d_{st}} (T_i^j - T_a) \quad (1a)$$

within the boundary defined by:

$$r_{st} = R_0 \quad (1b)$$

$$T_{st} \geq T_a \quad (1c)$$

where, h^* is the combined convective and radiative heat transfer coefficient.

The finite difference form of Eq. (1a) is:

$$T_i^{j+1} = \left[1 - \frac{2\delta K_{st}}{d_{st}^2 \rho_{st} C_{st}} - \frac{2\delta h^*}{d_{st} \rho_{st} C_{st}} \right] T_i^j + \frac{2\delta K_{st}}{d_{st}^2 \rho_{st} C_{st}} T_{i+1}^j + \frac{2\delta h^*}{d_{st} \rho_{st} C_{st}} T_a^j \quad (1d)$$

2.1.2. In the steel mould region, $I = 2$ to $(G - 1)$

The heat transfer within the steel mould material is by conduction for the region within ($I = 2$) to $(G - 1)$ and is expressed as:

$$\frac{\partial T_{st}}{\partial t} = \frac{K_{st}}{\rho_{st}C_{st}} \frac{\partial^2 T_{st}}{\partial r^2} + \frac{K_{st}}{\rho_{st}C_{st}r_{st}} \frac{\partial T_{st}}{\partial r} \quad (2a)$$

This equation is applicable within the region defined by:

$$R \leq r_{st} \leq R_0$$

The finite difference form of Eq. (2a) within the steel mould is:

$$T_i^{j+1} = \left[1 - \frac{2\delta \alpha_{st}}{d_{st}^2} - \frac{\delta \alpha_{st}}{d_{st} r_{st}} \right] T_i^j + \left[\frac{\delta \alpha_{st}}{d_{st}^2} + \frac{\delta \alpha_{st}}{d_{st} r_{st}} \right] T_{i+1}^j + \frac{\delta \alpha_{st}}{d_{st}^2} T_{i-1}^j \quad (2b)$$

where

$$\alpha_{st} = \frac{K_{st}}{\rho_{st}C_{st}} \quad (2c)$$

2.1.3. First time analysis, $I = G$ to N

For the first time analysis, the aluminium specimen is considered to be in the molten stage and at super heat temperature, $T_L = 720^\circ\text{C}$. The semi-empirical equation at the first time analysis is introduced into the governing equation thus:

$$\rho_L C_L r_{Li} \Delta r_L \frac{\partial T_L}{\partial t} = h_{L(i+1)}(r_{Li} - \Delta r_L)(T_{i+1}^j - T_i^j) - h_{L(i-1)}(r_{Li} + \Delta r_L)(T_i^j - T_{i-1}^j) \quad (3a)$$

where, h_L is the empirical equation, relating the heat transfer coefficients to temperature, T_i^j , and at applied pressure, P on the molten metal as presented by Aweda and Adeyemi (2009).

The empirical equations at the first time analysis within the cast temperature above 660°C are expressed as:

$$h_{L(i+1)} = 2.769T_{i+1}^j + 2.518P + 988.921 \quad (3b)$$

$$h_{L(i-1)} = 2.769T_{i-1}^j + 2.518P + 988.921 \quad (3c)$$

The finite difference form of Eq. (3a) is expressed as:

$$T_i^{j+1} = \left[1 - \frac{\delta h_{L(i+1)}(r_{Li} - d_L)}{\rho_L C_L r_{Li} d_L} - \frac{\delta h_{L(i-1)}(r_{Li} + d_L)}{\rho_L C_L r_{Li} d_L} \right] T_i^j + \frac{\delta h_{L(i+1)}(r_{Li} - d_L)}{\rho_L C_L r_{Li} d_L} T_{i+1}^j + \frac{\delta h_{L(i-1)}(r_{Li} + d_L)}{\rho_L C_L r_{Li} d_L} T_{i-1}^j \quad (3d)$$

This equation is subjected to the boundary conditions with:

$$T_L = T_P = 720^\circ\text{C} \quad (3e)$$

$$r = R \quad (3f)$$

2.1.4. In the solidified metal/steel mould interface, $I = G$

The governing equation at the solidified metal/steel mould interface is expressed while incorporating the semi-empirical equations at this interface thus:

$$h_{S(i+1)}(r_{Si} - \Delta r_S)(T_{i+1}^j - T_i^j) + \frac{1}{2} \rho_S C_S r_{Si} \Delta r_S \frac{\partial T_S}{\partial t} = \frac{1}{2} \rho_{St} C_{St} r_{Sti} \Delta r_{St} \frac{\partial T_{St}}{\partial t} + K_{St} r_{St} \frac{\partial T_{St}}{\partial t} \quad (4a)$$

The equation is subjected to the boundary condition within

$$T_S \leq 660^\circ\text{C} \quad (4b)$$

The finite difference formulation of Eq. (4a) is expressed as:

$$T_i^{j+1} = a \left[2\delta h_{S(i+1)}(r_{Si} - d_S) + \rho_S C_S r_{Si} d_S - \rho_S C_S r_{Si} d_S \frac{I - G}{M - G} \frac{X_{Si}^{j+1} - X_{Si}^j}{d_S} - \rho_{St} C_{St} r_{Sti} d_{St} + \frac{2\delta K_{St} r_{St}}{d_{St}} \right] T_i^j - \left[2\delta h_{S(i+1)}(r_{Si} - d_S) - \rho_S C_S r_{Si} d_S \frac{I - G}{M - G} \frac{X_{Si}^{j+1} - X_{Si}^j}{d_S} \right] T_{i+1}^j - \frac{2a\delta K_{St} r_{St}}{d_{St}} T_{i-1}^j \quad (4c)$$

where,

$$a = \frac{1}{\rho_S C_S r_{Si} d_S - \rho_{St} C_{St} r_{Sti} d_{St}} \quad (4d)$$

h_S is the semi-empirical equation relating heat transfer coefficients to temperature, T_{i+1}^j and at applied pressure, P on the solidified metal.

$$h_{S(i+1)} = 3.081T_{i+1}^j + 1.303P - 232.942 \quad (4e)$$

2.1.5. In the solidified metal region, $I = (G + 1)$ to $(M - 1)$

The heat transfer equation at the solidified portion is expressed as:

$$h_{S(i+1)}(r_{Si} - \Delta r_S)(T_{i+1}^j - T_i^j) - h_{S(i-1)}(r_{Si} + \Delta r_S)(T_i^j - T_{i-1}^j) = \rho_S C_S r_{Si} \Delta r_S \frac{\partial T_S}{\partial t} \quad (5a)$$

Eq. (5a) is obtainable within the boundary condition:

$$T_S \leq 500^\circ\text{C} \quad (5b)$$

and applicable within the region defined by:

$$(R - X_{Si}^j) \leq r_S \leq R$$

The finite difference formulation of Eq. (5a) is expressed thus:

$$T_i^{j+1} = b \left[\rho_S C_S r_{Si} d_S - \rho_S C_S r_{Si} d_S \frac{I-G}{M-G} \frac{X_{Si}^{j+1} - X_{Si}^j}{d_S} \right. \\ \left. - \delta h_{S(i+1)}(r_{Si} - d_S) T_i^j - \delta h_{S(i-1)}(r_{Si} + d_S) \right] T_i^j + b \\ \times \left[\delta h_{S(i+1)}(r_{Si} - d_S) + \rho_S C_S r_{Si} d_S \frac{I-G}{M-G} \frac{X_{Si}^{j+1} - X_{Si}^j}{d_S} \right] \\ \times T_{i+1}^j + b \delta h_{S(i-1)}(r_{Si} + d_S) T_{i-1}^j \quad (5c)$$

$$b = \frac{1}{\rho_S C_S r_{Si} d_S} \quad (5d)$$

where

$$h_{S(i-1)} = 3.081 T_{i-1}^j + 1.303 P - 232.942 \quad (5e)$$

2.1.6. In the liquid/solid phase change boundary condition, $I = MM$

For the phase change boundary condition, the equation monitoring the moving front in the liquid/solidified metal phase is expressed as:

$$\rho_L L_f \frac{dX_{SL}^j}{dt} = K_L \frac{\partial T_L}{\partial r} - K_S \frac{\partial T_S}{\partial r} \quad (6a)$$

within the region defined by:

$$r = R - (X_{Si}^j + X_{SLi}^j) \quad (6b)$$

and applicable at melting temperature defined by:

$$T_{SL} \leq 660^\circ\text{C} \quad (6c)$$

The finite difference formulation of Eq. (6a) is:

$$X_{SL}^{j+1} = X_{SL}^j - \left[\frac{\delta K_L}{\rho_L L_f h_{PL}} + \frac{\delta K_S}{\rho_L L_f h_{PS}} \right] T_i^j + \frac{\delta K_L}{\rho_L L_f h_{PL}} T_{i+1}^j + \frac{\delta K_S}{\rho_L L_f h_{PS}} T_{i-1}^j \quad (6d)$$

2.1.7. In the amorphous liquid/solid metal zone, $I = (M+1)$ to $(MM-1)$,

The heat transfer equation at this interface with empirical equation introduced into it is given as:

$$h_{SL(i+1)}(r_{SLi} - \Delta r_{SL})(T_{i+1}^j - T_i^j) - h_{SL(i-1)}(r_{SLi} + \Delta r_{SL})(T_i^j - T_{i-1}^j) \\ = \rho_{SL} C_{PSL} r_{SLi} \Delta r_{SL} \frac{\partial T_{SL}}{\partial t} \quad (7a)$$

This equation is applicable within the region defined by:

$$[R - (X_{Si}^j + X_{SLi}^j)] \leq r_{SL} \leq (R - X_{Si}^j)$$

Over the temperature range of:

$$500^\circ\text{C} \leq T_{SL} \leq 660^\circ\text{C}$$

The finite difference form is:

$$T_i^{j+1} = C \left[\rho_{SL} C_{PSL} r_{SLi} d_{SL} - \rho_{SL} C_{PSL} r_{SLi} d_{SL} \frac{X_{SLi}^{j+1} - X_{SLi}^j}{d_{SL}} \right. \\ \left. - \rho_{SL} C_{PSL} r_{SLi} d_{SL} \frac{I-M}{MM-M} \frac{X_{SLi}^{j+1} - X_{SLi}^j}{d_{SL}} - \delta h_{SL(i+1)}(r_{SLi} - d_{SL}) \right. \\ \left. - \delta h_{SL(i-1)}(r_{SLi} + d_{SL}) \right] T_i^j + C \left[\rho_{SL} C_{PSL} r_{SLi} d_{SL} \frac{X_{SLi}^{j+1} - X_{SLi}^j}{d_{SL}} \right. \\ \left. + \rho_{SL} C_{PSL} r_{SLi} d_{SL} \frac{I-M}{MM-M} \frac{X_{SLi}^{j+1} - X_{SLi}^j}{d_{SL}} + \delta h_{SL(i+1)}(r_{SLi} - d_{SL}) \right] \\ \times T_{i+1}^j + C \delta h_{SL(i-1)}(r_{SLi} + d_{SL}) T_{i-1}^j \quad (7b)$$

$$C = \frac{1}{\rho_{SL} C_{SL} r_{SLi} d_{SL}} \quad (7c)$$

The empirical equations for heat transfer coefficients, h_{SL} are defined by:

$$h_{SL(i+1)} = 10.420 T_{i+1}^j + 5.641 P - 4176.022 \quad (7d)$$

$$h_{SL(i-1)} = 10.420 T_{i-1}^j + 5.641 P - 4176.022 \quad (7e)$$

2.1.8. In the liquid portion, $I = (MM+1)$ to $(N-1)$

The rate of internal energy generated within the liquid region equals the difference in the rate of energy dissipated at the boundary of the liquid. Therefore,

$$h_{L(i+1)}(r_{Li} - \Delta r_L)(T_{i+1}^j - T_i^j) - h_{L(i-1)}(r_{Li} + \Delta r_L)(T_i^j - T_{i-1}^j) \\ = \rho_L C_{PL} r_{Li} \Delta r_L \frac{\partial T_L}{\partial t} \quad (8a)$$

Eq. (8a) is applicable within the region defined by:

$$0 \leq r_L \leq [R - (X_{Si}^j + X_{SLi}^j)]$$

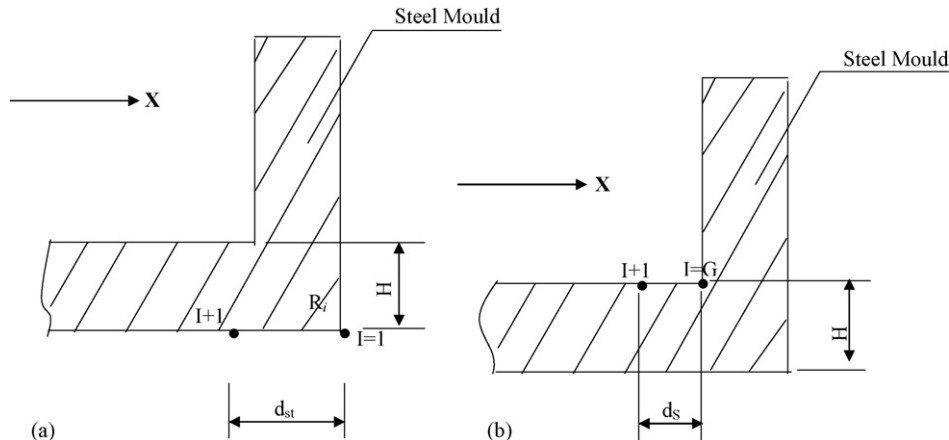


Fig. 2. a. Heat balance at the steel mould/atmosphere interface ($I = 1$). b. Heat balance at the solidified metal/steel mould interface ($I = G$).

and within the temperature boundary condition:

$$660^{\circ}\text{C} \leq T_L \leq 720^{\circ}\text{C}$$

The finite formulation of Eq. (8a) is thus:

$$T_i^{j+1} = d \left[\rho_L C_{PL} r_{Li} d_L - \rho_L C_{PL} r_{Li} d_L \frac{I - MM}{N - MM} \frac{X_{Si}^{j+1} - X_{Si}^j}{d_L} \right. \\ \left. + \rho_L C_{PL} r_{Li} d_L \frac{I - MM}{N - MM} \frac{X_{SLi}^{j+1} - X_{SLi}^j}{d_L} \right. \\ \left. - \delta h_{L(i+1)}(r_{Li} - d_L) - \delta h_{L(i-1)}(r_{Li} + d_L) \right] T_i^j + d \\ \times \left[\delta h_{L(i+1)}(r_{Li} - d_L) + \rho_L C_{PL} r_{Li} d_L \frac{I - MM}{N - MM} \frac{X_{Si}^{j+1} - X_{Si}^j}{d_L} \right. \\ \left. - \rho_L C_{PL} r_{Li} d_L \frac{I - MM}{N - MM} \frac{X_{SLi}^{j+1} - X_{SLi}^j}{d_L} \right] T_{i+1}^j + d \delta h_{L(i-1)}(r_{Li} + d_L) T_{i-1}^j \quad (8b)$$

$$d = \frac{1}{\rho_L C_{PL} r_{Li} d_L} \quad (8c)$$

2.1.9. At the completion of solidification, $I = G$ to N

At the completion of solidification, the cast region becomes solidified metal, and with applied pressure, P , the governing heat transfer equation becomes equation of solidified metal. The whole region becomes a single phase and the governing heat transfer equation is expressed as:

$$\rho_S C_S r_{Si} \Delta r_S \frac{\partial T_S}{\partial t} = h_{S(i+1)}(r_{Si} - \Delta r_S)(T_{i+1}^j - T_i^j) \\ - h_{S(i-1)}(r_{Si} + \Delta r_S)(T_i^j - T_{i-1}^j) \quad (9a)$$

This equation is subjected to the boundary condition within,

$$0 \leq r_S \leq R$$

over a temperature range of:

$$T_a \leq T_S \leq 500^{\circ}\text{C}$$

the finite difference forms of Eq. (9a) becomes:

$$T_i^{j+1} = F[\rho_S C_S r_{Si} d_{CS} - \delta h_{S(i+1)}(r_{CS} - d_{CS}) - \delta h_{S(i-1)}(r_{CS} + d_{CS})] T_i^j \\ + F \delta h_{S(i+1)}(r_{CS} - d_{CS}) T_{i+1}^j + F \delta h_{S(i-1)}(r_{CS} + d_{CS}) T_{i-1}^j \quad (9b)$$

where

$$F = \frac{1}{\rho_S C_S r_{CS} d_{CS}} \quad (9c)$$

2.1.10. Effects of corner boundaries

The usual one dimensional heat transfer analysis does not take into consideration heat losses at the corners. Heat loss at both the external and internal corner nodes of the steel mould have been considered and analysed as represented in Fig. 2a and b.

i. External corner effect ($I = 1$):

$$\frac{K_{st}}{d_{st}} \frac{\partial T_{st}}{\partial r} - \frac{1}{2} \rho_{st} C_{st} \frac{\partial T_{st}}{\partial t} = \frac{H^*}{d_{st}} (T_i^j - T_{\infty}^j) + \frac{H^*}{H} (T_i^j - T_a^j) \quad (10)$$

the finite difference formulation of Eq. (10) is:

$$T_i^{j+1} = \left[1 - \frac{2\delta K_{st}}{\rho_{st} C_{st} d_{st}^2} - \frac{2\delta H^*}{\rho_{st} C_{st} d_{st}} - \frac{2\delta H^*}{\rho_{st} C_{st} H} \right] T_i^j + \frac{2\delta K_{st}}{\rho_{st} C_{st} d_{st}^2} T_{i+1}^j \\ + \left[\frac{2\delta H^*}{\rho_{st} C_{st} d_{st}} + \frac{2\delta H^*}{\rho_{st} C_{st} H} \right] T_a^j \quad (10a)$$

ii. Internal corner effect ($I = G$):

$$\frac{K_S}{d_{st}} \frac{\partial T_S}{\partial r} + \frac{1}{2} \rho_S C_S \frac{\partial T_{st}}{\partial t} = \frac{K_{st}}{d_S} \frac{\partial T_{st}}{\partial r} + \frac{K_{st}}{H} \frac{\partial T_{st}}{\partial r} \quad (11)$$

The finite difference formulation of Eq. (11) is:

$$T_i^{j+1} = \left[1 + \frac{2\delta K_S}{\rho_S C_S d_S^2} + \frac{(I - G)}{(M - G)} \frac{(X_{Si}^{j+1} - X_{Si}^j)}{d_S} + \frac{2\delta K_{st}}{\rho_S C_S d_S d_{st}} + \frac{2\delta K_{st}}{\rho_S C_S H d_{st}} \right] \\ \times T_i^j - \frac{2\delta K_S}{\rho_S C_S d_S^2} T_{i+1}^j - \left[\frac{2\delta K_{st}}{\rho_S C_S d_S d_{st}} + \frac{2\delta K_{st}}{\rho_S C_S H d_{st}} \right] T_{i-1}^j \quad (11a)$$

2.2. Stability equations

The stability criteria of the finite difference equations are defined such that the coefficients of T_i^j should not contribute negatively to the finite difference equations. The stability equations are:

2.2.1. In the mould/atmosphere Interface, $I = 1$

From Eq. (1d):

$$\delta \leq \frac{\rho_{st} C_{st}}{2[(K_{st}/d_{st}^2) + (h^*/d_{st})]} \quad (12a)$$

2.2.2. In the steel mould region, $I = 2$ to $(G - 1)$

From Eq. (2b):

$$\delta \leq \frac{1}{2\alpha_{st}[(1/d_{st}^2) - (1/2 d_{st} r_{st})]} \quad (12b)$$

2.2.3. First time analysis, $I = G$ to N

From Eq. (3d)

$$\delta \leq \frac{\rho_L C_{PL} r_{Li} d_L}{[h_{L(i+1)}(r_{Li} - d_L) - h_{L(i-1)}(r_{Li} + d_L)]}$$

2.2.4. In the solidified metal/steel mould interface, $I = G$

From Eq. (4c):

$$\delta \leq \frac{[\rho_S C_S r_{Si} d_S (I - G/M - G)(X_{Si}^{j+1} - X_{Si}^j/d_S) + \rho_{st} C_{st} r_{sti} d_{st} - \rho_S C_S r_{Si} d_S]}{[2h_{S(i+1)}(r_{Si} - d_S) + (2K_{st} r_{st}/d_{st})]} \quad (12d)$$

2.2.5. In the solidified metal region, $I = (G + 1)$ to $(M - 1)$

From Eq. (5c)

$$\delta \leq \frac{\rho_S C_S r_{Si} d_S - \rho_S C_S r_{Si} d_S (I - G/M - G)(X_{Si}^{j+1} - X_{Si}^j/d_S)}{[h_{S(i+1)}(r_{Si} - d_S) + h_{S(i-1)}(r_{Si} + d_S)]} \quad (12e)$$

2.2.6. In the amorphous liquid/solid metal zone, $I = (M - 1)$ to $(MM - 1)$

From Eq. (7b):

$$\rho_{SL} C_{PSL} r_{SLi} d_{SL} - \rho_{SL} C_{PSL} r_{SLi} d_{SL} (X_{Si}^{j+1} - X_{Si}^j/d_{SL}) \\ - \rho_{SL} C_{PSL} r_{SLi} d_{SL} (I - M/MM - M)(X_{SLi}^{j+1} - X_{SLi}^j/d_{SL}) \\ \delta \leq \frac{[h_{SL(i+1)}(r_{SLi} - d_{SL}) + h_{SL(i-1)}(r_{SLi} + d_{SL})]}{\quad} \quad (12f)$$

2.2.7. In the liquid portion, $I = (MM + 1)$ to $(N - 1)$

From Eq. (8b):

$$\delta \leq \frac{\rho_L C_{PL} r_{Li} d_L - \rho_L C_{PL} r_{Li} d_L (I - MM/N - MM)(X_{Si}^{j+1} - X_{Si}^j/d_L) + \rho_L C_{PL} r_{Li} d_L (I - MM/N - MM)(X_{SLi}^{j+1} - X_{SLi}^j/d_L)}{[h_{L(i+1)}(r_{Li} - d_L) + h_{L(i-1)}(r_{Li} + d_L)]} \quad (12g)$$

2.2.8. At the completion of solidification, $I = G$ to N

From Eq. (9b):

$$\delta \leq \frac{\rho_S C_{CS} r_{CSi} d_{CS}}{[h_{S(i+1)}(r_{CS} - d_{CS}) + h_{S(i-1)}(r_{CS} + d_{CS})]} \quad (12h)$$

2.2.9. From Eq. (10a) external corner

$$\delta \leq \frac{1}{(2/\rho_{st} C_{st})[(K_{st}/d_{st}^2) + (H^*/d_{st}) + (H^*/H)]} \quad (12i)$$

2.2.10. From Eq. (11a) internal corner

$$\delta \leq \frac{1 - ((I - G)/(M - G))(X_r^{j+1} - X_r^j/d_s)}{2/\rho_S C_S [(K_{st}/d_{st}^2) - (K_{st}/H d_{st}) - (K_S/d_S^2)]} \quad (12j)$$

2.3. Numerical solution

The heat governing equations generated at the cast portion and interfaces were converted to finite difference forms while the steel mould, the solidified layer and the mushy portions were discretized appropriately. Solving the finite difference equations with the appropriate boundary conditions the fixed grid enthalpy approach as used by Voller and Prakash (1987) was adopted. Half the casting domain was considered as shown in Fig. 1 owing to the axisymmetry of the problem. The die cavity was assumed to have been filled with the liquid metal at the start of simulation. The time of filling was very small and heat loss was assumed to be negligible. The molten aluminium was poured into the die cavity observing superheat temperature of 60 °C, enough to remain molten throughout the pouring period. It is noted that Santos et al. (2001) reported that higher super heat temperature produces increase metal fluidity and smoother surface, which results in increase in interfacial contact. Hu and Yu (2002) observed a dependency of interfacial heat transfer coefficients on the values of applied pressure and casting temperature. The semi-empirical equations relating the applied pressure and casting temperature on the interfacial heat transfer coefficients within each of the solidus, liquidus–solidus amorphous portion and liquidus phases were inserted into the heat transfer equation. These semi-empirical equations for each of the distinct phase changes were incorporated into the appropriate heat transfer equations and solved by finite difference technique using the explicit method. This problem simulated a cylindrical geometry ($\phi 125 \times 40$) for the effect of squeeze casting parameters on the temperature distribution and heat transfer coefficients of commercially pure aluminium. The die wall temperature was assumed to be at pre-heat temperature with convection at the outer surface of the mould. The stability criteria were met by Eqs. (12a)–(12j).

3. Experimental methods

3.1. Materials used

The material used for the present investigation was commercially pure aluminium, of percentage composition of 99.5% of aluminium and traces of other elements.

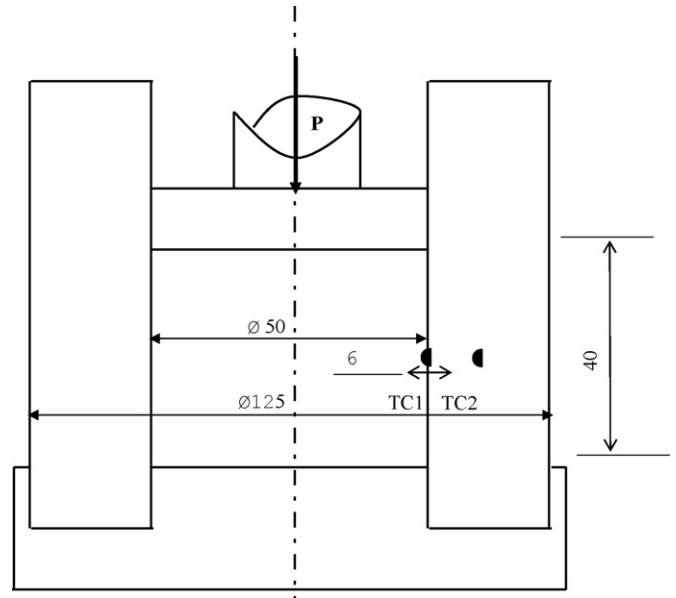


Fig. 3. Schematic diagram of squeeze casting test rig.

3.2. Squeeze casting rig

The rig used for the measurement of casting temperature is made up of steel metal and is shown in Fig. 3. The steel mould was constructed to accommodate two chromel–alumel thermocouples positioned on the side of the cylindrical steel mould as TC2, and TC1 positioned in the cast aluminium metal in the cylindrical surfaces.

3.3. Temperatures versus time curves determinations

Two thermocouples of chromel–alumel type, 3 mm in diameter were used to determine the solidifying temperatures of the cast molten metal and heating temperatures of the steel mould at the cylindrical surface with time, see Fig. 3. The solidifying temperatures at the cylindrical surface of the cast molten aluminium metal were monitored at a position 2 mm into the cast molten aluminium metal, TC1 (from the surface of the steel mould/cast aluminium metal interface) while the heating temperature of the steel mould was at a position 4 mm, TC2 (from the surface of the cast metal into the steel mould). The thermocouples were connected through a cold junction (thermo flask) maintained at the temperature of melting ice, 0 °C to a chart recorder. The millivolt readings versus time curves, from where the temperatures versus time curves were deduced, were plotted on the chart recorder graph paper. For temperature versus time curves in the cylindrical steel container (i.e. position TC2) the voltage setting and speed of the chart recorder used were 50 mV and 2 mm/s, while the voltage setting and speed of 100 mV and 10 mm/s were used for monitoring the temperatures against times for solidifying aluminium metal (i.e. position TC1, Fig. 3).

4. Results

4.1. Temperature/time curves under pressure application

Fig. 4 shows the comparison of temperature versus time curves of solidifying squeeze cast aluminium metal measured at a position of 2 mm into the cast metal. The graph shows the comparison of results obtained by the experimental measurements with the numerical results obtained by differential equations and by semi-empirical equations' methods. The maximum temperatures

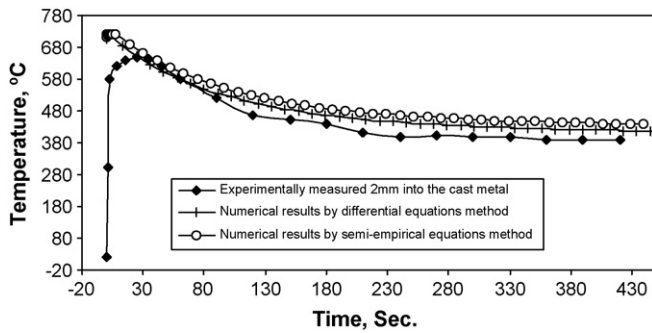


Fig. 4. Typical comparison of the effects of applied pressure on the solidification temperature of aluminium metal ($P=85.86$ MPa).

obtained are 658°C , 718.59°C and 719.84°C for experimental, differential and semi-empirical methods respectively. The temperature values obtained by the semi-empirical equations' methods are slightly higher as compared with the two other methods. This is because the semi-empirical equations' method uses the average sum of all the heat transfer coefficients values to obtain the semi-empirical equation.

4.2. Temperature/time curves with die heating

Fig. 5 is a typical graph, showing the effect of pre-heat temperature ($T_{st} = 150^{\circ}\text{C}$) on the solidification temperatures with time of aluminium metal, measured at a position 2 mm into the cast metal without pressure application. The graph shows the results as obtained by experimental, heat differential equations and semi-empirical equations' methods. There is a good agreement between the values obtained by the three methods with the semi-empirical equation method giving the highest values.

Fig. 6 is the comparison of heating and cooling temperatures of the steel mould (measured at the steel mould/cast metal inter-

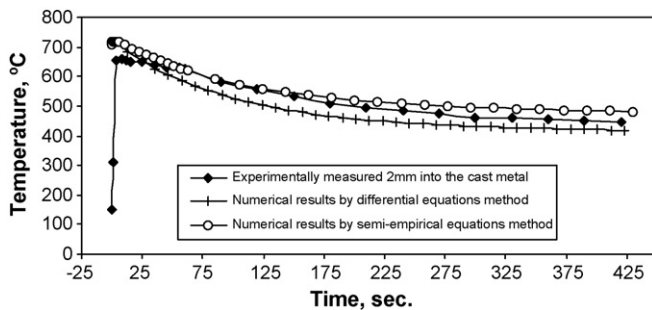


Fig. 5. Typical effect of die pre-heat temperature on solidification temperature of aluminium of aluminium metal without pressure application ($T_{st} = 150^{\circ}\text{C}$, $P=0$).

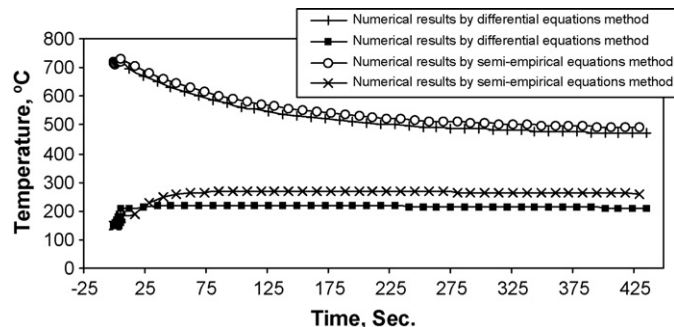


Fig. 6. Typical effect of die pre-heat temperature and pressure on solidification temperature of aluminium metal ($T_{st} = 150^{\circ}\text{C}$, $P=85.86$ MPa).

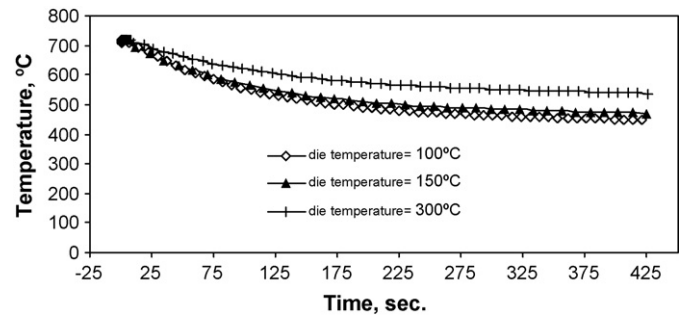


Fig. 7. Typical effect of die pre-heat temperature and pressure on solidification temperature of aluminium metal ($P=85.86$ MPa).

face) and solidifying aluminium metal (measured 2 mm into the cast metal) respectively with the application of pressure. The heating temperatures with time at the steel mould/cast metal interface as obtained by these methods are in close agreements with semi-empirical equation method giving higher values than the heat differential equations' method. The peak temperatures obtained 2 mm into the cast metal after 5 s are 719.96°C and 728.87°C respectively for differential and semi-empirical equations' methods. Fig. 7 shows the effects of die pre-heat temperature on the solidification time of cast metal with pressure application by differential method. With increase in die pre-heat temperature, there is a corresponding increase in the solidification time.

4.3. Effects of delay times on solidifying temperatures

The effects of delay times (time between die closure and pressure application) on the solidification temperatures with time of aluminium metal without die heating as obtained by differential and semi-empirical equations' methods are shown in Figs. 8–10. Both methods show close relationship in the values of solidifying temperatures with time obtained when the delay and retention

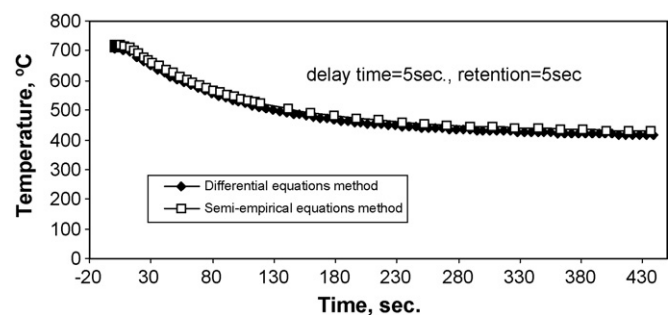


Fig. 8. Typical effect of time on the solidification temperature of aluminium metal without die pre-heating ($T_{st} = 30^{\circ}\text{C}$, $P=85.86$ MPa).

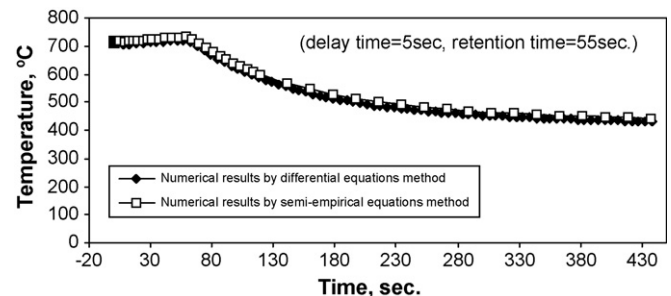


Fig. 9. Typical effect of delay time on the solidification temperature of aluminium metal without heating ($T_{st} = 30^{\circ}\text{C}$, $P=85.86$ MPa).

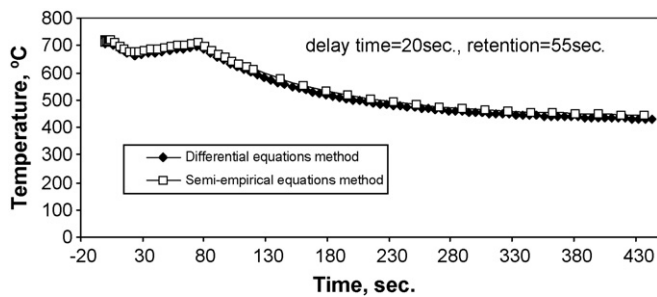


Fig. 10. Typical effect of delay time on the solidification temperature of aluminium metal without die pre-heating ($T_{st} = 30^\circ\text{C}$, $P = 85.86\text{ MPa}$).

times of pressure are both 5 s as portrayed in Fig. 8. The peak temperatures obtained with delay time of 20 s and retention time of 55 s are 695.54°C and 710.04°C respectively for heat differential and semi-empirical equations' methods under a delay time of 20 s and retention time of 55 s as seen from Fig. 9. From Fig. 10, the peak temperatures obtained are 720.91°C and 723.14°C respectively for heat differential and semi-empirical equations' methods under an applied pressure of 85.86 MPa observing a delay time of 5 s and retention time of 55 s. From the graph, both methods show close relationship in the values of solidifying temperatures with time obtained.

5. Discussion

5.1. Temperature/time curve with pressure and die heating

Fig. 4 is the typical effect of pressure on the cast metal without die heating while Fig. 5 represents the effect of die heating without pressure application. Figs. 6 and 7 display the combined effects of pressure and die pre-heat on the temperature distribution of solidifying aluminium metal. Chattopadhyay (2007) reported that a pressure of 60–100 MPa is considered to be optimal beyond which little improvement is observed in terms of casting quality. Therefore, the pressure of 85.86 MPa as used in this work is adequate to induce improve mechanical properties on the cast aluminium metal when applied at the appropriate time. The effect of pressure from Fig. 6, on the cast metal increases the temperature as compared to situation without pressure application of Fig. 5. The combination of die pre-heat temperature and pressure as in Figs. 6 and 7 leads to higher solidifying temperature and consequently lengthens the total solidifying time of the cast metal thus in line with the submission of Hu and Yu (2002). Application of pressure brings about undercooling in an initially superheated metal and thus increases nucleation frequency so bringing about a finer grain. Ghomashchi and Vikhrov (2000) noted that the effect of undercooling is appreciable if pressure is applied at the beginning of crystallisation temperature. The rise in temperature with higher mould pre-heat temperature as observed in Fig. 7 could be attributed to the improved or better contact at the casting/mould interface. Gafur et al. (2003) noted it and attributed it to the relative low initial temperature difference between the cast metal and die internal surface. Therefore, reducing the die pre-heat temperature would reduce the total solidification time and enhance a better cooling rate of the cast metal.

5.2. Effects of delay and retention times on the temperature distribution

Fig. 8 illustrates the application of pressure on the solidifying metal 5 s after pouring of molten metal and retained for a period of 5 s. The period of pressure retention was short with maximum

temperature obtained as 715.31°C and 702.95°C in 8 s for semi-empirical and differential methods respectively. Longer retention time raises the maximum temperature to 724.38°C and 713.95°C respectively for semi-empirical and differential methods at the removal of pressure as reflected in Fig. 9. The effect of longer retention time was not very significant on the temperature distribution of the cast metal comparing results obtained from Figs. 8 and 9. The reason for this behaviour as pointed out in the work of Hu and Yu (2002) was that once the air gap formed between the cast metal and internal surface of the die was eliminated, neither increase in pressure nor length of application of pressure could lead to any improvement in the cast metal.

The graph of Fig. 10 is the situation with pressure application 20 s after pouring and retained for 55 s while the metal solidified, indicating maximum temperature of 697.42°C and 695.54°C for semi-empirical and differential methods respectively. It is noticed from the graph that shortly after pouring and before pressure application, the metal temperature begins to drop from 720°C to 660.50°C in 20 s. This indicates the process of heat abstraction or metal solidification is in progress immediately the molten metal is introduced into the mould cavity even without the application of pressure. The temperature rises again with pressure application to 695.54°C in 75 s at the withdrawal of pressure see Fig. 10. Upon the removal of pressure, the cast metal temperatures drop sharply in a similar manner for both methods. Hu and Yu (2002) opined that the sharp drop in temperature after the removal of pressure suggests the start of solidification with corresponding decrease in the heat flux with the outer portion cooling faster than the interior. The finding of the present authors confirms that timing of pressure application is critical to the product of squeeze cast aluminium metal. There is good agreement between the other methods and the semi-empirical equations' method. The minor disparity noticed may be due partly to assumptions made in the computations which are results derived from the experimental data of squeeze cast of aluminium as presented by Aweda and Adeyemi (2009).

6. Conclusion

The following conclusions could be made from this work:

1. Applied pressures, time of application and duration of application of the pressure have significant effects on the temperature distributions and cooling rates of the squeeze cast aluminium metal. Appropriate timing of pressure application is crucial in squeeze cast of aluminium to obtain improved properties.
2. A reduction in the die pre-heat temperature would reduce the solidification time and enhance cooling rates thus producing better cast products. This agrees with the submission of Chattopadhyay (2007) that cooling rate of squeeze cast material is an important parameter as it affects the mechanical properties induced in the final product. Low die pre-heat temperature coupled with pressure application subsequently leads to a reduction in the solidification time produces products with enhance properties.
3. The semi-empirical equations generated can be used to predict the casting temperatures of other metals, if the empirical equations of heat transfer coefficient values related to the applied pressure and solidifying temperatures at the three phase changes are known. The temperature distributions obtained by semi-empirical equations and other methods are close in values, and hence can be used to determine temperature distributions of squeeze casting of metals.
4. Regardless of the delay and retention times, the effect of applied pressure becomes less significant on the temperature after 200 s of solidification for both methods.

Acknowledgement

The authors express their appreciation to Prof. B.F. Sule, Director, National Centre for Hydropower Research and Development, University of Ilorin, Nigeria for making facilities in his office available towards the final preparation of this work.

References

- Aweda, J.O., Adeyemi, M.B., 2009. Experimental determinations of heat transfer coefficients of squeeze cast aluminium. *J. Mater. Proc. Technol.* 209, 1477–1483.
- Beck, J.V., 1970. Nonlinear estimation applied to the nonlinear inverse heat conduction problem. *Int. J. Heat Mass Transfer* 13, 703–716.
- Chattopadhyay, H., 2007. Simulation of transport process in squeeze casting. *J. Mater. Proc. Technol.* 186, 174–178.
- Ghomashchi, M.R., Vikhrov, A., 2000. Squeeze casting: an overview. *J. Mater. Proc. Technol.* 101, 1–9.
- Gafur, M.A., Nasrul Haque, K., Narayan Prabhu, 2003. Effects of chill thickness and superheat on casting/chill interfacial heat transfer during solidification of commercially pure aluminium. *J. Mater. Proc. Technol.* 133, 257–265.
- Hu, H., Yu, A., 2002. Numerical simulation of squeeze cast magnesium alloy AZ91D. *Model. Simul. Mater. Sci. Eng.* 10, 1–11.
- Maleki, A., Niroumand, B., Shafyei, A., 2006. Effects of squeeze casting parameters on density, macrostructure and hardness of LM13 alloy. *Mater. Sci. Eng. A* 428, 135–140.
- Santos, C.A., Quaresma, J.M.V., Garcia, A., 2001. Determination of transient interfacial heat transfer coefficients in chill mold castings. *J. Alloys Compd.* 319, 174–186.
- Voller, V.R., Prakash, C., 1987. A fixed numerical moulding methodology for convection-diffusion mushy region phase-change problems. *Int. J. Heat Mass Transfer* 30, 1709–1720.
- Yang, L.J., 2003. The effect of casting temperature on the properties of squeeze cast aluminium and zinc alloys. *J. Mater. Proc. Technol.* 140, 391–396.
- Yang, L.J., 2007. The effect of solidification time in squeeze casting of aluminium and zinc alloys. *J. Mater. Proc. Technol.* 192–193, 114–120.