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Parametric study of natural convection heat transfer from an inclined rectangular cylinder embedded in a square enclosure

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ABSTRACT

This paper presents the effects of aspect ratio ($0.1 \leq AR \leq 0.7$), Rayleigh number ($10^2 \leq Ra \leq 10^6$) and inclination angle ($0^\circ \leq \gamma \leq 90^\circ$) of the heated rectangular cylinder on heat transfer and fluid flow characteristics due to natural convection in air around heated rectangular cylinders of different sizes inside a cold square enclosure. Galerkin finite element method was adopted for the solution of the developed model. The walls of the enclosure were maintained at an isothermal cold temperature while the boundaries of the rectangular cylinder were kept at a constant hot temperature. The results are presented in the form of isothermal contours, stream functions, local and average Nusselt numbers. It has been found that for the range of aspect ratio and Rayleigh number considered, the rate of heat transfer increases with increasing aspect ratio and Rayleigh number. Furthermore, the orientation angle of the rectangular cylinder was found to generally augment heat transfer rate except at AR value of 0.1, where the average Nusselt number declines beyond $\gamma = 45^\circ$ for all the range of Rayleigh numbers considered. Results from the present study could be applied to nuclear reactor technology and cooling of electronic chips system optimisation.

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Natural convection; rectangular cylinder; aspect ratio; inclination angle; square enclosure

1. Introduction

Over the years, natural convection in enclosures has been extensively investigated both numerically and experimentally due to its varied engineering applications such as in solar collector technology, cooling of electronic devices, space heating, nuclear and chemical reactors (Habibis, Ammar, and Ishark 2015; Zhang and Yang 2009; Bég et al. 2016). Furthermore, in a lot of engineering applications, the configurations that are of interest are more intricate than the simple enclosures; it involves an object enclosed in an enclosure (Yoon et al. 2009). Because of the varied engineering applications of such configurations, a lot of researchers have examined the fluid flow and heat transfer dynamics in such geometries some of which includes Souayeh et al. (Souayeh, Alfannakh, and Al Mutairi 2019); they numerically investigated natural convection in a square enclosure with adiabatic horizontal walls and isothermal vertical boundaries having a rectangular heat-conducting cylinder situated at different horizontal positions, they opined that both the rectangular cylinder location and Rayleigh number influence the fluid flow and heat transfer characteristics. Habib (Habib 2020) conducted a numerical investigation of natural convection heat transfer from an unbounded horizontal isothermal cylinder having

various cross-sections. It was submitted that heat transfer rate increased with increment in Rayleigh number; and the highest heat transfer rate was recorded for the elliptical cylinder with slender orientation.

Gangawane and Manikandan (Gangawane and Manikandan 2017) analysed laminar natural convection characteristics in an enclosure with a heated hexagonal block. The vertical walls of the enclosure were kept at isothermal cold temperature and its horizontal walls were perfectly insulated while the block was maintained at either a constant temperature or at a uniform heat flux condition. It was submitted that higher heat transfer rate was achieved when the block was maintained at uniform heat flux than when at constant temperature. Additionally, the heat transfer rate decreased with power law index. Karki and Yadav (Karki, Yadav, and Arumuga Perumal 2019) investigated the implication of the location of adiabatic obstacles on natural convection heat transfer in a square cavity by leveraging the Lattice Boltzmann approach; the vertical walls of the domain were maintained at different constant temperatures while the horizontal walls were adiabatic. The study revealed that an increase in the size of the obstacles enhanced the heat transfer rate along the hot wall up to a critical value beyond which a further increase in the obstacle

size inhibited the rate of heat transfer. However, outside the vicinity of the conduction-dominated zone, the presence of obstacles did not result in heat transfer augmentation.

Sannad et al. (Sannad et al. 2019) conducted a 3-D numerical analysis of natural convection heat transfer in a square cavity containing water-Cu nanofluid. They studied the effects of the locations of the heated block on heat transfer augmentation and reported that higher heat transfer rate was recorded when the heated block was located mid-point on the upper wall of the cavity compared to when the block was situated in the middle of the bottom wall. The results also showed that the heat transfer rate increased as the volume fraction of nanofluid and Rayleigh number increased.

Hassanzadeh et al. (Hassanzadeh et al. 2020) investigated natural convection heat transfer in a square enclosure fitted with inner rotating cylinders of different surface conditions. They reported that the rate of heat transfer was highest in the case of the smooth inner cylinder compared with cylinders of other surface conditions. Şahin (Şahin 2020) found out that as the centre of the linear heating was moved higher, heat transfer increased while the rate of fluid flow reduced. Rui et al. (Rui et al. 2020) employed the enthalpy-based Lattice Boltzmann model and Pseudo-potential Lattice-Boltzmann model in the study of natural convection melting rate in a square cavity. They showed that heating all the walls of the square cavity gave a 54% improvement in heat transfer than heating two opposite walls.

Adegun et al. (Adegun, Ibitoye, and Bala 2020) reported that the orientation angle of the elliptical cylinder situated concentrically in a square cavity had no tangible effect on the mean Nusselt number at low Rayleigh numbers. While at very high Rayleigh numbers, the mean Nusselt number increases as the angle of elliptical orientation increases. Leporini et al. (Leporini et al. 2018) analysed natural convection in square enclosures filled with air while tilting the square at various angles. They showed that tilt angle greatly influenced the heat transfer in the enclosure. Wang and Qing (Wang and Qin 2018) used the time-splitting method to study natural convection heat transfer inside a square enclosure. Their results showed that the higher the Rayleigh number went, the better the rate of heat transfer near the heat sources. Raisi and Arvinb (Raisi and Arvin 2018) studied the time-dependent natural convection heat transfer mode in a square enclosure filled with air based on the effects of fluid-structure interaction. They insulated the top and bottom walls while placing a baffle in the middle of the enclosure. They showed that the

higher the Rayleigh number, the higher the rate of heat transfer.

Cho et al. (Cho, Park, and Ha 2018) reported the implication of the aspect ratio of an elliptical cylinder on natural convection in a square enclosure containing a vertical array of a circular cylinder and an elliptic cylinder. The walls of the square enclosure were fixed at a constant cold temperature while that of the elliptical and circular cylinders were maintained at an isothermal hot temperature. It was submitted that in the interval of Rayleigh number of $10^4 \leq Ra \leq 10^5$, the attainment of steady state by the numerical solution is independent of the aspect ratio of the elliptic cylinder. Furthermore, when Ra was 10^6 , the numerical solution attained unsteady state for all the aspect ratios considered when the elliptic cylinder was located either close to the base of the enclosure or close to the top of the enclosure except for aspect ratios of 2 and 4. El Moutaouakil et al. (El Moutaouakil, Zrikem, and Abdelbaki 2017) carried out a Lattice Boltzmann simulation of natural convection in an annulus between a hexagonal cylinder and a square enclosure. In their estimation, flow intensity is dependent on Rayleigh number and cylinder size.

Wan et al. (Wan, Patnaik, and Wei 2001) employed a high-accuracy discrete numerical method to investigate coupled convective heat transfer problems in a square enclosure; the quasi-wavelet discrete singular convolution and the Galerkin finite element methods were used. One of the deductions made was that further study of flow in cavities was necessary due to the disparities observed in literature. Wang et al. (Wang et al. 2019) studied the influence of several parameters including Rayleigh number on the natural convection in a partially heated square enclosure filled with nano-fluids. They showed that the heat transfer improvement at low Rayleigh numbers increases with increase in nano-particle volume fraction. However, at high Rayleigh number, there exist an optimum value of volume fraction that resulted in the highest heat transfer rate.

In a study of natural convection in a square cavity filled with an inner elliptic cylinder, Zhang et al. (Zhang et al. 2016) numerically investigated the effects of Rayleigh number ranging from 10^3 to 10^6 and the inclining of outer square cavity from angle 0° to 45° . The outer square was maintained at a cold temperature and the inner cylinder was hot. They showed that the angle of inclination has a marginal effect on the mean Nusselt number of the inner cylinder. However, the angle of inclination and the Rayleigh number have significant effects on the heat transfer rate along the outer square.

Elsherbiny et al. (Elsherbiny, Teamah, and Moussa 2017) analysed steady-state natural convection in a square enclosure subjected to horizontal isothermal conditions for Rayleigh number ranging from 10^3 to 10^6 . Their results showed that local Nusselt number was higher at the left and right corners. However, it increased with Rayleigh number on all the walls except at the top wall. Husain and Siddiqui (Husain and Siddiqui 2018) investigated transient natural convection in a high aspect ratio vertical enclosure filled with distilled water. They inferred that the transient period increased as the height of the enclosure increased. Esfe et al. (Esfe et al. 2018) while studying the heat transfer rates of various nanofluids using data science and machine learning methods showed that increasing the Reynolds number and volume fraction resulted in increasing Nusselt number. Khatamifar et al. (Khatamifar et al. 2017) also examined natural convection in a differentially heated square enclosure split by a partition. They submitted that the mean Nusselt number increased as Rayleigh number increased. However, as the thickness of the partition increased, the mean Nusselt number decreased.

Kumar et al. (Kumar et al. 2020) in their work on flow in cavity with respect to nuclear reactors found out that radiation effects improved the overall heat transfer from the cavity walls and influenced the temperature distribution of the gas inside the cavity. Gao et al. (Gao, Pitz, and Chew 2020) studied buoyancy-driven flows in sealed rotating enclosures with Rayleigh number ranging from 10^7 to 10^9 and they submitted that buoyancy was more pronounced in the core. Singh and Gohil (Singh and Gohil 2019) investigated the effect of electrical properties on buoyancy-induced fluid flow and heat transfer. They reported that heat transfer rate improved as the flow became more unsteady at low Hartmann number. Sebilleau et al. (Sebilleau et al. 2018) examined the heat transfer in square enclosures with Rayleigh numbers 10^8 to 10^{11} . They reported that flow turbulence moved downwards towards the bottom of the cavity as Rayleigh number increased while the mean Nusselt number kept fluctuating. Mousa (Mousa 2016) in his analysis of steady natural convection in a square enclosure having an obstacle concluded that the rate of heat transfer increased with both Rayleigh number and the obstacle size. Saha (Saha 2020) in his investigation of nanofluid-filled trapezoidal cavity posited that an applied external magnetic field resulted in changes in the flow and temperature distributions in the cavity.

Izadi et al. (Izadi et al. 2018) in their study of hybrid nanofluid inside an inverted cavity stated that Nusselt number reduced based on the obstruction ratio of the cavity. But increasing the aspect ratio of the source of heat provided an improved cooling condition. Ahmed and Elshehabey (Ahmed and Elshehabey 2018)

analysed buoyancy heat transfer and nanofluid fluid flow within inclined cavities. They found out that an enclosure with a square obstacle gave a better rate of heat transfer compared to a circular enclosure. Additionally, their results showed that an inclined cavity gives a higher rate of heat transfer. Wang and Qin (Wang and Qin 2018) in their investigation of natural convection in a square enclosure by Legendre spectral element approximation showed that heat transfer is usually improved at areas close to the heat source in the square. Anejaa et al. (Aneja, Chandra, and Sharma 2020) examined natural convection process in a porous square enclosure to determine the effects of Darcy number and Rayleigh number on the heat transfer process. They concluded that conduction heat transfer mode was more significant for low Darcy number for the range of Rayleigh number between 10^2 and 10^6 . Kolsi et al. (Kolsi et al. 2017) considered the flow, entropy and heat transfer processes in three-dimensional cavities filled with Al_2O_3 nanofluid for the Rayleigh number of 10^5 . They reported that increasing the volume fraction of the nanoparticles enhanced the heat transfer and fluid flow significantly over the range of Marangoni numbers of -10^3 to 10^3 .

Chowdhury et al. (Chowdhury, Alim, and Hossen 2020) investigated the convection processes that occurred in a partially heated and cooled square cavity fitted with a diamond-shaped heated block. Their results revealed that mean Nusselt number is inversely proportional to the block size. Lu et al. (Lu et al. 2009) examined heat transfer in a square cavity fitted with a heated rectangular block. Their study focused mainly on buoyancy regime classification. Altaee et al. (Altaee, Ali, and Mahdi 2017) in their analysis of free convection inside square cavities fitted with equilateral triangles of different orientations. They observed that heat transfer rate was at a maximum when the Rayleigh number was 10^6 and the angle of orientation of the triangle was 30° . Nagarajana and Periyasamy (Nagarajan and Periyasamy 2015) in their examination of the steady-state free convection process inside a tilted square cavity considered the effect of aspect-ratio and angle of inclination of the cavity. They concluded that for high value of Rayleigh numbers, when the value of the heat generation, $\Delta T^* = 4$, the buoyancy force accelerated the fluid flow hence convection mode was the dominant mode of heat transfer. Munshi et al. (Munshi, Bhuiyan, and Alim 2015) studied natural convection in a square cavity containing an inner square block and a non-uniformly heated bottom wall. They maintained the inner block at a hot temperature and the side walls at a cold temperature. They showed that magnetic field intensity reduced with a higher intensity of free convection. Kalyana et al. (Kalyana Raman, Arul Prakash, and Vengadesan 2012) considered natural convection in a square cavity containing a heated inner cylinder with

different axis ratios. They found out that aspect ratio had a more significant effect on heat transfer rate and fluid flow at high Rayleigh numbers.

Based on the above literature review, a numerical investigation of natural convection fluid and heat transfer from a tilted rectangular cylinder embedded in a square enclosure has not been previously studied. A unique feature of the current investigation is the fact that finite element method was used to comprehensively analyse the combined effects of Rayleigh number ($10^3 \leq Ra \leq 10^6$), aspect ratio of the geometry ($0.1 \leq AR \leq 0.7$), and cylinder orientation angle ($0^\circ \leq \gamma \leq 90^\circ$) on fluid flow and heat transfer dynamics inside the domain investigated. This work contributes to heat transfer enhancement without adopting extra energy. As a result, it will provide useful insight into the scientific industry. Furthermore, the heat transfer characteristics of the current investigation elucidate on the import of non-dimensional parameters such as the inclination angle of rectangular cylinder and barrier aspect ratio in the regimes of natural convection.

2. Methodology

2.1. Description of the physical model

Figure 1 shows a 2D physical model and the coordinate system used for the present study. The model comprises a square enclosure with side length L , and an inclined rectangular cylinder with aspect ratio ($AR = \frac{W}{L}$) embedded in it. The height of the rectangular cylinder is assumed to be constant ($H = 0.1L$). The annular space between the enclosure and the cylinder contains air. It is assumed that the end effect in the z -direction is negligible; hence, the fluid flow and heat transfer are 2D. The flow in the annuli is also

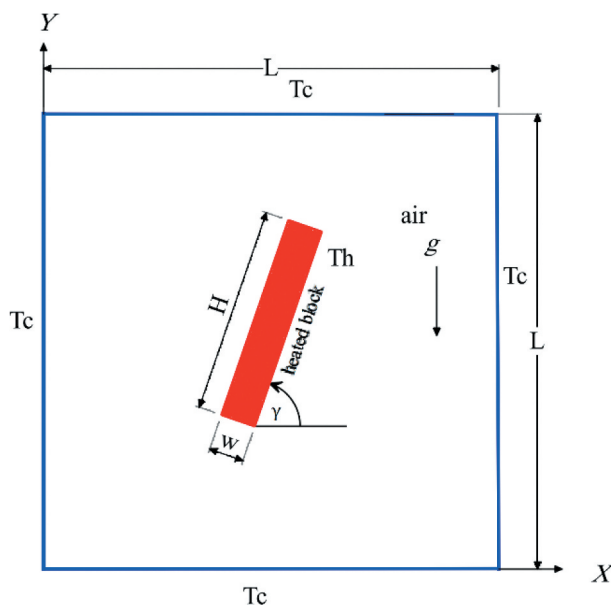


Figure 1. Physical Model.

assumed to be laminar with natural convection flow mechanism. The walls of the enclosure are at a constant cold temperature (T_c). The embedded cylinder is also assumed to be at an isothermal hot temperature, (T_h). The fluid properties are assumed to be constant except density, which follows the Boussinesq approximation. No slip condition is assumed at both the enclosure walls and the cylinder.

2.2. Governing equations

The non-dimensional continuity, momentum and energy equations are as follows

Continuity:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (6)$$

Momentum transport equations:

X-direction

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr \left[\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right] \quad (7)$$

Y-

direction:-

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr \left[\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right] + RaPr\phi \quad (8)$$

Energy transport equation:

$$U \frac{\partial \phi}{\partial X} + V \frac{\partial \phi}{\partial Y} = \frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Y^2} \quad (9)$$

2.3. Transformation parameters

$$X = \frac{x}{L}, Y = \frac{y}{L}, AR = \frac{W}{L}, U = \frac{uL}{\alpha}, V = \frac{vL}{\alpha}, \phi = \frac{T - T_c}{T_h - T_c}, P = \frac{pL^2}{\rho\alpha^2}, \text{GEAC_A_1912336}$$

2.4. Applicable boundary conditions

At all the solid boundaries, $U = V = 0$; all the walls of the cold enclosure are maintained at a constant temperature ($\phi_c = 0$), while the walls of the heated rectangular cylinder are at an isothermal temperature ($\phi_h = 1$).

3. Solution techniques

The numerical procedure used in the present investigation relies on the Galerkin weighted residual method of finite element formulation. The annular space between the square cavity and the rectangular cylinder was produced using the Boolean operation and the working fluid (air) was added to the model. The appropriate boundary conditions were then imposed on the respective boundaries; and the computational domain was then discretised into distinct finite element meshes. The mesh of the cold and the heated walls was generated with the aid of the extremely fine grid size option and free triangular mesh option was used to produce the mesh of the domain

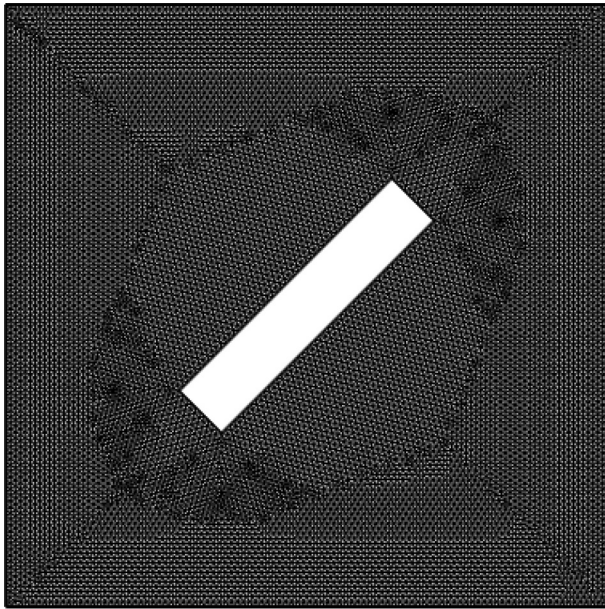


Figure 2. Mesh distribution.

Table 1. Grid independence test of the average Nusselt number ($\overline{Nu}$) on the heated cylinder for $Ra = 10^6$, $AR = 0.7$, $\gamma = 90^\circ$.

size of Mesh	Mesh elements	Average Nusselt number	% Error
Coarse	3490	27.05	–
Normal	4228	24.61	9.02
Fine	4482	23.78	3.37
Finer	5608	22.42	5.72
Extra fine	9152	21.74	3.03
Extremely fine	23,538	21.61	0.597

studied as shown in Figure 2. To check the quality of the selected mesh, a grid independence test has been done for different mesh sizes in terms of the average Nusselt number ($\overline{Nu}$) as shown in Table 1. The results show that the solution is mesh independent for the case of the extremely fine grid size. The continuity equation, momentum and energy transport equations were converted into a set of integral equations with the aid of the Galerkin residual method; and Gauss quadrature method was used to perform the needed integration. The appropriate boundary conditions were then imposed on the integrated equations and the resulting equations were converted into linear equations by Newton-Raphson iteration; and the linear equations were then solved by the triangular factorisation method (TFM).

4. Evaluation of Nusselt number

Nusselt number is a dimensionless parameter indicative of the ratio of heat convection to heat conduction over a given thickness of fluid. According to (Adegun, Ibitoye, and Bala 2020; Yoon et al. 2009) the local heat transfer coefficients are given on the rectangular cylinder as:

Table 2. Comparison of the predicted average Nusselt number on the right wall of the cavity with those of references (De Vahl Davis and Jones 1983; Al-Balushi, Uddin, and Rahman 2019; Tiwari and Das 2007) when $Pr = 0.71$.

Ra	Present Study	De Vahl Davis and Jones (De Vahl Davis and Jones 1983)	Al-Balushi et al. (Al-Balushi, Uddin, and Rahman 2019)	Tiwari and Das (Tiwari and Das 2007)
2×10^2	1.006	NIL	NIL	NIL
1×10^3	1.118	1.118	1.118	1.087
1×10^4	2.245	2.243	2.274	2.195
1×10^5	4.522	4.519	4.964	4.450
1×10^6	8.832	NIL	NIL	8.803

$$h_i = -\left[k \frac{\partial \phi}{\partial n}\right]_i / (\phi_h - \phi_c) \quad (12)$$

where, n' is the normal to wall surface.

Therefore, the local and average Nusselt numbers of the left wall are given respectively by:

$$Nu = h_i \frac{L_c}{k} \quad (13)$$

$$\overline{Nu} = \overline{h_i} \frac{L_c}{k} \quad (14)$$

Where L_c is the characteristic length and h is the heat transfer coefficient.

5. Stream function

The Streamline contours represent the flow fields within the cavity and formed from the velocity components (U and V) as follows: (Abdulkadhim, Abed, and Al-Farhany 2018; Abdulkadhim, Al-Farhany, and Abed 2018)

$$U = \partial \Psi / \partial Y \text{ and } V = -\partial \Psi / \partial X \quad (15)$$

The dimensional formula of the stream function equation is derived from the relationship between the velocity components of the two-dimensional flow:

$$\partial^2 \Psi / \partial X^2 + \partial^2 \Psi / \partial Y^2 = \partial U / \partial Y - \partial V / \partial X \quad (16)$$

6. Validation

To validate the numerical code used in the present study, the results of a differentially heated square cavity whose horizontal walls are adiabatic and without the rectangular cylinder are compared in Table 2 with previous works and the comparison shows a very good agreement.

7. Results and discussion

In this section, the results are presented for a wide range of parameters; Rayleigh number ($10^3 \leq Ra \leq 10^6$), aspect ratio of the geometry ($0.1 \leq AR \leq 0.7$), and the cylinder orientation angle ($0^\circ \leq \gamma \leq 90^\circ$).

Figure 3 shows the plots of isothermal contours and stream function for $AR = 0.3$ at Rayleigh numbers of $10^4, 10^5$ and 10^6 . For $Ra = 10^4$, the contours of the

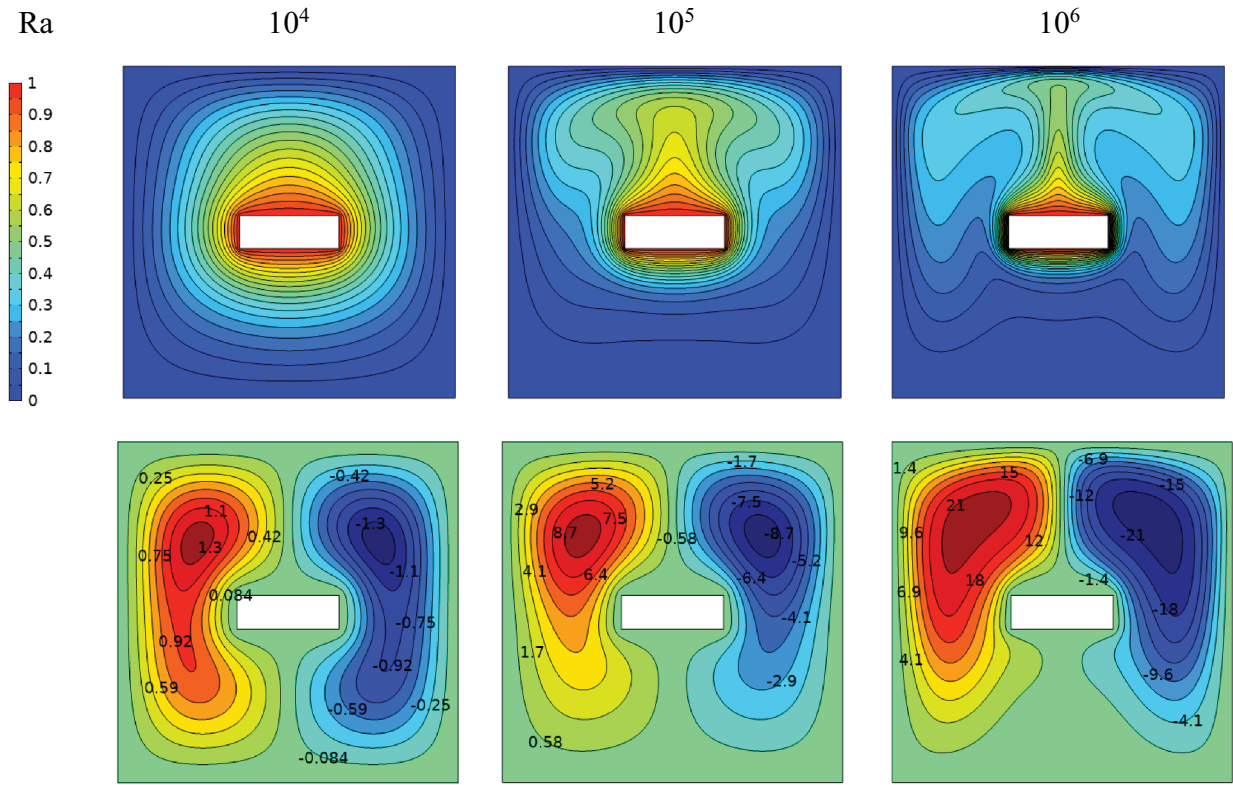


Figure 3. The effects of Rayleigh number (Ra) on isothermlines (top) and stream function (bottom) at $AR = 0.3$, and $\gamma = 0$.

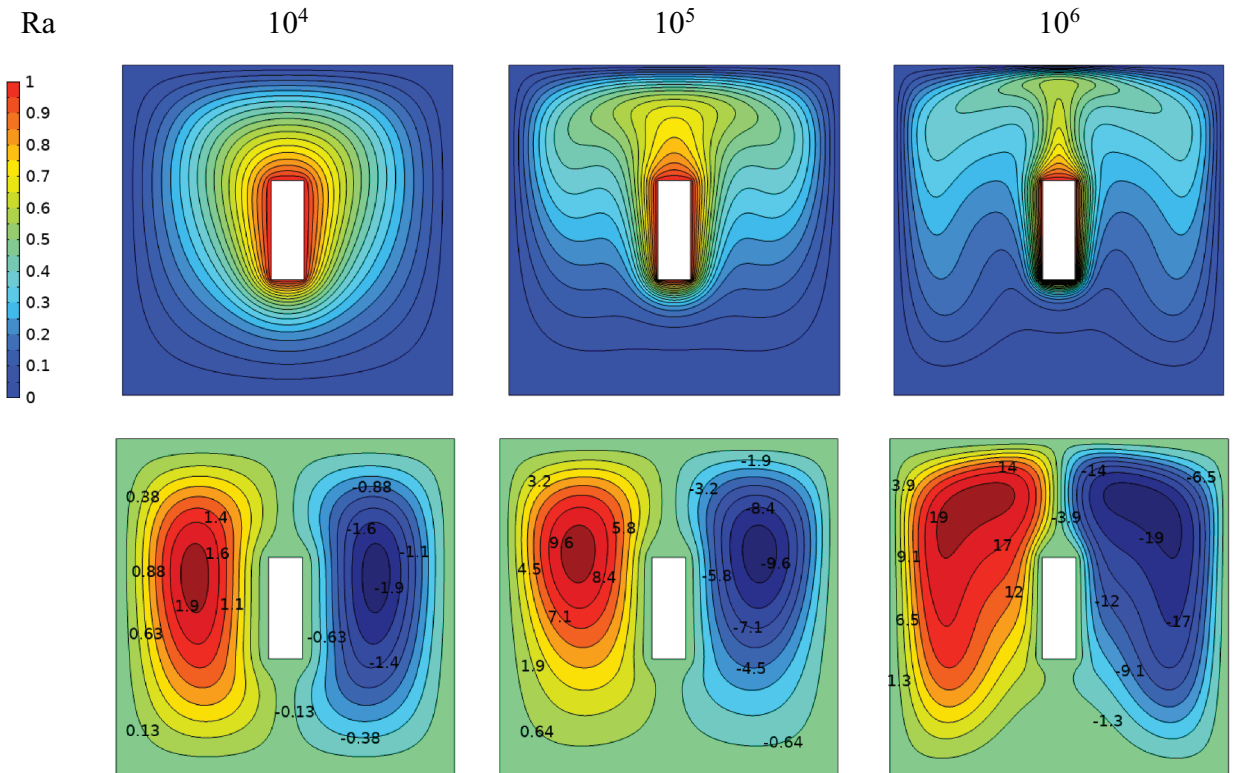


Figure 4. The effects of for Rayleigh number (Ra) on isothermlines (top) and stream function (bottom) at $AR = 0.3$, and $\gamma = 90^\circ$.

isotherms are almost parallel to the enclosure walls which is an indication of pure conduction heat transfer. When Rayleigh number attained the value of 10^5 , the isotherms became more closely packed towards

the top right and left portions since heating had become more intense and convection had set in. When $Ra = 10^6$, the isotherms became more distorted on both sides which is an indication that buoyancy

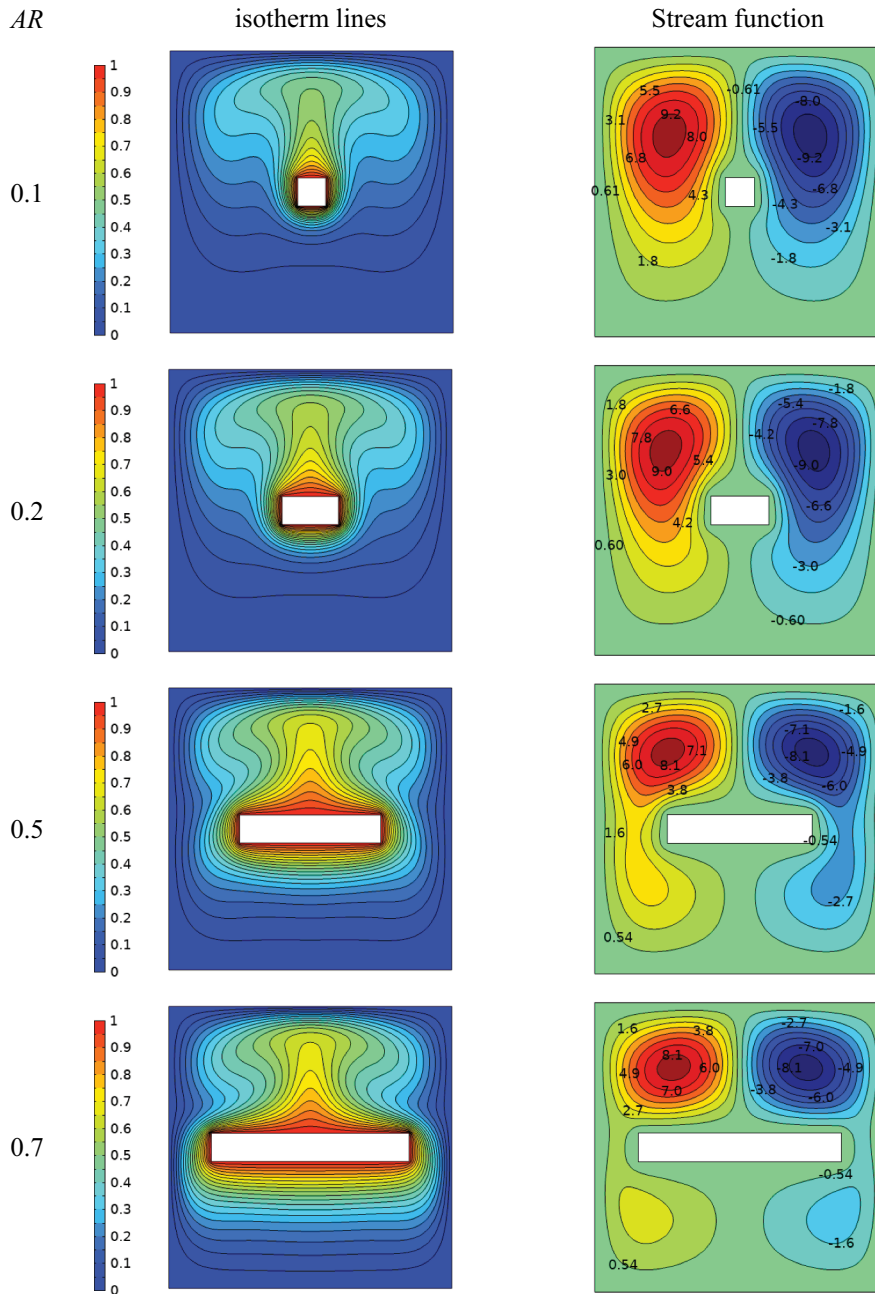


Figure 5. The effects of Aspect Ratio (AR) on isothermlines (left) and stream function (right) at $Ra = 10^5$, and $\gamma = 0$.

had become more vigorous. The maximum heat transfer can be seen near the hot rectangular block especially around the mid-section of the cavity, which is a consequence of the heated fluid rising due to buoyancy force while the vertical walls (short side) had a low-temperature gradient; therefore, they have less effect on fluid flow compared to the horizontal walls.

For the stream function plots, at $Ra = 10^4$, two symmetric vortices appeared in the enclosure due to heat convection, the left vortices rotating in clockwise while the right vortices rotating in counterclockwise directions. The stream function values on the top of the rectangular cylinder are higher than the stream function values on the lower part because of natural convection. This observation occurred because the

buoyancy force around the upper portion of the rectangular cylinder is higher than that below it.

At $Ra = 10^5$, the oval-shaped streamlines in the innermost cores increased in size and almost relapsed into circular shapes. Buoyancy and gravity effects had become more apparent and streamlines were observed to be less dense. When $Ra = 10^6$, the same behaviour can be seen, and hence gravity effect had become more pronounced in the flow. The maximum stream function (Ψ) is 21 while the minimum stream function can be seen at $Ra = 10^6$ which is 1.3.

As shown in Figure 4, when $AR = 0.3$ and $\theta = 90^\circ$ for $Ra = 10^4$ and 10^5 , the fluid velocity to the left of the rectangular cylinder had increased due to the increase in inclination angle. However, at $Ra = 10^6$, the

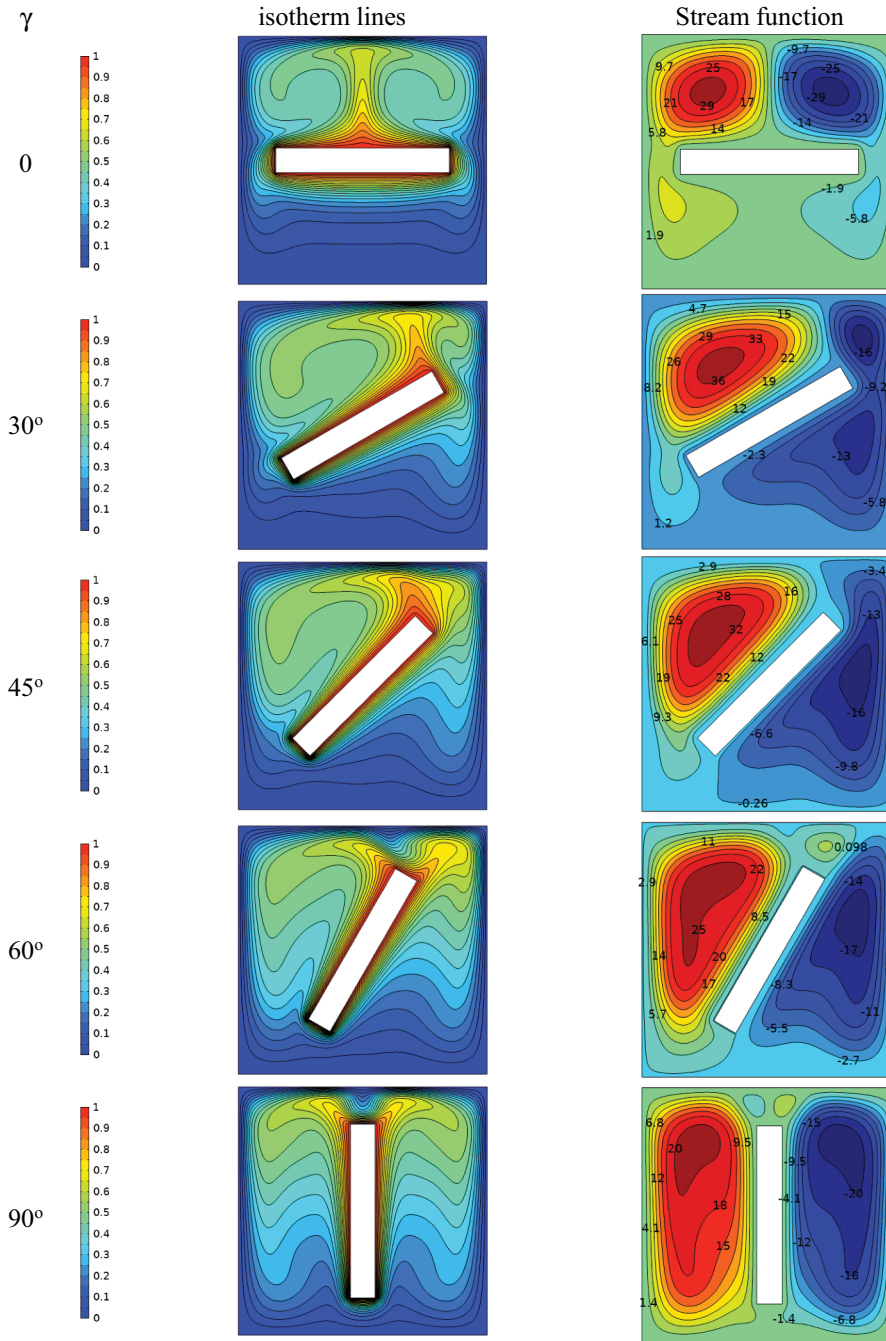


Figure 6. The effects of Inclination angle (γ) on isothermlines (left) and stream function (right) at $Ra = 10^6$, and $AR = 0.7$.

frictional effect on the cylinder became more dominant than the effect of inclination and therefore, the flow became slightly decelerated. The isothermal contours are denser than those of Figure 3 because of a change in orientation of the cylinder. This is because the natural convection on the vertical walls is more than that on the horizontal walls. Also, it can be seen that the stream function increases when Rayleigh number increases; and the maximum stream function ($\Psi = 19$) occurred when $Ra = 10^6$.

In Figure 5, as AR increases, the surface area over which heat transfer takes place increases, so heat transfer rate also increases. Furthermore, as AR increases, the stream function (Ψ) decreases from 9.2

at $AR = 0.1$ to 8.1 when $AR = 0.7$. This is because the rectangular cylinder will act as an obstacle and when AR increases, the obstacle volume increases, and that negatively affects the movement of the fluid. In addition, the density of the isothermal lines on the upper portion of the cylinder slightly increased while it decreased on the lower portion of the cylinder close to the base of the enclosure wall and that happened due to buoyancy force.

Figure 6 shows that when $\gamma = 0$, the isothermal contours are symmetric about the mid-point of the rectangular cylinder. At $\gamma = 30^\circ$, the flow pattern observed above the mid-point of the rectangular cylinder when $\gamma = 0$ had shifted to the top right corner of

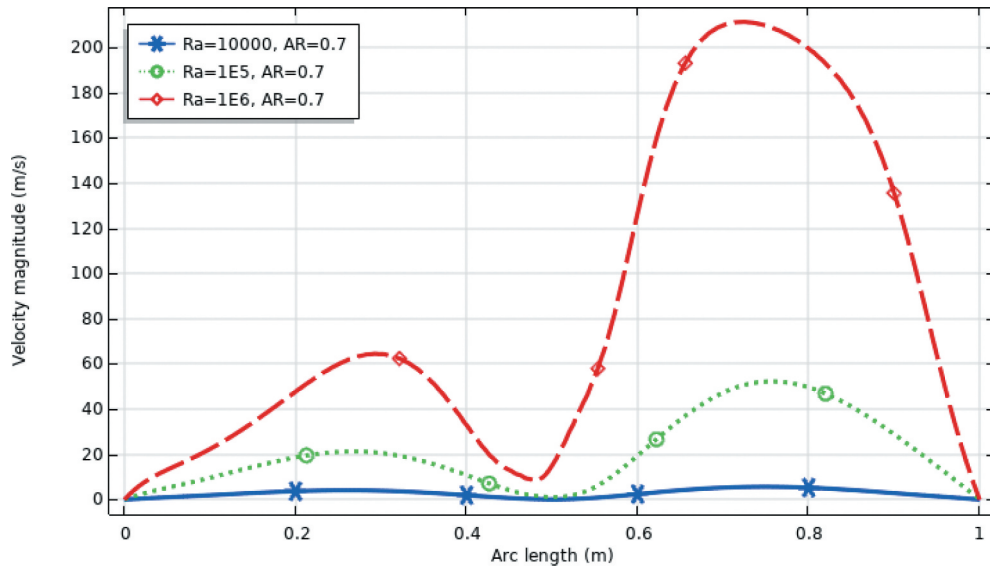


Figure 7. Velocity magnitude against Arc length at plane $x = 0.075$ when $AR = 0.7$ and $\gamma = 0$.

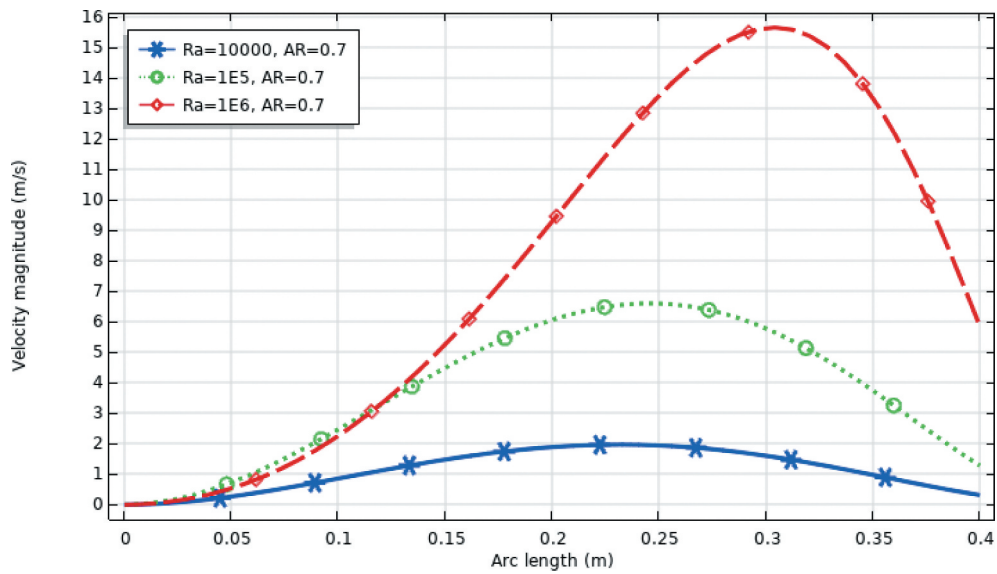


Figure 8. Velocity magnitude versus length at plane formed by $(0.5, 0.6)$ and $(0.5, 1)$ when $AR = 0.7$ and $\gamma = 0$.

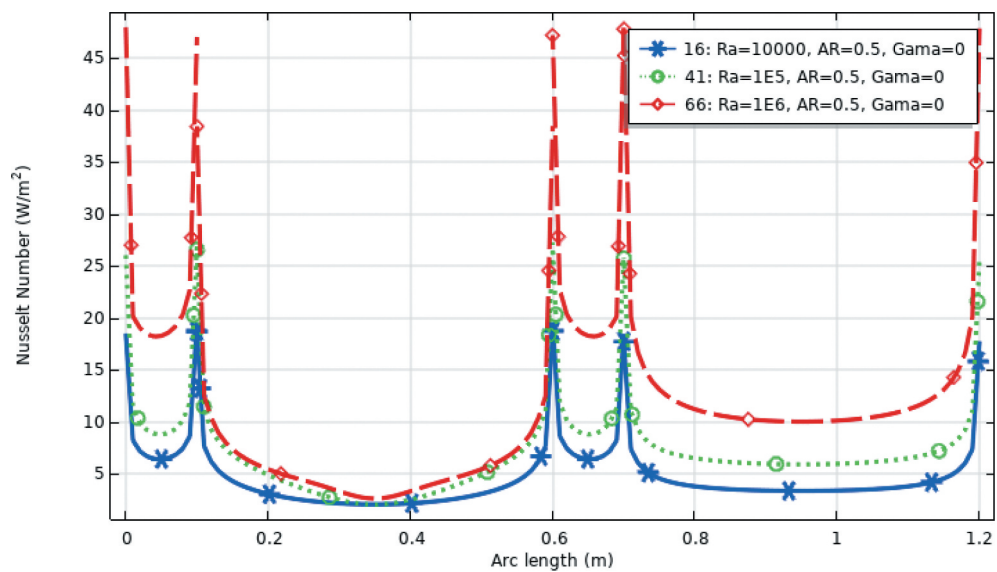


Figure 9. Local Nusselt number (Nu_L) with Rayleigh number (Ra) on the cylinder walls at.

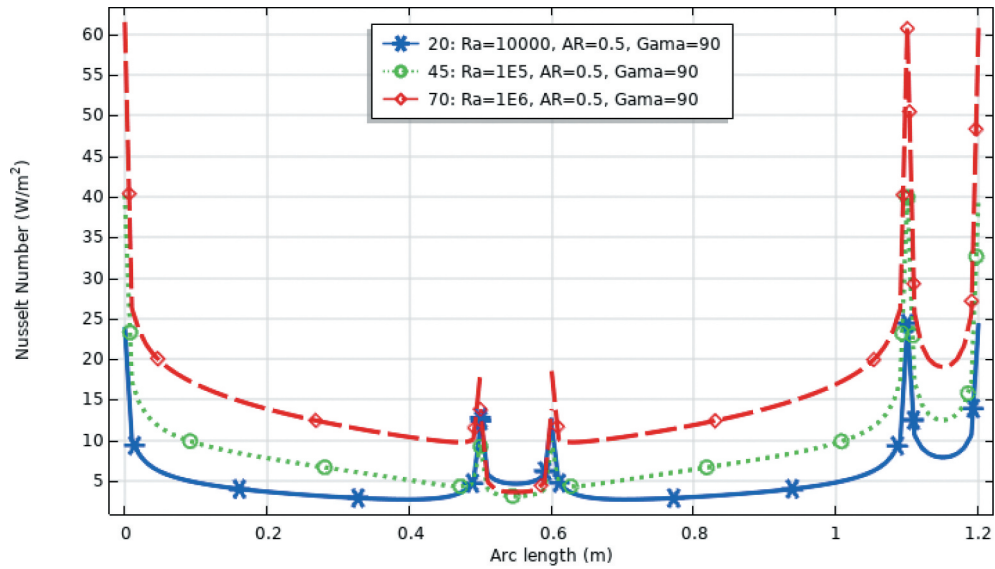


Figure 10. Local Nusselt number (Nu_L) with Rayleigh number (Ra) on the cylinder walls at $AR = 0.5$, and $\gamma = 90^\circ$.

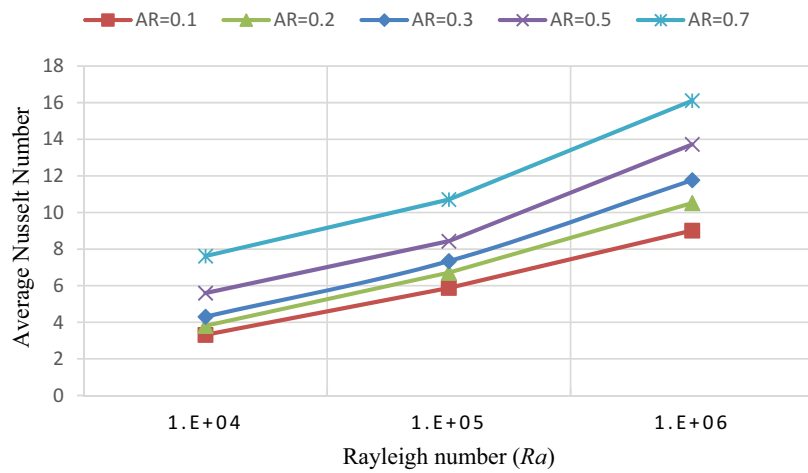


Figure 11. Average Nusselt Number on the heated cylinder versus Rayleigh number (Ra) with different Aspect ratio (AR) at $\gamma = 0$.

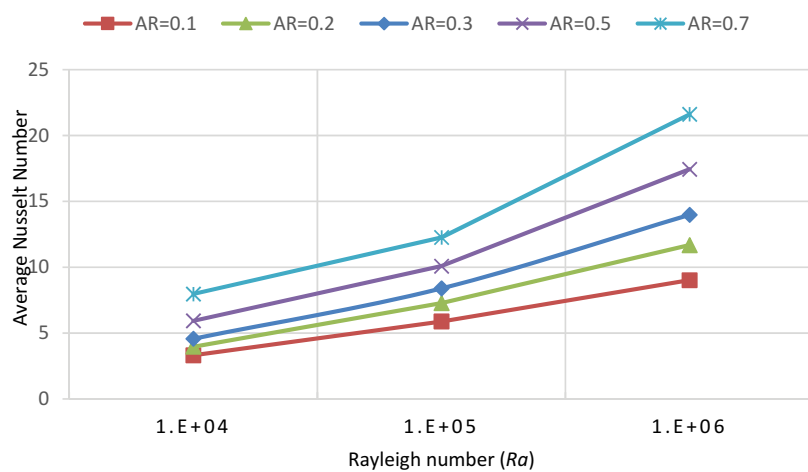


Figure 12. Average Nusselt Number on the heated cylinder versus Rayleigh number (Ra) with different Aspect ratio (AR) at $\gamma = 90$.

the rectangular cylinder and the streamlines had become denser at the same corner but less dense at the upper left corner of the rectangular cylinder and

the streamlines had spread out. Also, the isotherms on the lower right corner have become denser too. At $\gamma = 45^\circ$, the streamlines spread out the more. With

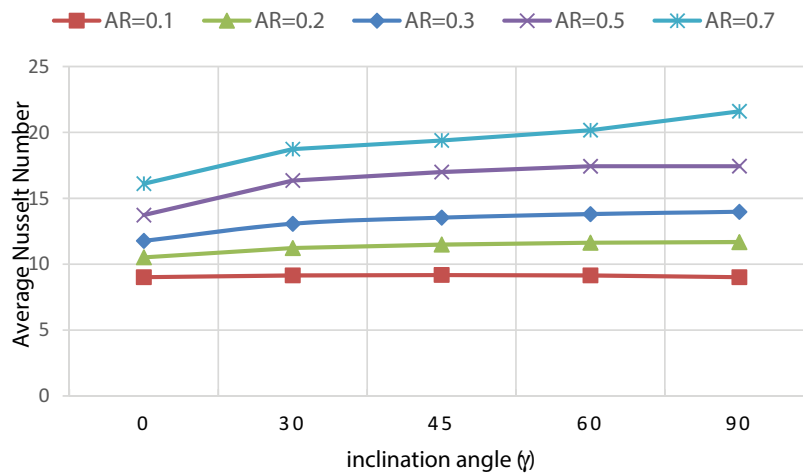


Figure 13. Average Nusselt Number on the heated cylinder versus inclination angle (γ) with different Aspect ratio (AR) at $Ra = 10^6$.

attendant increase in the rate of heat transfer. When $\gamma = 60^\circ$, the isothermal lines became denser. Around the top of the rectangular cylinder, the rate of heat transfer is apparently tending towards becoming symmetric. At $\gamma = 90^\circ$, the isothermal lines had become symmetrical about the vertical axis of the cylinder and denser to the left of the cylinder.

The velocity streamlines are symmetrical about the rectangular cylinder when $\gamma = 0$ and, there exists a vortex on each side of the cylinder. At $\gamma = 30^\circ$, the streamlines deviated from its symmetric nature. Furthermore, the velocity had become more intensified above the upper surface of the inclined rectangular cylinder. Below the cylinder, the velocity got retarded forming two secondary vortices towards the right side. At $\gamma = 45^\circ$, the velocity around the inclined surface of the rectangular cylinder was still high but less vigorous compared to when $\gamma = 30^\circ$. Below the rectangular cylinder, the velocity reduced and the two vortices earlier noticed at $\gamma = 30^\circ$ disappeared. At $\gamma = 60^\circ$, the velocity became less vigorous. At $\gamma = 90^\circ$, the flow had become symmetrical and two secondary vortices surfaced on the top horizontal wall of the cylinder.

Figure 7 is the plot of velocity magnitude at plane $x = 0.075$ for $AR = 0.7$. The plots show that for Rayleigh number (Ra) between 10^3 and 10^4 , the fluid velocity is almost zero signifying that buoyancy is yet to take place; beyond Ra of 10^4 , the velocity increases from almost zero, reaches a local maximum and then decreases again along the plane and picks up again till it reaches a maximum at about 0.8 length of the plane and then decreases to zero at the wall of the square cavity.

Figure 8 depicts the velocity magnitudes along the plane formed by the points (0.5, 0.6) and (0.5, 1). For Rayleigh number (Ra) between 10^3 and 10^4 , the plots show the same trend as Figure 7. For $Ra > 10^4$, velocity

increases along the plane to a maximum and then decreases as expected to zero on the wall.

The plots in Figure 9 correspond to the left, top, right, and bottom walls, respectively when $\gamma = 0$ and $AR = 0.5$. The plots show generally that the local Nusselt number increases for all the curves as Rayleigh number increases. Furthermore, the local Nusselt number variation is symmetric about the mid-point of the walls and the rates of heat transfer on the left and right walls are the same. However, due to increase in buoyancy force, the heat transfer rate at the bottom wall is higher than that of the top wall. The plots in Figure 10 correspond to the left, top, right and bottom walls, respectively when $\gamma = 0^\circ$ and $AR = 0.5$. Generally, the rate of heat transfer increases with increasing Rayleigh number except for the bottom wall. In addition, the rates of heat transfer on the left and right walls are a mirror image of each other and the rate of heat transfer on the top wall was observed to be significantly higher than that of the bottom wall.

$AR = 0.5$, and $\gamma = 0$

Figures 11 and 12 are the plots of average Nusselt number versus Rayleigh number for various aspect ratios when $\gamma = 0^\circ$ and $\gamma = 90^\circ$. The figures show that as Rayleigh number increases, the average Nusselt number increases with increasing aspect ratio. For aspect ratio in the range of $0.1 \leq AR \leq 0.3$, average Nusselt number increases linearly with increasing Rayleigh number but for aspect ratio in the interval of $0.4 \leq AR \leq 0.7$, average Nusselt number increases monotonically with Rayleigh number. Furthermore, the plots reveal that the rate of heat transfer when $\gamma = 90^\circ$ is higher than the heat transfer rate when $\gamma = 0^\circ$ for all the range of aspect ratios except at AR value of 0.1 where the rate of heat transfer for both orientation angles is the same.

Figure 13 describes the relationship between the average Nusselt number and inclination angle for different AR values. When $AR = 0.1$, the average Nusselt number

increases with increasing angle of inclination from $\gamma = 0$ and then begins to decrease when $\gamma = 60$. However, from $AR = 0.2$ to $AR = 0.7$, the average Nusselt number increases as the inclination angle increases.

8. Conclusions

A detailed analysis of the distribution of stream function, isotherms and Nusselt number was performed to investigate the effects of aspect ratio in the range of $0.1 \leq AR \leq 0.7$, Rayleigh number in the range of $10^2 \leq Ra \leq 10^6$ and inclination angle in the interval of $0^\circ \leq \gamma \leq 90^\circ$ on fluid flow and heat transfer characteristics in a square enclosure with a rectangular cylinder embedded in it. The study shows that the average Nusselt number for the rectangular cylinder at aspect ratio (AR) value of 0.1 was found to be the same for orientation angles of 0° and 90° , and, for 30° and 60° . Furthermore, the average Nusselt number for aspect ratio in the range of $0.2 \leq AR \leq 0.7$ increases as Rayleigh number ($10^2 \leq Ra \leq 10^6$) and orientation angle ($0^\circ \leq \gamma \leq 90^\circ$) increases.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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Data availability statement

The authors confirm that the data supporting the findings of this study are available within the article.

Nomenclature

AR	Aspect ratio, W/L	
H	dimensionless height of rectangular cylinder	Nu_l local Nusselt number
L	dimensionless Height of square cavity	P dimensionless pressure
$\overline{Nu}$	Average Nusselt number	Pr Prandtl number
q	heat flux, W/m^2	Ra Rayleigh number
T	Temperature, K	U, V dimensionless velocity components
W	dimensionless width of the cylinder	X, Y dimensionless Cartesian coordinates
ϕ	dimensionless temperature	γ The inclination angle
ψ	Stream function	

Subscript

$ccold/hHot/local$

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