Assessing the Flexibility of the Exponentiated Generalized Exponential Distribution.

Pelumi E. Oguntunde[1*], Adebowale O. Adejumo[2], and Kazeem A. Adepoju[3]

[1]Department of Mathematics, Covenant, University, Ota, Ogun State, Nigeria.
[2]Department of Statistics, University of Ilorin, Kwara State, Nigeria.
[3]Department of Statistics, University of Ibadan, Oyo State, Nigeria.

E-mail: peluemman@yahoo.com*

ABSTRACT

A three parameter probability model which serves as a generalization of the Exponential distribution was studied. The new model is named Exponentiated Generalized Exponential (EGE) distribution. The shape of the model could be increasing, decreasing or unimodal (depending on the value of the parameters). Explicit expressions are provided for the moments and generating functions, reliability function and failure rate. The method of maximum likelihood estimation (MLE) was proposed for the estimation of the parameters. An application to two real data sets was provided in order to assess the flexibility of the proposed model over some models in the literature.

(Keywords: exponential distribution, exponentiated generalized exponential distribution, generalization, maximum likelihood estimation)

INTRODUCTION

Exponential distribution has been widely used for the analysis of Poisson processes and it has also received appreciable use for problems in reliability. It is the only continuous distribution that has a constant failure rate. It has a unique property of being memoryless and its discrete analogue is the Geometric distribution.

Meanwhile, various generalizations of the Exponential distribution have been proposed in the literature and these models have been found to be better than the Exponential distribution when applied to data sets. The Beta-Exponential distribution; Nadarajah and Kotz (2006) and Weibull-Exponential distribution; Oguntunde et al. (2015) are examples of such.

Several classes of generalized distributions exist in the literature: for instance, The Beta-Generalized family of distributions; Eugene et al. (2002), The Kumaraswamy Generalized family of distributions; Cordeiro, et al. (2011); and the Exponentiated Generalized family of distributions; Cordeiro et al, (2013) (among many others) are examples; see Tahir et al (2014) for full details. All these classes were proposed to provide ways of increasing the flexibility of distributions.

We would like to note that Gupta and Kundu (1999) proposed and studied a generalization of the Exponential distribution called Generalized Exponential distribution by introducing a shape parameter to the Exponential distribution. The probability density function (pdf) of the Generalized Exponential (GE) distribution is given by:

$$f(x) = \alpha\lambda e^{-\lambda x}\left(1-e^{-\lambda x}\right)^{\alpha-1} \qquad \textbf{(1)}$$

For $x > 0, \lambda > 0$
where α is the shape parameter
λ is the scale parameter

The corresponding cumulative density function (cdf) of the Generalized Exponential distribution is given by:

$$F(x) = \left(1-e^{-\lambda x}\right)^{\alpha} \qquad \textbf{(2)}$$

For $x > 0, \lambda > 0$

But, of interest to us in this research is the Exponentiated Generalized family of distributions introduced by Cordeiro et al., 2013. This article seeks to extend the work of Cordeiro et al., 2013 in a view to defining and investigating a three parameter probability model named

Exponentiated Generalized Exponential distribution.

The Exponentiated Generalized family of distributions is an extension of the Exponentiated type distribution which can be widely applied in many areas of biology and engineering; see Cordeiro et al (2013) for details. It has been used to define the Exponentiated Generalized Inverse Weibull distribution; Elbatal and Muhammed (2014), Exponentiated Generalized Inverse Exponential distribution; Oguntunde et al. (2014), and Exponentiated Generalized Gumbel distribution Andrade et al (2015) successfully.
Given a non-negative continuous random variable X, the pdf of the Exponentiated generalized (EG) class of distributions is defined by;

$$f(x)=abg(x)\left\{1-G(x)\right\}^{a-1}\left[1-\left\{1-G(x)^{a}\right\}\right]^{b-1} \quad \textbf{(3)}$$

where $a,b>0$ are additional shape parameters
The corresponding cdf is given by:

$$F(x)=\left[1-\left\{1-G(x)\right\}^{a}\right]^{b} \quad \textbf{(4)}$$

where $G(x)$ is the cdf of the baseline distribution and $g(x)=\dfrac{dG(x)}{dx}$

The model in Equation (4) is considered to be more tractable than the beta generalized family of distributions introduced by Eugene et al., 2002 since Equation (4) does not involve any special function like the incomplete beta function. Equation (4) also has mild algebraic properties for simulation purposes because its quantile function takes a simple form; Cordeiro et al., 2013.

The rest of this article introduces the Exponentiated Generalized Exponential (EGE) distribution; deals with some basic statistical properties of the proposed model coupled with the estimation of the parameters; discusses the application of the proposed model to real life data sets; and ends with a concluding remark.

THE EXPONENTIATED GENERALIZED EXPONENTIAL (EGE) DISTRIBUTION

The pdf of the Exponential distribution with parameter λ is given by:

$$g(x)=\lambda e^{-\lambda x} \;; \qquad x\geq 0,\lambda>0 \quad \textbf{(5)}$$

where λ is the scale parameter

The cdf is given by:

$$G(x)=1-e^{-\lambda x}\;; \qquad x\geq 0,\lambda>0 \quad \textbf{(6)}$$

Also,

$$E[X]=\frac{1}{\lambda} \quad \textbf{(7)}$$

$$Var[X]=\frac{1}{\lambda^{2}} \quad \textbf{(8)}$$

The Quantile function is given by:

$$Q_{G(u)}=-\lambda^{-1}\log(1-u) \quad \textbf{(9)}$$

Hence, the proposed Exponentiated Generalized Exponential (EGE) distribution is derived by substituting Equations (5) and (6) into Equation (3). Therefore, if a continuous non-negative random variable X is such that:

$X \sim EGE(a,b,\lambda)$, its pdf is given by;

$$f(x)=ab\lambda e^{-\lambda x}\left\{1-\left(1-e^{-\lambda x}\right)\right\}^{a-1}\left[1-\left\{1-\left(1-e^{-\lambda x}\right)^{a}\right\}\right]^{b-1} \quad \textbf{(10)}$$

Equation (10) thus reduces to give:

$$f(x)=ab\lambda\left(e^{-\lambda x}\right)^{a}\left[\left\{1-e^{-\lambda x}\right\}^{a}\right]^{b-1} \quad \textbf{(11)}$$

For $x>0,a>0,b>0,\lambda>0$

where a and b are shape parameters
λ is the scale parameter

The corresponding cdf is given by:

$$F(x)=\left[1-\left\{1-\left(1-e^{-\lambda x}\right)\right\}^{a}\right]^{b} \quad \textbf{(12)}$$

Equation (12) reduces to give:

$$F(x)=\left[1-\left(e^{-\lambda x}\right)^{a}\right]^{b} \quad \textbf{(13)}$$

$$x>0, a>0, b>0, \lambda>0$$

<u>Expansions for the Cumulative Density Function</u>

Following Cordeiro et al., 2013, for any real non-integer $'b'$, they considered a power series expansion:

$$(1-z)^{b-1}=\sum_{k=0}^{\infty}\frac{(-1)^{k}\Gamma(b)}{\Gamma(b-k)k!}z^{k} \quad \textbf{(14)}$$

The expression in Equation (14) is valid for $|z|<1$. Using the binomial expansion for a positive real power, the resulting cdf is given by:

$$F(x)=\sum_{j=0}^{\infty}w_{j}G(x)^{j} \quad \textbf{(15)}$$

The coefficients $w_{j}=w_{j}(a,b)$ were given by:

$$w_{j}=\sum_{k=0}^{\infty}\frac{(-1)^{k+j}\Gamma(b+1)\Gamma(ak+1)}{\Gamma(b-k)k!j!}$$

With this understanding, the cdf of the proposed EGE distribution can therefore be written as:

$$F(x)=\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-1)^{k+j}\Gamma(b+1)\Gamma(ak+1)}{\Gamma(b-k)k!j!}\left(1-e^{-\lambda x}\right)^{j} \quad \textbf{(16)}$$

Equation (16) is an infinite power series of the Exponential distribution.

From the series expansion in Equation (14), Cordeiro et al., (2013) gave the pdf of the Exponentiated Generalized (EG) class of distributions (for $'a'$ real non-integer) as:

$$f(x)=abg(x)\sum_{j=0}^{\infty}t_{j}G(x)^{j} \quad \textbf{(17)}$$

The coefficients $t_{j}=t_{j}(a,b)$ were given as:

$$t_{j}=\frac{(-1)^{j}\Gamma(b)}{j!}\sum_{k=0}^{\infty}\frac{(-1)^{k}\Gamma(a(k+1))}{\Gamma(b-k)\Gamma((k+1)a-j)k!} \quad \textbf{(18)}$$

With this understanding, we re-write the pdf of the proposed EGE distribution as:

$$f(x)=ab\lambda e^{-\lambda x}\sum_{j=0}^{\infty}\frac{(-1)^{j}\Gamma(b)}{j!}\text{x}\sum_{k=0}^{\infty}\frac{(-1)^{k}\Gamma(a(k+1))}{\Gamma(b-k)\Gamma((k+1)a-j)k!}\left(1-e^{-\lambda x}\right)^{j} \quad \textbf{(19)}$$

<u>Special Cases</u>

We observe that some important models are sub-models of the proposed EGE distribution. For instance:

1. For $a=1$, Equation (10) reduces to give the Generalized Exponential (GE) distribution.
2. For $b=1$, Equation (10) reduces to give the Exponentiated Exponential distribution.
3. For $a=b=1$, Equation (10) reduces to give the Exponential distribution (which is the baseline distribution).

The possible shapes for the pdf of the proposed model at different values of parameters are given in Figures 1 - 3 as follows:

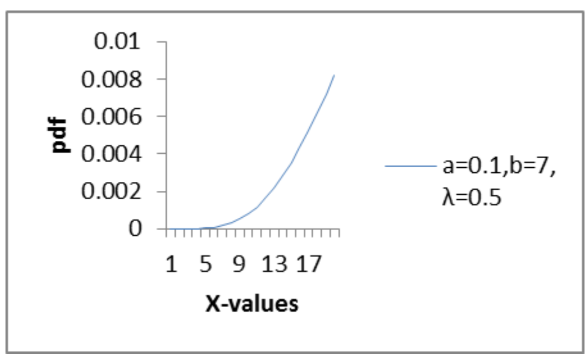

Figure 1: Plot for the pdf at $a=0.1, b=7, \lambda=0.5$

We observe from Figure 1 that the curve increases as the value of 'x' increases. Hence, the shape of the proposed EGE distribution could be increasing.

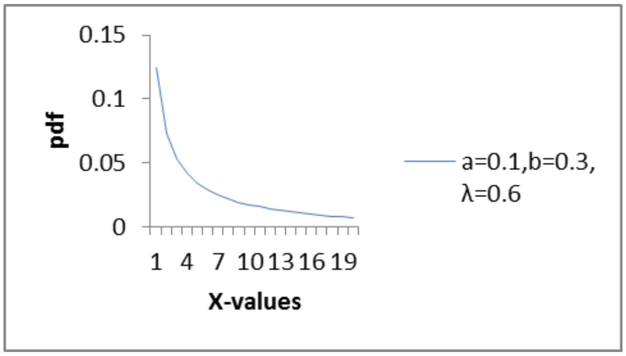

Figure 2: Plot for the pdf at $a = 0.1, b = 0.3, \lambda = 0.6$

We observe from Figure 2 that the curve decreases as the value of 'x' increases. Therefore, the shape of the proposed EGE distribution could be decreasing.

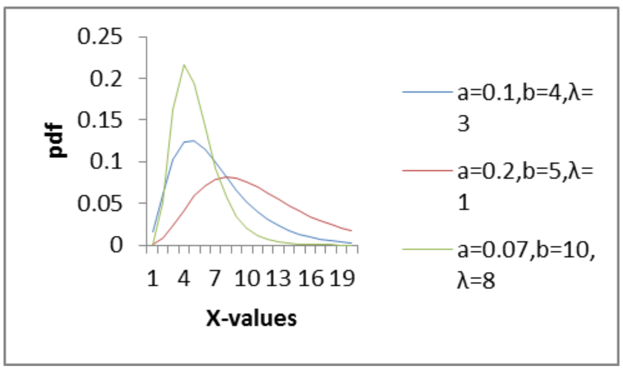

Figure 3: Plot for the pdf at varying values of a, b, λ

We observe from Figure 3 that the shape of the proposed EGE distribution could be unimodal. The plot for the cdf is shown in Figure 4 as:

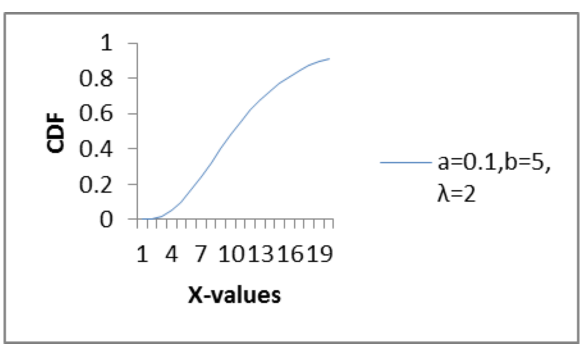

Figure 4: Plot for the cdf at $a = 0.1, b = 5, \lambda = 2$

SOME PROPERTIES OF THE EXPONENTIATED GENERALIZED EXPONENTIAL DISTRIBUTION

In this section, we present some basic properties of the proposed model starting with the asymptotic properties.

Asymptotic Behavior

We seek to investigate the behavior of the proposed model as given in Equation (10) as $x \to 0$ and as $x \to \infty$. This involves considering $\lim_{x \to 0} f(x)$ and $\lim_{x \to \infty} f(x)$.

As $x \to 0$;

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left[ab\lambda \left(e^{-\lambda x}\right)^a \left[\left\{1 - e^{-\lambda x}\right\}^a \right]^{b-1} \right]$$

$$= 0$$

As $x \to \infty$;

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \left[ab\lambda \left(e^{-\lambda x}\right)^a \left[\left\{1 - e^{-\lambda x}\right\}^a \right]^{b-1} \right]$$

$$= 0$$

These results confirm further that the proposed model has a unique mode.

Reliability Analysis

The reliability (survival) function is given by;

$S(x) = 1 - F(x)$

Hence, we present the reliability function of the Exponentiated Generalized Exponential distribution as:

$$S(x) = 1 - \left[1 - \left(e^{-\lambda x}\right)^a \right]^b \qquad \textbf{(20)}$$

where; $x > 0, a > 0, b > 0, \lambda > 0$

The probability that a system having age $'x'$ units of time will survive up to $'x + t'$ units of time for $x > 0, a > 0, b > 0, \lambda > 0$ and $t > 0$ is given by:

$$S_{EGE}(t|x)=\frac{S_{EGE}(x+t)}{S_{EGE}(x)}$$

$$S_{EGE}(t|x)=\frac{1-\left[1-\left(e^{-\lambda(x+t)}\right)^{a}\right]^{b}}{1-\left[1-\left(e^{-\lambda x}\right)^{a}\right]^{b}} \quad \textbf{(21)}$$

Hazard function (failure rate) is given by:

$$h(x)=\frac{f(x)}{1-F(x)}$$

We thus present the hazard function of the proposed model as:

$$h(x)=\frac{ab\lambda\left(e^{-\lambda x}\right)^{a}\left[\left\{1-e^{-\lambda x}\right\}^{a}\right]^{b-1}}{1-\left[1-\left(e^{-\lambda x}\right)^{a}\right]^{b}} \quad \textbf{(22)}$$

The plots for the failure rate of the proposed model at various parameter values are given in Figures 5 - 7 as:

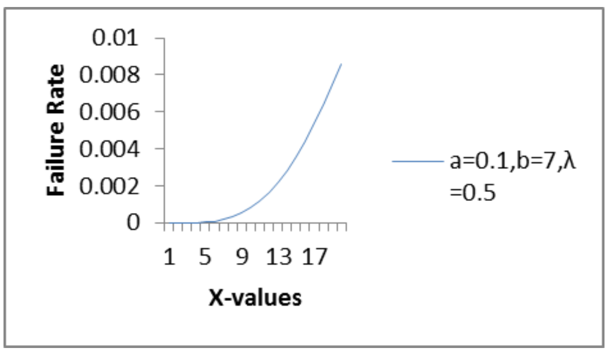

Figure 5: Failure rate for the EGE distribution at $a=0.1, b=7, \lambda=0.5$

Figure 5 shows that the failure rate increases as the value of 'x' increases. This shows that the model can be used for modeling problems where the risk is low at the initial stage but increases with time.

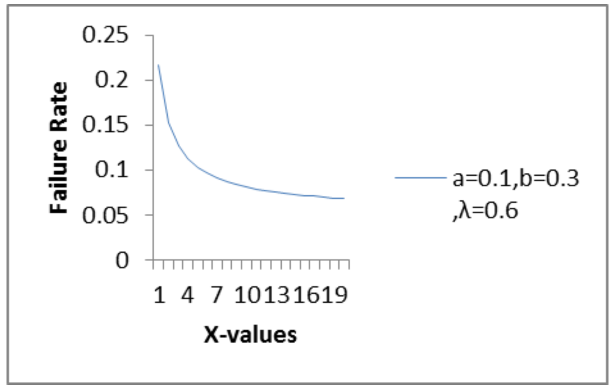

Figure 6: Failure rate for the EGE distribution at $a=0.1, b=0.3, \lambda=0.6$

Figure 6 shows that the failure rate decreases as the value of 'x' increases. This shows that the model can be used for modeling problems where the risk is high at the initial stage but decreases with time.

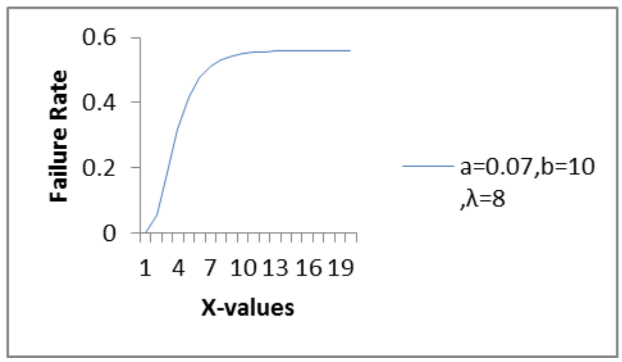

Figure 7: Failure rate for the EGE distribution at $a=0.07, b=10, \lambda=8$

Figure 7 shows that the failure rate increases as the value of 'x' increases but later remains constant at a point. This shows that the model can be used for modeling problems where the risk is low at the initial stage; increases with time but later remain constant.

Moments

The moments of any Exponential Generalized (EG) distribution can be expressed as an infinite weighted sum of the probability weighted sum of the parent distribution; (Cordeiro et al., 2013). Therefore, the rth moment for the EG distribution is given by:

$$\mu_r = E\left[X^r\right] = ab\sum_{j=0}^{\infty} t_j \tau_{rj} \quad \textbf{(23)}$$

Following Cordeiro et al., 2013,

$$\tau_{rj} = (-1)^r \lambda^{-r} \int_0^1 u^j \left[\log(1-u)\right]^r du = \sum_{m=0}^{\infty} \frac{(-1)^{m+r}\binom{j}{m}}{(m+1)^{r+1}} \quad \textbf{(24)}$$

Therefore, if a continuous random variable X is such that; $X \sim EGE(a,b,\lambda)$, we substitute Equations (18) and (24) into Equation (23). Then, the rth moment is presented as:

$$\mu_r = ab\sum_{j=0}^{\infty} \frac{(-1)^j \Gamma(b)}{j!} \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma\left(a(k+1)\right)}{\Gamma(b-k)\Gamma\left((k+1)a-j\right)k!} \times \sum_{m=0}^{\infty} \frac{(-1)^{m+r}\binom{j}{m}}{(m+1)^{r+1}} \quad \textbf{(25)}$$

The mean and the other higher-order r^{th} moments can be derived from Equation (25).

Generating Functions

Cordeiro et al., 2013 gave three formulae for the moment generating function (mgf) for the EG distribution. In this article, we shall present the mgf of the EGE distribution using the third formula:

$$M_X(t) = ab\sum_{j=0}^{\infty} t_j \rho_j(t) \quad \textbf{(26)}$$

$$\rho_j(t) = \int_0^1 e^{\left[tQ_G(u)\right]} u^j du \quad \textbf{(27)}$$

Hence, the mgf for the EGE distribution is given by:

$$M_X(t) = ab\sum_{j=0}^{\infty} \frac{(-1)^j \Gamma(b)}{j!} \times \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma\left(a(k+1)\right)}{\Gamma(b-k)\Gamma\left((k+1)a-j\right)k!} B(j+1, 1-\lambda t) \quad \textbf{(28)}$$

Order Statistics

The pdf of the ith order statistic for $i = 1, 2, ..., n$ for iid random variables $X_1, X_2, ..., X_n$ is given by:

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-i)!} f(x) F(x)^{i-1} \{1 - F(x)\}^{n-i} \quad \textbf{(29)}$$

Here, we take $f(x)$ and $F(x)$ in Equation (29) to be the pdf and cdf of the EGE distribution respectively. Now:

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-i)!} ab\lambda\left(e^{-\lambda x}\right)^a \left[\left\{1-e^{-\lambda x}\right\}^a\right]^{b-1} \times \left\{\left[1-\left(e^{-\lambda x}\right)^a\right]^b\right\}^{i-1} \left\{1-\left\{\left[1-\left(e^{-\lambda x}\right)^a\right]^b\right\}\right\}^{n-i}$$

In particular, the pdf of the minimum and maximum order statistics of the EGE distribution are respectively given by:

$$f_{X_{(1)}}(x) = nab\lambda\left(e^{-\lambda x}\right)^a \left[\left\{1-e^{-\lambda x}\right\}^a\right]^{b-1} \left\{1-\left\{\left[1-\left(e^{-\lambda x}\right)^a\right]^b\right\}\right\}^{n-1} \quad \textbf{(30)}$$

and:

$$f_{X_{(n)}}(x) = nab\lambda\left(e^{-\lambda x}\right)^a \left[\left\{1-e^{-\lambda x}\right\}^a\right]^{b-1} \left\{\left[1-\left(e^{-\lambda x}\right)^a\right]^b\right\}^{n-1} \quad \textbf{(31)}$$

Estimation of Parameters

We estimate the parameters of the proposed model using the method of maximum likelihood estimation (MLE) as follows; Let $X_1, X_2, ..., X_n$ be a random sample of size *n* from the $EGE(a,b,\lambda)$ distribution. The likelihood function is given by:

$$L\left(\underset{\sim}{X} \mid a,b,\lambda\right) = \prod_{i=1}^{n} \left[ab\lambda\left(e^{-\lambda x}\right)^a \left[\left\{1-e^{-\lambda x}\right\}^a\right]^{b-1}\right]$$

Let $l = \log L\left(\underset{\sim}{X} \mid a,b,\lambda\right)$

Therefore, the log-likelihood function is given by Equation (32):

$$l = n\log a + n\log b + n\log\lambda - a\lambda\sum_{i=1}^{n} x_i + a(b-1)\sum_{i=1}^{n}\log\left\{1-\left[e^{-\lambda x_i}\right]\right\} \quad \textbf{(32)}$$

Differentiating Equation (32) with respect to each of the model parameters and solving the resulting non-linear system of equations give the maximum likelihood estimate of the parameters.

REAL LIFE APPLICATION & RESULTS

In this section, the robustness of the EGE distribution is assessed using two real life data set. We provide an application of the EGE distribution in comparison to its sub-models: Generalized Exponential distribution and Exponential distribution.

First Data Set: The data set given by Lee and Wang (2003) which represent remission times (in months) of a random sample of 128 bladder cancer patients shall be used in this research. The data are as follows:

0.08, 2.09, 3.48,4.87, 6.94 , 8.66, 13.11, 23.63, 0.20, 2.23, 3.52, 4.98, 6.97, 9.02, 13.29, 0.40, 2.26, 3.57,5.06, 7.09, 9.22, 13.80, 25.74, 0.50, 2.46 , 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51, 2.54,3.70, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81, 2.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64,3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69, 4.18, 5.34, 7.59, 10.66, 15.96, 36.66, 1.05,2.69, 4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26,2.83, 4.33, 5.49, 7.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 1.40, 3.02,4.34, 5.71, 7.93, 11.79, 18.10, 1.46, 4.40, 5.85, 8.26, 11.98, 19.13, 1.76, 3.25, 4.50, 6.25,8.37, 12.02, 2.02, 3.31, 4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 6.76, 12.07, 21.73, 2.07,3.36, 6.93, 8.65, 12.63, 22.69.

Table 1: Descriptive Statistics on Remission Times.

Min.	Q_1	Q_2	Mean	Q_3	Max.	Var.	Skewness	Kurtosis
0.080	3.348	6.395	9.366	11.840	79.050	110.425	3.286569	18.48308

Table 2: Performance Ratings of Some Distributions.

Model	Estimates	Statistics	
		Log-likelihood	AIC
Exponential	$\hat{\lambda} = 0.1067734$	-414.3419	830.6838
Generalized Exponential	$\hat{b} = 1.217953$ $\hat{\lambda} = 0.121167$	-413.0776	830.1552
Exponentiated Exponential	$\hat{a} = 7.338231$ $\hat{\lambda} = 0.014550$	-414.3419	832.6838
Exponentiated Generalized Exponential	$\hat{a} = 0.00760888$ $\hat{b} = 0.30351482$ $\hat{\lambda} = 1.62037332$	-273.5548	553.1095

Second Data Set: The data presented here represents the Death times (in weeks) of patients with cancer of tongue with aneuploid DNA profile. The data has been used by Sickle-Santanello et al. (1988), Klein & Moeschberger (2003) and by Jain et al (2014). The data is as follows:

1, 3, 3, 4, 10, 13, 13, 16, 16, 24, 26, 27, 28, 30, 30, 32, 41, 51, **61**, 65, 67, 70, 72, 73, **74**, 77, **79**, **80**, **81**, **87**, **87**, **88**, **89**, 91, 93, **93**, 96, **97**, 100, **101**, 104, **104**, **108**, **109**, **120**, **131**, **150**, 157, 167, **231**, **240**, **400**

NOTE: The data written in bold represents censored observations.

Table 3: Descriptive Statistics on Death Times.

Min	Q_1	Q_2	Mean	Q_3	Max	Var.	Skewness	Kurtosis
1.00	30.00	78.00	81.76	100.20	400.00	4774.898	2.193221	10.35995

Table 4: Performance Ratings of Some Distributions.

Model	**Estimates**	**Statistics**	
		Log-likelihood	**AIC**
Exponential	$\hat{\lambda} = 0.012232$	-280.9938	563.9876
Generalized Exponential	$\hat{b} = 1.264407$ $\hat{\lambda} = 0.014129$	-280.236	564.4719
Exponentiated Exponential	$\hat{a} = 3.050e-04$ $\hat{\lambda} = 4.011e+01$	-280.9938	565.9876
Exponentiated Generalized Exponential	$\hat{a} = 2.858e-04$ $\hat{b} = 1.986e-01$ $\hat{\lambda} = 1.701e+00$	-184.2438	374.4876

NOTE: The model with the lowest Akaike Information Criteria (AIC) is ranked the best.

CONCLUSION

This paper introduces the probability density function of the Exponentiated Generalized-Exponential distribution and its application. The essence is to induce skewness into the baseline distribution to withstand strong asymmetry in the data that are heavily skewed. Statistical properties of the new distribution have been properly investigated. Exponentiated Generalized-Exponential distribution provides the best fit for the data under review as it poses the smallest AIC among the distributions considered.

REFERENCES

1. Andrade, T., H. Rodrigues, M. Bourguignon, and G. Cordeiro. 2015. “The Exponentiated Generalized Gumbel Distribution”. *Revista Colombiana de Estadistica*. 38(1):123-143.

2. Cordeiro, G.M. and M. de Castro. 2011. “A New Family of Generalized Distributions”. *Journal of Statistical Computation and Simulation*. 81(7): 883-898.

3. Cordeiro, G.M., E.M. Ortega, and D.C. da Cunha. 2013. “The Exponentiated Generalized Class of Distributions”. Journal of Data Science 11:1-27.

4. Elbatal, I. and H.Z. Muhammed. 2014. “Exponentiated Generalized Inverse Weibull Distribution”. *Applied Mathematical Sciences*. 8(81):3997-4012.

5. Eugene, N., C. Lee, and F. Famoye. 2002. “Beta-Normal Distribution and its Applications”. *Communication in Statistics: Theory and Methods*. 31:497-512.

6. Gupta, R.D and D. Kundu. 1999. “Generalized Exponential Distributions”. *Australian and New Zealand Journal of Statistics*. 41 (2):173-188.

7. Jain, K., N. Singla, and S.K. Sharma. 2014. “The Generalized Inverse Generalized Weibull Distribution and Its Properties”. *Journal of Probability*. 2014, Article ID 736101, 11 pages. http://dx.doi.org/10.1155/2014/736101

8. Klein J.P. and M.L. Moeschberger., 2003. *Survival Analysis: Techniques for Censored and Truncated Data*. Springer: New York, NY.

9. Lee, E.T. and J.W. Wang. 2003. *Statistical Methods for Survival Data Analysis*. 3rd Edition, Wiley: New York, NY.

10. Nadarajah, S. and S. Kotz. 2006. “The Beta-Exponential Distribution”. *Reliability Engineering and System Safety*. 91:689-697.

11. Oguntunde, P.E., A.O. Adejumo, and O.S. Balogun. 2014. “Statistical Properties of the Exponentiated Generalized Inverted Exponential Distribution”. *Applied Mathematics*. 4(2):47-55.

12. Oguntunde, P.E, O.S. Balogun, H.I. Okagbue, and S.A. Bishop. 2015. “The Weibull-Exponential Distribution: Its Properties and Applications”. *Journal of Applied Sciences*. 15(11):1305-1311.

13. Sickle-Santanello, B.J., W.B. Farrar, and S. Keyhani-Rofagha. 1988. “A Reproducible System of Flow Cytometric DNA Analysis of Paraffin Embedded Solid Tumors: Technical Improvements and Statistical Analysis. *Cytometry*. 9:594–599.

14. Tahir M. H., G.M. Cordeiro, A. Alzaatreh, M. Mansoor, and M. Zubair. 2014. “The Logistic-X Family of Distributions and Its Applications”. *Communication in Statistics-Theory and Methods*.

SUGGESTED CITATION

Oguntunde, P.E., A.O. Adejumo, and K.A. Adepoju. 2016. “Assessing the Flexibility of the Exponentiated Generalized Exponential Distribution”. *Pacific Journal of Science and Technology*. 17(1):49-57.

Pacific Journal of Science and Technology