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MHD free convective heat and mass transfer flow of dissipative Casson fluid with variable viscosity and thermal conductivity effects

Amos S. Idowu, Mojeed T. Akolade , Jos U. Abubakar  and Bidemi O. Falodun 

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ABSTRACT

In this paper, the Cattaneo–Christov heat flux relocation paradox on Casson fluid with MHD and dissipative effects was considered. The buoyancy and heat generation effects were believed to be responsible for the natural convection, while variable properties were perceived as temperature-dependent linear function. Under the given assumptions, the governing system of equations was formulated and transformed. Hence, the Chebyshev collocation spectral approach was therefore employed to achieve an approximate solution. However, the behaviour of temperature-dependent variability establishes the relationship between the boundary layer flow of plastic dynamic viscosity and the Casson fluid. Furthermore, it was observed that a corresponding increase in the stretching index (n) increases the skin friction and decreases the energy and mass gradient accordingly. The relocation phenomenon contributes to a decrease in the thermal process, while the temperature gradient attained maximum within (0.4 – 0.6) variation of the Casson parameter.

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stretching sheet; spectral
collocation method

1. Introduction

Generally, alloy structures like liquid metals, electrolytes and plasma demonstrate higher thermal conductivity, this is ascribed to its effectiveness in converting energy from a heat source into liquid and high possession of electrical conductivity. This application can be found in electronics, chemical industries, MHD pumps, power generation, among others [1]. Thus the dynamics of an electrically conducting fluid known as magneto fluid dynamics (i.e. the idea that produces its magnetic field from the caused electrical current, solely as a result of fluid movement into a magnetic field) is therefore essential [2]. To this, the attention of various researchers drawn into the MHD modelling in different physical properties was analysed, among which Mahanthesh et al. [3] presented their study on Cu- H_2O nanofluid, Nasir et al. [4] on a Carbon nanotube, Bilal et al. [5] on Carreau fluid, Idowu and Falodun [2] on Water's B viscoelastic fluid, Arthur et al. [6], Ajayi et al. [7] and Malik et al. [8] on Casson fluid accordingly. Combined MHD and heat transfer effects past a moving plate was studied by Ellahi et al. [9]. Recently, Kumar and Srinivas [10] highlighted the effects of unsteady MHD and thermal radiation past an inclined stretching sheet. They noticed a rise in fluid velocity with a higher magnitude of solutal and local Grashof number.

Recent attention on MHD provides more emphasis on its usefulness in industries, engineering, medical processes among others. Then, Asha and Sunitha

[11] present a study on MHD peristaltic blood flow of Eyring–Powell nanofluid through an irregular channel, Abubakar and Adeoye [12] on MHD porous tapered stenosed artery subject to radiation effect, Idowu and Falodun [13] on Soret–Dufour, thermophoresis and heat and mass transfer of Casson nanofluid over an inclined plate, Khan et al. [14] on peristaltic flow of MHD nanofluids in an asymmetric channel with different nanoparticle shapes. While El-Masry et al. [15] employed the variation iteration techniques on the flow of Eyring–Powell fluid to seek the solution of MHD variation on free convective heat transfer analysis across the wavy channel. These studies highlighted that the magnetic field effect gives rise to an opposing force which produces a reduction in the frictional force of MHD conducting fluid. However, Abubakar and Adeoye [12] thus concluded that the MHD effect gains a maximum momentum in the converging region relative to the diverging region of the stenosed artery.

An investigation over a stretching flat sheet is dated back to Crane [16]. For its enormous usages in industries and engineering operations (textile industry, cooling of papers, glass, to mention few) [17], several studies past a stretching/shrinking sheet were called for, among which Rao et al. [18] experimented heat transfer and slip effects over a stretching surface. Similar studies include, results on Casson fluid [19–21], MHD flow with power-law velocity in [22], Maxwell fluid in [23] and Nanofluid containing gyrotactic microorganisms by

Mehryan et al. [17]. Double stratifications and viscous dissipation effects were presented in [24]. And recently, Hussanan [25] presented the effects of a non-linear stretching sheet and Newtonian heating on MHD Casson fluid. While [26] presents its analysis on the isothermal sphere in MHD Casson nanofluid. And the effects of thermophysical properties on Casson nanofluid were discussed by Gbadeyan et al. [27], where variable properties were found to raise the velocity, thus, decreases energy and volume fraction respectively.

The physics of stratification in geophysical flows is paramount since it negates ambient heating from gaining access into the fluid region, thereby, minimize concentration and energy distributions [28–30]. To this, Omowaye and Animasaun [31] presented the differences in thermal properties over a melting sheet of an upper-converted Maxwell fluid. They deduced that the flow distributions declined to a rise in fluid stratification. In assuming the physical nature of problems, considering the thermophysical properties is essential. To this, a viscous variability of reacting flow over a heated plate was examined by Makinde et al. [32]. They found that for a temperature-dependent viscosity, skin friction decreases while the Nusselt number is appreciated. Animasaun et al. [20] examined the thermophysical properties on Casson fluid with stratification effects. While Agunbiade and Dada [33] reported an increase in resultant fluid velocity due to the rise in dissipation function and some other fluid properties. Ellahi et al. [34] studied the variable thermal properties on Kerosene- Al_2O_3 nano liquid and its application to the cooling system. While Zaib et al. [35] stated that viscous dissipation enhances heat absorption at the surface.

Marin [36] highlighted the contradiction between the theory of relativity and Fourier's law of heat conduction. The study is essential since each material exhibits a distinct time necessary for heat transfer. In an attempt to address these contradictions, a series of the modified laws of Fourier (Paradox of Heat Conduction (PHC)) were suggested, among which the Maxwell–Cattaneo model was found suitable [37]. However, the modified law is found useful in heat convection analysis. As a result, Malik et al. [38], Ali and Sandeep [39], Mustafa [40], Hayat et al. [41] and Arthur et al. [42] consider the modified model and observed an enhancement on heat transfer rate to rise in material time. Recently, Prasad [43] considered the theory of Cattaneo–Christov model on Williamson nanofluid. The theory was found to decelerate both concentration and energy profiles. Nandeppanavar and Shakunthala [44] analysed the modified theory on MHD carbon nanofluid past a stretching sheet. Zhah et al. [45] assumed an effective thermal conductivity on the modified model in a micropolar Casson ferrofluid. While Ijaz and Ayub [46] employed the modified heat flux in the analysis of dual diffusion and stratification with activation energy effects on waters-B viscoelastic fluid.

Among numerical techniques, Chebyshev Spectral Collocation Method (CSCM) based on spectral technique was found satisfactorily good for obtaining an approximate solution due to its potentials in rapid convergence approximations and ability to handle highly non-linear systems of equations [47], alongside its efficiency in the field of Computational Fluid Dynamics (CFD) [48]. Hence, CSCM is a strong numerical technique used in solving PDEs and ODEs systems [49]. Thus Ehrenstein and Peyret [50] solved the unsteady two-dimensional Navier–Stokes equations by applying CSCM. Elbarbary [51] utilized a new spectral differentiation matrix to reduce the round-off error. And recently, Babatin [52] utilized this method in the modelling of an unsteady stretching surface. While in an attempt to solve the fractional-order advection–dispersion equation, Malawi et al. [53] stated that this numerical scheme performed better than the existing schemes.

From the literature, the effects of stratifications and applied magnetic field over different flow assumptions past the stretching sheet had been carried out. While, to the best knowledge of authors, no studies reported yet on chemically reacting MHD free convective heat and mass transfer flow of dissipative Casson fluid with variable viscosity and thermal conductivity effects, using Cattaneo–Christov flux phenomena, thus this research work.

2. Mathematical formulation

Figure 1 described the physical coordinate and model system of free convective, MHD dissipative Casson fluid encompassed two-dimensional, incompressible, laminar, and modified flux model of viscous flow past a variable stretching sheet. Thus the following flow assumptions were made:

- the usual Boussinesq approximation hold, the stretching velocity is $u = U_w(x + b)^n$ such that $n \neq 1$, this is illustrated with the profile $y = A(x + b)^{(1-n)/2}$. n denote the power index, b a positive constant, U_w is the reference velocity and A is a constant assumed minimal,
- flow field is MHD conducting with magnetic field strength $B(x)$. While magnetic Reynolds number is considered so small compared to that of applied magnetic field and thus neglected,
- Cattaneo–Christov heat flux model was assumed over Fourier's law,
- the fluid variability are taking to be a linear function of temperature. $\mu(T) = \mu^*[i_1 + j_1(T_w - T)]$ and $\kappa(T) = \kappa^*[i_2 + j_2(T - T_\infty)]$ where j_1 and j_2 are temperature-dependent viscous parameter and temperature-dependent thermal conductivity parameter respectively,

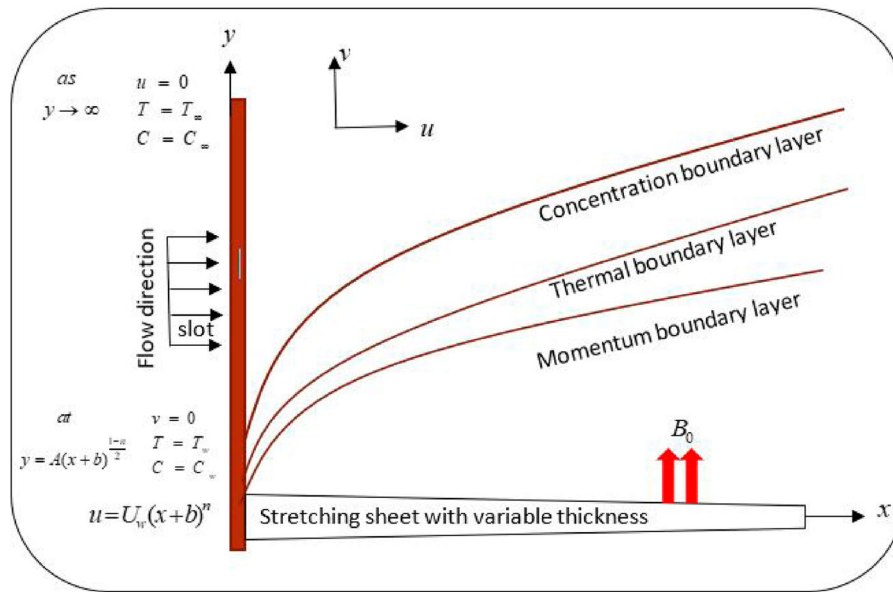


Figure 1. The physical coordinate system of the model.

- Casson fluid

$$\tau_{ij} = \begin{cases} \left(\mu_b + \frac{P_y}{\sqrt{2\pi}} \right) 2e_{ij} & \text{if } \pi > \pi_c \\ \left(\mu_b + \frac{P_y}{\sqrt{2\pi_c}} \right) 2e_{ij} & \text{if } \pi < \pi_c \end{cases}$$

was considered as the base fluid,

- variations in temperature and concentration (stratifications) were assume and,
- space-dependent heat generation/absorption viscous dissipation were therefore considered.

Considering the above assumptions, the governing equations; continuity, momentum, energy, and concentration were given as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_b(T)}{\rho} \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho} \left(1 + \frac{1}{\beta} \right) \frac{\partial u}{\partial y} \frac{\partial \mu_b(T)}{\partial y} - \frac{\sigma \beta_o^2}{\rho} u, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + h_i \left[u \frac{\partial u}{\partial x} \frac{\partial T}{\partial x} + v \frac{\partial v}{\partial y} \frac{\partial T}{\partial y} + u \frac{\partial v}{\partial x} \frac{\partial T}{\partial y} + v \frac{\partial u}{\partial y} \frac{\partial T}{\partial x} + 2uv \frac{\partial^2 T}{\partial y \partial x} + u^2 \frac{\partial^2 T}{\partial x^2} \right] = \frac{1}{\rho C_p} \frac{\partial}{\partial y} \left(k(T) \frac{\partial T}{\partial y} \right) + \left(1 + \frac{1}{\beta} \right) \frac{\mu_b(T)}{\rho C_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{Q_0(T - T_\infty)}{\rho C_p}, \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_p \frac{\partial^2 C}{\partial y^2} - K_r(C - C_\infty). \quad (4)$$

the boundary conditions are:

$$u = U_w(x+b)^n, v = 0, T = T_w, C = C_w \text{ at } y = A(x+b)^{(1-n)/2} \quad (5)$$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty.$$

Following the work presented in [31], the double stratification of thermal and solutal are given accordingly,

$$\begin{aligned} T_w - T_0 &= z_1(x+b)^{(1-n)/2}, \\ T_\infty - T_0 &= z_2(x+b)^{(1-n)/2}, \\ C_w - C_0 &= z_3(x+b)^{(1-n)/2}, \\ C_\infty - C_0 &= z_4(x+b)^{(1-n)/2}. \end{aligned} \quad (6)$$

Hence, these relations hold:

$$\begin{aligned} j_1(T_w - T_0) &= j_1 z_1(x+b)^{(1-n)/2}, \\ j_1(T_\infty - T_0) &= j_1 z_2(x+b)^{(1-n)/2}. \end{aligned} \quad (7)$$

Thus produces double differences in temperature: first by stratification that take place for all x at a point $y = A(x+b)^{(1-n)/2}$ and the other for all x as $y \rightarrow \infty$. To this, temperature-dependent thermal conductivity (γ) and plastic dynamic viscosity (δ) is defined as $\gamma = j_2(T_\infty - T_0)$ and $\delta = j_1(T_\infty - T_0)$ respectively along with the relation $\gamma\varphi = j_2(T_w - T_0)$ and $\delta\varphi = j_1(T_w - T_0)$. The non-dimensional stratification parameters are produced by taking the ratio of the two term in (7) (Thermal and Solutal) $\varphi = T_w - T_0/T_\infty - T_0 = z_1/z_2$ and $\varrho = C_w - C_0/C_\infty - C_0 = z_3/z_4$ respectively. Transforming the governing PDEs in (1)–(4) together with (5) into an ODEs, using similarity transformations

below as presented in [7, 8].

$$\begin{aligned} u &= \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \\ \psi &= \left(\frac{2\nu U_w(x+b)^{n+1}}{n+1} \right)^{\frac{1}{2}} f(\eta), \\ \eta &= \left(\frac{(n+1)U_w(x+b)^{n-1}}{2\nu} \right)^{\frac{1}{2}} y, \\ \theta(\eta) &= \frac{T - T_\infty}{T_w - T_0}, \\ \phi(\eta) &= \frac{C - C_\infty}{C_w - C_0}. \end{aligned} \quad (8)$$

Equation (1) is satisfied, invoking (8), the governing Equations (2)–(4) along with (5) are reduced to;

$$\begin{aligned} \left(1 + \frac{1}{\beta}\right) [Q_1 f'''(\eta) - \delta \theta'(\eta) f''(\eta)] \\ + f(\eta) f''(\eta) - \left(\frac{2}{n+1}\right) f'(\eta) [nf'(\eta) + M] = 0, \end{aligned} \quad (9)$$

$$\begin{aligned} Q_2 \theta''(\eta) + \gamma (\theta'(\eta))^2 \\ - Pr \left(\frac{1-n}{n+1}\right) \theta(\eta) f'(\eta) - Pr \varphi \left(\frac{1-n}{n+1}\right) f'(\eta) \\ - Pr h_o \left[\left(\frac{n+1}{2}\right) f(\eta) \theta'(\eta) f'(\eta) - \left(\frac{(n-1)^2}{2(n+1)}\right) \right. \\ \left. (f(\eta))^2 \theta(\eta) - \varphi \left(\frac{(n-1)^2}{2(n+1)}\right) (f'(\eta))^2 \right. \\ \left. + \left(\frac{n-1}{2}\right) f(\eta) \theta(\eta) f''(\eta) \right. \\ \left. + \varphi \left(\frac{n-1}{2}\right) f(\eta) f''(\eta) \right. \\ \left. + \left(\frac{n+1}{2}\right) f^2(\eta) \theta''(\eta) \right] \\ + Pr \theta'(\eta) f(\eta) + Ec Pr Q_1 \left(1 + \frac{1}{\beta}\right) (f''(\eta))^2 \end{aligned} \quad (1)$$

$$+ Pr H_0 \left(\frac{2}{n+1}\right) \theta(\eta) = 0, \quad (10)$$

$$\begin{aligned} \phi''(\eta) + Sc f(\eta) \phi'(\eta) \\ + \left(\frac{n-1}{n+1}\right) [Sc f'(\eta) \phi(\eta) + Sc \varphi f'(\eta)] \\ - Sc \lambda \left(\frac{2}{n+1}\right) \phi(\eta) = 0. \end{aligned} \quad (11)$$

where $Q_1 = [i_1 + \delta - \delta \theta(\eta) - \delta \varphi]$ and $Q_2 = [i_2 + \gamma \theta(\eta)]$. Since the sheet is stretching, we choose $\eta = A((n+1)U_w/2\nu)^{\frac{1}{2}}$ as the least similarity variable. Then, Equation (5) becomes

$$f(\eta) = \chi \left(\frac{1-n}{1+n}\right), \quad f'(\eta) = 1, \quad \theta(\eta) = 1,$$

$$\phi(\eta) = 1 \quad \text{at } \eta = \chi,$$

$$f'(\eta) \rightarrow 0, \quad \theta(\eta) \rightarrow 0, \quad \phi(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty. \quad (12)$$

where $\chi = A((n+1)U_0/2\nu)^{\frac{1}{2}}$ represent the wall surface. Equations (9)–(11) with (12) are in the domain

$[\chi, \infty)$. Dimensionalizing into $[0, \infty)$ we set $g(\tau) = g(\eta - \chi) = f(\eta)$, $h(\tau) = h(\eta - \chi) = \theta(\eta)$ and $\Phi(\tau) = \Phi(\eta - \chi) = \phi(\eta)$ then we have

$$\begin{aligned} \left(1 + \frac{1}{\beta}\right) [Q_3 g'''(\tau) - \delta h'(\tau) g''(\tau)] \\ + g(\tau) g''(\tau) - \left(\frac{2}{n+1}\right) g'(\tau) [ng'(\tau) + M] = 0, \end{aligned} \quad (13)$$

$$\begin{aligned} Q_4 h''(\tau) + \gamma (h'(\tau))^2 - Pr \left(\frac{1-n}{n+1}\right) h(\tau) g'(\tau) \\ - Pr \varphi \left(\frac{1-n}{n+1}\right) g'(\tau) \\ - Pr h_o \left[\left(\frac{n+1}{2}\right) g(\tau) h'(\tau) g'(\tau) - \left(\frac{(n-1)^2}{2(n+1)}\right) \right. \\ \left. (g(\tau))^2 h(\tau) - \varphi \left(\frac{(n-1)^2}{2(n+1)}\right) (g'(\tau))^2 \right. \\ \left. + \left(\frac{n-1}{2}\right) g(\tau) h(\tau) g''(\tau) \right. \\ \left. + \varphi \left(\frac{n-1}{2}\right) g(\tau) g''(\tau) \right. \\ \left. + \left(\frac{n+1}{2}\right) g^2(\tau) h''(\tau) \right] \\ + Pr h'(\tau) g(\tau) + Ec Pr Q_3 \left(1 + \frac{1}{\beta}\right) (g''(\tau))^2 \\ + Pr H_0 \left(\frac{2}{n+1}\right) h(\tau) = 0, \end{aligned} \quad (14)$$

$$\begin{aligned} \Phi''(\tau) + Sc g(\tau) \Phi'(\tau) \\ + \left(\frac{n-1}{n+1}\right) [Sc g'(\tau) \Phi(\tau) + Sc \varphi g'(\tau)] \\ - Sc \lambda \left(\frac{2}{n+1}\right) \Phi(\tau) = 0. \end{aligned} \quad (15)$$

and Equation (12) gives

$$g(\tau) = \chi \left(\frac{1-n}{1+n}\right), \quad g'(\tau) = 1, \quad h(\tau) = 1,$$

$$\Phi(\tau) = 1 \quad \text{at } \tau = 0,$$

$$g'(\tau) \rightarrow 0, \quad h(\tau) \rightarrow 0, \quad \Phi(\tau) \rightarrow 0 \quad \text{as } \tau \rightarrow \infty. \quad (16)$$

where $Q_3 = [i_1 + \delta - \delta h(\tau) - \delta \varphi]$ and $Q_4 = [i_2 + \gamma h(\tau)]$, $M = \sigma B_0^2 / \rho U_w (x+b)^{n-1}$, $Pr = C_p \mu / k^*$, $h_o = h_i U_w (x+b)^{n-1}$, $\delta = j_1 (T_\infty - T_0)$, $\gamma = j_2 (T_\infty - T_0)$, $H_0 = Q_0 (x+b)^{1-n} / \rho C_p$, $Sc = \nu / D_p$, $\lambda = K_r / \rho C_p U_w (x+b)^{1-n}$, $\varphi = z_1 / z_2$, $\varrho = z_3 / z_4$, $\chi = A((n+1)U_w/2\nu)^{\frac{1}{2}}$, $Ec = U_w^2 (x+b)^{2n} / C_p z_2 (x+b)^{(1-n)/2}$, and i_1, i_2 are taken to be one.

3. Method of solution

To solve the formulated problem, the Chebyshev Spectral Collocation Method (CSCM) was employed. Due to its global nature (i.e. it approximate value, at any point in space depends on the entire domain) and its ability

to handle linear/nonlinear, PDEs/ODEs systems of equations [54]. Define the n th-order Chebyshev polynomial denoted by $T_i(x)$; $i \geq 0$ as

$$T_i(\xi) = \cos(N \cos^{-1} \xi); \quad -1 \leq \tau \leq 1, \quad (17)$$

and the recursive formula is given by $T_{i+1} = 2xT_i(x) - T_{i-1}(x)$; $i \geq 1$. To apply CSCM, the flow domain $[0, \infty)$ is approximated to $[0, L]$. L is the scaling parameter, the choice of L determines the convergence of the result at infinity. And initial trial would be taken to guide the right choice L by observing the convergence. Thus the domain $[0, L]$ is transformed to the domain $[-1, 1]$ by applying the mapping in Equation (18).

$$\xi = \frac{2\tau}{L} - 1, \quad \xi \in [-1, +1]. \quad (18)$$

Thus, it is assumed that the unknown functions, $g(\tau)$, $h(\tau)$ and $\Phi(\tau)$ are approximated by the sum of basis function $T_k(\xi)$.

$$\begin{aligned} g(\tau) &\approx g_N(\tau) = \sum_{k=0}^N a_k T_k(\tau), \\ h(\tau) &\approx h_N(\tau) = \sum_{k=0}^N b_k T_k(\tau), \\ \Phi(\tau) &\approx \Phi_N(\tau) = \sum_{k=0}^N c_k T_k(\tau) \end{aligned} \quad (19)$$

Using the basis function in Equation (17) where a_k , b_k and c_k are unknown to be determined, in order to compute the residue, substituting Equation (19) into the governing equations (13)–(15) the non-zero residue is obtained. For the purpose of minimizing error, residues are equated to zero at N th collocation points. Chebyshev collocation points used is defined by Kayvan et al. [54].

$$\tau_j = \cos\left(\frac{\pi j}{N}\right), \quad j = 0, \dots, N. \quad (20)$$

Owing to this, a system of $3N + 3$ algebraic equations with $3N + 3$ unknown coefficient expansion a_k , b_k and c_k were obtained. Newton iteration method Finlayson [55] was imposed on the system of derived equations. The constants a_k , b_k , and c_k are obtained using MATHEMATICA and substituted into Equation (19) to get the required approximate solution for the flow distributions.

4. Results and discussion

The following pertinent parameters were used $Pr = 0.72$, $\beta = 0.2$, $M = 1$, $E_c = \delta = \gamma = 0.5$, $Sc = 0.62$, $n = H_0 = \lambda = h_0 = \varrho = \varphi = \chi = 0.1$. These values are kept constant throughout the study unless otherwise stated in the respective tables and figures. The analysis shows the influence of the flow parameters on momentum $g'(\tau)$, temperature $h(\tau)$ and concentration $\Phi(\tau)$ boundary layers.

To authenticate the method used, we compared the obtained results with the existing results in the literature. Table 1 presents the skin friction result obtained by Nadeema et al. [56], where Adomian Decomposition Method (ADM) along with Pade approximation was employed. And the heat transfer coefficient of the work presented by Pramanik [57] for various values of Prandtl number, and the results show a good agreement with the existing literature. The simulation presented in Figures 2 and 3 highlighted the effectiveness of the method used. And was carried out by analysing graphically the velocity and temperature profile obtained using the Chebyshev Spectral Collocation Method (CSCM) and Galerkin Weighted Residual Method (GWRM).

Table 3 shows the effects of some flow parameters on the flow model. Bearing in mind, other pertinent parameters remain the same as stated earlier. It was observed that an increase in the parameters β , n and γ increases the skin friction and decreases the energy as well as mass gradient close to the wall. Where β tends to attain

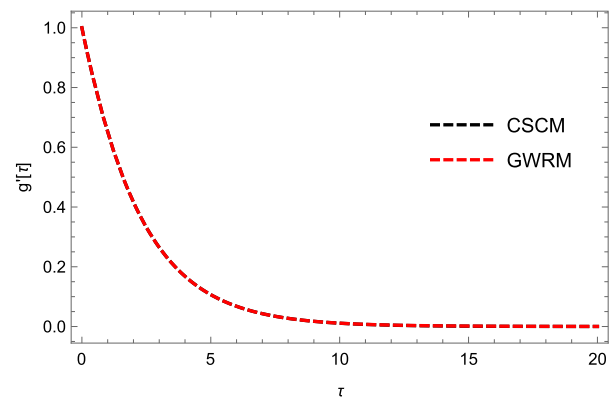


Figure 2. Comparison of the velocity profiles obtained using CSCM and GWRM.

Table 1. Comparison of the Skin friction coefficient [$g''(0)$] Nadeema et al. [56] and heat transfer coefficient [$-h'(0)$] Pramanik [57], where $\Gamma = (1 + 1/\beta)$.

For $n = 2$, $\beta = 1$ and varying χ & M				For $n = 1$, $\beta \rightarrow \infty$ varying Pr		
χ	M	Present study $\Gamma g''(0)$	Nadeema et al. [56] $\Gamma g''(0)$	Pr	Present study $-h'(0)$	Pramanik [57] $-h'(0)$
-3	2 ²	1.366679	1.36668	1	0.955216	0.9547
-3	3 ²	2.184183	2.184183	2	1.471361	1.4714
- $\frac{3}{2}$	2 ²	1.215503	1.215503	3	1.868998	1.8691
				5	2.500074	2.5001
				10	3.660627	3.6603

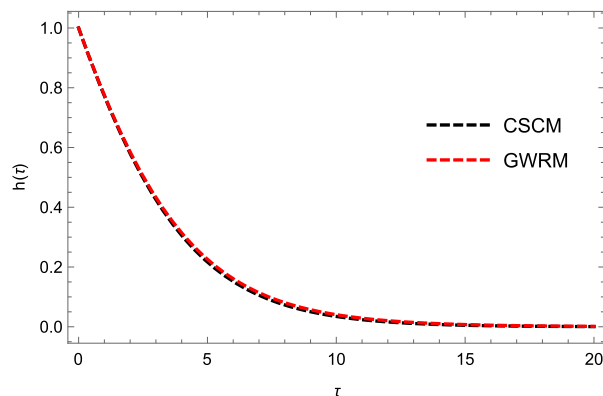


Figure 3. Comparison of the temperature profiles obtained using CSCM and GWRM.

Table 2. Comparison, skin friction coefficient $[g''(0)]$ and heat transfer coefficient $[-h'(0)]$ for different values of n and χ when $Pr = 0.7$, $\beta \rightarrow \infty$ and $Ec = \delta = \gamma = Sc = H_0 = \lambda = h_0 = \varrho = \varphi = 0$.

n	χ	Present study		Nadeema et al. [56]	
		$[g''(0)]$	$[-h'(0)]$	$[g''(0)]$	$[-h'(0)]$
-0.5	0.5	-1.16666855	1.90452127	-1.16668224	1.90459332
-0.5	0.25	-0.08338680	1.73607330	-0.08365155	1.73611251
$-\frac{1}{3}$	0.25	-0.50009300	1.34269600	-0.50041742	1.34290940

the maximum value of temperature gradient within the range of 0.4–0.6 and declined afterwards. Meanwhile, the addition of M , h_0 and δ decreases the Sherwood number, Nusselt number and skin friction accordingly. Also, a rise in χ and Pr presented an increase in the temperature and concentration while it decreases the skin friction.

Figure 4 presents the effect of thermal relaxation time (h_0) on energy distribution, it found decelerating the temperature field. Due to thermal relaxation enhancement in temperature, the material particle (Casson fluid) needed extra time for heat transfer to its surrounding particles. However, a higher Pr is expected to bring down energy and velocity profiles due to the heat generated. Pr the ratio of mass diffusion to thermal

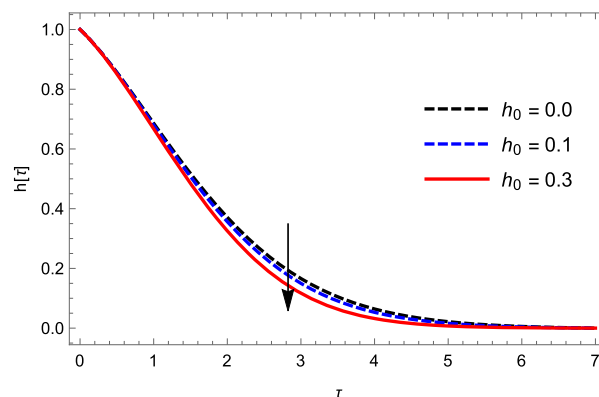


Figure 4. Effect of h_0 on temperature profile.

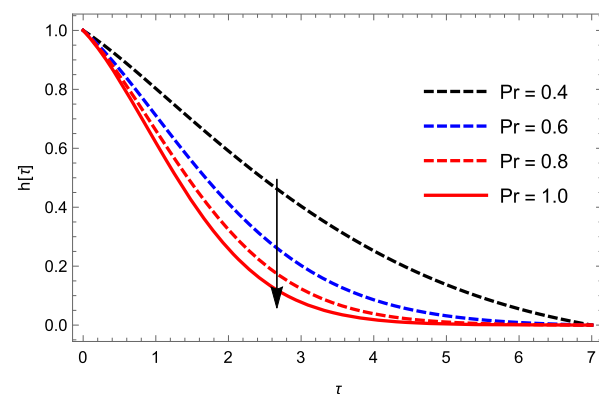


Figure 5. Effect of Pr on temperature profile.

diffusion enhances the thinning thermal boundary layer while it reduces thermal diffusion. Thus found decelerating the temperature penetration depth as shown in Figure 5.

The effect of variable plastic dynamic viscosity (δ) on flow distributions is presented in Figures 6 and 7. The rise in δ reduces the fluid velocity near the stretching surface $\tau \leq 2$ but increases as it moves far away. Hence, it gave rise to the heat transfer rate and produces no significant effect on mass distribution across the flow field. Physically, this effect described the flow resistance

Table 3. Computed effects of β , M , δ , γ , n , h_0 , Pr and χ on skin friction $[\Gamma g''(0)]$, heat transfer $-h'(0)$ and mass transfer coefficients $-\Phi'(0)$, where $\Gamma = (1 + 1/\beta)$.

Values		$[\Gamma g''(0)]$	$-h'(0)$	$-\Phi'(0)$	Values		$[\Gamma g''(0)]$	$-h'(0)$	$-\Phi'(0)$
β	0.1	-5.59247486	0.1770193178	0.957091728	h_0	0	-4.06291010	0.2302894562	0.918161778
	0.5	-2.835951590	0.233680834	0.858556713		0.1	-4.06665031	0.2301594690	0.918120515
	1	-2.299385061	0.2065882874	0.816728263		0.3	-4.07437670	0.230036910	0.91803718
	1.5	-2.093466865	0.1874745755	0.796565990		0.5	-4.08223560	0.229846146	0.917954250
M	0	-2.159398840	0.5189922000	0.992106653	n	0.1	-4.06665032	0.2301594689	0.918120515
	1	-4.06665032	0.2301594689	0.918120515		0.3	-3.945226384	0.1566960229	0.806440287
	2	-5.28096019	0.01342416390	0.870878306		0.5	-3.853767675	0.0973804777	0.717744511
	3	-6.22960757	-0.1632299379	0.835962125		0.7	-3.782215666	0.0480339083	0.645157108
δ	0	-3.7454662	0.2639588253	0.923280391	γ	0	-4.10156775	0.279676751	0.918589690
	1	-4.39056771	0.1980186777	0.91226014		0.5	-4.06665031	0.2301594689	0.918120515
	2	-5.06014418	0.1385759489	0.898627054		1	-4.03991428	0.1986482341	0.917935200
	3	-5.78079501	0.0856307559	0.882334776		1.5	-4.01847970	0.1757166117	0.917886024
χ	0.1	-4.06665032	0.2301594689	0.918120515	Pr	0.5	-4.03034794	0.1976834674	0.918116368
	0.3	-4.17792814	0.2684542540	0.979868056		0.7	-4.063781109	0.2277012294	0.918116097
	0.5	-4.290386850	0.309263052	1.044307037		1	-4.09944173	0.2558173921	0.918228520
	0.7	-4.40397472	0.35257511	1.111261349		2	-4.15966601	0.2773167879	0.918746799

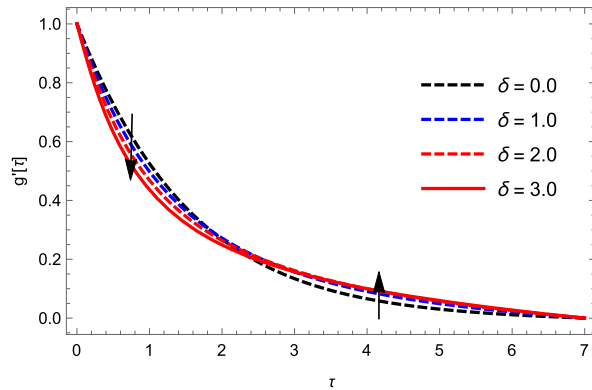


Figure 6. Effect of δ on velocity profile.

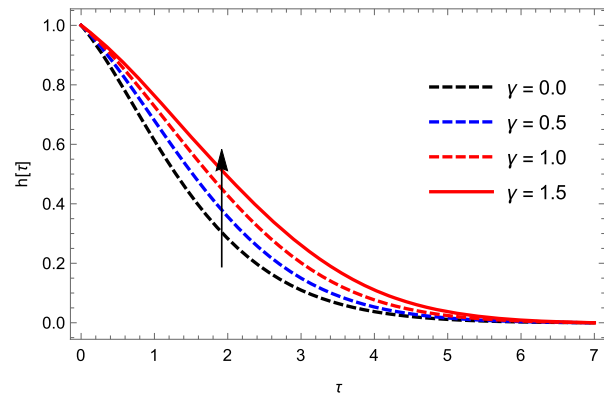


Figure 8. Effect of γ on temperature profile.

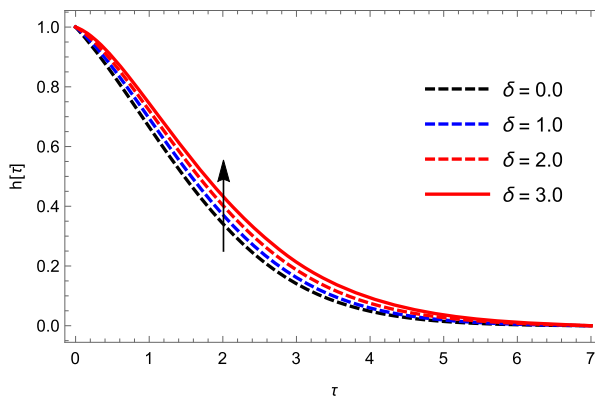


Figure 7. Effect of δ on temperature profile.

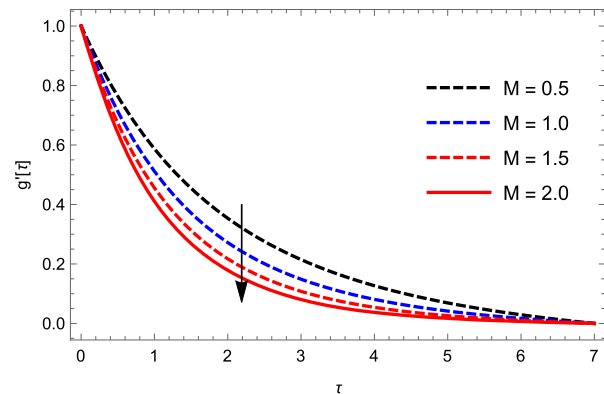


Figure 9. Effect of M on velocity profile.

to fluids which are helpful in many applications, such as lubricants that perform well under variable temperature conditions.

Figure 8 displayed the influence of thermal conductivity (γ) known as random movement of molecular motion across the temperature gradient on energy distribution. This influenced the thermal boundary layer to generate heat and aided the temperature profile. Interestingly, high thermal conductivity helps in heat conduction which is used in heat sink applications. Meanwhile, rise in magnetic parameter (M) decreases $g'(\tau)$ profile, this allowed the perpendicular magnetic field to create an electromagnetic force corresponding to the resistance force that acts as a retarding force, hence, provides resistance to the momentum boundary layer as displayed in Figure 9. Consequently, an increase in M produces a significant rise in the temperature field as shown in Figure 10.

The variation of Casson parameter (β) increases both the energy and concentration profile across the flow field while it decelerates the flow field (see Figures 11–13). Physically, a larger and smaller value of β parameter corresponds to Newtonian and non-Newtonian fluids respectively (i.e. β reduces the yield stress). However, the result obtained in Figure 12 contradicts the stated physical analogy which is due to the temperature injected that affects the fluid temperature

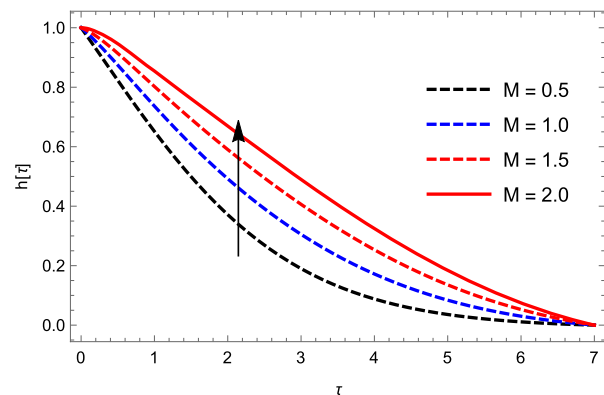


Figure 10. Effect of M on temperature profile.

thus behave contrarily. Moreover, this behaviour of Casson fluid with variable viscosity was also observed by Ajayi et al. [7] and other related literature.

Figures 14–16 analysed the variation of wall thickness parameter χ on flow, energy and mass transfer respectively. Literarily, materials with variable thickness exhibit a lower magnitude towards the flow distributions for free convective flow. Thus an increase in χ results to a decrease in boundary layer thickness, velocity and other flow distributions. Meanwhile, a rise in velocity power index (n) gives rise to the stretching velocity which produces distortion in the fluid region, thereby gave rise to $g'(\tau)$ and $h(\tau)$ distributions as shown in Figures 17 and 18 respectively. Practically, a

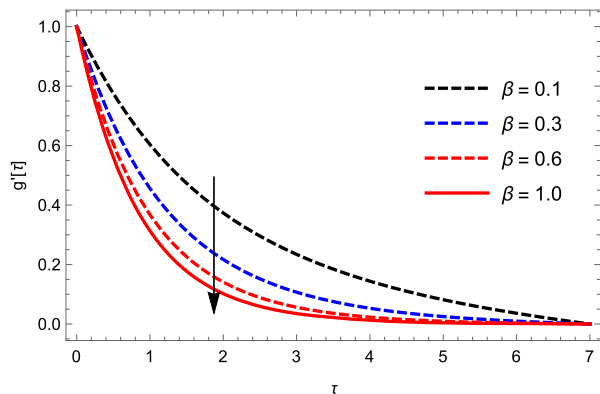


Figure 11. Effect of β on velocity profile.

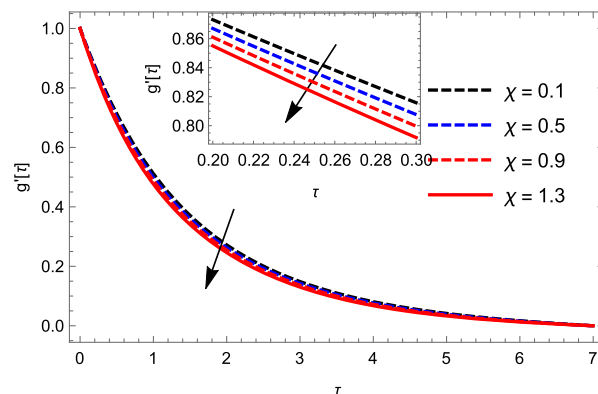


Figure 14. Effect of χ on velocity profile.

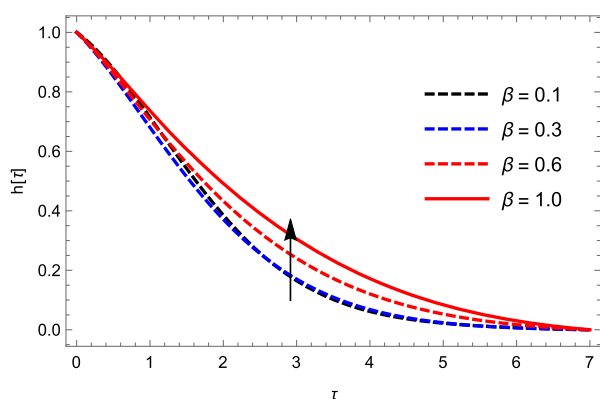


Figure 12. Effect of β on temperature profile.

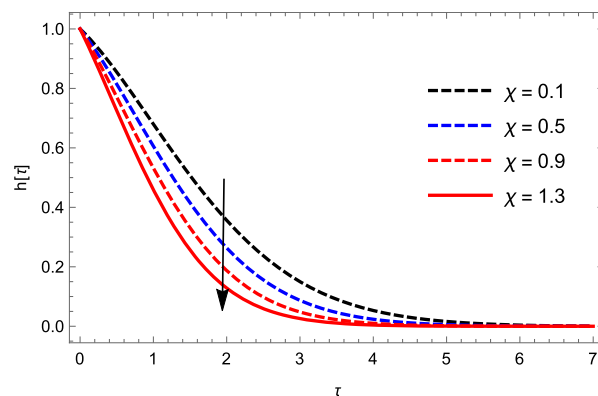


Figure 15. Effect of χ on temperature profile.

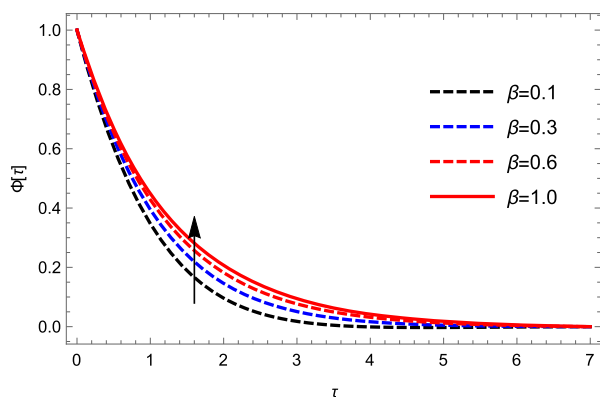


Figure 13. Effect of β on concentration profile.

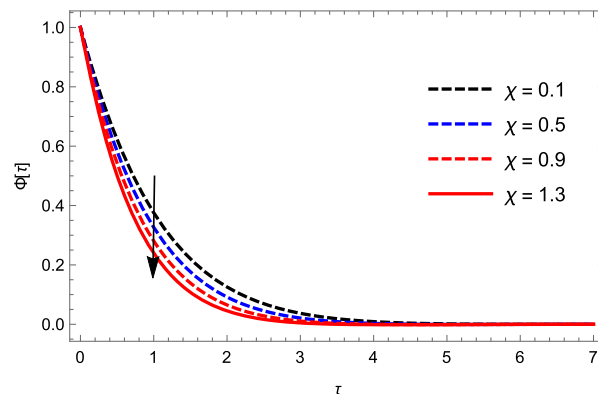


Figure 16. Effect of (χ) on concentration profile.

rise in n generates more force in the same direction, hence, appreciate the flow distributions.

Figure 19 displays the variations in physical quantities Eckert number (Ec). Ec gives rise to the temperature profile due to heat energy stored in the liquid as a result of frictional heating, physically, resulting in a reduction in the velocity distribution. In Figure 20, behaviour of H_0 on temperature distribution for both heat generation ($H_0 < 0$) and absorption ($H_0 > 0$) result to higher magnitude of heat, thus enhanced the fluid temperature. And Figure 21 accounts for the magnitude of λ which decelerates the fluid concentration.

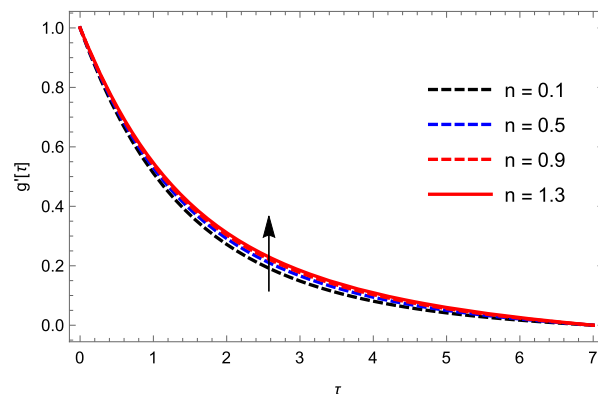


Figure 17. Effect of (n) on velocity profile.

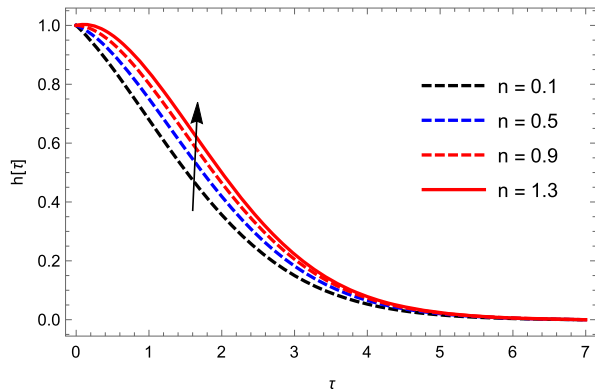


Figure 18. Effect of (n) on temperature profile.

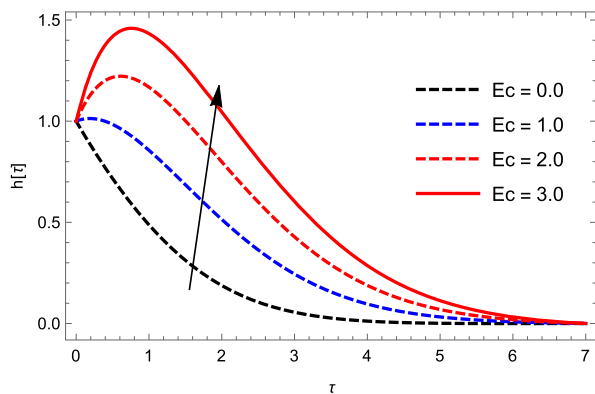


Figure 19. Effect of (Ec) on temperature profile.

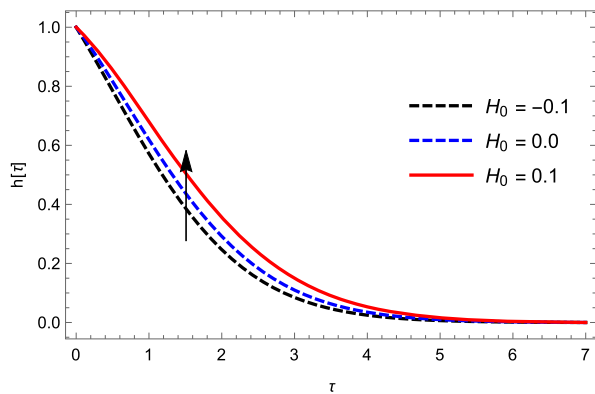


Figure 20. Effect of (H_0) on temperature profile.

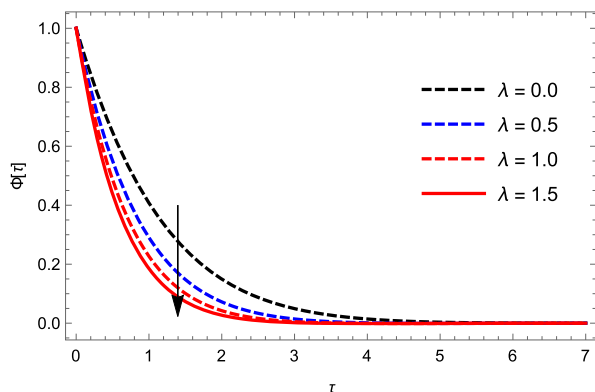


Figure 21. Effect of (λ) on concentration profile.

5. Conclusion

A study on the boundary layer flow of MHD dissipative Casson fluid has been investigated. The fluid is assumed to encompass variable viscosity, variable thermal conductivity along with thermal and solutal stratifications. The effects of a stretching sheet with a variable thickness on free convective heat and mass transfer were examined. While discovering heat relocation, the phenomena employed was the Cattaneo–Christov heat flux model. The governing nonlinear coupled ordinary differential equations were solved using the Spectral Method based on Chebyshev Collocation (CSCM) and the following conclusions were drawn.

- (i) Existence of thermal relaxation time and Prandtl number brings down the temperature distribution accordingly
- (ii) Introduction of magnetic field effect reduces the velocity field across the flow region while it appreciates the temperature field
- (iii) Higher magnitude of Casson parameter results to the reduction in velocity and concentration fields while it increases temperature field
- (iv) Variable viscosity parameter decreases the fluid velocity near the wall and gives rise to both velocity and temperature distributions as it approaches the free stream region. While temperature-dependent thermal conductivity appreciates the temperature distribution through the entire boundary region
- (v) Wall thickness parameter (χ) slows down the flow distributions (velocity, energy and concentration), while these distributions were appreciated on varying the velocity power index (n)
- (vi) Heat generation/absorption increases the velocity distribution, and a hike in Eckert number appreciates the temperature profiles accordingly.

The effect of thermal relaxation on material time in this study helps in model decision making. In particular, Casson fluid is considered the most preferable model for rheological data in comparison to viscoelastic fluid models, which found useful in the model of blood oxygenators and haemodialysers. It is, however, applicable in the area of magnetic drug targeting, where the blood flow can be represented by Casson fluid while the magnetic nanoparticles are considered as drug carriers. External MHD field will be employed to control the drug to reach the target, and thermal relaxation time will be utilized in determining the nature of the material used. This study can as well address the minimization of heat in an electrical voice coil and engine coolant by employing higher thermal conductivity (radiators) which leads to a reduction in aerodynamic drag thus reduces the consumption of the radiators.

Nomenclature

χ	Wall thickness parameter (—)	π	Product of the component deformation rate (—)
T_∞	Free stream temperature (K)	T_w	Temperature at the wall (K)
T	Fluid temperature (K)	E_c	Eckert number (—)
C_p	Specific heat $\left(\frac{\text{J}}{\text{kg K}}\right)$	λ	Chemical reaction parameter (—)
Sc	Schmidt number (—)	Pr	Prandtl number (—)
ϕ	Dimensional concentration $\left(\frac{\text{mol}}{\text{L}}\right)$	h_0	Dimensionless thermal relaxation (—)
h_i	Relaxation time of heat flux (—)	δ	Dimensionless variable viscosity (—)
ρ	Fluid density $\left(\frac{\text{kg}}{\text{m}^3}\right)$	ϵ	Temperature-dependent thermal conductivity $\left(\frac{\text{W}}{\text{m K}}\right)$
B_0	Magnetic field parameter (tesla)	K^*	Absorption coefficient (m^{-1})
ϱ	Solutal stratification (—)	α	Thermal diffusivity $\left(\frac{\text{m}^2}{\text{s}}\right)$
M	Dimensionless magnetic field parameter (—)	θ	Dimensional temperature (K)
σ^*	Stefan–Boltzmann constant $\left(\frac{\text{W}}{\text{m}^2 \text{K}^4}\right)$	σ	Fluid electrical conductivity $\left(\frac{\text{s}}{\text{m}}\right)$
n	Velocity power index parameter (—)	μ_0	Dynamical viscosity $\left(\frac{\text{kg}}{\text{m s}}\right)$
τ_w	Skin-friction coefficient (—)	H_0	Dimensionless heat generation/absorption (—)
$g'(\tau)$	Dimensionless fluid velocity (—)	$h(\tau)$	Dimensionless fluid temperature (—)
ν	Kinematic viscosity $\left(\frac{\text{m}^2}{\text{s}}\right)$	$\kappa(T)$	Variable thermal conductivity $\left(\frac{\text{W}}{\text{m K}}\right)$
P_y	Yield stress of the fluid (—)	γ	Dimensionless thermal conductivity (—)
$\Phi(\tau)$	Dimensionless concentration (—)	C	Fluid concentration (mol)
L	Scaling parameter (—)	D_p	Mass diffusivity term (—)
K_r	Chemical reaction $\left(\frac{\text{mol}}{\text{L s}}\right)$	Q_o	Dimensionless internal heat generation (—)
β	Casson parameter (—)	μ_b	Plastic dynamic viscosity $\left(\frac{\text{kg}}{\text{m s}}\right)$
u, v	Velocity components along x and y direction $\left(\frac{\text{m}}{\text{s}}\right)$	φ	Dimensionless thermal stratification (—)

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